Quantum computing of chirality imbalance in SU(2) gauge theory

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Introduction

- Spontaneous chiral symmetry breaking
  - One of the key features of QCD
  - Origin of mass
  - Chiral magnetic effect, chiral vortical effect…
  - Non-perturbative, high baryon chemical potential
    - Challenging for traditional methods

Introduction

- Quantum computing: a promising new method
- Topics of interest:
  - Real-time evolution
  - Non-perturbative physics
  - Non-Abelian gauge theory
  - Thermal states
  - …

Model

- 1+1D SU(2) model: simplest non-Abelian model

$$H = -i\bar{\psi}\gamma^1(\partial_1 + igA_1^a t^a)\psi + m\bar{\psi}\psi + \mu\psi^\dagger\psi + \frac{1}{2}\sum_a \left(L^a\right)^2$$

- Chiral condensate: $\sigma = \bar{\psi}\psi$
- Discritization: Staggered fermion

$$\psi_1(x) \to \phi_{2n}, \quad \psi_2(x) \to \phi_{2n+1}$$

$$H = \frac{1}{2\Delta}\sum_{n=0}^{N-2}\left(\phi_n^\dagger U_n \phi_{n+1} + H.C.\right) + m\sum_{n=0}^{N-1}(-1)^{n+1}\phi_n^\dagger\phi_n + \mu\sum_{n=0}^{N-1}\phi_n^\dagger\phi_n + \frac{\Delta g^2}{2}\sum_{n=0}^{N-2}\mathbf{L}_n^2$$

Eliminating Gauge Field

- Gauss's Law

$$\mathrm{L}_n^a - \mathrm{R}_{n-1}^a = Q_{n-1}^a = \phi_n^\dagger t^a \phi_n \to \mathrm{L}_n^a = \sum_{i<n} Q_i^a$$

- Local gauge transformation

$$\Theta = \prod_k \exp\left(i\theta_k \cdot \sum_{j>k} \mathbf{Q}_j \right)$$

$$H = \frac{1}{2}\sum_{n=0}^{N-2}\left(\phi_n^\dagger \phi_{n+1} + \mathrm{H.C.}\right) + \Delta m \sum_{n=0}^{N-1} (-1)^{n+1} \phi^\dagger \phi_n + \Delta\mu \sum_{n=0}^{N-1} \phi^\dagger \phi_n + \frac{\Delta^2 g^2}{2} \sum_{n=0}^{N-2} \left(\sum_{k\le n} \mathbf{Q}_k \right)$$

Jordan-Wigner transformation

$$\phi_n^r \to \varphi_{2n}, \quad \phi_n^g \to \varphi_{2n+1}$$

$$\varphi_n = \sigma_n^- \prod_{l=0}^{n-1} (-i\sigma_l^z)$$

$$H = -\frac{1}{2} \sum_{n=0}^{N-2} \left(\sigma_{2n}^+ \sigma_{2n+1}^z \sigma_{2n+2}^- + \sigma_{2n+1}^+ \sigma_{2n+2}^z \sigma_{2nn+3}^- + \mathrm{H.C.} \right) + \Delta m \sum_{n=0}^{N-1} \left[(-1)^{n+1} \frac{\sigma_{2n}^z + \sigma_{2n+1}^z}{2} + 1 \right] + \frac{\Delta^2 g^2}{2} \sum_{n=0}^{N-2} \left(\sum_{k \le n} \mathbf{Q}_k \right)$$

$$Q_n^x = \frac{1}{2}(\sigma_{2n+1}^+ \sigma_{2n}^- + \mathrm{H.C.})$$

$$Q_n^y = \frac{i}{2}(\sigma_{2n+1}^+ \sigma_{2n}^- - \mathrm{H.C.})$$

$$Q_n^z = \frac{1}{4}(\sigma_{2n}^z - \sigma_{2n+1}^z)$$

Algorithm: Variational Method

- Finite temperature: the Gibbs state

$$\rho(\beta) = \frac{1}{Z(\beta)} e^{-\beta H}, \quad Z(\beta) = \mathrm{Tr}(e^{-\beta H})$$

$$\rho(\alpha) = \sum_i \boxed{P_i(\beta)} \boxed{|\varphi_i\rangle\langle\varphi_i|}$$

