

Quarkonium Energy Correlator



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Based on: Chen, Liu, Ma, Phys.Rev.Lett. 133 (2024) 19, 191901

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I. Motivation

II. Quarkonium energy correlator

III. e^+e^- annihilation

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Motivation



➤ Heavy quarkonium

- Bound state of $Q\bar{Q}$ pair under strong interaction
- Production is ideal to understand hadronization: why and how quarks become hadrons?
- NRQCD factorization Bodwin, Braaten, Lepage, 9407339

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} = \sum_n d\hat{\sigma}_n(P_H) \langle \mathcal{O}_n^H \rangle$$

Production of a heavy quark pair
Expansion in: α_s

Hadronization (LDMEs)
Expansion in: v

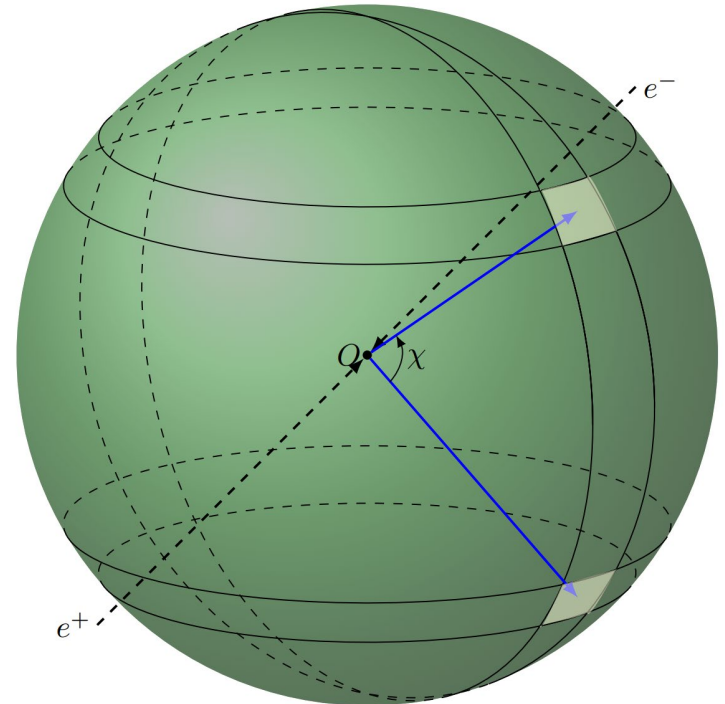
- n : quantum numbers of the pair: $^{2S+1}L_J^{[c]}$
- Hadronization: remains largely unknown: amount of energy released?
Energy Distribution?

➤ Energy-energy correlator (EEC)

□ Angular correlation weighted by energy deposit
in the Calorimeter Basham, Brown, Ellis and Love, Phys. Rev. Lett. 41, 1585 (1978).

$$\frac{d\Sigma}{d \cos \chi} = \sum_{i,j} \int \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos \chi) d\sigma,$$

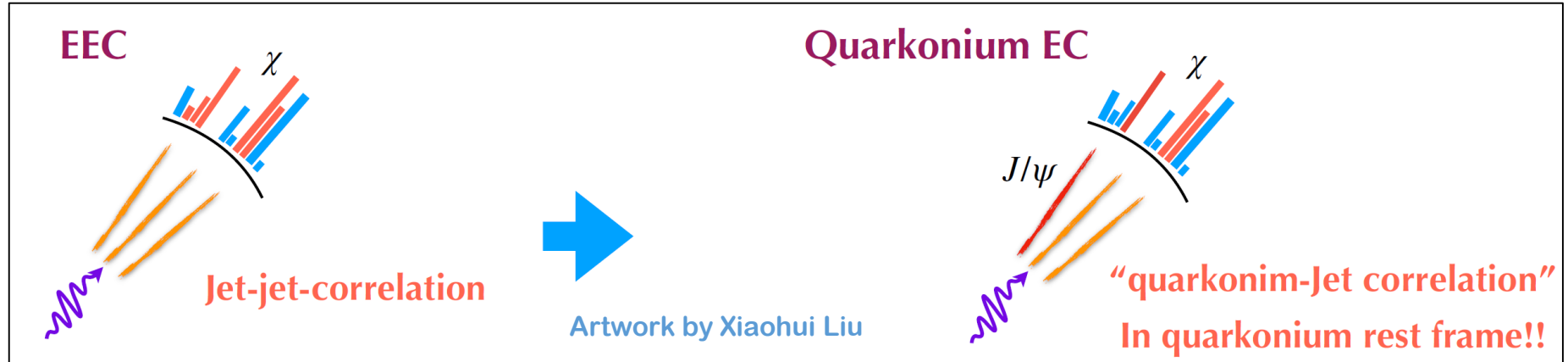
- An infrared safe observable
- Nonperturbative effect is supposed to be small
- Calculable in perturbative QCD



