Bottomonium potential from deep learning

施舒哲 清华大学

refs:

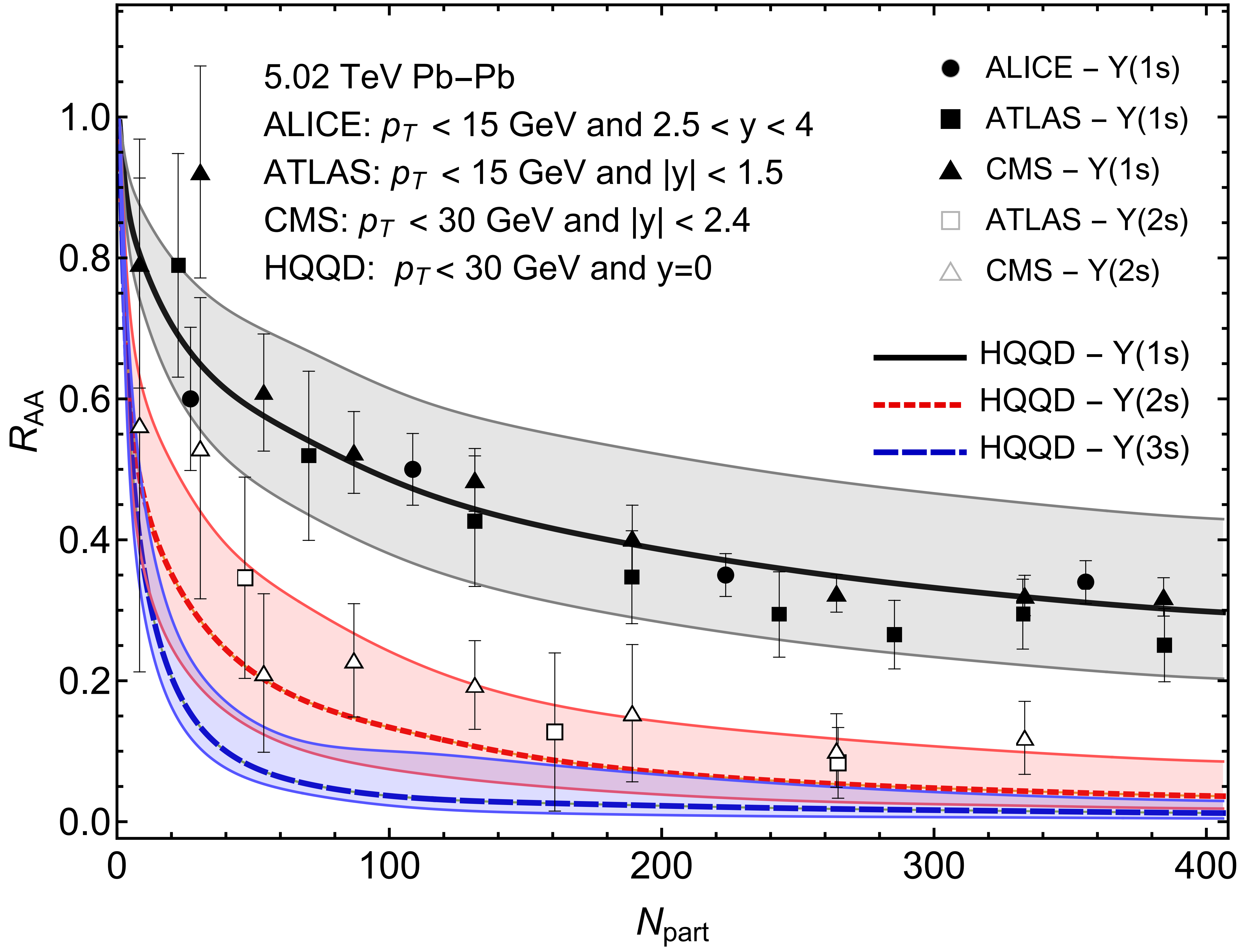
_SS_, Zhou, Zhao, Mukherjee, Zhuang, Phys. Rev. D 105, 014017;
Wang, **_SS_**, Zhou, Phys. Rev. D 106, L051502;
_SS_, Wang, Zhou, Comput.Phys.Commun. 282 (2023) 108547.

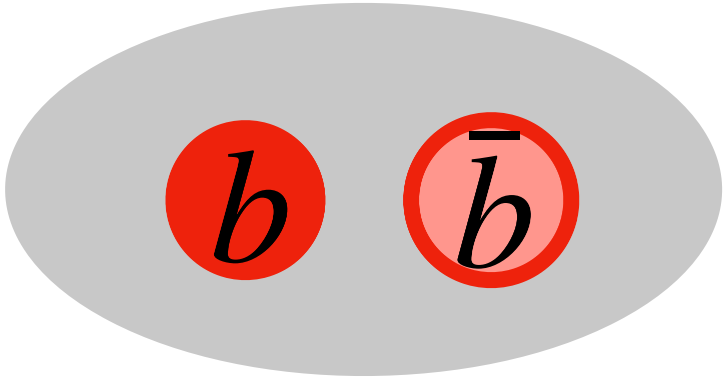
a recent review:

Zhou, Wang, Pang, **_SS_** Prog.Part.Nucl.Phys. 104084(2023)[2303.15136].

Quarkonium in the QGP

- In heavy-ion collisions, quarkonium production serves as a probe of the QGP.





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- Accurate understanding of the in-medium heavy-quark interaction?
    - Real potential modified by color-screening
    - Imaginary potential arises due to $(Q\bar{Q})_1 \to (Q\bar{Q})_8$, Landau damping, ...

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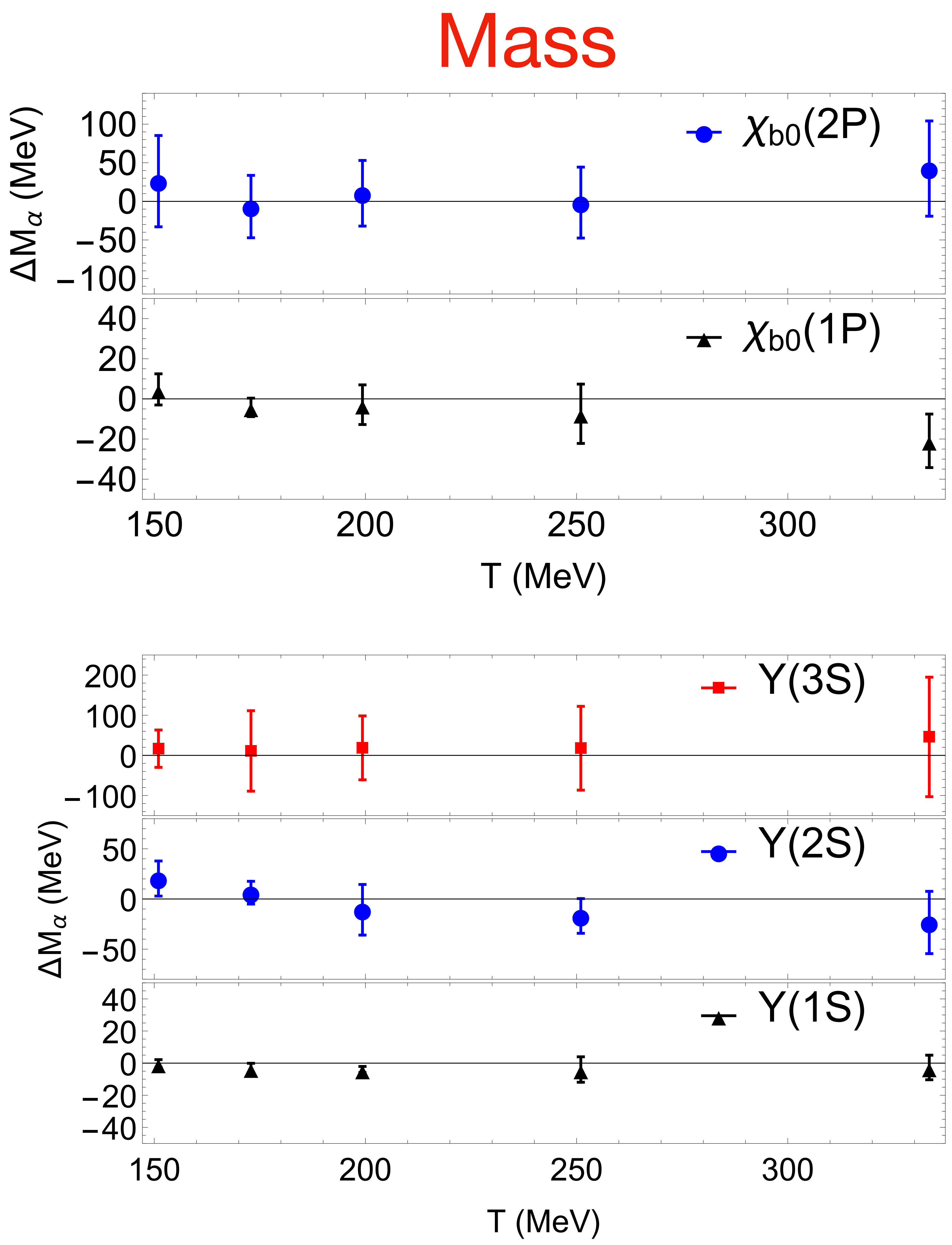
Hard Thermal Loop potentials

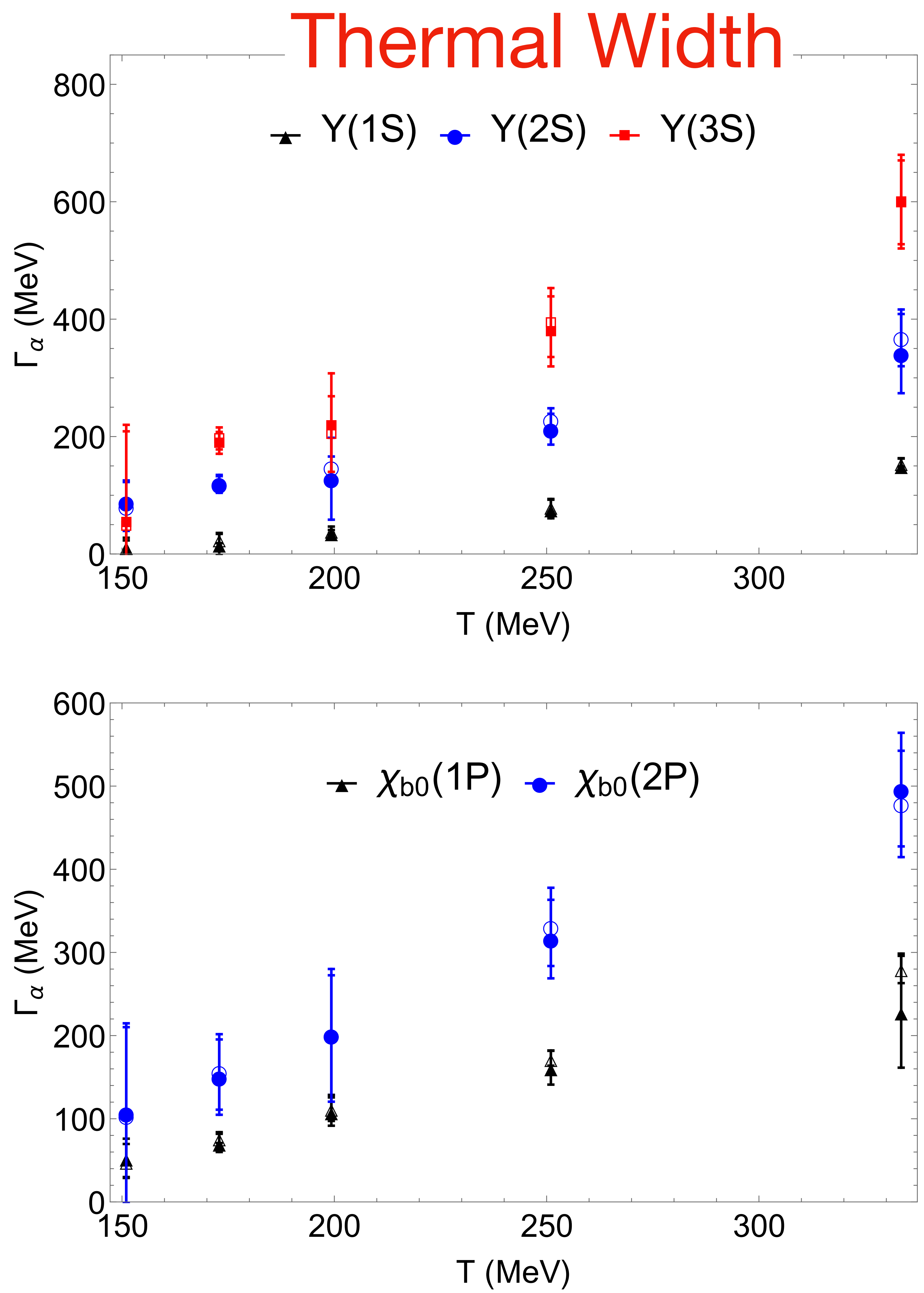
$$V_R(T,r) = \frac{\sigma}{\mu_D}\left(2 - (2+\mu_D r)e^{-\mu_D r}\right) - \alpha\left(\mu_D + \frac{e^{-\mu_D r}}{r}\right) + B\,,$$

$$V_I(T,r) = -\frac{\sqrt{\pi}}{4}\mu_D T\sigma r^3 G^{2,2}_{2,4}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{2} \\ \frac{1}{2},\frac{1}{2},-\frac{3}{2},-1 \end{matrix}\middle|\,\frac{\mu_D^2 r^2}{4}\right) - \alpha\, T\,\phi(\mu_D r)\,.$$

see e.g., Laine, Philipsen, Romatschke, and Tassler, JHEP 03, 054 (2007)

Bottomonium mass and thermal width, lattice QCD with finite m_Q

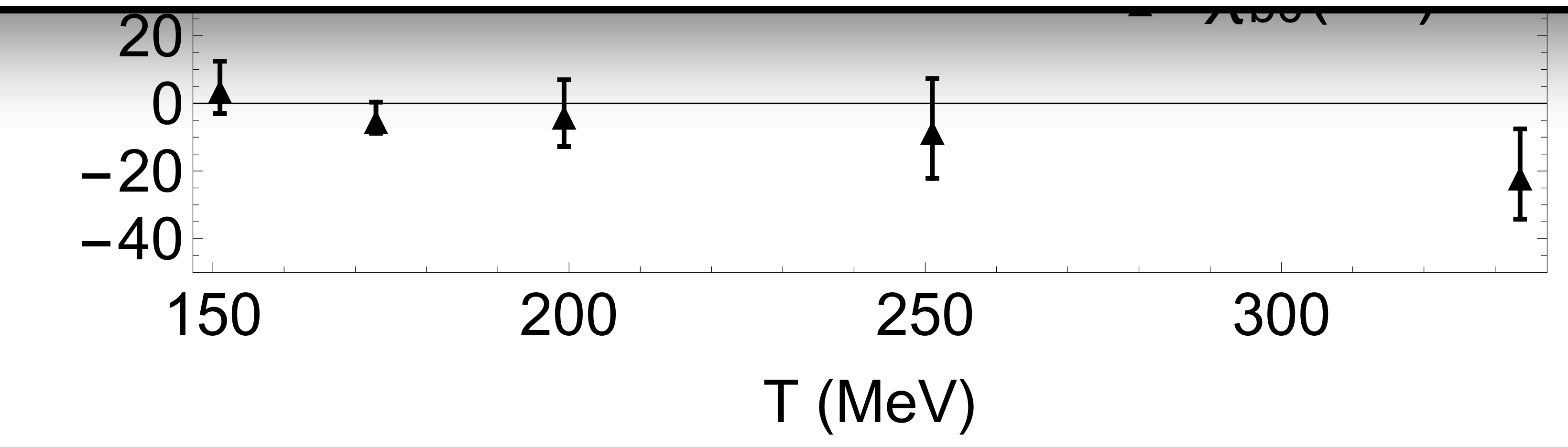


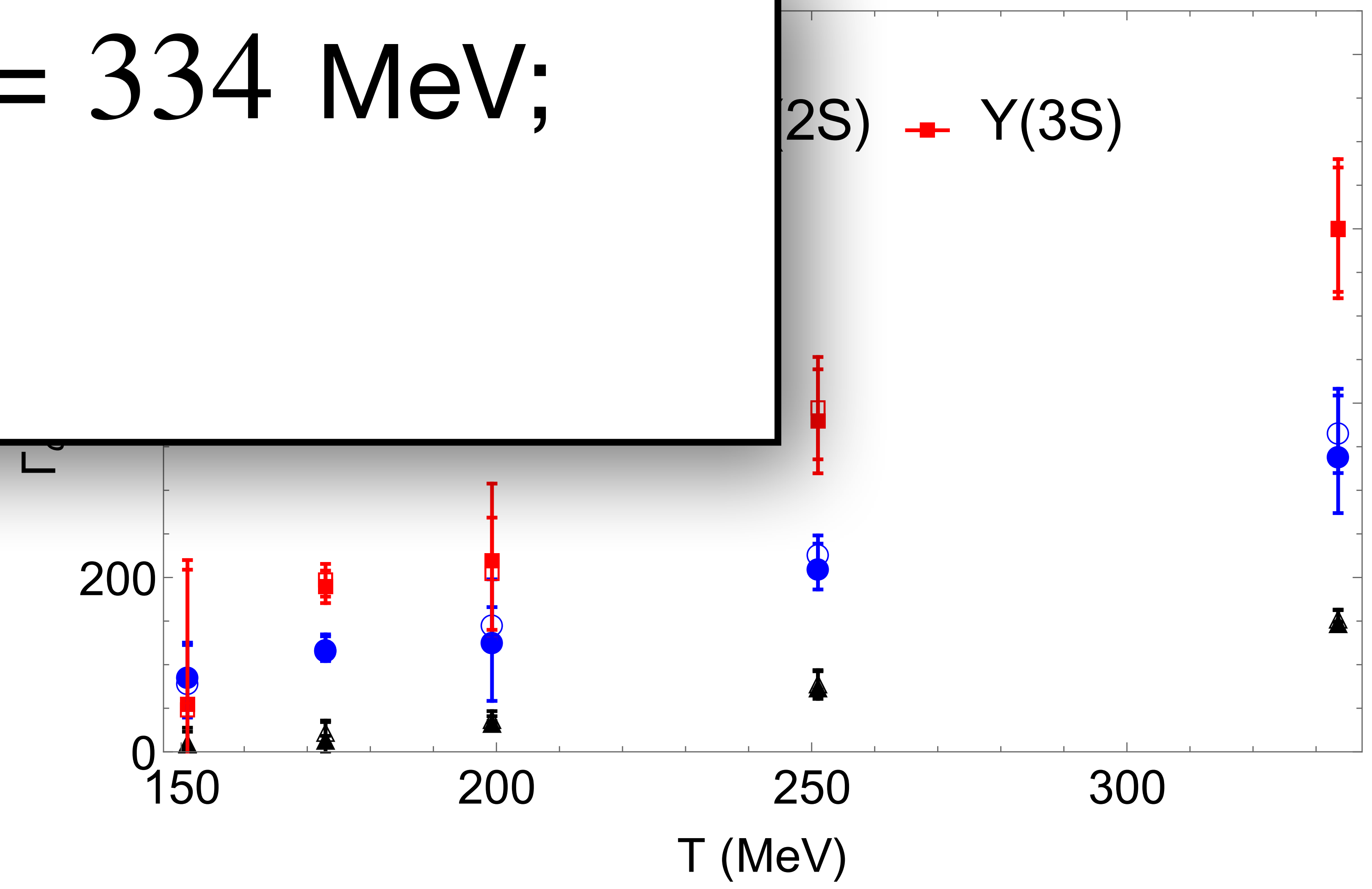


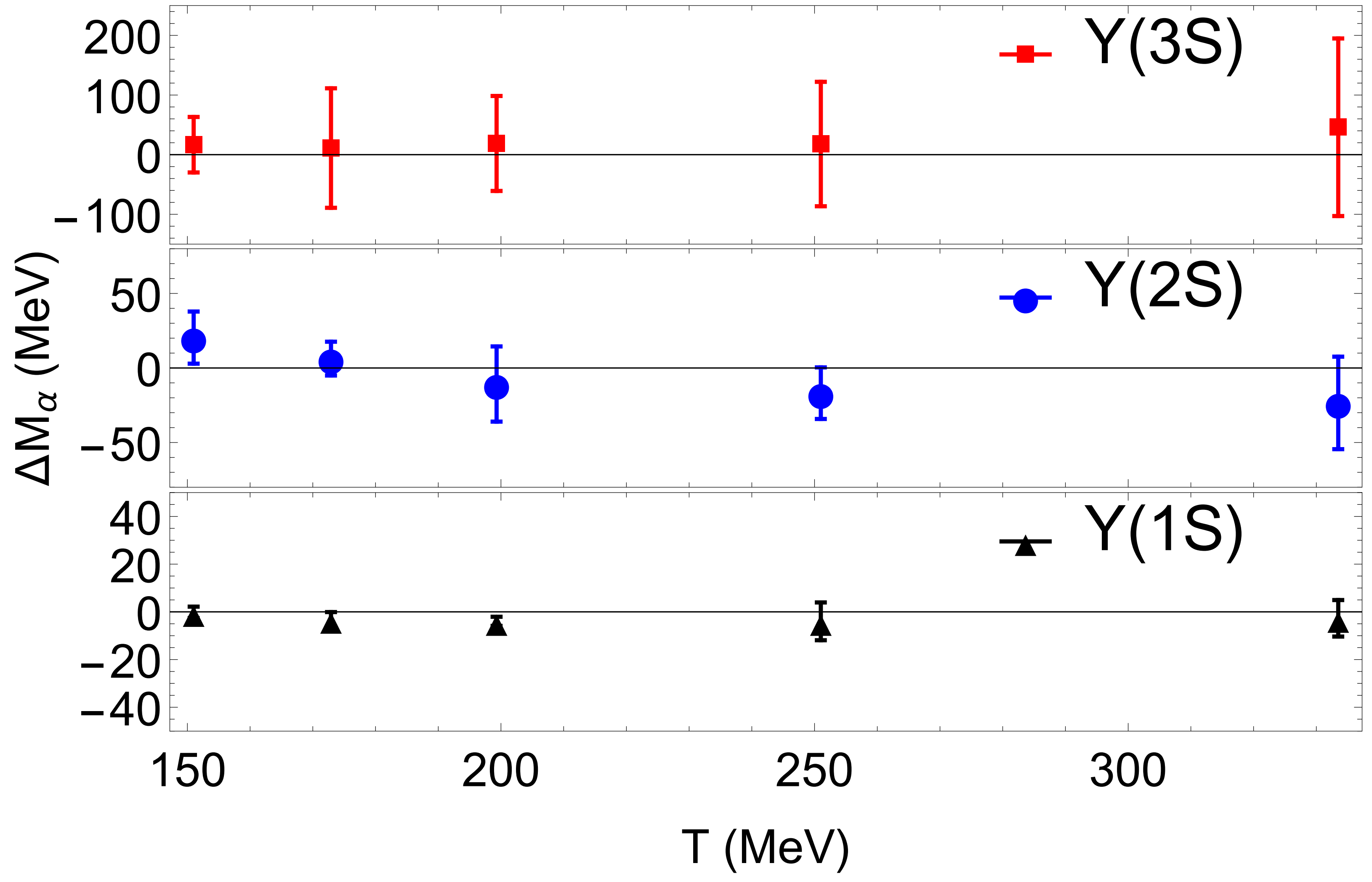
R. Larsen, S. Meinel, S. Mukherjee, and P. Petreczky:
Phys.Rev.D100,074506(2019), Phys.Lett.B800,135119(2020), Phys.Rev.D102,114508(2020)

