The 9th International Symposium on Heavy Flavor Production in Hadron and Nuclear Collisions

Heavy flavor Hadronization in relativistic heavy ion collisions

Jiaxing Zhao (HFHF)

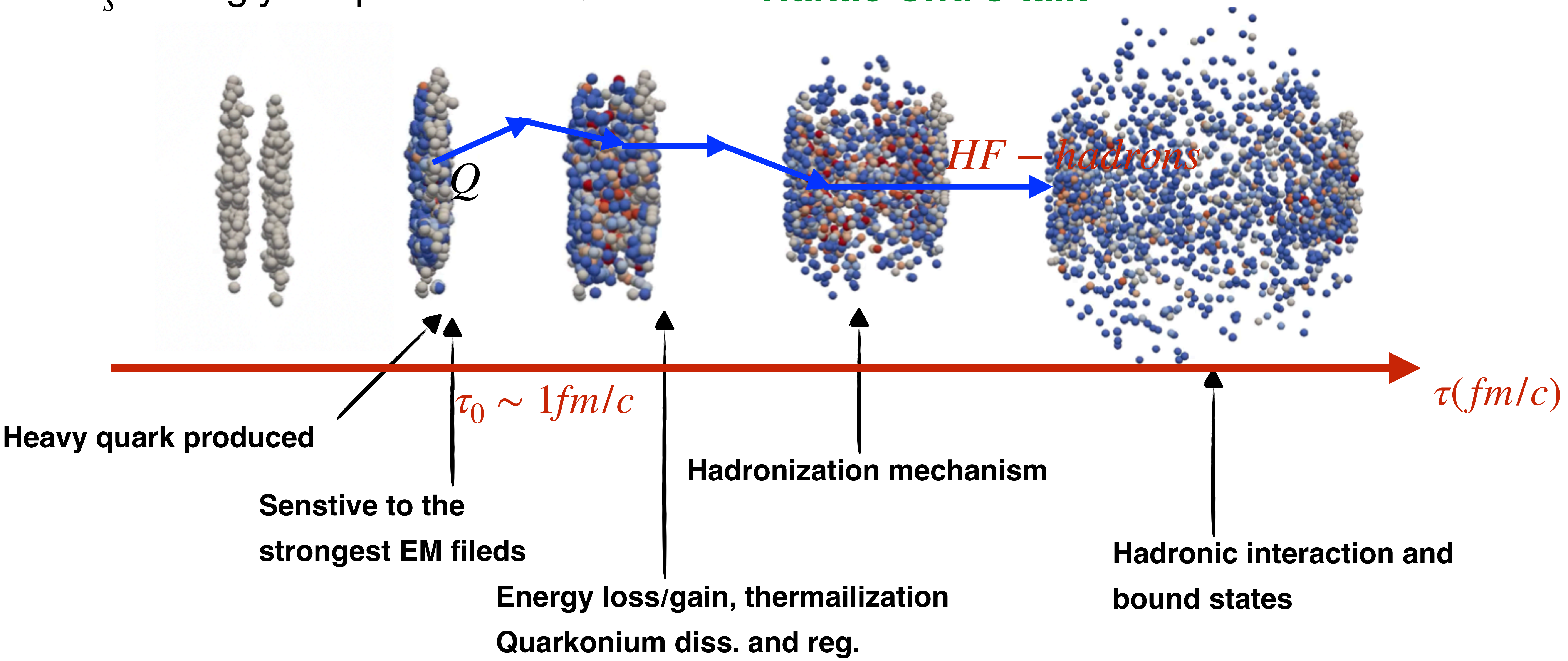
08/12/2024

Heavy flavor probes

$m_c \sim 1.5\mathrm{GeV}, \mathrm{m_b} \sim 4.7\mathrm{GeV}$

- ✦ $\tau_c \sim 1/m_c, \tau_b \sim 1/m_b < \tau_0 \sim 1fm/c$, "see" full system evolution.
- ✦ $\tau_c, \tau_b < \tau_B \approx R/\gamma \sim 0.1fm/c$, feel strong electromagnetic fields in HICs.
- ✦ $m_c, m_b \gg \Lambda_{QCD}$, produced by hard scattering, pQCD.
- ✦ $m_c, m_b \gg T$, number is conserved during the evolution (thermal production can be neglected).
- ✦ $m \gg T \sim q$, can be treated as a Brownian particle.
- ✦ Small $\mathscr{D}_s$. strongly coupled to the QGP.

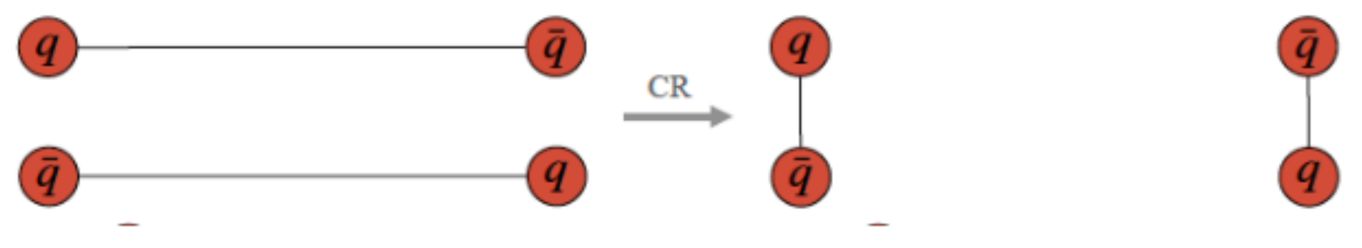
Haitao Shu's talk



Hadronization of open heavy flavor

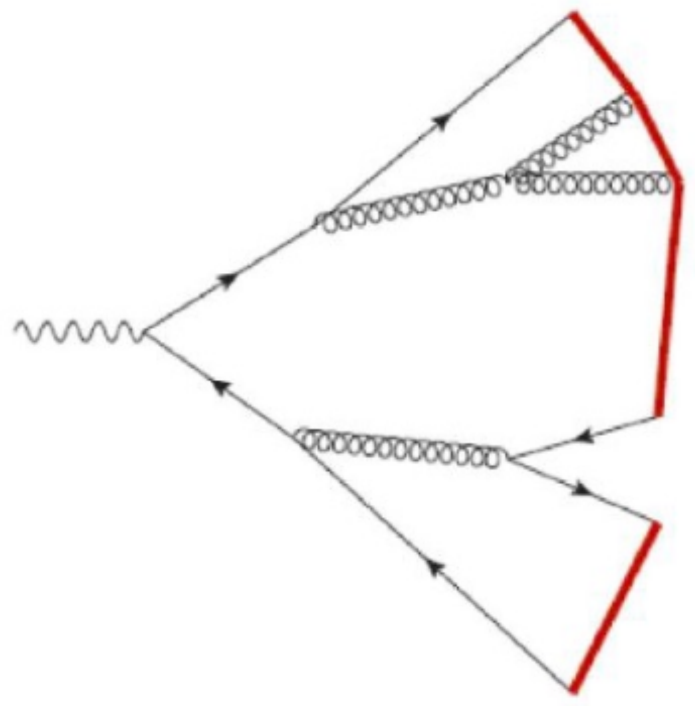
Whst's the hadronization?

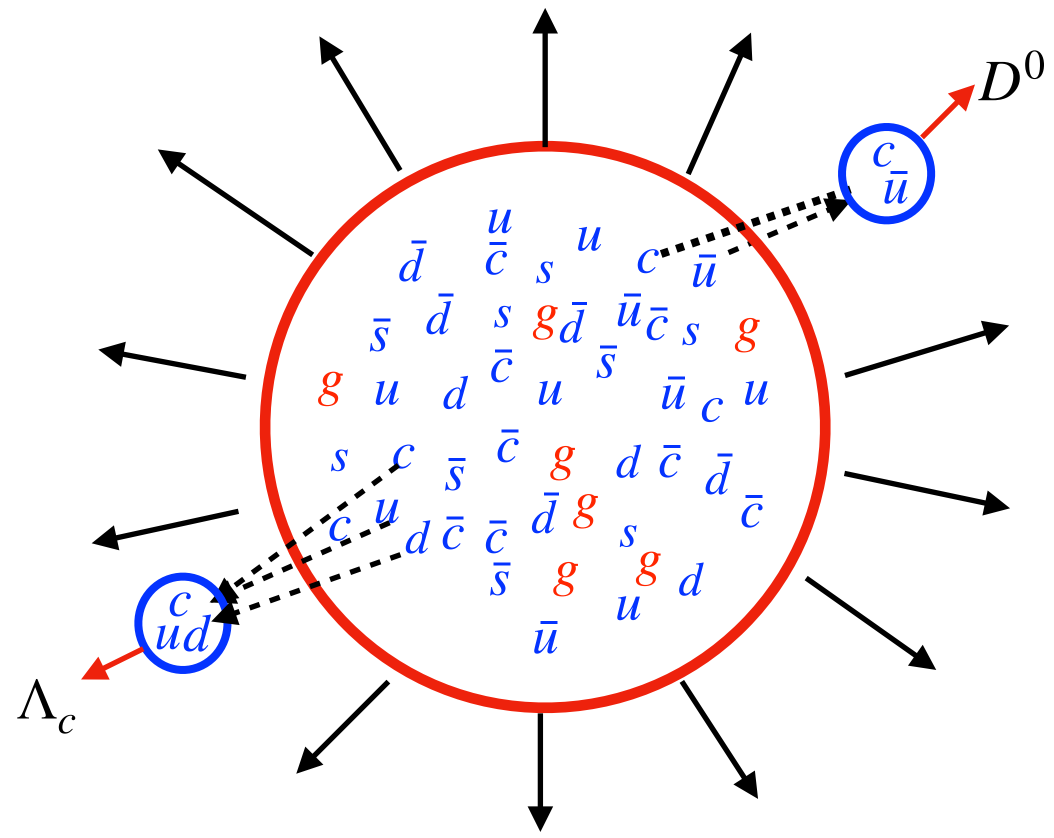
➡ Colorful quark to colorless hadron; non-perturbative due to soft gluon emission

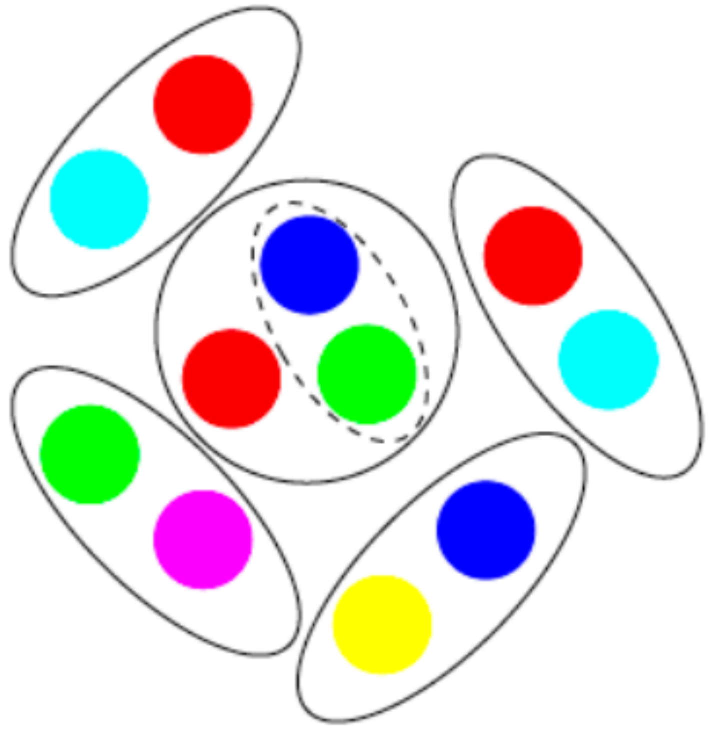


(a) Dipole-type reconnection.

(b) Junction reconnection.







Color Reconnection (CR); Fragmentation; Coalescence; Local color recombination,….

- ✦ Hadronization itself reflect the non-perturbative properties of the QCD;
- ✦ Cruticial to investigate no matter the heavy flavor production or energy loss in the QGP;
- ✦ Prespective to search new hadrons and dectect their inner structures (diquark,…).

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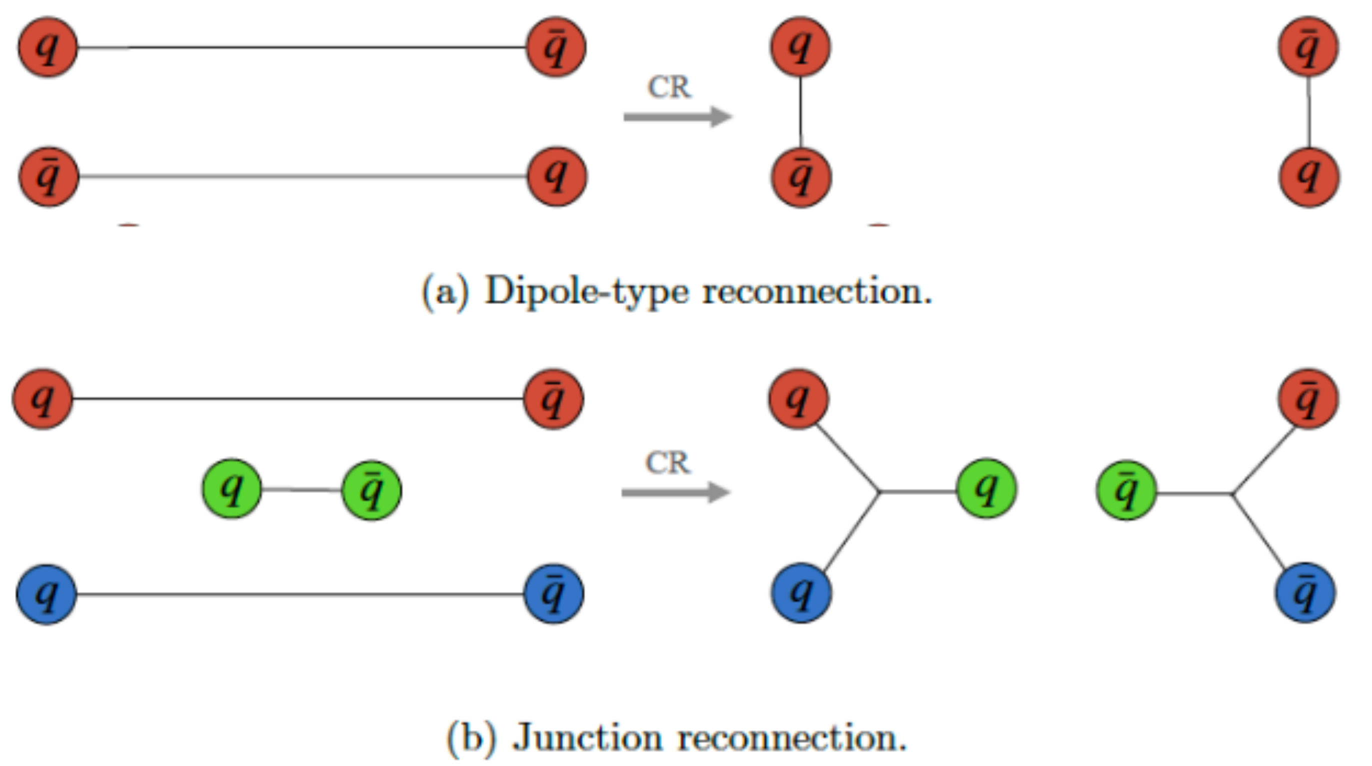

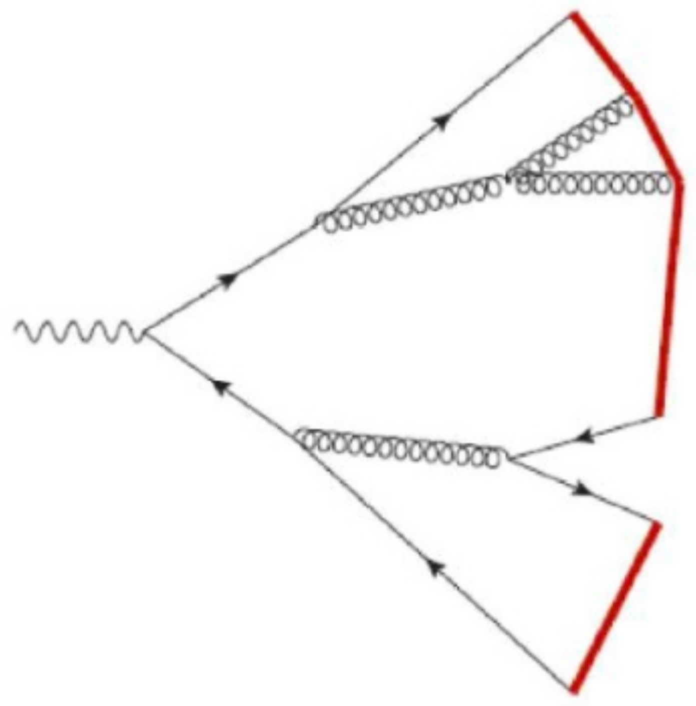

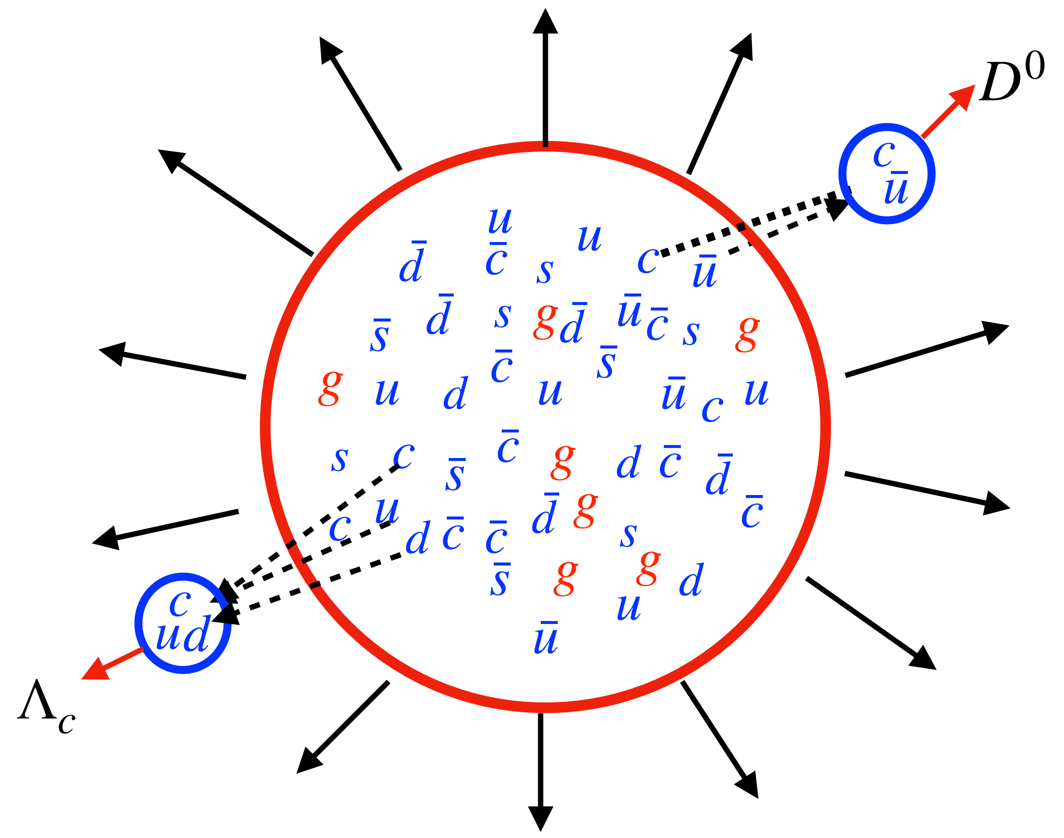

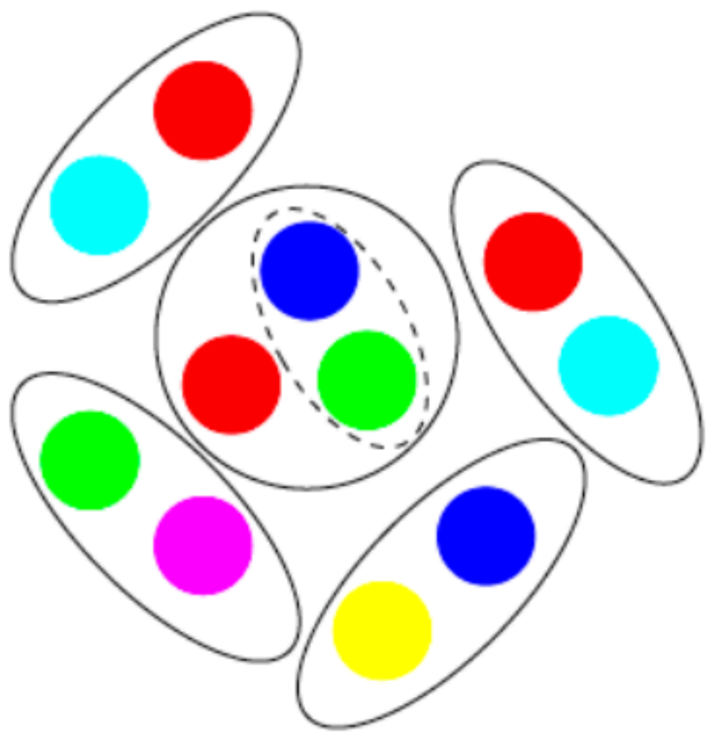

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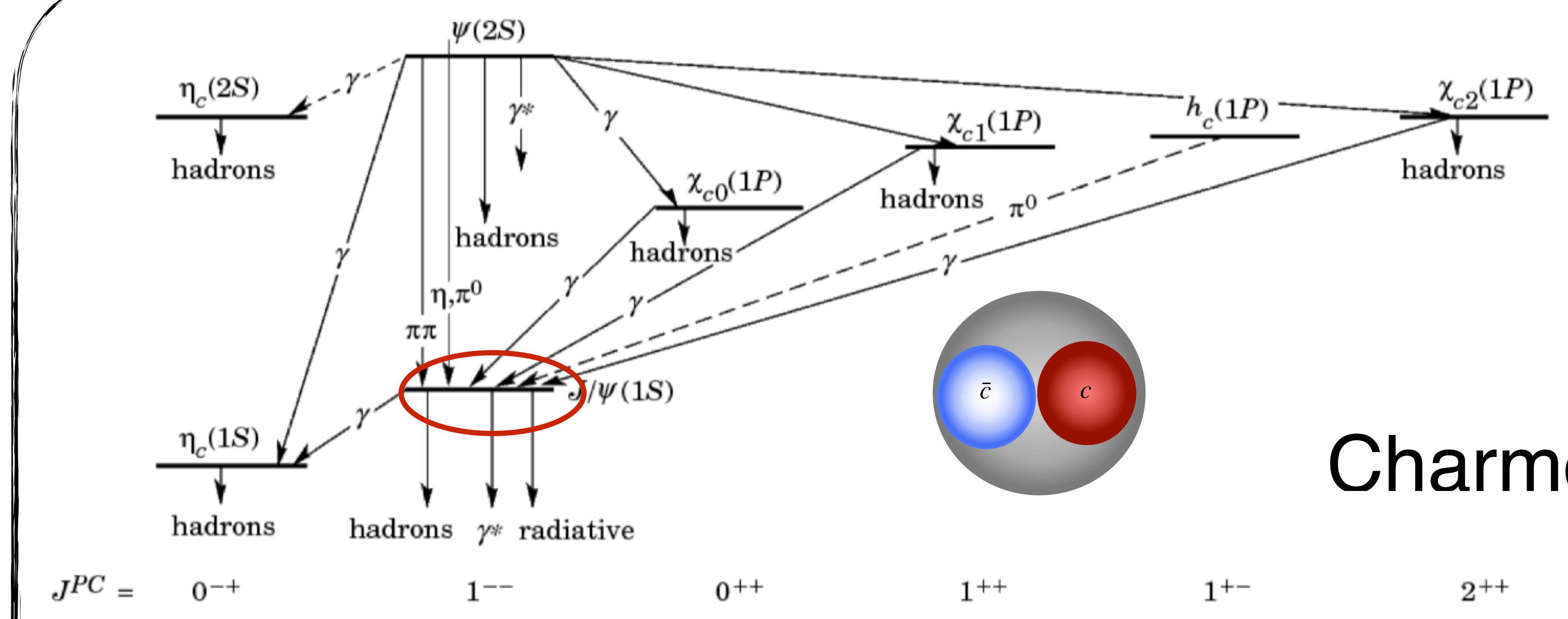