arXiv:1801.02627

➤ Quarkonium Energy Correlator

Chen, Liu, Ma, arXiv:2405.10056



$$\Sigma_{\text{EEC}} = \frac{1}{\sigma} \int d\sigma \sum_{i,j} \frac{E_i E_j}{Q^2} \delta(\cos \theta_{ij} - \cos \chi) \Rightarrow \Sigma_{\text{QEC}} \propto \frac{1}{\sigma_{J/\psi}} \int d\sigma_{J/\psi} \sum_i \frac{E_i}{M} \delta(\cos \theta_i - \cos \chi)$$

~ average energy at the angle χ

- $\Sigma_{\text{QEC}} = \Sigma_{\text{QEC,P.T.}} + \Sigma_{\text{QEC,N.P.}}$
- Hadronization could be large
- Probe the average energy emitted during the hadronization
- Distinguish between different production mechanisms (CS or CO mechanisms in the NRQCD)

Quarkonium energy correlator



➤ Definition

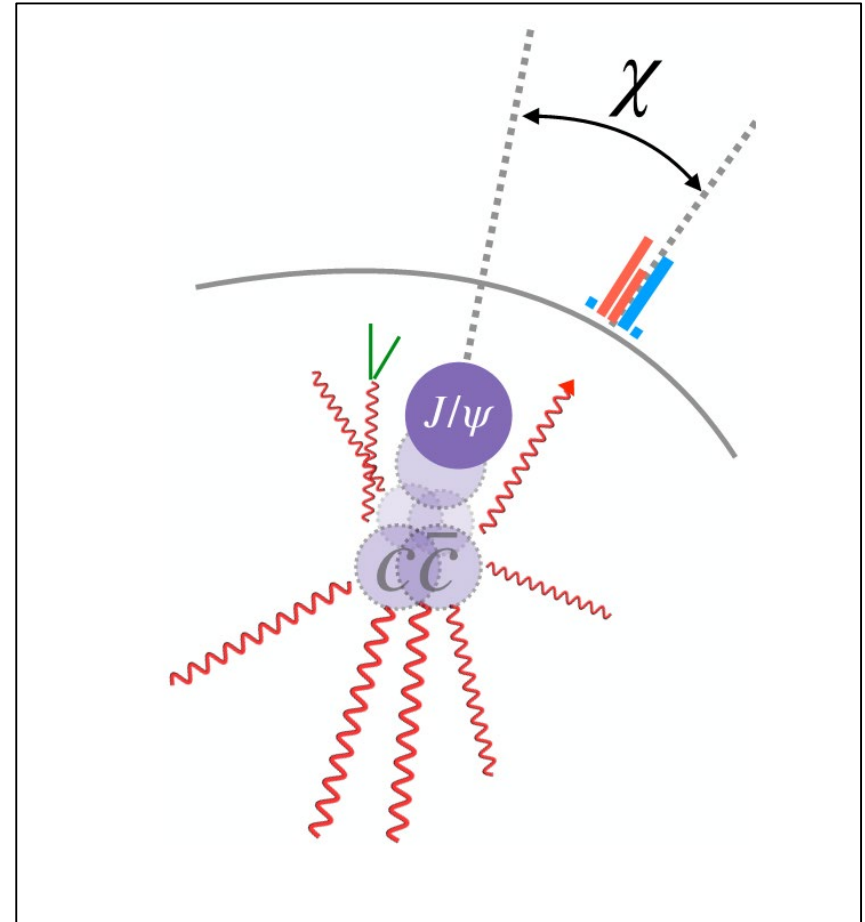
$$\Sigma(\cos \chi) = \int d\sigma \sum_i \frac{E_i}{M} \delta(\cos \chi - \cos \theta_i),$$

