Collisional Energy Loss of a Heavy Fermion in the Quark-Gluon Plasma

Yun Guo

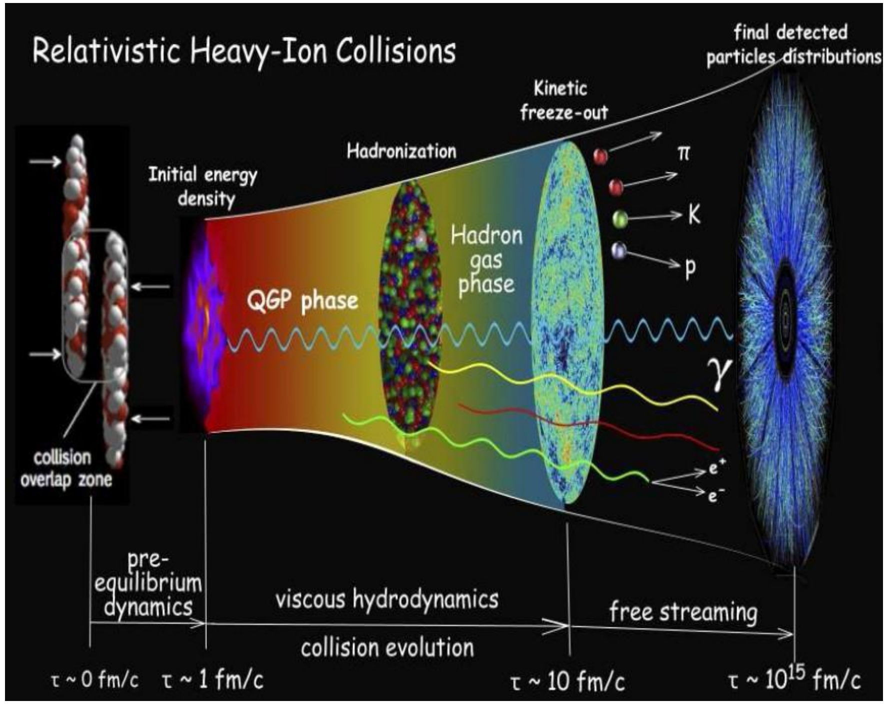
Physics Department, Guangxi Normal University

- **Introduction**
- **Theoretical framework to compute HQ energy loss**
- **HQ energy loss in QGP:**
  - the influence of a nontrivial Polyakov loop
  - the influnece of the collisions among medium partons
- **Summary and outlooks**

References: Phys.Rev.D 109 (2024) , 114025 & Phys.Rev.D 110 (2024) , 034011

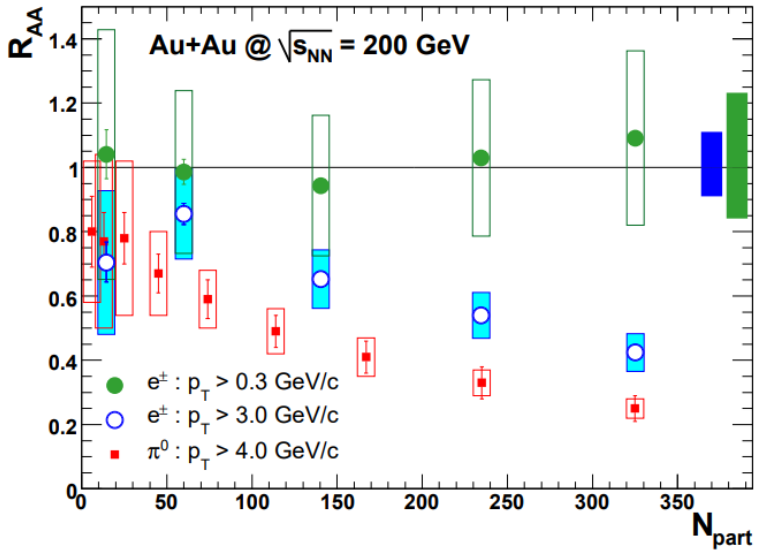
The 9th International Symposium on HF Production, Guangzhou, 08-Dec.-2024

Introduction



probes that used to study the QGP:

- ***Heavy Flavors(HF)*;**
- **Jets;**
- **Strangeness… …**

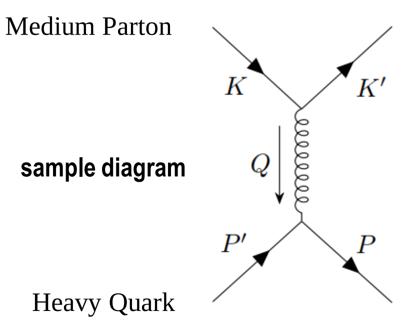


(PHENIX Collaboration, PRL 2007)

HF spectra retain a memory of their interactions

- **produced at the early stage in the HIC**
- **relaxation time comparable with the life time of the QGP**

HF can interact with quarks and gluons (partons) when traversing through the QGP which leads to the energy loss



- **inelastic collision, loss energy due to gluon radiation, dead cone effect**

Dokshitzer and Kharzeev, PLB 2001; Armesto, Dainese, Salgado, *et.al.* PRD 2005; Zhang, Wang and Wang, PRL 2004

- **elastic collision**

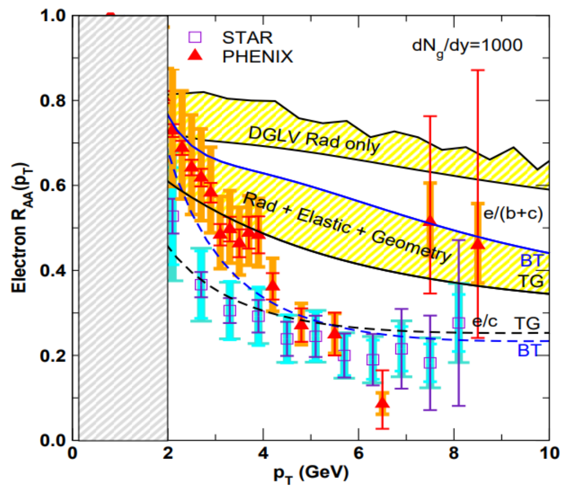
M. Thoma and M. Gyulassy, NPB 1991; Braaten and Thoma, PRD 1991; Peigne and Peshier, PRD 2008

- **Definition: the rate of energy loss per distance travelled**

$$-\frac{dE}{dx} = \frac{1}{v}\int_{M_Q}^{\infty} dE'(E-E')\frac{d\Gamma}{dE'}$$

Γ: interaction rate between the heavy quark and medium partons.

