Comparison of Standard Deviation Intervals in Gaussian and q-Gaussian Distributions

A Statistical Analysis 20.06.2024

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Introduction

This presentation explores the comparison between standard deviation intervals in Gaussian and q-Gaussian distributions. We aim to find the equivalent interval for a q-Gaussian distribution that encompasses 68% of the probability mass, analogous to the standard deviation in a Gaussian distribution.

Gaussian Distribution

For a perfectly Gaussian distribution, the standard deviation (σ) contains 68% of the probability mass within the interval $\mu \pm \sigma$.

$$P(\mu - \sigma \leq X \leq \mu + \sigma) = 0.68$$

q-Gaussian Distribution (Part 1)

The q-Gaussian distribution is a generalization of the Gaussian distribution, used in non-extensive statistical mechanics. As $q \to 1$, the q-Gaussian distribution converges to the standard Gaussian distribution.

$$f_{qg}(x) = A\left[1 - (1-q)\left(\frac{x-\mu}{\beta}\right)^2\right]^{\frac{1}{1-q}}$$

q-Gaussian Distribution (Part 2)

- μ: Mean (location parameter)
- β: Scale parameter (related to the width)
- q: Shape parameter (determines the tail behavior)
- A: Normalization constant

Normalization Constant (Part 1)

The normalization constant A ensures that the integral of the PDF over the entire space equals 1. It can be computed as follows:

- For $q < 1$:

$$A = \frac{\sqrt{\pi}\Gamma\left(\frac{3-q}{2(1-q)}\right)}{\beta\Gamma\left(\frac{1}{1-q}\right)}$$

- For $q = 1$:

$$A = \frac{1}{\beta\sqrt{\pi}}$$

Normalization Constant (Part 2)

- For $1 < q < 3$:

$$A = \frac{\Gamma\left(\frac{1}{q-1}\right)}{\beta\sqrt{(q-1)\pi}\,\Gamma\left(\frac{3-q}{2(q-1)}\right)}$$

Integral for q-Gaussian Interval

To find the interval σ_{qg} for the q-Gaussian distribution that corresponds to 68% probability, we solve the following equation:

$$\frac{\int_{-\sigma_{qg}}^{\sigma_{qg}} f_{qg}(x)\,dx}{\int_{-\infty}^{\infty} f_{qg}(x)\,dx} = 0.68$$

This involves numerical integration and potentially root-finding methods to solve for σ_{qg}.

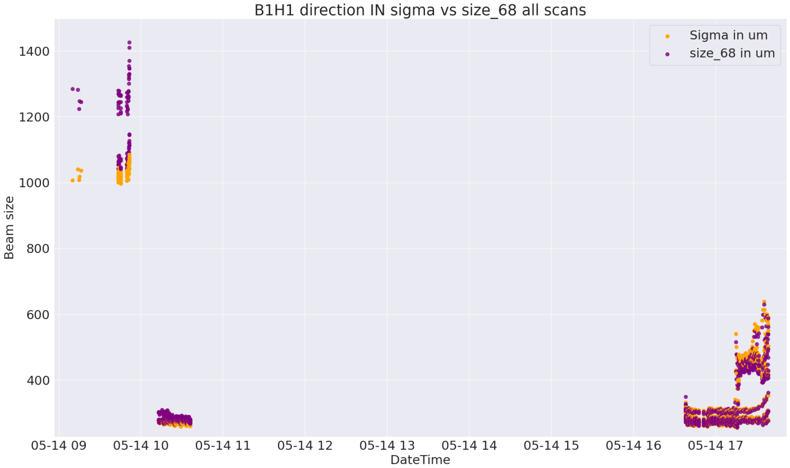
Special Cases

When $q = 1$, the q-Gaussian distribution reduces to the standard Gaussian distribution:

$$f(x) = \frac{1}{\beta\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\beta^2}\right)$$

Where β is equal to σ.