$d\sigma$: the differential cross section for generating J/ψ

χ : the angular of detector relative to the flying direction of J/ψ

E_i : the total energy carried by particles propagating at the angle χ

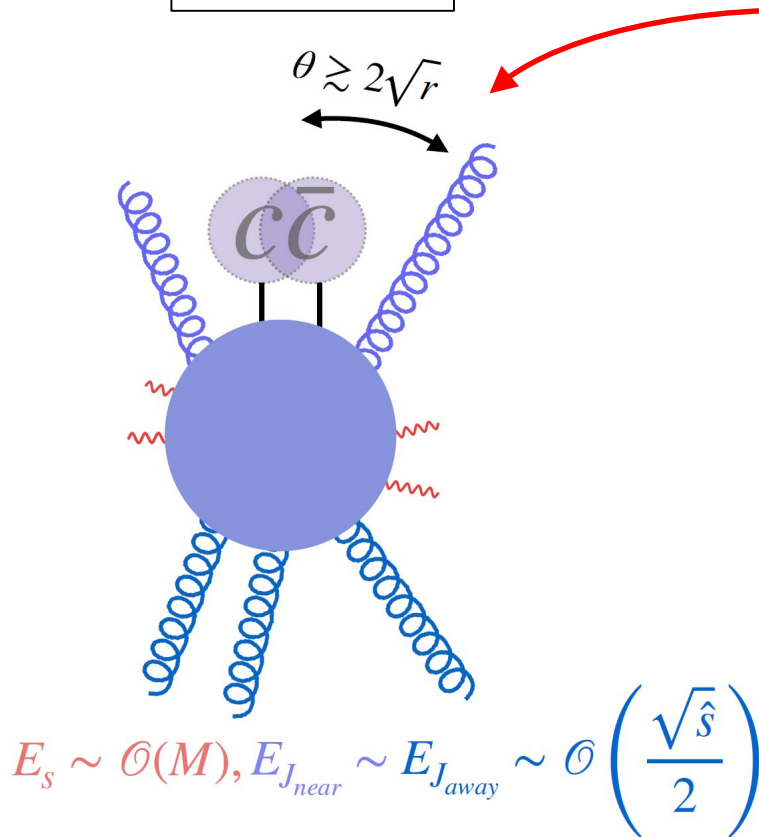
M : the mass of the J/ψ



➤ Generic J/ψ production configuration

COM frame

$$r \equiv \frac{M^2}{\hat{s}} \ll 1$$



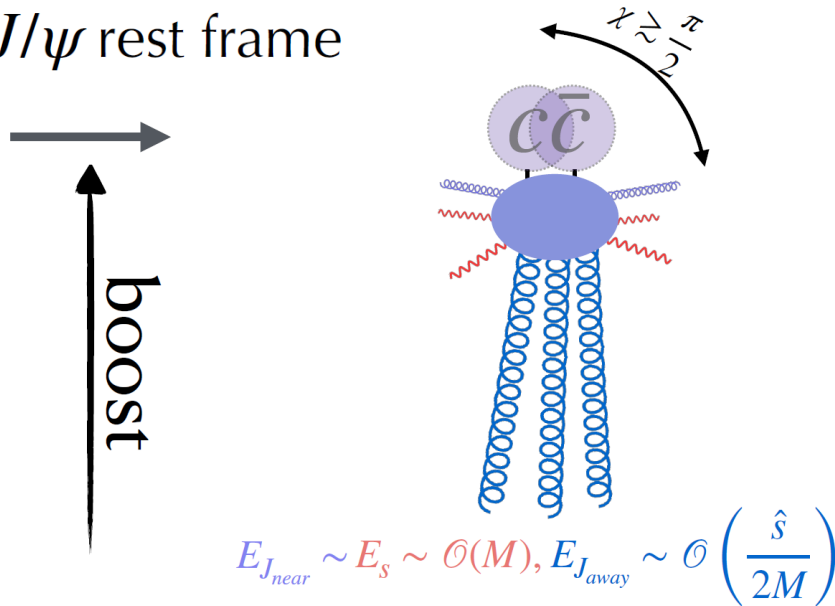
- dead-cone effects

Dokshitzer, Khoze and Troian, J. Phys. G 17 (1991)

$$d\sigma_{Q \rightarrow Qg} \sim \frac{\alpha_s C_F}{\pi} \frac{dE_g}{E_g} \frac{\theta^2 d\theta^2}{[\theta^2 + \theta_0^2]^2}$$

