Exploring hadronic quarkonium production in QCD factorization formalism

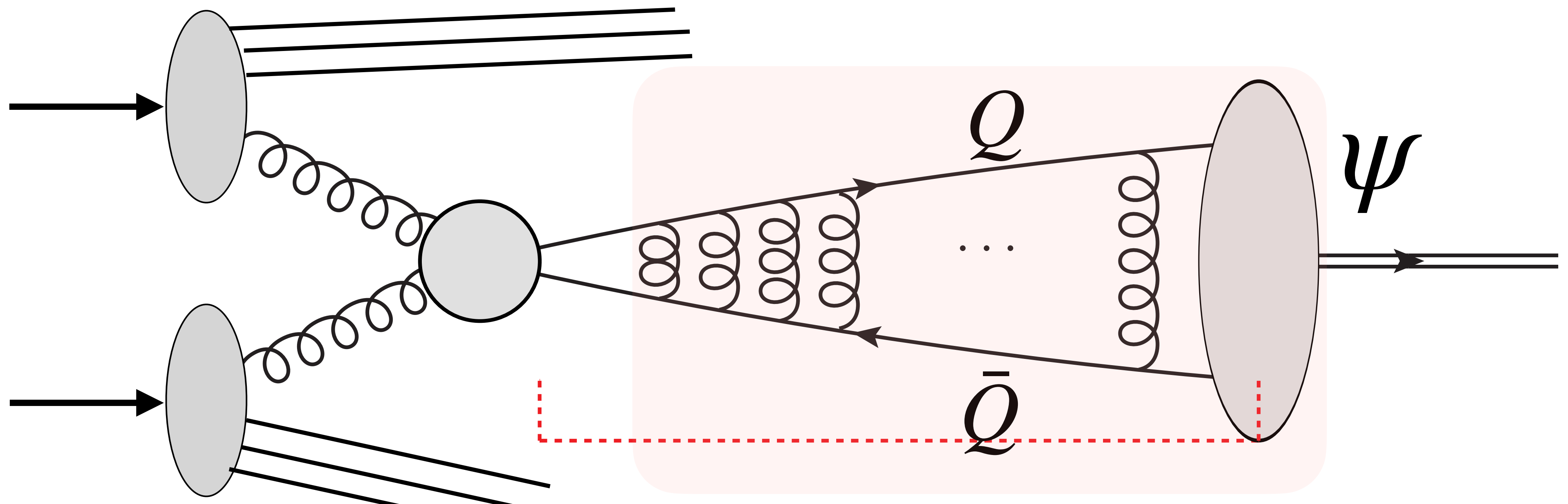
Kazuhiro Watanabe
Tohoku University

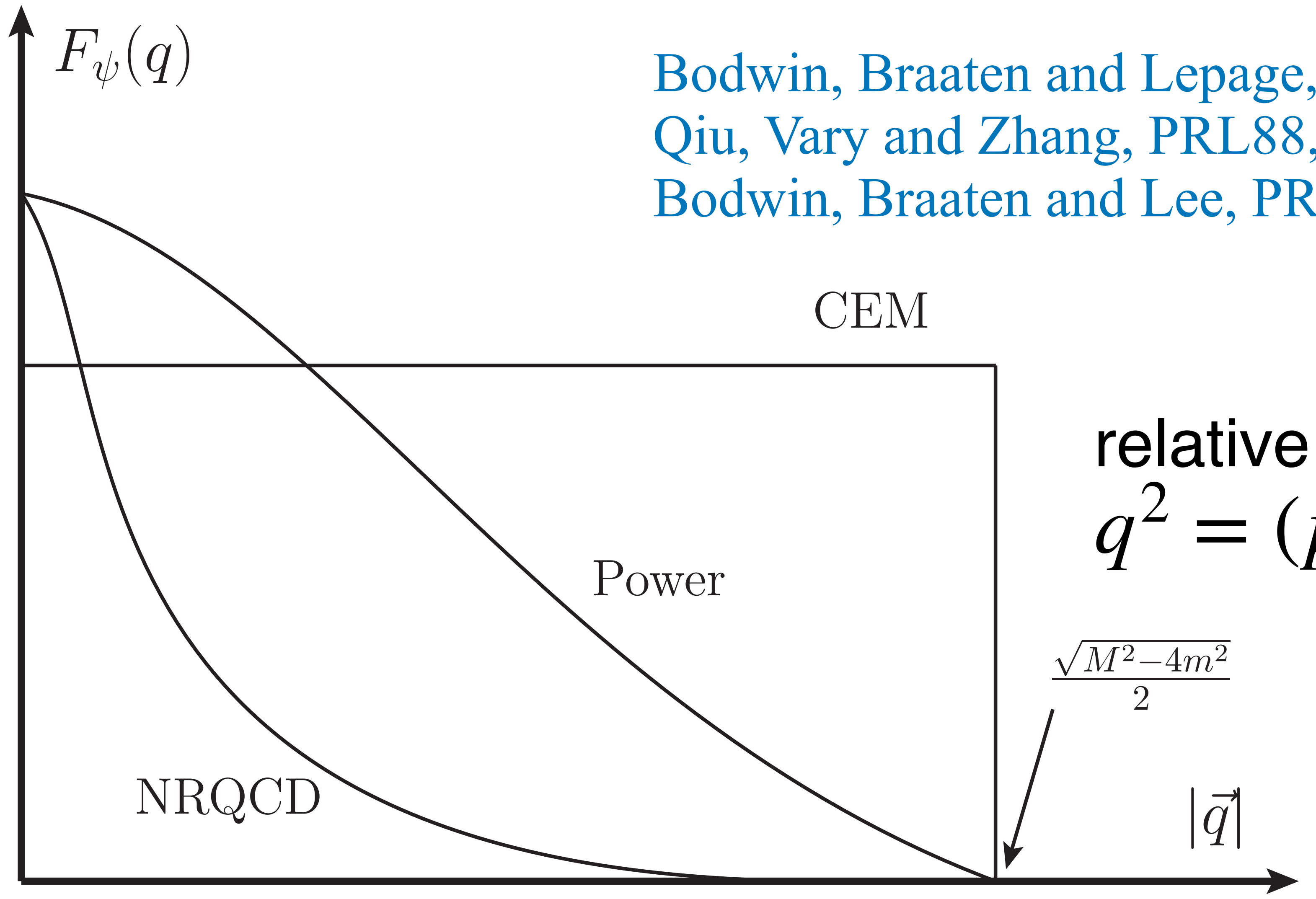
In collaboration with Kyle Lee, Jianwei Qiu, George Sterman

HF-HNC2024, 12/09/2024, Guangzhou, China

TOHOKU UNIVERSITY

Emergence of heavy quarkonium in pp collisions





Bodwin, Braaten and Lepage, PRD51, 1125 (1995)
Qiu, Vary and Zhang, PRL88, 232301 (2002)
Bodwin, Braaten and Lee, PRD72, 014004 (2005)

relative momentum
$q^2 = (p_Q - p_{\bar{Q}})^2$

Inclusive quarkonium production x-section may read:

$$d\sigma_{A+B\to\psi+X} \approx \sum_n \int dq^2\, d\hat{\sigma}_{A+B\to Q\bar{Q}[n]+X}(q^2)\, F_{Q\bar{Q}[n]\to\psi}(q^2)$$

Transition distribution: yet unknown well…

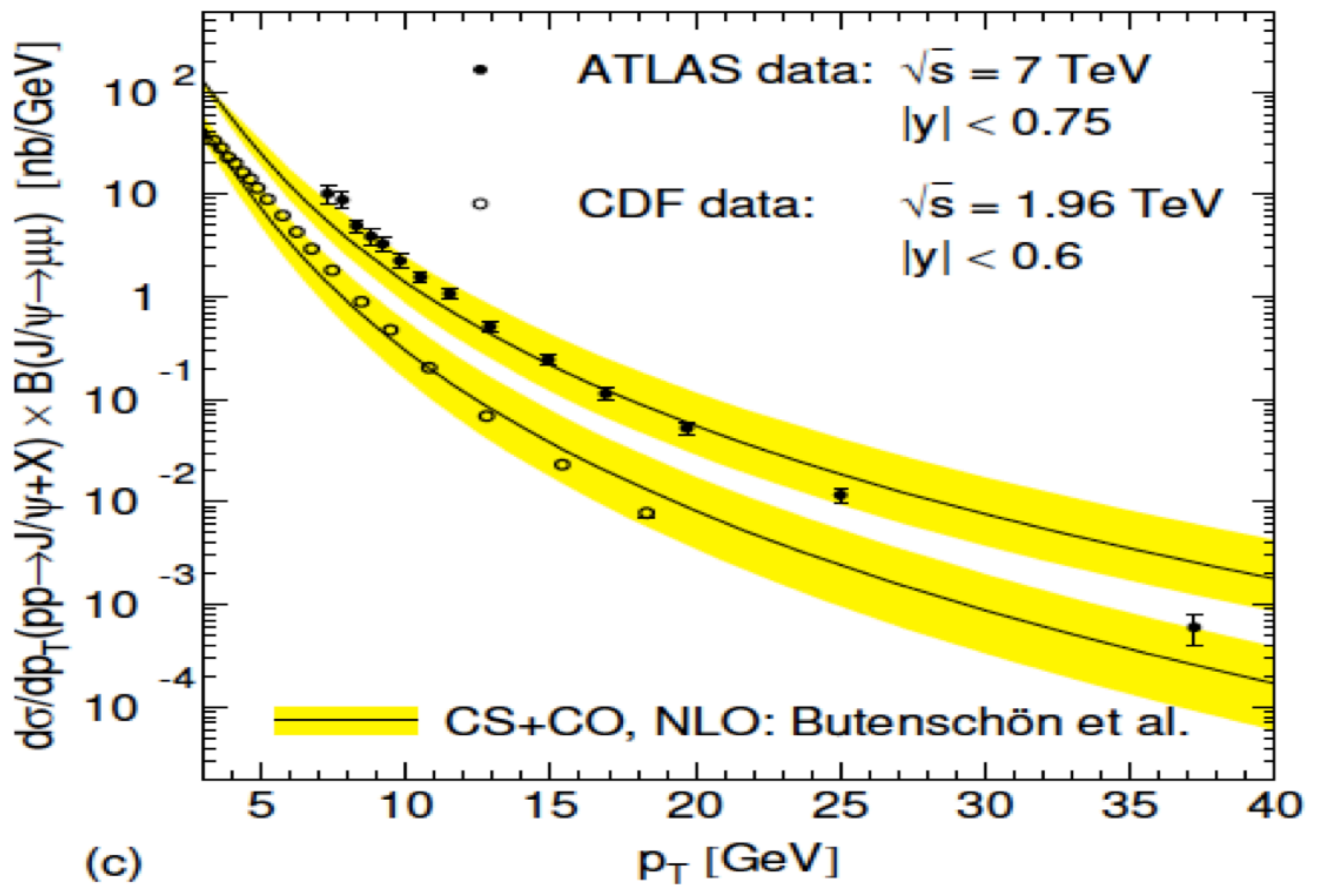
❖ The nonperturbative emergence of a heavy quarkonium from a heavy quark pair in hadronic collisions has been an exciting challenge to the study of QCD.

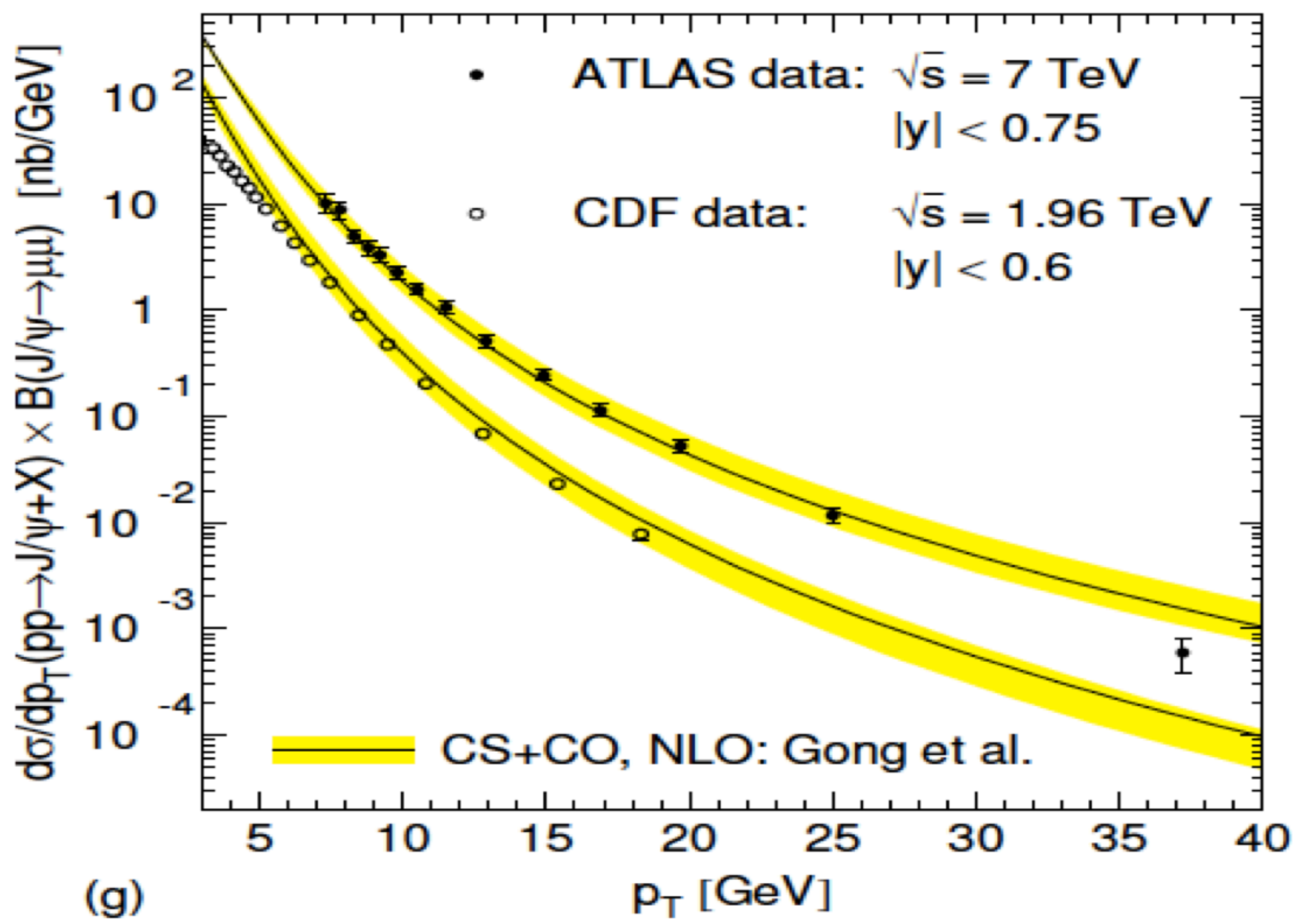
❖ $F_{Q\bar{Q}\to\psi}$: Color Singlet Model, (improved) Color Evaporation Model, NRQCD factorization.

