Non perturbative effect in charm diffusion from Gribov-Zwanziger approach

Reporter: Xueqiang Zhu

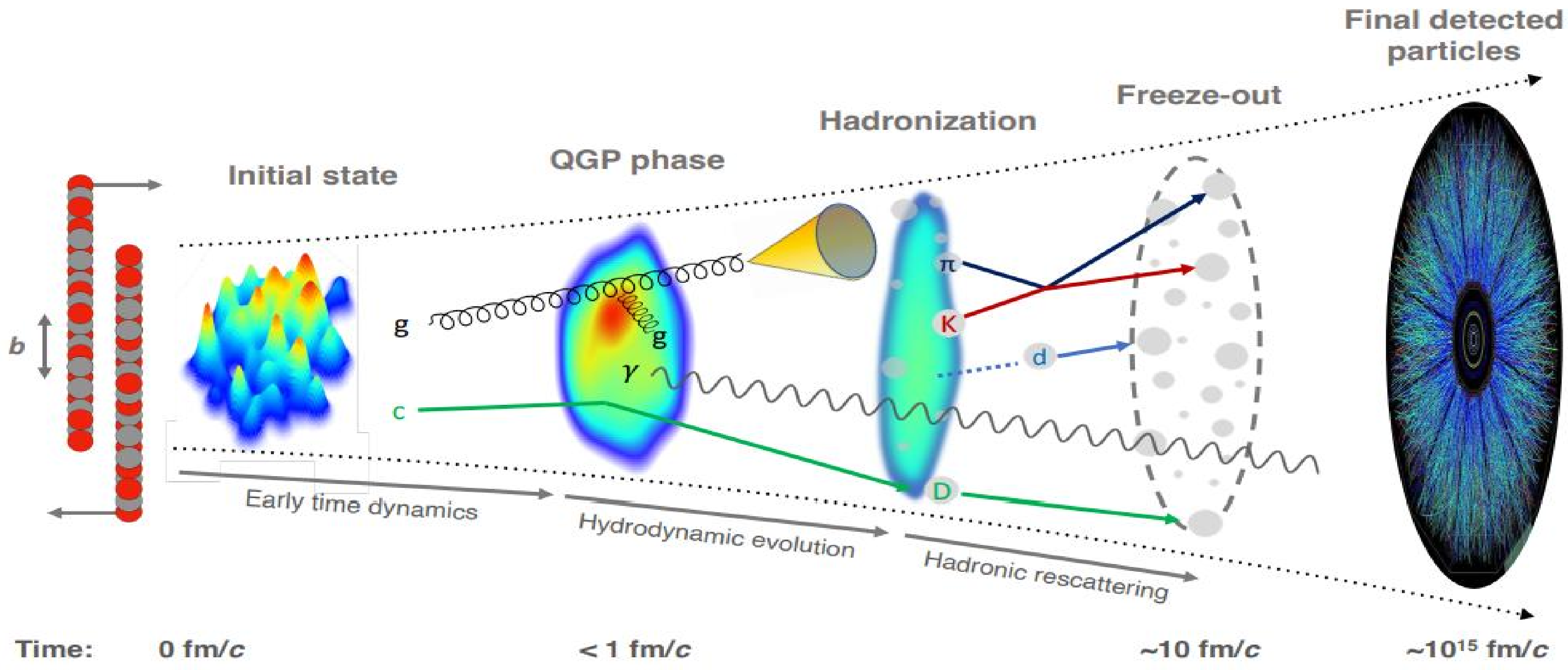
Supervisor: Guangyou Qin, Weiyao Ke

Outline

- **Background**
- **Gribov-Zwanziger prescription in scattering matrix**
- **Numerical results of the anisotropic diffusion coefficients**
- **Summary**

Background

- Heavy-Ion Collision



- Different stages in a heavy-ion collision

Due to the longer relaxation time of heavy quarks (HQ) compared to light quarks in QGP, HQ are good probes for the strongly interacting system.

Langevin equation for heavy quark

A Brownian motion with uncorrelated momentum kicks has been a well-accepted description of the dynamics of the HQ.

- The non-relativistic Langevin equation for HQ:

$$\frac{dp_i}{dt} = -\eta p_i + \xi_i, \langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$$

drag coefficient η , stochastic force $\xi_i(t)$, correlation coefficient κ

- Momentum diffusion coefficient κ_{tot} obtained from the mean squared momentum transfer per unit time.

$$\kappa_{\text{tot}} = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \int \frac{d^3\boldsymbol{p'}}{(2\pi)^3} \int \frac{d^3\boldsymbol{k'}}{(2\pi)^3} (\boldsymbol{k}-\boldsymbol{k'})^2 \omega(P,K|P',K') f(\boldsymbol{k})[1 \pm f(\boldsymbol{k'})]$$

$$\omega(P,K|P',K') = \frac{(2\pi)^4\delta^4(P+K-P'-K')|\mathcal{M}|^2}{2p^0 2p'^0 2k^0 2k'^0}$$