- **for light quarks: radiated energy loss dominates**
- **for heavy quarks: elastic energy loss becomes important**



(Wicks, Horowitz, Djordjevic and Gyulassy, NPA 2007)

Theoretical framework to compute HQ energy loss

Braaten & Yuan, PRL 1991

for the t-channel: divergent when the exchanged momentum is infinitely small

$\theta(q-q^*)$ **+** $\theta(q^*-q)$

defines the hard process, no infrared divergence	defines the soft process, regulate the divergence with HTL resummation

- **Hard contributions :** $p \gg T$ **and** $M \gg T$

$$-\left(\frac{dE}{dx}\right)^{Qq}_{\text{hard}} = \frac{8g^4 N_f \pi}{3v} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} n_F(k) \frac{\omega}{kk'} \frac{\delta(\omega - \mathbf{v}\cdot\mathbf{q})}{(\omega^2-q^2)^2} \left[2(k-\mathbf{v}\cdot\mathbf{k})^2 + \frac{1-v^2}{2}(\omega^2-q^2)\right] \theta(q-q^*)$$

- **Soft contributions :** HTL approximated resummed propagator

$$-\left(\frac{dE}{dx}\right)_{\text{soft}} = \frac{g^2 m_D^2}{8\pi v^2} \int_0^{q^*} dq \int_{-vq}^{vq} d\omega\, \omega^2 \left[|\Delta_L(Q)|^2 + \frac{1-\hat{\omega}^2}{2}(v^2-\hat{\omega}^2)|\Delta_T(Q)|^2\right]$$

Theoretical framework to compute HQ energy loss

- **about the cutoff dependence**

✓ in general, the energy loss depends on the cutoff and needs to be evaluated numerically

✓ the energy loss becomes more sensitive to the cutoff as the coupling constant increases

Romatschke and Strickland, PRD 2004

> the cutoff q^* can be determined by using the minimal sensitivity, *i.e.,*
> by minimizing the total energy loss with respect to q^*

Braaten and Thoma, PRD 1991

✓ the cancellation of the cutoff dependence happens in the limit $T \gg q^* \gg m_D$ and the energy loss can be obtained analytically

> hard: logarithmic divergence ~log(T/q^*) if $q^* \ll T$
> soft: logarithmic divergence ~log(q^*/m_D) if $q^* \gg m_D$

✓ this is possible in the weak coupling limit since $m_D \sim gT$

Theoretical framework to compute HQ energy loss

- **a new approach without introducing a cutoff** Djordjevic, PRC 2006

  - **instead of a bare gluon propagator, using an effective gluon propagator, eg., the HTL resumed propagator to compute the squared matrix element (result is still gauge-invariant for Qq scattering)**

$$\Gamma(E) = \frac{1}{2E}\int \frac{d^3\mathbf{p}'}{(2\pi)^3}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int \frac{d^3\mathbf{k}'}{(2\pi)^3}\frac{n_F(k)(1-n_F(k'))}{8E'kk'}(2\pi)^4\delta^4(P+K-P'-K')\left(\frac{1}{2}\sum_{\text{spins}}|\mathcal{M}|^2\right)$$

  - **there is no cutoff needed, therefore, don't differ the hard and soft scatterings**
  - **there is NO divergence appears in the energy loss**

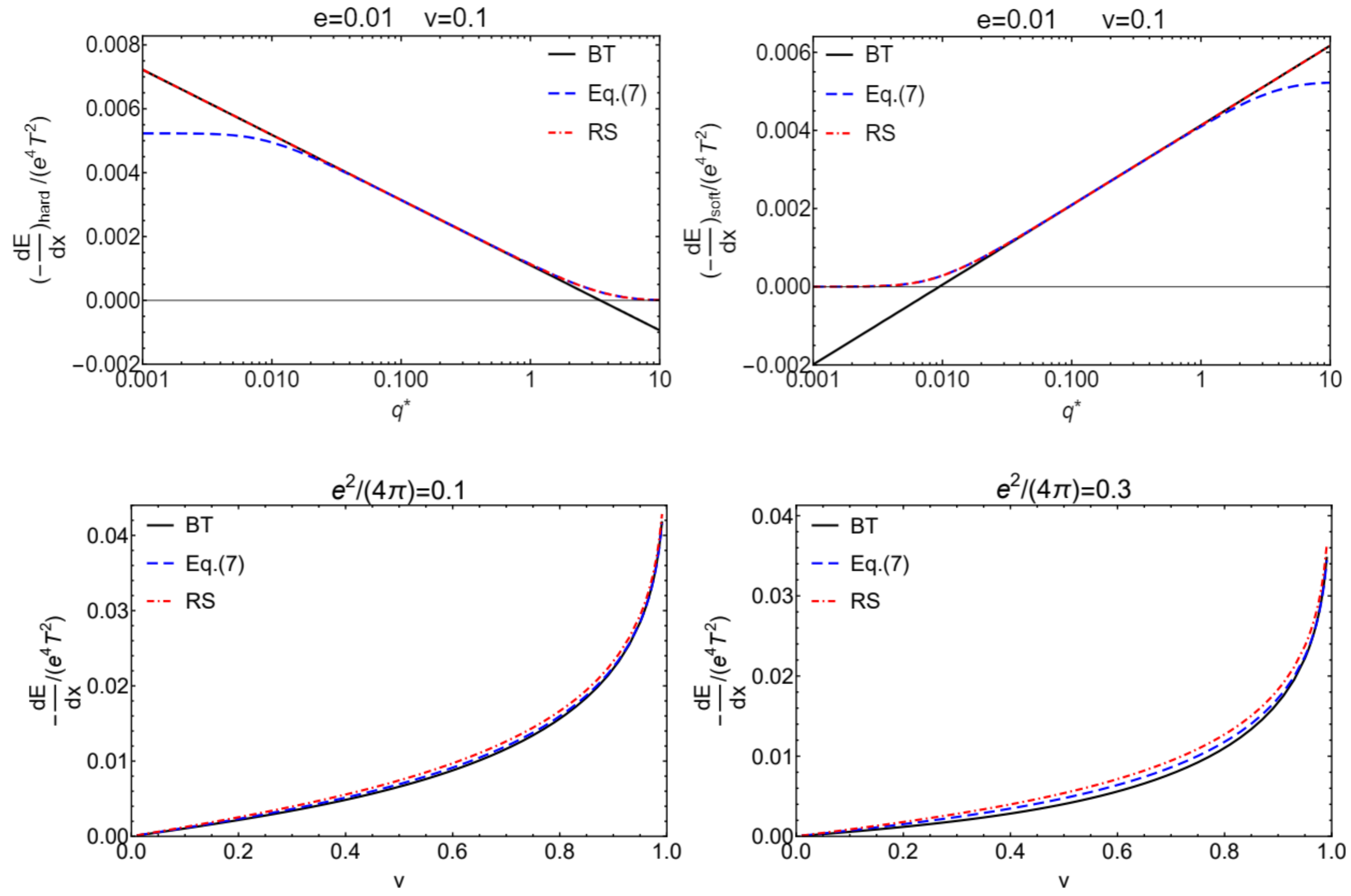
$$-\left(\frac{dE}{dx}\right) = \frac{e^4}{4\pi^3 v^2}\int_0^\infty dk\, n_F(k)\Big(\int_0^{2k/(1+v)} dq\, q^2\int_{-vq}^{vq} d\omega\,\omega + \int_{2k/(1+v)}^{2k/(1-v)} dq\, q^2\int_{q-2k}^{vq} d\omega\,\omega\Big) \times\left[|\Delta_L(Q)|^2 f_1(k,q,\omega) + \frac{Q^4}{q^4}|\Delta_T(Q)|^2(f_2(k,q,\omega)-f_1(k,q,\omega))\right], \quad (7)$$