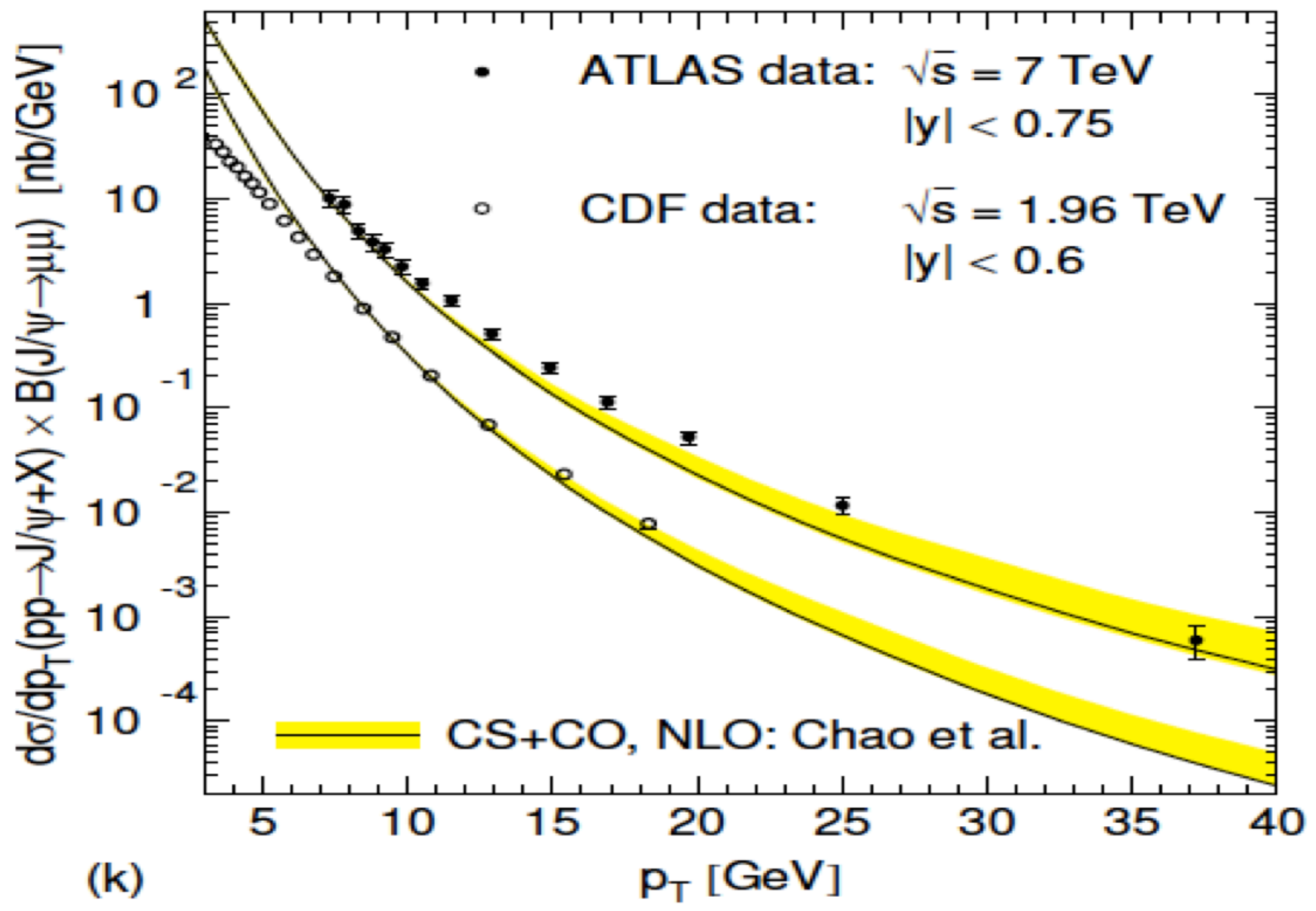
NRQCD at NLO vs. Experimental data

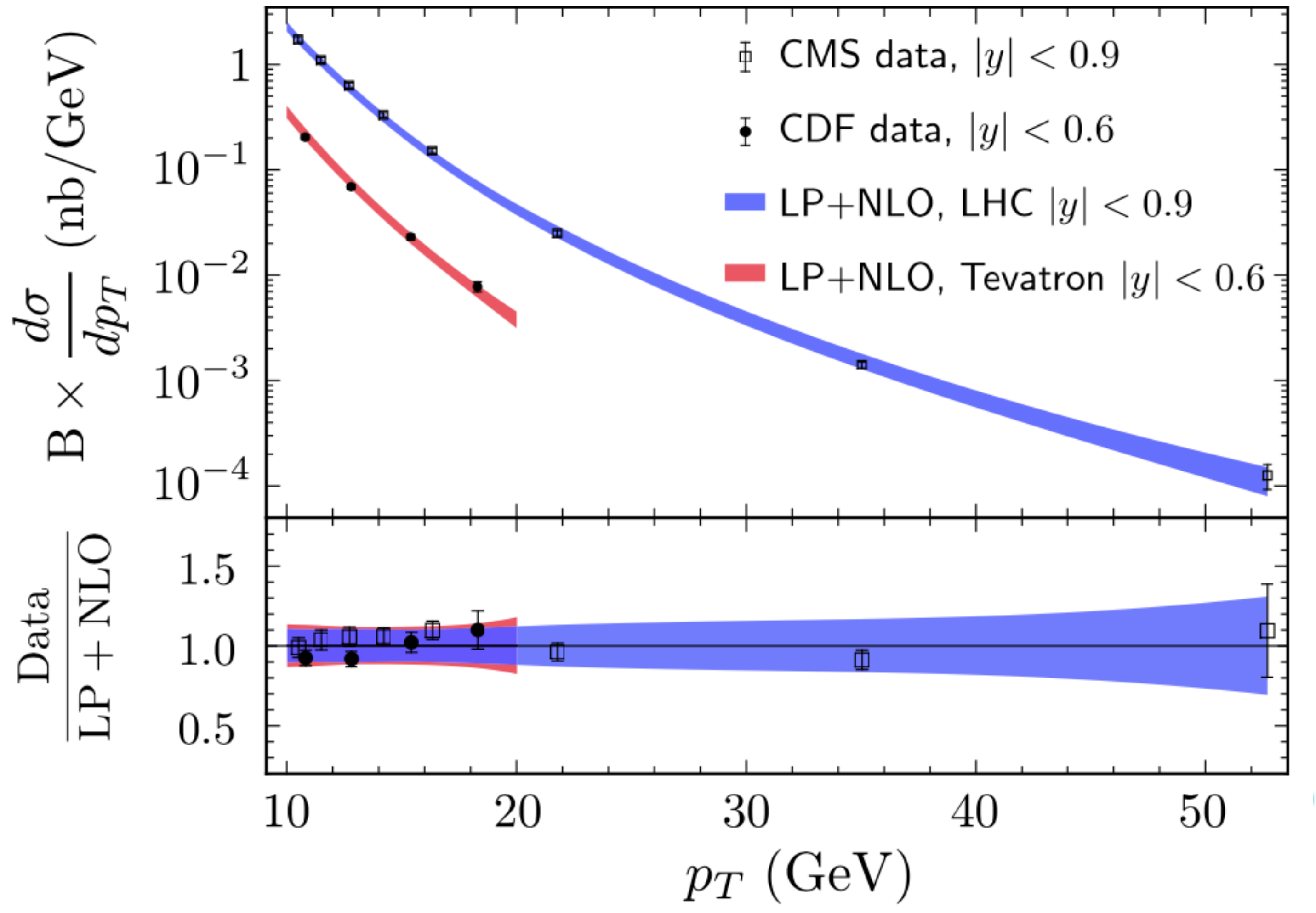
$$d\sigma_{pp\to\psi+X} \approx \sum_{\kappa} d\hat{\sigma}_{pp\to Q\bar{Q}[\kappa]+X} \langle \mathcal{O}_{Q\bar{Q}[\kappa]\to\psi} \rangle$$

α_s & v expansion — Hadronization









4-leading channels in v-expansion

$$\kappa = {}^3S_1^{[1]},\ {}^1S_0^{[8]},\ {}^3S_1^{[8]},\ {}^3P_J^{[8]}$$

Long-Distance Matrix Elements (LDMEs)

J/ψ	$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle$ 10^{-2} GeV^5
Set I (Butenschoen *et al.*)	1.32	3.04	0.16	−0.91
Set II (Chao *et al.*)	1.16	8.9	0.30	1.26
Set III (Gong *et al.*)	1.16	9.7	−0.46	−2.14
Set IV (Bodwin *et al.*)	-	9.9	1.1	1.1

LDMEs should be universality, however:

- Numbers are not the same.
- Not even the sign.

More work is needed!

NRQCD

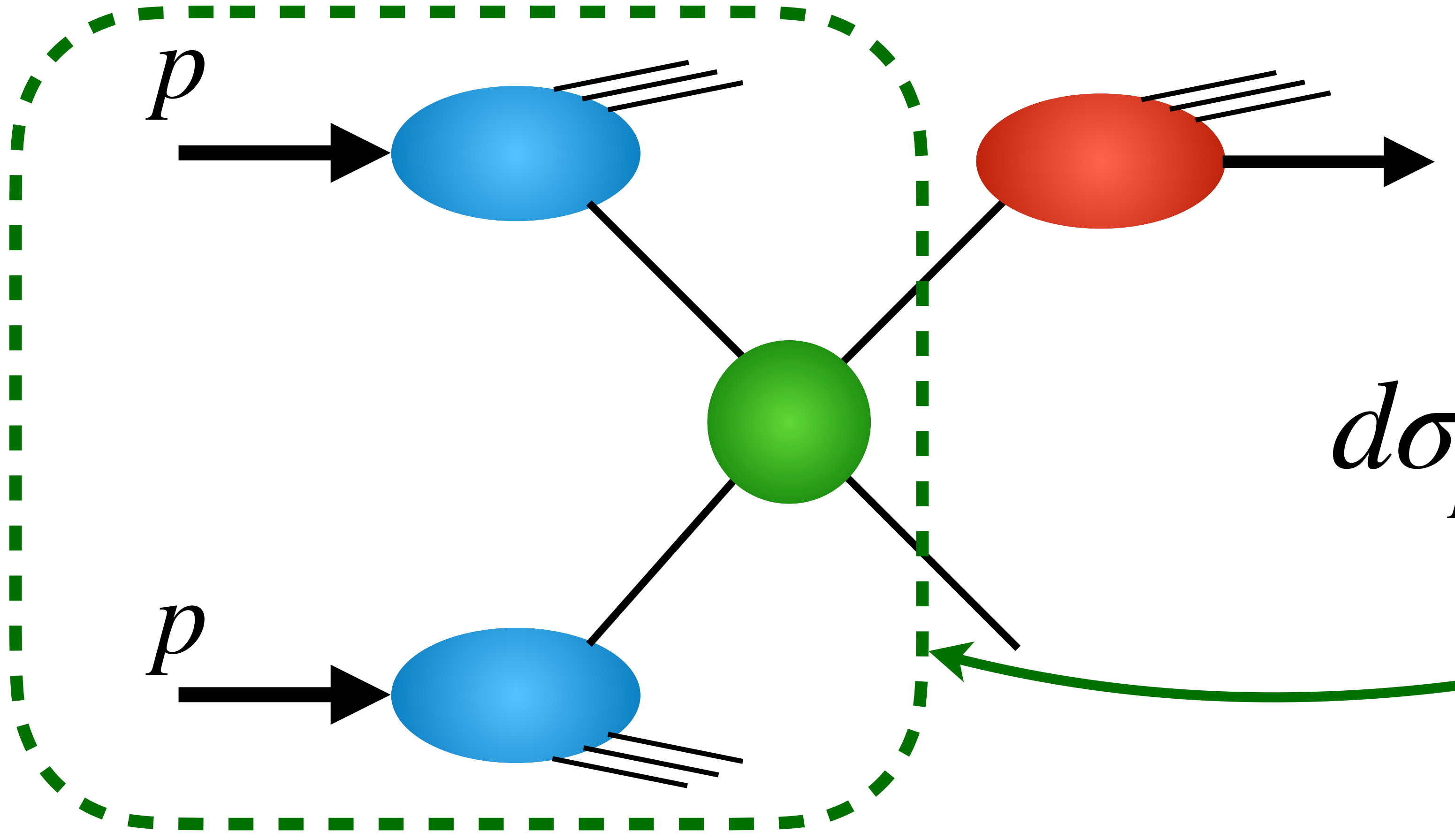
Butenschoen, Kniehl, PRD84, 051501 (2011).
Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).
Gong, Wan, Wang, Zhang, PRL110, 042002 (2013).
Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014).
…

pNRQCD

Brambilla, Chung, Vairo, Wang, PRD105, no.11, L111503 (2022).