Analytically solvable

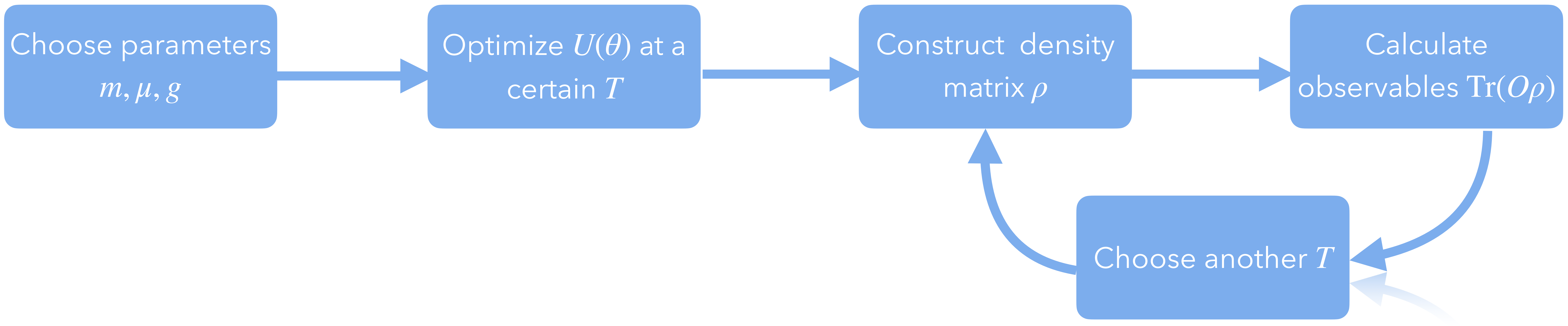
Eigenstate of H
independent of β

$$P_i = \frac{e^{-\beta E_i}}{\left(\sum_n e^{-\beta E_i}\right)}$$

- Variational method
  - Parametrization

$$\rho(\alpha) = \sum_i P_i(\beta) U(\alpha) | i \rangle \langle i | U^\dagger(\alpha)$$

Algorithm Workflow



Algorithm: Variational Method

- Loss Function : Free energy

$$
\begin{aligned}
F &= E - TS \\
&= \mathrm{Tr}\left[\rho H\right] + T\,\mathrm{Tr}\left[\rho \log \rho\right] \\
&= \sum_i P_i \left[E_i + T \ln P_i\right]
\end{aligned}
$$

- The variational method is only used once for all different temperatures.
- Many part of the calculation is analytical.

Algorithm: Variational Method

- Construction of $U(\alpha)$: QAOA ansatz

$$U(\alpha) = \prod_{i=1}^{p} \prod_{j=1}^{n} \exp(i\alpha_{ij} H_j)$$

$$H = H_1 + H_2 + \cdots + H_n$$

$$[H_i, H_j] \neq 0$$

- Each H_i perserves the same symmetries as H, so that $U(\alpha)$ also preserves the symmetries.

