Heavy quark thermalization using quantum search algorithm

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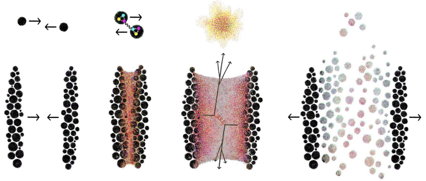
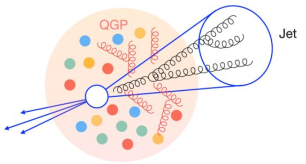
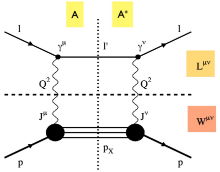
Experiments

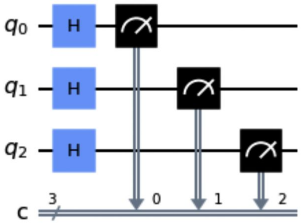
Theory
+
Phenomenology

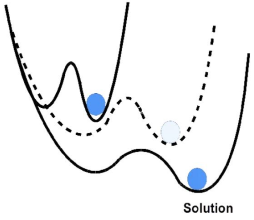
New
Technology

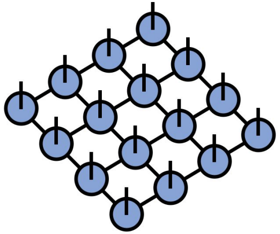






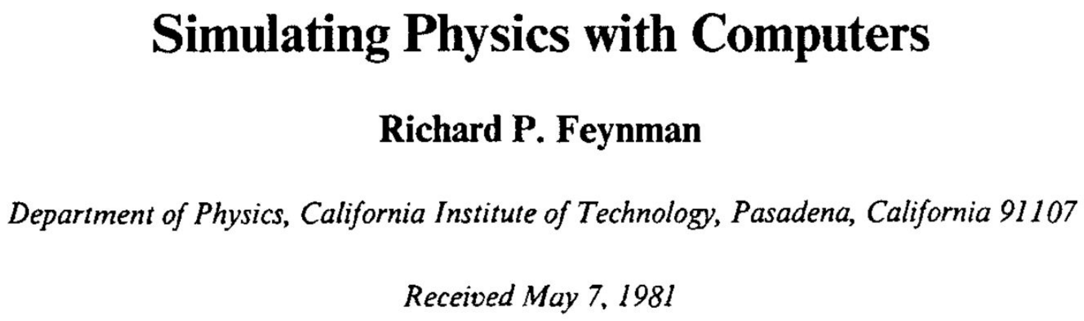






Quantum computing

The Idea: *"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical"*


Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

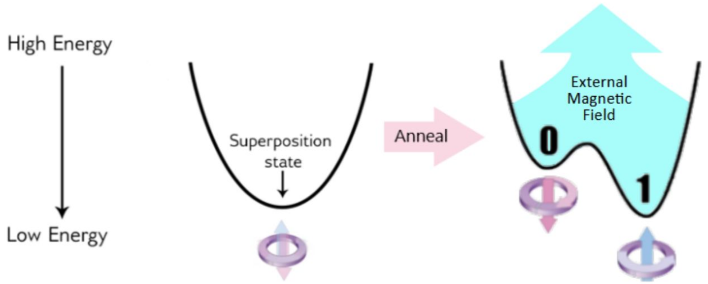
Quantum computing (QC) is a rapidly-emerging technology that harnesses the laws of quantum mechanics to solve problems.

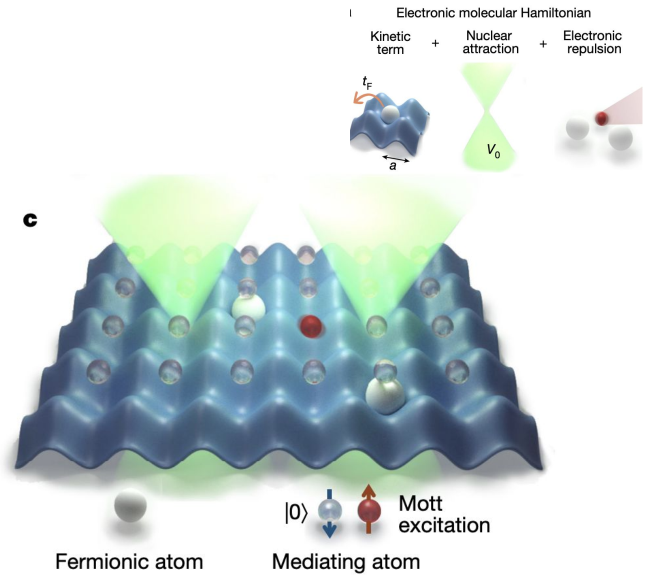
After 40+ years, we are almost there with various emerging QC technology…

Analog quantum computer

Basic idea: Mimic with physical systems using continuous variables

Quantum annealing





Successfully for optimization problems, ~5000 quantum bits (qubits)

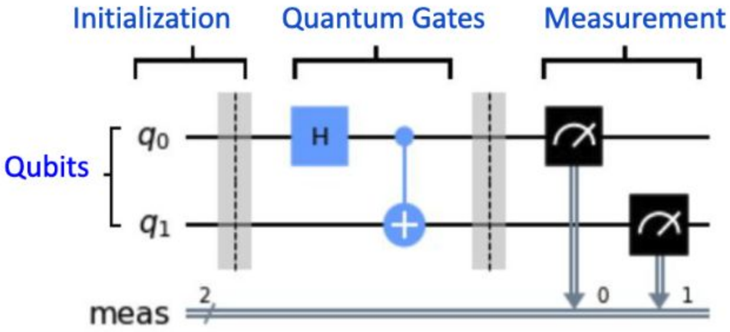
Argüello-Luengo et al, 1807.09228
ultracold atoms in optical lattices + cavity QED