See Xiangpeng Wang's talk: Dec 9, 11:10am

Heavy quarkonium production of high p_T

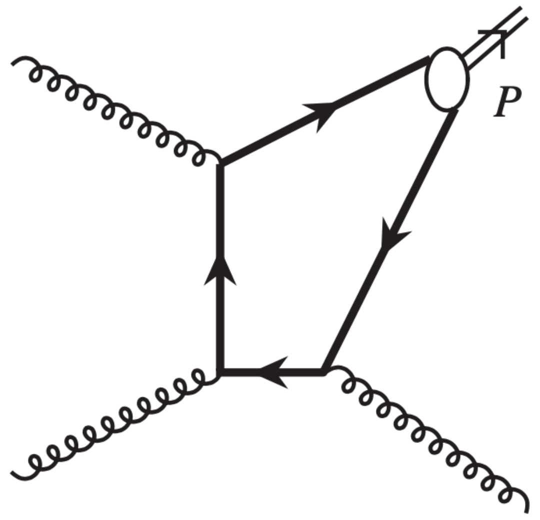


$p_T^2 \gg (2m)^2 \gg \Lambda_{\rm QCD}^2$: separation between $Q\bar{Q}$ production and the bound state formation.

$$d\sigma_{pp\to f+X} = f_{a/p} \otimes f_{b/p} \otimes d\hat{\sigma}_{ab\to f+X}$$

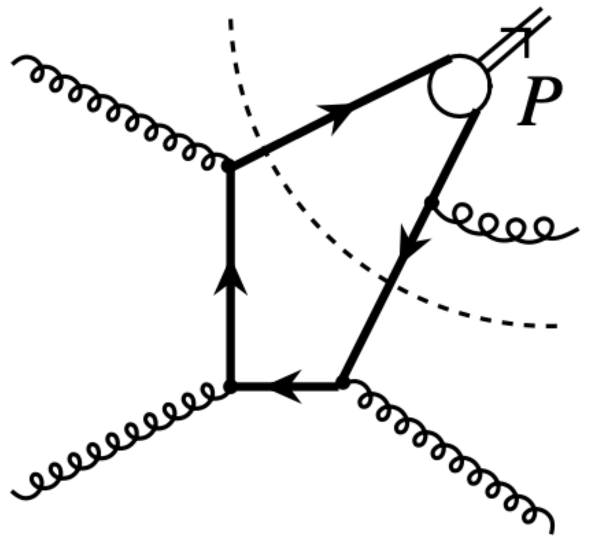
partonic x-section in collinear factorization

LO in CSM:



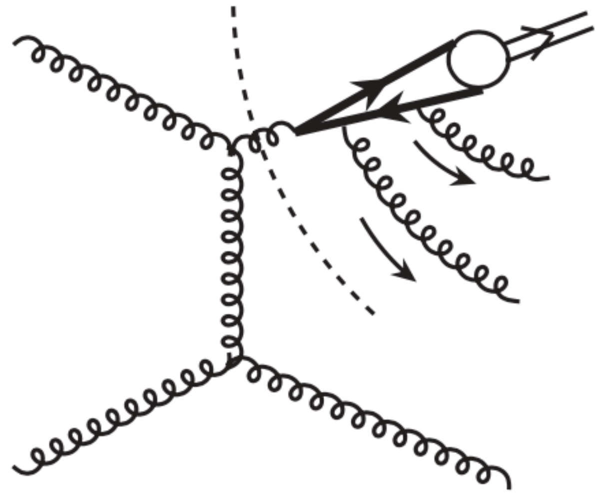
$$d\sigma(Q\bar{Q}[^3S_1^{[1]}]) \propto \frac{\alpha_s^3 m^4}{p_T^8}$$

Higher order corrections



$$d\sigma(Q\bar{Q}[^3S_1^{[1]}]) \propto \frac{\alpha_s^3 m^4}{p_T^8} \times \frac{\alpha_s p_T^2}{m^2} = \frac{\alpha_s^4 m^2}{p_T^6}$$

Gluon jet fragmentation



$$d\sigma \propto \frac{\alpha_s^2}{p_\perp^4} \times \alpha_s^3 \ln\left(\frac{p_T^2}{m^2}\right)$$

We may not obtain reliable predictions by considering only diagrams in α_s & v expansion.

We will discuss QCD factorization with fragmentation functions (FFs) in more detail.

QCD factorization + NRQCD

Nayak, Qiu, Sterman, PRD72 (2005) 114012
Kang, Qiu, Sterman, PRL108 (2012) 102002
Kang, Ma, Qiu, Sterman, PRD90 (2014) 3, 034006, PRD91 (2015) 1, 014030

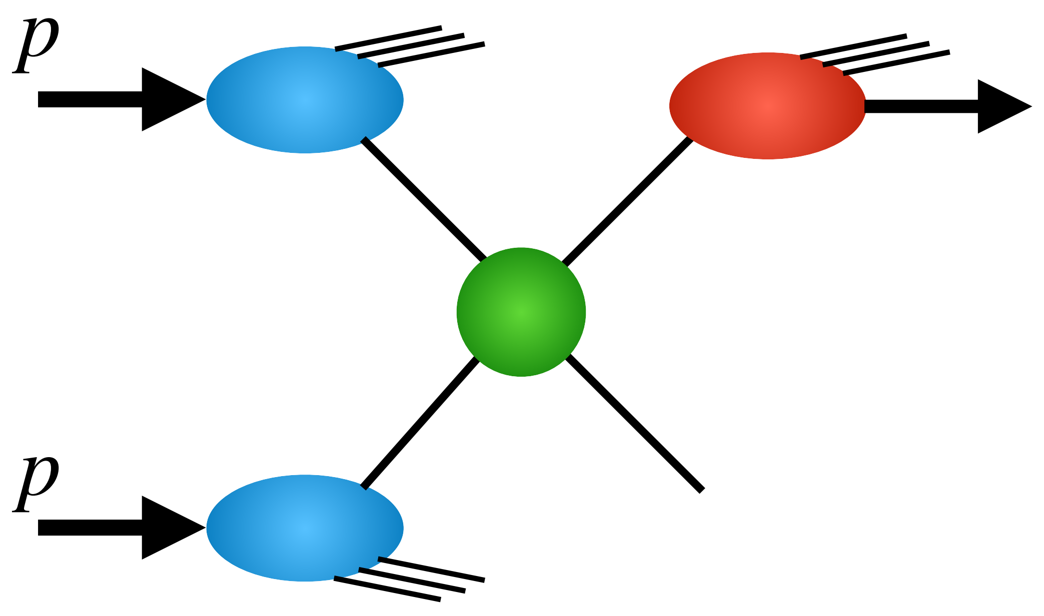
$$d\sigma_{A+B\to H+X}(m\neq 0) = d\sigma^{\rm Res}_{A+B\to H+X}(m=0) + d\sigma^{\rm NRQCD}_{A+B\to H+X}(m\neq 0) - d\sigma^{\rm Asym}_{A+B\to H+X}(m=0)$$

Leading power (LP) up to NLO

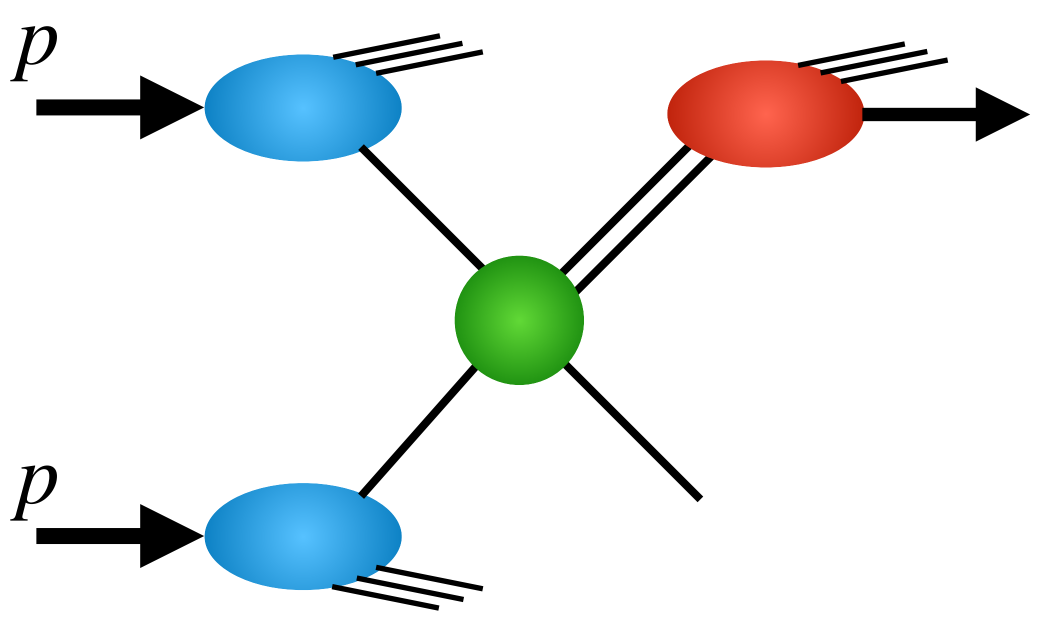
$$d\sigma^{\rm Res}_{A+B\to H+X}(\mu) = \sum_{f=q,\bar{q},g} C^{\rm LP}_{A+B\to[f]+X}(\mu)\otimes D_{[f]\to H}(\mu)$$

Subleading power (NLP) at LO

$$+\frac{1}{p_\perp^2}\left[\sum_n C^{\rm NLP}_{A+B\to[Q\bar{Q}(n)]+X}(\mu)\otimes \mathscr{D}_{[Q\bar{Q}(n)]\to H}(\mu)\right]$$



Twist-2 FFs D



Twist-4 FFs $\mathscr{D}$

One must expand amplitudes in $1/p_T$ first, then expand each contribution in α_s.

When $p_T \gg m$, $d\sigma^{\rm NRQCD}_{A+B\to H+X}$ cancels $d\sigma^{\rm Asym}_{A+B\to H+X}$. ➡ $d\sigma_{A+B\to H+X} \approx d\sigma^{\rm Res}_{A+B\to H+X}$

Resums $\ln(p_T^2/m^2)$

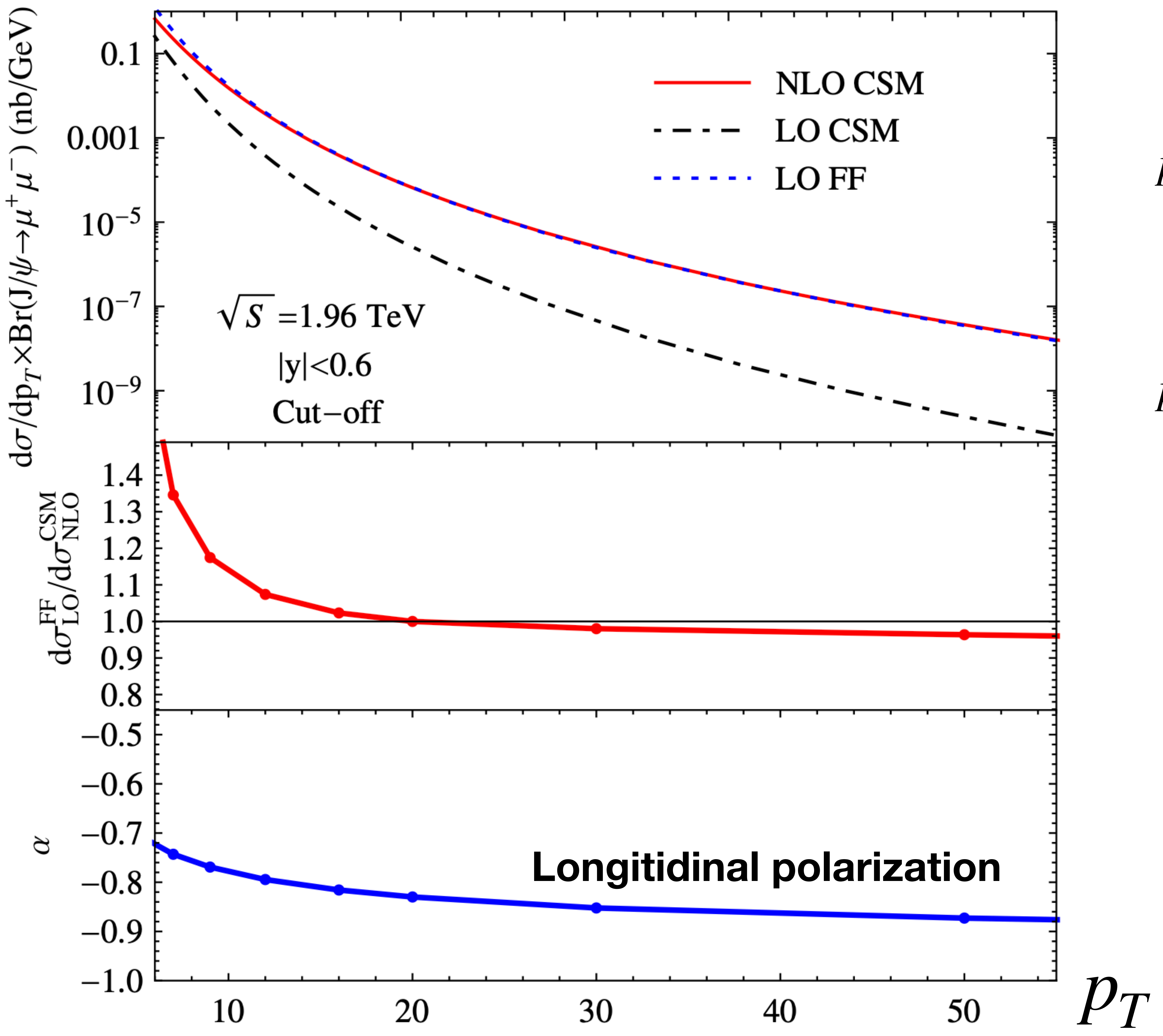
When $p_T \sim m$, $d\sigma^{\rm Res}_{A+B\to H+X}$ cancels $d\sigma^{\rm Asym}_{A+B\to H+X}$. ➡ $d\sigma_{A+B\to H+X} \approx d\sigma^{\rm NRQCD}_{A+B\to H+X}$