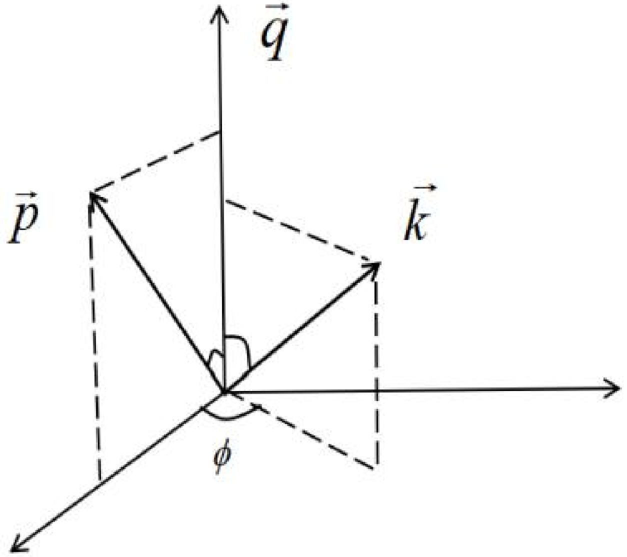
The anisotropic momentum coefficients

By using the properties of delta function to perform the integration in momentum space, we simplify as

$$\kappa_{\text{tot}} = \frac{1}{(2\pi)^4}\frac{1}{16pp^0}\int_0^{2\pi} d\phi \int_0^{\infty} dq \int_{\text{I}_1}^{\text{I}_2} d\omega \int_{\frac{\omega+q}{2}}^{\infty} dk q^2$$
$$\times \{2N_f|\mathcal{M}|^2_{\text{cq}} f_{\text{F}}(k)[1-f_{\text{F}}(k-\omega)] + |\mathcal{M}|^2_{\text{cg}} f_{\text{B}}(k)[1+f_{\text{B}}(k-\omega)]\}$$

$$\text{I}_1 = \sqrt{q^2 - 2pq + (p^0)^2} - p^0,$$
$$\text{I}_2 = \sqrt{q^2 + 2pq + (p^0)^2} - p^0$$

C donates charm quark and we fix its mass at 1.3 GeV.
q and g respectively correspond to quark and gluon



by concise Writing Style $\kappa_{\text{tot}} = \langle q^2 \rangle$

$$2\kappa_{\text{T}} = \langle q_{\text{T}}^2 \rangle = \langle (q\sin\theta_{pq})^2 \rangle = \langle q^2 \rangle - \langle q_{\text{L}}^2 \rangle$$

$$\kappa_{\text{L}} = \langle q_{\text{L}}^2 \rangle - \frac{\langle q_{\text{L}}^2 \rangle}{\langle 1 \rangle}$$

The two body elastic scattering amplitude $|\mathcal{M}|^2$

The lowest-order Feynman diagrams : only the t-channel is considered in matrix element calculations.

(a) cq (b) cg

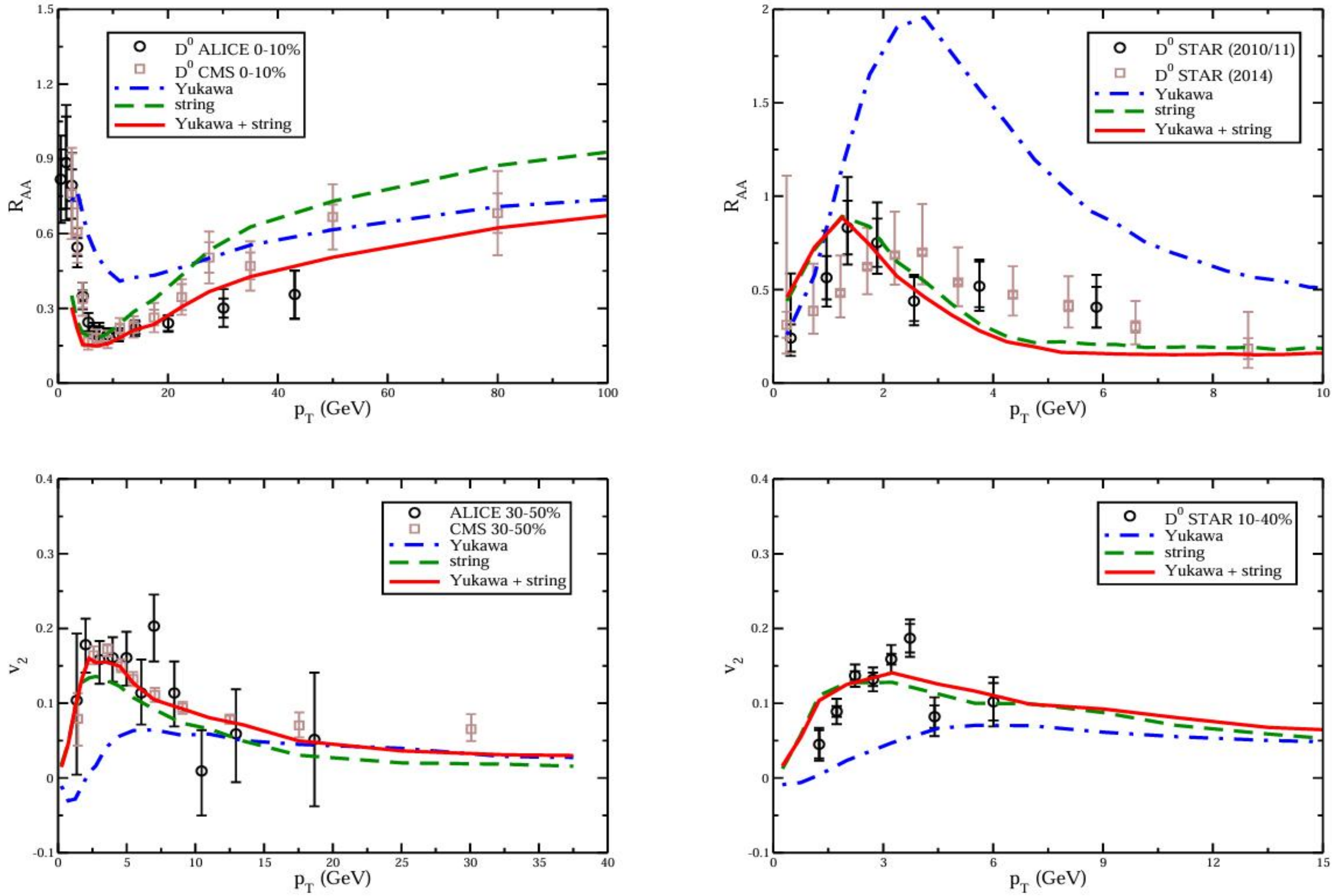
- According to Feynman rules:

$$|\mathcal{M}|^2_{\mathrm{cq}} = |\mathcal{M}|^2_{\mathrm{c\bar{q}}} = \frac{1}{3}g^4\frac{1}{(P-P')^4}[-32M^2(K\cdot K') + 32(P'\cdot K)(P\cdot K') + 32(P\cdot K)(P'\cdot K')]$$

$$|\mathcal{M}|^2_{\mathrm{cg}} = 2g^4\frac{1}{(P-P')^4}[32M^2(K\cdot K') + 32(P'\cdot K)(P\cdot K') + 32(P\cdot K)(P'\cdot K') - 32(K\cdot K')(P\cdot P')]$$

Some methods for non-perturbative effects

Results of Cornell type potential [1]



- CMS,arXiv:1708.03497
- CMS,arXiv:1708.04962
- ALICE,arXiv:1707.01005
- ALICE,arXiv:1804.09083

- Cornell type HQ potential
- T-matrix

[1] W. Xing, G. Qin, and S. Cao, Phys. Lett. B 838, 137733(2023).

[2]M. Djordjevic, Phys. Rev. Lett. 112, 042302 (2014)

Gribov-Zwanziger prescription in scattering matrix

- Gluon propagator in Gribov-Zwanziger (GZ) prescription :

$$iD_{\mu\nu}(Q) = \frac{-iQ^2}{Q^4+\gamma_G^4}[g_{\mu\nu} - (1-\frac{1}{\lambda})\frac{Q_\mu Q_\nu}{Q^2}]$$

- The partition function in Gribov prescription [3] :

$$Z = \int [\mathcal{D}A][\mathcal{D}\bar{c}][\mathcal{D}c]\, V(\Omega)\delta(\partial_\mu A_\mu^a) \times \exp\Big[- S_{\mathrm{YM}} - \int d^4x \bar{c}^a(x)\partial_\mu D_\mu^{ab} c^b(x)\Big],$$

- The gap equation for the Gribov parameter γ_G :

$$\frac{d-1}{d} N_c g^2 \int \frac{d^d q}{(2\pi)^d}\frac{1}{q^4+\gamma_G^4} = 1$$

  - Matsubara frequencies sum
  - Renormalization scheme

$$\rightarrow \quad \frac{3N_c g^2}{64\pi^2}[\frac{5}{6} - \ln\frac{\gamma_G^2}{\mu_0^2} + \frac{4}{i\gamma_G^2}\int_0^\infty dp p^2 (\frac{f_B(\omega_-)}{\omega_-} - \frac{f_B(\omega_+)}{\omega_+})] = 1$$

- $T \to \infty$ limit $\longrightarrow$ $\gamma_G^{\mathrm{asy}}(T) = \frac{d-1}{d}\frac{N_c}{4\sqrt{2}\pi} g^2(T) T\,.$

[3] N. Vandersickel, D. Zwanziger, Phys. Rep. 520 (2012) 175–251,

The value of Gribov parameter

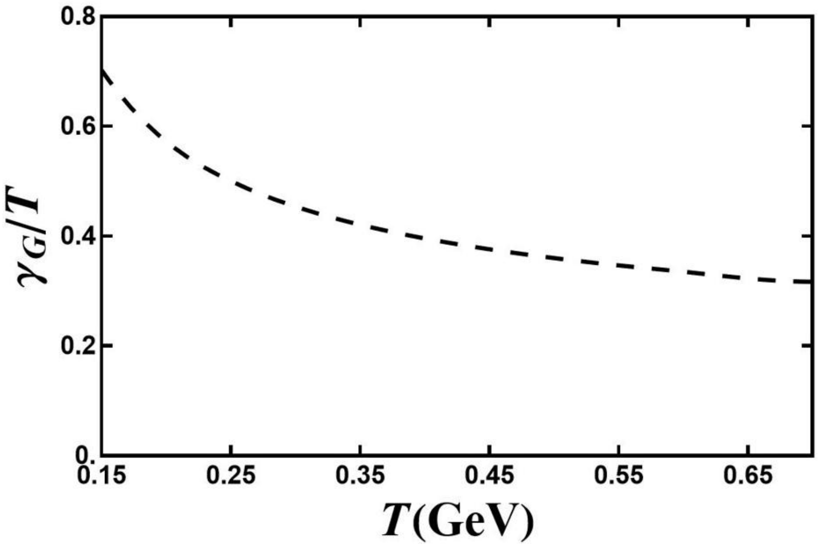
- The value of Gribov parameter γ_G is extracted by solving the Gap equation:

$$\frac{3N_c g^2}{64\pi^2}[\frac{5}{6} - \ln\frac{\gamma_G^2}{\mu_0^2} + \frac{4}{i\gamma_G^2}\int_0^\infty dp p^2 (\frac{f_B(\omega_-)}{\omega_-} - \frac{f_B(\omega_+)}{\omega_+})] = 1$$

the one-loop perturbation running coupling: $\alpha_s^{per} = \dfrac{6\pi}{(11N_c - 2N_f)\ln[2\pi T/\Lambda_{\overline{MS}}]}$

$$\mu_0 = 1.69\text{GeV},$$
$$\Lambda_{\overline{MS}} = 0.176\text{MeV}$$

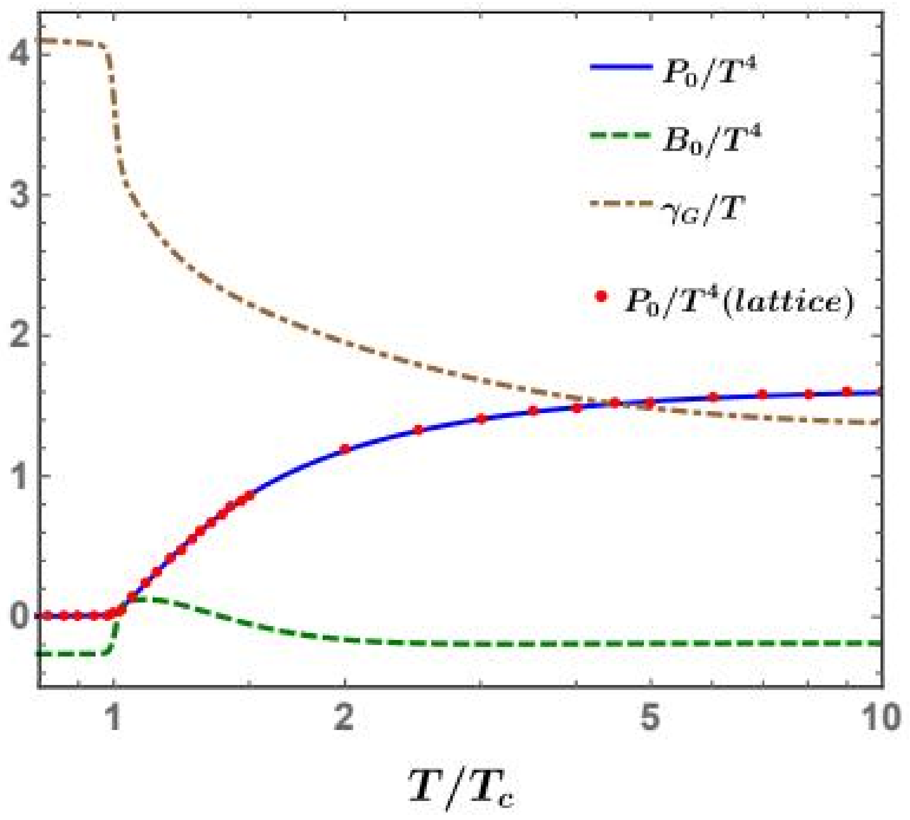
Numerical results of γ_G :

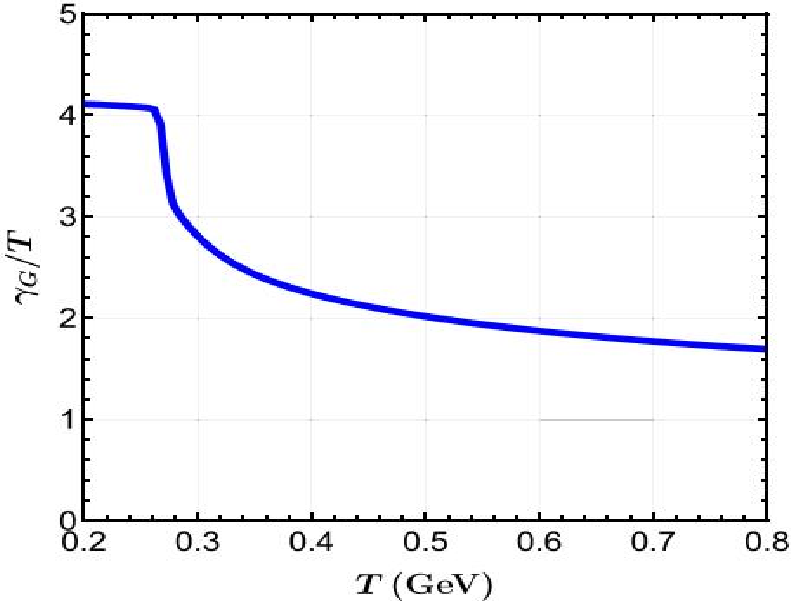


The scale Gribov parameter decreases with temperature.