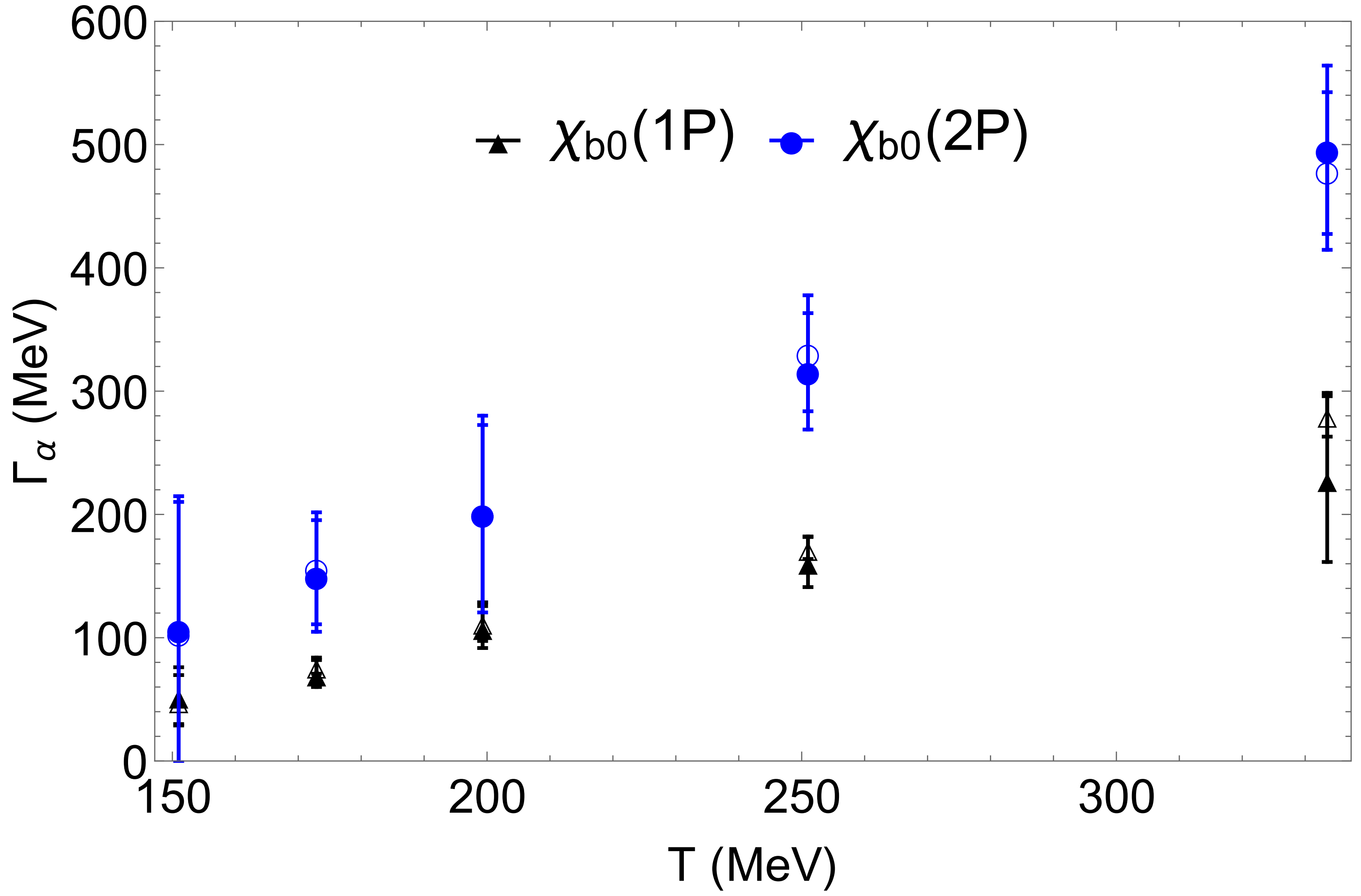
Bottomonium mass and thermal width, lattice QCD with finite m_Q

High excitations (2P, 3S) can survive at $T = 334$ MeV;
Mass - mild temperature dependence;
Thermal width - quantitatively larger.









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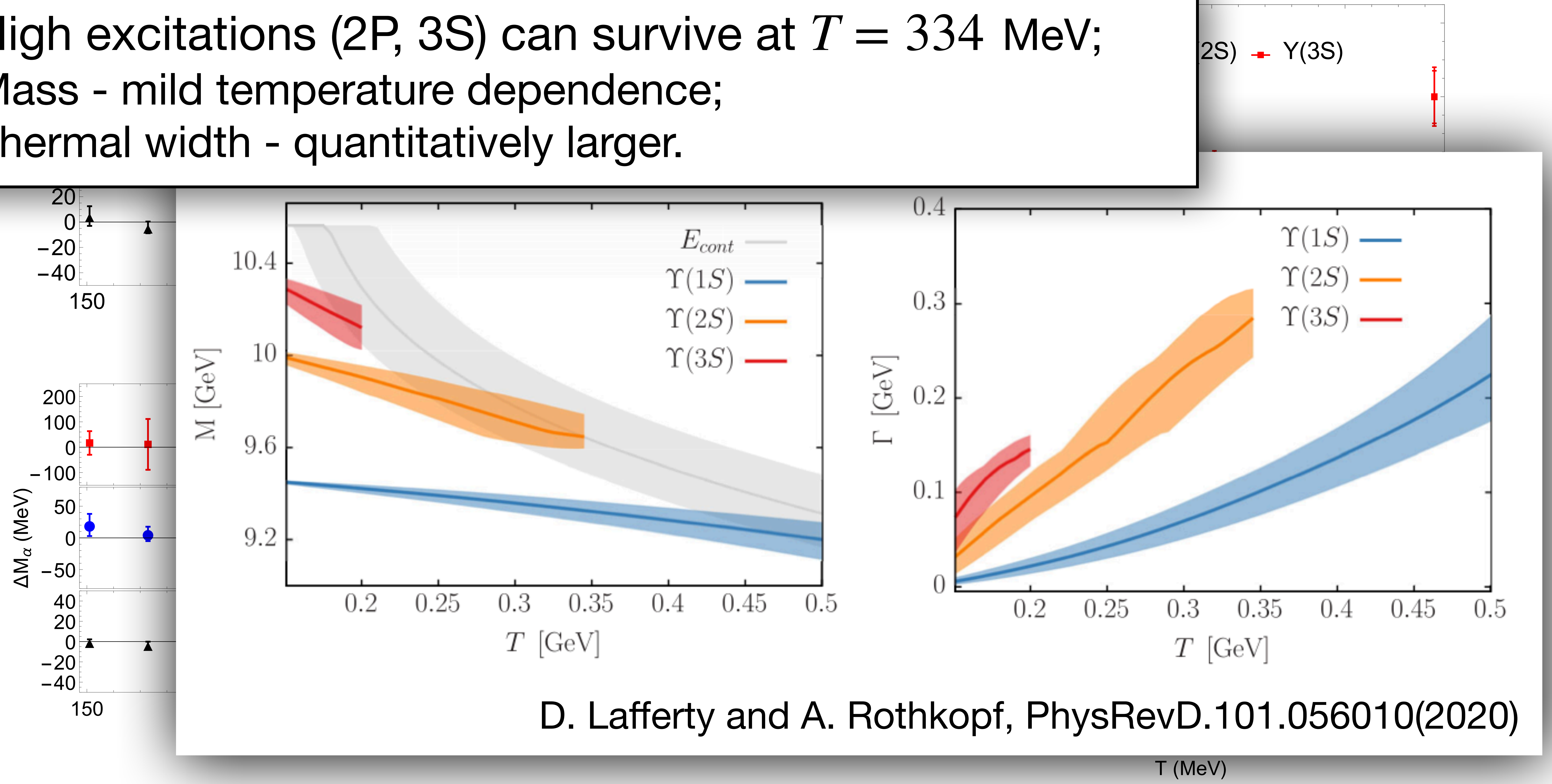

D. Lafferty and A. Rothkopf, PhysRevD.101.056010(2020)

R. Larsen, S. Meinel, S. Mukherjee, and P. Petreczky:
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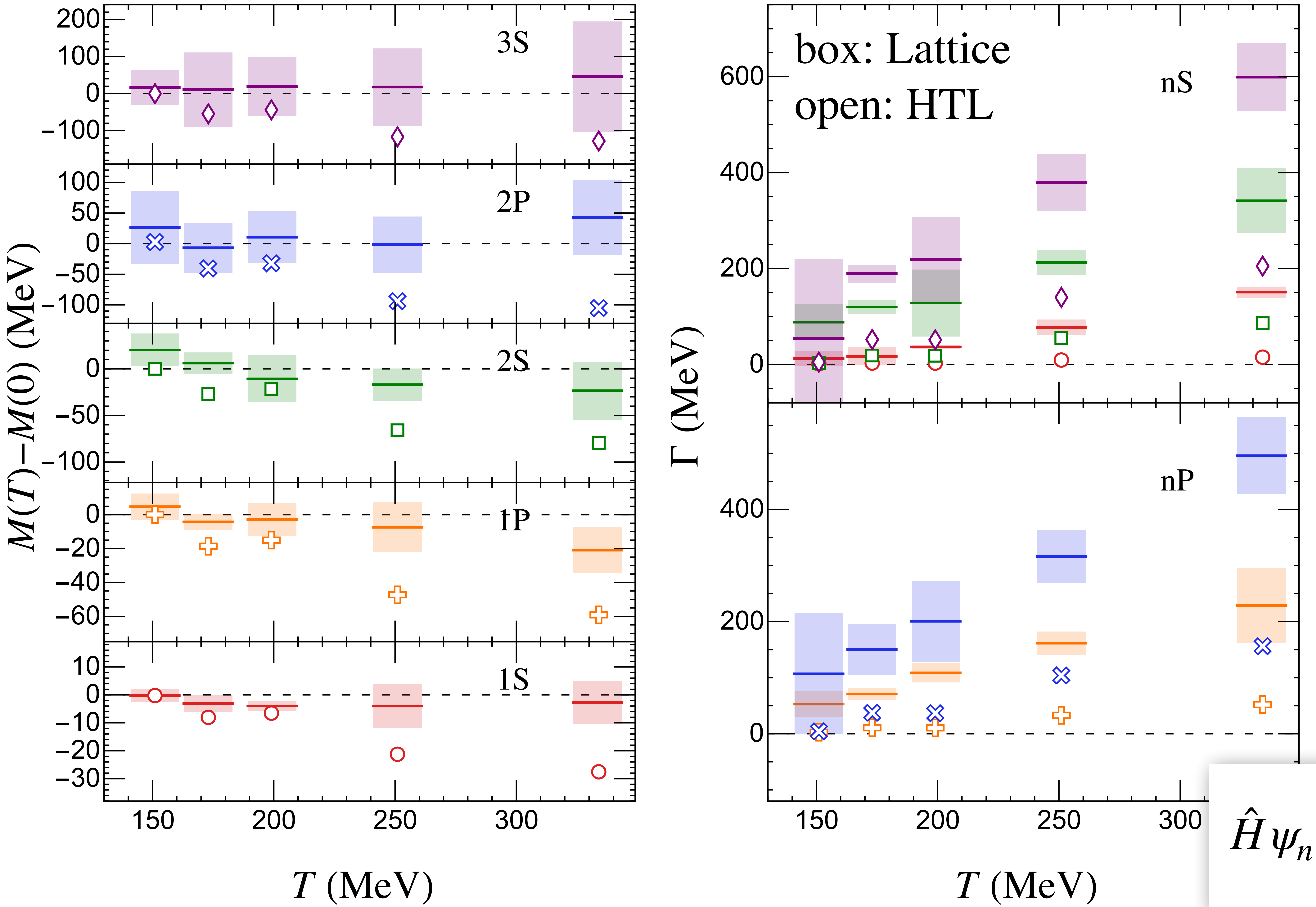
Can we understand the new lattice result using Hard Thermal Loop potential?



$m_D(T)$ fitted by LQCD

$$V(T,r) = V_R(T,r) + i\, V_I(T,r)$$

$$\hat{H}\psi_n = -\frac{\nabla^2}{2m_\mu}\psi_n + V(r)\,\psi_n = E_n\psi_n$$

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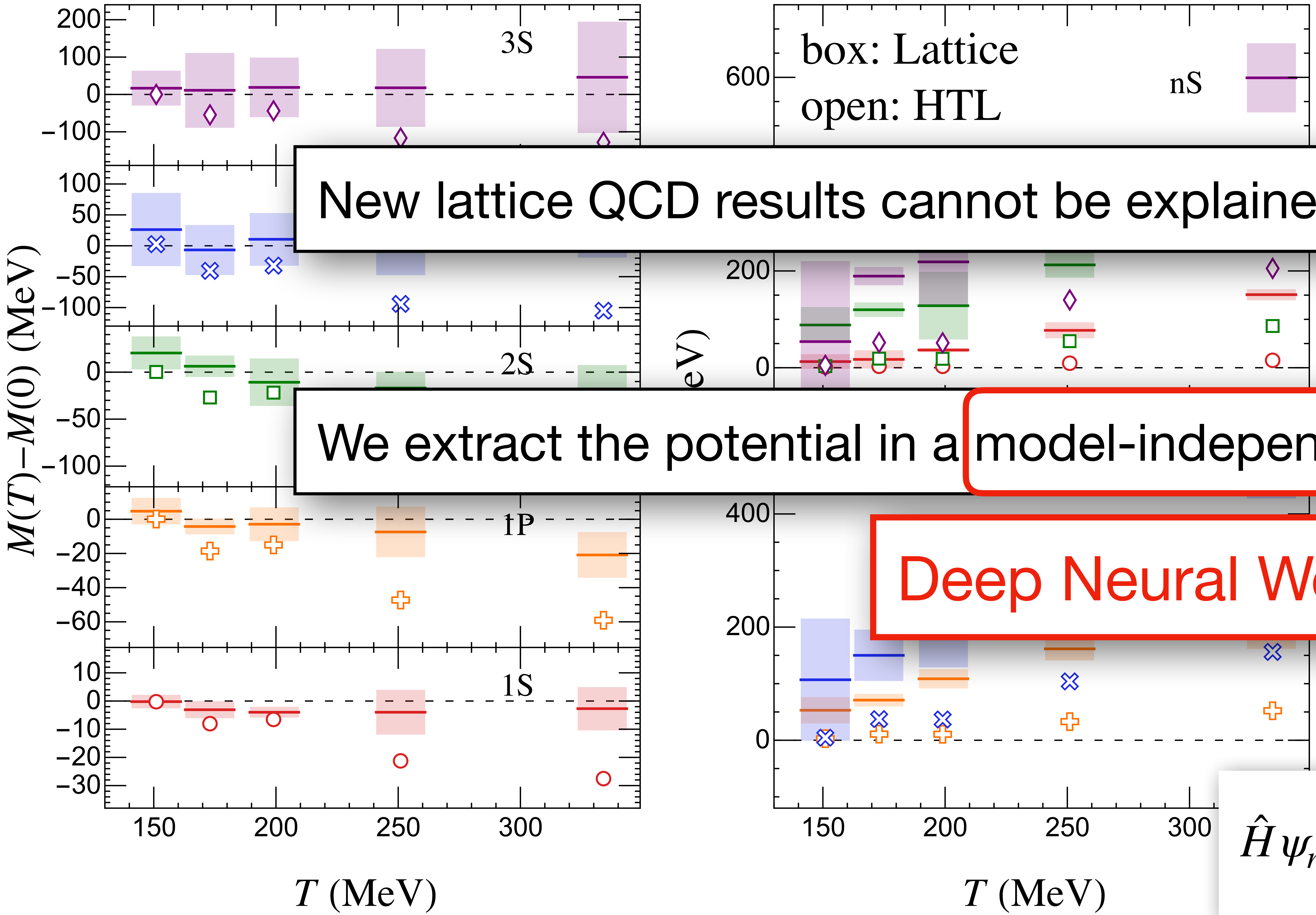


New lattice QCD results cannot be explained by the HTL potential

We extract the potential in a $\boxed{\text{model-independent}}$ way

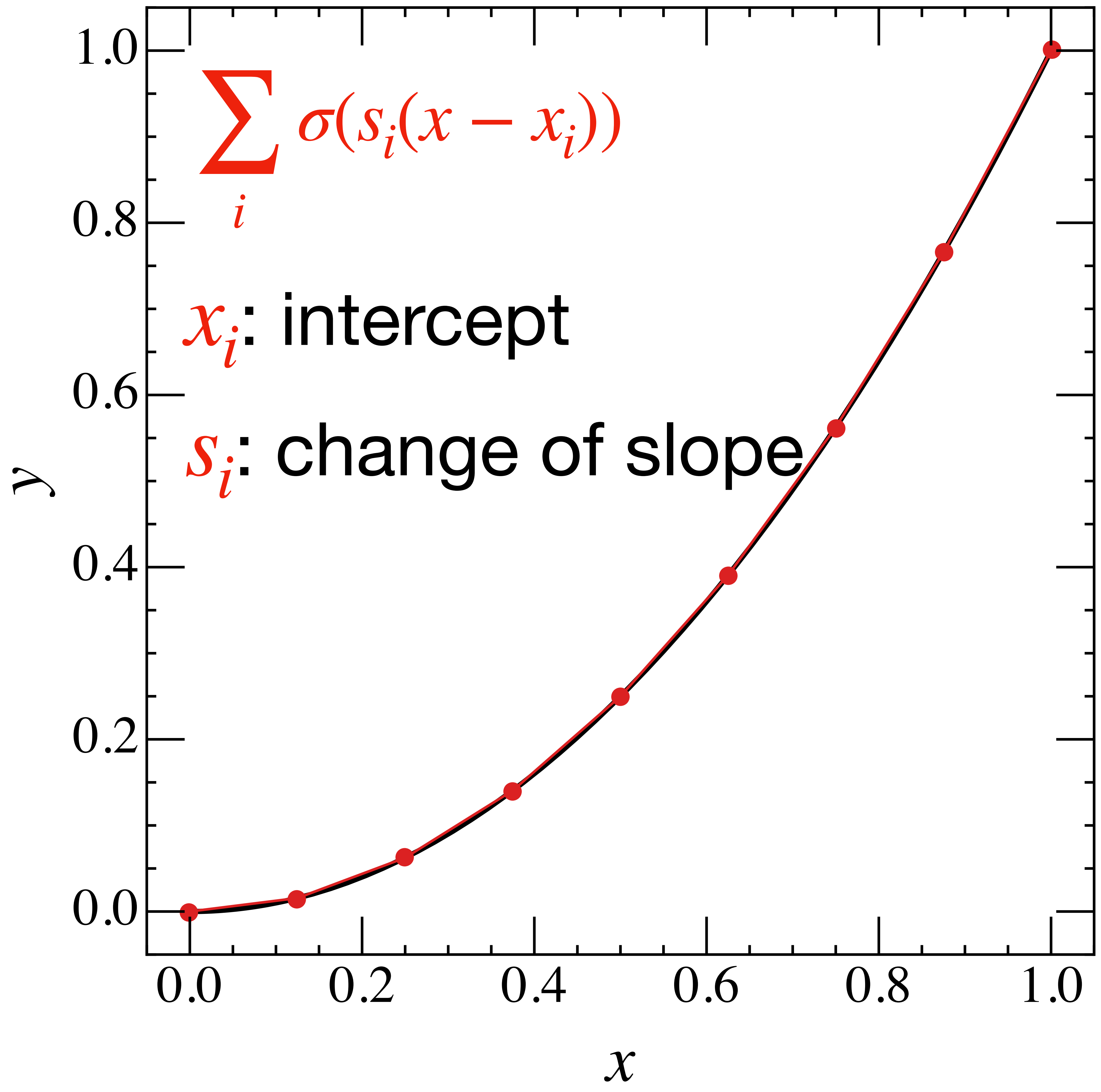
$\boxed{\text{Deep Neural Works (DNN)}}$