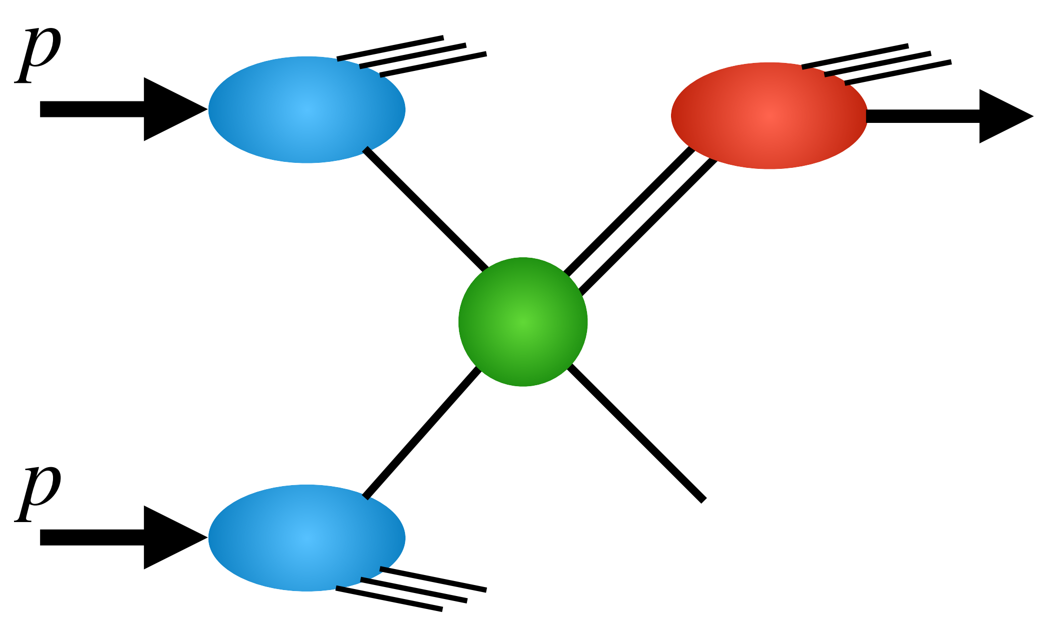
Fixed order

QCD factorization reproduces NRQCD

Kang, Ma, Qiu, Sterman, PRD91 (2015) 1, 014030

$$\alpha = \frac{d\sigma^T_{J/\psi} - d\sigma^L_{J/\psi}}{d\sigma^T_{J/\psi} + d\sigma^L_{J/\psi}}$$





Perturbative FFs without quantum evolution or resummation is used.

$$D_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(^3S_1^{[1]})]\to H}(z;m,\mu_0) = \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(^3S_1^{[1]})]}(z;m,\mu_0)\frac{\langle \mathcal{O}^H_{[Q\bar{Q}(^3S_1^{[1]})]}\rangle}{m}$$

Renormalization group improvement

Kang, Ma, Qiu, Sterman, PRD 90, 3, 034006 (2014)

❖ **Twist-2 evolution equation: DGLAP + quark pair power corrections:**

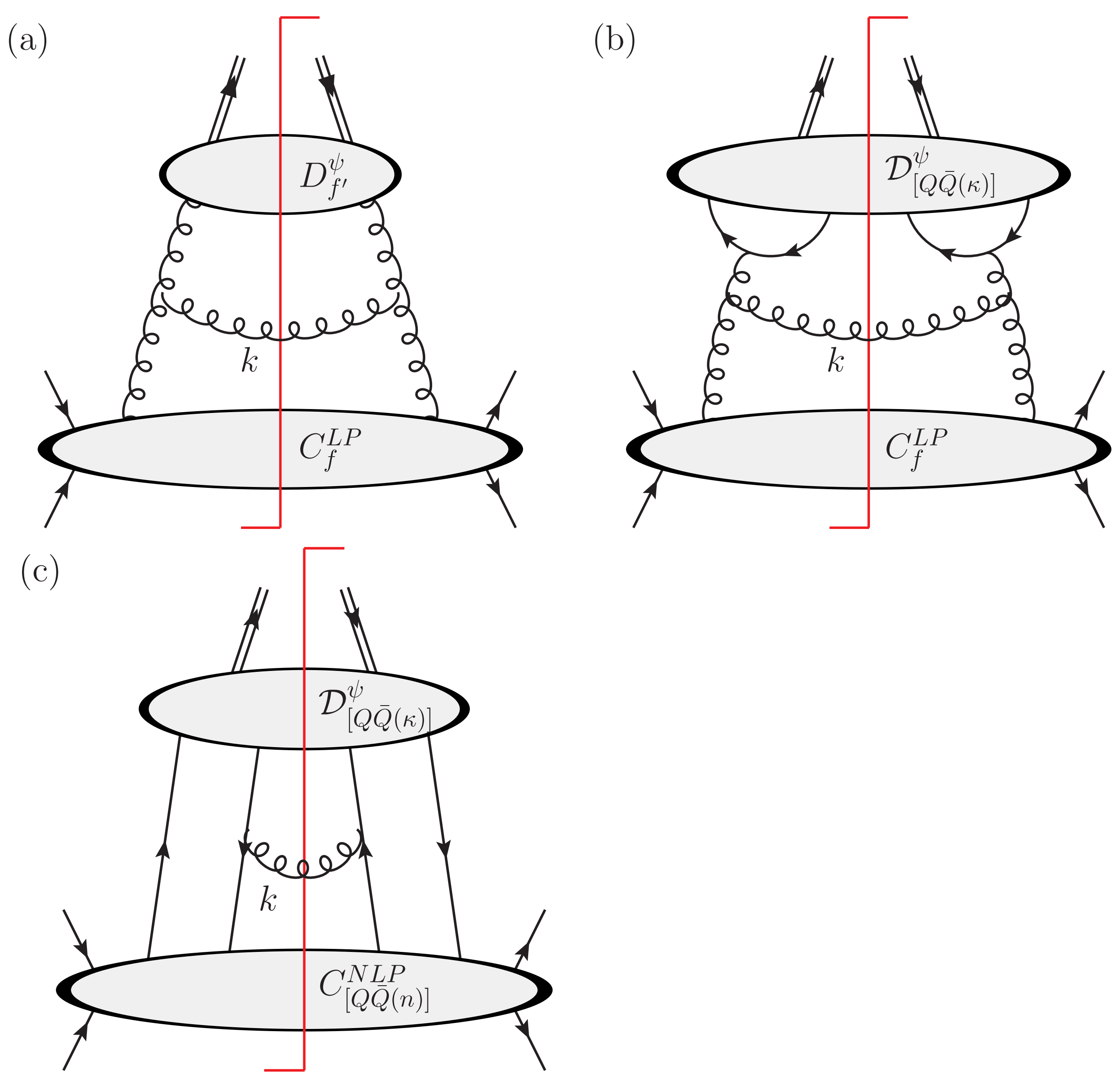
$$\frac{\partial D_{[f]\to H}}{\partial \ln \mu^2} = \gamma_{[f]\to[f']} \otimes D_{[f']\to H} + \frac{1}{\mu^2}\gamma_{[f]\to[Q\bar{Q}(\kappa)]} \otimes \mathscr{D}_{[Q\bar{Q}(\kappa)]\to H}$$

Driving force term

❖ **Twist-4 "DGLAP like" evolution equation:**

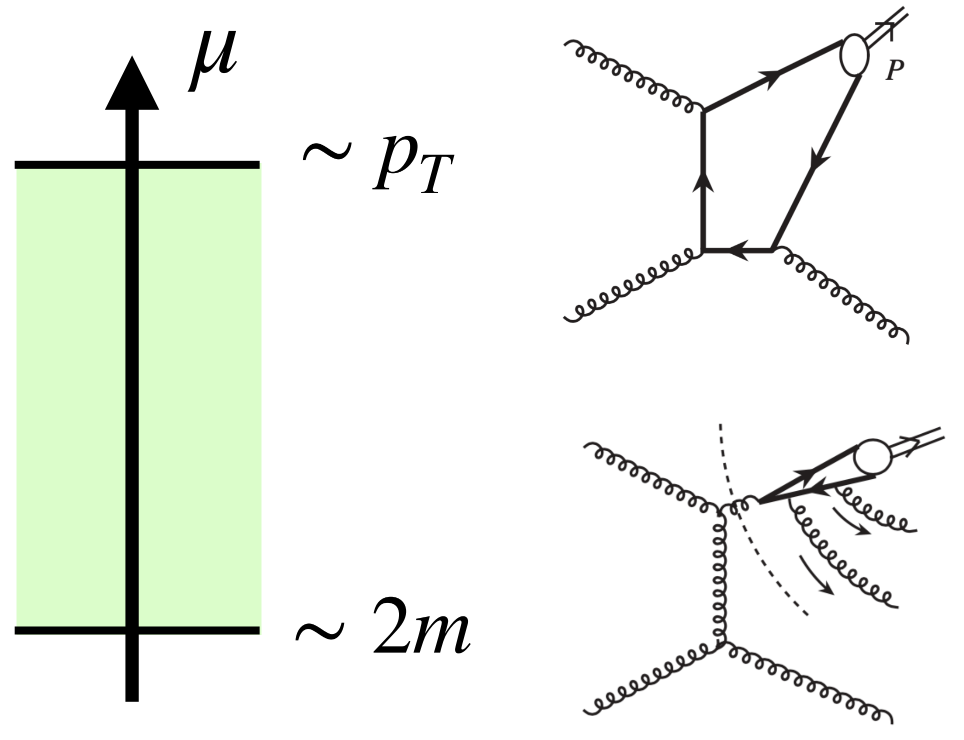
$$\frac{\partial \mathscr{D}_{[Q\bar{Q}(n)]\to H}}{\partial \ln \mu^2} = \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]} \otimes \mathscr{D}_{[Q\bar{Q}(\kappa)]\to H}$$

$f, f' = q, Q, g \qquad \kappa, n = v^{[1,8]}, a^{[1,8]}, t^{[1,8]}$



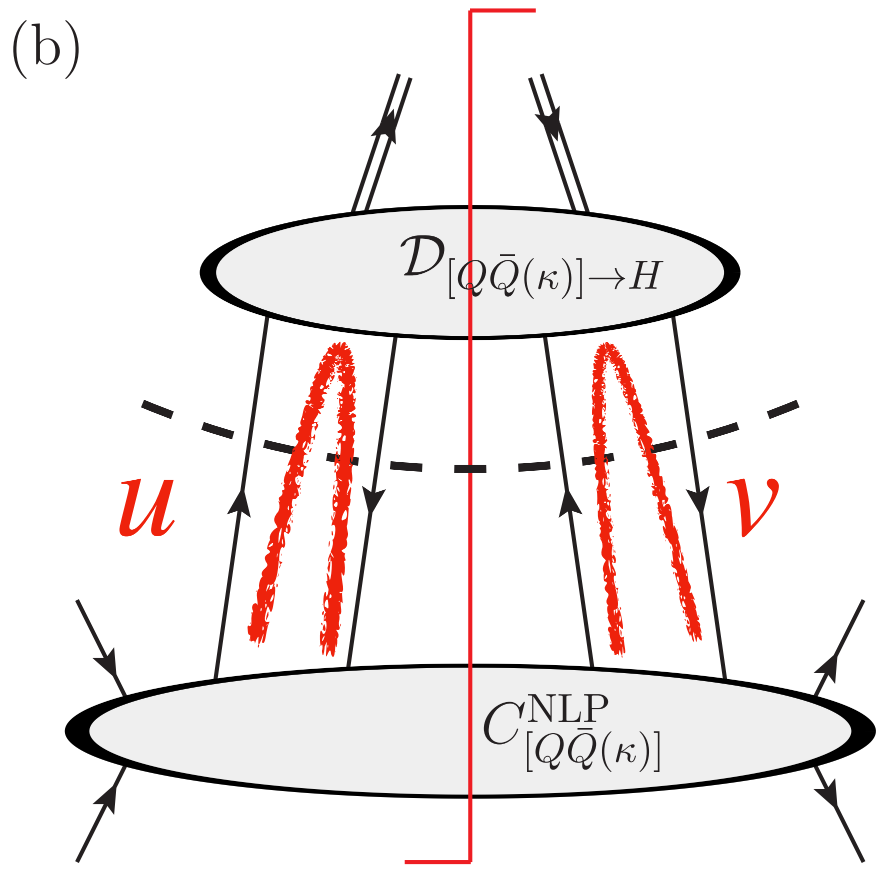
The RG improved factorized cross section covers all events in which the heavy quark pair can be produced:

1. at the short-distance ($\mu \sim p_T$): **NLP**
2. from a single parton at a lower scale ($\mu \sim 2m$): **LP**
3. in-between: **Quark pair power corrections**



Momentum flow between Q and $\bar{Q}$

Lee, Qiu, Sterman, **KW**, arXiv:2211.12648 [hep-ph]