Other methods for fixed parameters

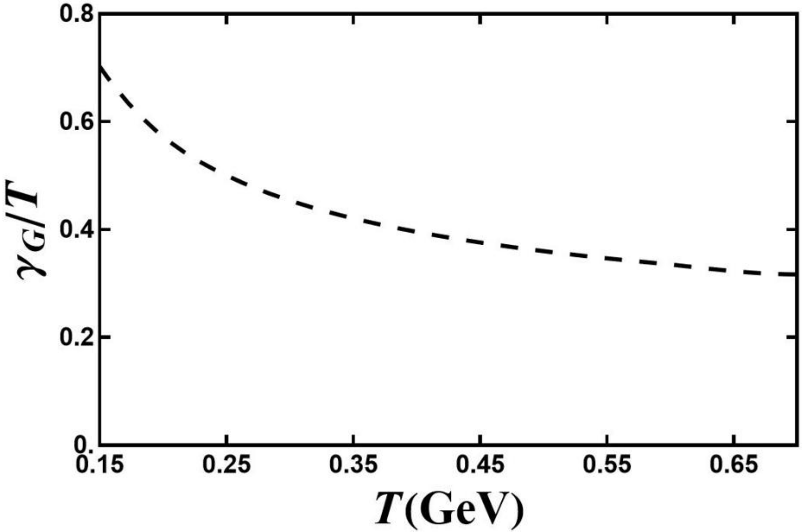
By matching the thermodynamics of the medium with the pure gauge lattice data [4]





vs

Our result



[4] A. Jaiswal and N. Haque, Phys. Lett. B 811, 135936 (2020).

Non-perturbation effect of the Gribov plasma modification

In order to investigate the non-perturbation effect of the Gribov plasma modification, the gluon propagator modulus in the scattering matix above is replaced by GZ propagater modulus.

$1/(P-P')^4 = 1/Q^4$ replaced by $1/(Q^2+\gamma_G^4/Q^2)^2$

$$|\mathcal{M}|^2_{\mathrm{cq}} = |\mathcal{M}|^2_{\mathrm{c\bar{q}}} = \frac{1}{3}g^4\frac{1}{(P-P')^4}[-32M^2(K\cdot K') + 32(P'\cdot K)(P\cdot K') + 32(P\cdot K)(P'\cdot K')]$$

$$|\mathcal{M}|^2_{\mathrm{cg}} = 2g^4\frac{1}{(P-P')^4}[32M^2(K\cdot K') + 32(P'\cdot K)(P\cdot K') + 32(P\cdot K)(P'\cdot K') - 32(K\cdot K')(P\cdot P')]$$

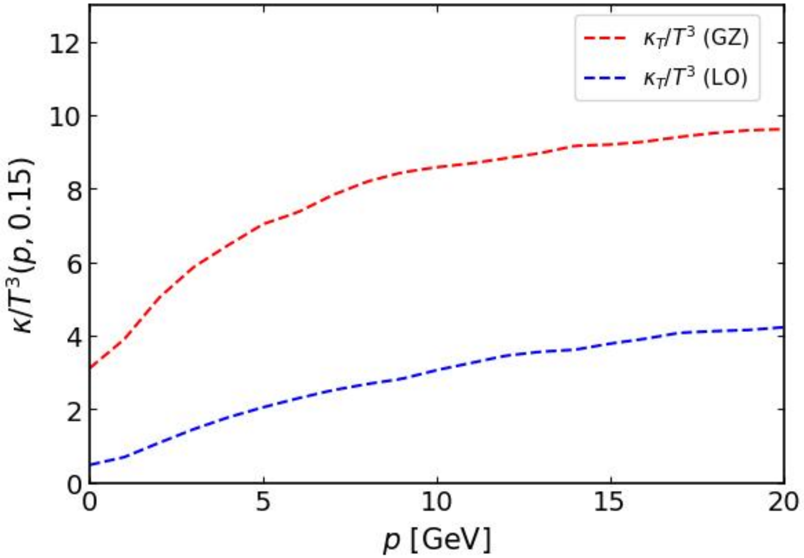
In order to compare the results of leading order (LO) :

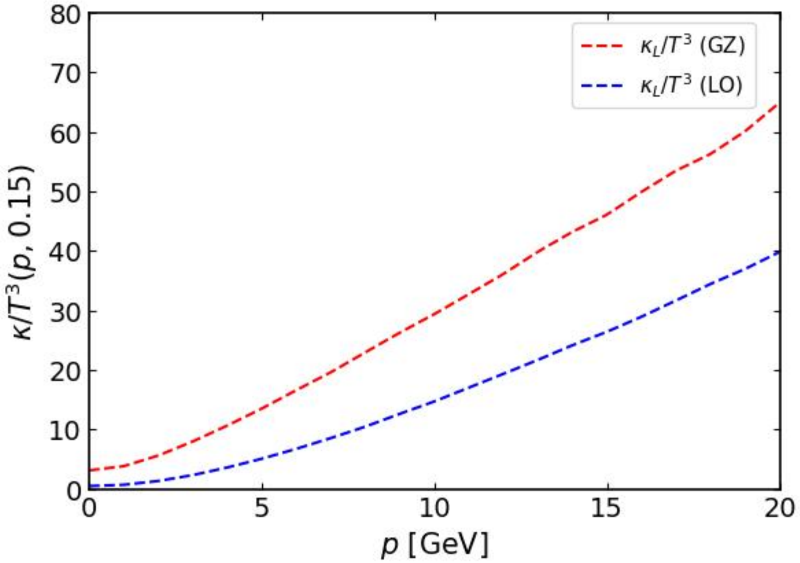
$1/(P-P')^4 = 1/Q^4$ replaced by $1/(Q^2+m_\mathrm{D}^2)^2$

Debye screened mass : $m_\mathrm{D}^2 = (1/3)(N_\mathrm{c}+N_\mathrm{f}/2)g^2T^2$