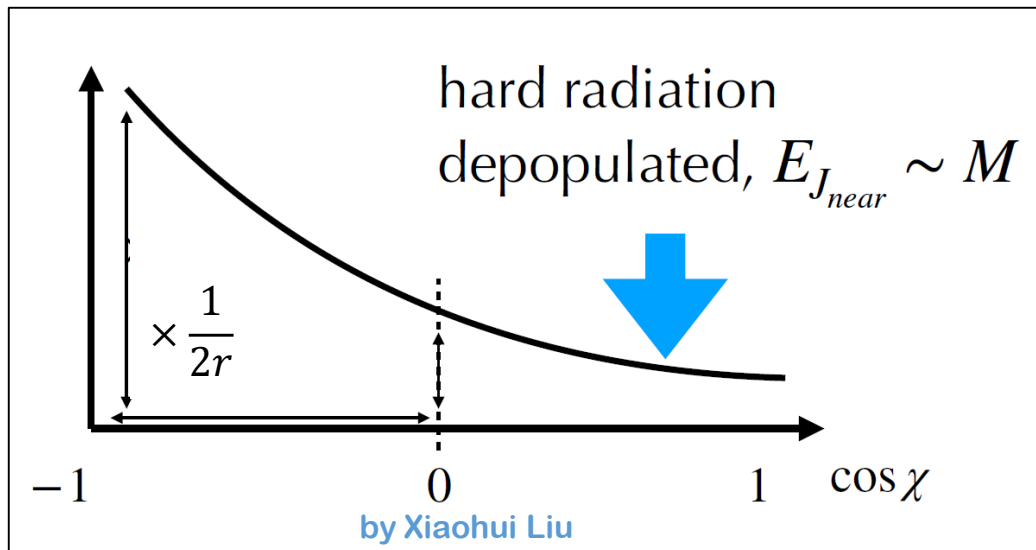
$$\theta_0 \sim \frac{M}{E_{J/\psi}} \sim \frac{2M}{\sqrt{\hat{s}}} = 2\sqrt{r}$$

J/ψ rest frame



- **boost factor** $a \sim \frac{\sqrt{\hat{s}}}{M} = \sqrt{\frac{1}{r}}$
- $E_{J_{away}} \sim a \frac{\sqrt{\hat{s}}}{2} = \frac{\hat{s}}{2M}$
- $E_{J_{near}} \sim \mathcal{O}(M)$
- $\chi \geq \mathcal{O}\left(\frac{\pi}{2}\right)$

COM frame



- $\Sigma_{\text{QEC,P.T.}}$ drops rapidly by a factor of

$$O\left(E_{J_{away}}/E_{J_{near}}\right) \sim O\left(\frac{1}{2r}\right)$$

e^+e^- annihilation



➤ $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + X$

□ Leading $\Sigma_{P.T.}$: $e^+e^- \rightarrow \gamma^* \rightarrow c\bar{c}[{}^3S_1^{[1]}] + g + g$

Defining: $r = \frac{4m_c^2}{s}$, $z = \frac{2p_\psi \cdot q_{\gamma^*}}{s}$, $x_i = \frac{2k_i \cdot q_{\gamma^*}}{s}$, $z_i = \frac{2k_i \cdot p_\psi}{4m_c^2}$,

⇒

$$\Sigma_{P.T.}^{3S_1^{[1]}}(\cos \chi) = \frac{256\pi e_c^2 \alpha^2 \alpha_s^2}{81s} \frac{\mathcal{O}^{J/\psi}({}^3S_1^{[1]})}{(2m_c)^3} r^2$$
$$\times \int_{2\sqrt{r}}^{1+r} dz \int_{x_-}^{x_+} dx_1 f(z, x_1) \sum_{i=1}^2 \frac{z_i}{2} \delta(\cos \chi - \cos \theta_i),$$

with $z_1 = \frac{1}{2r}(x_1 - x_2 + z - 2r)$, $z_2 = \frac{1}{2r}(x_2 - x_1 + z - 2r)$,