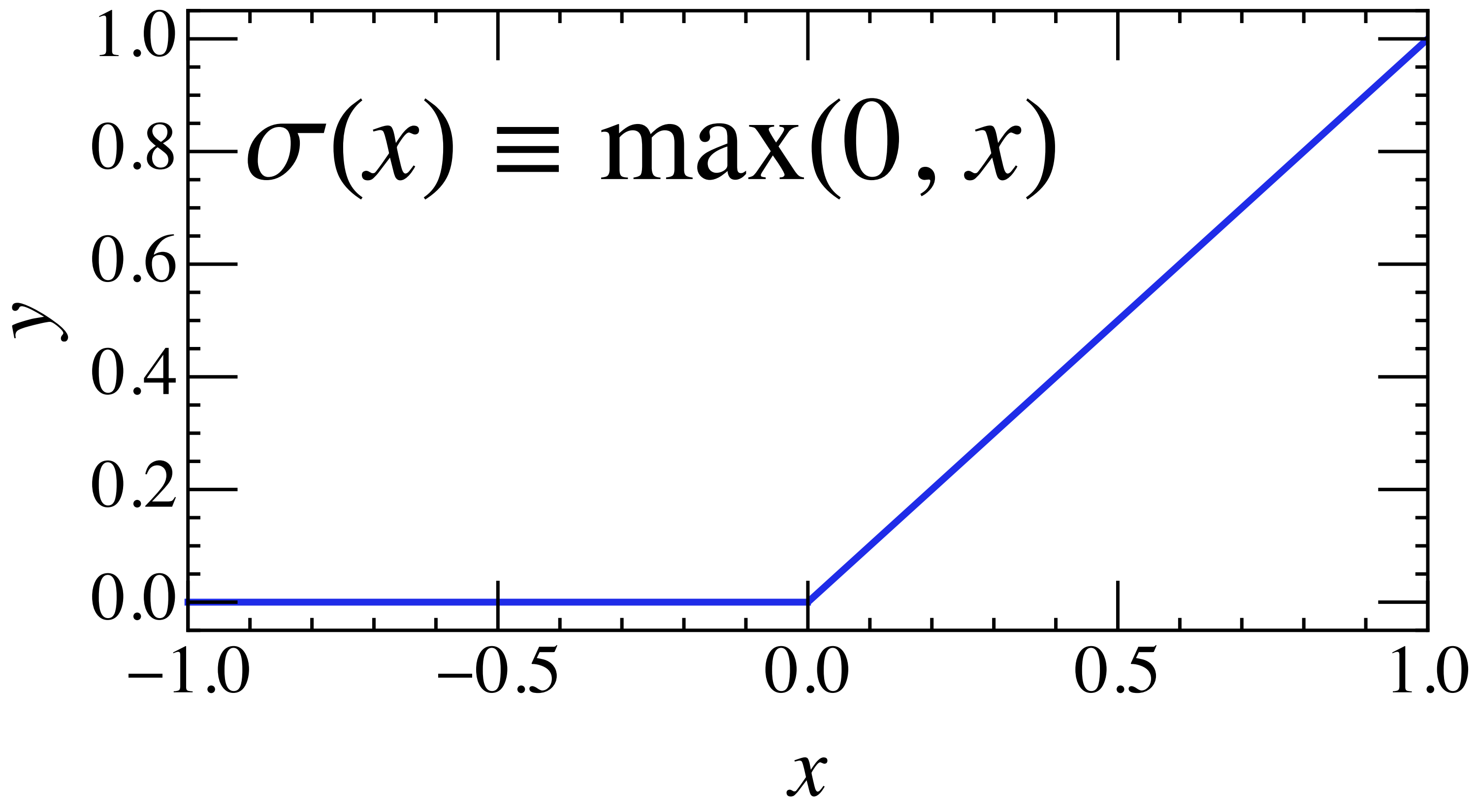
$$V(T,r) = V_R(T,r) + i\,V_I(T,r)$$

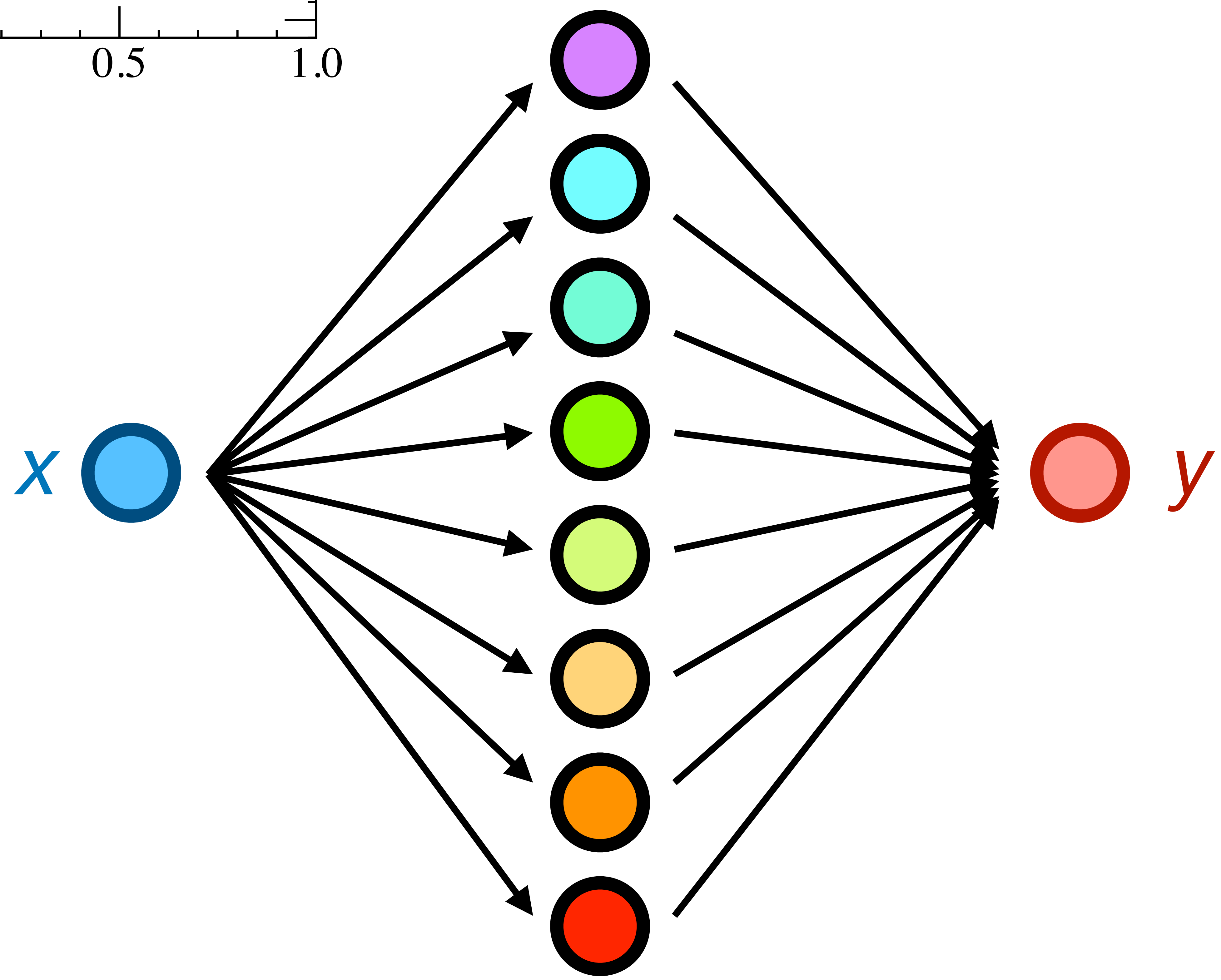
$$\hat{H}\psi_n = -\frac{\nabla^2}{2m_\mu}\psi_n + V(r)\,\psi_n = E_n\psi_n$$

What are Deep Neural Networks?

--- a general parameterization scheme to approximate continuous functions.



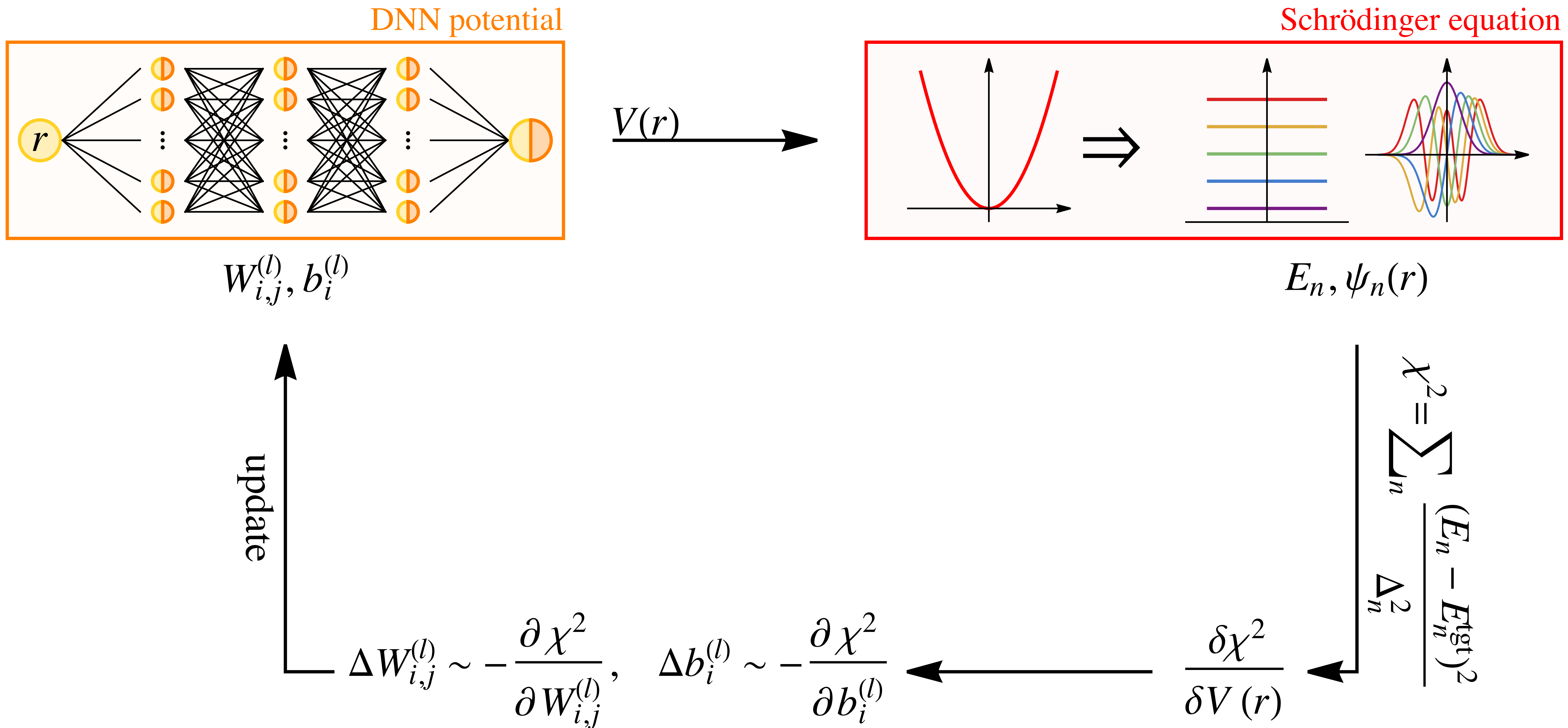




each represents one of $\sigma(s_i(x - x_i))$

How to learn $V(r)$ from $\{E_n\}$?

- parameterize the potential $V(r\,|\,\boldsymbol{\theta})$, minimize $\chi^2 \equiv \sum_i \left(\frac{E_{\boldsymbol{\theta},i} - E_i}{\delta E_i}\right)^2$



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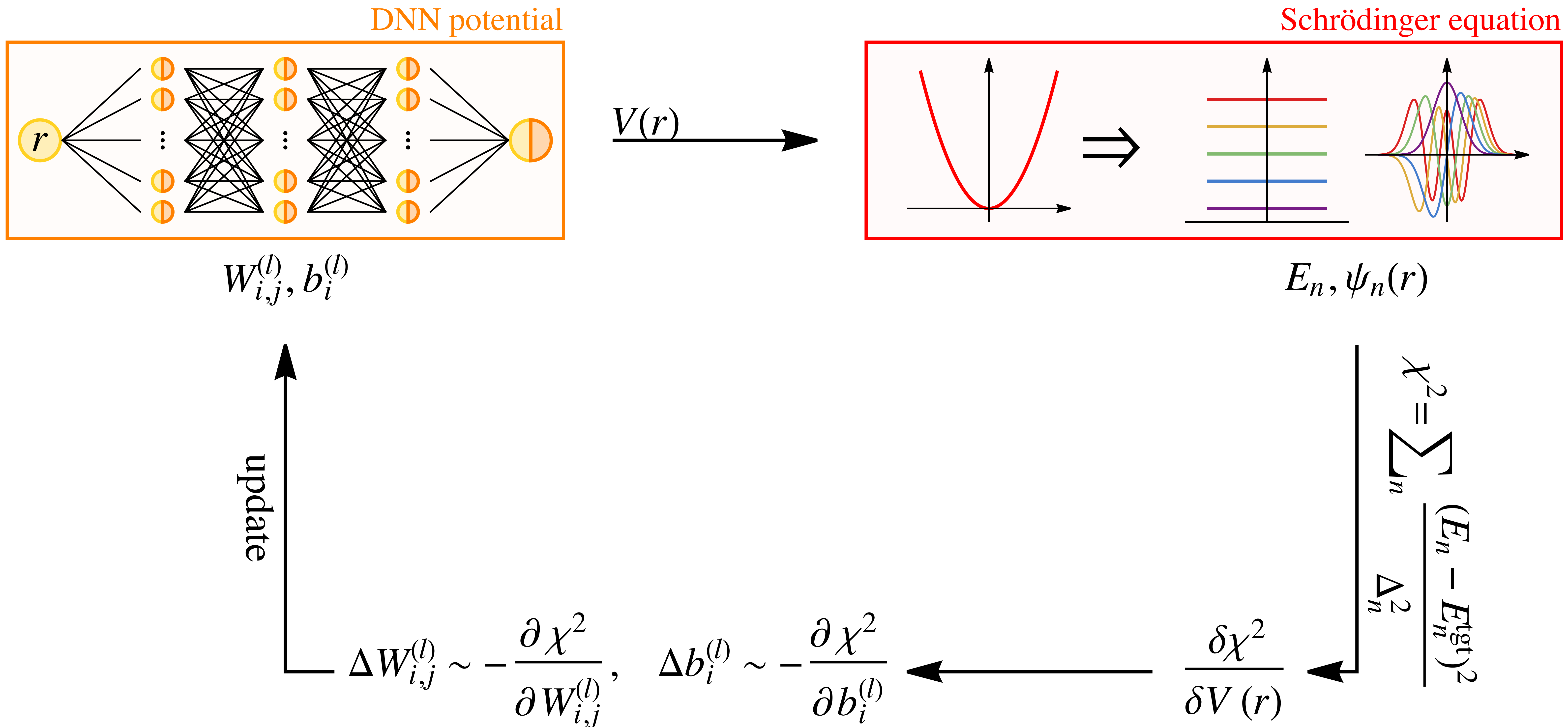
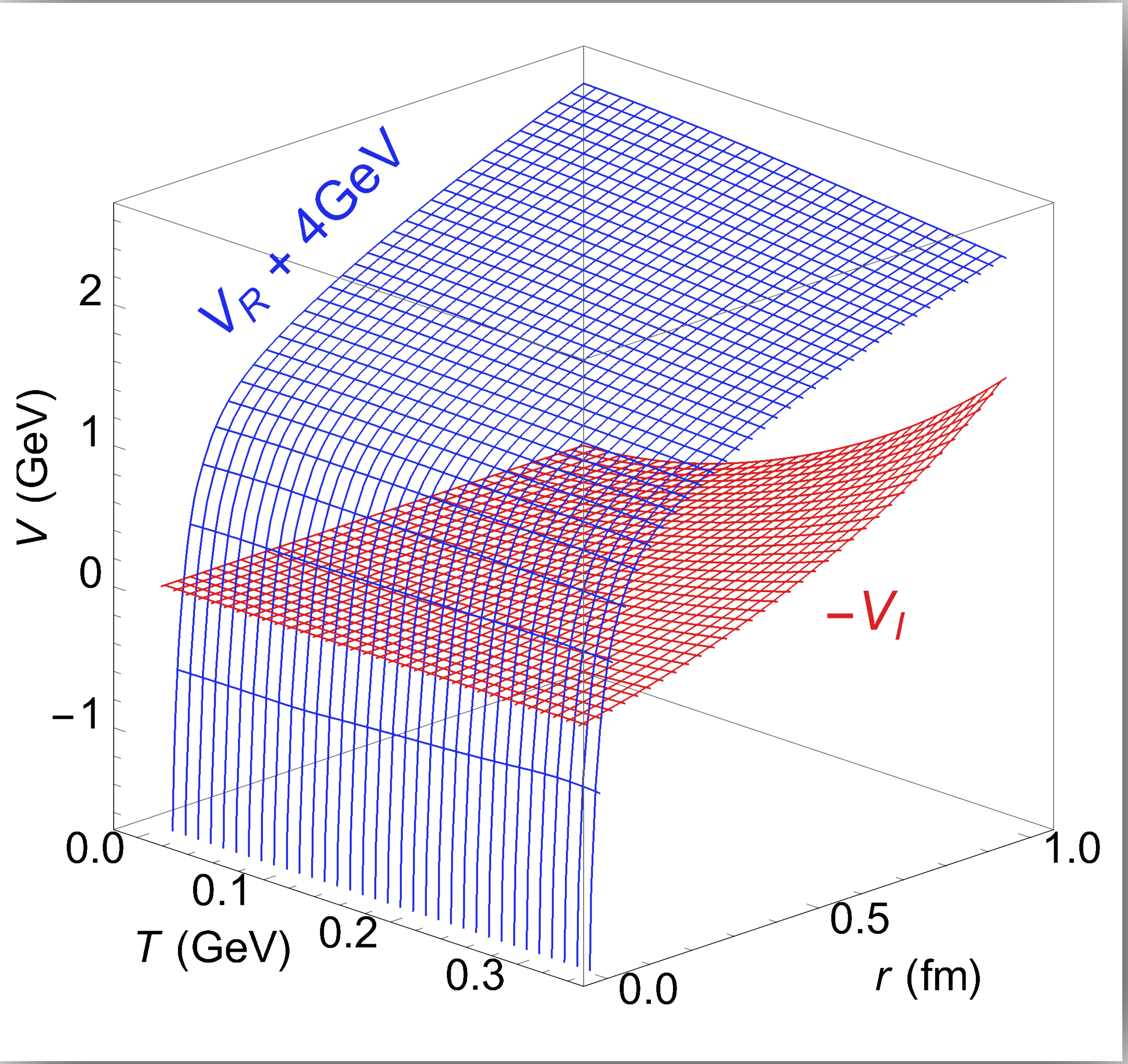
- parameterize the potential $V(r\,|\,\boldsymbol{\theta})$, minimize $\chi^2 \equiv \sum_i \left(\frac{E_{\boldsymbol{\theta},i} - E_i}{\delta E_i}\right)^2$
- a ***gradient-descent*** based method:
    - goal -- find the $\boldsymbol{\theta}$-point that $\nabla_{\boldsymbol{\theta}}\chi^2 = 0$
    - update $\boldsymbol{\theta}$ iteratively according to $\Delta\boldsymbol{\theta} \propto \nabla_{\boldsymbol{\theta}}\chi^2$