Algorithm: Monte-Carlo

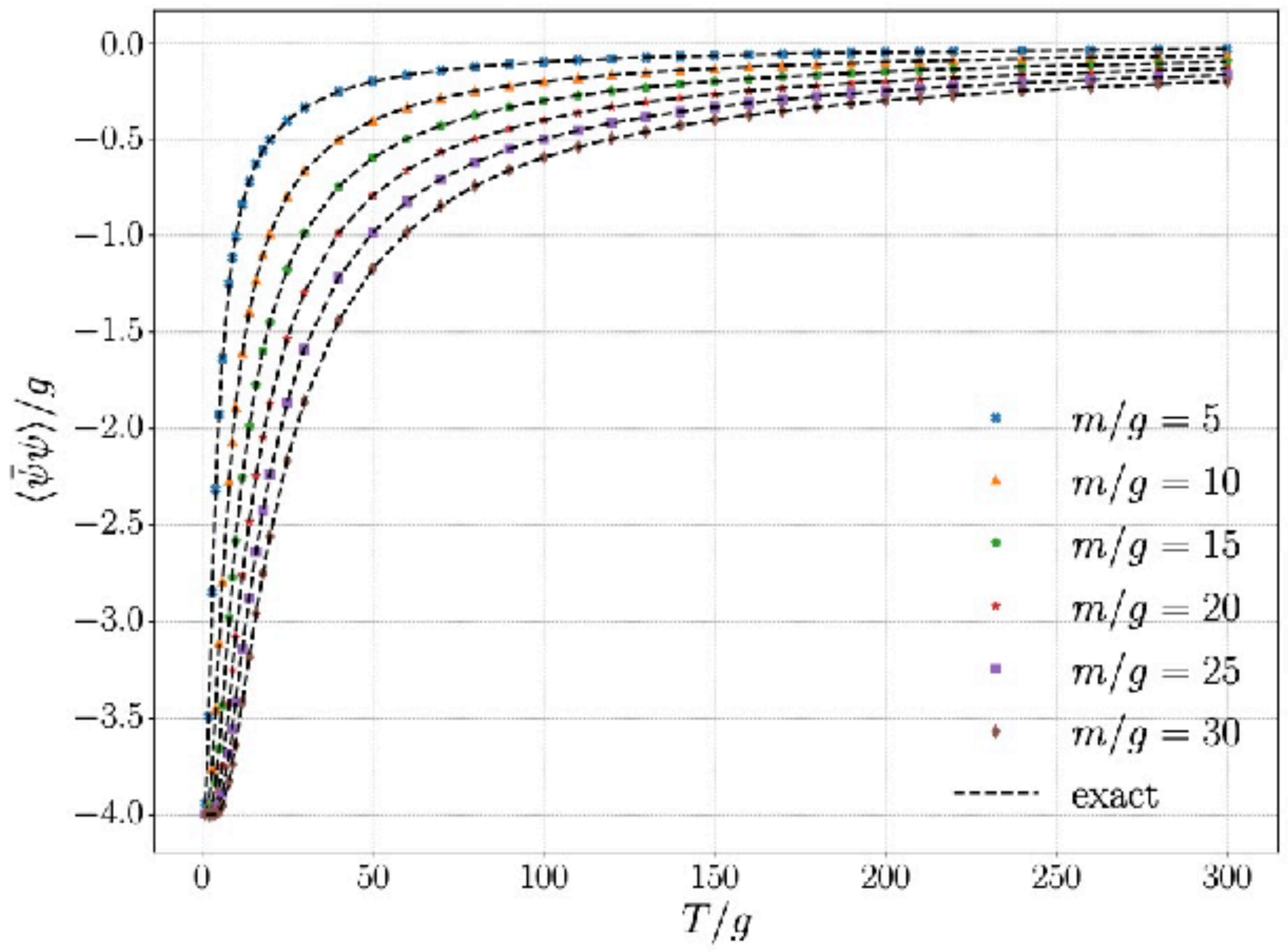
- Monte-Carlo in optimization
  - Randomly select a small set of states, do the optimization with this set until finished.
  - Select another set, continue the optimization with the new set.
  - Repeat until parameters convergence.
- In practical, a set of 20 qubits is used for each step.

Algorithm: Monte-Carlo

- Monte-Carlo in thermal state construction
  - Start from $|i\rangle$ such that $U|i\rangle$ is the ground state.
  - Randomly flip one qubit of $|i\rangle$ to get a new state $|j\rangle$
  - Calculate the energy expectation value $E_j\langle i|U^\dagger HU|i\rangle$
  - If $E_j < E_i$, accept the new state, otherwise, accept it with the probability $e^{-(E_j-E_i)/T}$
    - If the new state is rejected, the old state is added into the mixed state again.
  - Repeat until number of states reaches a predetermined limit.