with

$$\Delta_T(Q) = \frac{1}{Q^2-\Pi_T(\hat\omega)}, \qquad \Delta_L(Q) = \frac{1}{q^2-\Pi_L(\hat\omega)}$$

Theoretical framework to compute HQ energy loss

- **comparisons among the different methods**



HQ energy loss in QGP (I): with a nontrivial Polyakov loop

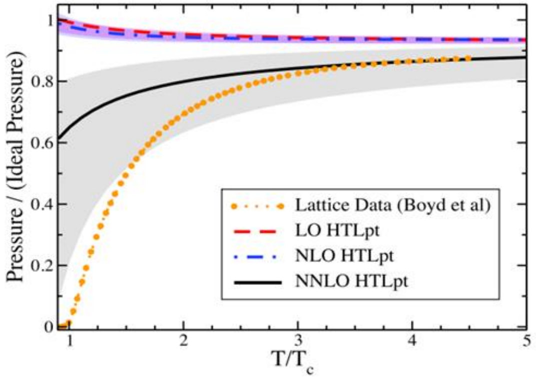
- **medium properties are very important to determine the collisional energy loss**

***Hard process*:**	***Soft process*:**
depends on the hard parton distribution functions	**depends on the screening strength, *i.e.*, the Debye mass**

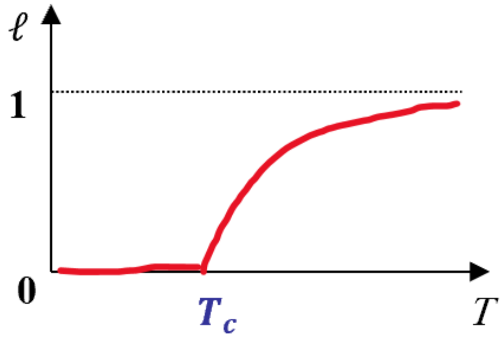
- ***how to describe the semi-QGP?***

✓ **rapid change of the thermodynamic properties corresponds to a phase transition from the normal hadronic phase to the QGP phase.**

✓ **for pure gauge theory, the order parameter of the deconfining phase transition is the Polyakov loop which has a non-trivial dependence on the temperature**



HTL resummation theory can describe the lattice data down to $\sim 3T_c$.



- **the background field effective theory**

  ✓ **to generate a T-dependent Polyakov loop in the "semi"-QGP region, introduce a classical background field for the gauge potential.**

$$A_\mu = A_\mu^{\rm cl} + gB_\mu \quad \textit{with} \quad (A_\mu^{\rm cl})_{ab} = \mathcal{Q}^a \delta_{ab}\delta_{\mu 0}$$

  ✓ **the effective potential from the background field effective theory**

Hidaka and Pisarski, PRD 2021

Meisinger, Miller and Ogilvieeldon, PRD 2002

$$\mathcal{V}(\mathrm{q}) = \sum_{a,b=1}^{N} \mathcal{P}_{ab}^{ab}\left(\frac{2\pi^2 T^4}{3} B_4(|\mathrm{q}^{ab}|) + \frac{CT^2}{2} B_2(|\mathrm{q}^{ab}|)\right)$$

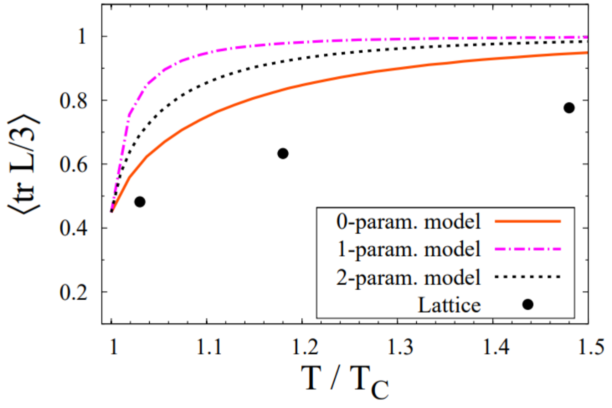
$$\mathrm{q}^{ab} = \mathrm{q}^a - \mathrm{q}^b \qquad \mathrm{q}^a = \mathcal{Q}^a/(2\pi T)$$

$$\mathcal{P}_{cd}^{ab} \equiv \delta_{ac}\delta_{bd} - \frac{1}{N}\delta_{ab}\delta_{cd}$$

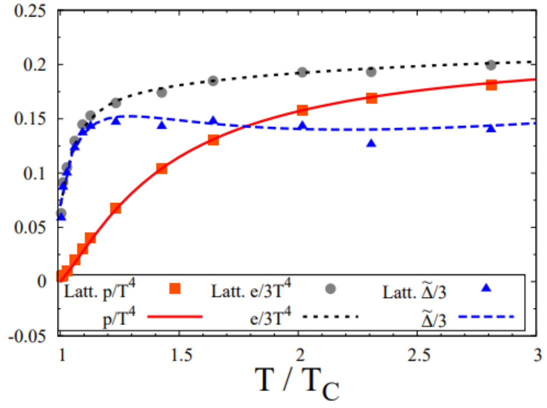
for SU(3) $\mathcal{Q} = (\mathrm{q}, 0, -\mathrm{q})$

$$\begin{cases} \mathrm{q}_{\rm decon} = \frac{1}{36}\left(9 - \sqrt{81 - 80(T/T_d)^2}\right) \\ \mathrm{q}_{\rm con} = \frac{1}{3} \end{cases}$$

non-pert. contribution is necessary to generate a non-zero BF from the eom



Dumitru, YG, Hidaka, Korthals Altes and Pisarski, PRD 2012



YG, JHEP 2014

HQ energy loss in QGP (I): with a nontrivial Polyakov loop

- **ingredients needed to compute the HQ energy loss**

  ✓ **the distribution functions in the BF effective theory**

$$n_B(q) \Longrightarrow \frac{n_B^{+ab}(q) + n_B^{-ab}(q)}{2} \quad \textit{with} \quad n_B^{\pm ab}(q) = \frac{1}{e^{\beta q \mp i\beta Q^{ab}} - 1}$$