$$x_2 = 2 - x_1 - z, \quad x_{\pm} = \frac{1}{2} (2 - z \pm \sqrt{z^2 - 4r})$$

$$f(z, x_1) = \frac{1}{(2-z)^2} \left(\frac{1}{r} + \frac{(2+x_1)x_1}{(1-x_2-r)^2} + \frac{(2+x_2)x_2}{(1-x_1-r)^2} \right. \\ \left. + \frac{2(1-z)(1-r)}{r(1-x_2-r)(1-x_1-r)} + \frac{(2-z)^2((z-r)^2-1)}{(1-x_2-r)^2(1-x_1-r)^2} \right. \\ \left. + \frac{6(1+r-z)^2}{(1-x_2-r)^2(1-x_1-r)^2} \right), \quad \text{Yuan, Qiao, Chao, 9703438}$$

□ Leading $\Sigma_{N.P.}$: $e^+e^- \rightarrow \gamma^* \rightarrow c\bar{c}[^1S_0^{[8]}, ^3P_J^{[8]}] + g$

$$\Rightarrow \Sigma_{N.P.}^{ch}(\cos \chi) = \int d\Phi_g \frac{k^0}{M} \delta(\cos \chi - \cos \theta) \sum_{\lambda} \hat{\sigma}_{\lambda}^{ch} A_{\lambda}^{ch}(\theta, \phi),$$

$A_{\lambda}^{ch}(\theta, \phi)$: the transition amplitude for $c\bar{c}[ch] \rightarrow J/\psi + g$

Φ_g : the gluon phase space

$$d\Phi_g = \frac{d^4k}{(2\pi)^4} (2\pi) \delta_+(k^2) = \frac{k^0 dk^0 d^2\Omega}{2(2\pi)^3},$$

$\hat{\sigma}_{\lambda}^{ch}$: the total cross section that produces the $c\bar{c}[ch]$ pair with polarization λ

For $1S_0^{[8]}$ channel

- The intermediate $c\bar{c}[1S_0^{[8]}]$ state is unpolarized

⇒ • The soft gluon emission should be isotropic

⇒ • $A^{1S_0^{[8]}}(\theta, \phi) = \frac{1}{4\pi} G_0$, $\frac{1}{4\pi} \int d\Phi_g G_0 \sim \mathcal{O}(\langle \mathcal{O}^{J/\psi}(1S_0^{[8]}) \rangle)$

⇒ $\Sigma_{N.P.}^{1S_0^{[8]}}(\cos \chi) = \hat{\sigma}^{1S_0^{[8]}} \int d\Phi_g \frac{k_0}{M} \frac{G_0}{4\pi} \delta(\cos \chi - \cos \theta) = \hat{\sigma}^{1S_0^{[8]}} \langle \mathcal{O}^{J/\psi}(1S_0^{[8]}) \rangle \frac{m_c \bar{v}_0}{2M}$

$$= \bar{v}_0 \frac{\sigma^{1S_0^{[8]}}}{4},$$

average energy emitted during hadronization

$$\sigma^{1S_0^{[8]}} = \frac{32\pi^2 \alpha^2 \alpha_s e_c^2}{3s^2 m_c} (1 - r) \langle \mathcal{O}^{J/\psi}(1S_0^{[8]}) \rangle$$

For ${}^3P_J^{[8]}$ channel

- The intermediate $c\bar{c}[{}^3P_J^{[8]}]$ state with $S = 1$ and $L = 1$ is transmitted into the J/ψ through one electrical dipole (E1) interaction,



$$\Sigma_{N.P.}^{3P_J^{[8]}}(\cos \chi) = \int d\Phi_2 \frac{k^0}{M} \delta(\cos \theta - \cos \chi) \sum_{\lambda, L_z} \hat{\sigma}_{\lambda, L_z}^{3P_J^{[8]}} [\{S = 1, L = 1\}^{(8)}] \\ \times A_{\lambda, L_z}^{3P_J^{[8]}} [\{S = 1, L = 1\}^{(8)}](\theta, \phi).$$

$$A_{\lambda, L_z}^{3P_J^{[8]}} [\{S = 1, L = 1\}^{(8)}](\theta, \phi)$$

$$\propto \sum_{S_z} A[c\bar{c}[(1L_z; 1\lambda)] \rightarrow J/\psi(S_z) + g] A^*[c\bar{c}[(1L_z; 1\lambda)] \rightarrow J/\psi(S_z) + g],$$