Results: Full Gibbs State

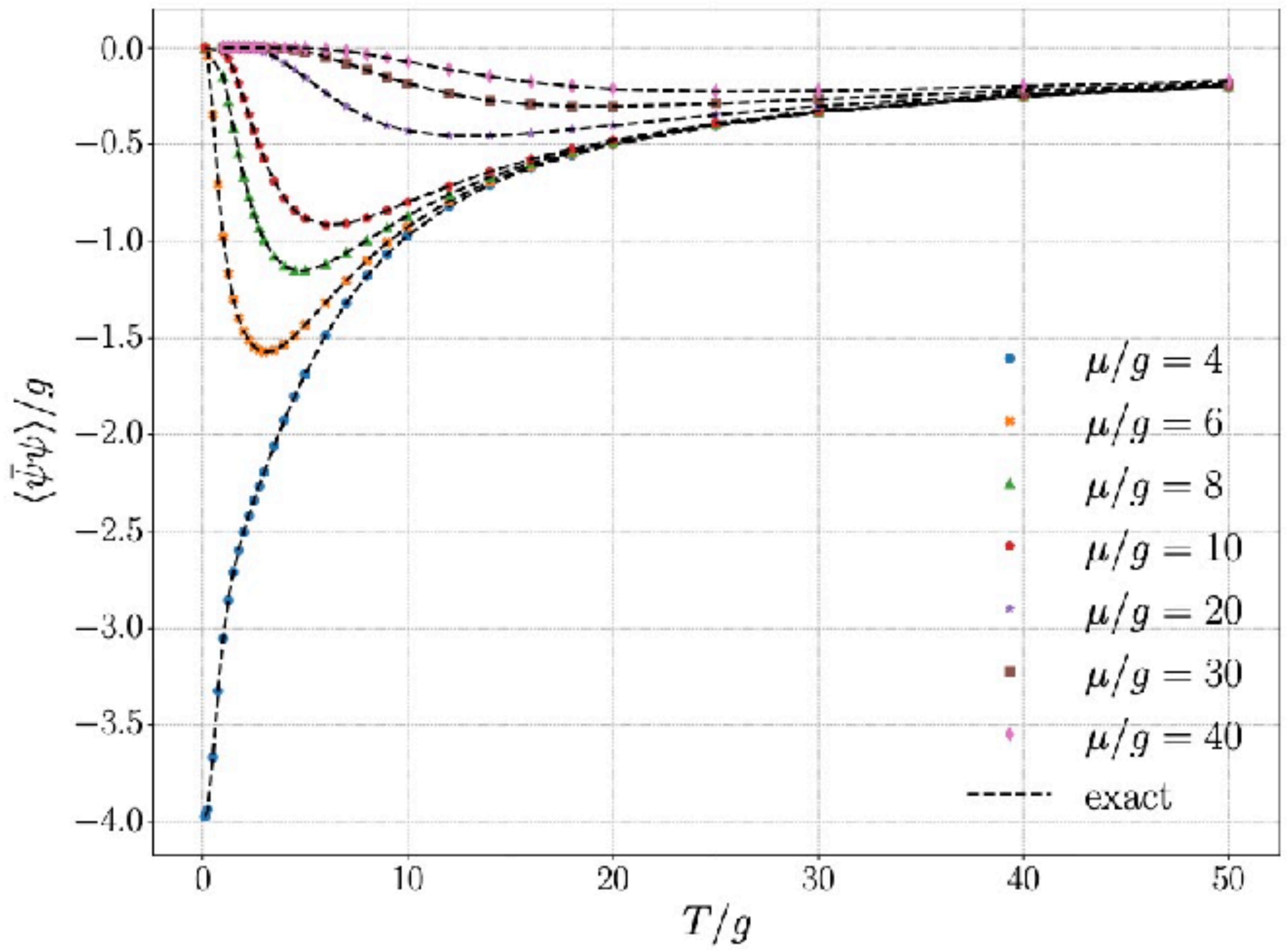
- 8 qubits, $\mu = 0$.
- Optimization done at highest temperature.
- All 256 sates are used to construct the Gibbs state.
- The VQE method produces the Gibbs state very accurately.