Not a universal approach

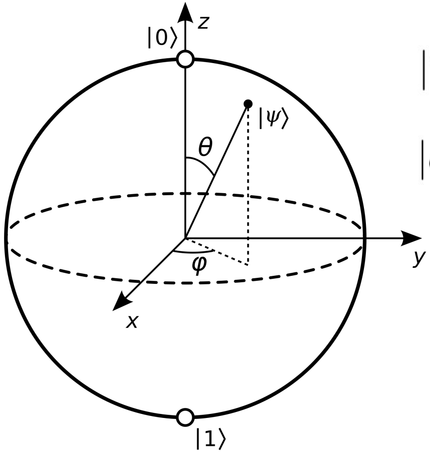
Schematic: D-Wave

Digital quantum computer

Basic idea: Spin chain = qubits (lines) + unitary gates (operators)



Qubits = Digitalization



$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

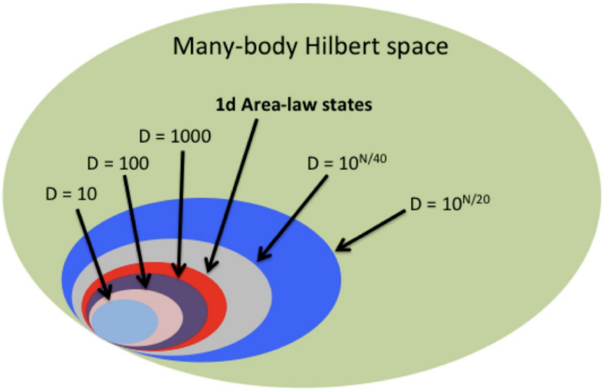
Conceptually clean for universal simulation

Noisy, intermediate-scale (NISQ) era, ~100 qubits

Tensor network (classical)

Basic idea: Efficient local representation of Hilbert space obeying the area-law

$S \sim \log D$

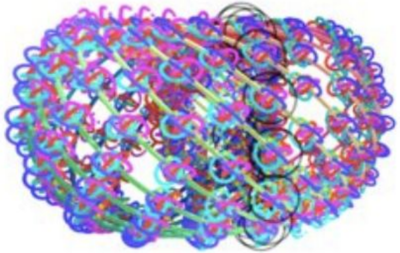


Very suitable for 1+1 problems

Fail for long time simulation or high entanglement

Why is QC important

QC really **outclass** CC in multiple qubit states due to entanglement!



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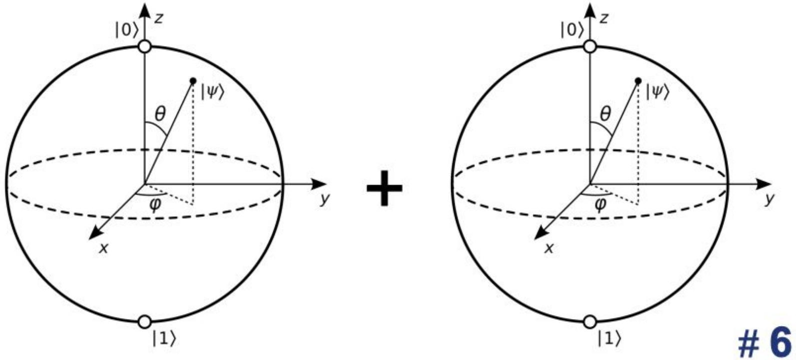

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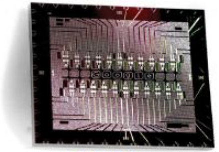
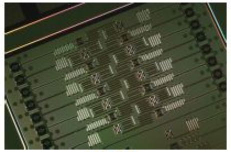
D.o.f. for n qubits:

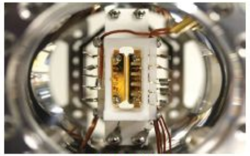
$$2^n\text{(variables)} \times 2\text{(complex)} - 1\text{(normalization constraint)} = \mathbf{2^{n+1}-1} \gg 3n$$

Quantum computing progress

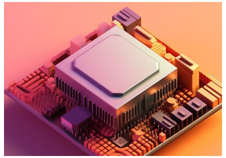
State-of-the-art: Noisy intermediate-scale quantum (NISQ) era = substantially imperfect and insufficient qubits. However, this can change fast!


Google


IBM


Rigetti


Intel


USTC

Qiskit

Cirq

XANADU

…

…

Optimist's prediction:

Neven's law = Double-exponential scaling

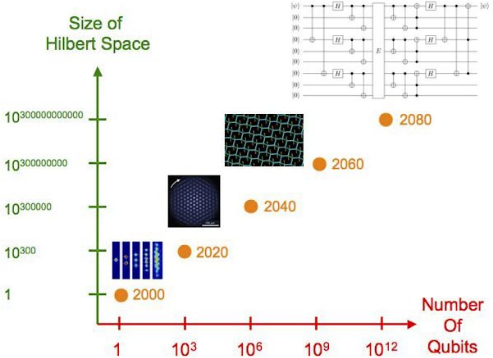
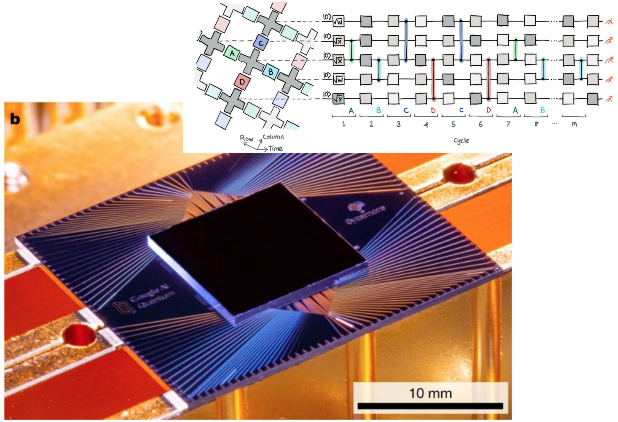