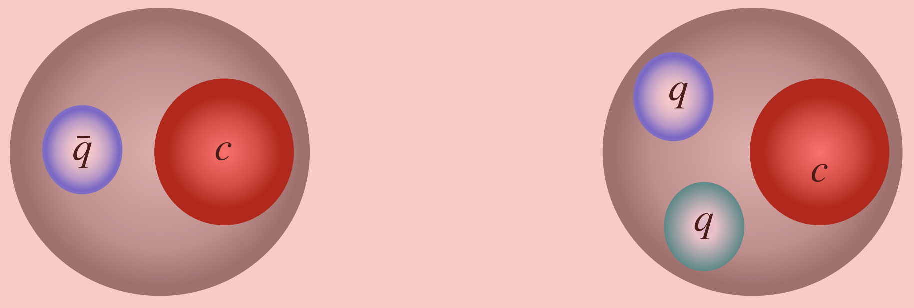
Charmonium

Zebo Tang's Talk

Theoretical review: SQM Plenary talk by **Jiaxing Zhao**

$D^0, D^+, D_s^+, \Lambda_c, \Xi_c, \Omega_c, \ldots$

$B^0, B^-, B_s^0, \Lambda_b, \Xi_b, \Omega_b, \ldots$



Meson   Baryon

Hadronization in the vacuum

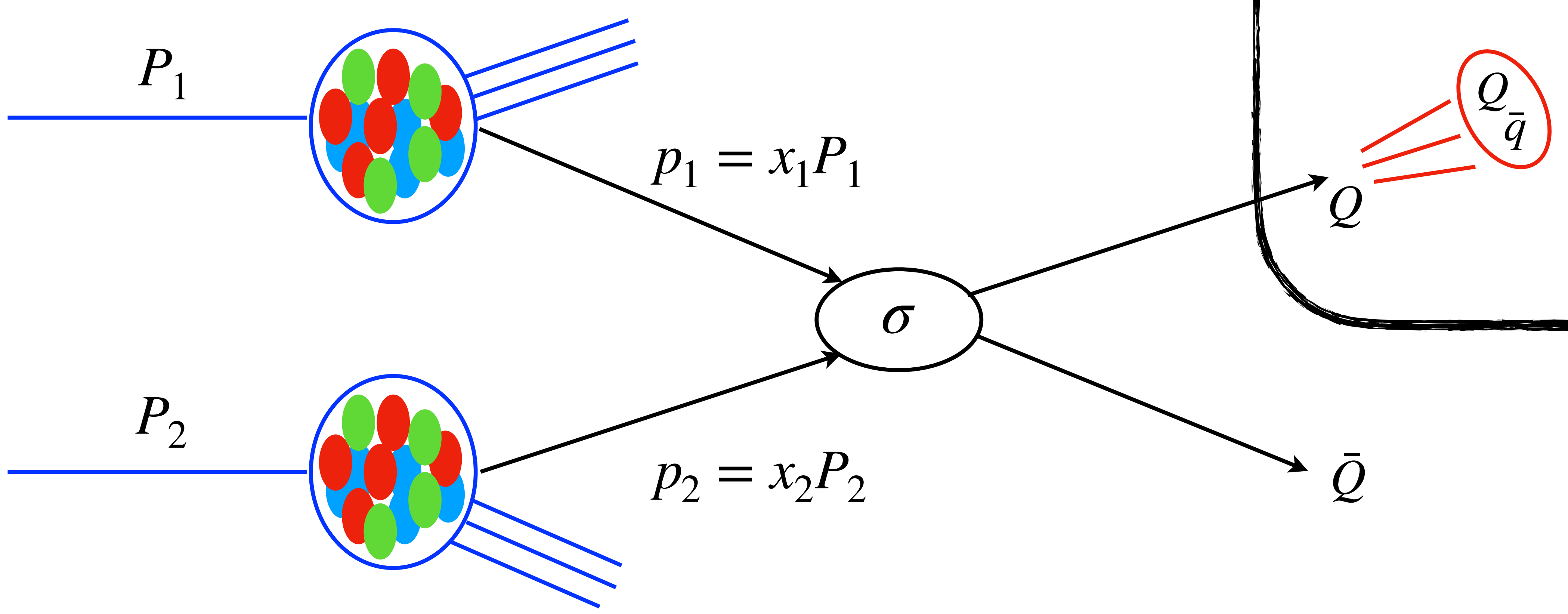
QCD factorization & Fragmentation：

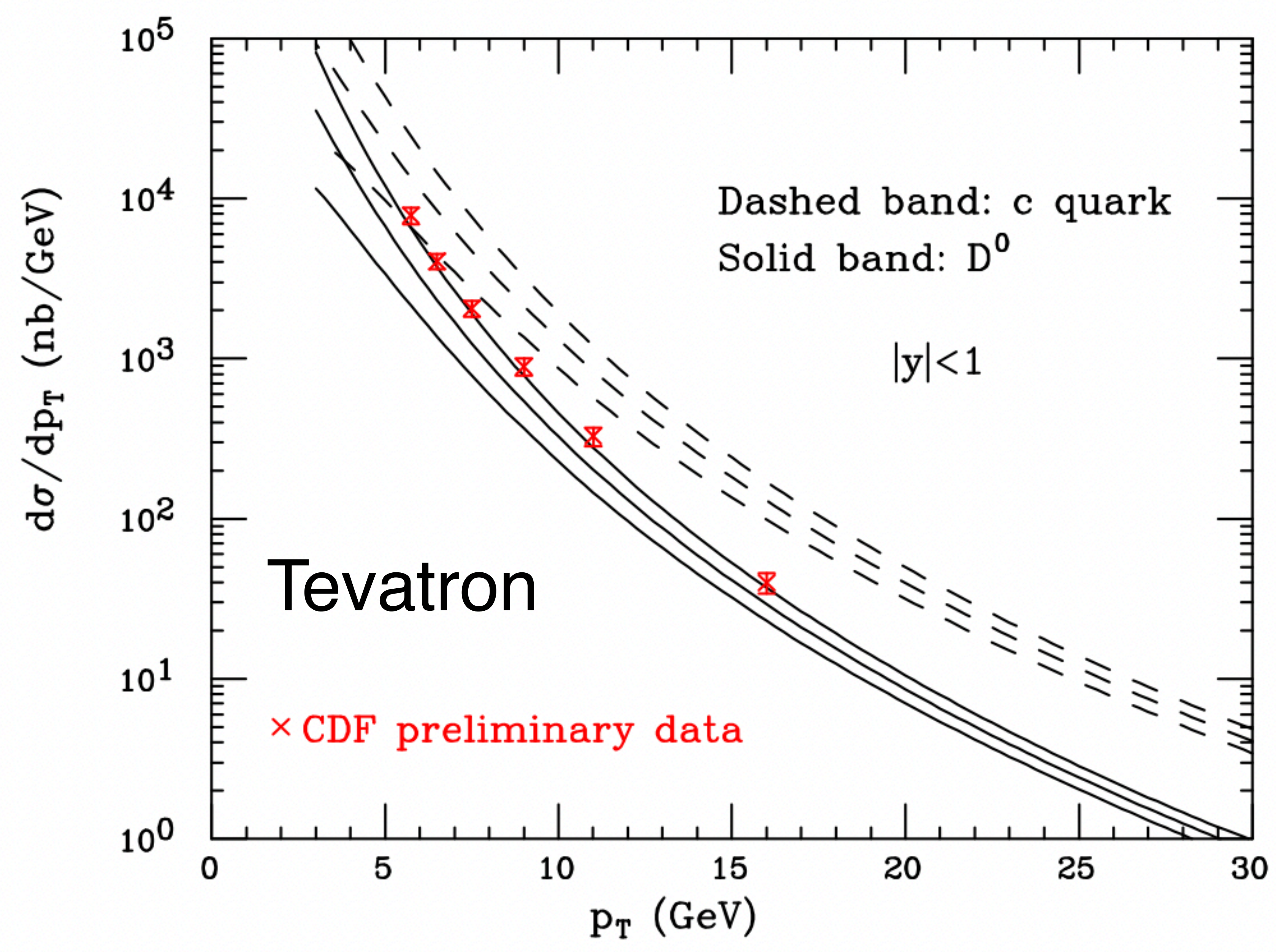
$$\sigma_H \propto f_i^A(x_1,\mu_F) f_j^B(x_2,\mu_F) \otimes \sigma_{ij\to Q\bar{Q}+X} \otimes \mathscr{D}_{Q\to H}$$

Parton distribution (PDFs)

Hard scattering (pQCD)

Fragmentation function





M. Cacciari, P. Nason, JHEP 01 (2003) 006

Fragmentation functions can be determined by fitting the experimental data ($e^+e^-, pp, p\bar{p},\ldots$)

Peterson fragmentation;

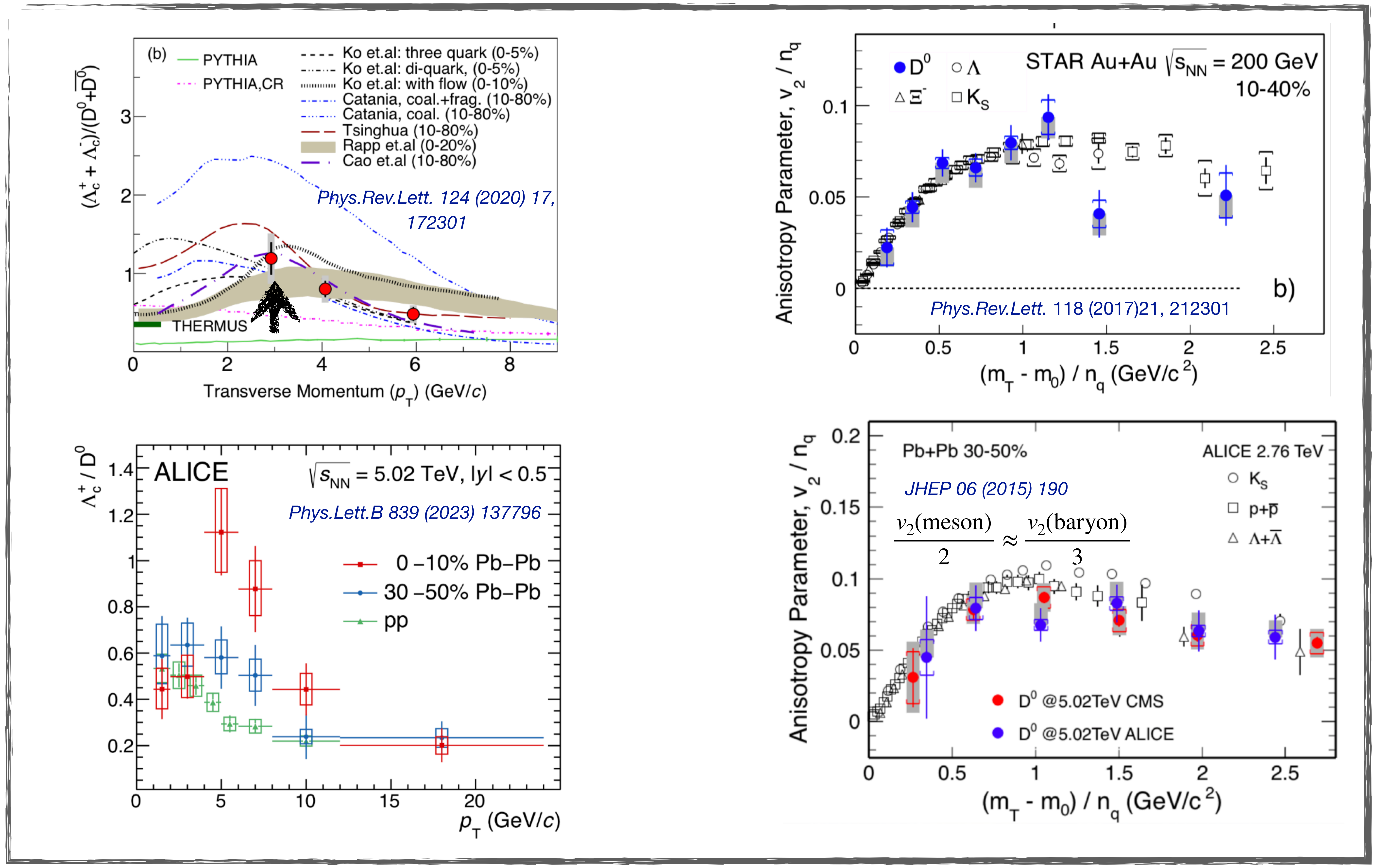
String fragmentation;

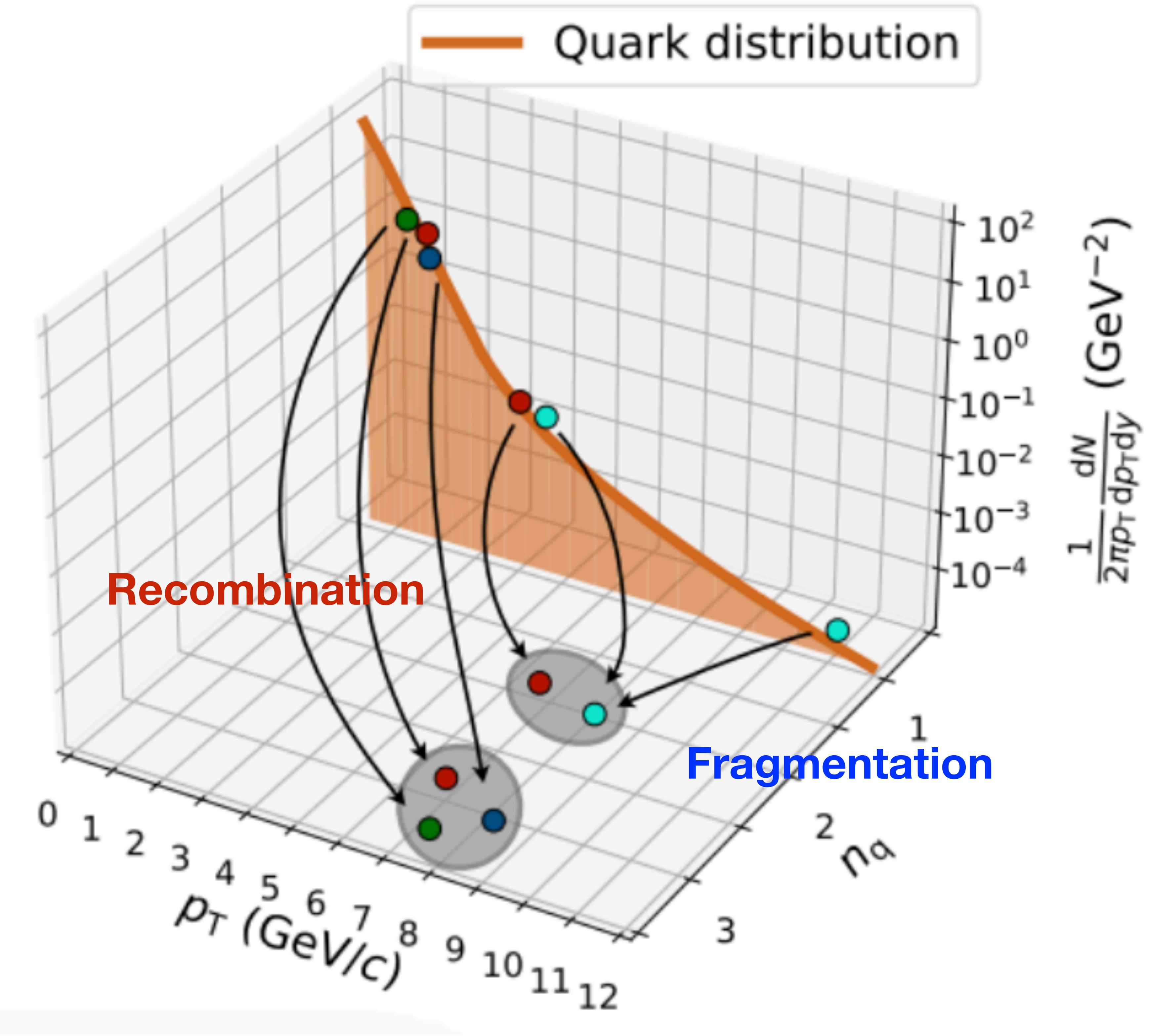
HQET fragmentation function;

…

Hadronization in the hot QCD medium

Hadronization in the hot medium shows a huge difference compared to the vacuum case (Fragmentation).





- Enhancement Baryon / Meson Ratio
- Quark Number Scaling of Elliptic flow

The heavy quark combines with the light quark(s), which are close together in phase space, to form a hadron.

State of art

SOA: low p_T heavy quark hadronizes by recombination while high p_T hadronizes by fragmentation in heavy ion collisions. Still controversial in pp. **Min He's talk**

In the theoretical side, there are many models:

Catania (coalescence+fragmentation; pp and AA),
Duke (coalescence+fragmentation; only AA),
EPOS4 (coalescence+fragmentation: pp and AA),
LBT (coalescence+fragmentation; only AA),
Nantes (coalescence+fragmentation; only AA),
PHSD (coalescence+fragmentation; pp and AA),
POWLANG (local color neutralization; pp and AA),
PYTHIA (fragmentation/color reconnection; only pp),
Qufu (equal-velocity combination; only AA),
TAMU (pp-fragmentation; AA-resonance recombination+fragmentation; pp and AA),
Tsinghua (coalescence; only AA).
….

They give a more or less good description of the expermential data. **experimential talks**

What we want is not just to explain the exp. data!