Chiral condensate at finite temperature

Results: Full Gibbs State

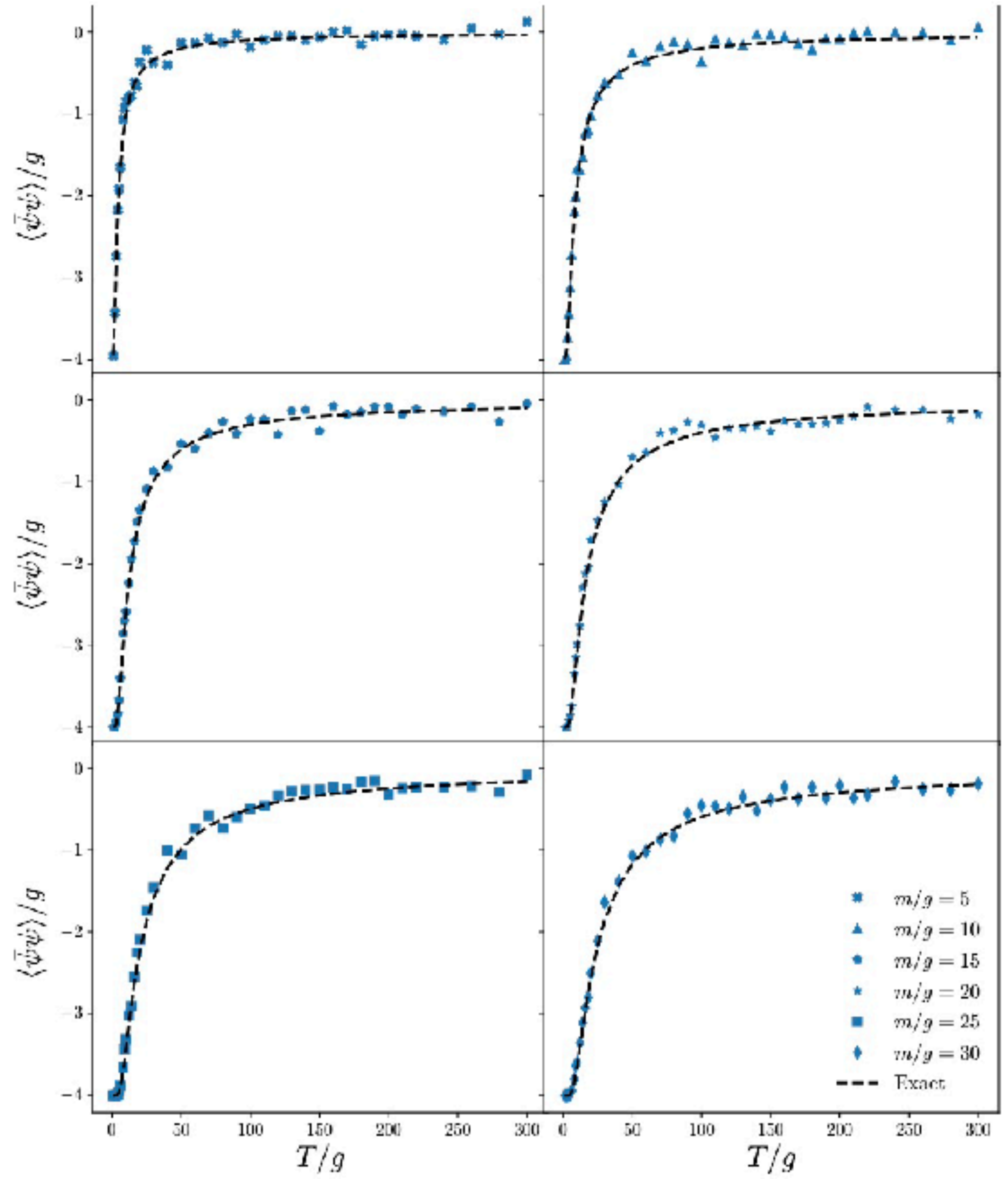
- 8 qubits, $m/g = 5$.
- The grond state changes at high chemical potential.
- Consistent with theoretical prediction.