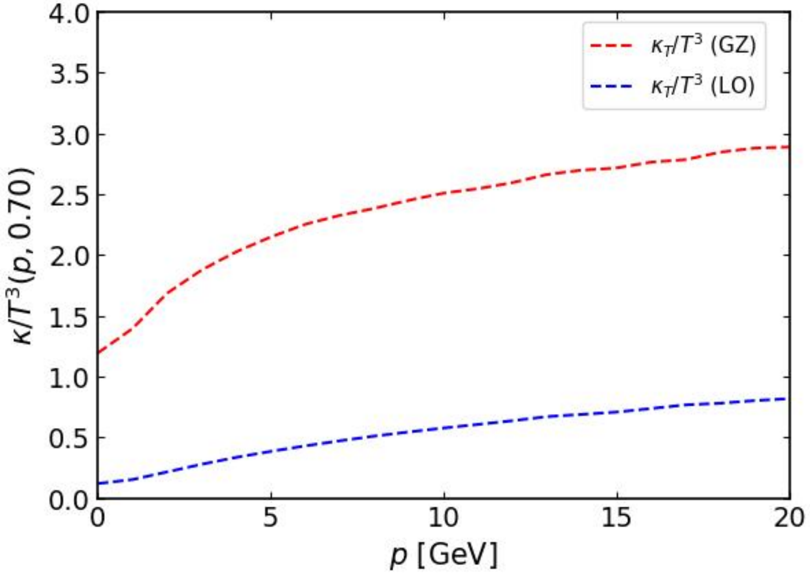
Numerical results of the diffusion coefficient of the charm