Chart: Quantum Pundit

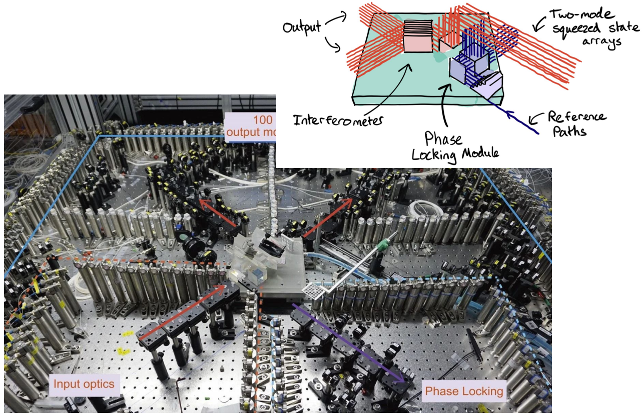
Quantum supremacy

Quantum supremacy = **anything** with a quantum device “cannot” be performed classically

Specific evidence for supremacy are found in sampling distributions!



random circuit 53 qubits, Google Quantum, 1910.11333

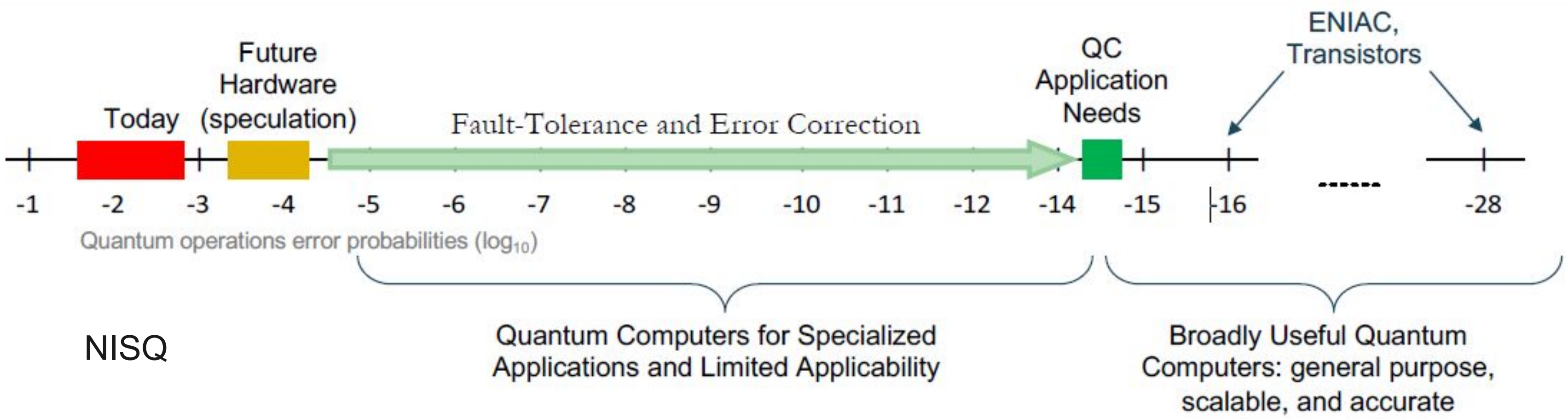


GBS 76 qubits, UTSC, 2012.01625
Schematic: Pennylane

Quantum advantage = **sth useful** involving a quantum device “cannot” be performed classically

Quantum supremacy

Simply a matter of time before QC revolutionizes the modern research (**sth useful**)



NISQ

Proof of principles calculations

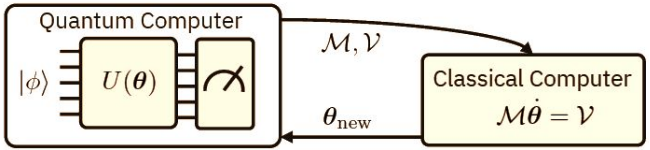
Schematic: De Jong

Applications in HEP/NP

Roadmap: Bauer et al, 2204.03381; Meglio et al, 2307.03236

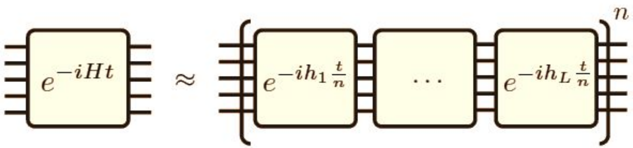
Two main directions:

- **Variational Approaches**: Variational Quantum Eigensolver (VQE) to obtain hadronic spectrum and observables, Partonic structure functions, Hadronic observables, etc



Peruzzo et al, 1304.3061 (2013)
Kreshchuk et al, 2011.13443
WQ et al, 2112.01927
Li et al, 2106.03865 & 2301.04179
Gallimore & Liao et al, 2202.03333
…