$$\mathscr{D}_{[Q\bar{Q}(\kappa)]\to H}(z, u, v)$$

amplitude : $p_Q = u p_{Q\bar{Q}}$

c . c . : $p_Q = v p_{Q\bar{Q}}$

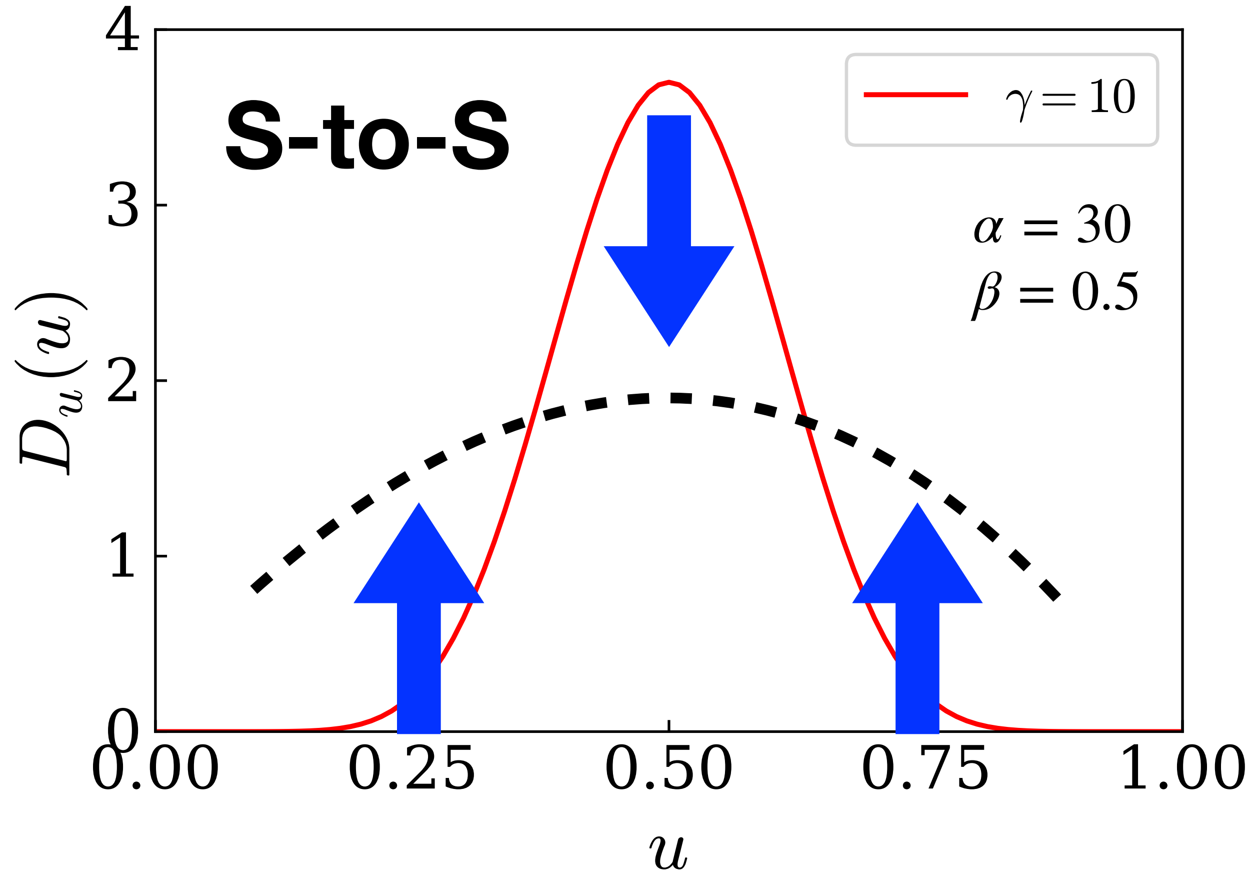
with $z\, p^+_{Q\bar{Q}} = p^+_\psi$

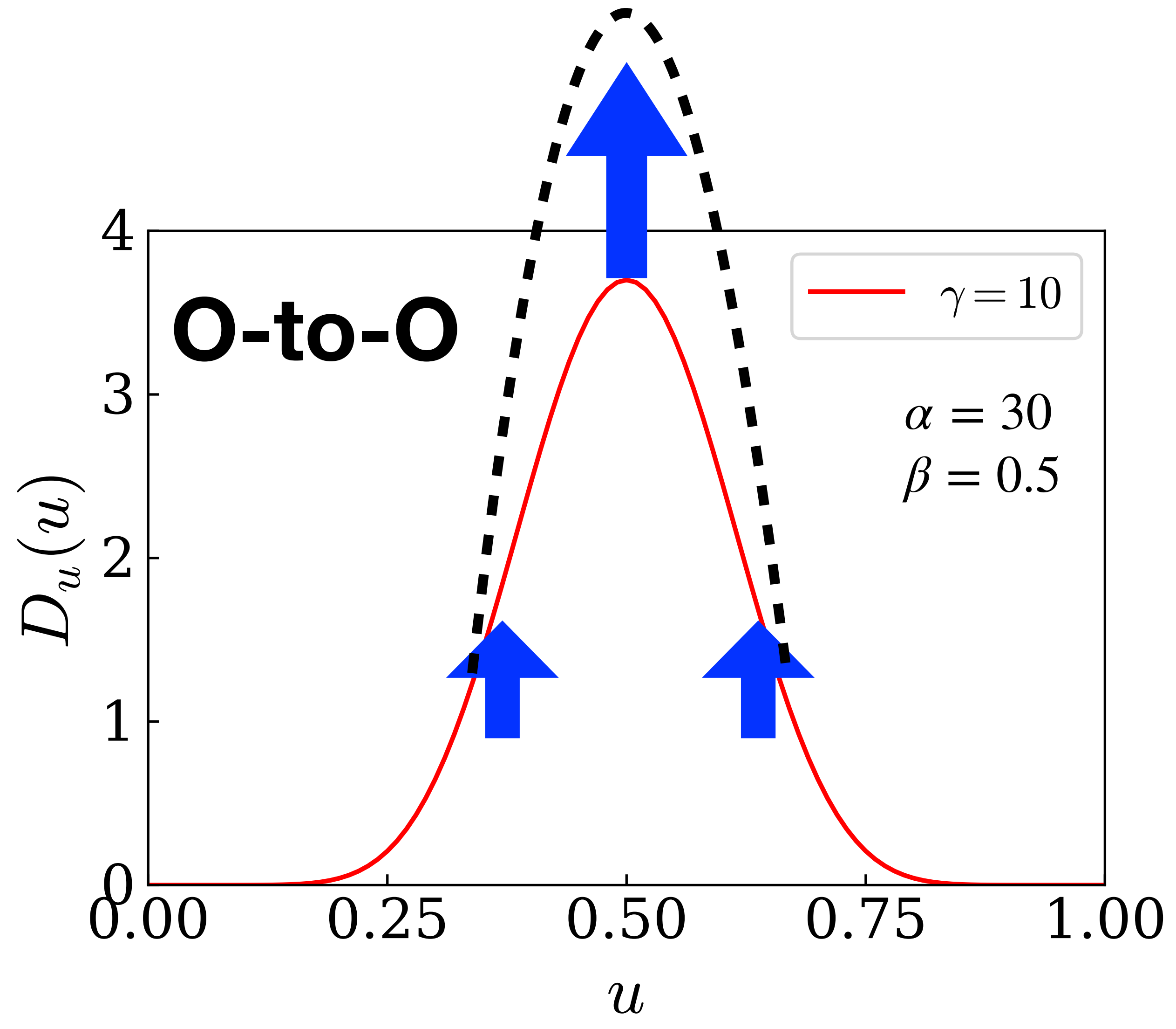
Test function

$$D(z,u,v) \to D_z(z) D_u(u) D_v(v)$$

$$D_z(z) = \frac{z^\alpha (1-z)^\beta}{B[1+\alpha, 1+\beta]}$$

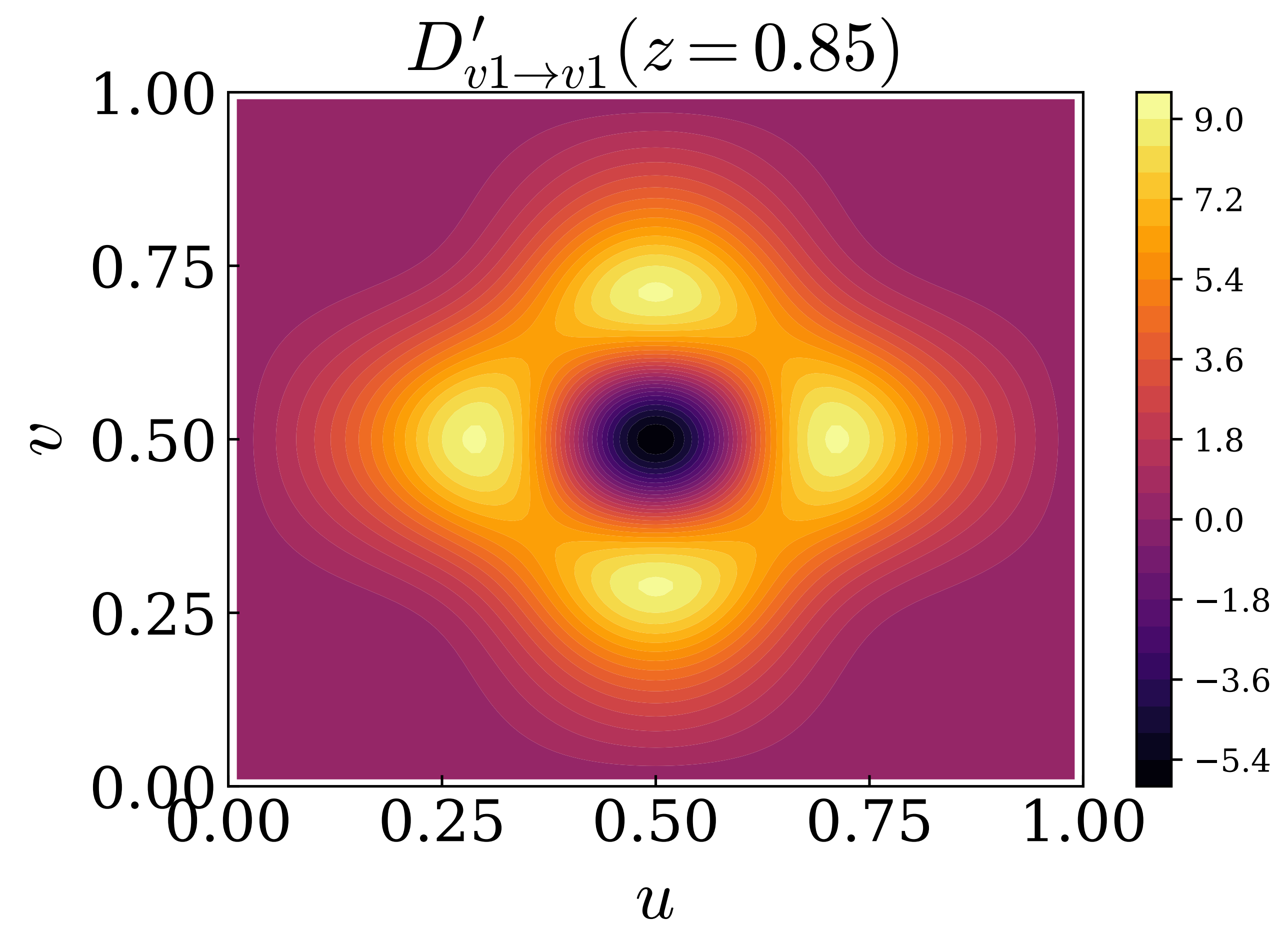
$$D_u(u) = \frac{u^\gamma (1-u)^\gamma}{B[1+\gamma, 1+\gamma]}$$



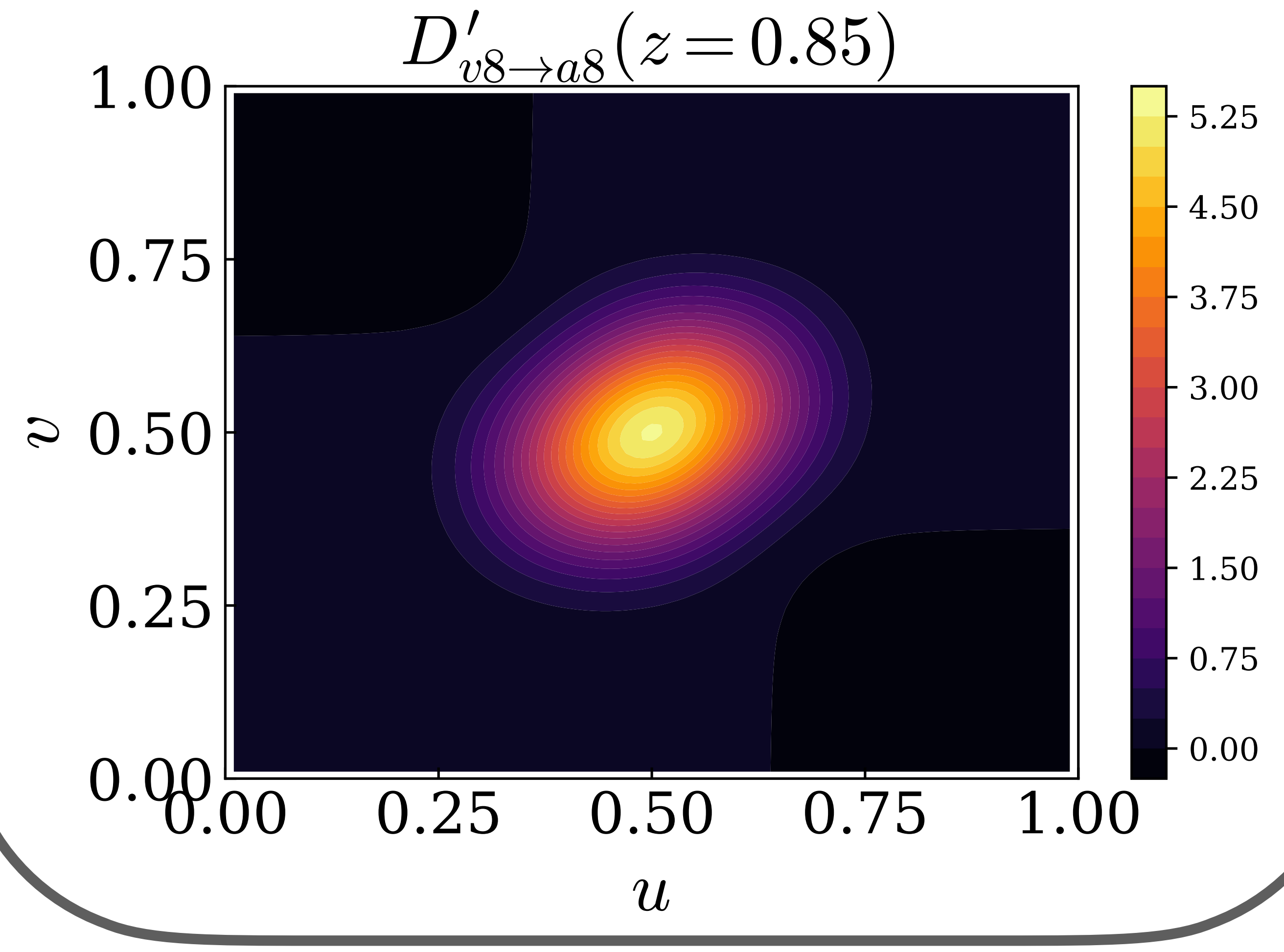


$$D'_{\kappa\to n}(z,u,v) \equiv \frac{2\pi}{\alpha_s} \frac{dD_{\kappa\to n}(z,u,v)}{d\ln\mu^2}$$