Chiral condensate at finite temperature

Results: Monte-Carlo

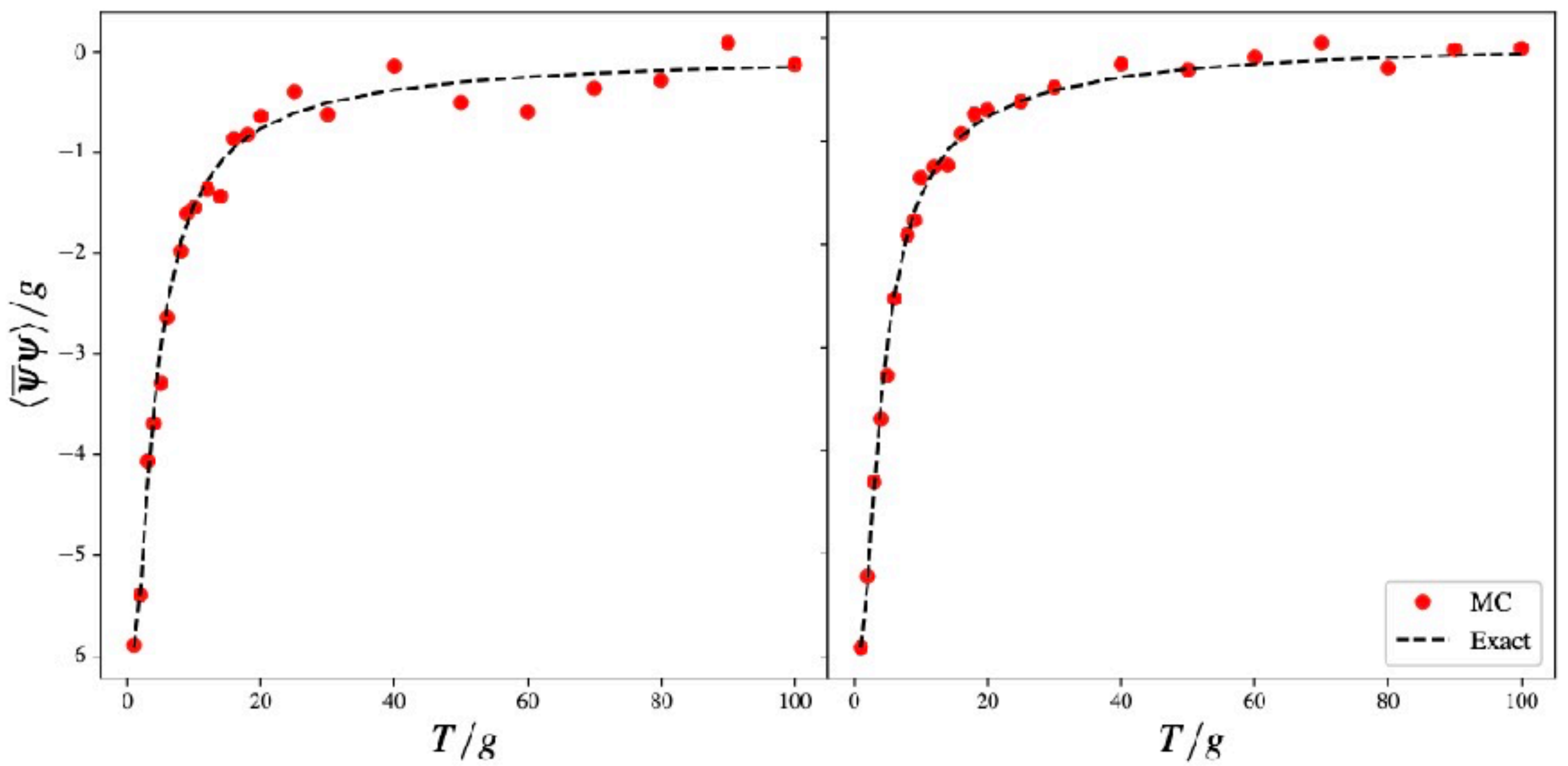
- 8 qubits, 1000 states for each sampling.
- The accuracy is good.
- Monte-Carlo method is not effective for small system. (1000 vs. 256)