Many related concerns

There are many hidden problems:

1, Simultaneous or sequential hadronization

2, How many states involved -> heavy flavor conservation

3, Recombination probablity: Wigner density or other Criterions

4, Fragmentation function and ratio

5, Off-shell -> on-shell -> strong/EM decay

6, Hadronization of fully heavy states & exotic states

7, Equilibrium limit

…

Pengfei Zhuang's talk

Jinfeng Liao's talk

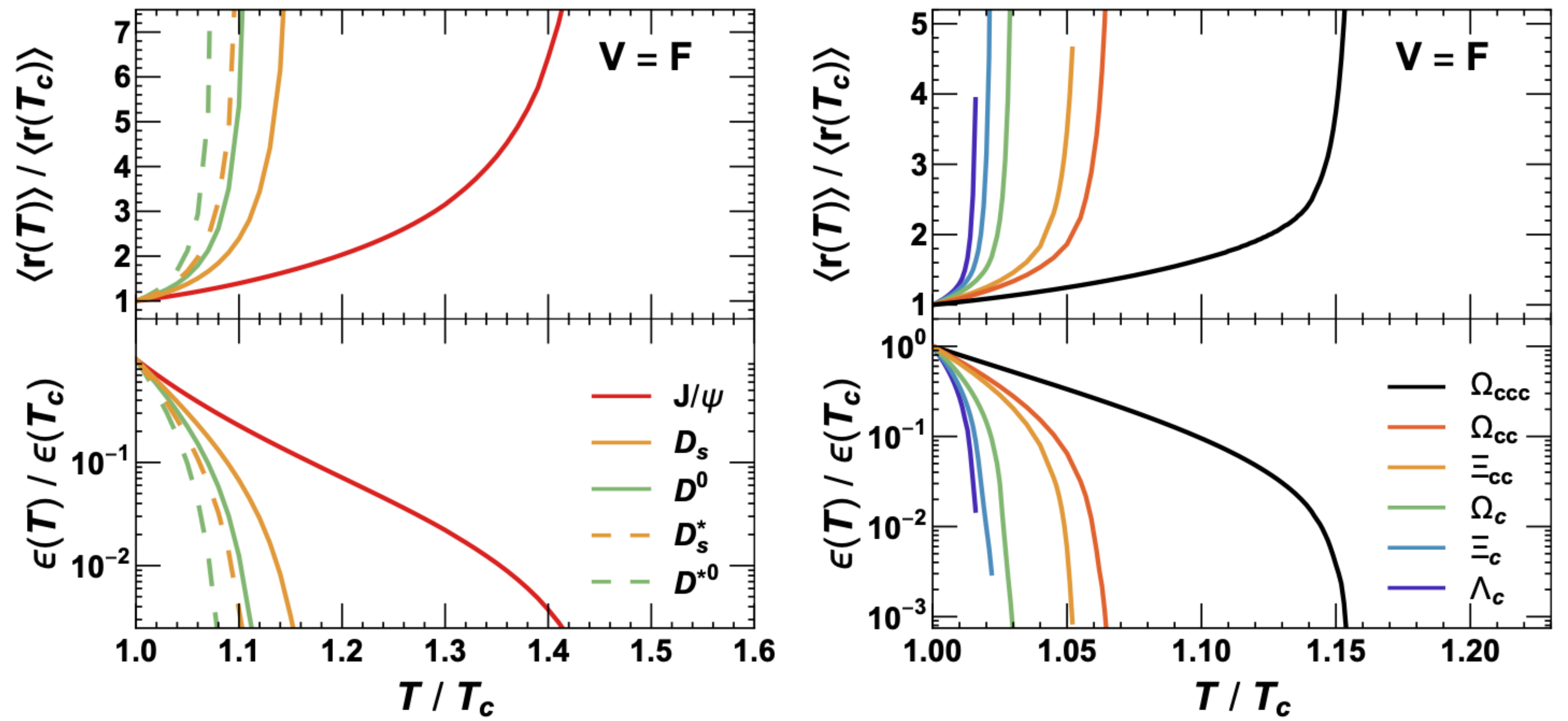
1. simultaneous vs. sequential

The dissociation temperature of various charmed hadrons are solved by 2-/3-body Dirac equation with F & U.

*S. Shi, **JX**, P. Zhuang, Chin.Phys.C 44 (2020) 8, 084101*

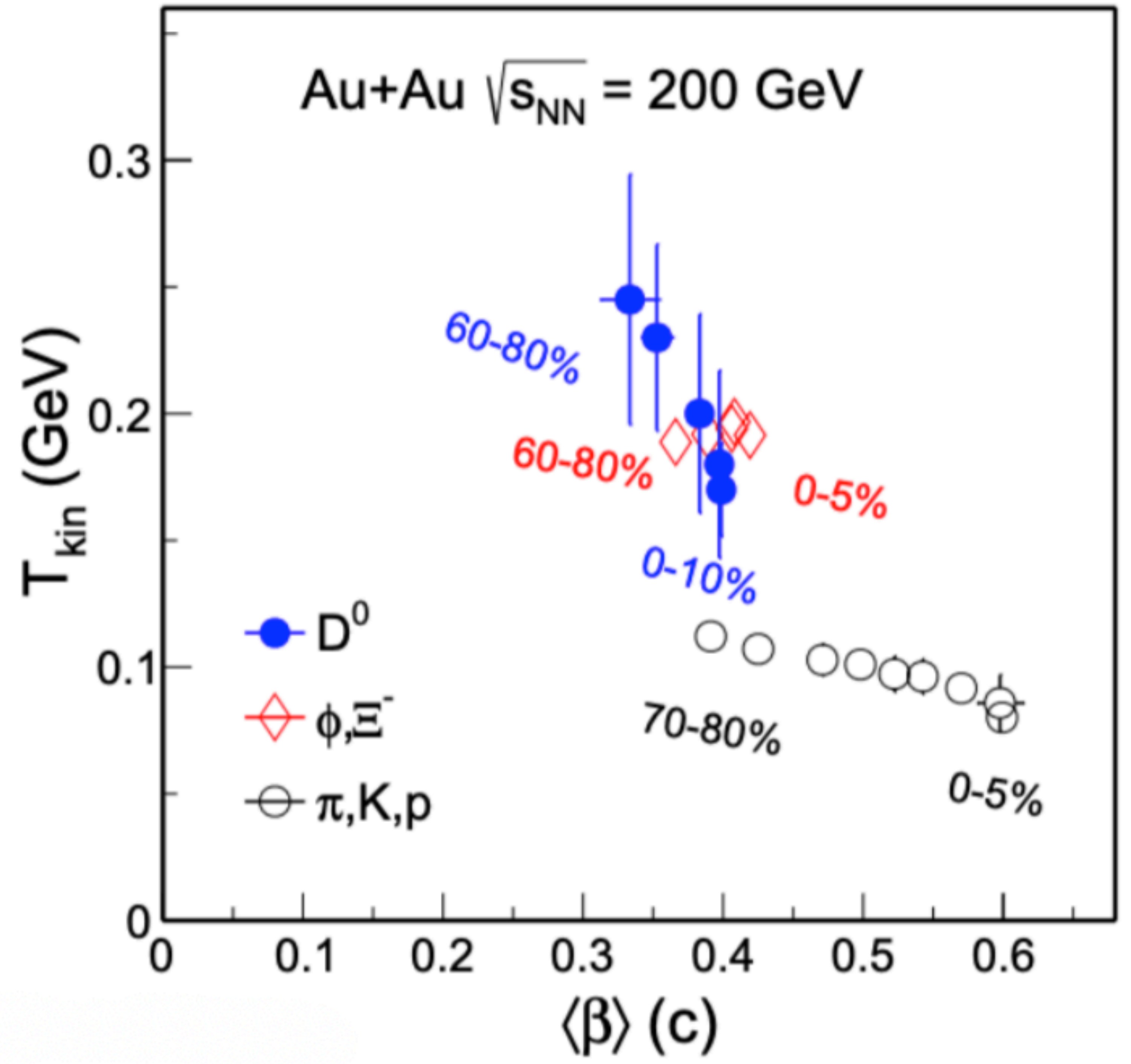
D^0 seem to decouple from the system earlier compared with light hadrons in Blast-Wave model! **Dandan Shen's talk**

STAR Collaboration. Phys. Rev. C99, 034908 (2019)



	J/ψ	χ_c	ψ'	D_s	D_s^*	D^0	D^{*0}
$V=F$	1.42	-	-	1.14	1.10	1.10	1.08
$V=U$	3.09	1.30	1.24	2.50	1.98	2.35	1.80

	Ω_{ccc}	Ω_{cc}	Ξ_{cc}	Ω_c	Ξ_c	Λ_c
$V=F$	1.15	1.06	1.05	1.03	1.02	1.02
$V=U$	2.18	1.63	1.54	1.41	1.39	1.37



$$T_{J/\psi} > T_{D_s} > T_{D^0} > T_{\Lambda_c} > T_{\pi,K,N}$$

Hadrons with larger binding energy can survive at higher temperature and are earlier produced !

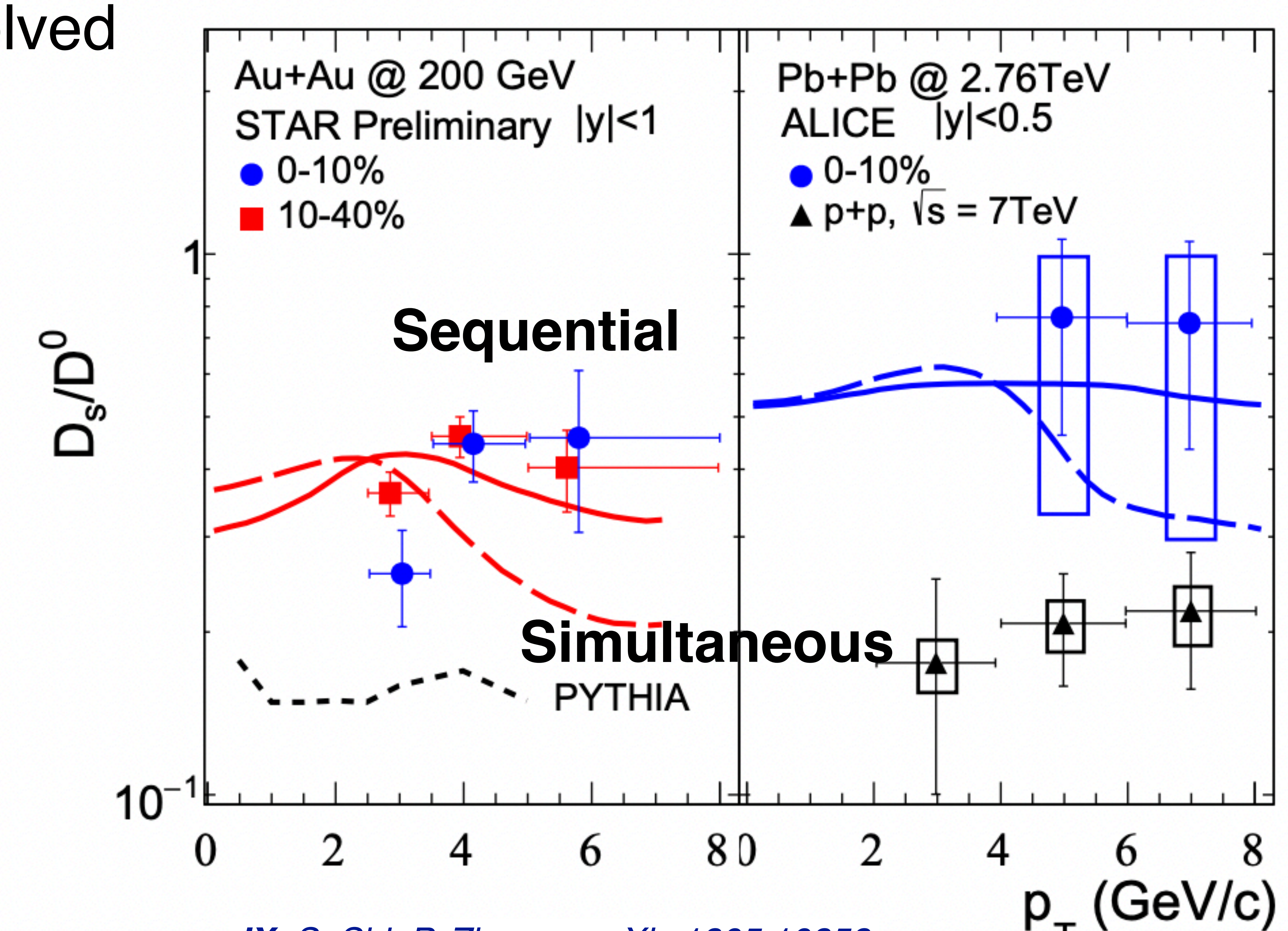
1. simultaneous vs. sequential

Simultaneous or sequential is related to **heavy quark number conservation**.

If all HF hadrons are simultaneously produced, the HF conservation contributes only a overall normalization constant, as used in most hadronization models.

If HF hadrons are sequentially produced, more heavy quarks are involved in the earlier production and less in the later production.

$$r_h = \frac{involved\ charm\ quarks}{total\ charm\ quarks\ N_c} = \begin{cases} 1 & for\ \ h = D_s \\ 1 - \frac{N_{D_s}}{N_c}\ \ (\sim 90\%) & for\ \ h = D^0 \\ 1 - (N_{D_s} + N_{D^0})/N_c\ \ (\sim 60\%) & for\ \ h = \Lambda_c \end{cases}$$



JX, S. Shi, P. Zhuang, arXiv:1805.10858.

Enhances the earlier produced hadrons;
Suppresses the later produced hadrons.

Still a challenge to confirm this point in the experiment due to the many other uncertainties involved.

2. how many states? PDG vs. RQM

Relativistic quark model (also the lattice QCD)predicted many HF hadrons but most of them have not yet been found.

e.g. charmed hadrons

PDG and D. Ebert, R. Faustov, and V. Galkin, Phys.Rev.D 84 (2011) 014025.

D

Meson	M(MeV)	J^P	Meson	M(MeV)	J^P
$D^\pm$	1869.66 ± 0.05	0^-	$D_s^\pm$	1968.35 ± 0.07	0^-
D^0	1864.84 ± 0.05	0^-	$D_s^{*\pm}$	2112.2 ± 0.4	1^-
$D^{*0}(2007)$	2006.85 ± 0.05	1^-	$D_{s0}^{*\pm}(2317)$	2317.8 ± 0.5	0^+
$D^{*\pm}(2010)$	2010.26 ± 0.05	1^-	$D_{s1}^\pm(2460)$	2459.5 ± 0.6	1^+
$D_0^*(2300)$	2343 ± 10	0^+	$D_{s1}^\pm(2536)$	2535.11 ± 0.06	1^+
$D_1(2420)$	2422.1 ± 0.6	1^+	$D_{s2}^*(2573)$	2569.1 ± 0.8	2^+
$D_1^0(2430)$	2412 ± 9	1^+	$D_{s0}^\pm(2590)$	2591 ± 6	0^-
$D_2^*(2460)$	2461.1 ± 0.8	2^+	$D_{s1}^{*\pm}(2700)$	2714 ± 5	1^-
$D_0^0(2550)$	2549 ± 19	0^-	$D_{s1}^{*\pm}(2860)$	2859 ± 12	1^-
$D_1^{*0}(2600)$	2627 ± 10	1^-	$D_{s3}^{*\pm}(2860)$	2860.5 ± 0.6	3^-
$D^{*\pm}(2640)$	2637 ± 2	?	$D_{sJ}^\pm(3040)$	3044 ± 8	?
$D_2^0(2740)$	2747 ± 6	2^-			
$D_3^*(2750)$	2763.1 ± 3.2	3^-			
$D_1^{*0}(2760)$	2781 ± 18	1^-			
$D_0(3000)$	3214 ± 29	?			

Λ_c

Baryon	M(MeV)	J^P	Meson	M(MeV)	J^P
Λ_c^+	2286.46 ± 0.14	$1S(1/2)^+$	Λ_c	3747	$4D(3/2)^+$
$\Lambda_c^+(2765)$	2766.6 ± 2.4	$?2S(1/2)^+$	$\Lambda_c^+(2880)$	2881.63 ± 0.24	$1D(5/2)^+$
Λ_c	3130	$3S(1/2)^+$	Λ_c	3209	$2D(5/2)^+$
Λ_c	3437	$4S(1/2)^+$	Λ_c	3500	$3D(5/2)^+$
Λ_c	3715	$5S(1/2)^+$	Λ_c	3767	$4D(5/2)^+$
Λ_c	3973	$6S(1/2)^+$	Λ_c	3097	$1F(5/2)^-$
$\Lambda_c^+(2595)$	2592.25 ± 0.28	$1P(1/2)^-$	Λ_c	3375	$2F(5/2)^-$
$\Lambda_c^+(2910)$	2913.8 ± 5.6	$?2P(1/2)^-$	Λ_c	3646	$3F(5/2)^-$
Λ_c	3303	$3P(1/2)^-$	Λ_c	3900	$4F(5/2)^-$
Λ_c	3588	$4P(1/2)^-$	Λ_c	3078	$1F(7/2)^-$
Λ_c	3852	$5P(1/2)^-$	Λ_c	3393	$2F(7/2)^-$
$\Lambda_c^+(2625)$	2628.00 ± 0.15	$1P(3/2)^-$	Λ_c	3667	$3F(7/2)^-$
$\Lambda_c^+(2940)$	2939.6 ± 1.4	$2P(3/2)^-$	Λ_c	3922	$4F(7/2)^-$
Λ_c	3322	$3P(3/2)^-$	Λ_c	3270	$1G(7/2)^+$
Λ_c	3606	$4P(3/2)^-$	Λ_c	3546	$2G(7/2)^+$
Λ_c	3869	$5P(3/2)^-$	Λ_c	3284	$1G(9/2)^+$
$\Lambda_c^+(2860)$	2856.1 ± 2.0	$1D(3/2)^+$	Λ_c	3564	$2G(9/2)^+$
Λ_c	3189	$2D(3/2)^+$	Λ_c	3444	$1H(9/2)^-$
Λ_c	3480	$3D(3/2)^+$	Λ_c	3460	$1H(11/2)^-$

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Σ_c

Baryon	M(MeV)	J^P	Meson	M(MeV)	J^P
Σ_c^+	2443.97 ± 0.14	$1S(1/2)^+$	Σ_c	3161	$2P(5/2)^-$
Σ_c	2901	$2S(1/2)^+$	Σ_c	3475	$3P(5/2)^-$
Σ_c	3271	$3S(1/2)^+$	Σ_c	3757	$4P(5/2)^-$
Σ_c	3581	$4S(1/2)^+$	Σ_c	3041	$1D(1/2)^+$
Σ_c	3861	$5S(1/2)^+$	Σ_c	3370	$2D(1/2)^+$
$\Sigma_c(2520)$	2518.41 ± 0.22	$1S(3/2)^+$	Σ_c	3043	$1D(3/2)^+$
Σ_c	2936	$2S(3/2)^+$	Σ_c	3366	$2D(3/2)^+$
Σ_c^+	3293	$3S(3/2)^+$	Σ_c	3040	$1D(3/2)^+$
Σ_c	3598	$4S(3/2)^+$	Σ_c	3364	$2D(3/2)^+$
Σ_c	3873	$5S(3/2)^+$	Σ_c	3038	$1D(5/2)^+$
$\Sigma_c(2800)$	2801 ± 5	$1P(1/2)^-$	Σ_c	3365	$2D(5/2)^+$
Σ_c	3172	$2P(1/2)^-$	Σ_c	3023	$1D(5/2)^+$
Σ_c	3488	$3P(1/2)^-$	Σ_c	3349	$2D(5/2)^+$
Σ_c	3770	$4P(1/2)^-$	Σ_c	3013	$1D(7/2)^+$
Σ_c	2713	$1P(1/2)^-$	Σ_c	3342	$2D(7/2)^+$
Σ_c	3125	$2P(1/2)^-$	Σ_c	3288	$1F(3/2)^-$
Σ_c	3455	$3P(1/2)^-$	Σ_c	3283	$1F(5/2)^-$
Σ_c	3743	$4P(1/2)^-$	Σ_c	3254	$1F(5/2)^-$
Σ_c	2798	$1P(3/2)^-$	Σ_c	3253	$1F(7/2)^-$
Σ_c	3172	$2P(3/2)^-$	Σ_c	3227	$1F(7/2)^-$
Σ_c	3486	$3P(3/2)^-$	Σ_c	3209	$1F(9/2)^-$
Σ_c	3768	$4P(3/2)^-$	Σ_c	3495	$1G(5/2)^+$
Σ_c	2773	$1P(3/2)^-$	Σ_c	3483	$1G(7/2)^+$
Σ_c	3151	$2P(3/2)^-$	Σ_c	3444	$1G(7/2)^+$
Σ_c	3469	$3P(3/2)^-$	Σ_c	3442	$1G(9/2)^+$
Σ_c	3753	$4P(3/2)^-$	Σ_c	3410	$1G(9/2)^+$
Σ_c	2789	$1P(5/2)^-$	Σ_c	3386	$1G(11/2)^+$