Diagonal singlet channel: S-to-S

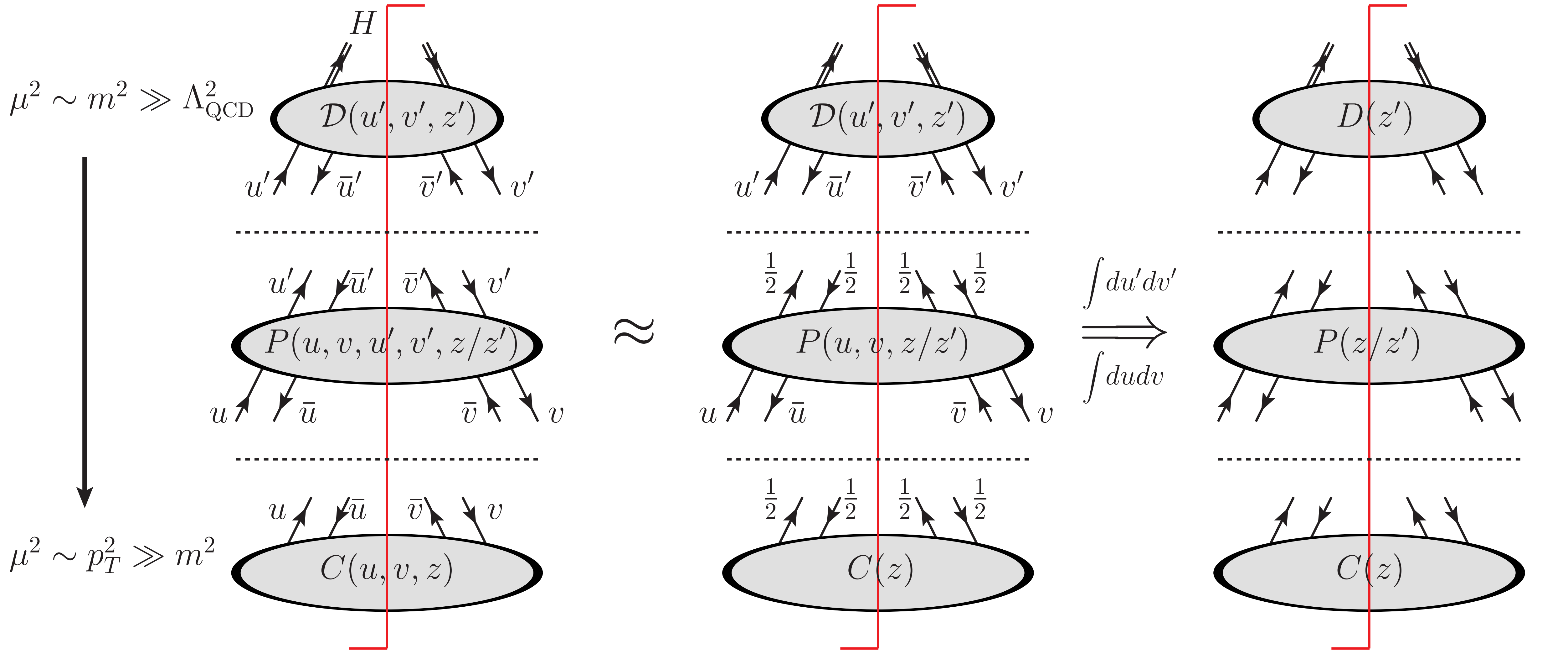


Diagonal octet channel: O-to-O



- **S-to-S**: $\mathscr{D}_{[Q\bar{Q}](k)}$ get **broader** in u, v-space after evolution.
- **O-to-O**: $\mathscr{D}_{[Q\bar{Q}](k)}$ become **narrower** with a large peak around $u = v = \dfrac{1}{2}$.
- Off-diagonal channels (**O-to-S**): similar to **O-to-O**.

Modified evolution equations at Twist-4



(3) **Heavy quark pair fragments into quarkonium**

(2) **Heavy quark pair's quantum state changes**

(1) **Heavy quark pair production from the hard scattering**

❖ The produced heavy quark pair is dominated by its on-shell state at high p_T.

❖ We may expand the SDCs and evolution kernels on lower virtuality sides at each evolution step around $u = v = 1/2$: a reasonable approximation.

$$\frac{\partial D_{[f]\to H}}{\partial \ln \mu^2} = \gamma_{[f]\to[f']} \otimes D_{[f']\to H} + \frac{1}{\mu^2}\tilde{\gamma}_{[f]\to[Q\bar{Q}(\kappa)]} \otimes D_{[Q\bar{Q}(\kappa)]\to H}$$

$$\frac{\partial D_{[Q\bar{Q}(n)]\to H}}{\partial \ln \mu^2} = \tilde{\Gamma}_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]} \otimes D_{[Q\bar{Q}]\to H}$$

Modified FFs

$$D_{[Q\bar{Q}]\to H}(z) \equiv \int dudv\, \mathscr{D}_{[Q\bar{Q}]\to H}(u, v, z)$$

Input FFs and NRQCD

Ma, Qiu, Zhang, PRD89, no.9, 094029, ibid. 094030 (2014)
Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)

The NRQCD factorization is capable of giving a good conjecture for input FFs!

$$D_{f\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \alpha_s \hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \alpha_s^2 \hat{d}^{(2)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^3) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+3}}$$

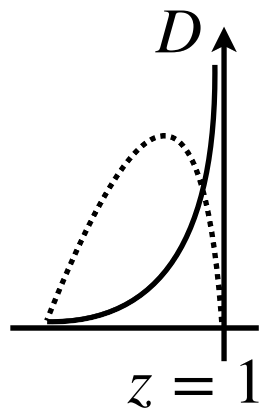
LDMEs

$$D_{[Q\bar{Q}(\kappa)]\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \alpha_s \hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+1}}$$

$\mu_0 = \mathcal{O}(2m)$: input scale, $\mu_\Lambda = \mathcal{O}(m)$: NRQCD factorization scale $\quad \kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = {}^{2S+1}L_J^{[c]}$

Caveat: Perturbatively calculated SDCs $\hat{d}(z)$ for the input FFs (**not SDCs for hard parts**) are meaningful only after the convolution. $\hat{d}(z) = \mathcal{O}(1)$ due to δ-function at LO, so the z-distribution of the input FFs is unreliable.

Once we allow gluon radiation, the probability for $D(z=1) \to 0$.



$$D_f(z) = C_f(\alpha_s) \frac{z^{\alpha_f}(1-z)^{\beta_f}}{B[1+\alpha_f, 1+\beta_f]}$$

first moments including LDMEs (abs value)

$(\alpha_f \gg 1,\ 1 > \beta_f > 0)$

→ to be tuned, imitating δ-function at LO

Similar problems are seen in Jet functions.

T. Kaufmann, A. Mukherjee and W. Vogelsang, PRD92, no.5, 054015 (2015)
Z.B. Kang, F. Ringer and I. Vitev, JHEP10, 125 (2016)

The QCD evolution of the Twist-2 FFs

Lee, Qiu, Sterman, **KW**, in preparation.

$\alpha = 30,\ \beta = 0.5$ for all channels

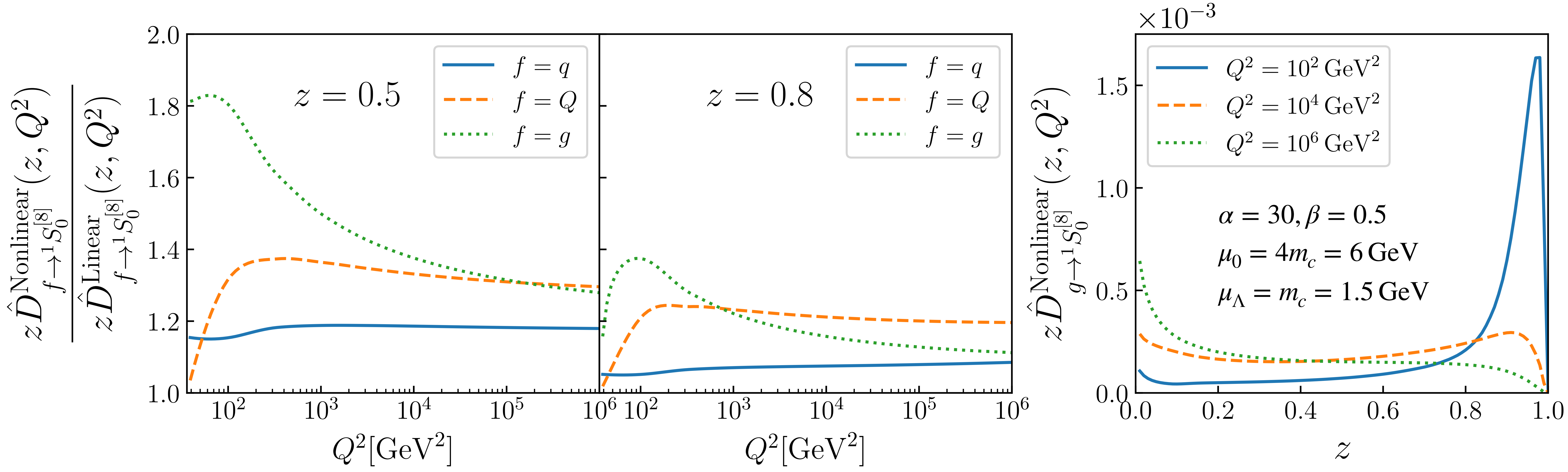
The QCD evolution of the Twist-4 FFs

Lee, Qiu, Sterman, **KW**, in preparation.

$\alpha = 30,\ \beta = 0.5$ for all channels

Quark pair corrections to Twist-2 FFs

Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)

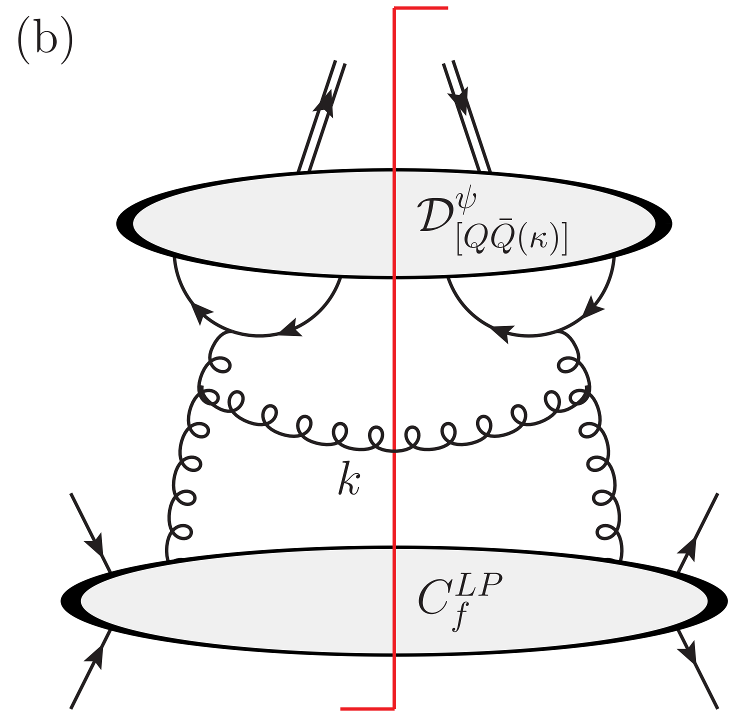


The quark pair corrections to DGLAP evolution remain significant even at high $\mu^2 \sim p_T^2$.

$$\frac{\partial D_{f\to H}}{\partial \ln \mu^2} = \gamma_{f\to f'} \otimes D_{f'\to H} + \frac{1}{\mu^2}\tilde{\gamma}_{f\to[Q\bar{Q}(\kappa)]} \otimes D_{[Q\bar{Q}(\kappa)]\to H}$$

$\mu^2 \to \infty$

$$\frac{\partial D^{\text{Inhomogeneous}}_{f\to H}}{\partial \ln \mu^2} \sim \frac{\partial D^{\text{Homogeneous}}_{f\to H}}{\partial \ln \mu^2}$$

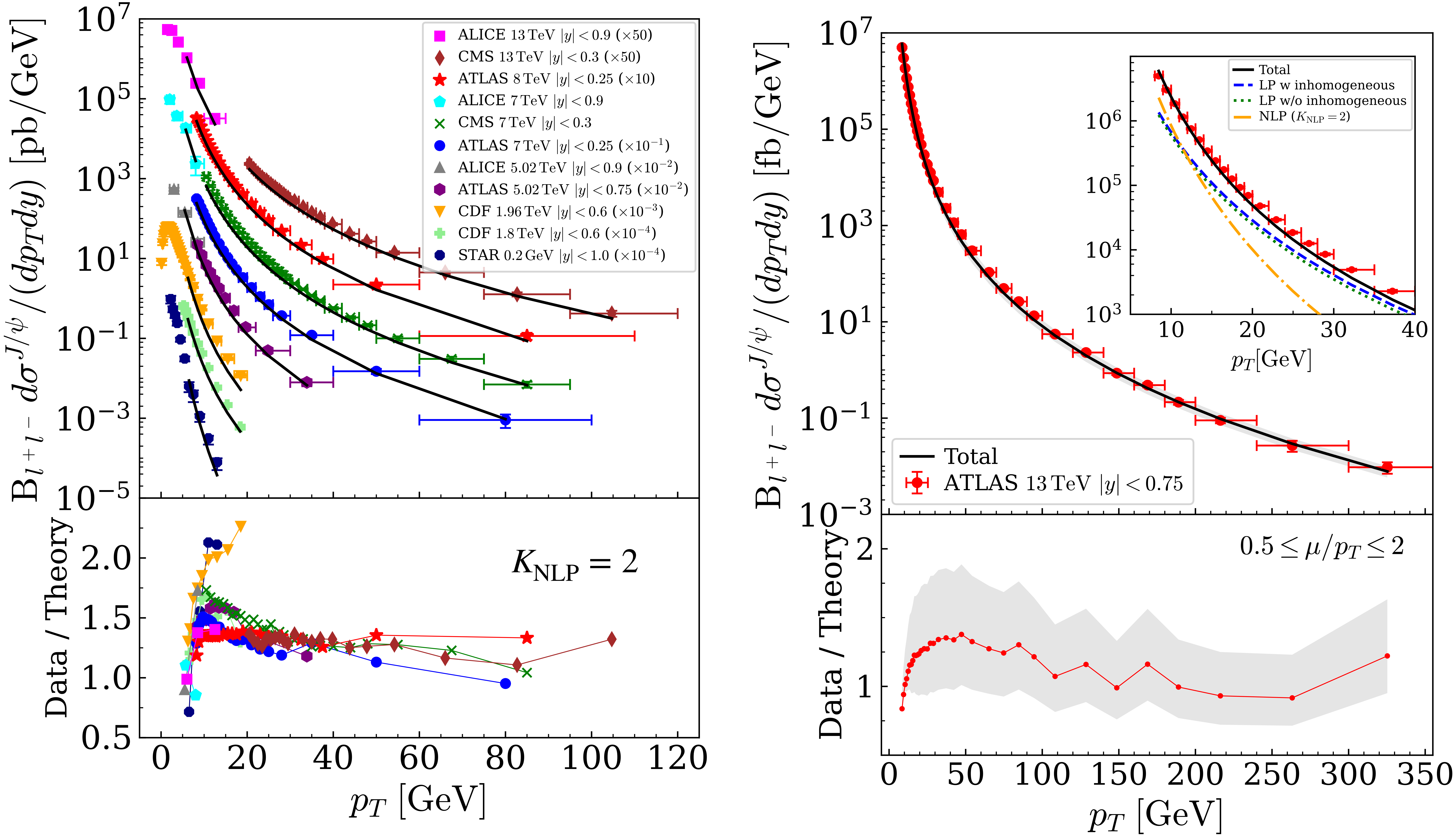


The power corrections effect at low μ^2 does not go away fast: **analogous to nonlinear gluon recombination effects to gluon PDF at small-x and large μ^2.**

Mueller and Qiu, NPB268, 427 (1986)
Qiu, NPB291, 746 (1987)
Eskola, Honkanen, Kolhinen, Qiu and Salgado, NPB660, 211 (2003)

Phenomenology: prompt J/ψ production

Lee, Qiu, Sterman, **KW**, arXiv:2211.12648
Lee, Qiu, Sterman, **KW**, in preparation.