Finite Temperature Heavy-Quark Potential



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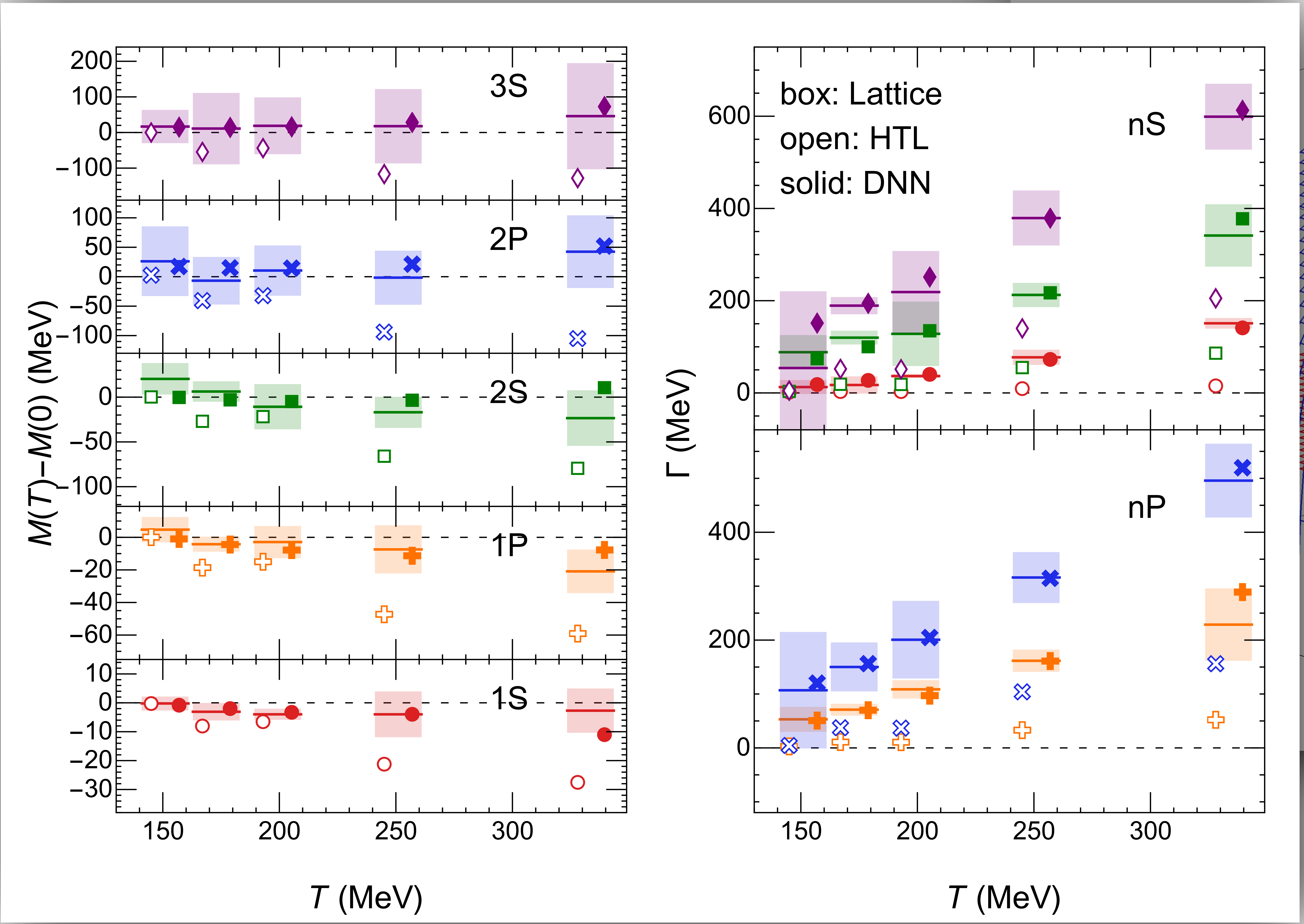
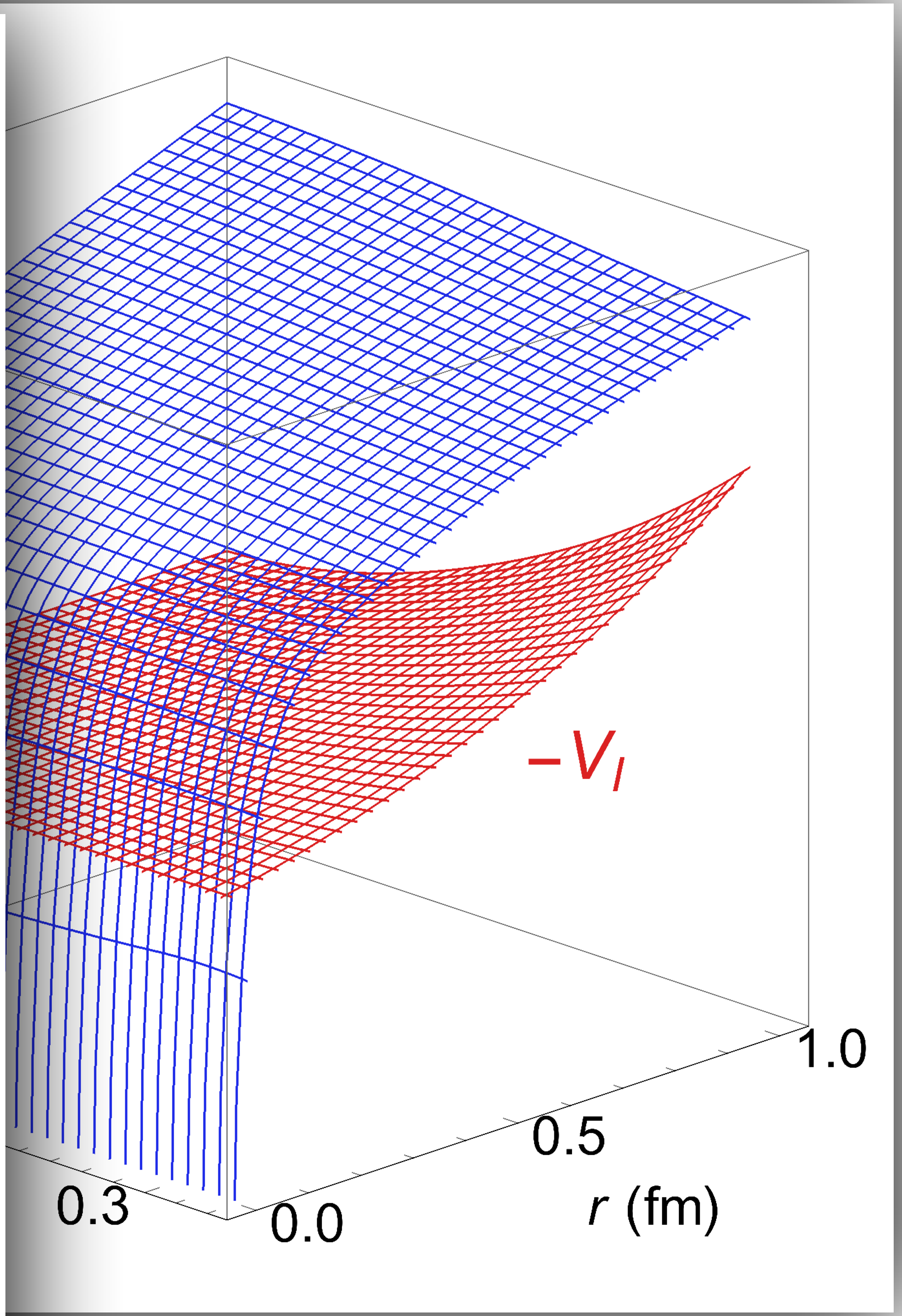
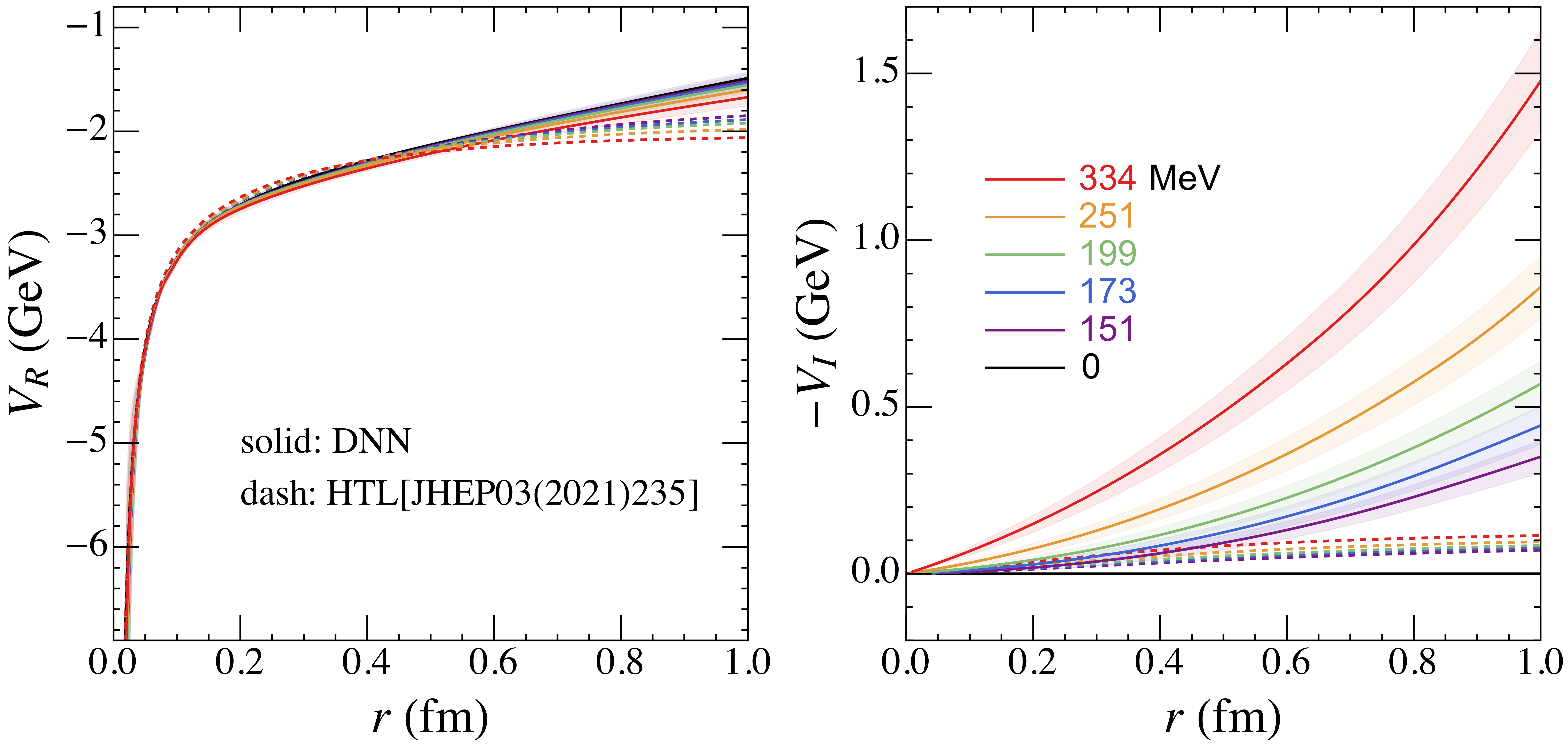
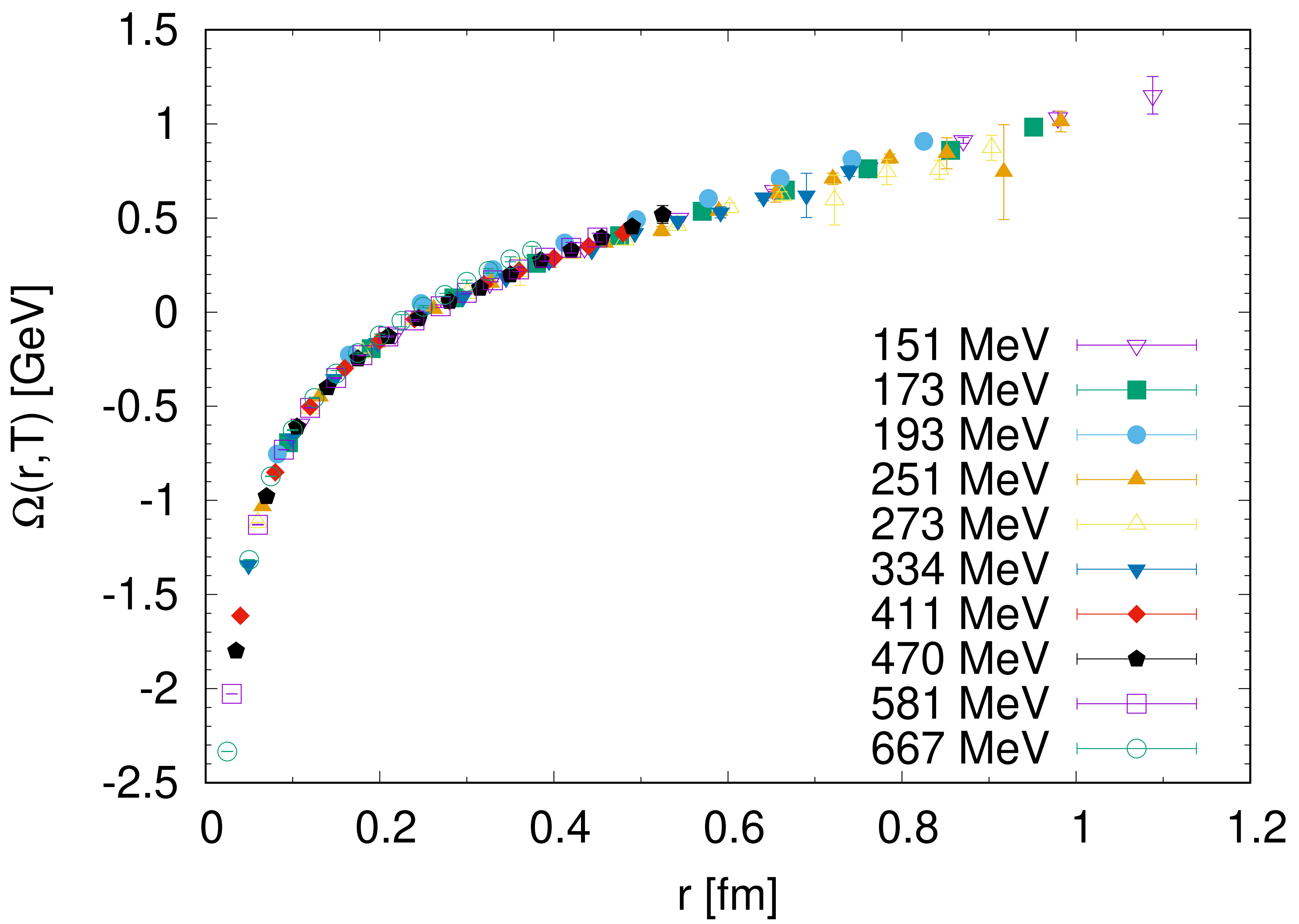


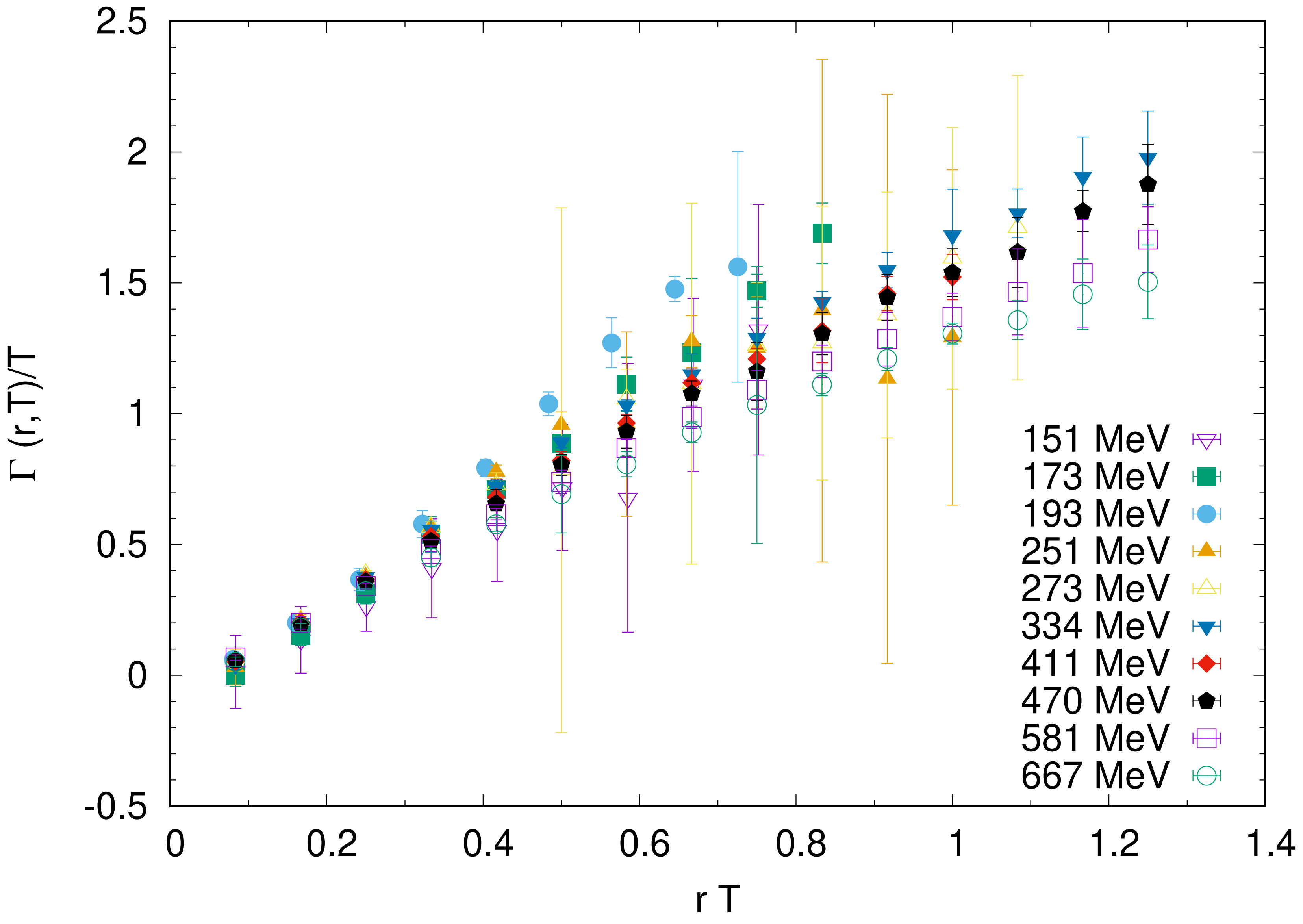


What physics we have learned from $V_{\text{DNN}}(T, r)$?



Reason of the difference?





PHYSICAL REVIEW D **105,** 054513 (2022)

Static quark-antiquark interactions at nonzero temperature from lattice QCD

Dibyendu Bala,[1] Olaf Kaczmarek,[1] Rasmus Larsen,[2] Swagato Mukherjee,[3] Gaurang Parkar,[2] Peter Petreczky,[3] Alexander Rothkopf,[2] and Johannes Heinrich Weber[4]

(HotQCD Collaboration)

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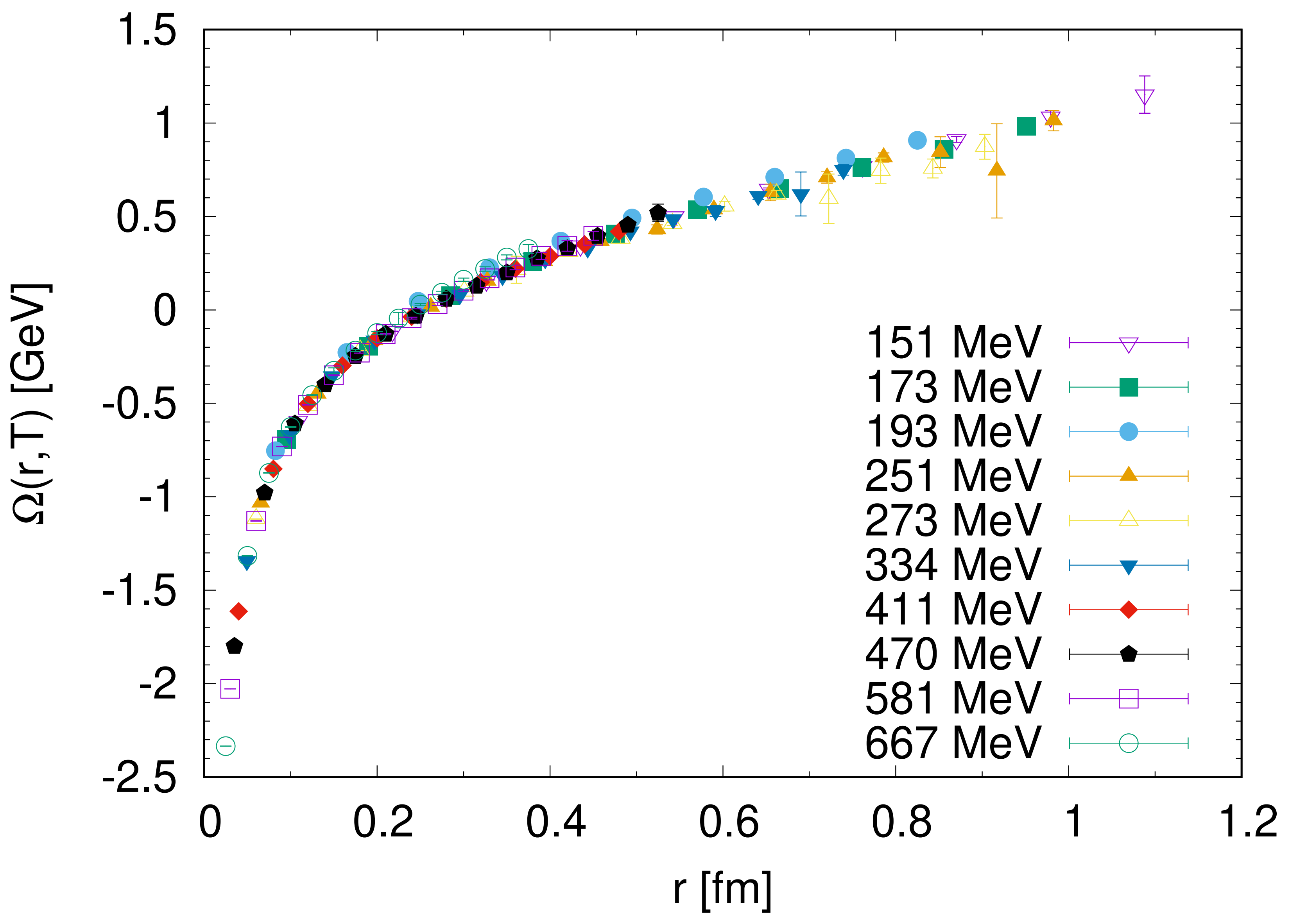