Ξ_c

Baryon	M(MeV)	J^P	Meson	M(MeV)	J^P
Ξ_c'	2578.2 ± 0.5	$1S(1/2)^+$	Ξ_c	3303	$2P(5/2)^-$
$\Xi_c(2970)$	2964.3 ± 1.5	$2S(1/2)^+$	Ξ_c	3619	$3P(5/2)^-$
Ξ_c	3377	$3S(1/2)^+$	Ξ_c	3902	$4P(5/2)^-$
Ξ_c	3695	$4S(1/2)^+$	Ξ_c	3163	$1D(1/2)^+$
Ξ_c	3978	$5S(1/2)^+$	Ξ_c	3505	$2D(1/2)^+$
$\Xi_c(2645)$	2645.10 ± 0.3	$1S(3/2)^+$	Ξ_c	3167	$1D(3/2)^+$
Ξ_c	3026	$2S(3/2)^+$	Ξ_c	3506	$2D(3/2)^+$
Ξ_c	3396	$3S(3/2)^+$	Ξ_c	3160	$1D(3/2)^+$
Ξ_c	3709	$4S(3/2)^+$	Ξ_c	3497	$2D(3/2)^+$
Ξ_c	3989	$5S(3/2)^+$	Ξ_c	3166	$1D(5/2)^+$
Ξ_c	2936	$1P(1/2)^-$	Ξ_c	3504	$2D(5/2)^+$
Ξ_c	3313	$2P(1/2)^-$	Ξ_c	3153	$1D(5/2)^+$
Ξ_c	3630	$3P(1/2)^-$	Ξ_c	3493	$2D(5/2)^+$
Ξ_c	3912	$4P(1/2)^-$	Ξ_c	3147	$1D(7/2)^+$
Ξ_c	2854	$1P(1/2)^-$	Ξ_c	3486	$2D(7/2)^+$
Ξ_c	3267	$2P(1/2)^-$	Ξ_c	3418	$1F(3/2)^-$
Ξ_c	3598	$3P(1/2)^-$	Ξ_c	3408	$1F(5/2)^-$
Ξ_c	3887	$4P(1/2)^-$	Ξ_c	3394	$1F(5/2)^-$
Ξ_c	2935	$1P(3/2)^-$	Ξ_c	3393	$1F(7/2)^-$
Ξ_c	3311	$2P(3/2)^-$	Ξ_c	3373	$1F(7/2)^-$
Ξ_c	3628	$3P(3/2)^-$	Ξ_c	3357	$1F(9/2)^-$
Ξ_c	3911	$4P(3/2)^-$	Ξ_c	3623	$1G(5/2)^+$
Ξ_c	2912	$1P(3/2)^-$	Ξ_c	3608	$1G(7/2)^+$
Ξ_c	3293	$2P(3/2)^-$	Ξ_c	3584	$1G(7/2)^+$
Ξ_c	3613	$3P(3/2)^-$	Ξ_c	3582	$1G(9/2)^+$
Ξ_c	3898	$4P(3/2)^-$	Ξ_c	3558	$1G(9/2)^+$
Ξ_c	2929	$1P(5/2)^-$	Ξ_c	3536	$1G(11/2)^+$

Baryon	M(MeV)	J^P	Meson	M(MeV)	J^P
Ξ_c	2467.71 ± 0.23	$1S(1/2)^+$	Ξ_c	3945	$4D(3/2)^+$
Ξ_c	2959	$2S(1/2)^+$	Ξ_c	3076	$1D(5/2)^+$
Ξ_c	3323	$3S(1/2)^+$	Ξ_c	3407	$2D(5/2)^+$
Ξ_c	3632	$4S(1/2)^+$	Ξ_c	3699	$3D(5/2)^+$
Ξ_c	3909	$5S(1/2)^+$	Ξ_c	3965	$4D(5/2)^+$
Ξ_c	4166	$6S(1/2)^+$	Ξ_c	3278	$1F(5/2)^-$
$\Xi_c^+(2790)$	2791.9 ± 0.5	$1P(1/2)^-$	Ξ_c	3575	$2F(5/2)^-$
Ξ_c	3179	$2P(1/2)^-$	Ξ_c	3845	$3F(5/2)^-$
Ξ_c	3500	$3P(1/2)^-$	Ξ_c	4098	$4F(5/2)^-$
Ξ_c	3785	$4P(1/2)^-$	Ξ_c	3292	$1F(7/2)^-$
Ξ_c	4048	$5P(1/2)^-$	Ξ_c	3592	$2F(7/2)^-$
$\Xi_c(2815)$	2816.51 ± 0.25	$1P(3/2)^-$	Ξ_c	3865	$3F(7/2)^-$
Ξ_c	3201	$2P(3/2)^-$	Ξ_c	4120	$4F(7/2)^-$
Ξ_c	3519	$3P(3/2)^-$	Ξ_c	3469	$1G(7/2)^+$
Ξ_c	3804	$4P(3/2)^-$	Ξ_c	3745	$2G(7/2)^+$
Ξ_c	4066	$5P(3/2)^-$	Ξ_c	3483	$1G(9/2)^+$
Ξ_c	3059	$1D(3/2)^+$	Ξ_c	3763	$2G(9/2)^+$
Ξ_c	3388	$2D(3/2)^+$	Ξ_c	3643	$1H(9/2)^-$
Ξ_c	3678	$3D(3/2)^+$	Ξ_c	3658	$1H(11/2)^-$

Ω_c

Baryon	M(MeV)	J^P	Meson	M(MeV)	J^P
Ω_c^0	2695.2 ± 1.7	$1S(1/2)^+$	Ω_c	3427	$2P(5/2)^-$
$\Omega_c^0(2770)$	3088	$2S(1/2)^+$	$\Omega_c^+(2880)$	3744	$3P(5/2)^-$
Ω_c	3489	$3S(1/2)^+$	Ω_c	4028	$4P(5/2)^-$
Ω_c	3814	$4S(1/2)^+$	Ω_c	3287	$1D(1/2)^+$
Ω_c	4102	$5S(1/2)^+$	Ω_c	3623	$2D(1/2)^+$
$\Omega_c^0(2770)$	2765.9 ± 2	$1S(3/2)^+$	Ω_c	3298	$1D(3/2)^+$
Ω_c	3123	$2S(3/2)^+$	Ω_c	3627	$2D(3/2)^+$
Ω_c	3510	$3S(3/2)^+$	Ω_c	3282	$1D(3/2)^+$
Ω_c	3830	$4S(3/2)^+$	Ω_c	3613	$2D(3/2)^+$
Ω_c	4114	$5S(3/2)^+$	Ω_c	3297	$1D(5/2)^+$
$\Omega_c^0(3000)$	3000.46 ± 0.25	$?1P(1/2)^-$	Ω_c	3626	$2D(5/2)^+$
Ω_c	3435	$2P(1/2)^-$	Ω_c	3283	$1D(5/2)^+$
Ω_c	3754	$3P(1/2)^-$	Ω_c	3614	$2D(5/2)^+$
Ω_c	4037	$4P(1/2)^-$	Ω_c	3283	$1D(7/2)^+$
Ω_c	2966	$1P(1/2)^-$	Ω_c	3611	$2D(7/2)^+$
Ω_c	3384	$2P(1/2)^-$	Ω_c	3533	$1F(3/2)^-$
Ω_c	3717	$3P(1/2)^-$	Ω_c	3522	$1F(5/2)^-$
Ω_c	4009	$4P(1/2)^-$	Ω_c	3515	$1F(5/2)^-$
Ω_c	3054	$1P(3/2)^-$	Ω_c	3514	$1F(7/2)^-$
Ω_c	3433	$2P(3/2)^-$	Ω_c	3498	$1F(7/2)^-$
Ω_c	3752	$3P(3/2)^-$	Ω_c	3485	$1F(9/2)^-$
Ω_c	4036	$4P(3/2)^-$	Ω_c	3739	$1G(5/2)^+$
Ω_c	3029	$1P(3/2)^-$	Ω_c	3721	$1G(7/2)^+$
Ω_c	3415	$2P(3/2)^-$	Ω_c	3707	$1G(7/2)^+$
Ω_c	3737	$3P(3/2)^-$	Ω_c	3705	$1G(9/2)^+$
Ω_c	4023	$4P(3/2)^-$	Ω_c	3685	$1G(9/2)^+$
Ω_c	3051	$1P(5/2)^-$	Ω_c	3665	$1G(11/2)^+$

2. how many states? PDG vs. RQM

	D	D_s	Λ_c
Catania	D^0, D^+, D^{*0}, D^{*+}	D_s, D_s^{*+}	Λ_c, $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Sigma_c(2455)$, $\Sigma_c(2520)$
Duke	D^0, D^+, D^{*0}, D^{*+}	-	-
LBT	All S and P-wave D^0 and D^+	All S and P-wave D_s	All S and P-wave Λ_c and Σ_c
Nantes	D^0	-	-
PHSD	Most S and P-wave D^0	Most S and P-wave D_s	-
TAMU	PDG	PDG	RQM
Torino	D^0	D_s	Λ_c
LANL	D^0, D^+, D^{*0}, D^{*+}	-	-

first introduced in the TAMU model (Min & Ralf)

Allso called POWLANG

How to include hundreds of excited states? Decay branching ratio?

Thermal model? $$n_i = \frac{g_i}{2\pi^2} T_{\rm FO} m_i^2 K_2\left(\frac{m_i}{T_{\rm FO}}\right)$$ -> TAMU & EPOS4

3. recombination probability: Wigner density vs. others

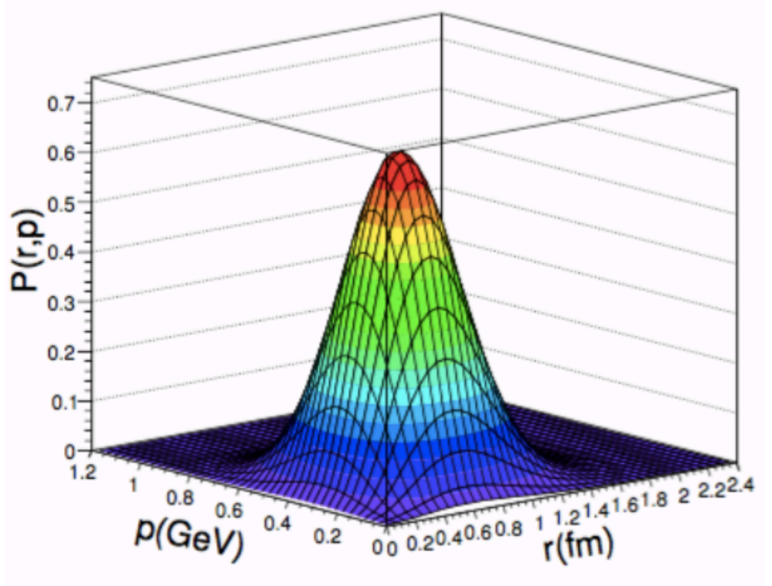
Hadronization in a hot QCD medium need to consider both statistics and dynamics.

$$N \propto f_Q(x_Q, p_Q) f_q(x_q, p_q) \times W$$

3. recombination probability: Wigner density vs. others

Hadronization in a hot QCD medium need to consider both statistics and dynamics.

$$N \propto f_Q(x_Q, p_Q) f_q(x_q, p_q) \times W$$



Wigner density (Breit-Wigner cross section) ---> the only QCD input to hadronization mechanism!

Separation of scales: $m_Q \gg m_Q v \gg m_Q v^2$ QCD -->NRQCD-->pNRQCD--> potential model

Wigner function (2-body): $$W(r,p) = \int d^4 y e^{-ipy} \psi(r + \frac{y}{2}) \psi(r - \frac{y}{2})$$

Wavefunctions are given by 2/3-body Schrödinger equation (Dirac equation, Bethe-Salpeter equation,…)

*S. Shi, **JX**, P. Zhuang, Chin.Phys.C 44 (2020) 8, 084101* ***JZ,*** *K. Zhou, S. Chen, P. Zhuang, PPNP.* 114 (2020) 103801.

➡ If assume that the potential is 3-D isotropic harmonic oscillator

$$W_S(r,p) = 8 e^{-\frac{r^2}{\sigma^2} - p^2 \sigma^2}$$

Catania, Nantes, Duke, LBT, EPOS4 …

Only LBT & PHSD used p-wave formula

To any states: ***JX**, et al, EPJ Web Conf. 296 (2024) 09014.*

➡ The width can be determined by the charge radius, mass radius, or geometry radius.

$$\langle r^2 \rangle_{charge} = \sum_i Q_i \langle (r_i - R)^2 \rangle \to \frac{3}{2} \frac{Q_1 m_2^2 + Q_2 m_1^2}{(m_1 + m_2)^2} \sigma^2$$

Catania, Duke, LBT,…

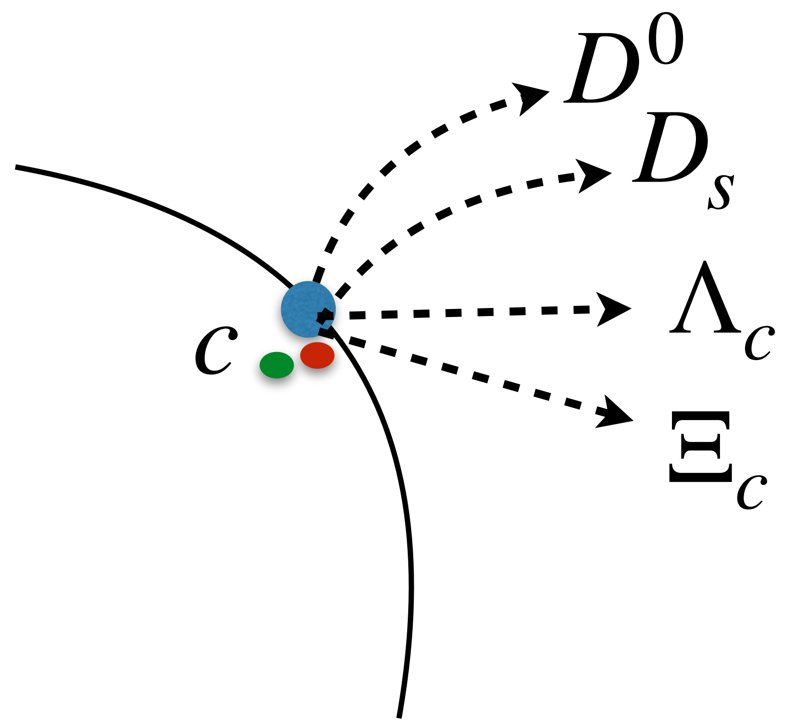
$$\langle r^2 \rangle_{geo.} = \sum_i \langle (r_i - R)^2 \rangle \to \frac{3}{2} \frac{m_2^2 + m_1^2}{(m_1 + m_2)^2} \sigma^2$$

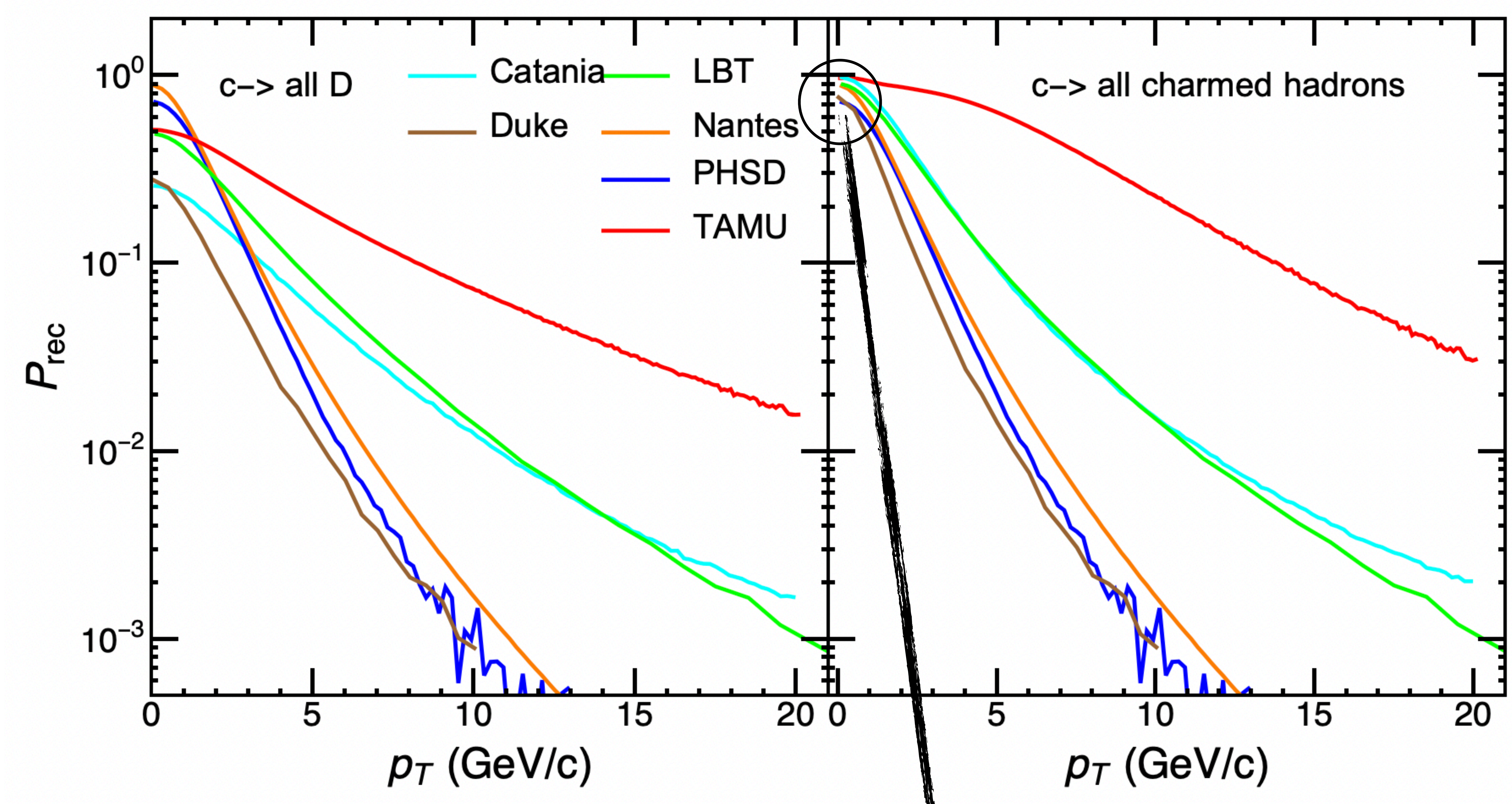
PHSD, EPOS4,…

$$\langle r^2 \rangle_{mass} = \sum_i m_i \langle (r_i - R)^2 \rangle / (2\mu) \to \frac{3}{4} \sigma^2$$

To restore the prophecy and explore non-perturbative QCD --> potential model, instead of an effective parameter.

3. recombination probability: Wigner density vs. others





The differences come from:

Light quark mass/distribution -> relative distance and momentum;
Number of excited states involved and the Wigner density formula of the excited states;
Width in the Wigner density (width in Breit-Wigner cross section)…

- Total recombination probability ~1.0 at zero p_T required by all charm quarks hadronize via recombination at $p_T \sim 0$.

3. recombination probability: Wigner density vs. others

There are two main kinds of recombination criterions:

- ➡ Phase space criterion:

  Catania, Nantes, Tsinghua, and PHSD model, phase-space Wigner function. $W(r,p) = 8e^{-\frac{r^2}{\sigma^2}-p^2\sigma^2}$

- ➡ Momentum space criterion:

  Duke, LBT, EPOS4 model, momentum-space Wigner function. $W(p) = \frac{(2\sqrt{\pi}\sigma)^3}{V}e^{-p^2\sigma^2}$

  TAMU model, resonance amplitude. $\sigma(s) = g_\sigma \frac{4\pi}{k^2}\frac{(\Gamma m)^2}{(s-m^2)^2+(\Gamma m)^2}$

  Torino model, invariant mass. $M_D < M_{Cluster} < M_{max.}$

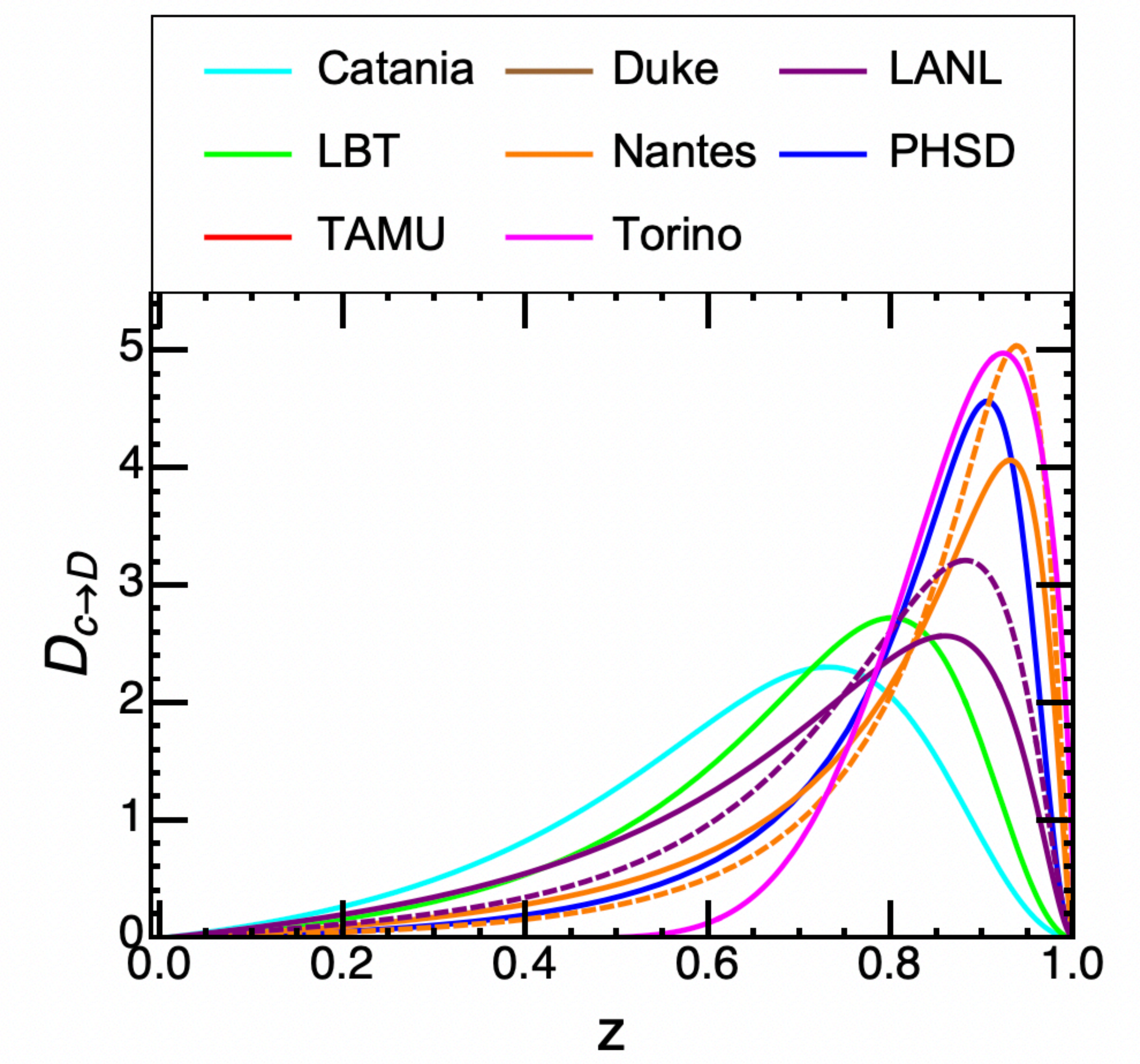
  Qufu model, equal-velocity combination. $W = A \times \frac{m_Q + m_q}{m_Q m_q}\delta\left(\frac{p_Q}{m_Q} - \frac{p_q}{m_q}\right)$

- Huge difference when p_T > 3 GeV; Phase space criterions give a steep recombination probability.