$$\begin{aligned} \Sigma_{N.P.}^{3P_0^J}(\cos \chi) &= \int d\Phi_g \frac{k^0}{M} \frac{G_1}{2\pi} \delta(\cos \chi - \cos \theta) \times 3 \left(\frac{1 + \cos^2 \theta}{8} \hat{\sigma}_T^{3P_0^{[8]}} + \frac{1 - \cos^2 \theta}{4} \hat{\sigma}_L^{3P_0^{[8]}} \right) \\ &\equiv \bar{v}_1 \times 3 \left(\frac{1 + \cos^2 \chi}{16} \sigma_T^{3P_0^{[8]}} + \frac{1 - \cos^2 \chi}{8} \sigma_L^{3P_0^{[8]}} \right). \end{aligned}$$

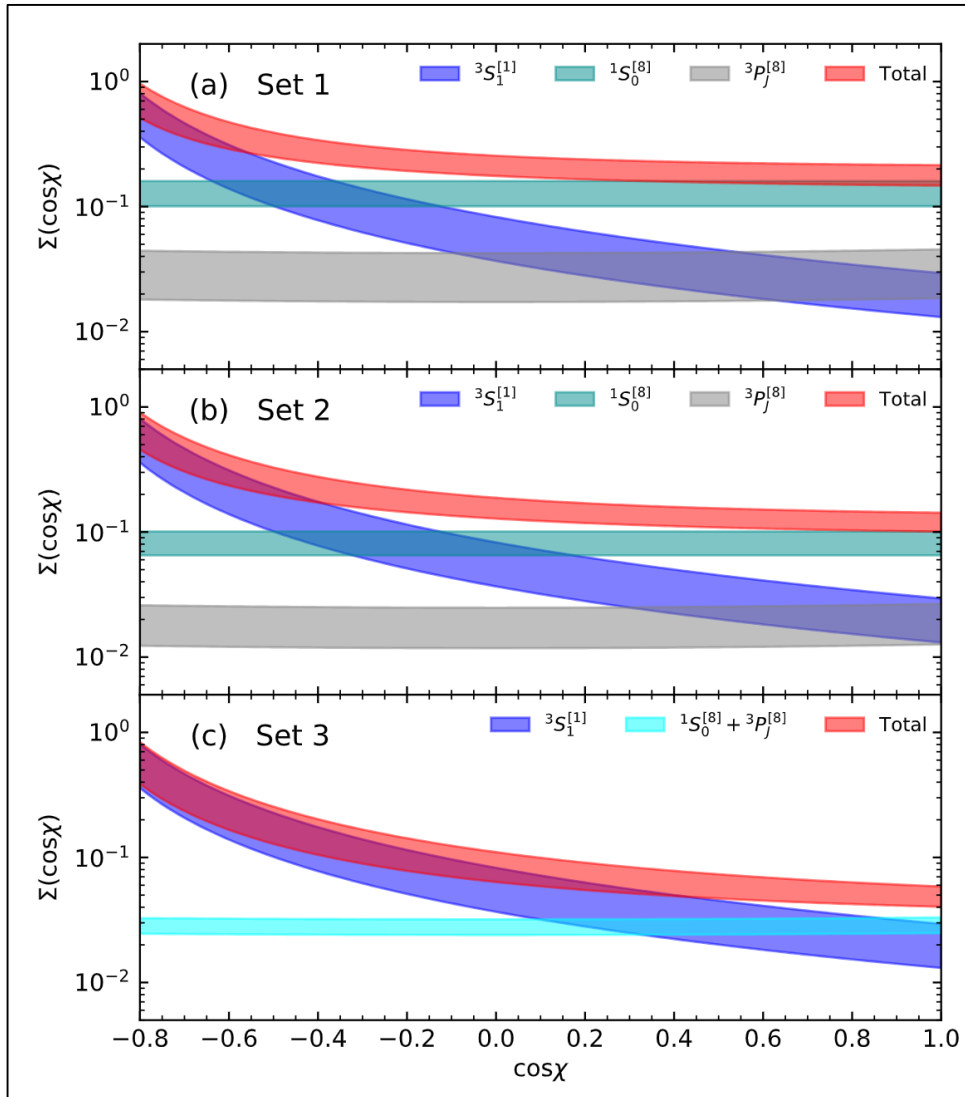
- with \bar{v}_1 → average energy emitted during hadronization