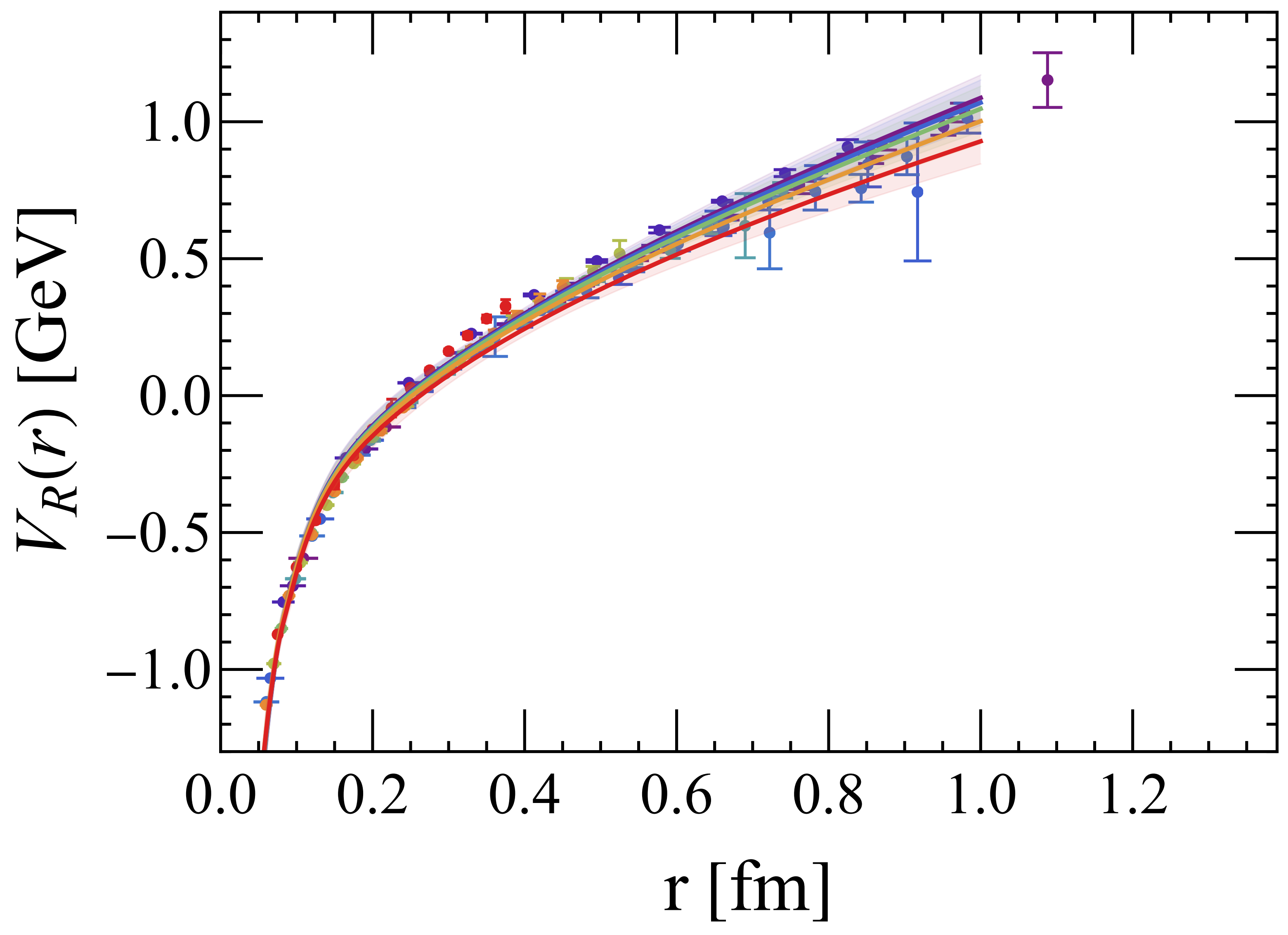

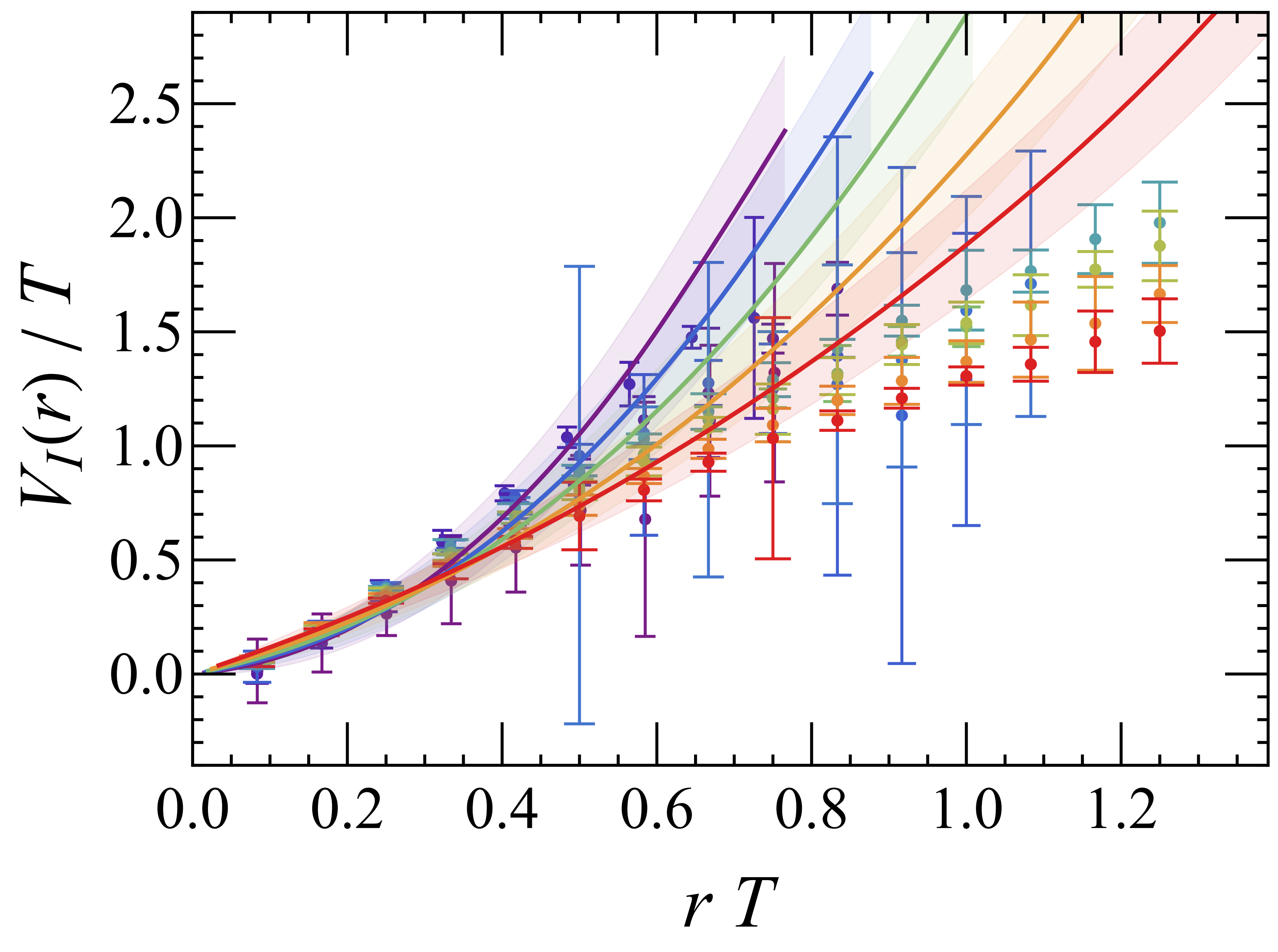

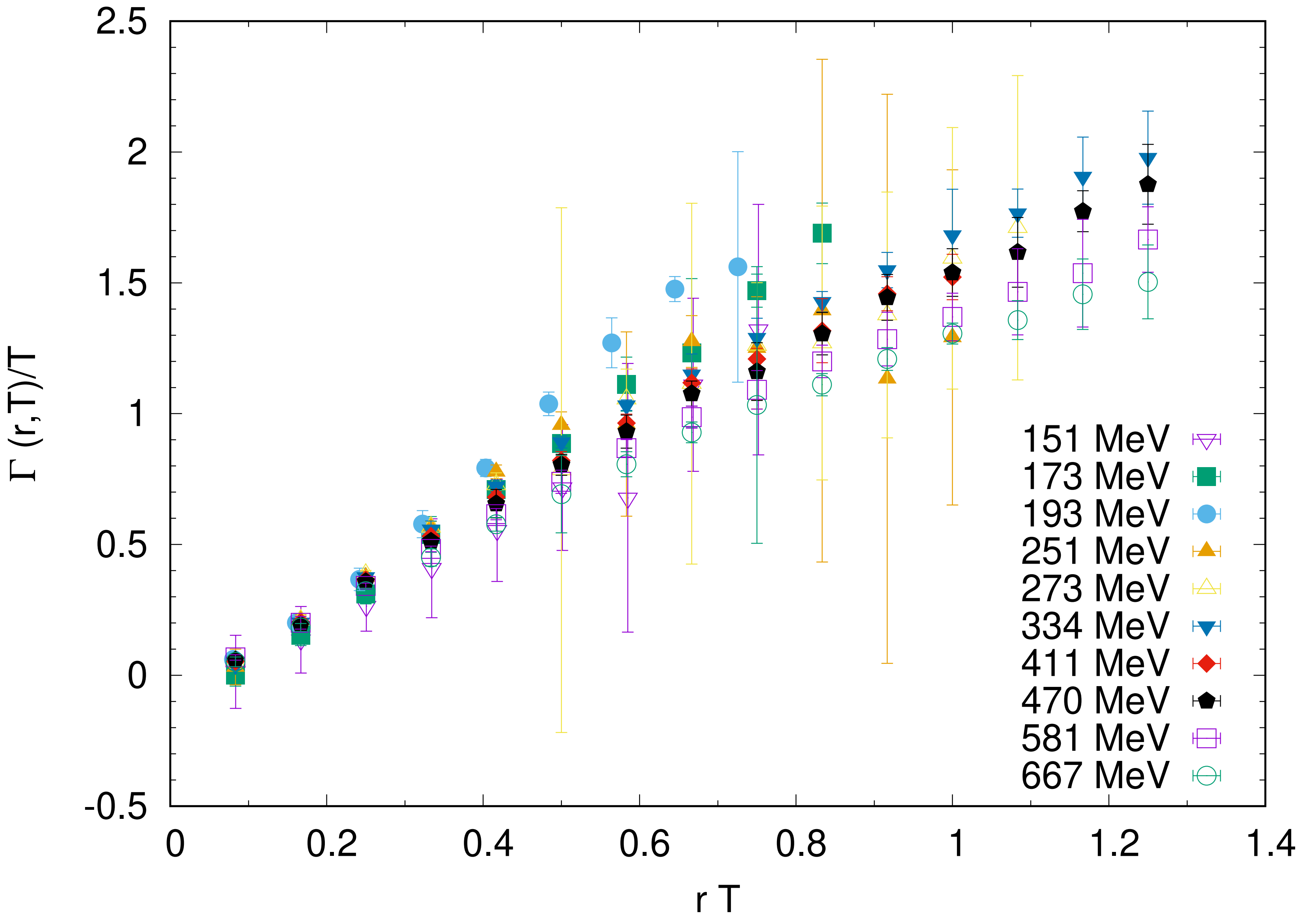

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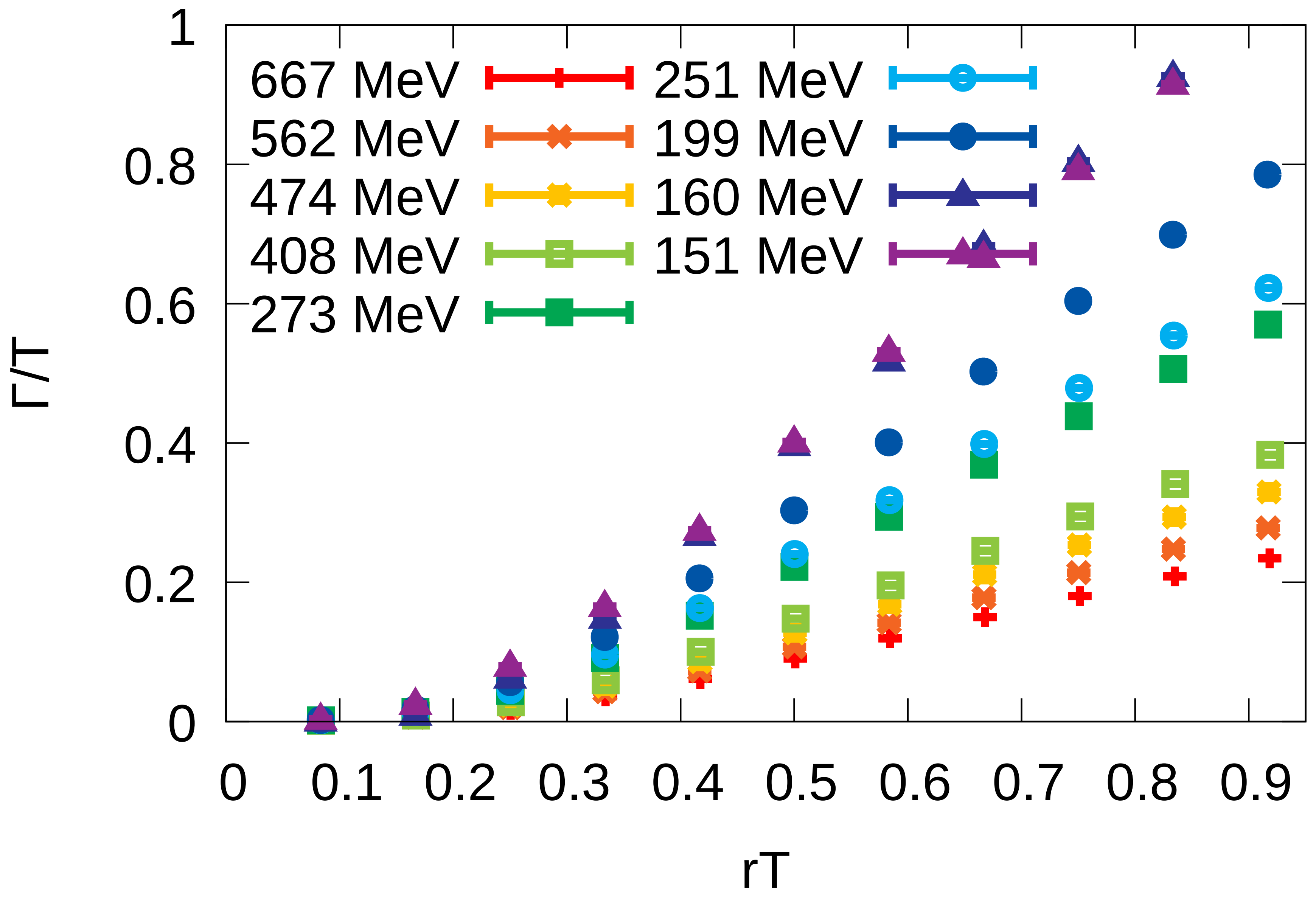

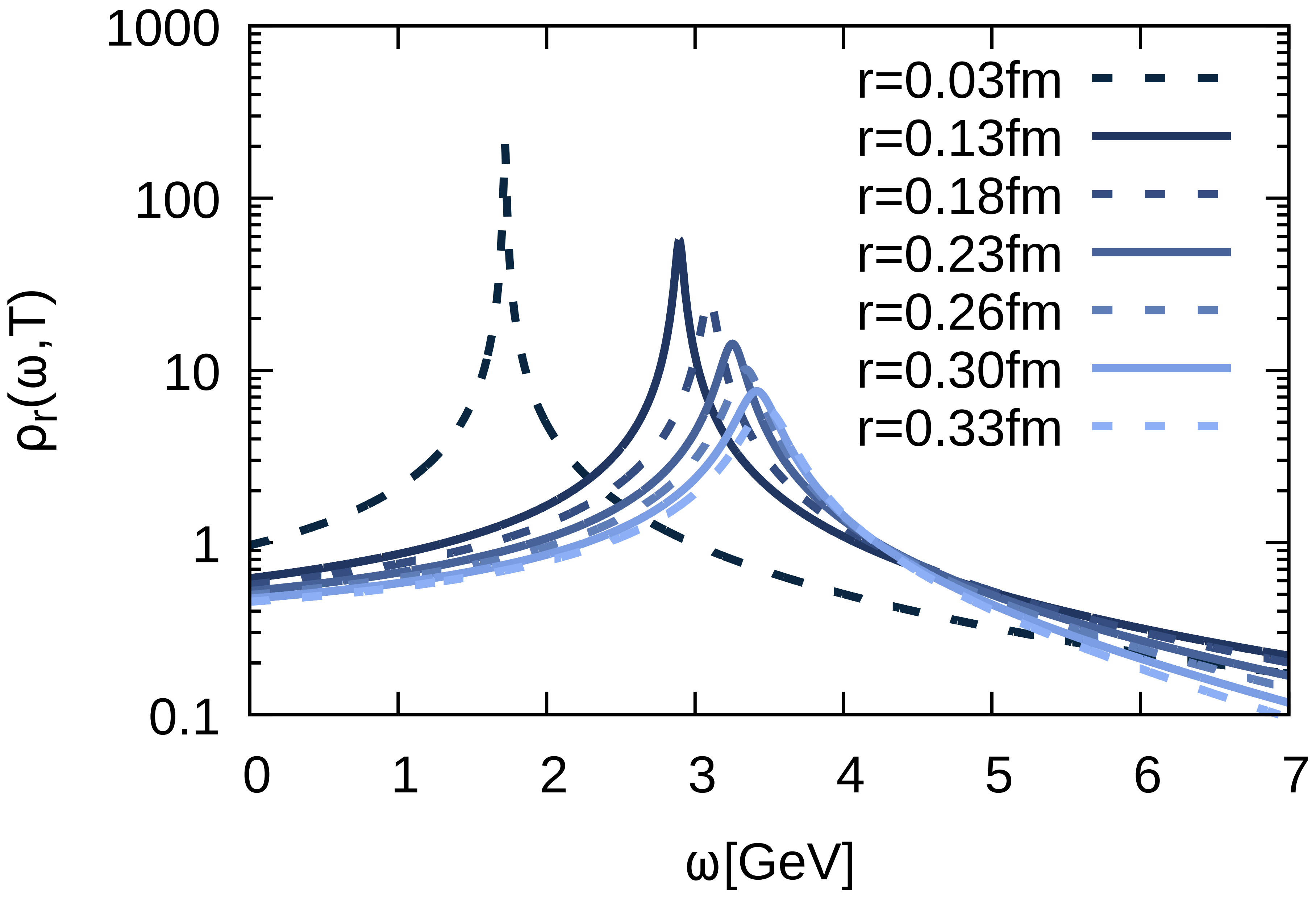

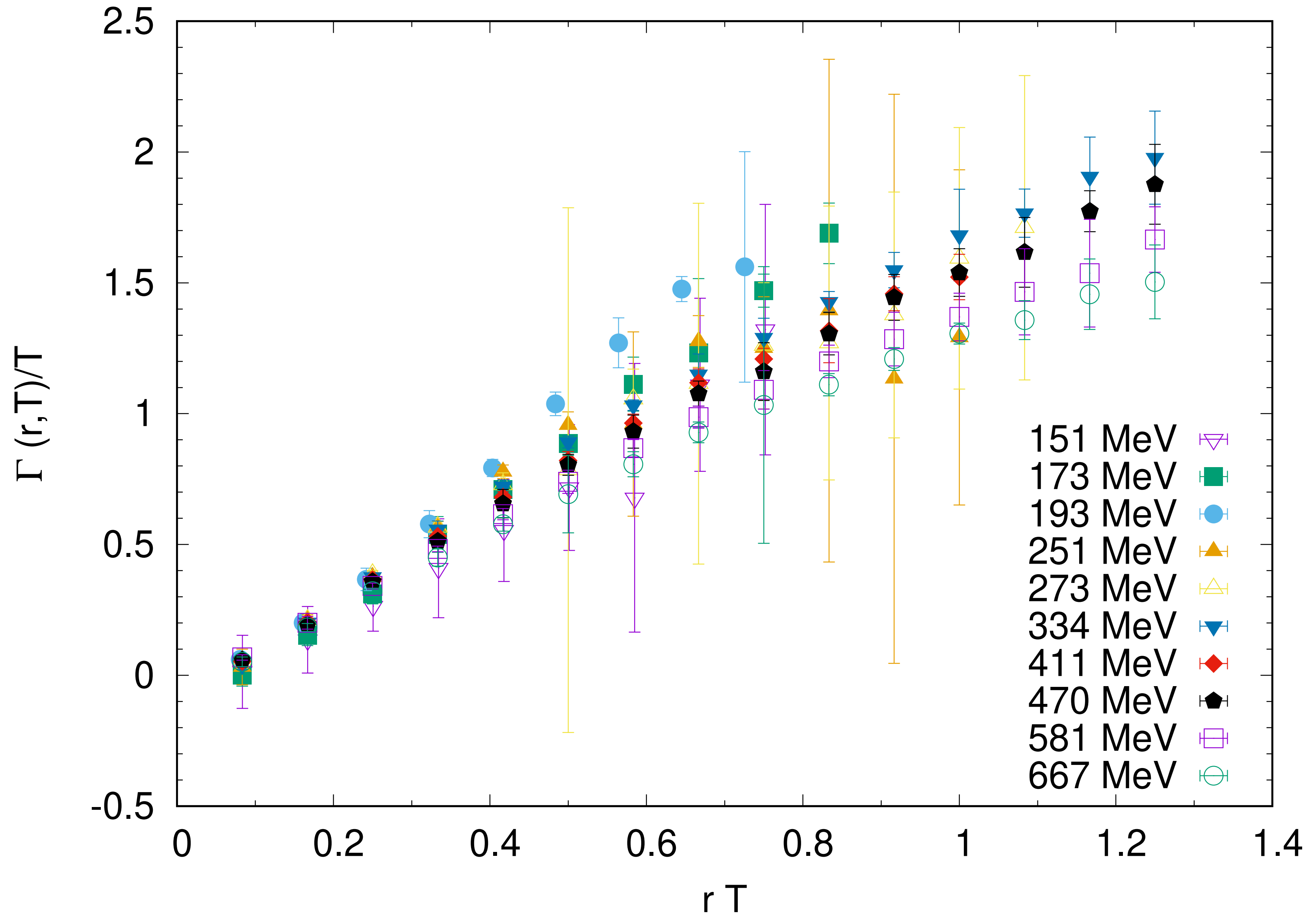

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Problem # II:

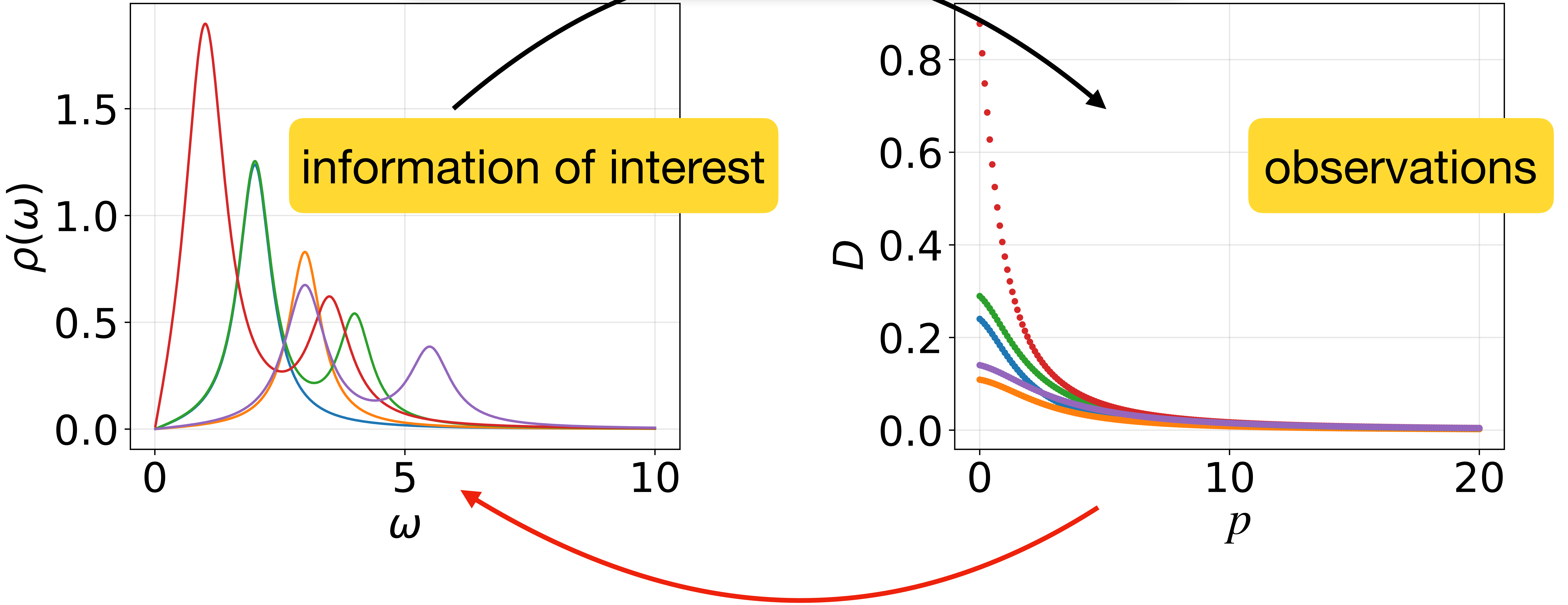
spectral function reconstruction

refs:

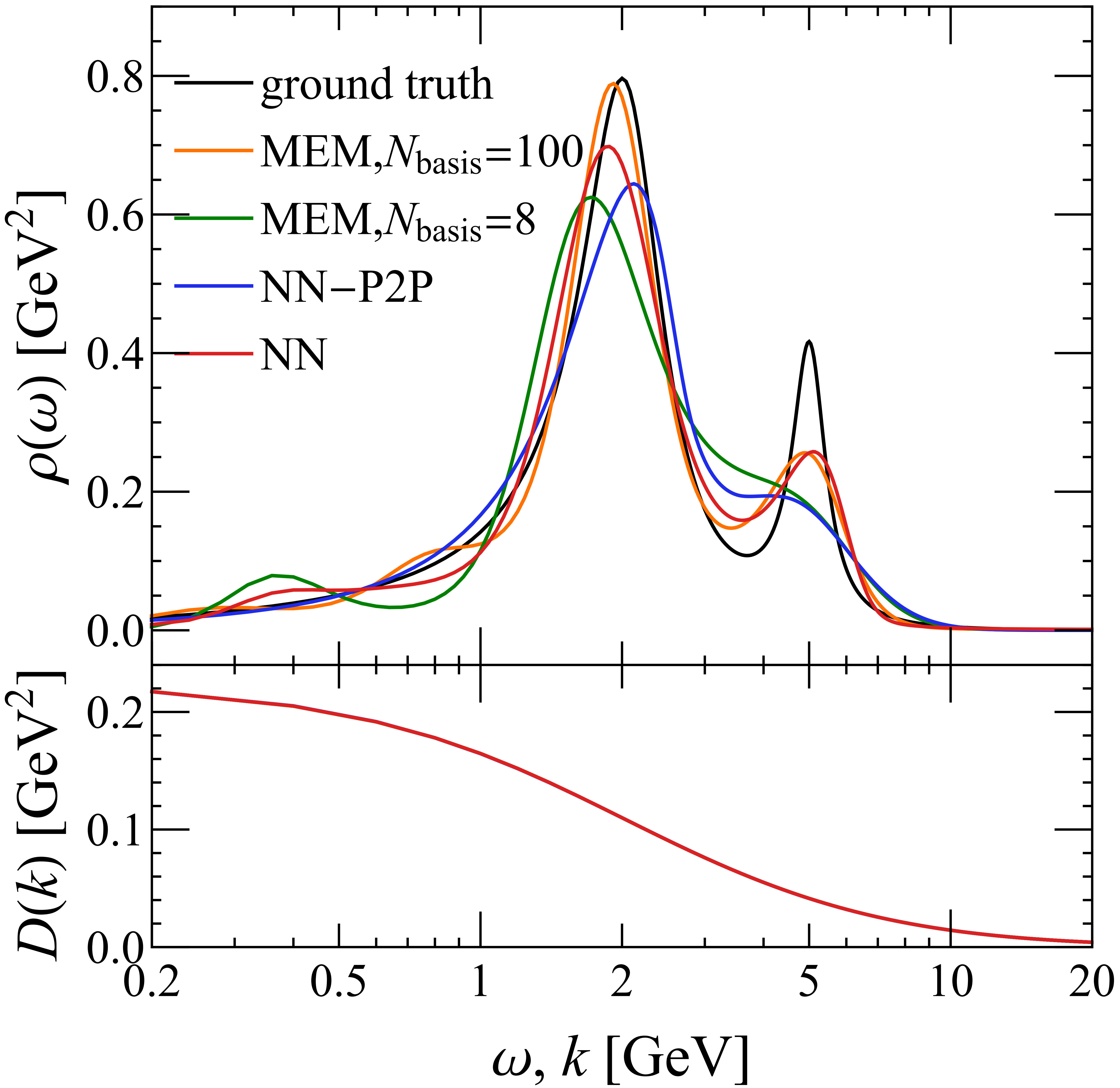
SS, Wang, Zhou, Comput.Phys.Commun. 282 (2023) 108547;
Wang, ***SS***, Zhou, Phys. Rev. D **106**, L051502;

3. spectral function <=> correlation

$$D(p) = \int_0^\infty K(p,\omega)\,\rho(\omega)\,d\omega \qquad K(p,\omega) \equiv \frac{\pi^{-1}\,\omega}{\omega^2 + p^2}$$



Can one learn the spectral function from correlation?