4. fragmentation function and ratio

$$P_{frag.}(p_T) = 1 - P_{rec.}(p_T)$$

There are three main fragmentation functions used:



- Peterson fragmentation;

$$\mathscr{D}_{c\to H} \propto \frac{1}{z[1 - \frac{1}{z} - \frac{\epsilon}{1-z}]^2}$$

Catania, LBT, PHSD model with different ϵ

- String fragmentation in PYTHIA and pure EPOS;

$$\mathscr{D}_{c\to H} \propto \frac{1}{z^{1+rbm_Q^2}} z^{a_\alpha} \left(\frac{1-z}{z}\right)^{a_\beta} \exp\left(-\frac{bm_T^2}{z}\right)$$

Duke, Torino model

- HQET fragmentation function (for the pseudoscalar and vector meson):

$$\mathscr{D}_{c\to P} \propto \frac{rz(1-z)^2}{[1-(1-r)z]^6}\left[6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2 - 2(1-r)(6 - 19r + 18r^2)z^3 + 3(1-r)^2(1 - 2r + 2r^2)z^4\right]$$

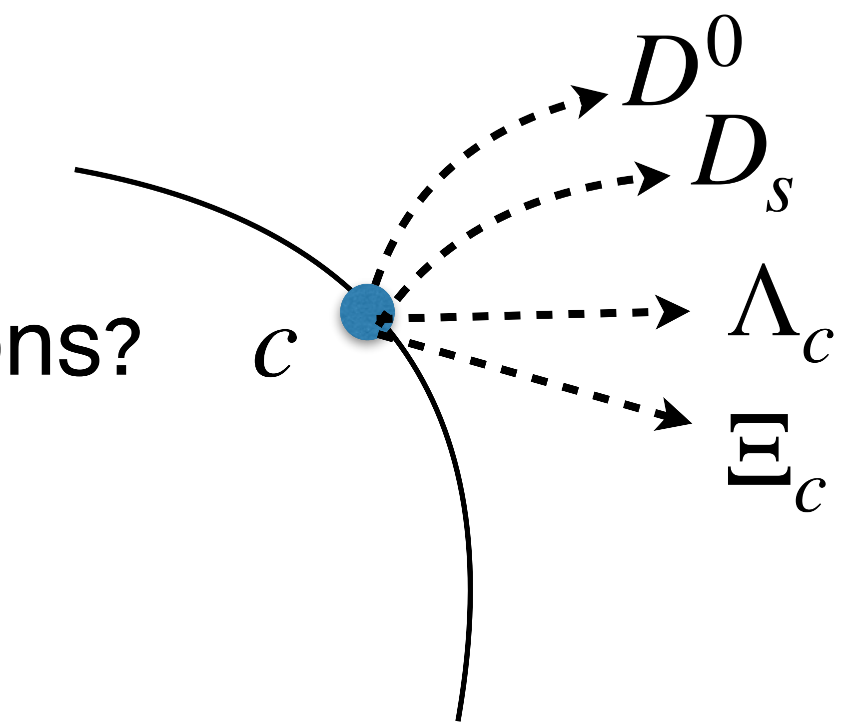
$$\mathscr{D}_{c\to V} \propto \frac{rz(1-z)^2}{[1-(1-r)z]^6}\left[2 - 2(3-2r)z + 3(3 - 2r + 4r^2)z^2 - 2(1-r)(4 - r + 2r^2)z^3 + 3(1-r)^2(3 - 2r + 2r^2)z^4\right]$$

$r = 0.1$ *in Nantes and TAMU models;* $r = 0.2$ *in Los Alamos model.*

4. fragmentation function and ratio

HQ with a given momenta → **Recombination** - - - → Which HF hardons?

HQ with a given momenta → **Fragmentation** → Which HF hardons?

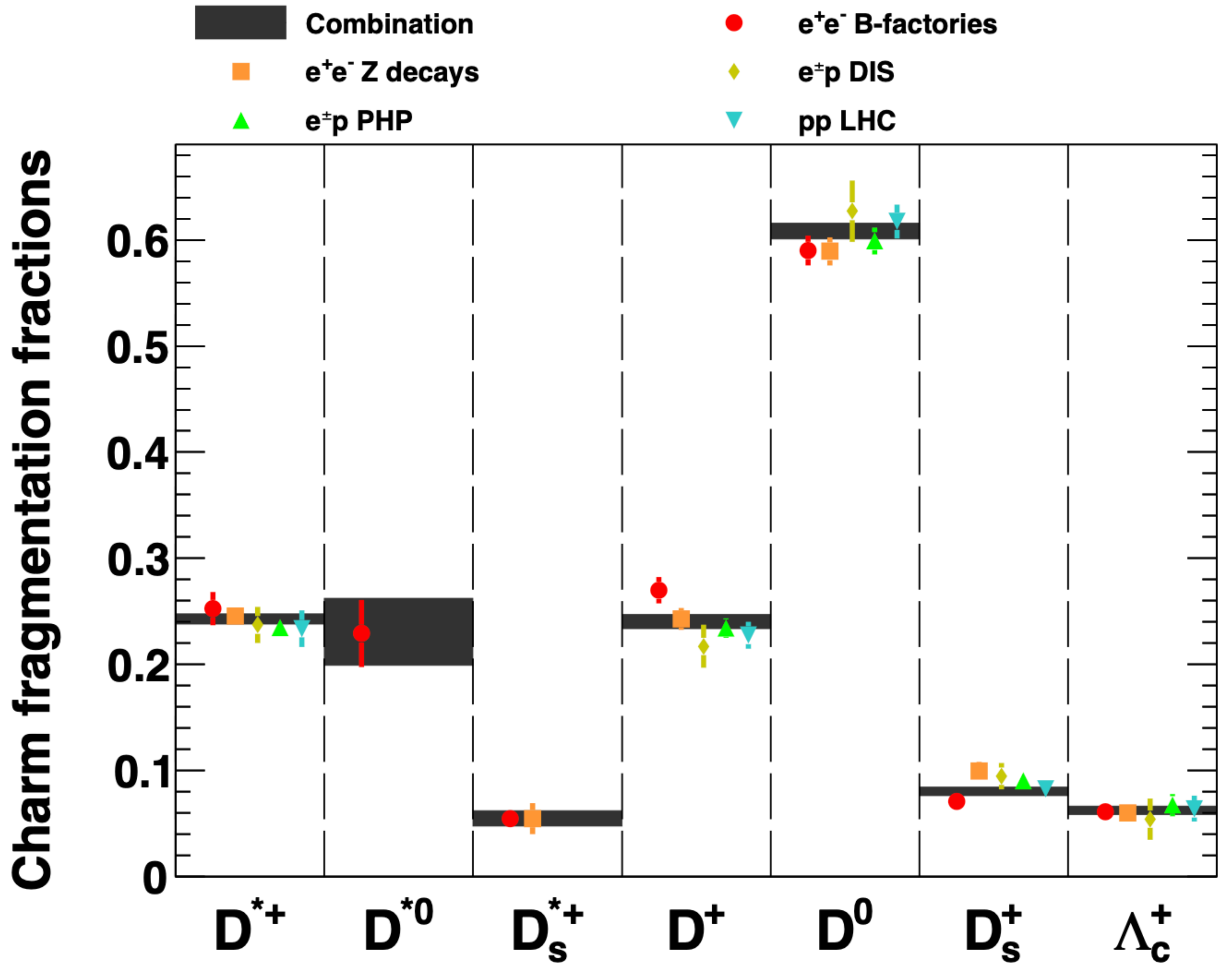


The fragmentation ratio is given by e^+e^-, ep, pp

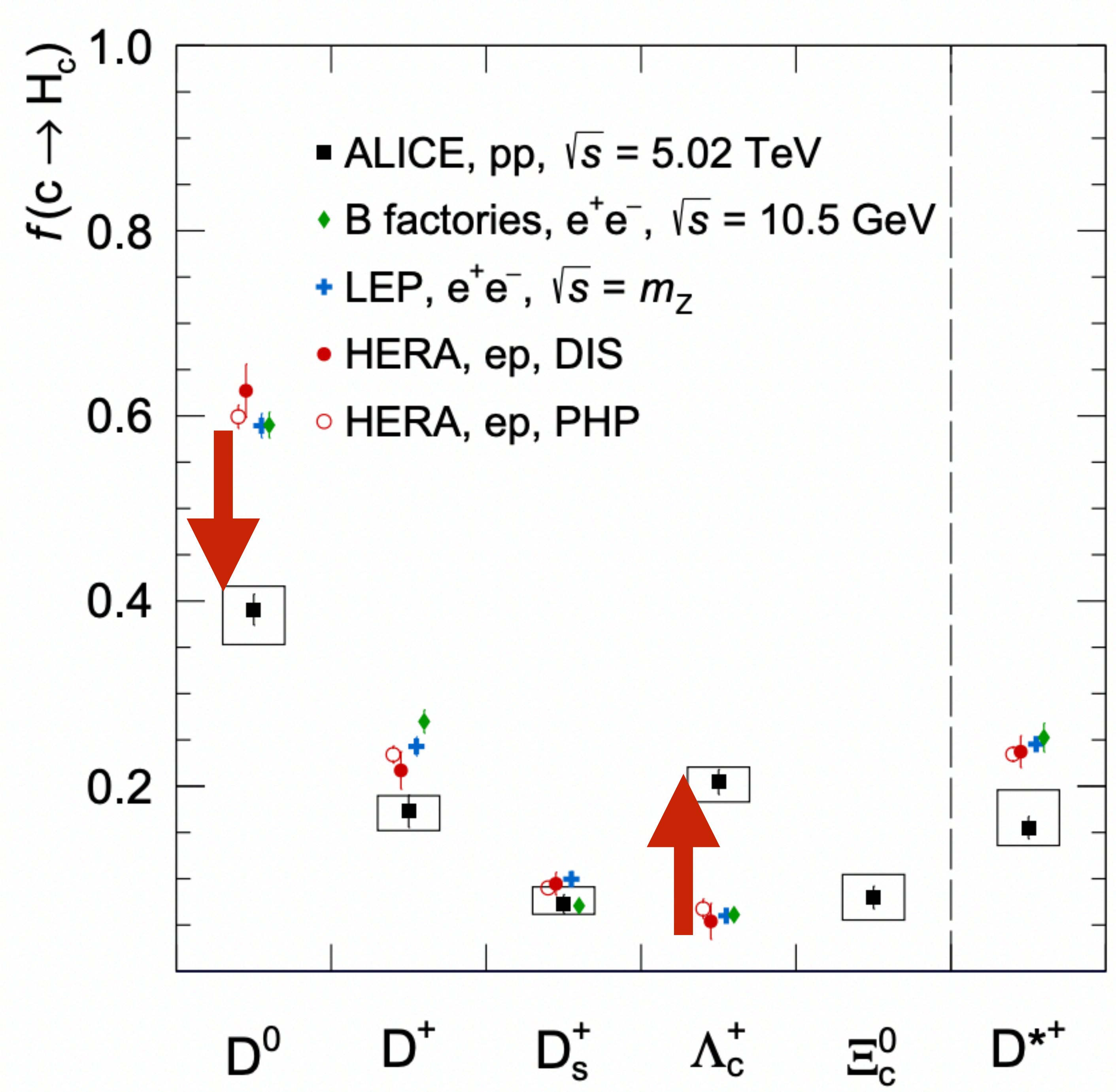
The high energy/multiplicity pp collisions show different ratios

-> a small QGP droplet & the ratio is modified by the recombination probability

Jing Wang's talk



M. Lisovyi et al, Eur.Phys.J.C 76 (2016) 7, 397

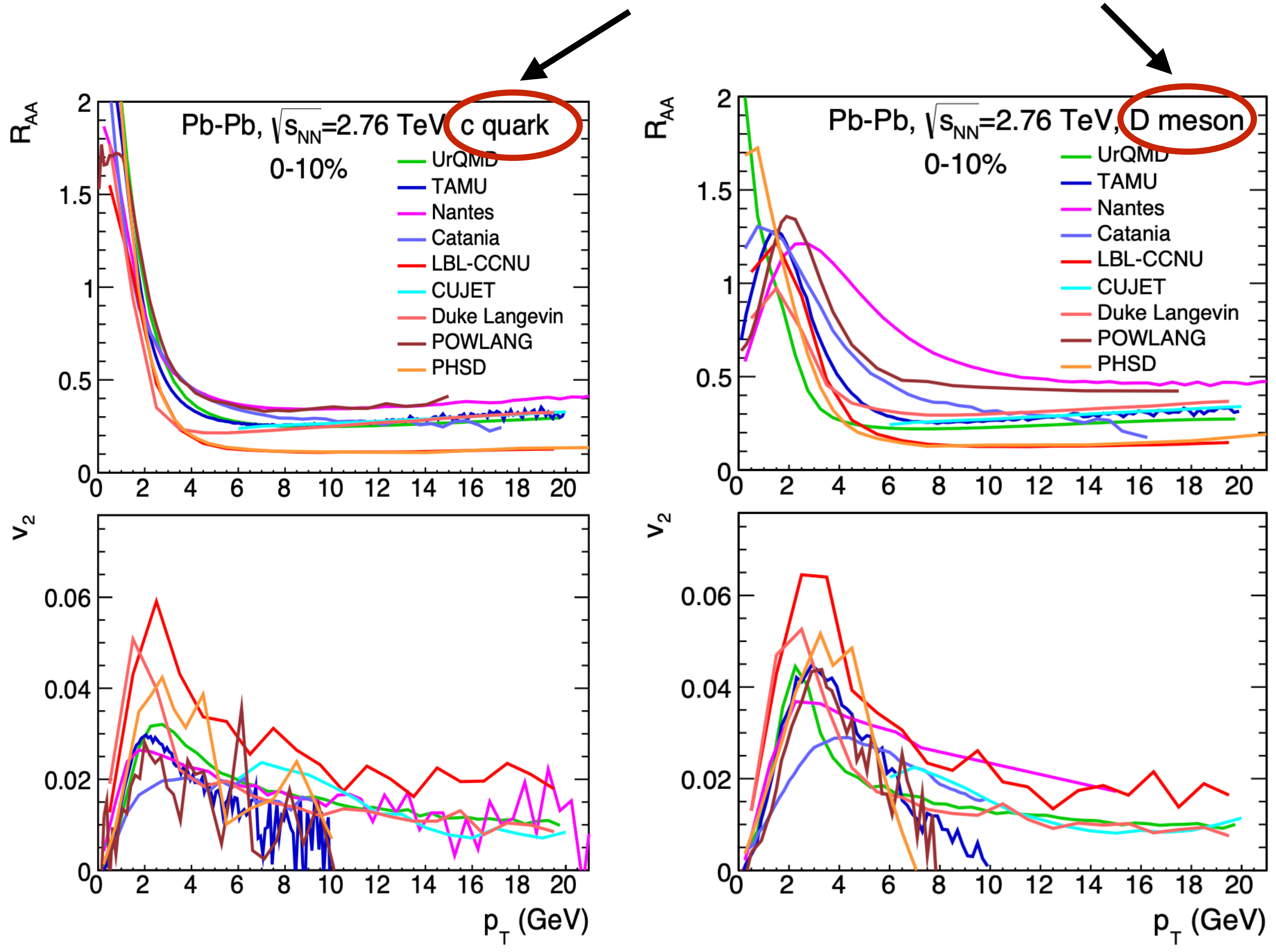


ALICE, Phys.Rev.D 105 (2022) 1, L011103

Model comparison I

❖ Use a common the initial charm distribution & charm quark transport coefficient

Differece comes from the bulk evolution & hadronization mechanism (inlcuding temperature, chemistry,…)



Hadronization makes a larger difference!

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Nuclear Physics A 979 (2018) 21–86

NUCLEAR PHYSICS A

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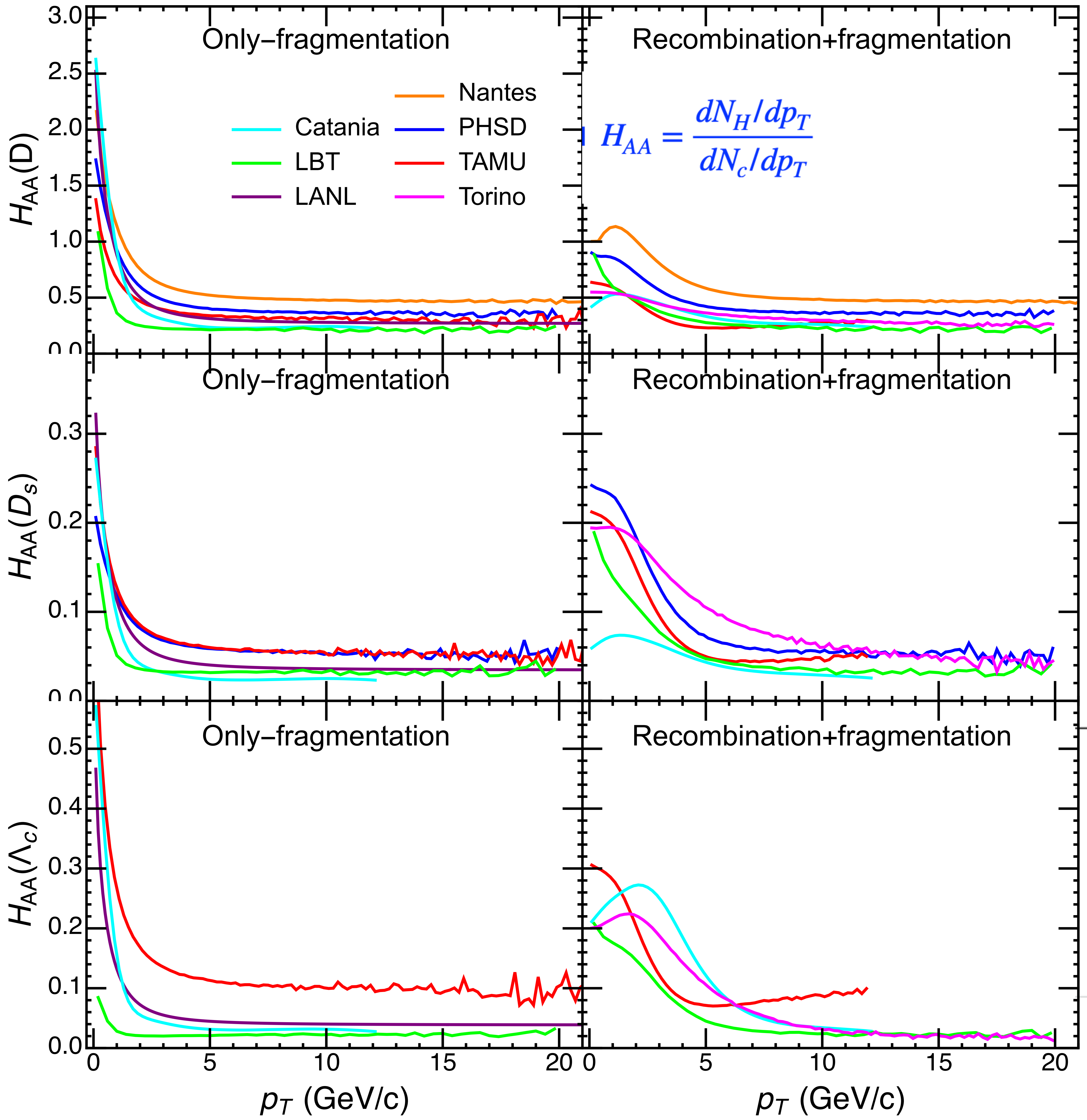
Extraction of heavy-flavor transport coefficients in QCD matter

R. Rapp [a,1], P.B. Gossiaux [b,*,1], A. Andronic [c,d,1], R. Averbeck [c,1], S. Masciocchi [c,1], A. Beraudo [e], E. Bratkovskaya [c,f], P. Braun-Munzinger [c,g], S. Cao [h], A. Dainese [i], S.K. Das [j,k], M. Djordjevic [l], V. Greco [k,m], M. He [n], H. van Hees [f], G. Inghirami [c,f,o,p], O. Kaczmarek [q,r], Y.-J. Lee [s], J. Liao [t], S.Y.F. Liu [a], G. Moore [u], M. Nahrgang [b], J. Pawlowski [v], P. Petreczky [w], S. Plumari [k], F. Prino [e], S. Shi [t], T. Song [x], J. Stachel [g], I. Vitev [y], X.-N. Wang [r,z]

Model comparison II

❖ Fix the Hadronization hypersurface and charm and light quark distributions at hadronization hypersurface.