$$\sigma_T^{3P_0^{[8]}} = -F \frac{2}{(1-r)^4} \left(3r^5 + r^4 + 12r^3 - 16r^2 - 4(r^2 + 3r + 2)r^2 \ln r + r - 1 \right),$$

$$\sigma_L^{3P_0^{[8]}} = -F \frac{1+r}{(1-r)^4} \left(r^4 - 22r^3 + 16r^2 + 8(r+2)r^2 \ln r + 6r - 1 \right),$$

$$F = \frac{32\pi^2 \alpha^2 \alpha_s e_c^2}{3s^2 m_c^3} \langle \mathcal{O}^{J/\psi}(3P_0^{[8]}) \rangle$$

Predictions



$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.16 \text{ GeV}^3$$

$$\bar{v}_0^2 = \bar{v}_1^2 = 0.25$$

TABLE I. NRQCD LDMEs used in this Letter.

	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle / m_c^2$ 10^{-2} GeV^3
Set 1 [41]	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
Set 2 [42]	5.66 ± 0.47	0.177 ± 0.058	0.342 ± 0.102
Set 3	1.0		0.25

Eichten, Quigg, 9503356,
Chao, Ma, Shao, Wang, Zhang, 1201.2675,
Feng, Gong, Chang, Wang, 1810.08989.

- Theoretical predictions for J/ψ energy correlator in e^+e^- collision with $\sqrt{s}=10.6$ GeV

➤ The gluon fragmentation mechanism

$$d\sigma_{A+B \rightarrow J/\psi+X}(p_\psi) \approx \int_0^1 dz d\hat{\sigma}_{A+B \rightarrow g+X}(\hat{p}/z, \mu_F) D_{g \rightarrow J/\psi}(z, \mu_F),$$

$d\hat{\sigma}$: partonic production cross section for hadrons A and B to produce parton g

D : gluon fragmentation function

$$D_{g \rightarrow J/\psi}(z, \mu_F) = \sum_n d_{g \rightarrow c\bar{c}[n]}(z, \mu_F) \langle \mathcal{O}^{J/\psi}(n) \rangle$$

- ❑ Leading $\Sigma_{P.T.}$: one hard gluon is emitted in $^1S_0^{[8]}$ and $^3P_J^{[8]}$ channels

$$\Sigma_{P.T.}(\cos \chi) = \sum_n \int_0^1 dz d\hat{\sigma}_{A+B \rightarrow g+X}(\hat{p}/z, \mu_F) \hat{D}_{g \rightarrow c\bar{c}[n]}(z, \cos \chi, \mu_F) \langle \mathcal{O}^{J/\psi}(n) \rangle,$$

- with

$$\hat{D}_{g \rightarrow c\bar{c}[^1S_0^{[8]}]}(z, y, \mu_F) = \frac{5\alpha_s^2}{16} \frac{1}{6m_c^3} \frac{(1-z)^2}{(y+1)((y-1)z+2)^2} \times \left((y-1)^2 z^2 + 2(y-1)z + 2 \right),$$

$$\hat{D}_{g \rightarrow c\bar{c}[^3P_J^{[8]}]}(z, y, \mu_F) = \frac{5\alpha_s^2}{16} \frac{1}{6m_c^5} \frac{1}{(y+1)((y-1)z+2)^2} \times \left(-2(y^3 - 10y + 9)z^3 \right. \\ \left. + (3y^2 - 14y + 25)z^2 + (y-1)^2(2y+5)z^4 + 2(5y-7)z + 6 \right),$$

$$y = \cos \chi$$

- Leading $\Sigma_{N.P.}$: the soft radiation in the $^3S_1^{[8]}$ channel

$$\Sigma_{N.P.}(\cos \chi) = \int_0^1 dz d\hat{\sigma}_{A+B \rightarrow g+X}(\hat{p}/z, \mu_F) \hat{D}_{g \rightarrow c\bar{c}[^3S_1^{[8]}]}(z, \cos \chi, \mu_F) \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle,$$