Chiral condensate at finite temperature

Results: Monte-Carlo

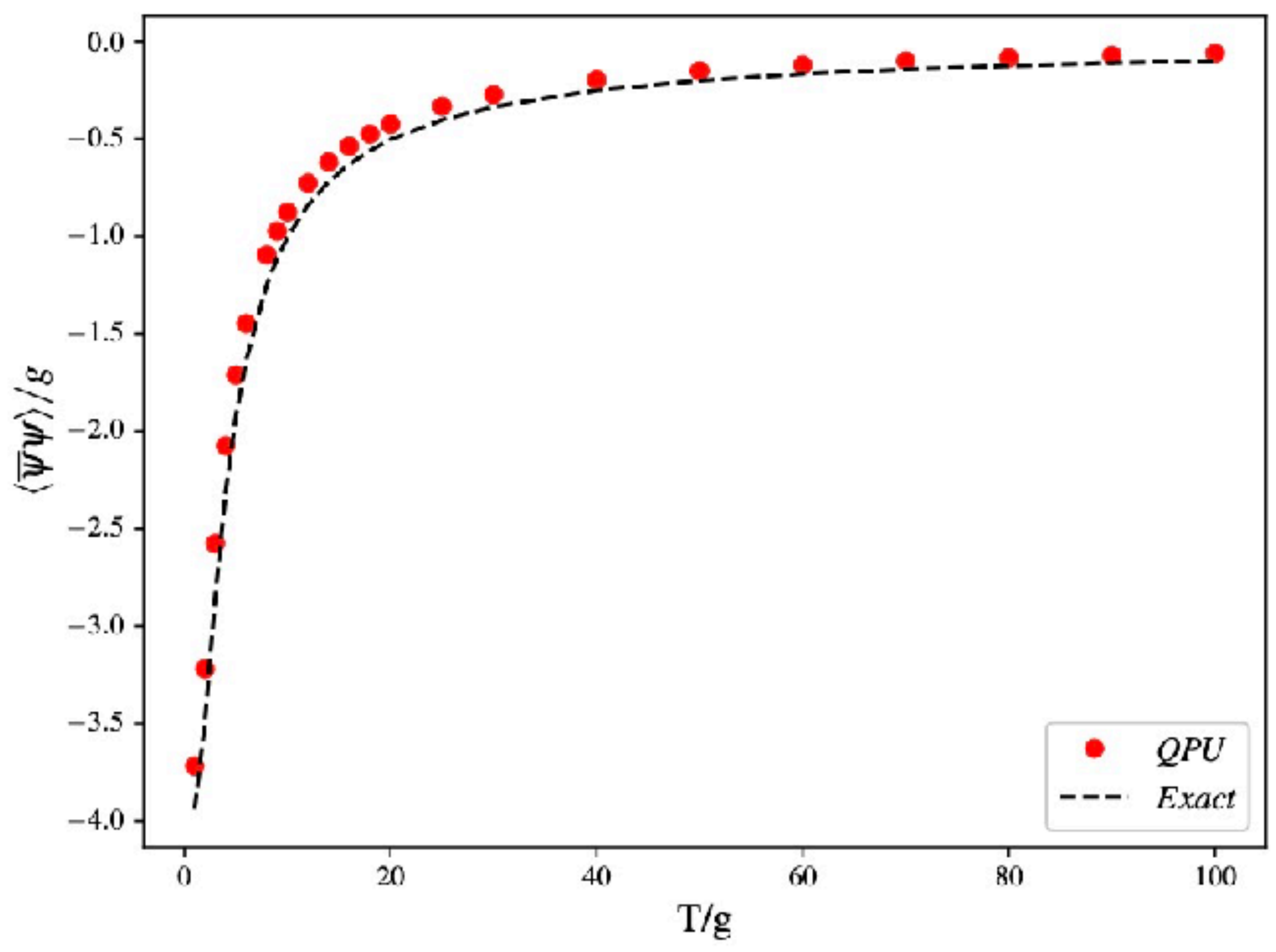
- 12 qubits, 1000 (left) and 2000 (right) states.
- Required number of sampling increases only as power law of number of qubits.
- 2000 vs. 4096 is already effective.