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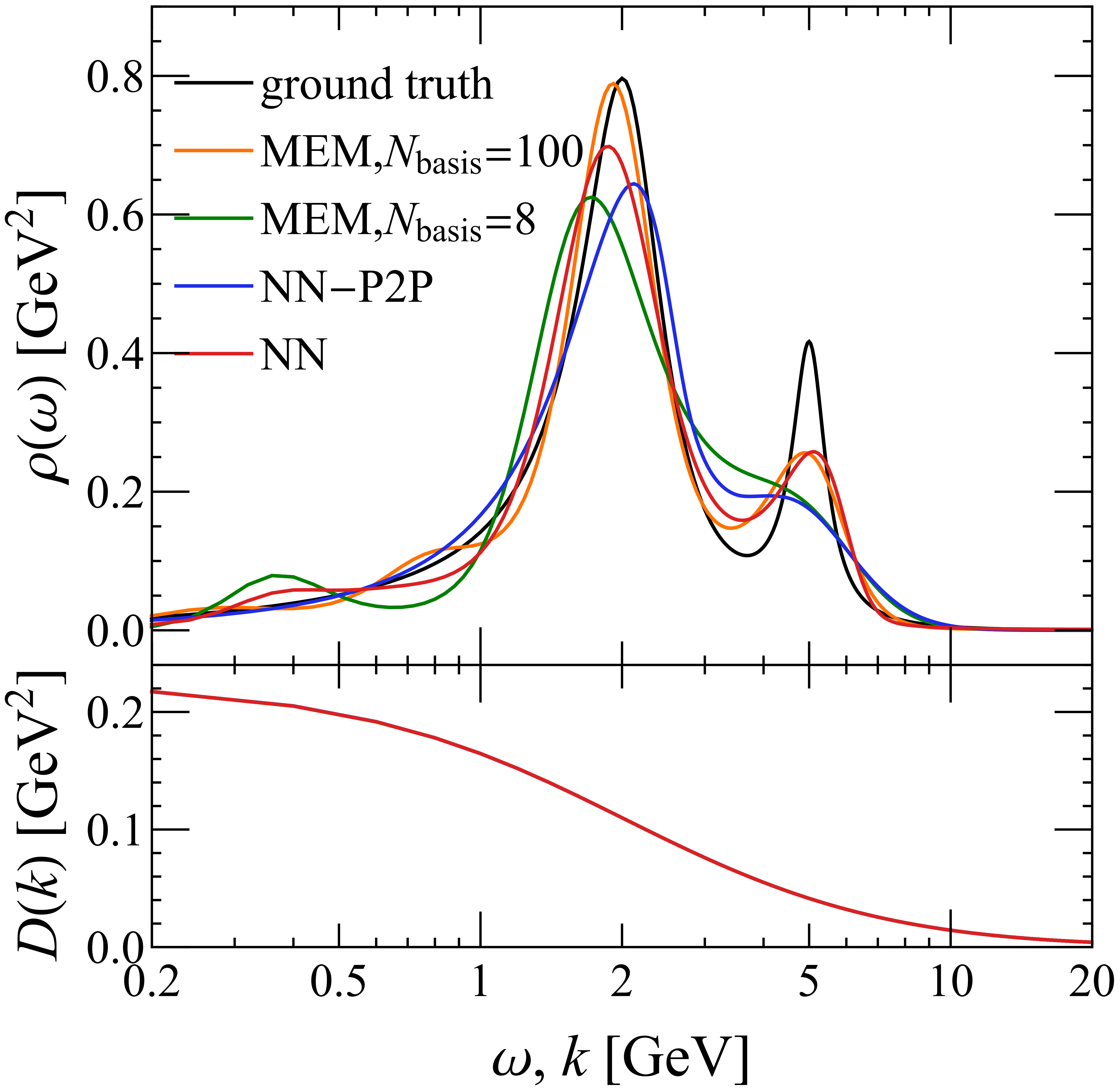

- ill-conditioned?
  - one needs more points in ω than that were given in k

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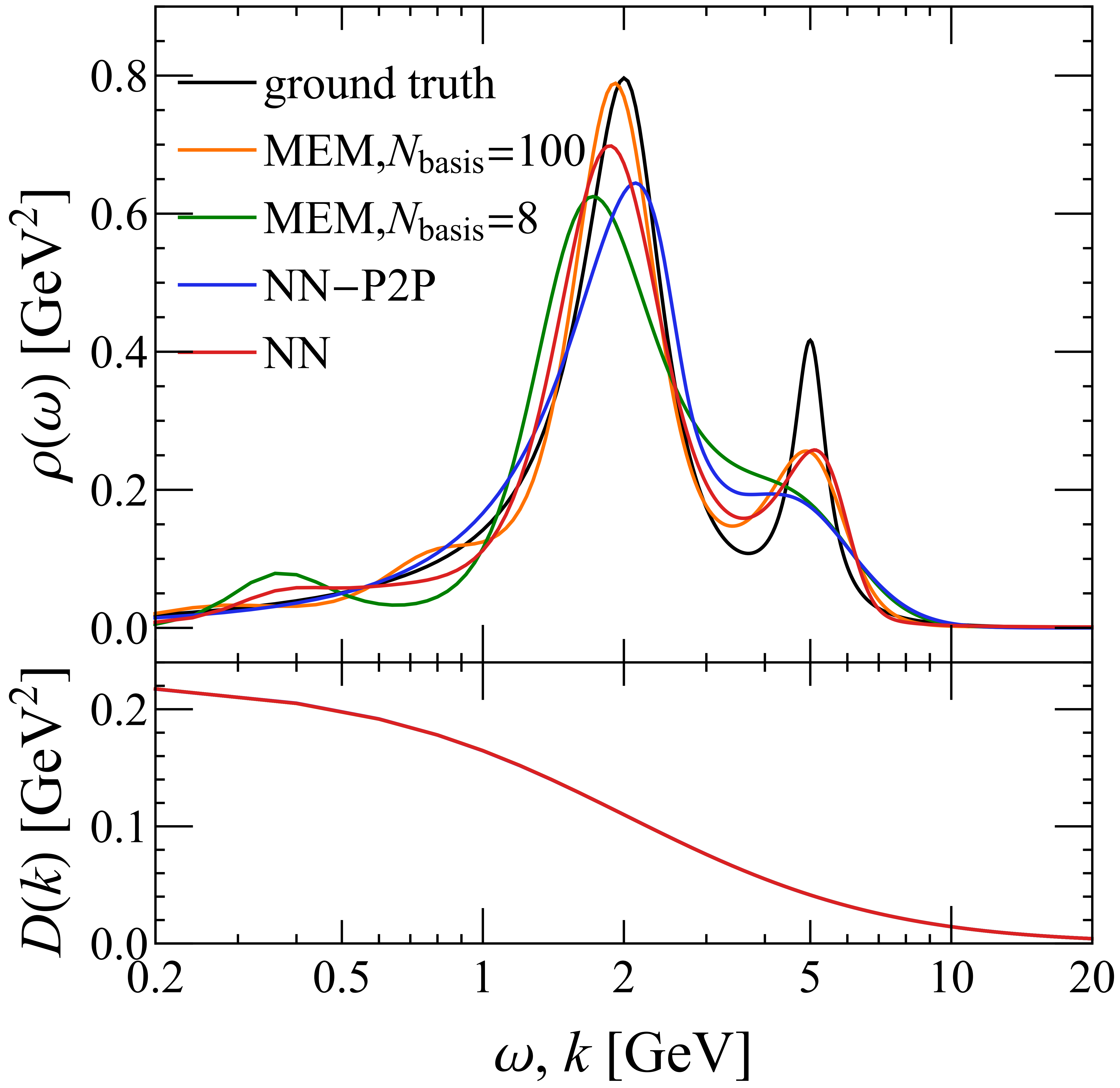

- ill-conditioned?
  - one needs more points in ω than that were given in k

resolved by increasing # of k-points?

- No!!! The problem is ill-posed!!!

ill-posedness of inverse KL convolution

$$D(k) = \frac{1}{\pi}\int_0^\infty \frac{\omega\,\mathrm{d}\omega}{\omega^2 + k^2}\rho(\omega)$$

Linear operator in continuous space,
maps $\mathbb{R}_{[0,+\infty)}$ to $\mathbb{R}_{[0,+\infty)}$.

$\mathbb{R}_{[0,+\infty)}$: Real function defined in the domain $[0, +\infty)$.

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Linear operator in continuous space,

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One can define its eigenfunctions and eigenvalues:

$$\boxed{\frac{1}{\pi}\int_0^\infty \frac{\omega\,\mathrm{d}\omega}{\omega^2+k^2}\psi(\omega) = \lambda\,\psi(k)\,,}$$

$\mathbb{R}_{[0,+\infty)}$: Real function defined in the domain $[0,+\infty)$.

eigenfunctions and eigenvalues of KL convolution:

$$\frac{1}{\pi}\int_0^\infty \frac{\omega\,\mathrm{d}\omega}{\omega^2+k^2}\psi(\omega) = \lambda\,\psi(k)\,,$$

infinite amount of solutions, labeled by (continuous) s:

$$\psi_{s,+}(x) = \frac{\cos\big(s\ln(x/a)\big)}{\sqrt{\pi\,x/a}}\,,$$

$$\psi_{s,-}(x) = \frac{\sin\big(s\ln(x/a)\big)}{\sqrt{\pi\,x/a}}\,,$$

$$\lambda_{s,\pm} = \frac{1}{2\cosh(\pi s/2)}\,.$$

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$$\widetilde{\rho}_\pm(s) = \widetilde{D}_\pm(s) \,/\, \lambda_s$$

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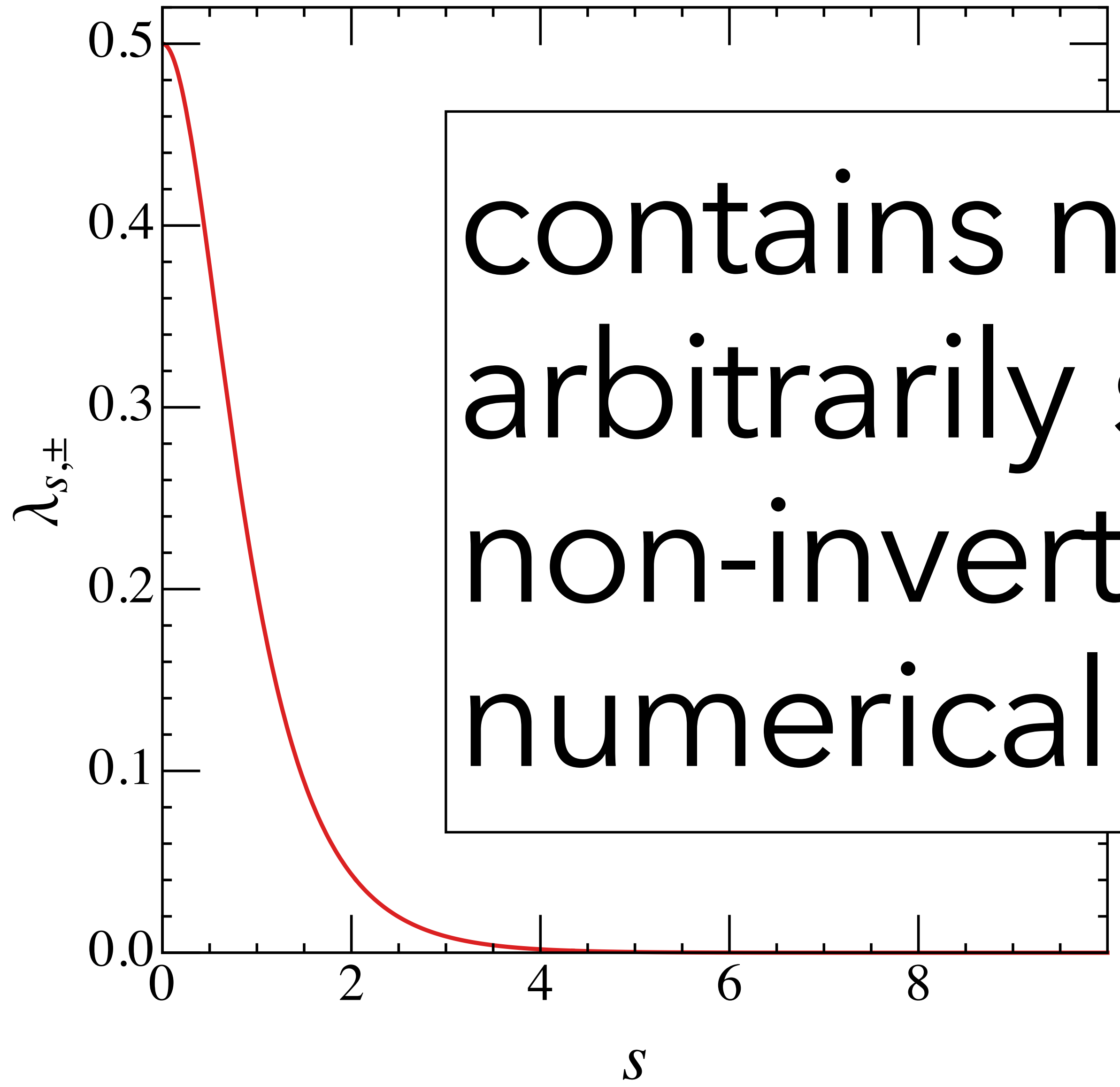

contains non-zero but arbitrarily small eigenvalues, non-invertible given finite numerical precision.

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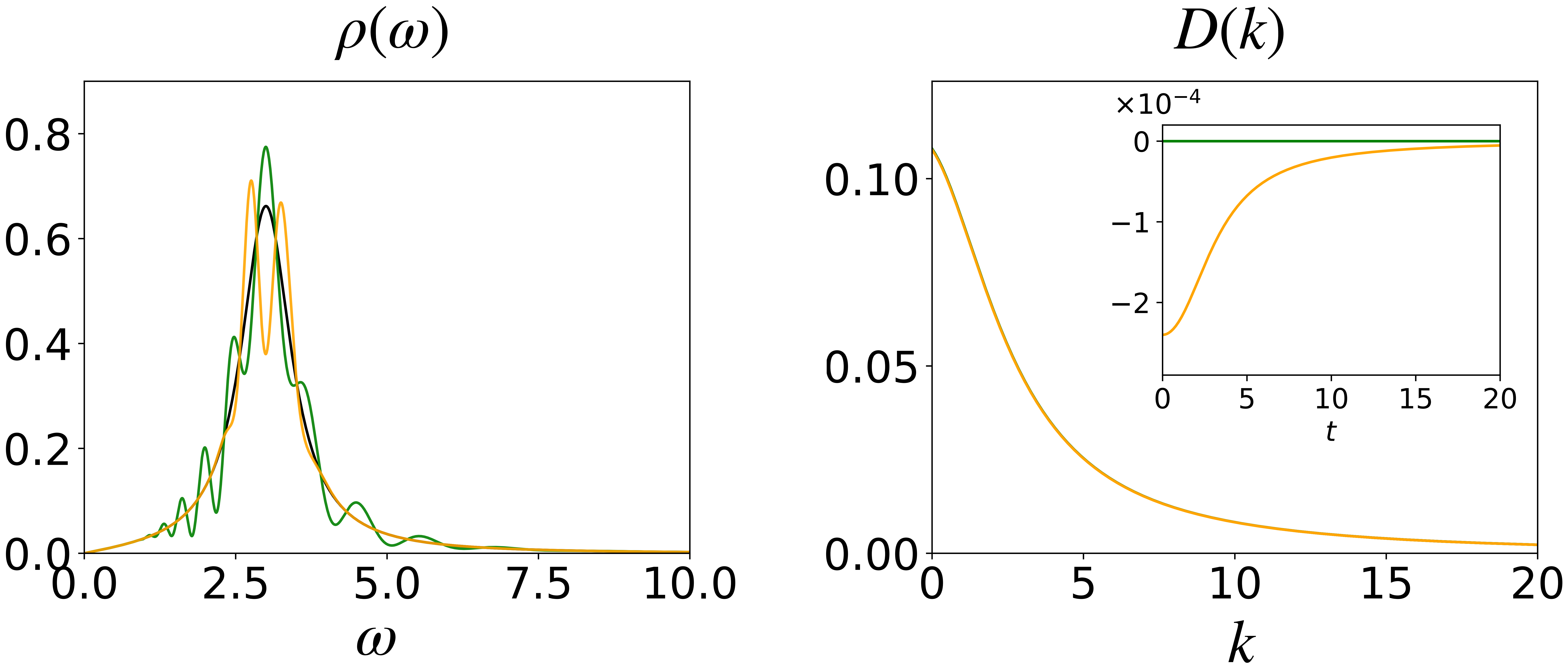
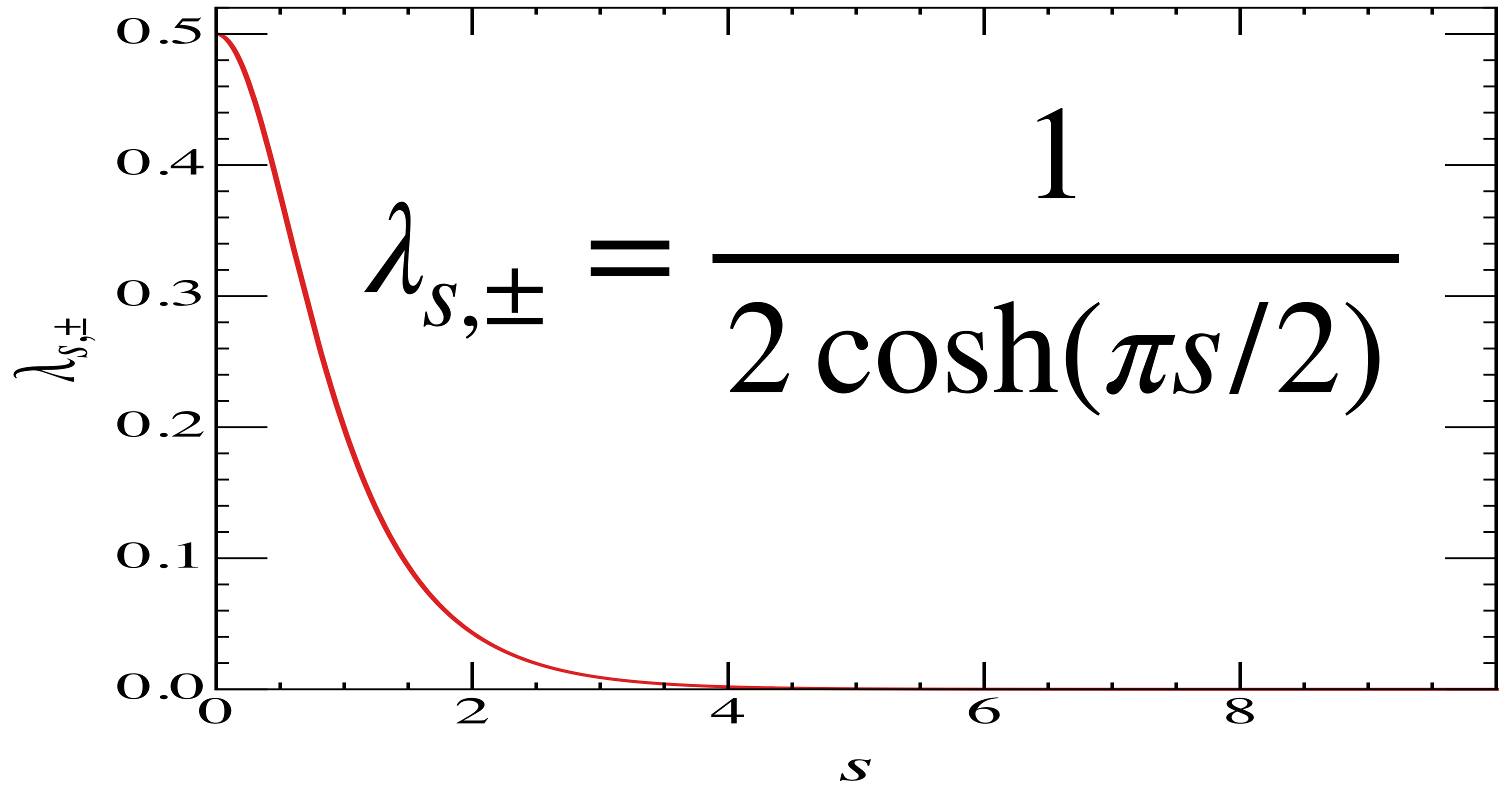