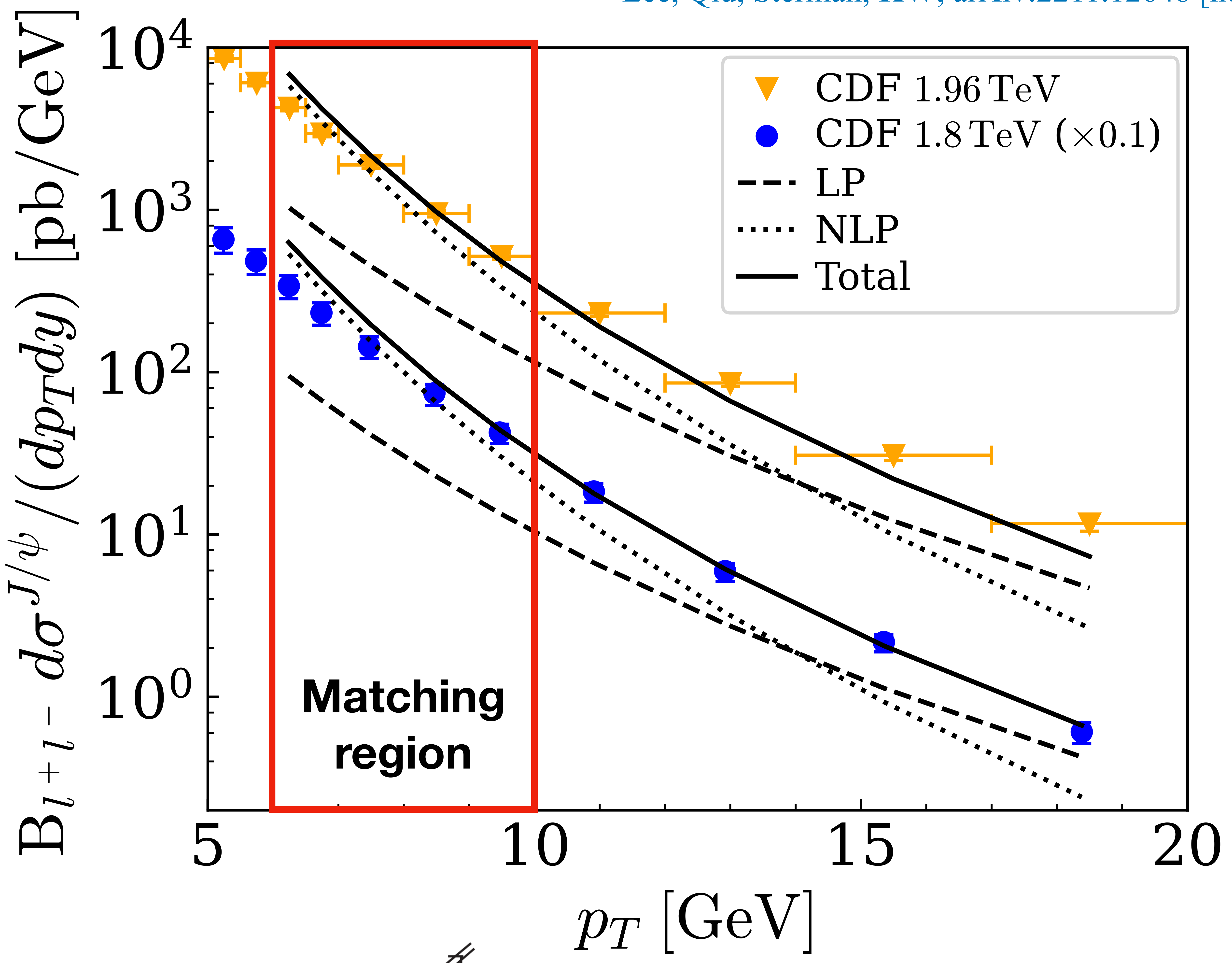


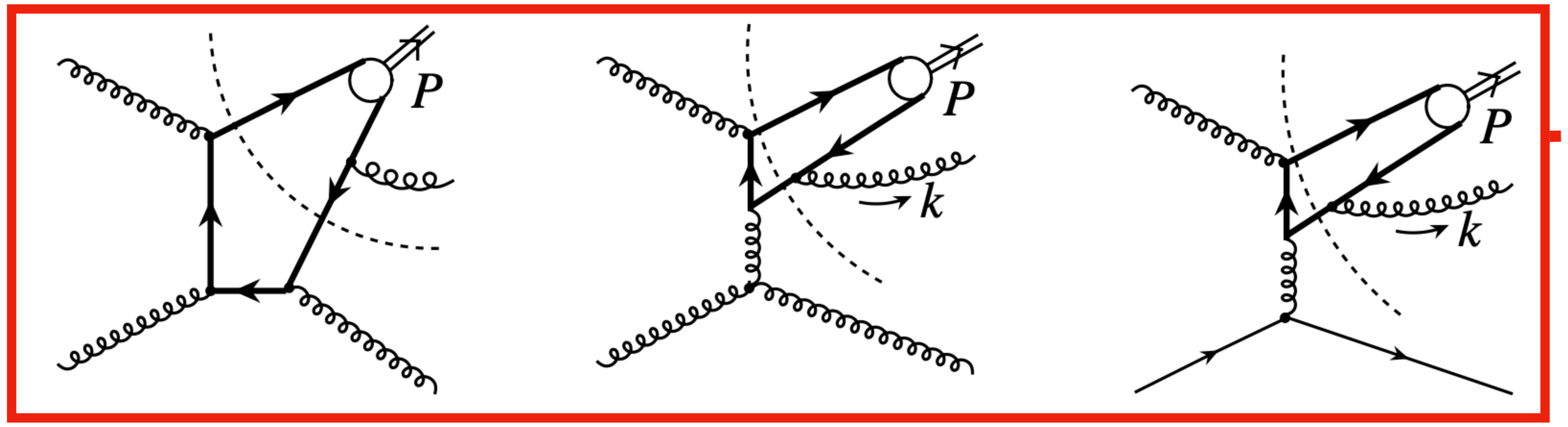
LDMEs are fitted to high p_T data (CMS 7,13TeV): ${}^1S_0^{[8]}$ is predominant.

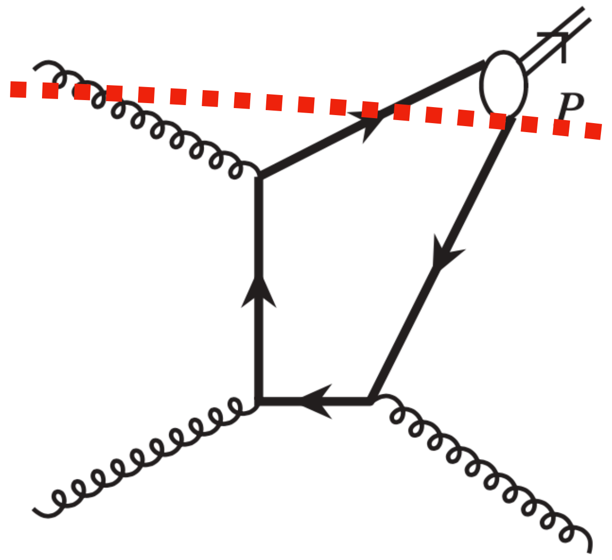
NLP corrections and matching to NRQCD

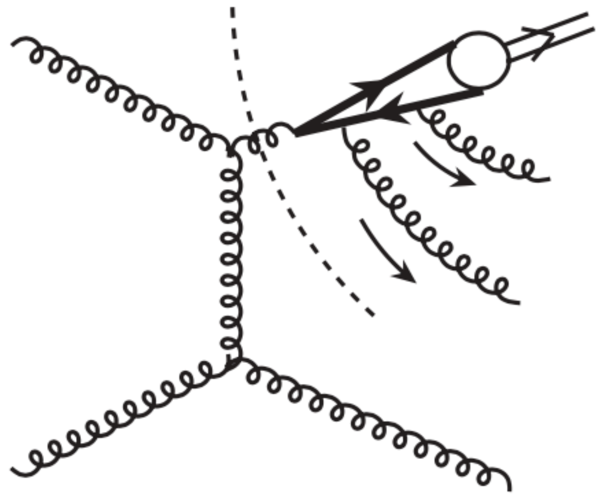
Lee, Qiu, Sterman, **KW**, arXiv:2211.12648 [hep-ph]

1. $\ln(p_T^2/m^2)$-type logarithmically enhanced contributions start to dominate when $p_T \sim 15\,\mathrm{GeV}$, where the LP is significant.
2. The NLP contribution dominates when $p_T \lesssim 10\,\mathrm{GeV} = \mathcal{O}(2m_c)$. The p_T shape is different from the LP contribution.
3. We need to match the asymptotic term to the fixed order NRQCD contribution when $p_T \to 2m_c$.

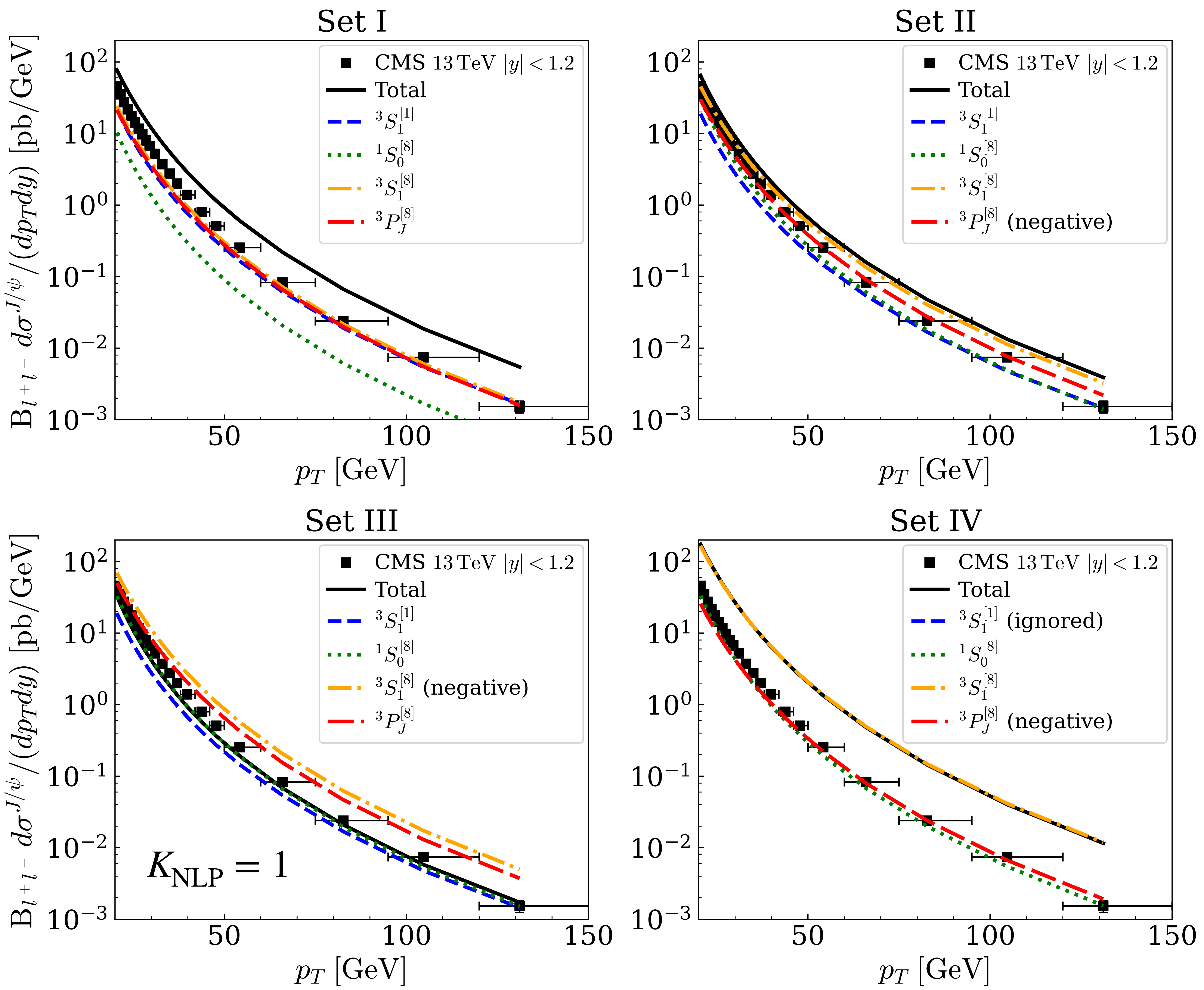








Exploiting LDMEs on the market



$\alpha = 30,\ \beta = 0.5$ for all channels

J/ψ	$\langle \mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle \mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}(^3P_0^{[8]})\rangle$ 10^{-2} GeV^5
Set I (Butenschoen *et al.*)	1.32	3.04	0.16	−0.91
Set II (Chao *et al.*)	1.16	8.9	0.30	1.26
Set III (Gong *et al.*)	1.16	9.7	−0.46	−2.14
Set IV (Bodwin *et al.*)	-	9.9	1.1	1.1

$$D_{f\to H}(\mu_0) = \sum_n \hat{D}_{f\to[Q\bar{Q}(n)]}(\mu_0)\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}\rangle$$

- We **do not** restrict the SDCs of the input FFs to positive:
- The SDCs for P-wave are **negative**.
- Uncertainty is big, but the theory calculations reproduce the order of magnitude.