- **Decomposition Approaches**: Quantum simulation algorithm, Lattice Gauge Theory, Light-front Hamiltonian simulation, Open quantum system, Thermal field theory, etc



Wiesner, 9603028 (1996); Zalka, 9603026; JLP, 1111.3633
Du et al, 2006.01369
De Jong et al, 2010.03571 & 2106.08394
Czajka et al, 2112.03944 & 2210.03062
Bañuls et al, 2409.16996
WQ et al., 2411.09762, 2307.01792, 2208.06750
…

Image: Miessen (2022)

Applications in HEP/NP

Roadmap: Bauer et al, 2204.03381; Meglio et al, 2307.03236

Promising third direction:

Speedup in extraction of quantum information

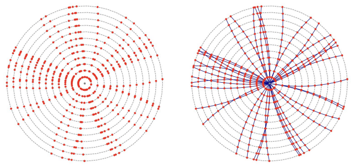
- **Quantum Search Algorithms**: Quantum Search, Quantum Amplitude Estimation — **Grover, 9605043 (1996)**

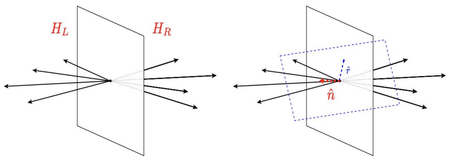
  Quantum speedup to recover charge particle trajectories — Magano et al, 2104.11583

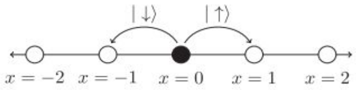
  Jet algorithm for thrust via Grover search — Wei et al, 1908.08949; Delgado & Thaler, 2205.02814

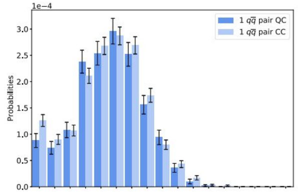
  Quantum walk approach to simulate parton showers — Williams et al, 2109.13975, 2207.10694

  Heavy quark thermalization using quantum search algorithm (this talk) — Du & WQ, arXiv:2312.16294

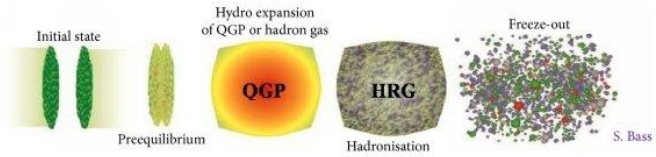

(a) Input. (b) Output.







Heavy quark thermalization

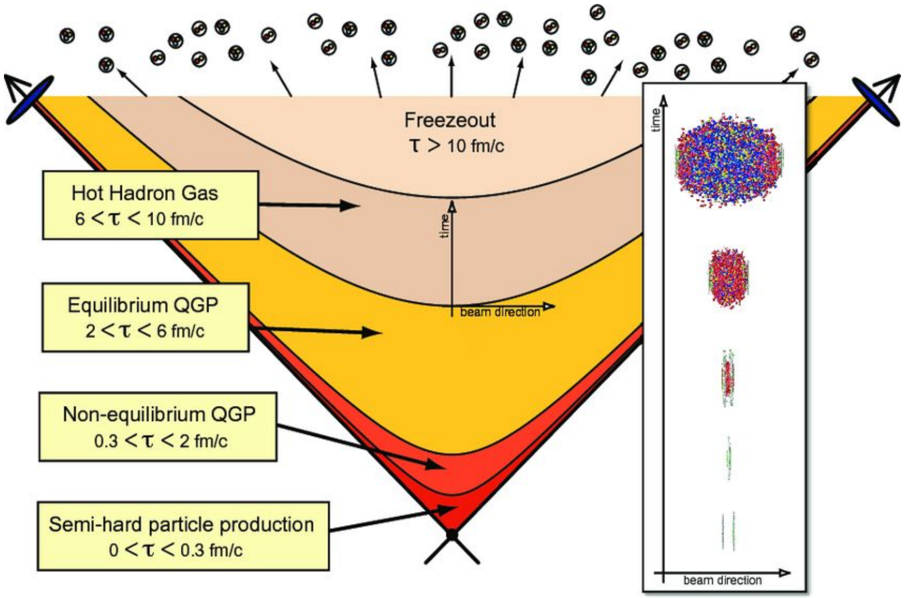


Distinguished separation of scales $E \gg T$

- Hard probe (jet, heavy quark) energy, E
- Medium temperature, T

Different time scales in thermalization (low to high)

- Heavy quark production
- QGP thermalization
- Heavy quark thermalization

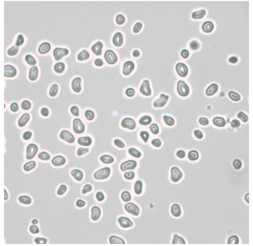


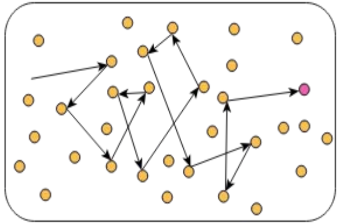
Heavy quark thermalization

Two dominant factors:

- Sudden change of momentum from radiation
- Slow change of momentum from environment

Model of Brownian motion





Brownian motion can be described by a stochastic differential equation (SDE):