- Given by the Fireball model for $\sqrt{s_{NN}} = 2.76TeV$ Pb+Pb with b=7fm and Tfo=180MeV.
- Uniform distribution in the coordinate space and momentum space is given by EMMI RRTF.

Catania, Duke, LBT, Los Alamos, Nantes, PHSD, TAMU, Torino groups/models

Still a clear difference, but smaller than before!
These behaviours can be explained by their recombination probabilities and fragmentation functions…

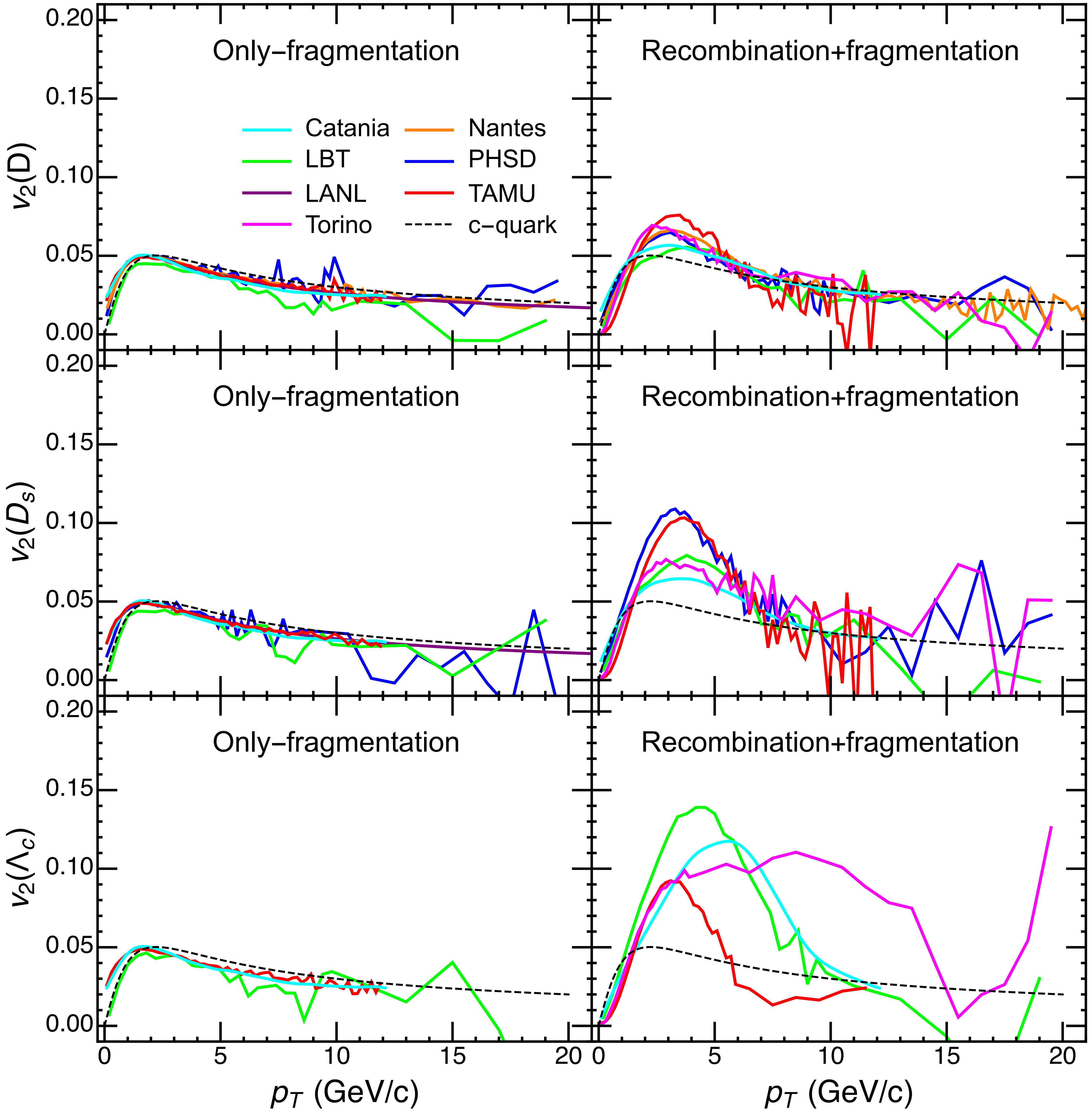
PHYSICAL REVIEW C **109**, 054912 (2024)

Hadronization of heavy quarks

Jiaxing Zhao,[1] Jörg Aichelin,[1] Pol Bernard Gossiaux,[1] Andrea Beraudo,[2] Shanshan Cao,[3] Wenkai Fan,[4] Min He,[5]
Vincenzo Minissale,[6,7] Taesoo Song,[8] Ivan Vitev,[9] Ralf Rapp,[10] Steffen Bass,[4] Elena Bratkovskaya,[8,11,12]
Vincenzo Greco,[6,7] and Salvatore Plumari[6,7]

Model comparison II

❖ Fix the Hadronization hypersurface and charm and light quark distributions at hadronization hypersurface.



The v_2 of charmed hadrons via only-fragmentation process gives the same v_2 as charm quark.

The existence of v_2 sequence: $v_2(\Lambda c) > v_2(Ds) > v_2(D)$.

Final v_2 comes from: charm quark v_2, light quark v_2, recombination probability, fragmentaion ratio, and also excited states…

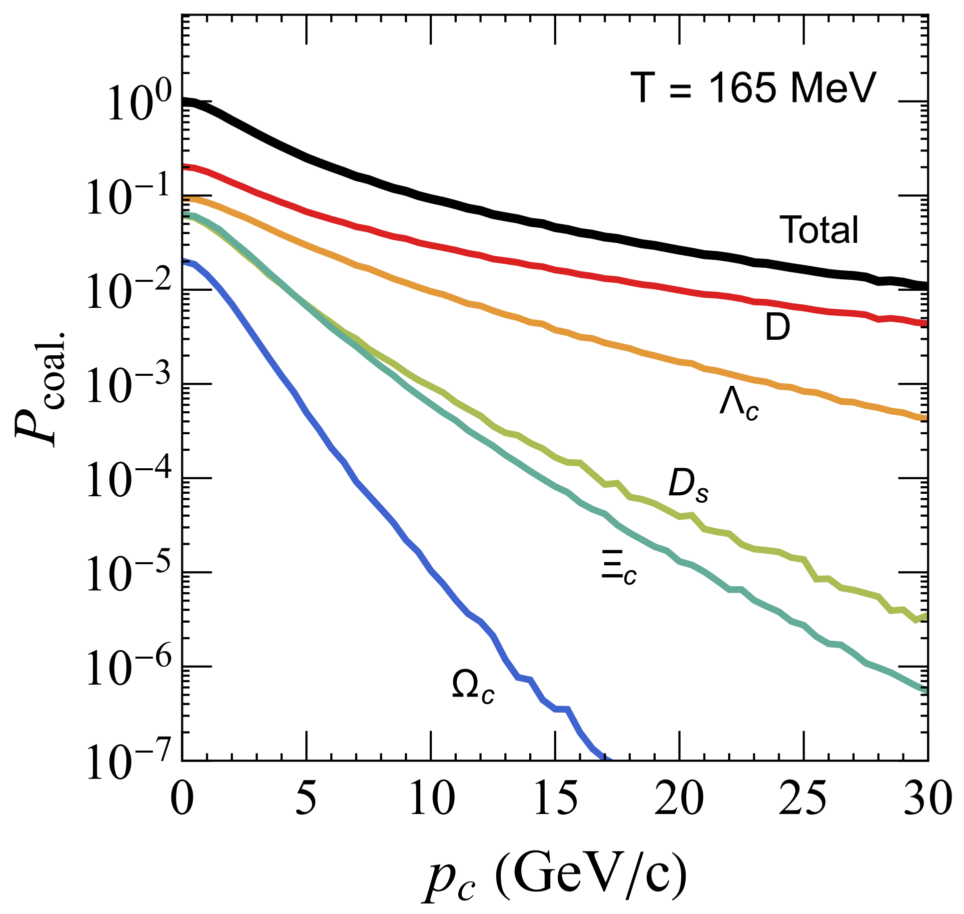
Model comparison II

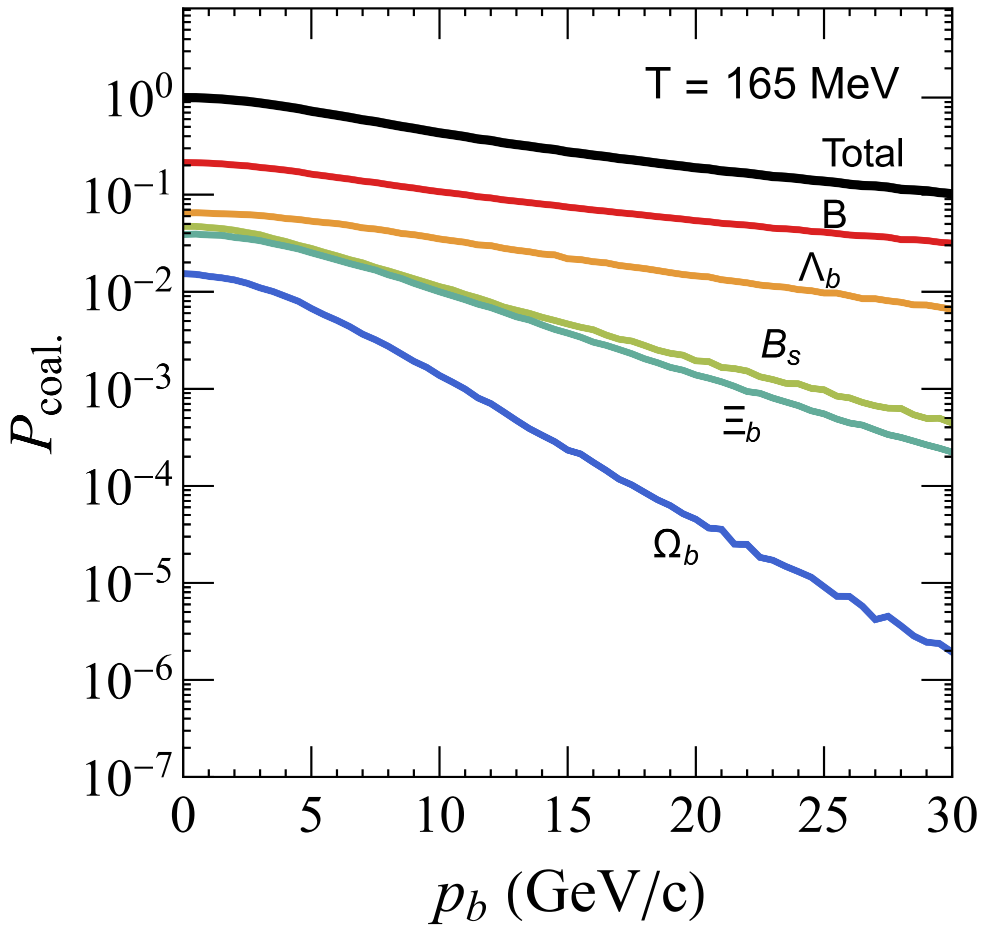
- A big difference shows in these theoretical model but finally give a similar results and comparable with the exp. data. Because there are many other uncontrolled things, such as, QGP background, heavy quark energy loss, hadronic evolution,…

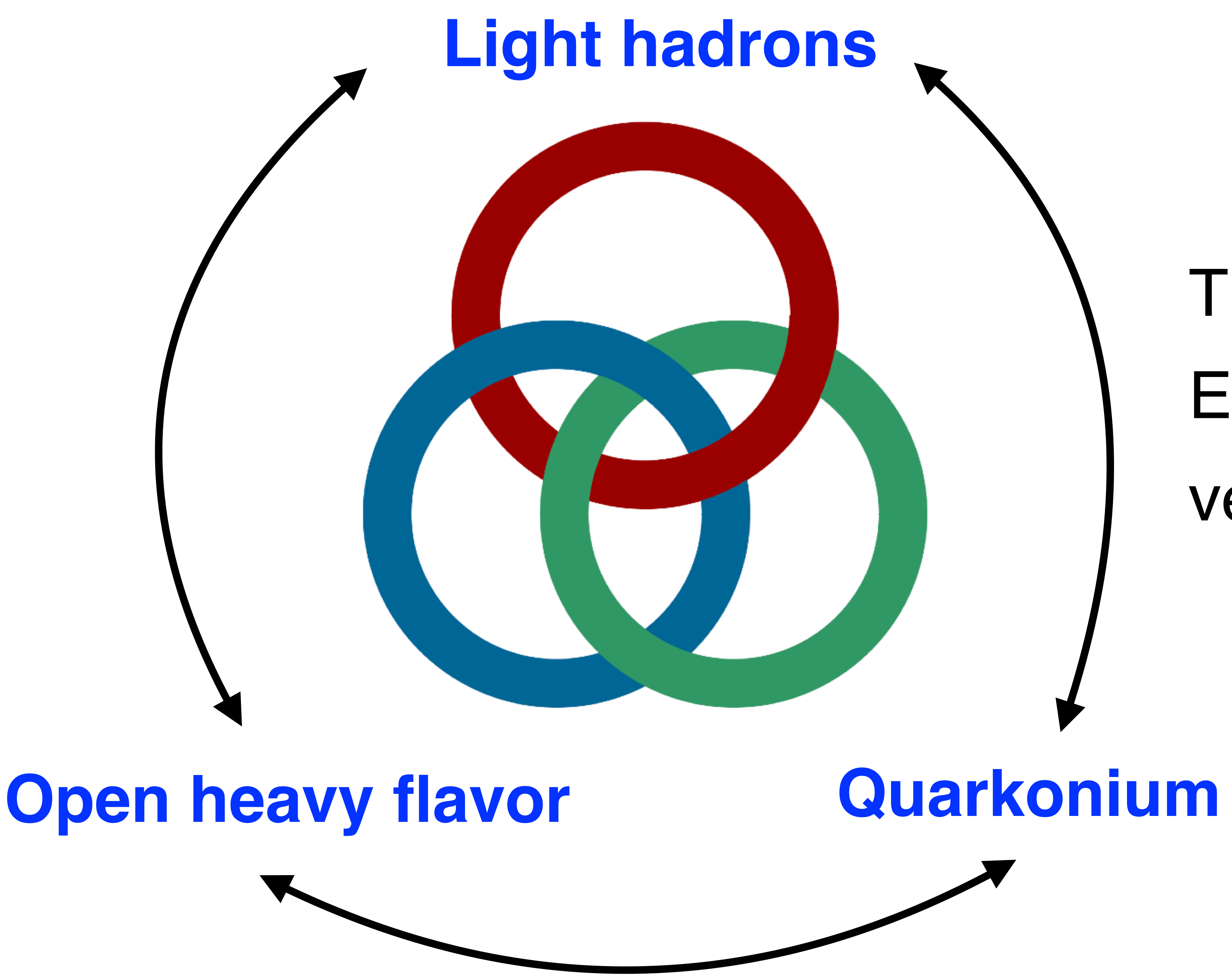
- With more and more precise data, it's still possible to study the mechanism of hadronization: simultaneous vs. sequential hadronisation; phase space recombination vs. momentum space recombination; excited states;…

Bulid a unified framework

To combine the light with heavy, open heavy flavor with quarkonium!







Thermal medium properties:
EOS, lifetime,temperature,
velocity, shear viscosity…

Heavy quark energy loss
Correlations
Quark number conservation
…

➡ **EPOS4** **gives a good description of both charm and bottom hadrons production in pp & heavy ion collisions, central and peripheral collisions, RHIC and LHC energies!**

will be released soon! *JZ, J.Aichelin, P.B. Gossiaux, K.Werner, Phys.Rev.D 109 (2024) 5, 054011; Phys.Rev.C 110 (2024) 2, 024909; arxiv: 2407.20919.*

Summary

- Due to its non-perturbative nature, there is less first-principle input for heavy quark hadronisation, so it is very complicated and model-dependent. But it's very important and cannot be overlooked when studying other heavy flavor-related physics.

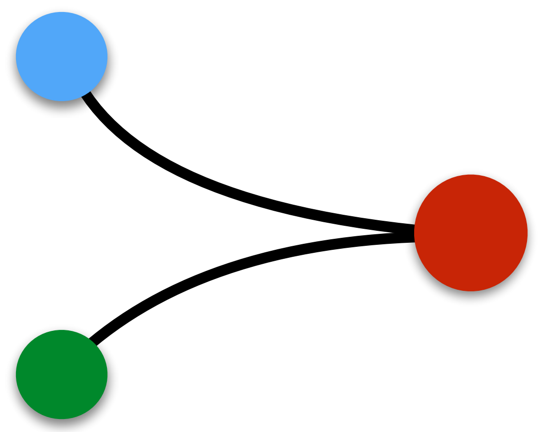
- A big difference shown in different hadronization models. We know where we are and how to impove based on the model comparison work.

- It is really worth combining the light and open heavy flavors and the quarkonium part together. Get constraints from all possible data to further the understanding.

Thanks for your attention!

backup

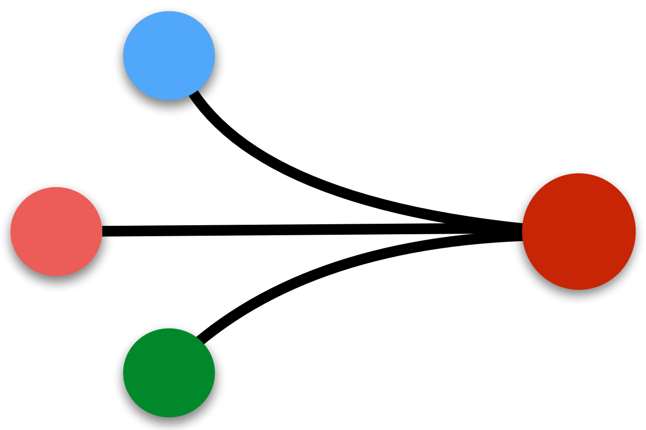
5. on-shell procedure

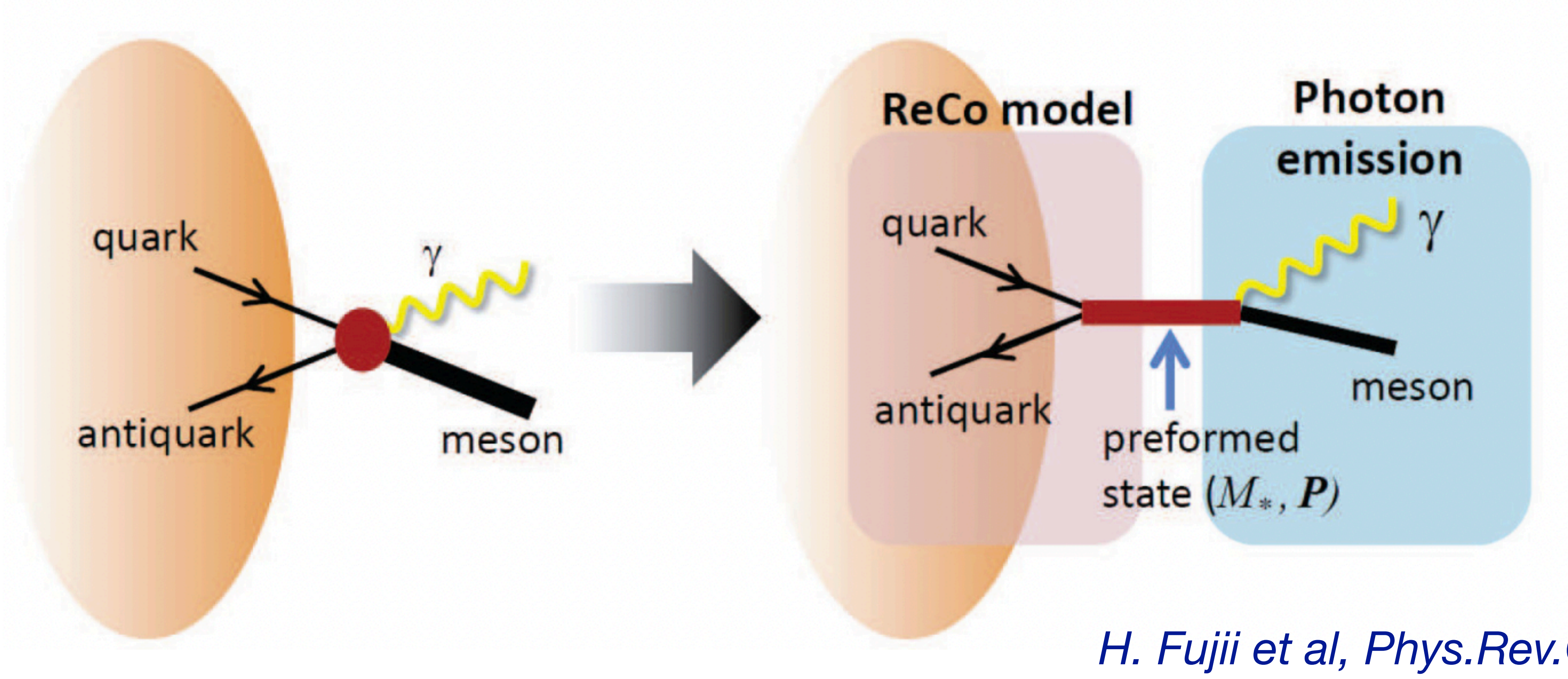


The four-momentum is not conserved !