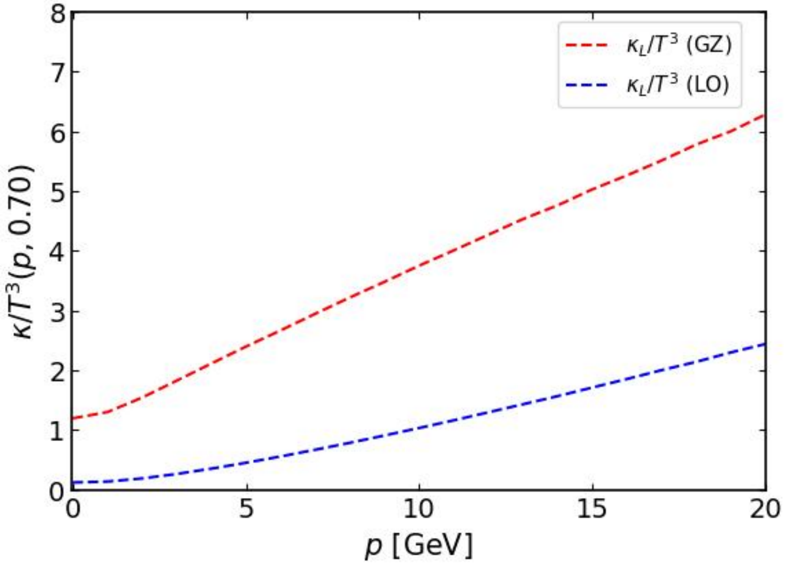
Low temperature



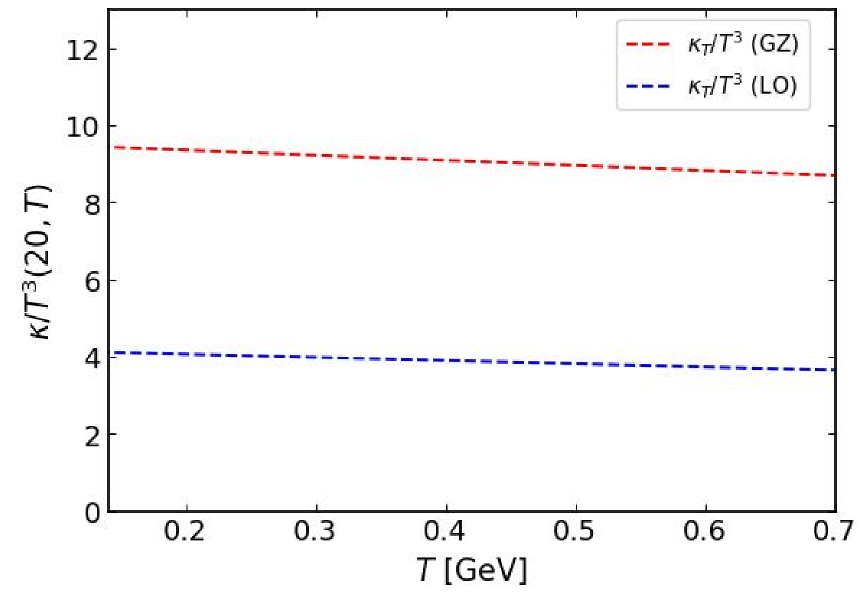


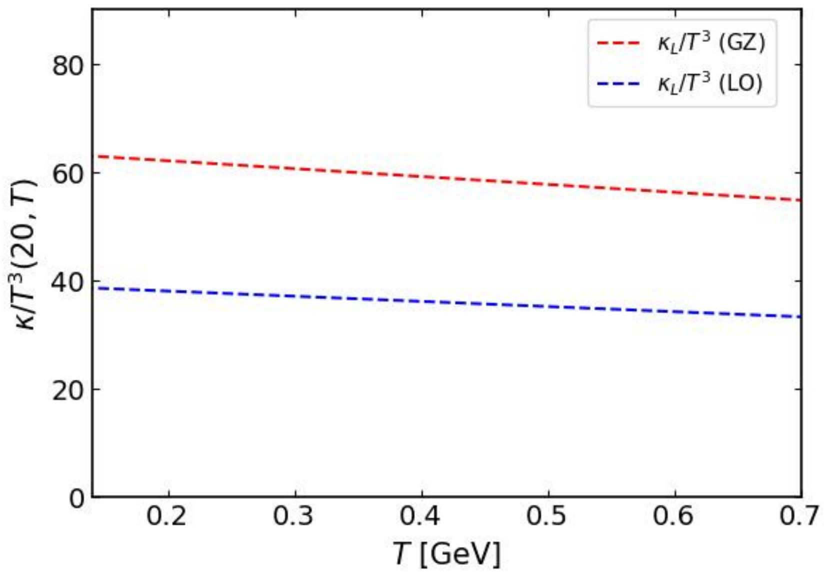
High temperature





The dependence of the diffusion coefficient on temperature

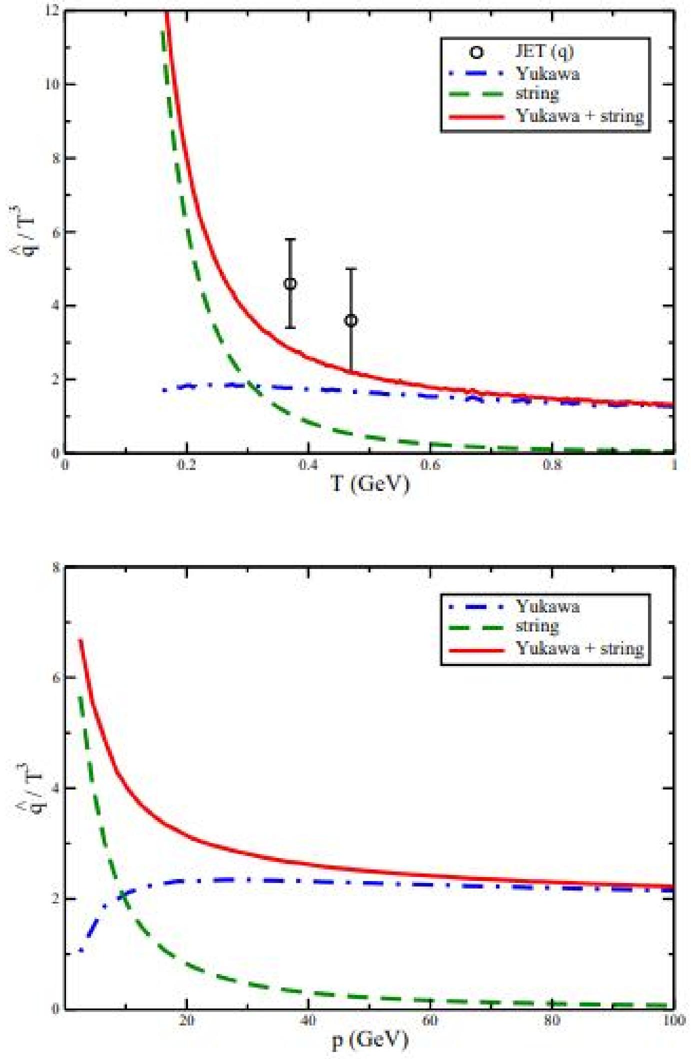




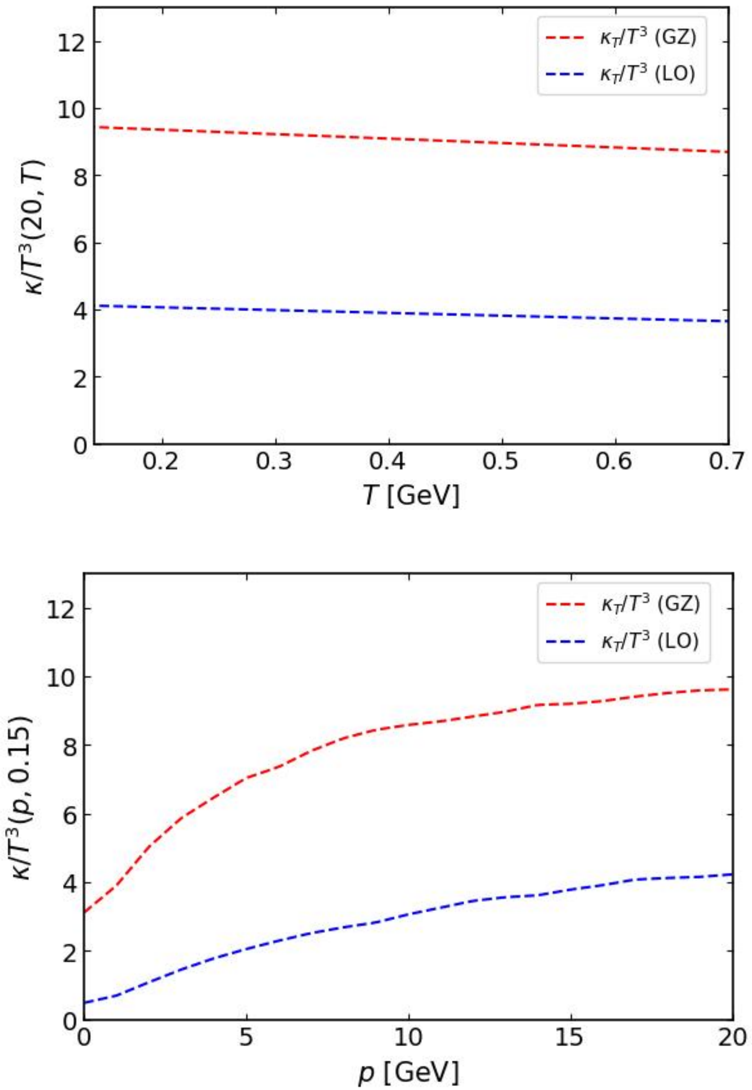
- The scaled diffusion coefficient, as a function of the incident momentum of charm and the background temperature, increases with the incident momentum and decreases with the temperature of the medium.
- Additionally, the HQ always exhibits a larger diffusion coefficient when traversing Gribov plasma compared to pQCD plasma.