  ✓ **the HTL resumed gluon propagator in the BF effective theory**

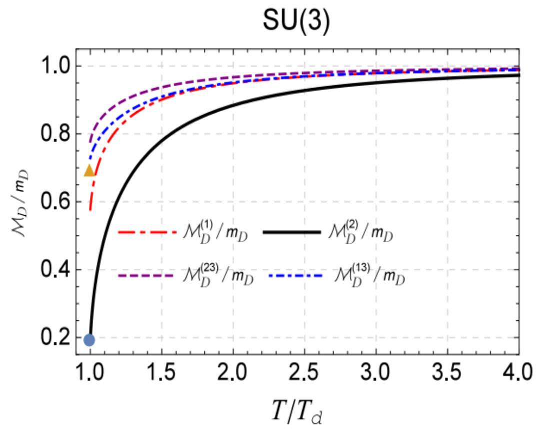
YG and Kuang, PRD 2021

off-diagonal components

$$\Delta_{\mu\nu}^{ab,ba}(Q,\mathrm{q}) = \frac{1}{Q^2 + F_T(Q,\mathrm{q}^a,\mathrm{q}^b)} A_{\mu\nu} + \frac{\omega^4/Q^4}{\omega^2 + F_L(Q,\mathrm{q}^a,\mathrm{q}^b)} B_{\mu\nu} + \frac{\xi}{Q^4} Q_\mu Q_\nu$$

diagonal components

$$\sum_{ab} \mathcal{P}^{aa,bb} \Delta_{\mu\nu}^{aa,bb}(Q,\mathrm{q}) = \sum_{i=1}^{N-1} \left[\frac{1}{Q^2 + \Lambda_T^i(Q,\mathrm{q})} A_{\mu\nu} + \frac{\omega^4/Q^4}{\omega^2 + \Lambda_L^i(Q,\mathrm{q})} B_{\mu\nu} + \frac{\xi}{Q^4} Q_\mu Q_\nu \right]$$



✓ **the $N^2 - 1$ gluons have the different screening masses**

✓ **the screening strength is weakened by the BF**

HQ energy loss in QGP (I): with a nontrivial Polyakov loop

- **analytical results of the t-channel contributions in the limit $T \gg q^* \gg m_D$**

  (using Braaten and Thoma's method)

➢ Hard contributions

$$-\left(\frac{dE}{dx}\right)^{Qg(t)}_{\mathrm{BF,hard}} = \frac{2\alpha_s^2 T^2}{\pi}\sum_{n=1}^{\infty}\frac{|\mathrm{Tr}\,\mathbf{L}^n|^2-1}{n^2}\left[f_1(v)\left(\ln\frac{T}{q^*}+\frac{1}{2}-\gamma-\ln n\right)+f_2(v)\right]$$

for SU(3) *the divergent part*

$$-\frac{2\alpha_s^2 T^2}{\pi}\sum_{n=1}^{\infty}\frac{|\mathrm{Tr}\,\mathbf{L}^n|^2-1}{n^2}f_1(v)\ln q^* = -\frac{8\pi\alpha_s^2 T^2}{3}f_1(v)(1-3\mathsf{q})^2\ln q^*$$

➢ Soft contributions

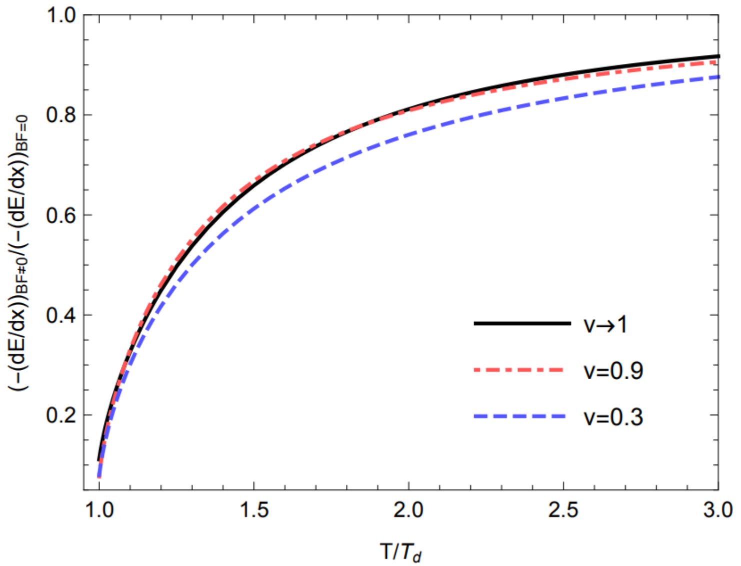
$$-\left(\frac{dE}{dx}\right)_{\mathrm{BF,soft}} = \frac{8\pi\alpha_s^2 T^2}{3}(1-3\mathsf{q})^2\left[f_1(v)\ln\left(q^*/m_D\right)-f_3(v)\right] - \frac{\alpha_s}{24}f_1(v)\sum_{ab/\sigma/s}(\mathcal{M}_D^2)^{ab/\sigma/s}\ln\left((\mathcal{M}_D^2)^{ab/\sigma/s}/m_D^2\right)$$

The cut-off dependence is cancelled between the hard and soft contributions which is independent on the explicit form of the BF

HQ energy loss in QGP (I): with a nontrivial Polyakov loop

- **numerical results q^* is determined by using the minimal sensitivity**

(using Romatschke and Strickland's method)



results for SU(3) without dynamical quarks with $T_d = 270$ MeV and $\alpha_s = 0.3$.

✓ the suppression is very significant in the entire semi-QGP region, the ratio is $\sim 10\%$ at the critical temperature.

✓ the effects of the BF vanish very quickly as the temperature increases. Above $\sim 3T_d$, suppression becomes very weak.