The energy is not conserved in Coalescence model (it's satisified in the TAMU model)

Physically, this off-shell hadron will emit gluons, **photons**, other light hadrons to become on-shell !





H. Fujii et al, Phys.Rev.C 106 (2022) 3, 034906.

In these heavy flavor models, there are two difference ways:

1, decay into an on-shell charmed hadron and a pion or photon *LBT, Torino*

2, modify the energy with the on-shell hadron mass and keep the 3-momentum unchanged.

Catania, PHSD, EPOS4

EPOS4

EPOS4: A Monte Carlo tool for simulating high-energy scatterings

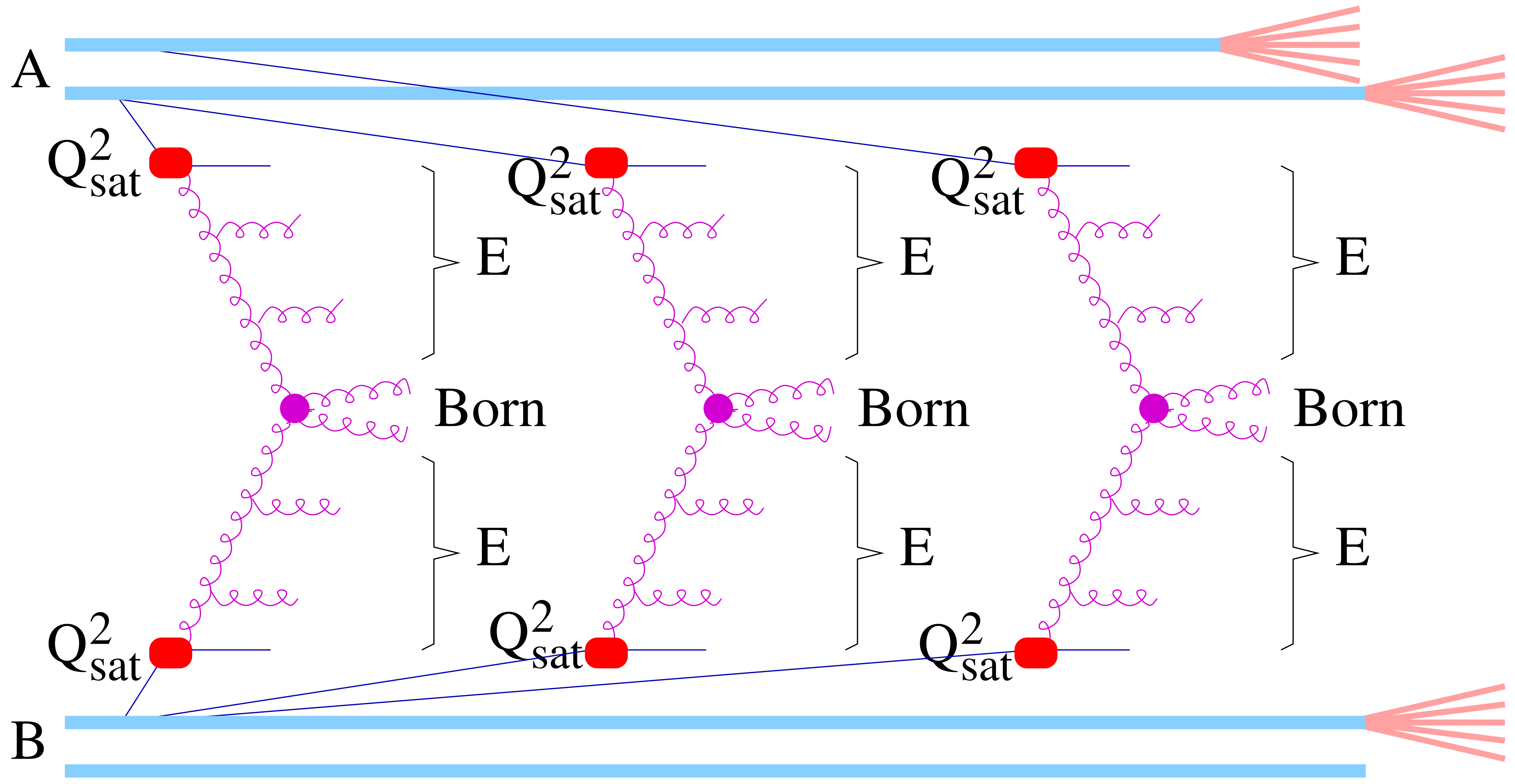
VENUS(1990)—>NEXUS(2000)—> EPOS1(2002)—>EPOS2(2010)—>EPOS3(2013)—>EPOS4(2020)

An abbreviatation of **E**nergy conserving quantum mechanical multiple scattering approach, based on **P**arton (parton ladders), **O**ff-shell remnants, and **S**aturation of parton ladders.

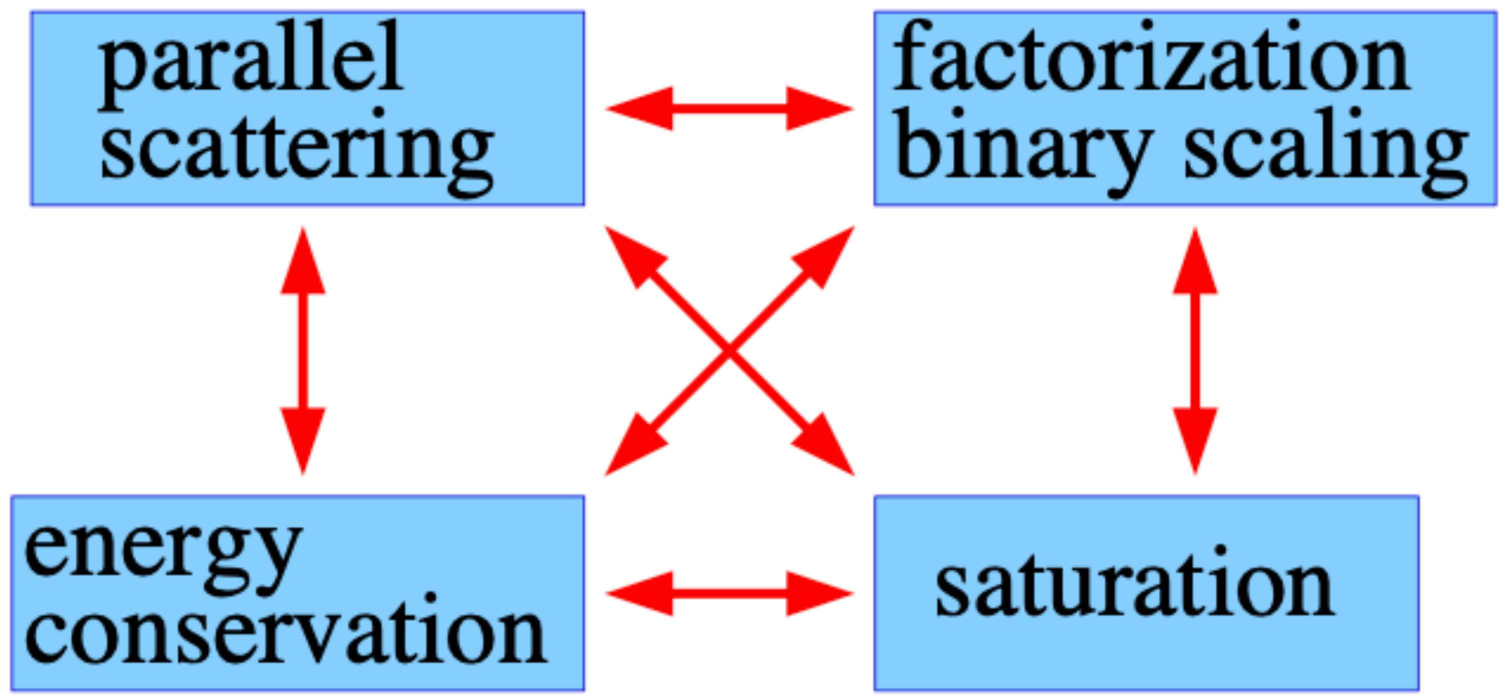
K. Werner. PRC 108 (2023) 6, 064903

K. Werner, B. Guiot, PRC 108 (2023) 3, 034904

K. Werner, PRC 109 (2024) 1, 014910



e.g. three parallel scatterings



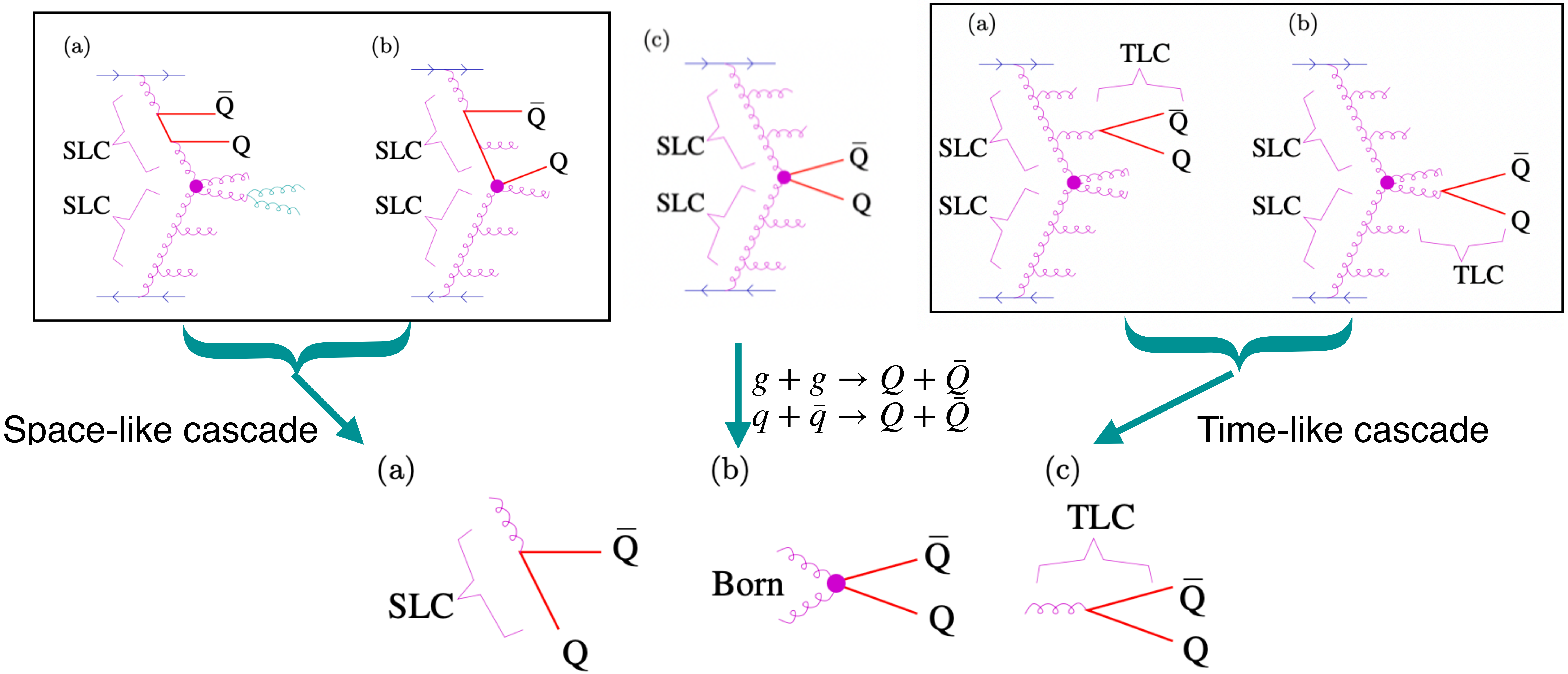
S-matrix theory (to deal with parallel scatterings happens in high energy collisions)

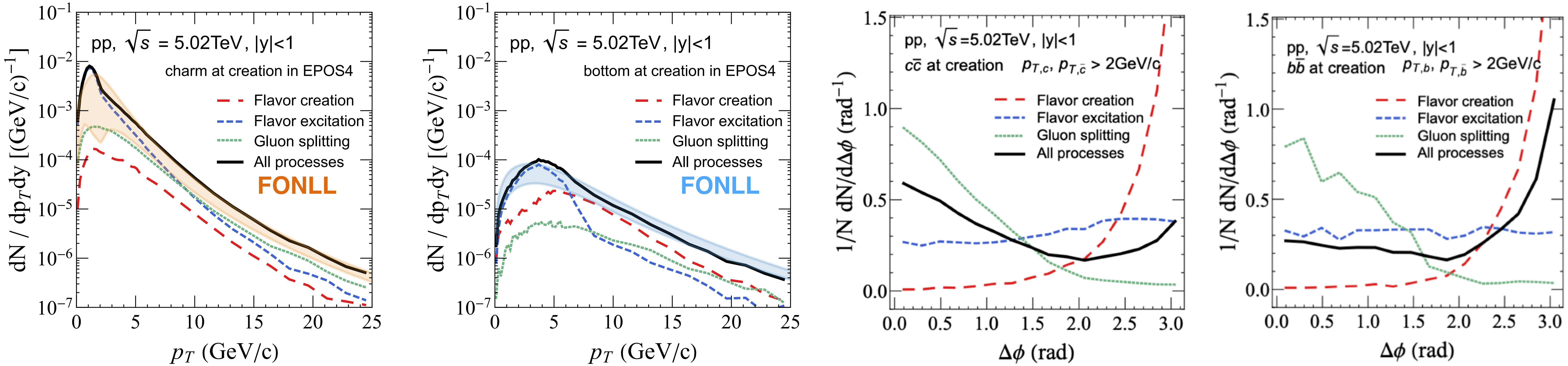
For each one we have a parton evolution according to the DGLAP .

Consistently accommodate these four crucial concepts is realized in the EPOS4!

EPOS4: heavy quark production

Heavy quarks are produced initially via:

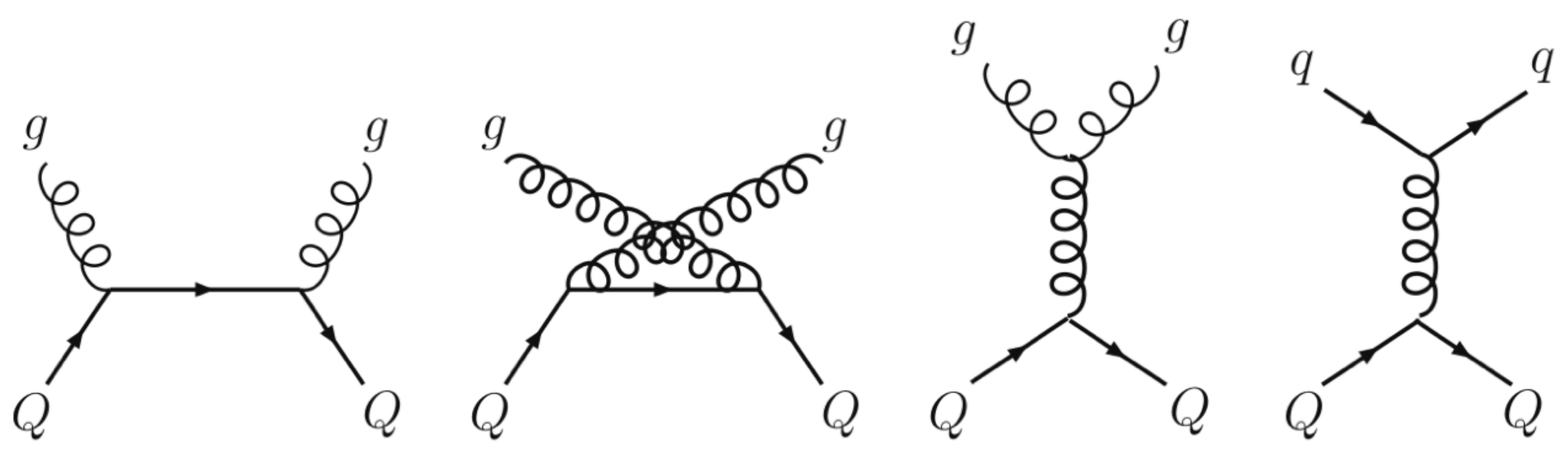


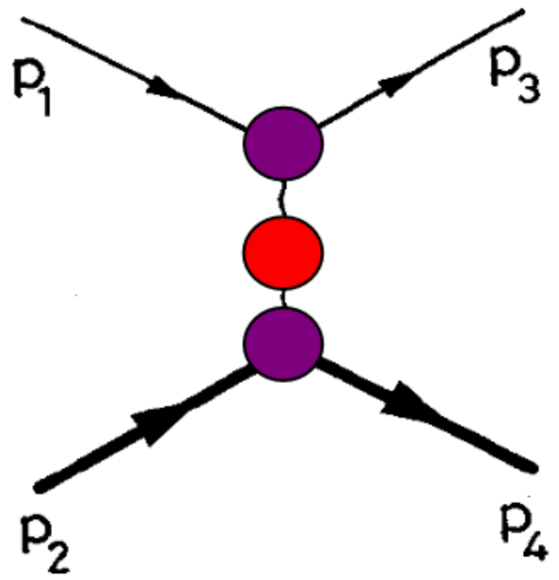


EPOS4: heavy quark energy loss

Heavy quark is treated as a Brownian particle and its evolution is described by the **Boltzmann equation**

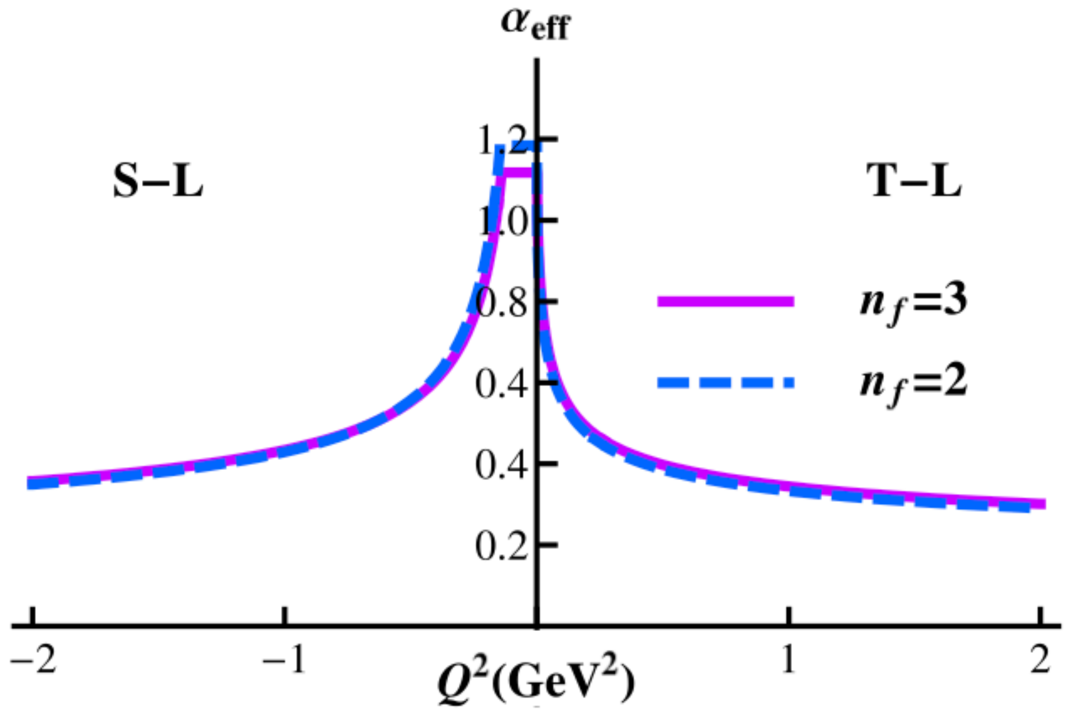
Both collisional and radiative energy loss are included





$$\frac{1}{q^2 - \kappa m_D^2(T)}$$

+ u and s channels

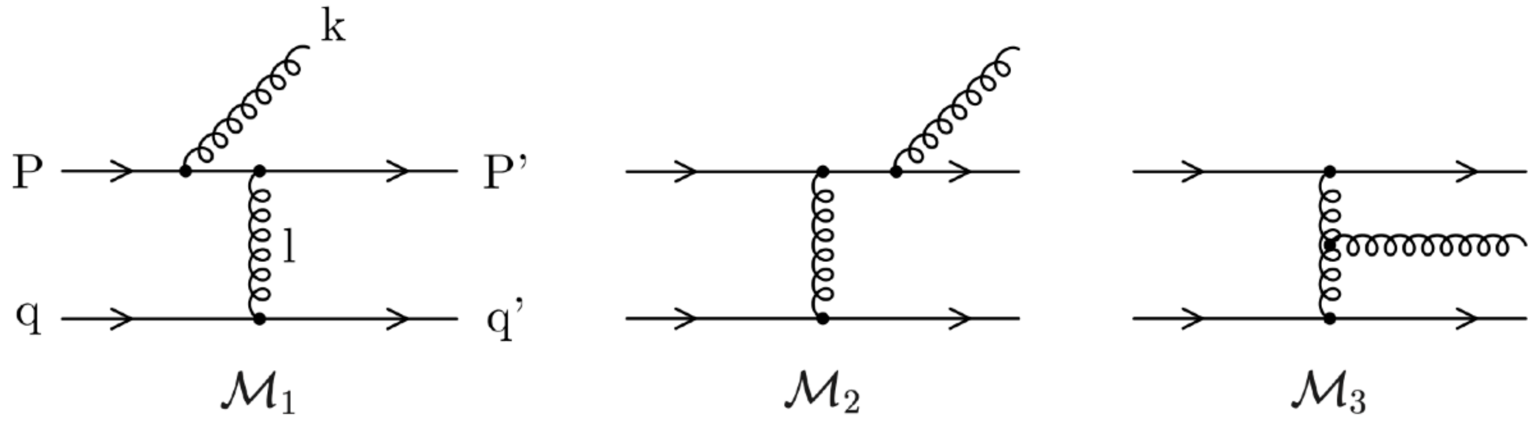


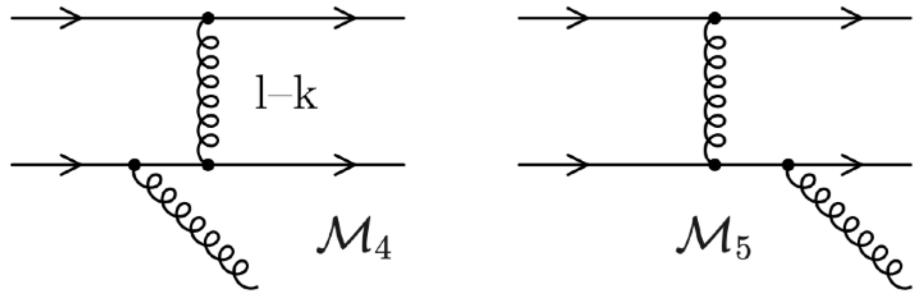
- ➡ IR regulator κm_D^2, where m_D given by HTL
- ➡ Running coupling

$$m_D^2(T) = (1 + \frac{N_f}{6})4\pi\alpha_s T^2$$

$$\alpha \rightarrow \alpha_{\text{eff}}(Q^2) = \frac{4\pi}{\beta_0} \begin{cases} L_-^{-1} \\ \frac{1}{2} - \pi^{-1}\text{atn}(L_+/\pi) \end{cases} \quad \text{for } Q^2 \lessgtr 0\,,$$