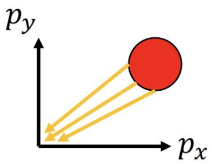
- Drag term: Energy loss
- Diffusion term: Momentum broadening

$$dp_i = -Ap_i dt + \sigma_{ij} dW_j$$

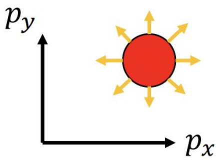
Stochastic term

Drag

Diffusion



Drag: Energy loss



Diffusion: Momentum broadening

Heavy quark thermalization

The Langevin equation

$$dx_i = \frac{p_i}{E(\vec{p})}dt, \quad i = x, y, z,$$

$$dp_i = -A(\vec{x},\vec{p},t)p_i dt + \sigma_{ij}(\vec{x},\vec{p},t)dW_j,$$

drag coefficient — diffusion coefficient

Reformulated as Fokker-Planck equation, with evolution of the heavy quark distribution $f(\vec{x},\vec{p},t)$

$$\frac{\partial}{\partial t}f(\vec{x},\vec{p},t) = \frac{\partial}{\partial p_i}\left[A(\vec{x},\vec{p},t)p_i f(\vec{x},\vec{p},t)\right] + \frac{\partial^2}{\partial p_i \partial p_j}\left[B_{ij}(\vec{x},\vec{p},t)f(\vec{x},\vec{p},t)\right]$$

No general solution but we know

- Relativistic limit: $f^{eq}(\vec{p}) \propto \exp(-E(\vec{p})/T)$ $\quad E(\vec{p}) = \sqrt{\vec{p}^2 + M^2}$
- Non-relativistic limit: $f^{eq}(\vec{p}) \propto \exp(-\vec{p}^2/(2MT))$ $\quad E(\vec{p}) \simeq \vec{p}^2/(2M) + M$

Boosting classical computation

Rebentrost et al, 1805.00109 (2018)
Stamatopoulos et al, 1905.02666v5 (2020).

The idea: use Grover-like operator to accelerate the extraction of quantum amplitude information (by a square root) over classical Monte Carlo methods. It has been first applied to quantum fiance to solve pricing options, involving stochastic differential equations (SDE).

Here, in heavy-ion physics, heavy quark thermalization with background QCD plasma can also be described by a SDE in phase space, as we introduced:

Du and Rapp, 2207.00065 (2022)
Du, arXiv:2306.02530 (2023)

$$dp^i(t) = -A^i(\vec{p},t)dt + \sigma^{ij}(\vec{p},t)dW^j$$

Drag coefficient

Stochastic Wiener process

We are interested in obtaining the physical observable at final step or “maturity”:

$$\mathbb{E}\left[F(\vec{p},T)\right] = \frac{1}{N}\sum_{i=1}^{N} F(\vec{p},T) \qquad F(\vec{p},T) = \sqrt{p^2+m_Q^2}$$

Simplified Langevin description

Du & WQ, arXiv:2312.16294

We work with simplified Langevin equation with dimensionless variables q and $d\tilde{t}$

$$dq_i = -q_i d\tilde{t} + d\tilde{W}_i \qquad q_i = p_i/M \qquad d\tilde{t} = A dt$$

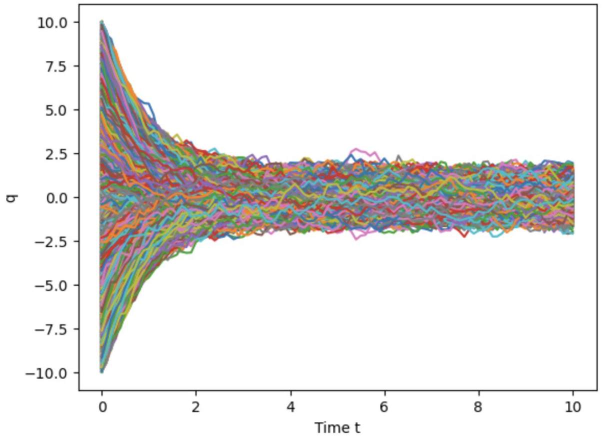
Drag Stochastic Dimensionless variables

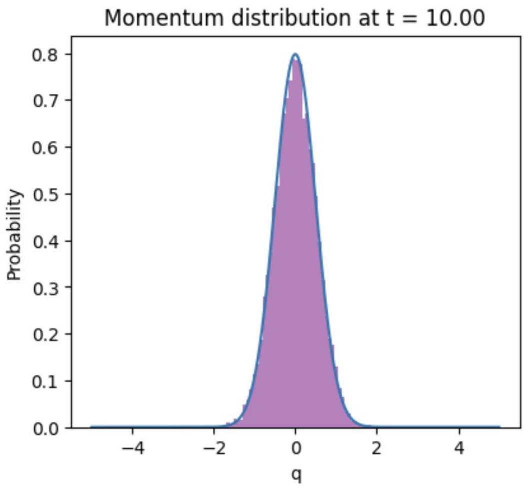
Anisotropic stochastic terms are

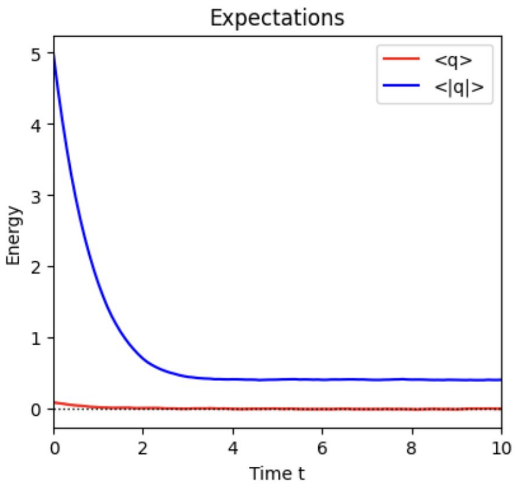
$$A = \sigma_{ii}^2 \chi_i^2 / (2MT) \qquad d\tilde{W}_i \sim \mathcal{N}(0, 2T d\tilde{t}/(M\chi_i^2))$$