✓ in the ultra-relativistic limit, $v \to 1$, as well as in the weak coupling limit, the energy loss ratio is simply given by $(1\text{-}3q)^2$

ignoring a weak dependence on the heavy-quark velocity, the ratio can be simply approximated by $(1-3\mathrm{q})^2$ **Could be important for phenomenological applications.**

HQ energy loss in QGP (II): with collisions among medium partons

✓ **most applications of the HTL perturbation theory are limited in their assumption that collisions among plasma constituents are ignorable**

the time scale between hard collisions:	$t_{\rm hard}T \sim 1/\alpha_s^2 \log \alpha_s^{-1}$
the time scale between soft collisions:	$t_{\rm soft}T \sim 1/\alpha_s \log \alpha_s^{-1}$

Schenke, Strickland, Greiner and Thoma, PRD 2006

In high T limit: small coupling constant ✓ **In semi-QGP: coupling increases** ✗

✓ **the systematic inclusion of collisions can be accomplished by using the effective kinetic theory**

linearized kinetic equations: ***a BGK-type collision term***

$$V\cdot\partial_X \delta f_a^i(\mathbf{p},X) + g\theta_i V_\mu F_a^{\mu\nu}\partial_\nu^{(P)} f^i(\mathbf{p}) = \mathcal{C}_a^i(\mathbf{p},X)$$

with

$$\mathcal{C}_a^i(\mathbf{p},X) = -\nu\left[f_a^i(\mathbf{p},X) - \frac{N_a^i(X)}{N_{\rm eq}^i} f_{\rm eq}^i(p)\right]$$

$$\nu/m_D \approx 1.27\alpha_s^{3/2}\ln(c+0.25/\alpha_s)$$

HQ energy loss in QGP (II): with collisions among medium partons

- ✓ solving the kinetic equations for the fluctuations gives the induced current which determines the self-energy for the gauge bosons

$$\Pi^{\mu\nu}(Q) = \frac{\delta J^{\mu}_{\rm ind}(Q)}{\delta A_{\nu}(Q)}$$

with

$$\Pi_T(\hat{\omega},\hat{\nu}) = \frac{m_D^2}{4}\hat{\omega}\left[2z + (z^2-1)\ln\frac{z-1}{z+1}\right]$$

$$\Pi_L(\hat{\omega},\hat{\nu}) = -\frac{m_D^2}{2}\frac{1}{\mathcal{W}(\hat{\omega},\hat{\nu})}\left(2 + z\ln\frac{z-1}{z+1}\right)$$

$$z = \hat{\omega} + i\hat{\nu} \equiv \omega/q + i\nu/q$$

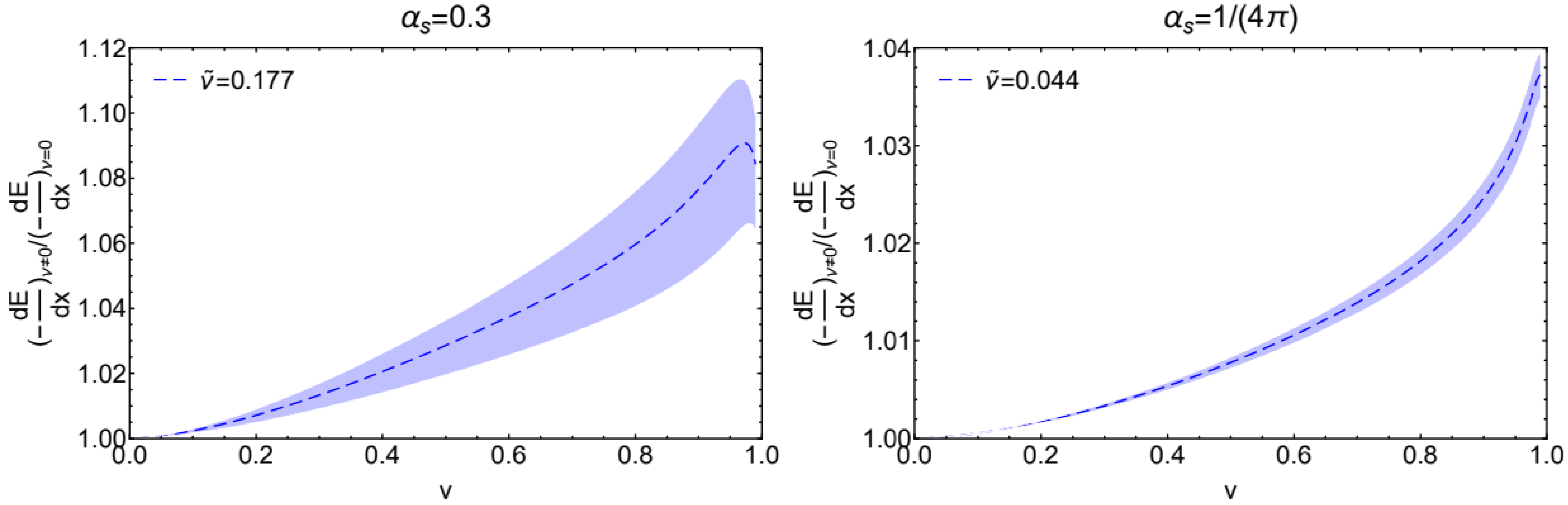
- ✓ computing the matrix element for the Qq scattering with the resumed propagator modified by the BGK collisional kernel , then evaluate the energy loss

One important reason to use this method to study energy loss:

> no sharp transition from the soft process (using resumed propagator) to the hard process (using bare propagator) where the collision effects are completely go away.

HQ energy loss in QGP (II): with collisions among medium partons

- **numerical results I**



✓ The heavy quark loses more energy in a collisional plasma and the enhancement is more significant when the incident velocity becomes large.