P.B. Gossiaux. J. Aichelin. Phys.Rev.C 78 (2008) 014904.





$$\frac{d\sigma_{II}^{Qq\rightarrow Qgq}}{dx d^2k_t d^2\ell_t} = \frac{d\sigma_{\text{el}}}{d^2\ell_t} P_g(x, \vec{k}_t, \vec{\ell}_t)\Theta(\Delta)\,.$$

$$P_g(x, \vec{k}_t, \vec{\ell}_t; M) = \frac{C_A\alpha_s}{\pi^2}\frac{1-x}{x}\left(\frac{\vec{k}_t}{\vec{k}_t^2 + x^2M^2} - \frac{\vec{k}_t - \vec{\ell}_t}{(\vec{k}_t - \vec{\ell}_t)^2 + x^2M^2}\right)^2.$$

- ➡ Extension of Gunion-Bertsch approximation (massless and high energy)
- ➡ LPM effect for moderate gluon energy

J. Aichelin, P. B. Gossiaux, and T. Gousset, Phys. Rev. D 89, 074018 (2014)

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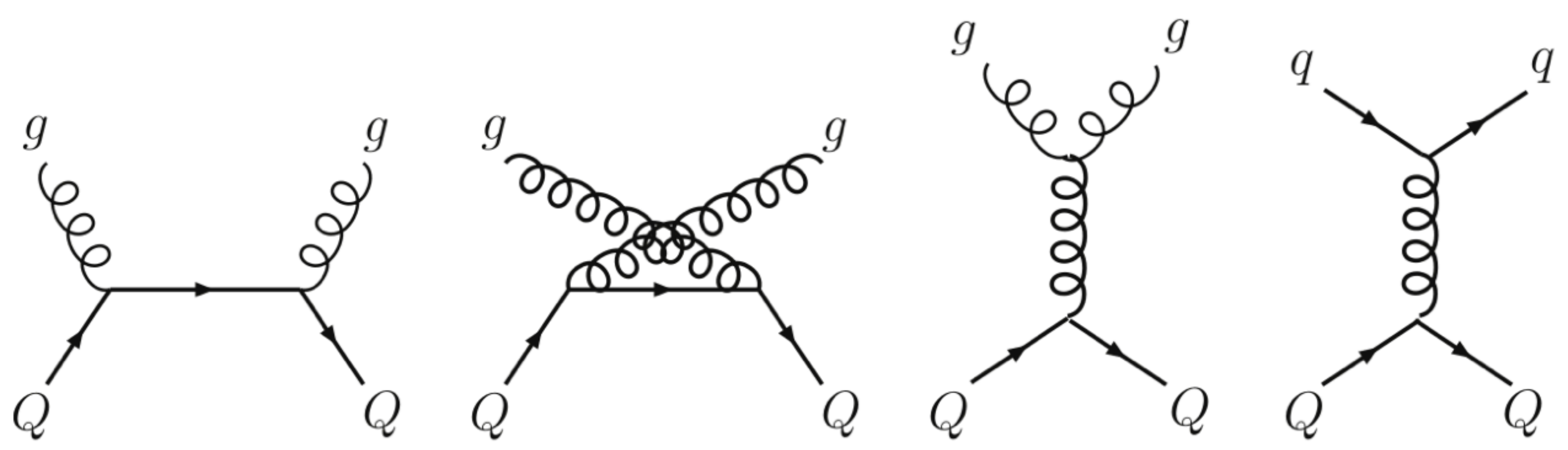
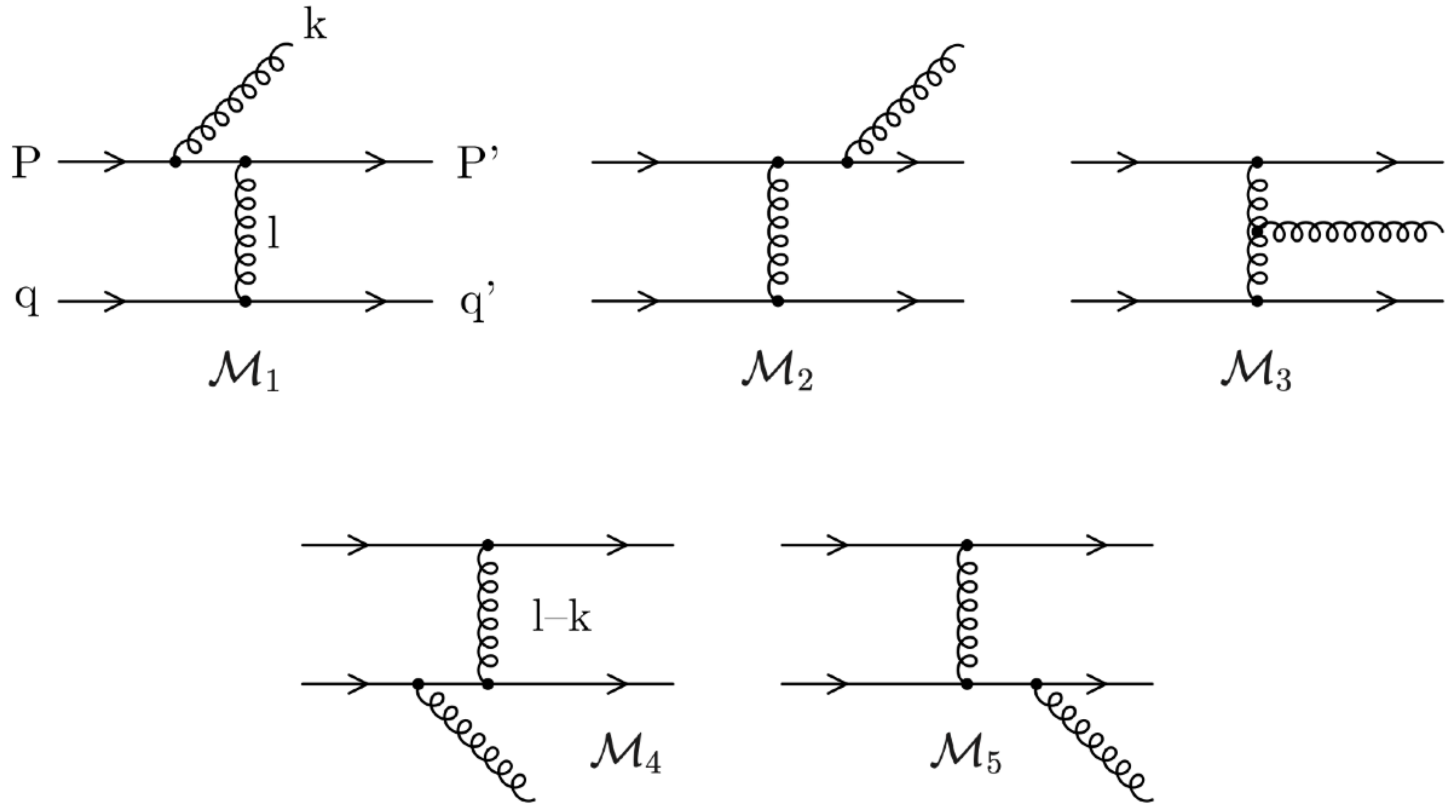
Heavy quark is treated as a Brownian particle and its evolution is described by the **Boltzmann equation**

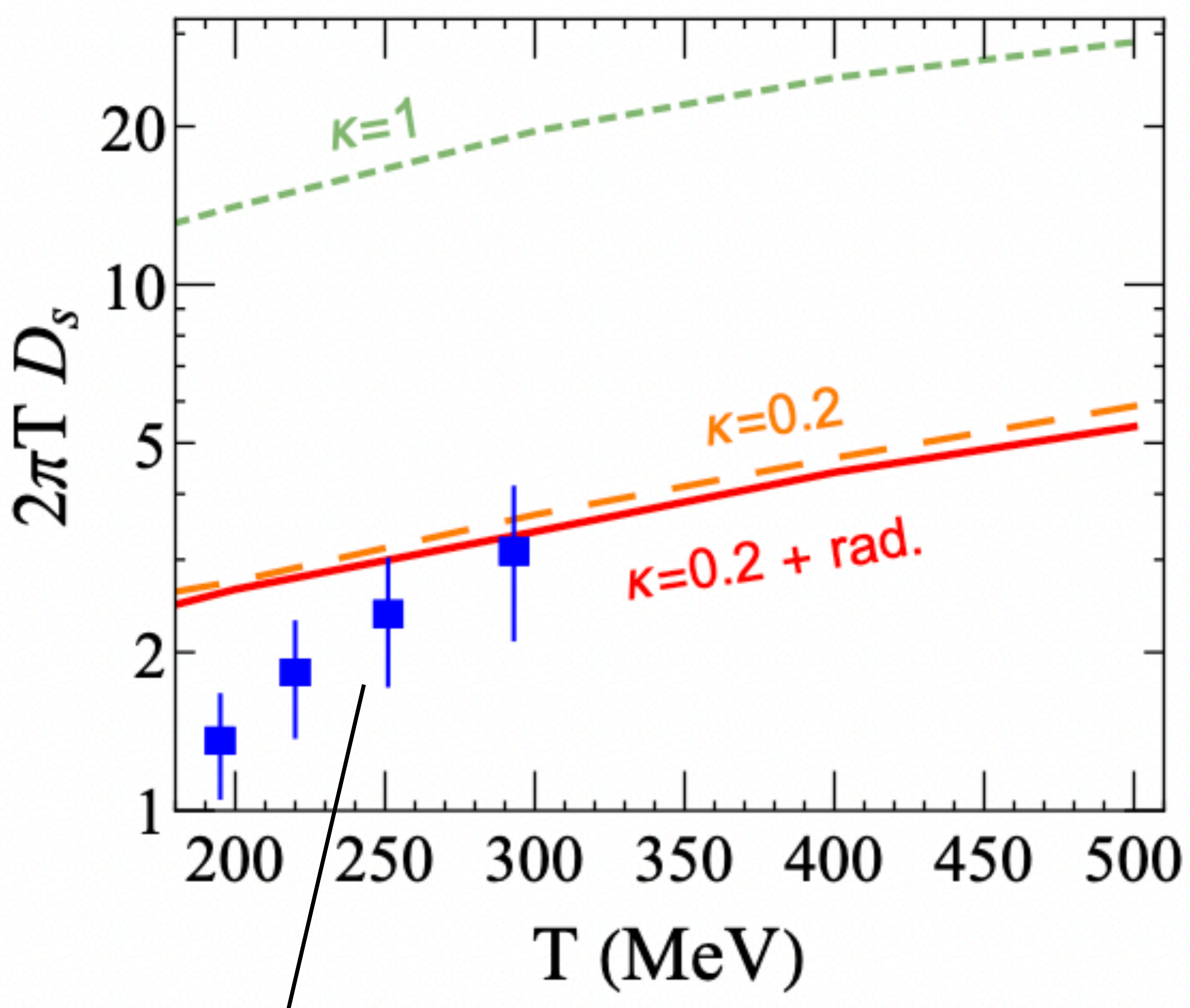
Both collisional and radiative energy loss are included



- IR regulator κm_D^2, where m_D given by HTL
- Running coupling

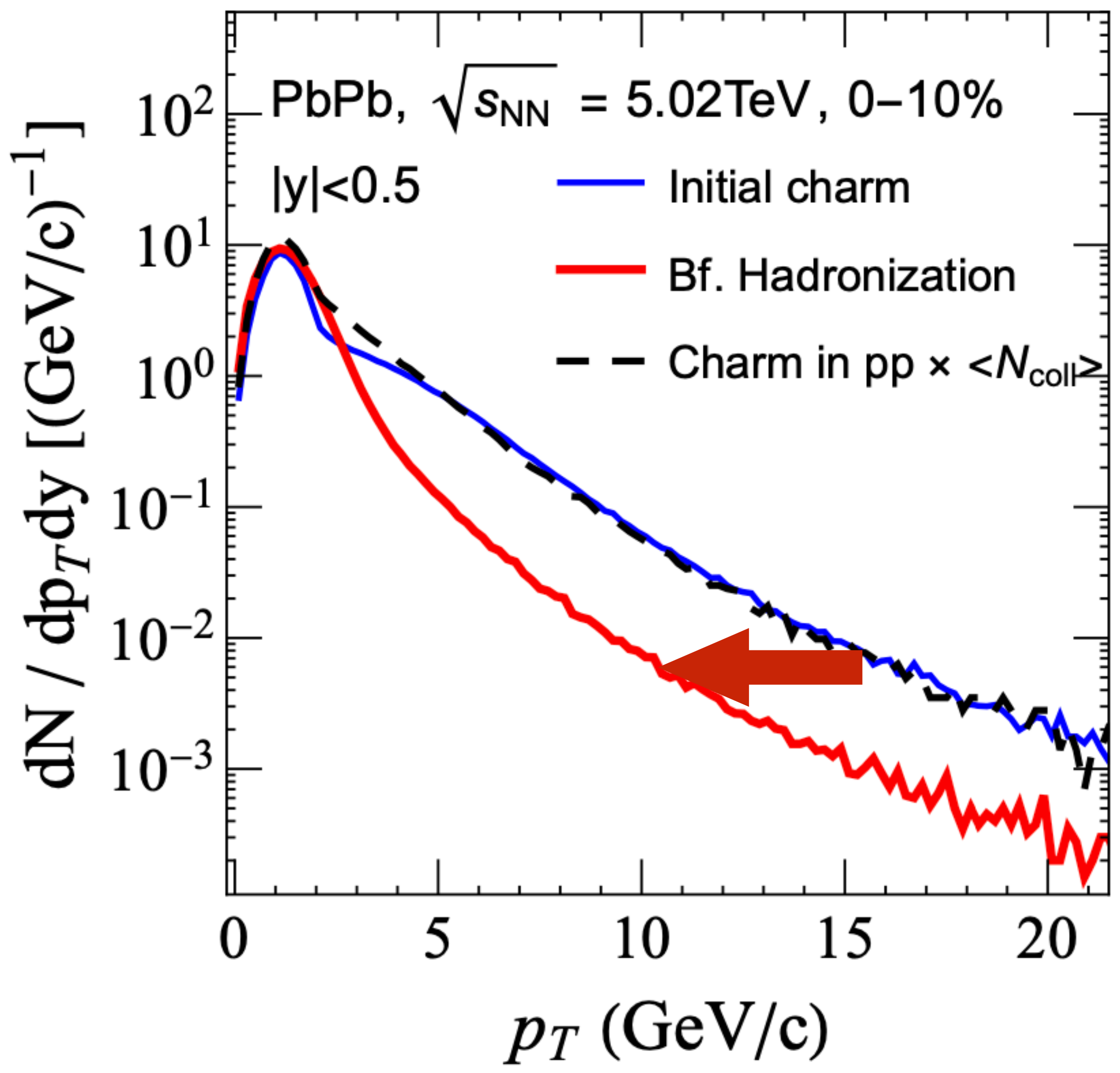


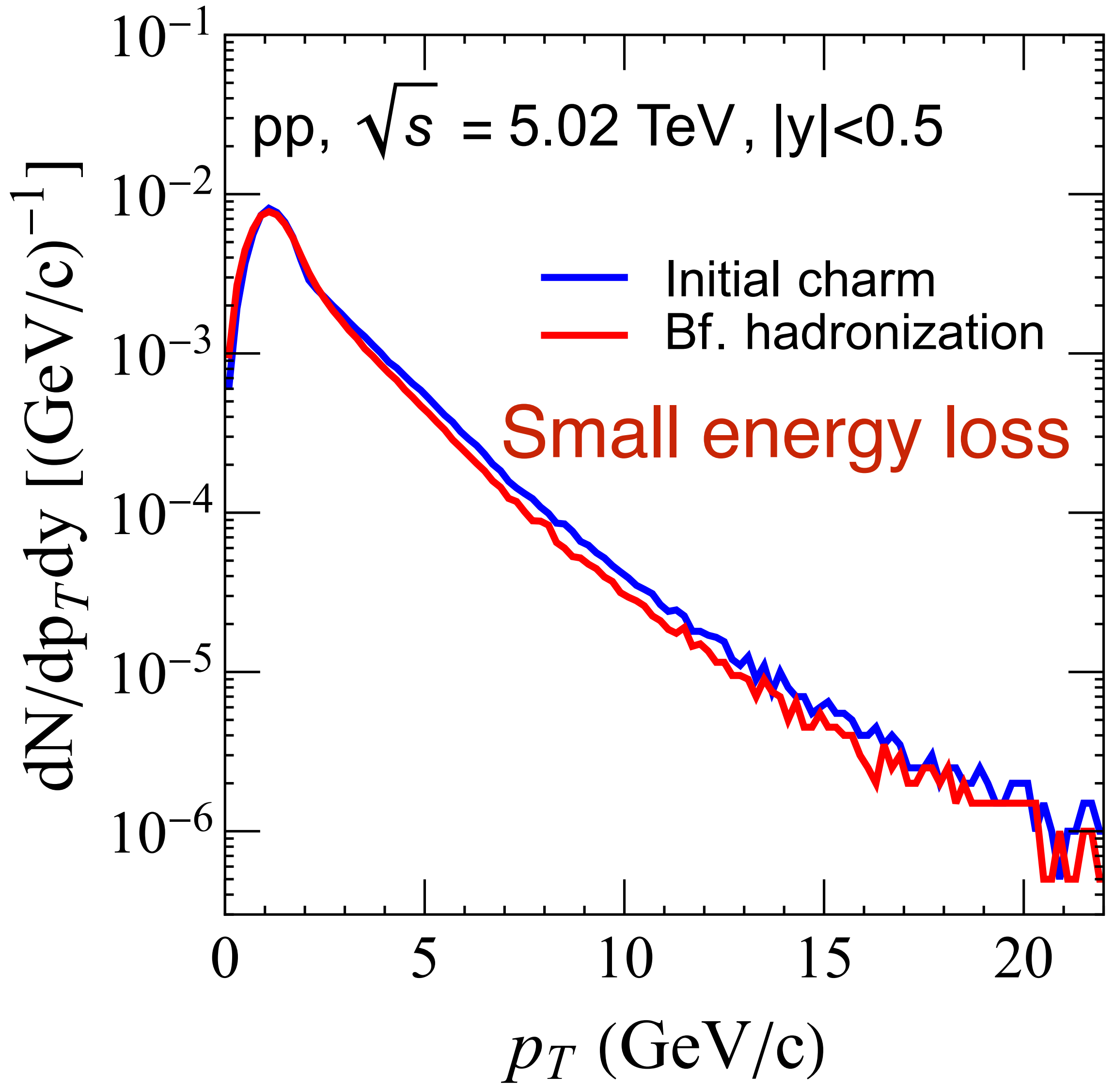
- Extension of Gunion-Bertsch approximation (massless and high energy)
- LPM effect for moderate gluon energy



Recent lattice results with dynamic quarks

Olaf Kaczmarek et al, Phys.Rev.Lett. 130 (2023) 23, 231902





The Influence of bulk evolution models on heavy-quark phenomenology #1

Pol Bernard Gossiaux (SUBATECH, Nantes), Sascha Vogel (SUBATECH, Nantes), Hendrik van Hees (Giessen U.), Joerg Aichelin (SUBATECH, Nantes), Ralf Rapp (Texas A-M, Cyclotron Inst. and Texas A-M) et al. (Feb, 2011)

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Hadronization of heavy quarks

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