Comparison of Cornell potential [1] for GZ prescription.

Cornell potential



GZ prescription



[1] W. Xing, G. Qin, and S. Cao, Phys. Lett. B 838, 137733(2023).

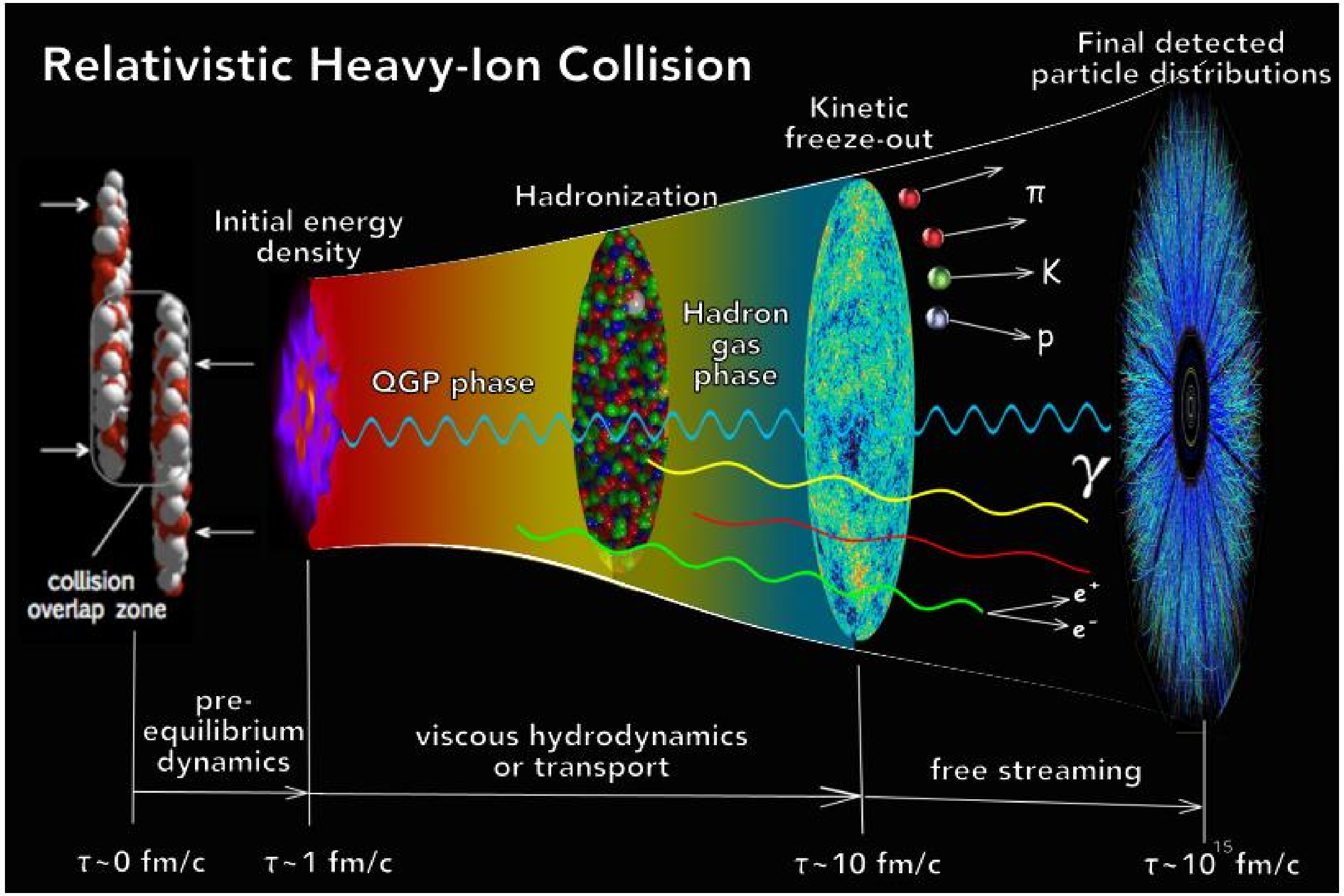
Summary

We investigated the momentum diffusion coefficient for the charm quark by employing the Gribov-Zwanziger framework.

- Nonperturbative effects increases the momentum diffusion coefficients compared to the perturbative LO results.
- Under the same conditions, the longitudinal coefficient has a larger value and changes more significantly.

We expect to gain a better understanding of the HQ' behaviour of the experimental quantities such as R_{AA} and v_2

Thanks



Relativistic Heavy-Ion Collision
Final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
π
K
p
Hadron gas phase
QGP phase
γ
collision overlap zone
e+
e-
pre-equilibrium dynamics
viscous hydrodynamics or transport
free streaming
τ~0 fm/c
τ~1 fm/c
τ~10 fm/c
τ~10^15 fm/c