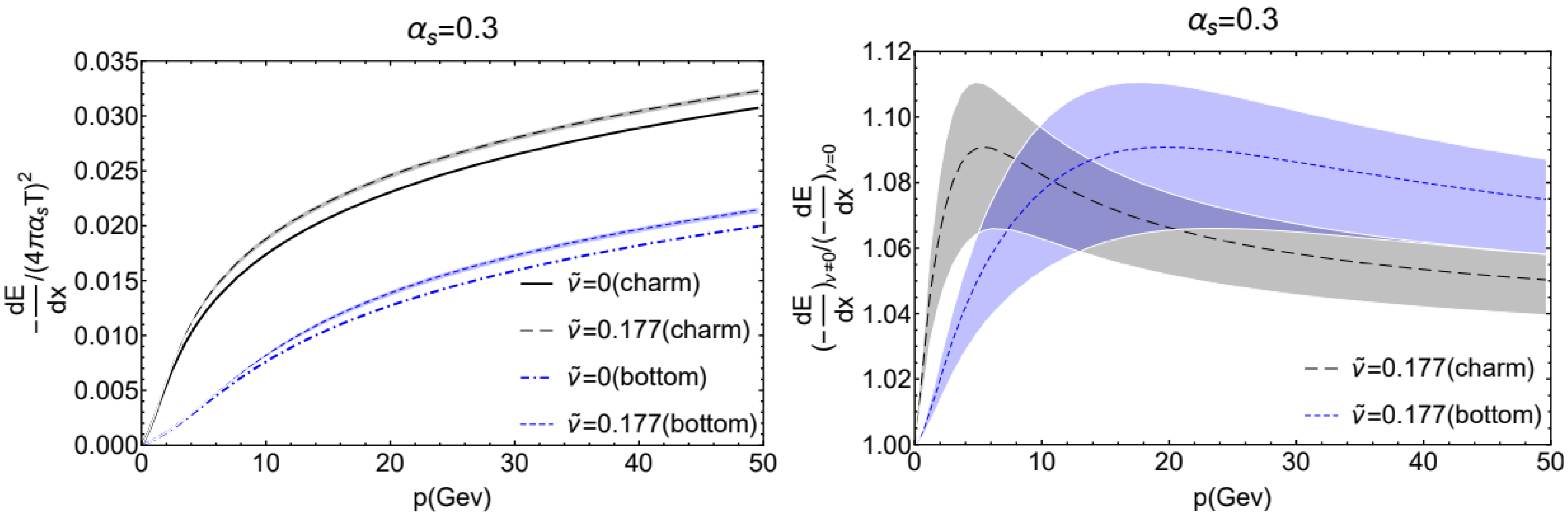
✓ For small coupling constant, the corrections to $-dE/dx$ are at most $\sim$ 5%

✓ For a realistic coupling constant, the influence becomes moderate, and the corrections can reach $\sim$ 10%

magnitudes are strongly dependent on the parameterization of the collision rate.

HQ energy loss in QGP (II): with collisions among medium partons

- **numerical results II**



✓ The mass hierarchy of the collisional energy loss is also observed in a collisional plasma.

✓ The increase in −dE/dx of a charm quark is comparable to that of a bottom quark at large momenta.

✓ A more significant correction for the bottom quark at large p, while the opposite is found for small p.

Summary and outlooks

- **We review the theoretical approaches to compute the HQ collisional energy loss.**
- **Our results show:**

- **HQ energy loss is reduced the semi-QGP. Near T_c, the reduction is very significant, as compared to the perturbation theory, only ~10% of the energy loss obtained in the BF effective theory.**
- **The BF suppression of the HQ energy loss is very sensitive to the temperature, beyond ~3 T_c, the influence of the BF turns out to be negligible.**

- **A moderate enhancement of the energy loss in a collisional plasma. Near T_c, , the corrections can reach ~10% for large incident velocities.**
- **A more significant correction due to collisions among thermal partons for bottom at large momenta, for charm at small momenta.**

- **New paths forward for future work**
  - **Consider other physical observables based on the BF effective theory，eg. heavy quarkonia, Arxiv: 2412.XXXX**
  - **Full QCD calculation of the HQ energy loss with a BGK collisional kernel, contributions from Qg scatterings.**

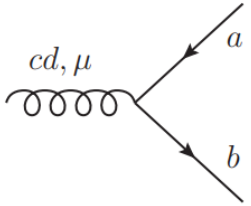
Thank You for Your Attention

- **double line notations**

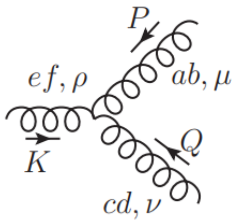
$$(t^{ab})_{cd} = \frac{1}{\sqrt{2}}\mathcal{P}^{ab}_{cd}\,, \qquad \mathcal{P}^{ab}_{cd} = \mathcal{P}^{ab,dc} = \mathcal{P}_{ba,cd} = \delta^a_c\delta^b_d - \frac{1}{N}\delta^{ab}\delta_{cd}\,.$$

For $SU(N)$ gauge theories, the color indices a, b, c and d run from 1 to N. There are $N^2 - N$ off-diagonal generators t^{ab} with $a \neq b$ which are the ladder operators of the Cartan basis. They are orthogonal to each other and normalized as $\mathrm{tr}(t^{ab}\, t^{ba}) = 1/2$ with fixed a and b. The N diagonal generators are not independent, satisfying $\sum_{a=1}^{N} t^{aa} = 0$ and the normalization becomes $\mathrm{tr}(t^{aa}\, t^{bb}) = (\delta^{ab} - 1/N)/2$ where no summation over a and b applies.

- **Feynman rules**



$$= -ig(t^{cd})_{ba}\gamma_\mu$$



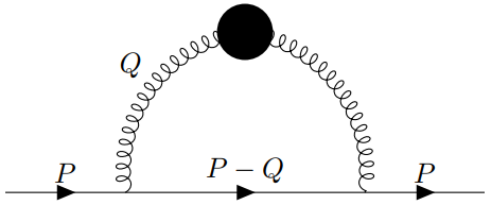
$$= -gf^{(ab,cd,ef)}[(P-Q)_\rho\, g_{\mu\nu} + (Q-K)_\mu\, g_{\nu\rho} + (K - P)_\nu\, g_{\mu\rho}]$$

$$f^{ab,cd,ef} = \frac{i}{\sqrt{2}}(\delta^{ad}\delta^{ad}\delta^{ad} - \delta^{ad}\delta^{ad}\delta^{ad})$$

- **a general definition from quantum field theory** Weldon, PRD 1983

$$\Gamma(E) = -\frac{1}{2E}\mathrm{Tr}\left[(\not{P}+M)\mathrm{Im}\,\Sigma(P)\right]$$
$$= -\frac{g^2}{4\pi^2 v}\int_0^\infty dq\, q\int_{-vq}^{vq} d\omega(1+n_B(\omega))\,\mathrm{Im}\Big[\Delta_L(Q)+(v^2-\hat{\omega}^2)\Delta_T(Q)\Big]$$

with $\Sigma(P) = ig^2C_F\int\frac{d^4Q}{(2\pi)^4}\Delta^{\mu\nu}(Q)\gamma_\mu\frac{1}{(P-Q)\cdot\gamma - M_Q}\gamma_\nu$



resummed propagator $\Delta_T(Q) = \frac{1}{Q^2-\Pi_T(\hat{\omega})}\,,\qquad \Delta_L(Q) = \frac{1}{q^2-\Pi_L(\hat{\omega})}\,,\qquad \hat{\omega}\equiv\omega/q$

HQ energy loss in perturbation theory

- **Soft contributions: basic formulas**

HTL approximated resummed propagator

$$-\left(\frac{dE}{dx}\right)_{\rm soft} = \frac{g^2 m_D^2}{8\pi v^2}\int_0^{q^\star} dq \int_{-vq}^{vq} d\omega\,\omega^2 \left[|\Delta_L(Q)|^2 + \frac{1-\hat{\omega}^2}{2}(v^2-\hat{\omega}^2)|\Delta_T(Q)|^2\right]$$