Though simplified, the setup proves sufficient to Monte Carlo simulation of heavy quark thermalization

Classical MC





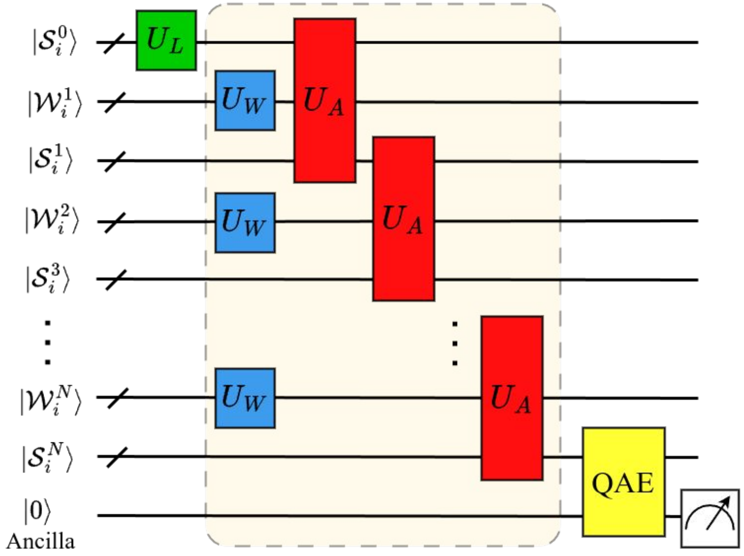


Float precision, 10000 “shots”, dt = 0.1

Quantum circuit MC

Du and WQ, arXiv:2312.16294

We proposed *accelerated quantum circuit Monte Carlo (aQCMC)* for heavy quark thermalization



Main components:

- Distribution loading gate (U_L)
- Stochastic Wiener gate (U_W)
- Quantum evolution gate (U_A)
- Quantum Amplitude Estimation (speed up)

The key is to make use of the phase information to extract desired observable

Qubitization of momenta

Discretize momenta to map onto qubits and apply periodic boundary condition (PBC)

State index i	0	1	2	3	$\cdots$	2^n-2	2^n-1
Quantum state $\|i\rangle$	$\|0...000\rangle$	$\|0...001\rangle$	$\|0...010\rangle$	$\|0...011\rangle$	$\cdots$	$\|1...110\rangle$	$\|1...111\rangle$
Physical momentum q	$-q_{\max}$	$-q_{\max}+\delta q$	$-q_{\max}+2\delta q$	$q_{\max}+3\delta q$	$\cdots$	$q_{\max}-2\delta q$	$q_{\max}-\delta q$
Positive momentum $\bar{q}$	0	δq	$2\delta q$	$3\delta q$	$\cdots$	$2q_{\max}-2\delta q$	$2q_{\max}-\delta q$

$q \in [-q_{\max}, q_{\max})$ $\quad\quad$ $\bar{q} = q + q_{\max} \in [0, 2q_{\max})$

Here, n is number of qubits, then there are $N = 2^n$ momentum modes, with spacing $\delta q = 2q_{\max}/N$

With PBC, assuming the momentum box size is N and stochastic box size is S,

$$\bar{q}_i' = d\tilde{W}_i + (1 - d\tilde{t})\bar{q}_i + (Ndt - S)$$

Quantum module arithmetic gates, asymptotically same cost as classical logic gates.

Quantum amplitude estimation

Brassard, Hoyer, Mosca,Tapp, 0005055 (2002)

Quantum amplitude estimation (QAE) is **key component to quantum speedup** in aQCMC.

Underlying principle: Generalization of Grover's algorithm

Quantum estimating expectation with $\epsilon = \mathcal{O}(1/N_q)$, quadratically faster than classical $\epsilon = \mathcal{O}(1/\sqrt{N_q})$

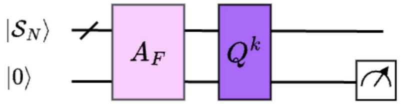
Two parts:

1. Quantum amplitude loading of desire expectation function

$$A_F \left|\psi\right\rangle_n \left|0\right\rangle = \sqrt{1-a}\left|\psi_0^*\right\rangle_{n+1} + \sqrt{a}\left|\psi_1^*\right\rangle_{n+1} = \left|\psi^*\right\rangle_{n+1}$$

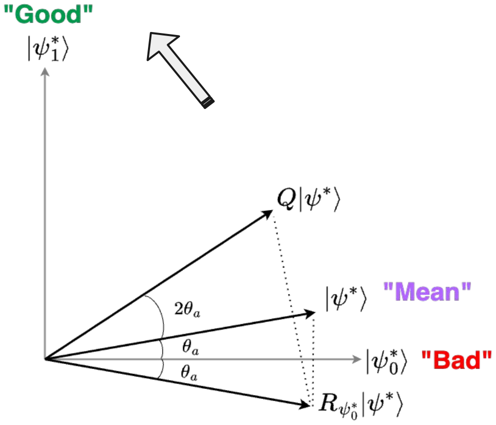
2. Quantum phase estimation (QPE)

$$a = \sum_{i=0}^{2^n-1} \mathcal{P}(i)F(i) = \langle F \rangle \qquad Q \sim A_F\left(2\left|\psi\right\rangle\left|0\right\rangle\left\langle 0\right|\left\langle\psi\right| - I_{n+1}\right)A_F^\dagger R_{\psi_0^*}$$



$$Q^k A_F \left|\psi\right\rangle_n \left|0\right\rangle = \cos\left((2k+1)\theta_a\right)\left|\psi_0^*\right\rangle\left|0\right\rangle + \sin\left((2k+1)\theta_a\right)\left|\psi_1^*\right\rangle\left|1\right\rangle$$