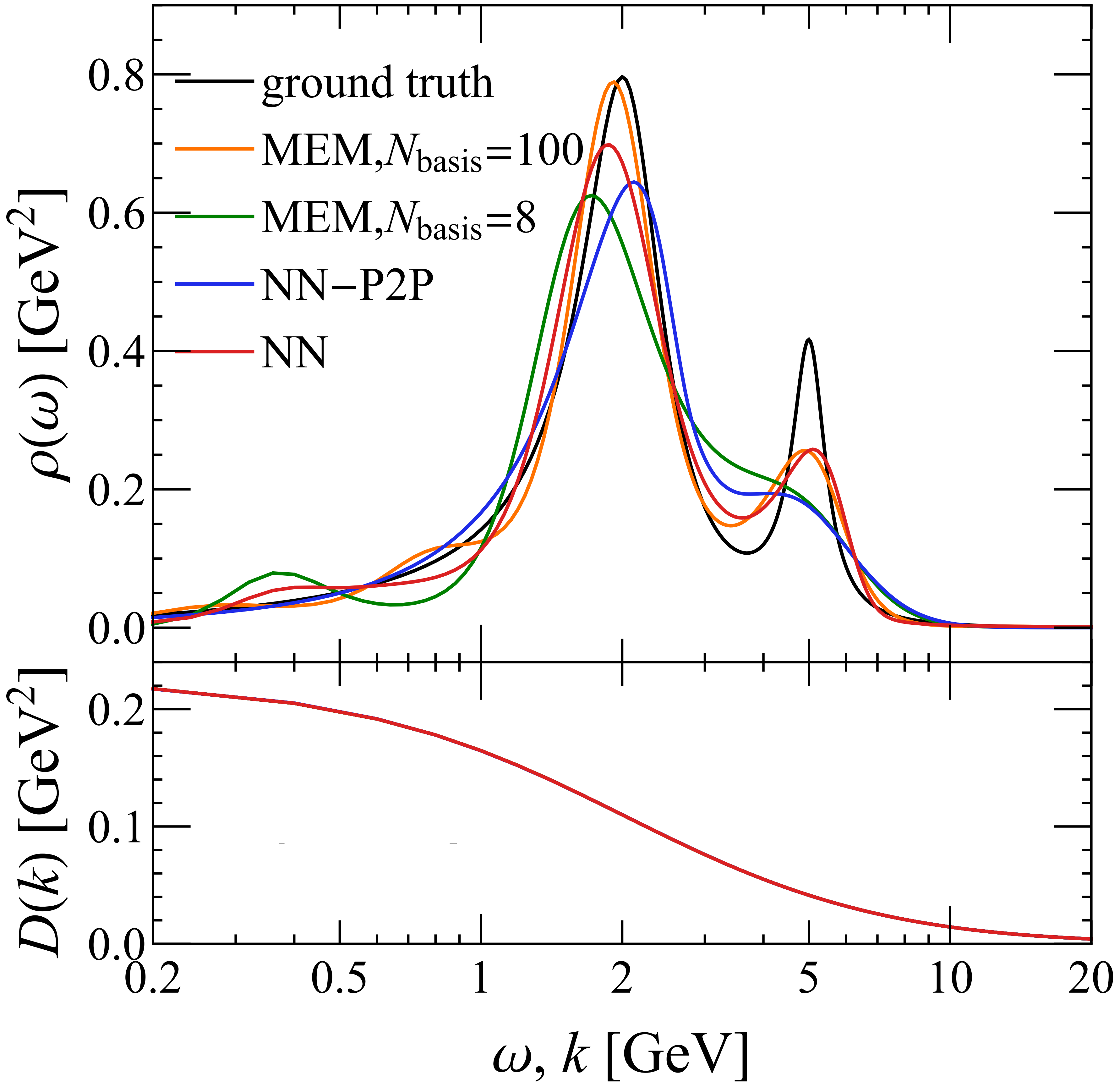
relook at the discrepancy

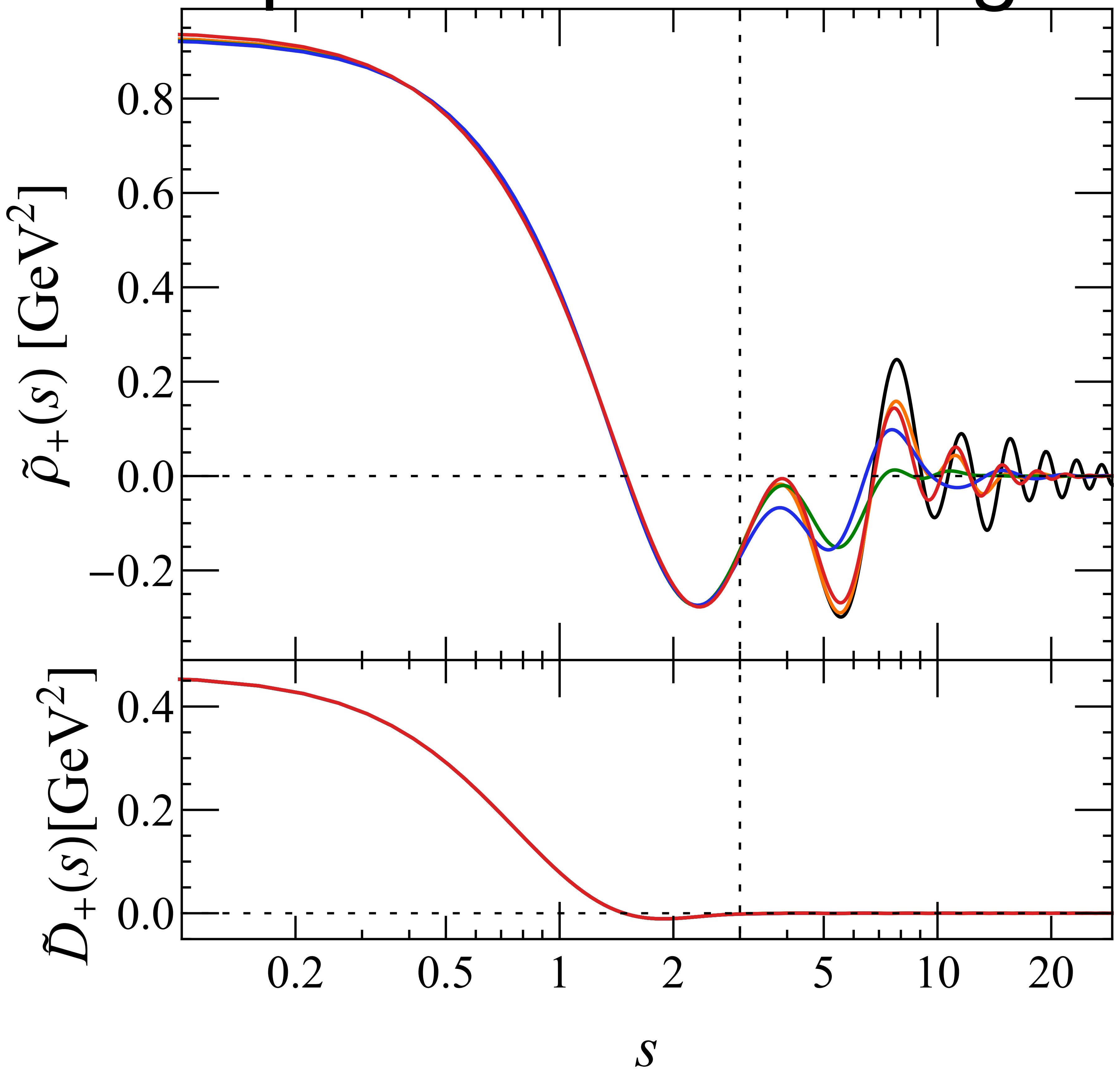


$$\lambda_{s,\pm} = \frac{1}{2\cosh(\pi s/2)}$$

$$\psi_{s,+}(x) = \frac{\cos\big(s \ln(x/a)\big)}{\sqrt{\pi}\, x/a}$$

decomposition into the eigen-space





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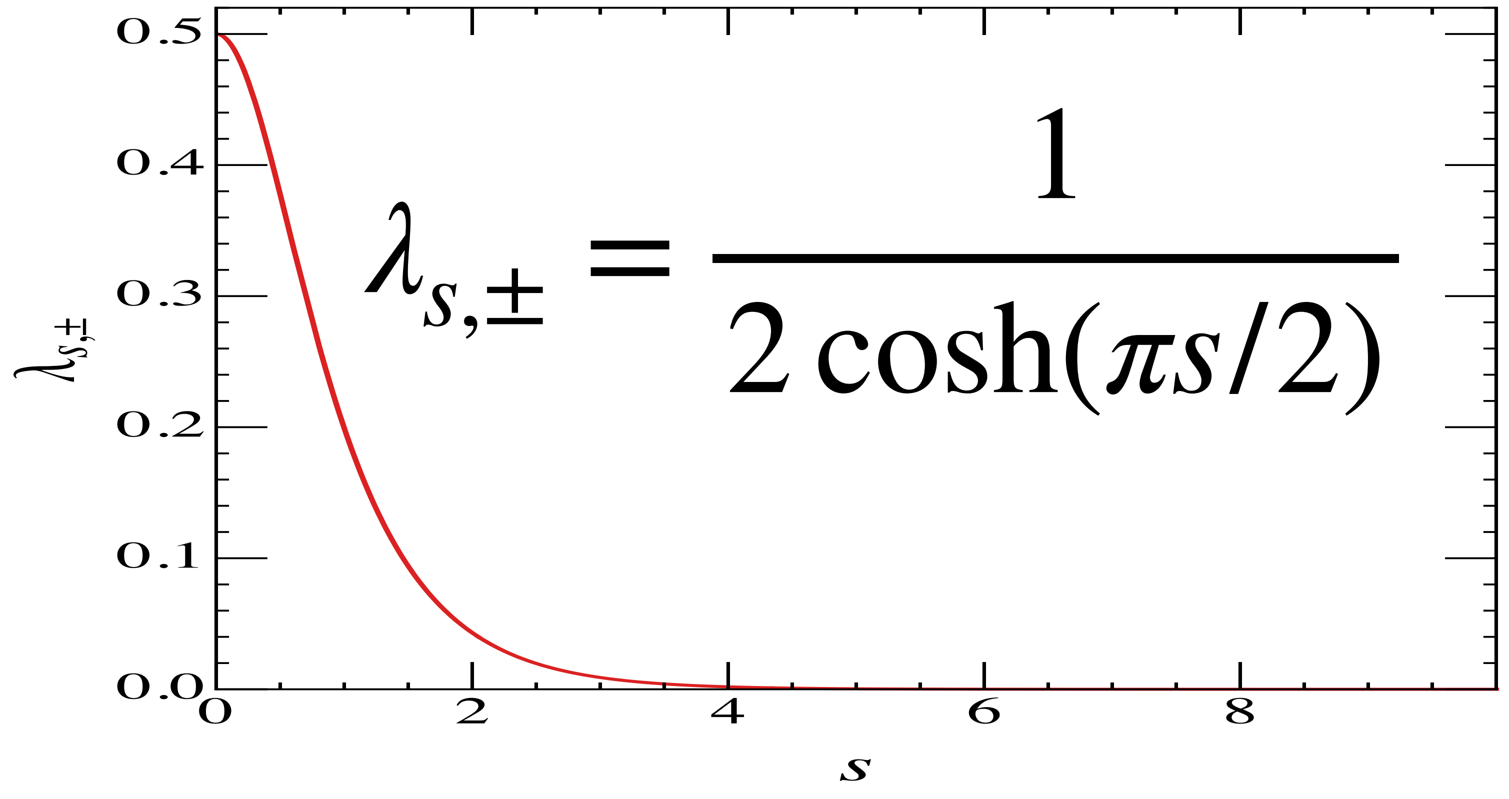
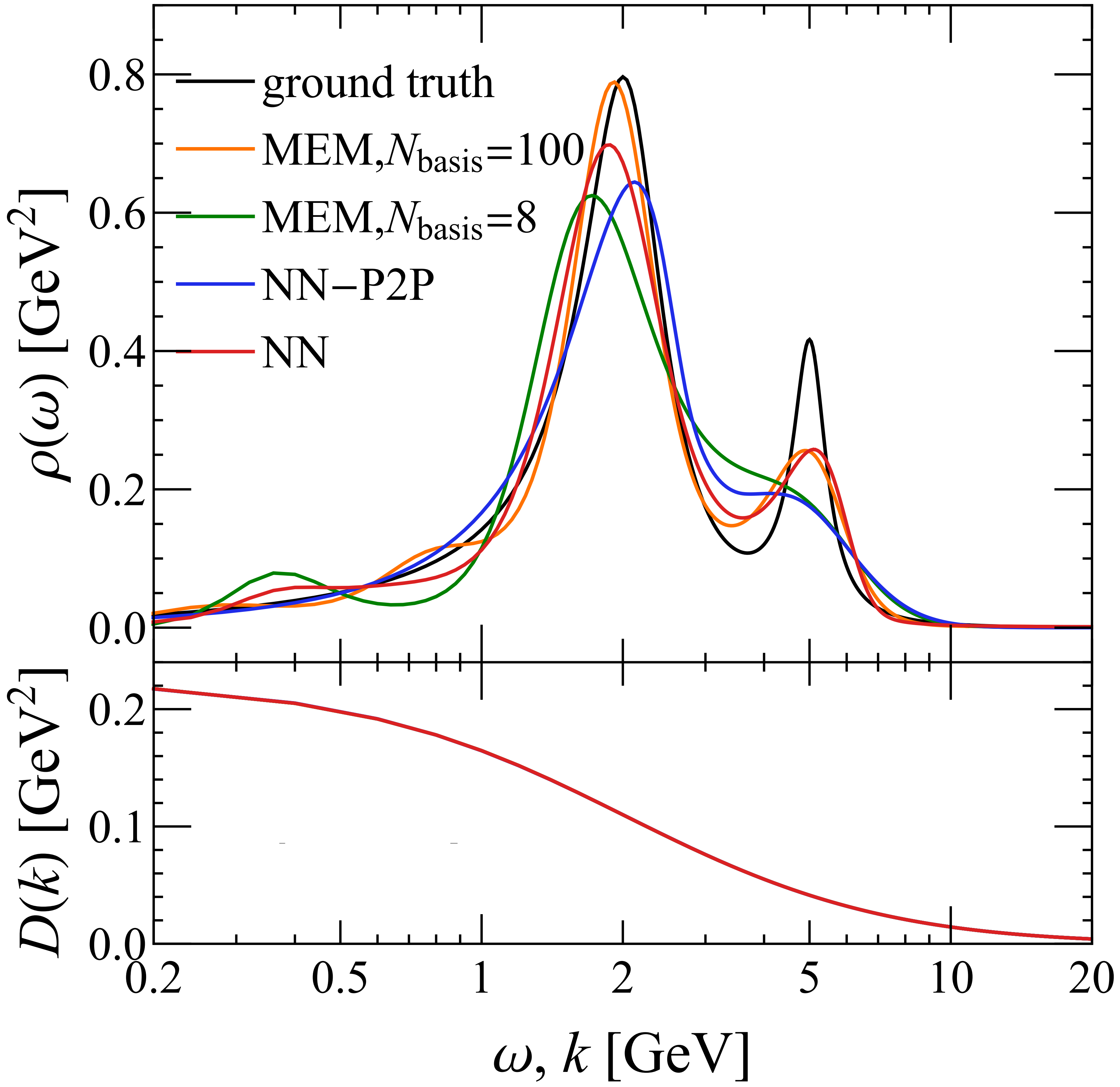
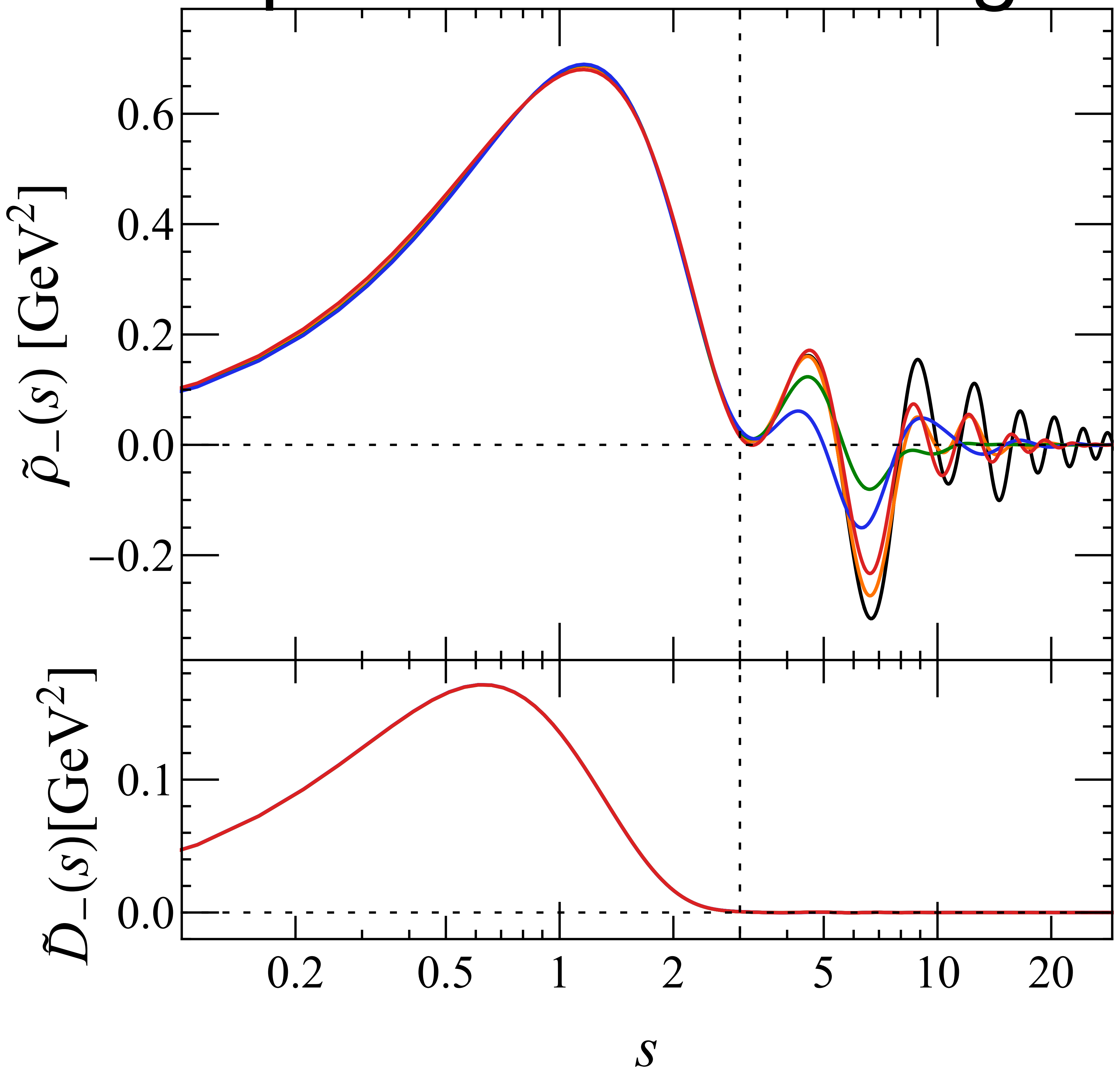


$$\psi_{s,-}(x) = \frac{\sin\left(s \ln(x/a)\right)}{\sqrt{\pi}\, x/a}$$

decomposition into the eigen-space





Then, why Deep Learning?

DNN is a natural implementation of smoothness regularization!