Alternatively,

Thoma and Gyulassy, NPB 1991

Romatschke and Strickland, PRD 2004

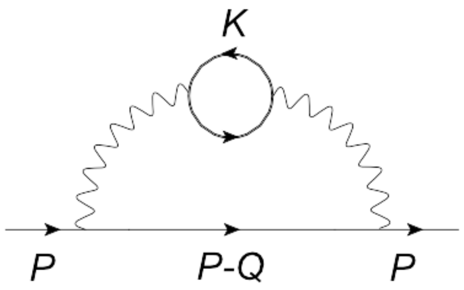
$$-\left(\frac{dE}{dx}\right)_{\rm soft} = \frac{g^2}{v} C_F\,{\rm Im}\int \frac{d^3\mathbf{q}}{(2\pi)^3}(\mathbf{v}\cdot\mathbf{q})v^i\left[\Delta_{ij}(Q) - (\Delta_0)_{ij}(Q)\right]v^j\theta(q^\star - q)\Big|_{\omega=\mathbf{v}\cdot\mathbf{q}}$$

- ✓ **for soft gluon exchange, it is necessary to use the resummation theory**
- ✓ **the HTL resumed gluon propagator should be used which regulates the infrared divergence in the t-channel**

HTL

- **hard contribution from the Weldon's formula**

LO for large *Q*:



beyond HTL approximation $\mathrm{Im}\,\Pi(K) \sim n_F(k) - n_F(k+\omega)$

$$(1 + n_B(\omega))(n_F(k) - n_F(k+\omega)) = n_F(k)(1 - n_F(k+\omega))$$

⟹ **Hard contributions to the collisional energy loss**

- **Hard contributions from Qg scattering ($p \gg T$ and $M \gg T$) s- and u-channel**

$$-\left(\frac{dE}{dx}\right)^{Qg(s+u)}_{\rm hard} = \frac{2\pi g^4}{v}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\int\frac{d^3\mathbf{k}'}{(2\pi)^3}\frac{n_B(k)}{kk'}\delta(\omega-\mathbf{v}\cdot\mathbf{q})\frac{\omega(1-v^2)^2}{(k-\mathbf{v}\cdot\mathbf{k})^2}$$
$$= \frac{g^4T^2}{12\pi}\left(\frac{1}{v}-\frac{1-v^2}{2v^2}\ln\frac{1+v}{1-v}\right)$$

- **contributions suppressed by T/M or T/p are dropped in the above expressions**
- **in QED, suppressed by $(T/M)^2$ and can be ignored. No suppression in QCD.**
- **for s- and u-channel contributions, no infrared divergence**
- **interference between t and u/s channel can be neglected in the assumption $T \ll M$**

HQ energy loss in perturbation theory

- **Condition I: $q^* \gg m_D$ soft contribution**

$$-\left(\frac{dE}{dx}\right)_{\rm soft} = \frac{g^2 C_F}{2v^2}{\rm Im}\int_{-v}^{v}\frac{\hat\omega d\hat\omega}{(2\pi)^2}\left(\frac{v^2-\hat\omega^2}{(\hat\omega^2-1)^2}\Pi_T \ln\frac{q^{*2}(1-\hat\omega^2)+\Pi_T}{\Pi_T}+\Pi_L\ln\frac{q^{*2}-\Pi_L}{-\Pi_L}\right)$$

expand the above result in the limit $q^* \gg m_D$, it has a logarithmic divergence ~ $\log(q^*/m_D)$

- **Condition II: $q^* \ll T$ hard contribution**

$$\int d^3\mathbf{k}\int d^3\mathbf{k}' \longrightarrow 2(2\pi)^2\int dk\int d\omega\int dq$$

$$\int_{\frac{1+v}{2}q^*}^{\infty} dk\int_{q^*}^{\frac{2k}{1+v}} dq\int_{-vq}^{vq} d\omega + \int_{\frac{1+v}{2}q^*}^{\infty} dk\int_{\frac{2k}{1+v}}^{\frac{2k}{1-v}} dq\int_{q-2k}^{vq} d\omega + \int_{\frac{1-v}{2}q^*}^{\frac{1+v}{2}q^*} dk\int_{q^*}^{\frac{2k}{1-v}} dq\int_{q-2k}^{vq} d\omega$$

introducing q^* in $\int dq$, remove the infrared divergence
in the limit $q^* \ll T$ ($q^* \to 0$), the above result has a logarithmic divergence ~ $\log(T/q^*)$

→ *in the limit $T \gg q^* \gg m_D$, the cutoff dependence is cancelled between the hard & soft contributions, the total energy loss is finite and no cutoff dependence*

The background field effective theory

- one-loop result for the pressure with/without the BF

$$P_{\mathrm{BF}=0} = \sum_{ab} \frac{1}{3\pi^2} \int_0^\infty q^3 n_B(q) dq (1 - \delta_{ab}/N)$$

$$P_{\mathrm{BF}\neq 0} = \sum_{ab} \frac{1}{6\pi^2} \int_0^\infty q^3 (n_B^{+ab}(q) + n_B^{-ab}(q)) dq (1 - \delta_{ab}/N)$$

BF modified distribution function

$$n_B(q) \Longrightarrow \frac{n_B^{+ab}(q) + n_B^{-ab}(q)}{2}$$

$$n_F(q) \Longrightarrow \frac{n_F^{+a}(q) + n_F^{-a}(q)}{2}$$

with $n_B^{\pm ab}(q) = \dfrac{1}{e^{\beta q \mp i\beta Q^{ab}} - 1}$ $n_F^{\pm a}(q) = \dfrac{1}{e^{\beta q \mp i\beta Q^{a}} + 1}$ **and** $Q^{ab} \equiv Q^a - Q^b$

- ***however,*** the perturbation theory with BF

> ➢ predicts a system which always favors the complete QGP phase where BF=0, no phase transition could happen
> ➢ leads to a non-transverse gluon self-energy and an ill-defined screening mass

- to drive the system to the confined phase, one needs to consider effective theory with the background field.