Chiral condensate at finite temperature

Results: Real QC results

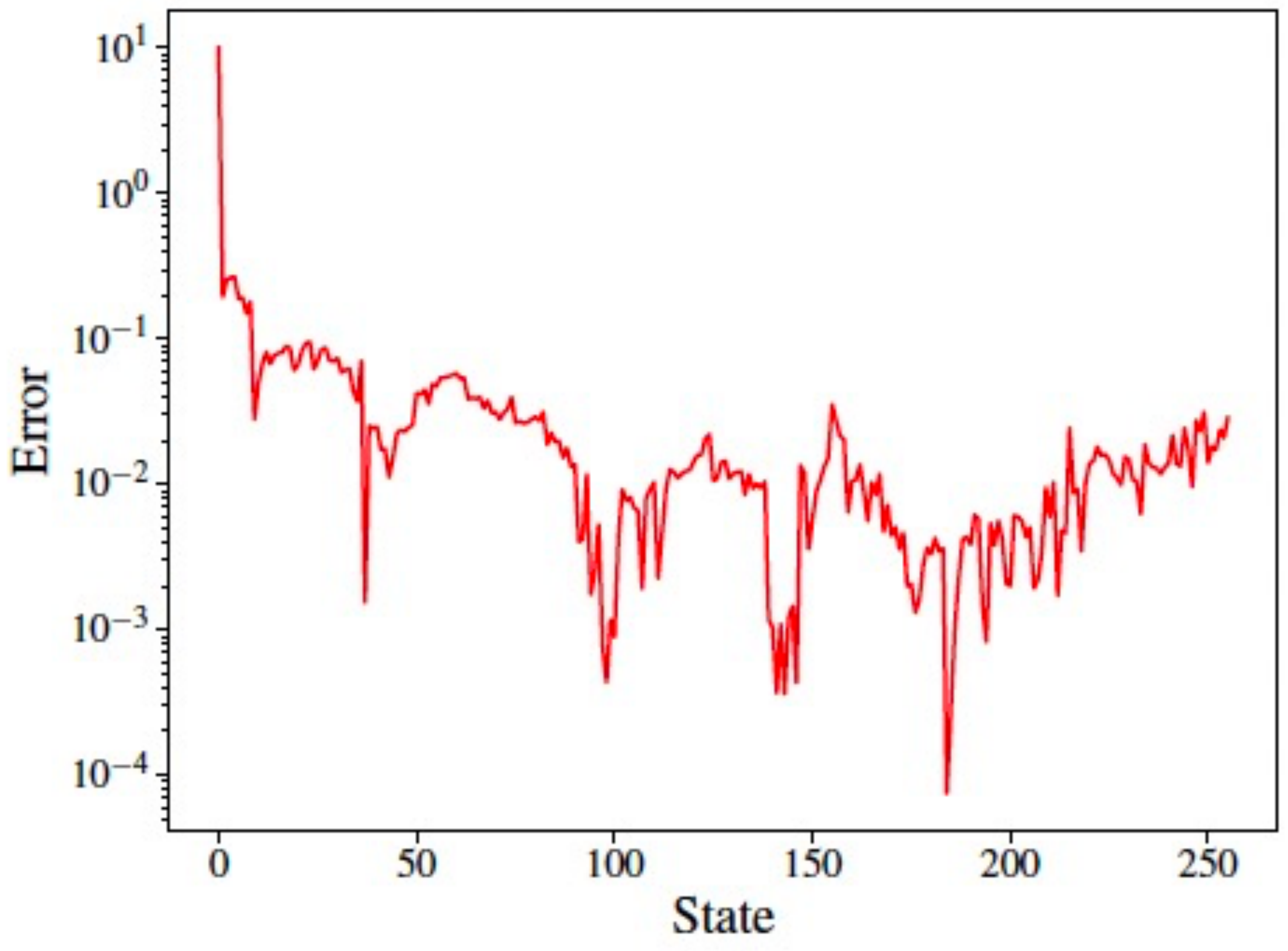
- 8 qubits, results from IBM's quantum hardware.
- A simpler ansatz for U is used.
- Optimization is still done classically.
- No error mitigation used.
- Our algorithm can achieve good precision on real QC.



Chiral condensate at finite temperature

Results: Real QC results

- Good accuracy for all the eigenenergy.
- Promising to apply to larger systems.



Relative error of the eigenenergy

Summary and Outlook

- We propose a framework with VQE and Monte-Carlo method to simulate thermal states on quantum computers.
- With this frame work, the chiral condensate of 1+1D SU(2) gauge model is studied.
- Our method is efficient and accurate in classical simulations as well as on real QCs.
- Apply to massless fermions: spontaneous symmetry breaking. (Paper to appear soon)

- Extend to larger systems or even higher dimensions.