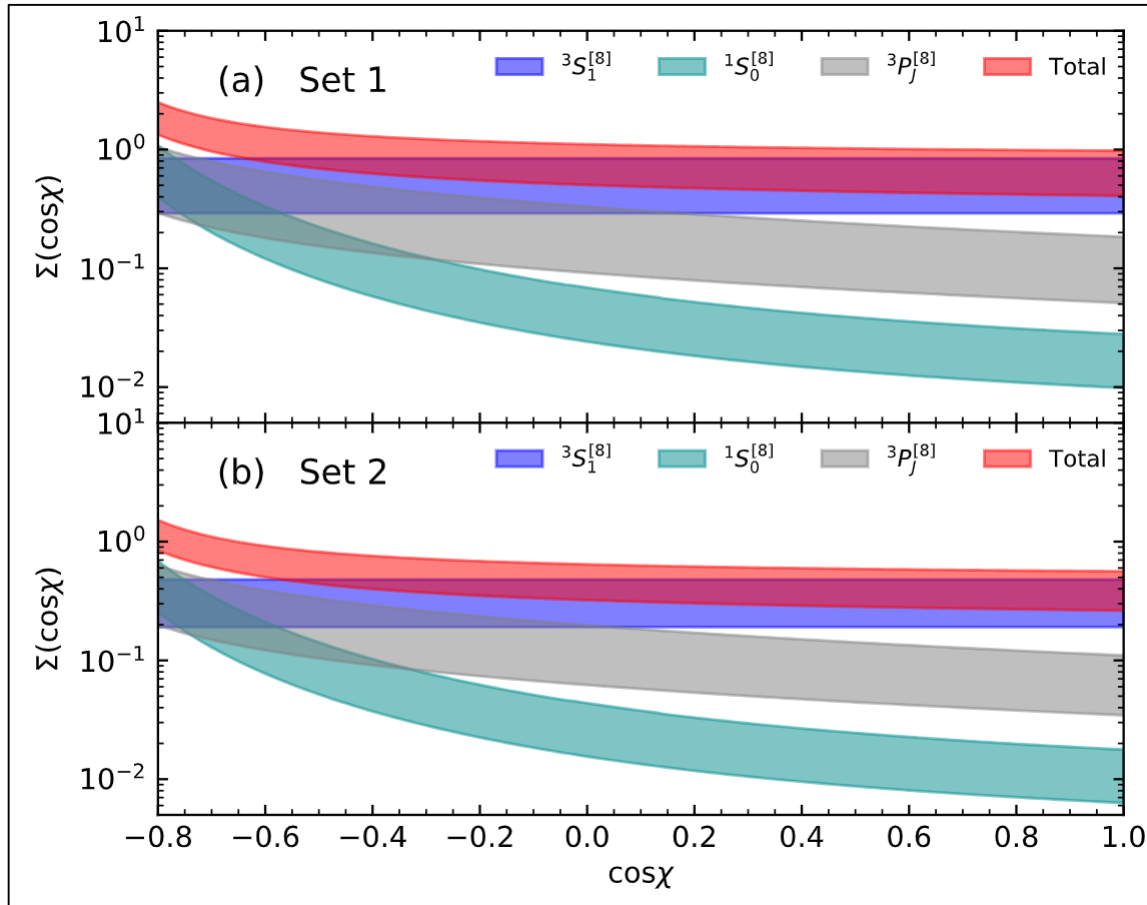
- with

$$\hat{D}_{g \rightarrow c\bar{c}[^3S_1^{[8]}]}(z, \cos \chi, \mu_F) = \int d\Phi_{gg} \frac{G_2}{4\pi} \frac{2k_1^0}{M} \delta(\cos \chi - \cos \theta) d_{g \rightarrow c\bar{c}[^3S_1^{[8]}]}^{\text{LO}}(z, \mu_F) \frac{1}{\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle}$$

$$\equiv \bar{v}_2 \times \frac{1}{2} d_{g \rightarrow c\bar{c}[^3S_1^{[8]}]}^{\text{LO}}(z, \mu_F),$$

average energy emitted during hadronization 17/19

➤ Predictions



- Theoretical predictions for J/ψ energy correlator in pp collision.

Summary



- The quarkonium energy correlator measures the energy flow at an angle distance χ from the quarkonium in its rest frame.
- The hadronization effect leads to a correction to the energy correlator comparable to that of the perturbative hard radiation in the region where $\cos \chi \gtrsim 0$.
- Quarkonium energy correlator provide a new insight to quarkonium hadronization.

Thank you!