Negative x-section problems are addressed using soft gluon factorization approach.

A. P. Chen, Y. Q. Ma and C. Meng, PRD108, no.1, 014003 (2023)

Summary

❖ We have reviewed hadronic quarkonium production of high p_T in the QCD factorization.

❖ The LP contributions for single inclusive J/ψ production are significant at $p_T \gtrsim 15\,\mathrm{GeV}$, while the power corrections dominate at $p_T \lesssim \mathcal{O}(2m_c)$.

❖ The p_T shape of the NLP contribution is different from the LP contribution.

❖ The QCD factorization formalism should enable a new global data analysis. The input FFs with LDMEs can be much improved, and theoretical uncertainty should be reduced.

❖ Implementing the evolution of FFs for polarized quarkonium is in progress.

❖ The asymptotic term should be matched to the fixed order contributions when $p_T \to 2m_c$.

Thank you!

Backup

SDCs of input FFs look like…

1. $\delta(1-z)$ at LO in α_s expansion
2. $f(z)\ln(1-z)$ with $f(z)$ being a regular function
3. $\frac{f(z)}{[1-z]_+}$, $f(z)\left[\frac{\ln(1-z)}{1-z}\right]_+$ due to the perturbative cancelation of IR divergences

- If "SDCs $\gg$1", it indicates that "uncalculated high order corrections" are very important!
- In our current analysis, we use analytic results if those vanish as $z \to 1$, otherwise, singular or negative SDCs are cast into:

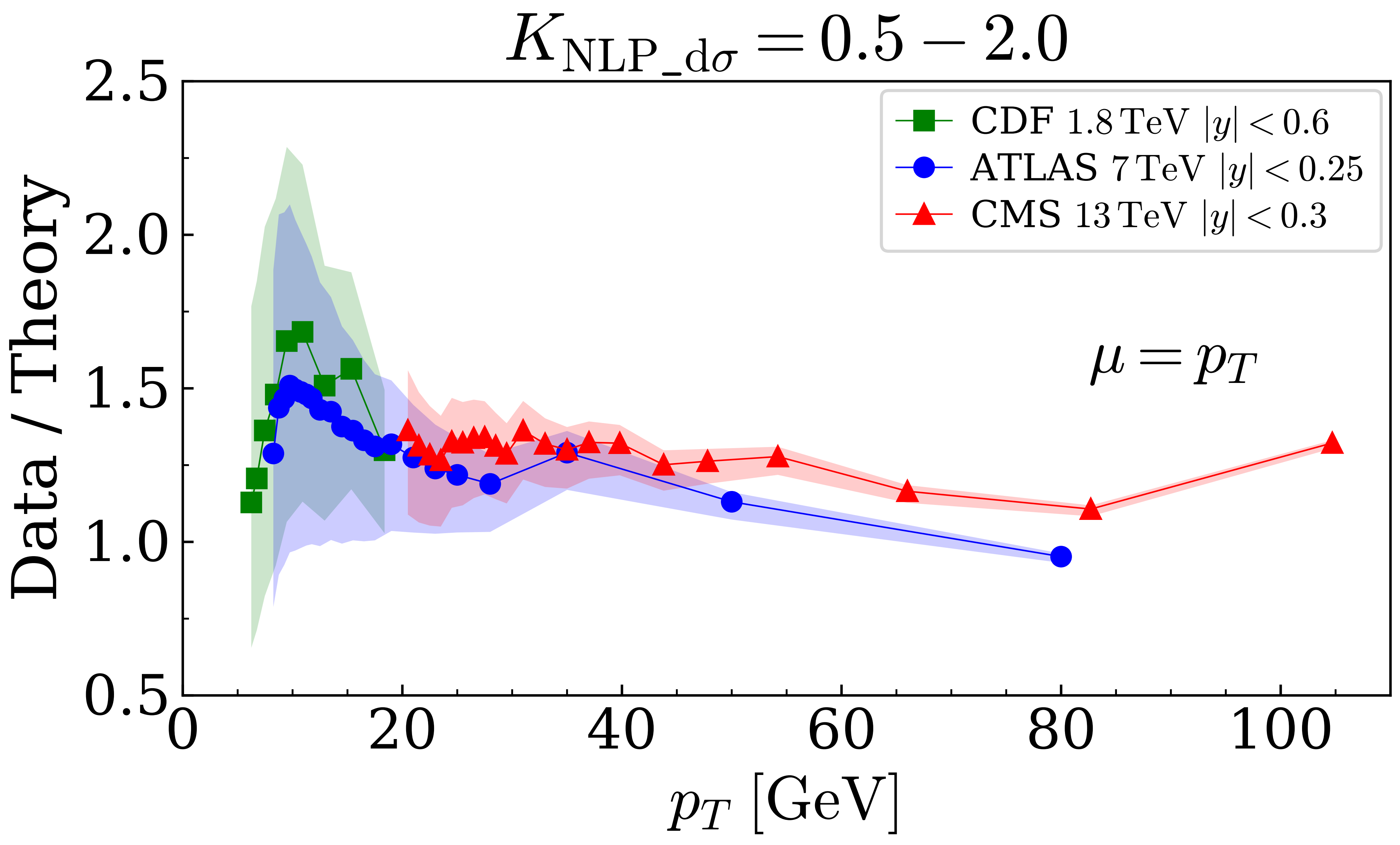
$$D_f(z) = C_f(\alpha_s)\frac{z^{\alpha_f}(1-z)^{\beta_f}}{B[1+\alpha_f, 1+\beta_f]} \qquad (\alpha_f \gg 1,\ 1 > \beta_f > 0)$$

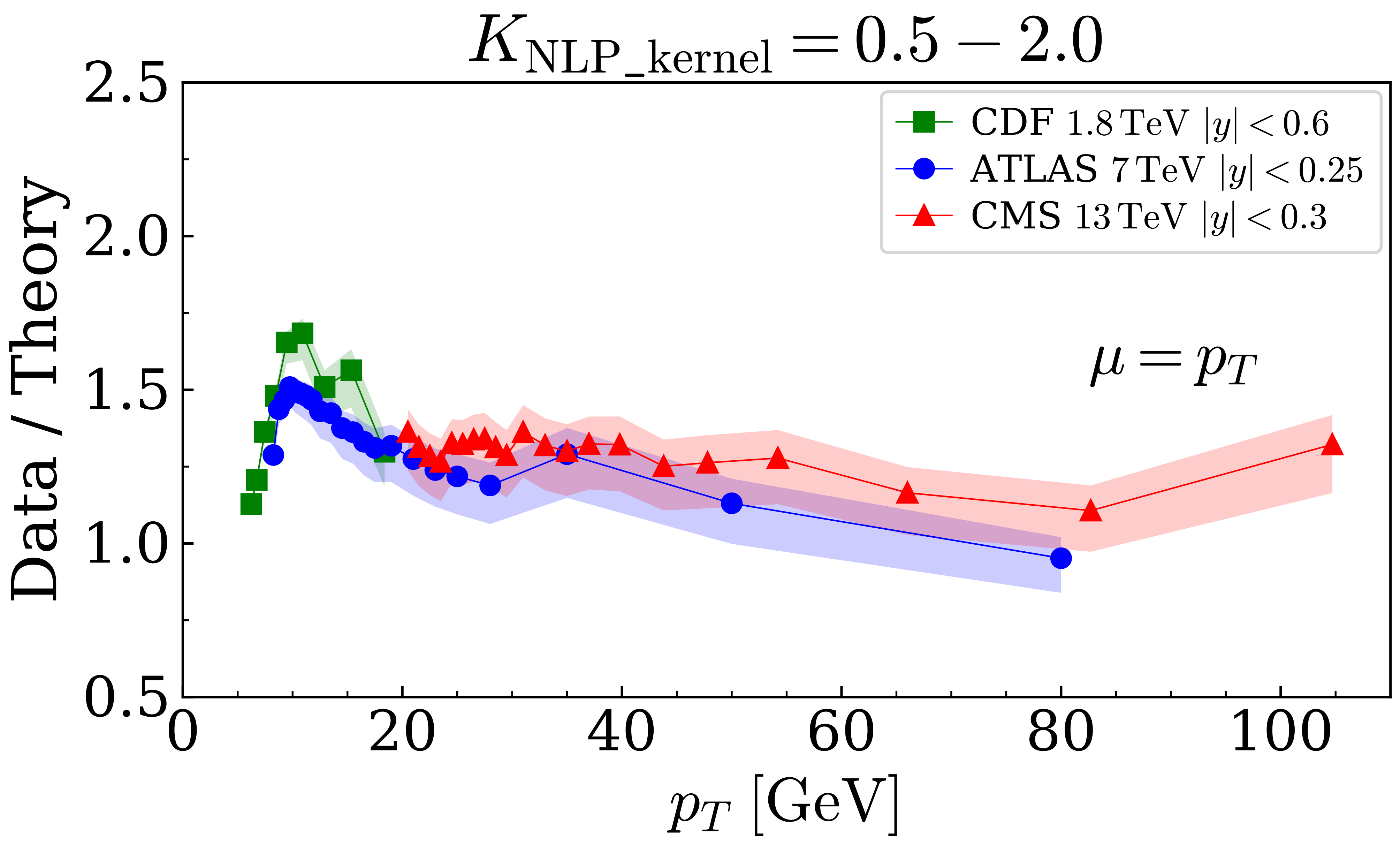
first moments

→ to be tuned, imitating δ-function at LO.

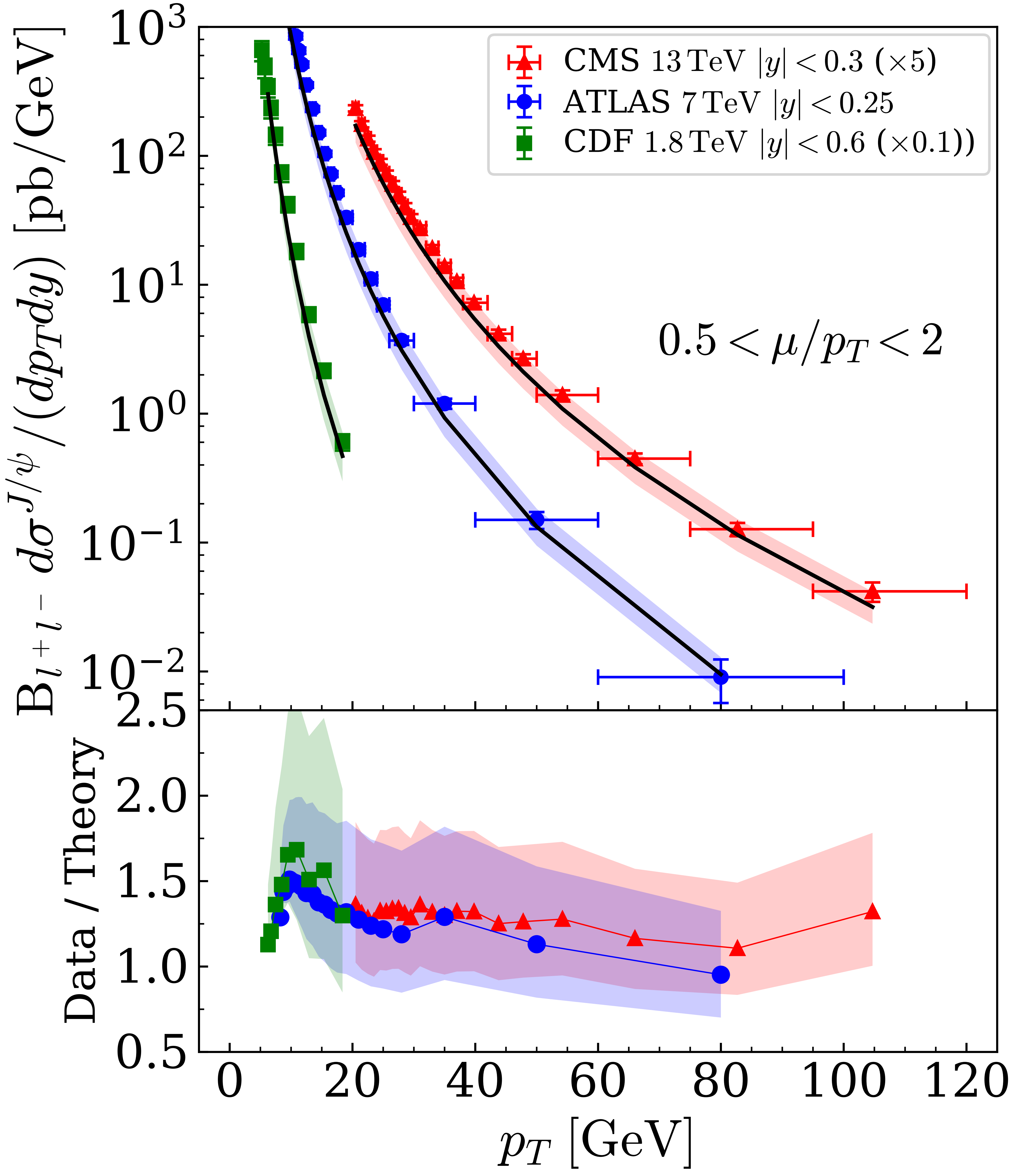
- Large α, small β mimics δ-function, which should be a significant piece.
- The first moments of plus-distributions are negative.

Uncertainty of theoretical calculations





Choice of factorization/renormalization scale



The use of fitted LDMEs at Tevatron