Many circuits, single auxiliary, classical post-processing
(Advantageous for today's QC)



Maximum likelihood Suzuki et al, 1904.10246
Iterative QAE Grinko et al, 1912.05559

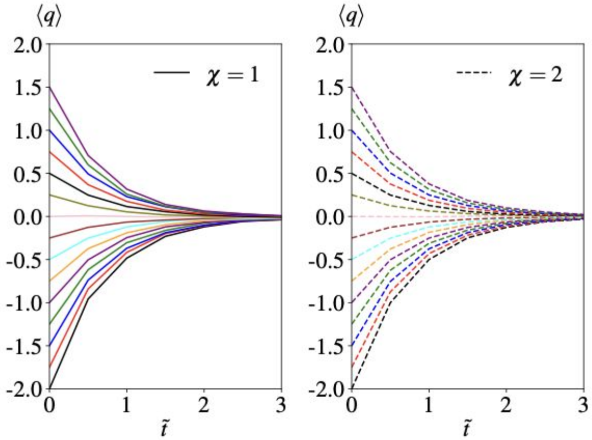
Simulation results (1D)

Proof of principles, equivalent to classical simulation

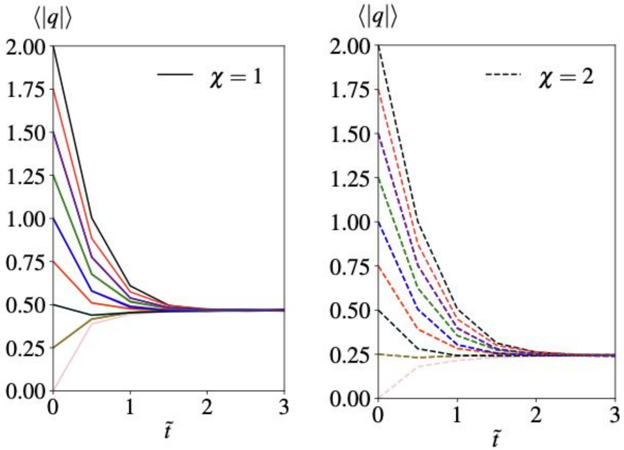
Du and WQ, arXiv:2312.16294

Large time thermalization, aQCMC without QAE but with reset gates

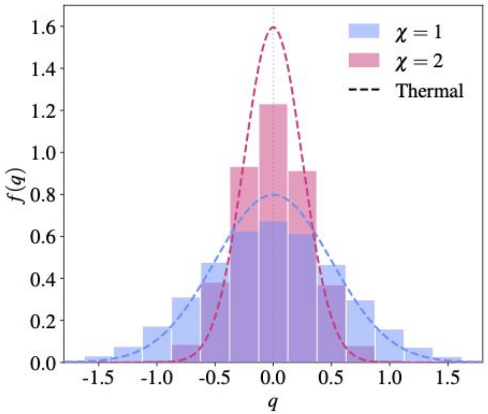
$$f^{\rm eq}(\vec{q}) \propto \exp\left[-\frac{q_x^2}{\tilde{\sigma}_x^2}-\frac{q_y^2}{\tilde{\sigma}_y^2}-\frac{q_z^2}{\tilde{\sigma}_z^2}\right]$$



Momentum



Abs Momentum



Thermal Distribution

$$M = 1.5\,\mathrm{GeV}$$

$$T \simeq 300 \text{ - } 500\,\mathrm{MeV}$$

$$\tilde{\sigma}_i^2 d\tilde{t} = 2T d\tilde{t}/(M\chi_i^2) \simeq d\tilde{t}/(2\chi_i^2).$$

n = 4, 20 qubits, dq = 0.25, dt = 0.5

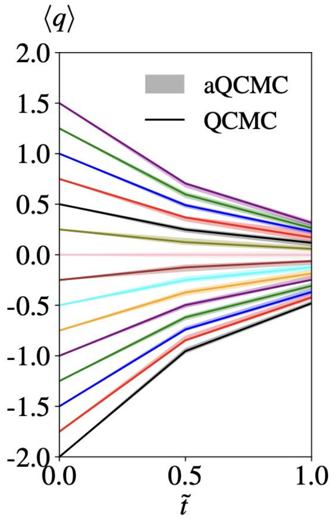
Qiskit

Simulation results (1D)

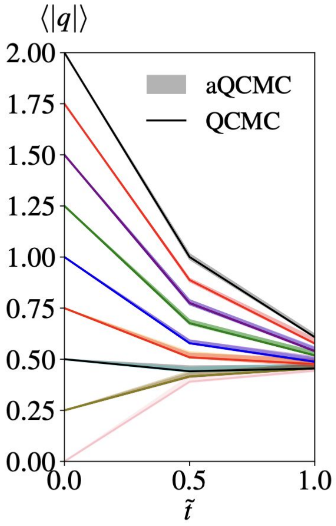
Proof of principles, quadratically reduce sampling shots