Results for the HQ energy loss in SU(3)

- **basic formulas for the HQ energy loss with the BF**
  - Hard contributions from Qg scattering ($p \gg T$ and $M \gg T$)

$$-\left(\frac{dE}{dx}\right)^{Qg(t)}_{\mathrm{BF,hard}} = \frac{2\pi g^4}{v}\sum_{a,b=1}^{3}\left(1-\frac{1}{3}\delta^{ab}\right)\int\frac{d^3\mathbf{k}}{(2\pi)^3 k}\frac{n_B^{+ab}(k)+n_B^{-ab}(k)}{2}\int\frac{d^3\mathbf{k}'}{(2\pi)^3 k'}\delta(\omega-\mathbf{v}\cdot\mathbf{q})$$
$$\times\frac{\omega}{(\omega^2-q^2)^2}\left[(k-\mathbf{v}\cdot\mathbf{k})^2+\frac{1-v^2}{2}(\omega^2-q^2)\right]\theta(q-q^*)$$

$$-\left(\frac{dE}{dx}\right)^{Qg(s+u)}_{\mathrm{BF,hard}} = \frac{\pi g^4}{4v}\sum_{a,b=1}^{3}\left(1-\frac{1}{3}\delta^{ab}\right)\int\frac{d^3\mathbf{k}}{(2\pi)^3 k}\frac{n_B^{+ab}(k)+n_B^{-ab}(k)}{2}\int\frac{d^3\mathbf{k}'}{(2\pi)^3 k'}\delta(\omega-\mathbf{v}\cdot\mathbf{q})\frac{\omega(1-v^2)^2}{(k-\mathbf{v}\cdot\mathbf{k})^2}$$
$$=\frac{g^4T^2}{16\pi^3}\sum_{n=1}^{\infty}\frac{|\mathrm{Tr}\,\mathbf{L}^n|^2-1}{n^2}\left(\frac{1}{v}-\frac{1-v^2}{2v^2}\ln\frac{1+v}{1-v}\right)$$

$$n_B^{\pm ab}(q)=\frac{1}{e^{\beta q\mp i\beta\mathcal{Q}^{ab}}-1}\qquad \mathcal{Q}^{ab}\equiv\mathcal{Q}^a-\mathcal{Q}^b$$

BF acts as an imaginary chemical potential

  - Soft contributions

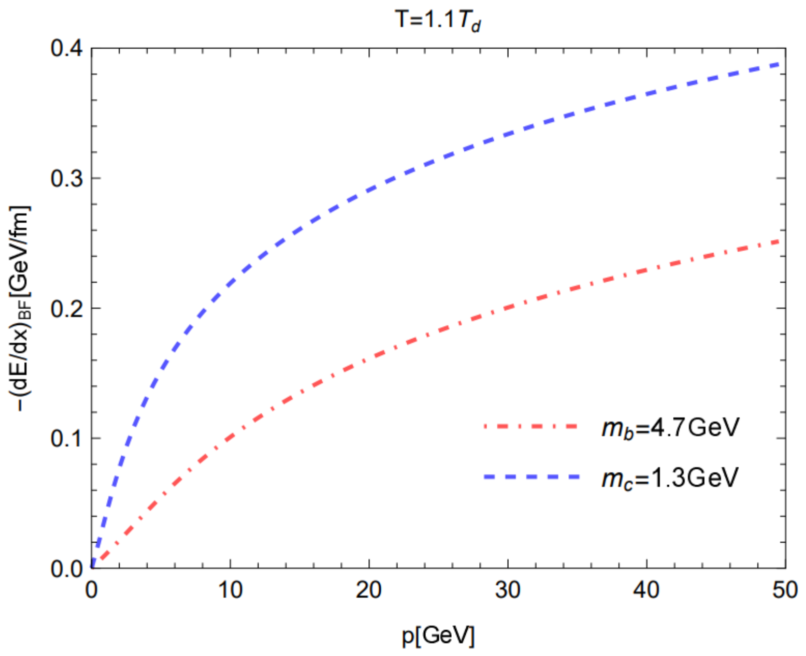
$$-\left(\frac{dE}{dx}\right)_{\mathrm{BF,soft}} = \frac{g^2}{6v}\sum_{abcd}\mathcal{P}^{ab,cd}\,\mathrm{Im}\int\frac{d^3\mathbf{q}}{(2\pi)^3}(\mathbf{v}\cdot\mathbf{q})v^i\left[\Delta^{ab,cd}_{ij}(Q,\mathsf{q})-(\Delta_0)^{ab,cd}_{ij}(Q)\right]v^j\theta(q^\star-q)\Big|_{\omega=\mathbf{v}\cdot\mathbf{q}}$$

⇓ *zero temperature limit of* $\Delta^{ab,cd}_{ij}(Q,\mathsf{q})$

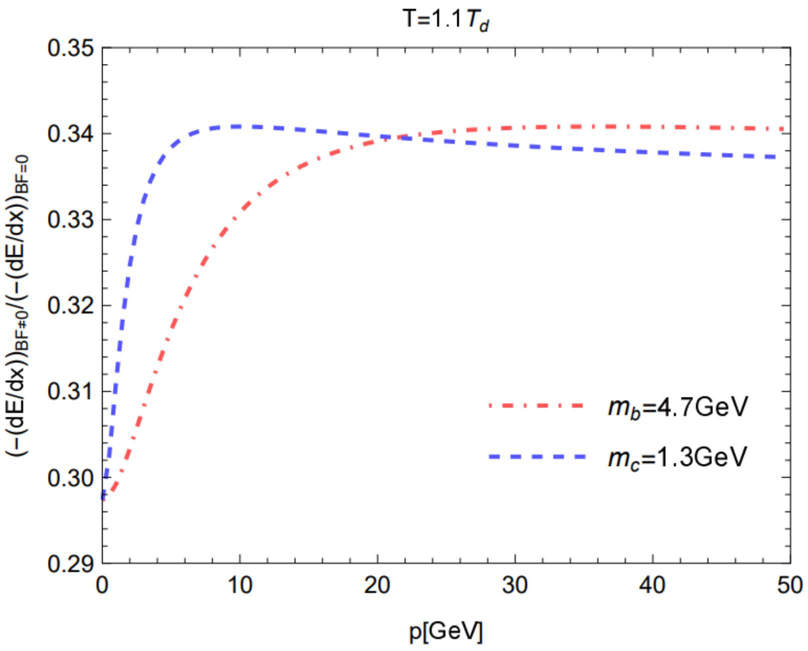
Numerical results for the HQ energy loss in SU(3)

- **results for SU(3) without dynamical quarks with $T_d = 270$ MeV and $\alpha_s = 0.3$.**

the flavor dependence



Energy loss of a charm and bottom quark as a function of the momentum p .



Energy loss ratio of a charm and bottom quark as a function of the momentum p.

$$p = v m_Q / \sqrt{1 - v^2}$$

The results at higher temperature, eg., $T = 2.0T_d$ is qualitatively similar as what we found at $T = 1.1T_d$.

charm quark: $p \sim 5$ GeV, $v \sim 0.95$ bottom quark: $p \sim 15$ GeV, $v \sim 0.95$