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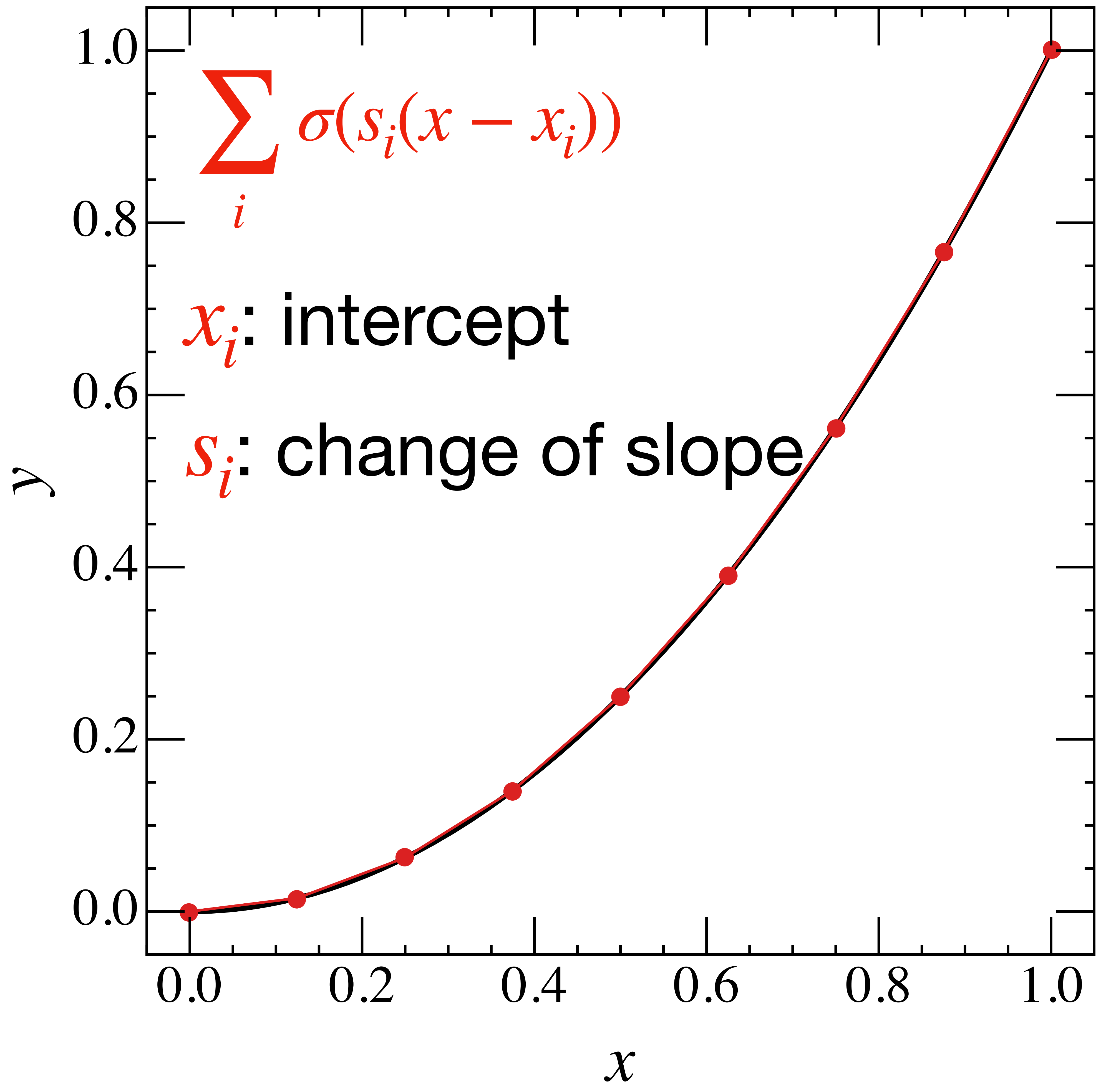
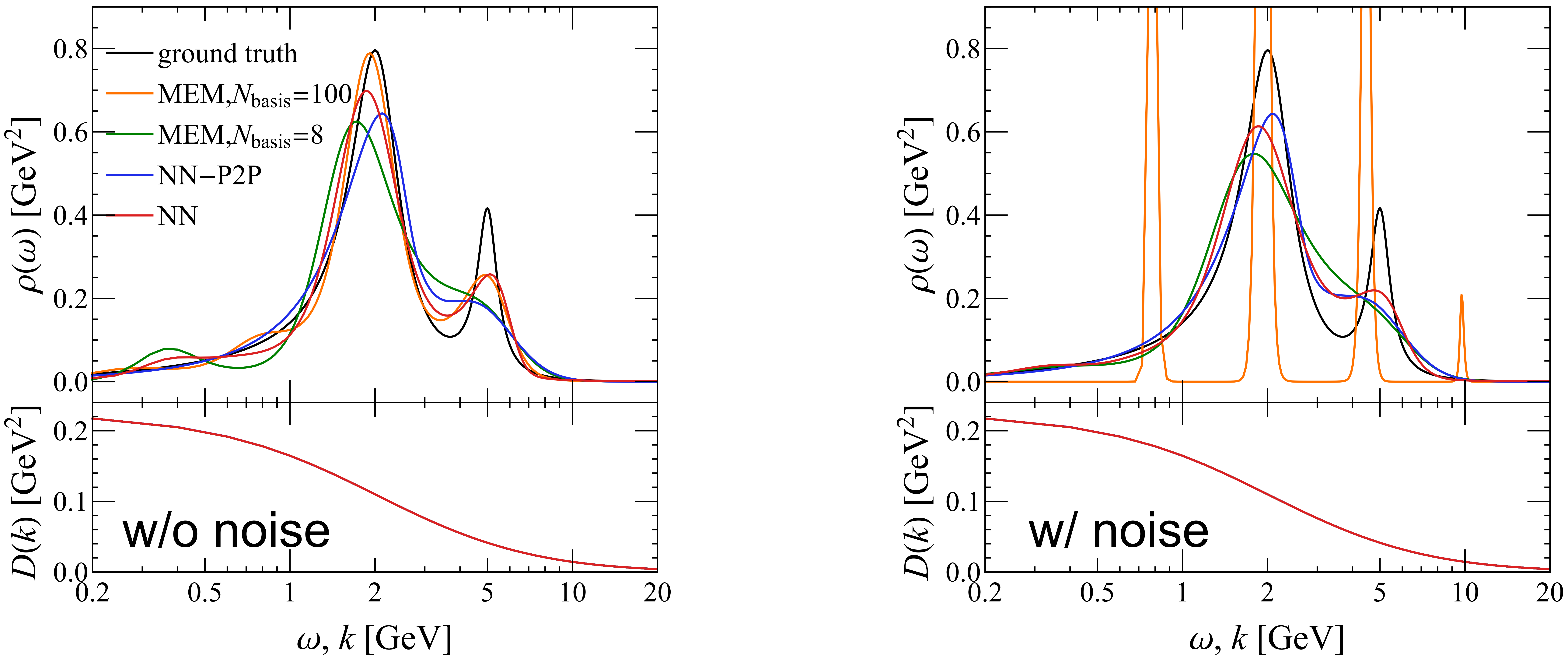


minimizing $\sum_i s_i^2 \leftrightarrow$ imposing smoothness

Robustness against larger noise



Summary

- New algorithm employing DNN to solve inverse problems;
- Extracted HF complex $V(T, r)$;
- Reconstructing spectral function … is ill-posed!!!
    - physics-based parameterization
    - real time evolution?
    - supervised learning?

How to learn $V(r)$ from $\{E_n\}$?

- parameterize the potential $V(r\,|\,\boldsymbol{\theta})$, ***scan*** the whole $\boldsymbol{\theta}$-space, minimize $\chi^2 \equiv \sum_i \left(\frac{E_{\boldsymbol{\theta},i} - E_i}{\delta E_i}\right)^2$

- a ***gradient-descent*** based method:
    - goal -- find the $\boldsymbol{\theta}$-point that $\nabla_{\boldsymbol{\theta}}\chi^2 = 0$
    - update $\boldsymbol{\theta}$ iteratively according to $\boldsymbol{\Delta\theta} \propto \nabla_{\boldsymbol{\theta}}\chi^2$

$$\nabla_{\boldsymbol{\theta}}\chi^2 = 2\sum_i \frac{E_{\boldsymbol{\theta},i} - E_i}{(\delta E_i)^2}\nabla_{\boldsymbol{\theta}}E_{\boldsymbol{\theta},i}$$

$$\nabla_{\boldsymbol{\theta}}E_{\boldsymbol{\theta},i} = \langle \psi_i \,|\, \nabla_{\boldsymbol{\theta}}V(\boldsymbol{\theta}) \,|\, \psi_i \rangle$$

- general unbiased parameterization scheme? Deep Neural Network!

How to compute the likelihood (density) distribution of $V_{\boldsymbol{\theta}}$

$$P(V_{\boldsymbol{\theta}})\mathrm{d}V = \text{Posterior}(\boldsymbol{\theta}\,|\,\text{data})\mathrm{d}^N\boldsymbol{\theta}$$

- Sample $\{\boldsymbol{\theta}_i\}$ according to a reference distribution: $P(\boldsymbol{\theta}) = \tilde{P}(\boldsymbol{\theta})$;
- Each data point corresponds to the element volume $\mathrm{d}^N\boldsymbol{\theta}_i = 1/\tilde{P}(\boldsymbol{\theta}_i)$;
- Compute $V_{\boldsymbol{\theta}_i}(r)$, $\chi^2_{\boldsymbol{\theta}_i}$, and Posterior$(\boldsymbol{\theta}_i\,|\,\text{data})$;
- For given *r*, histogram $V_{\boldsymbol{\theta}_i}(r)$ with weights

$$w_i = P(V_{\boldsymbol{\theta}_i})\mathrm{d}V_i = \text{Posterior}(\boldsymbol{\theta}_i)/\tilde{P}(\boldsymbol{\theta}_i)$$

- In practice:

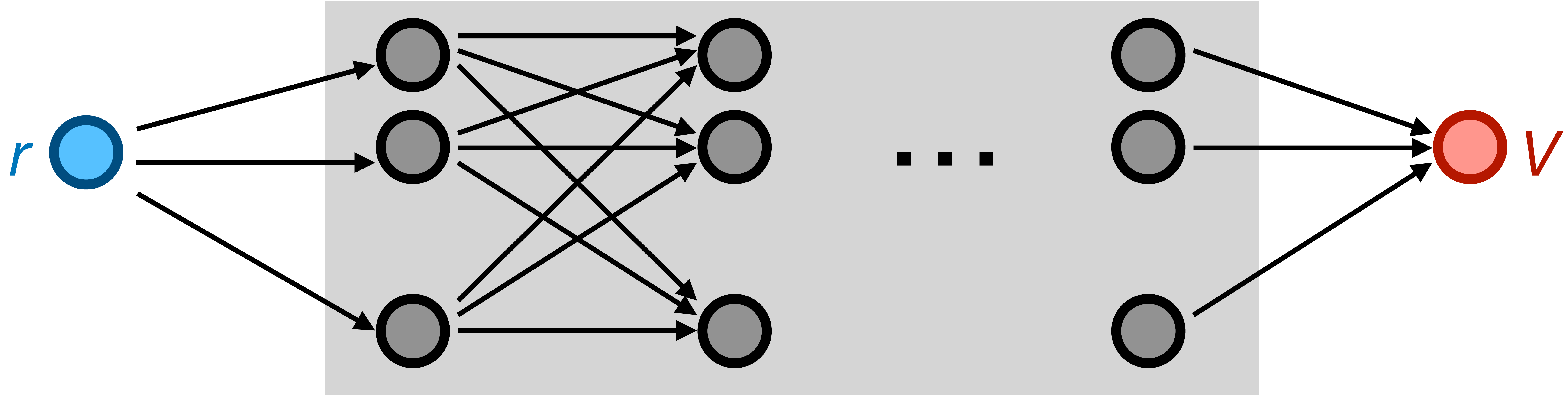
$$\tilde{P}(\boldsymbol{\theta}) = (2\pi)^{-N_\theta/2}\sqrt{\det[\Sigma^{-1}]} \times \exp\Big[-\frac{\Sigma^{-1}_{ab}}{2}(\theta_a - \theta_a^{\text{opt}})(\theta_b - \theta_b^{\text{opt}})\Big] \qquad \Sigma^{-1}_{ab} = \lambda\delta_{ab} + \frac{1}{2}\frac{\partial^2\chi^2(\boldsymbol{\theta})}{\partial\theta_a\partial\theta_b}$$

What are Deep Neural Networks?

--- a general parameterization scheme to approximate continuous functions.

$$V(r) \approx V_{\mathrm{DNN}}(r\,|\,\text{parameters})$$

(iterative function substitution)



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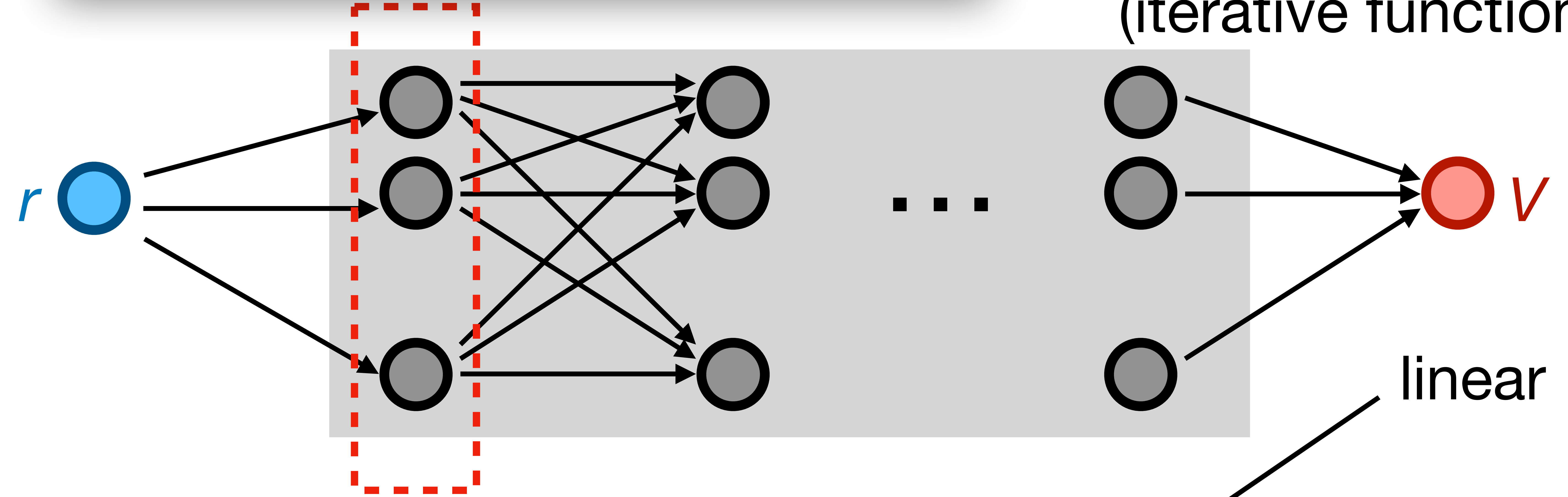
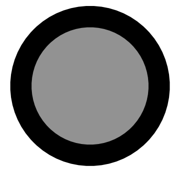
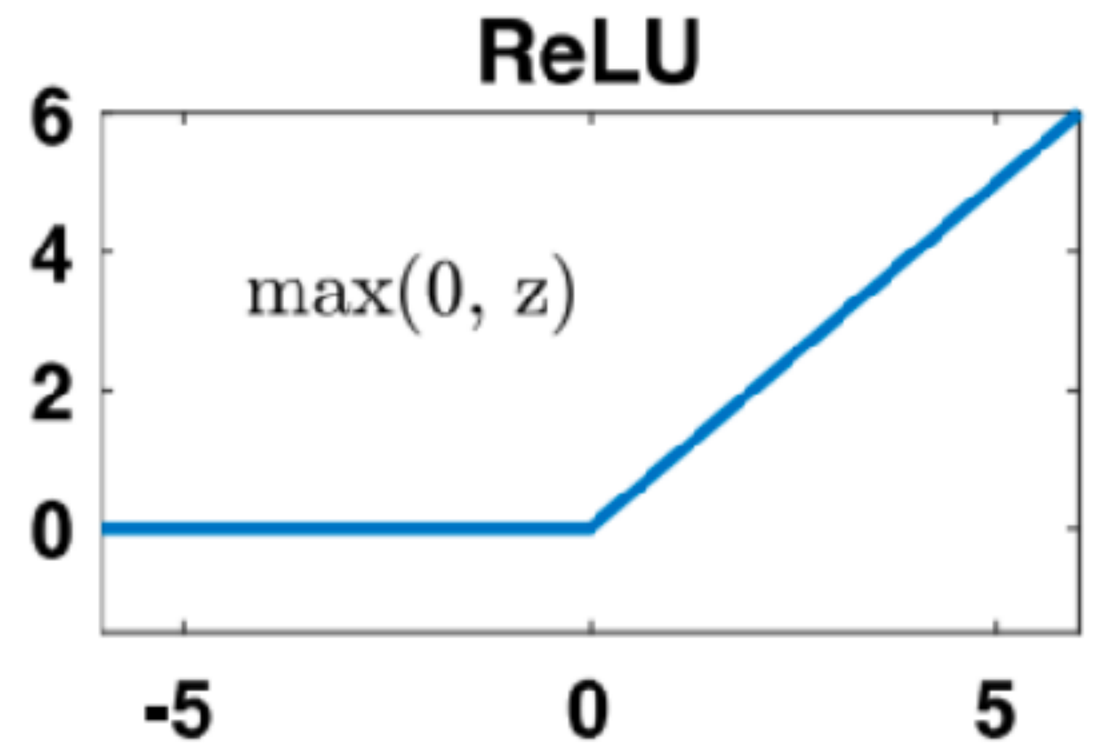


linear

nonlinear

Each  is an intermediate function $(a_i^{(l)})$:

- At the first layer: $z_i^{(1)} = b_i^{(1)} + W_{i,1}^{(1)}\, r,$ $\quad a_i^{(1)} = \sigma(z_i^{(1)})$



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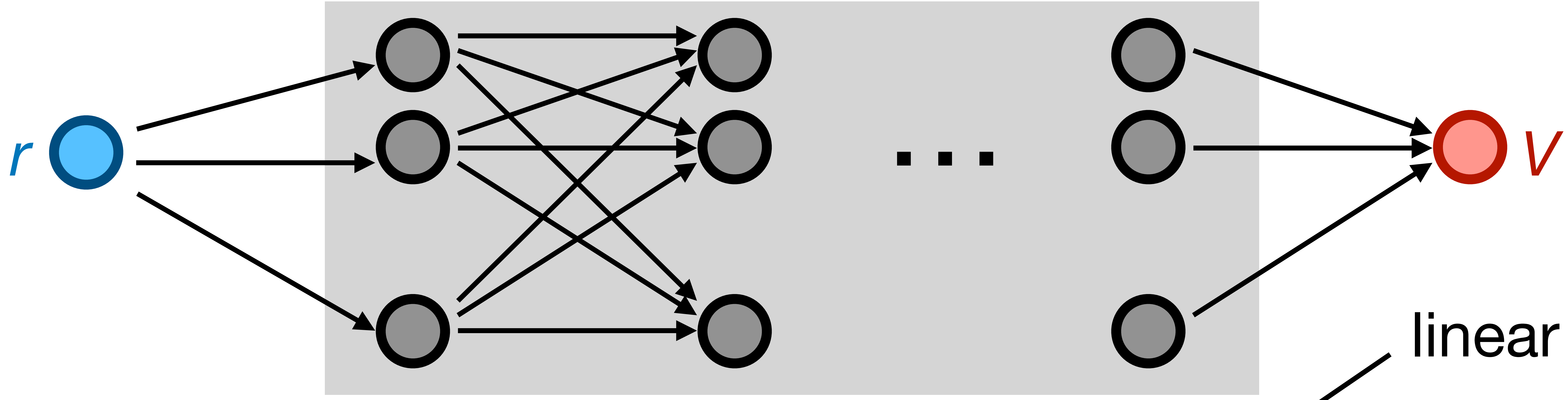

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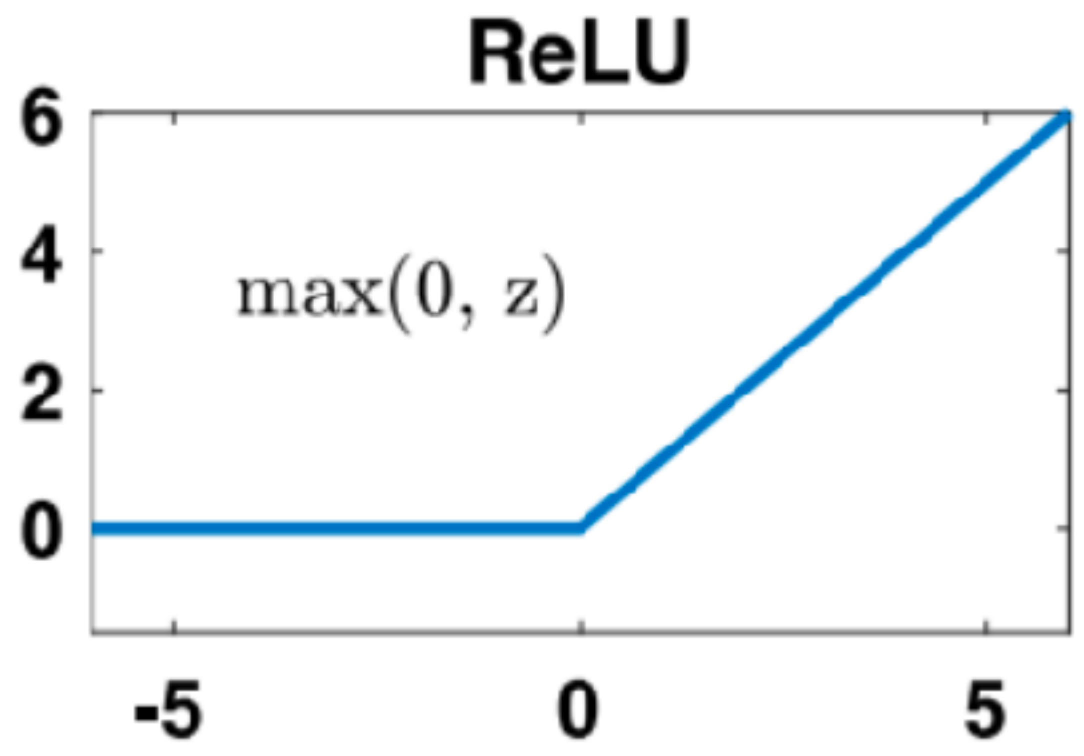
- At later layers: $z_i^{(l)} = b_i^{(l)} + \sum_j W_{i,j}^{(l)}\, a_j^{(l-1)}, \qquad a_i^{(l)} = \sigma(z_i^{(l)})$



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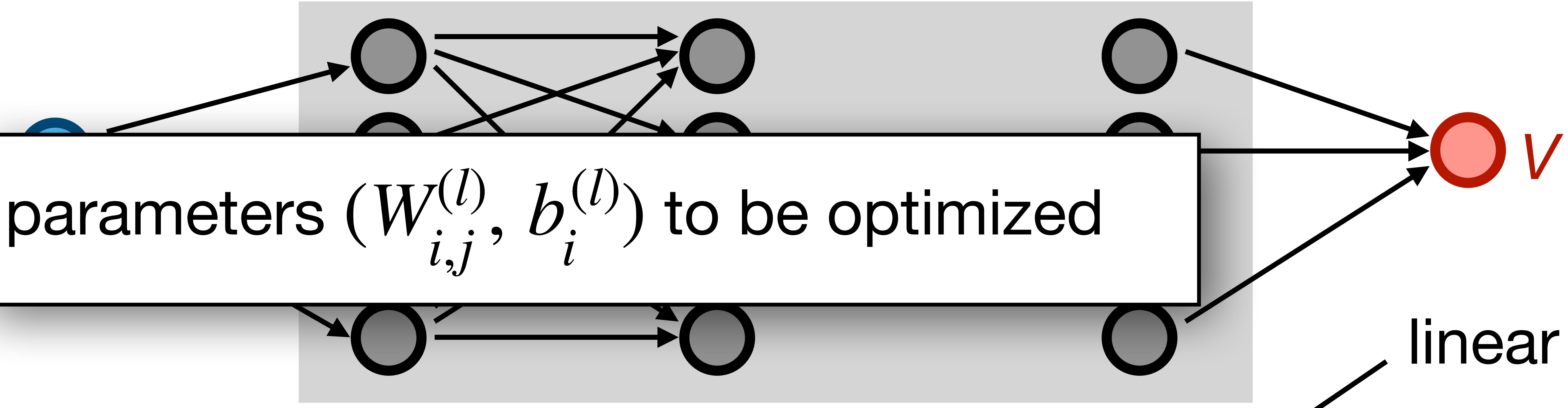

parameters $(W_{i,j}^{(l)}, b_i^{(l)})$ to be optimized

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