Du and WQ, arXiv:2312.16294

Early time thermalization, aQCMC with QAE



Momentum

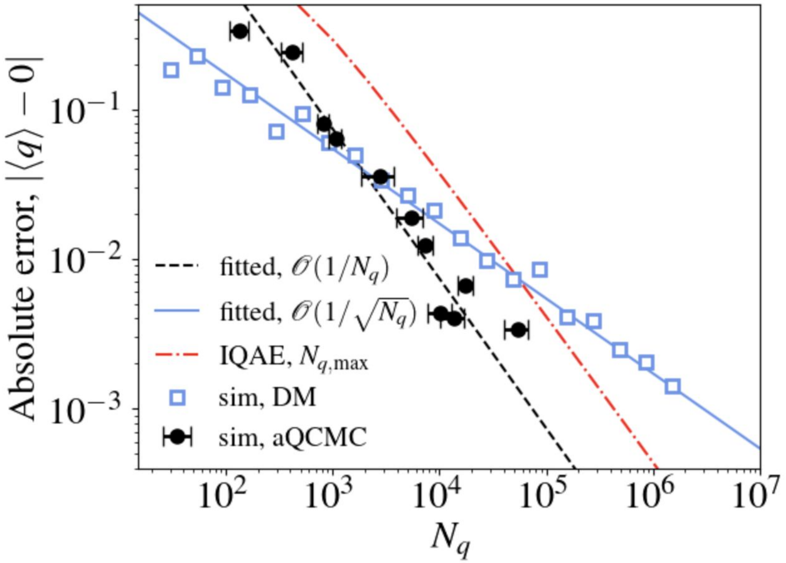


Abs Momentum

$\epsilon = 0.01$

$1 - \alpha = 95\%$

Quantum advantage

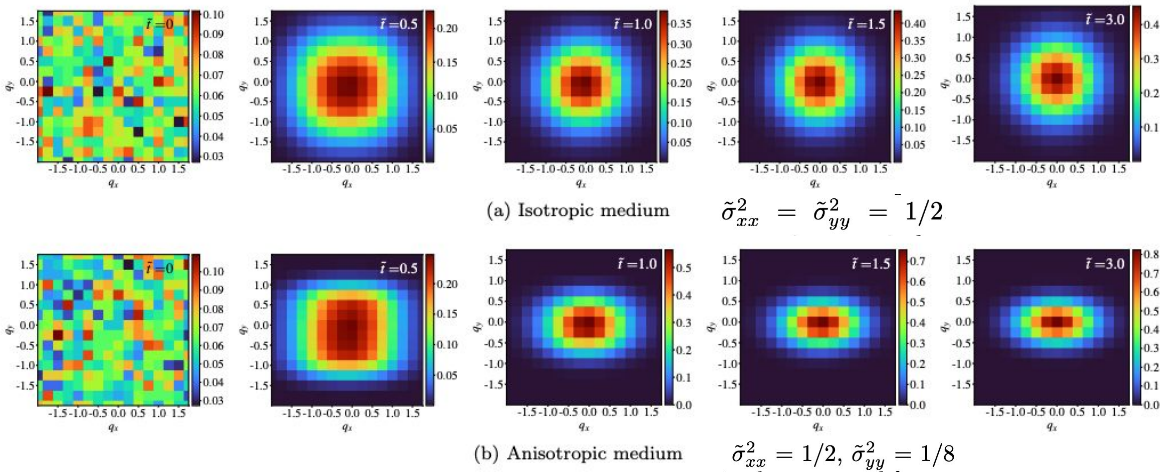


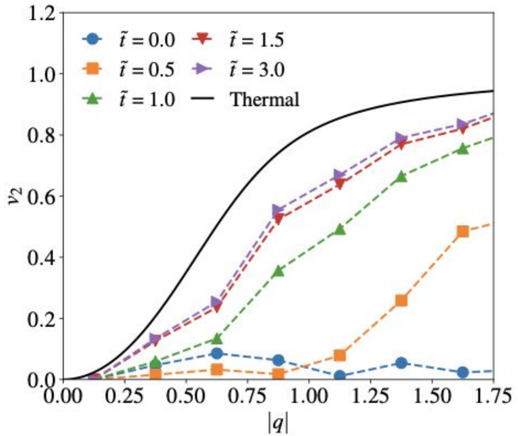
n = 4, 20 qubits, dq = 0.25, dt = 0.5

Qiskit

Simulation results (2D)

Du and WQ, arXiv:2312.16294





2D evolution with different medium profiles

Elliptic flow “v2” build up

$$v_2 = \frac{\int f(q,\cos(\phi),t)\cos(2\phi)\mathrm{d}\phi}{\int f(q,\cos(\phi),t)\mathrm{d}\phi} \overset{\text{thermal}}{=} \frac{I_1\left(\frac{1}{2q^2}\left|\frac{1}{\tilde{\sigma}_x^2} - \frac{1}{\tilde{\sigma}_y^2}\right|\right)}{I_0\left(\frac{1}{2q^2}\left|\frac{1}{\tilde{\sigma}_x^2} - \frac{1}{\tilde{\sigma}_y^2}\right|\right)}$$

Qiskit

Summary and outlook

- *Quantum search algorithm* provides an alternative (algorithmically proven) path toward simulating dynamics of quantum system efficiently.
- We proposed accelerated Quantum Circuit Monte-Carlo (aQCMC) method for heavy quark thermalization in a stochastic description to speed up calculation quadratically.
- Proof of principle simulation for heavy quark thermalization compared with exact and analytical thermal expectation in both 1D and 2D systems.
- Our framework may also be extended to non-Markovian process, quarkonium, and quantum walk.

Thanks for your attention!

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Acknowledgements