Beam Size Analysis

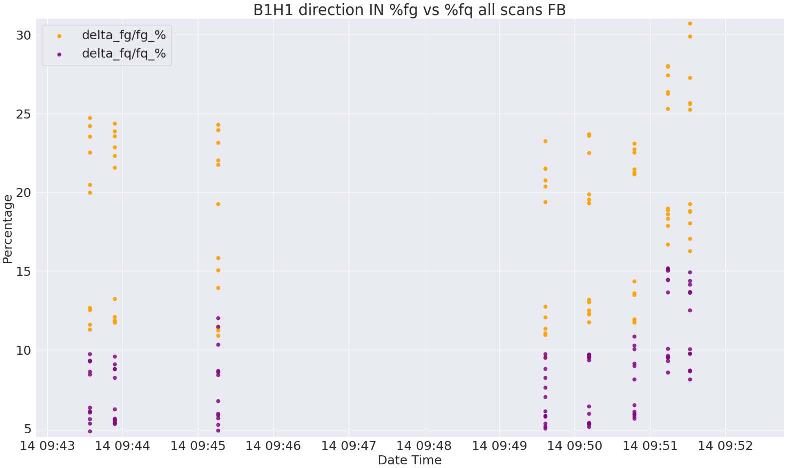


Percentage Analysis

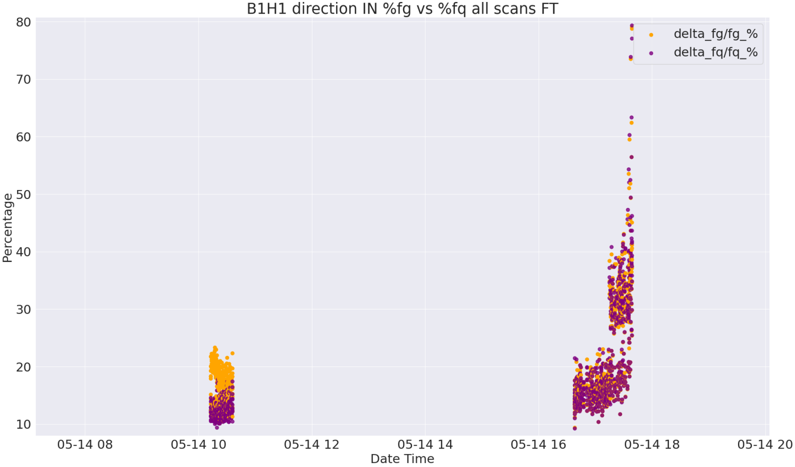
$$\delta_{fg}/fg\% = \frac{\int_{-\infty}^{+\infty}(\text{projDataSet} - fg)\,dx}{\int_{-\infty}^{+\infty} fg\,dx} * 100$$

$$\delta_{fq}/fq\% = \frac{\int_{-\infty}^{+\infty}(\text{projDataSet} - fq)\,dx}{\int_{-\infty}^{+\infty} fq\,dx} * 100$$

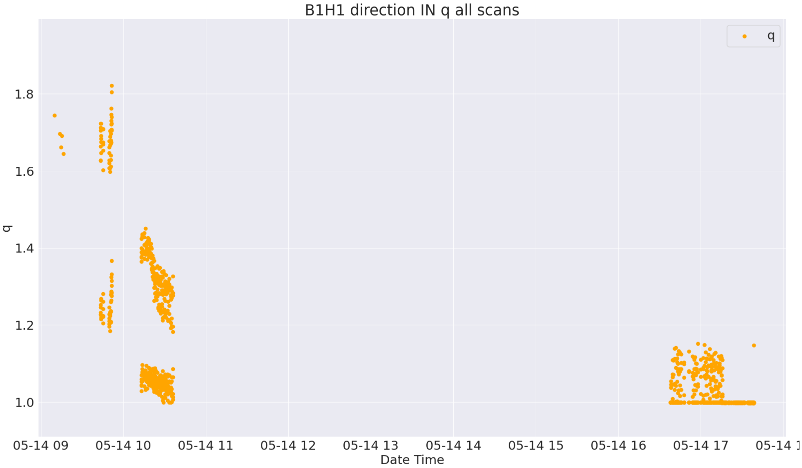
Percentage Analysis - FB



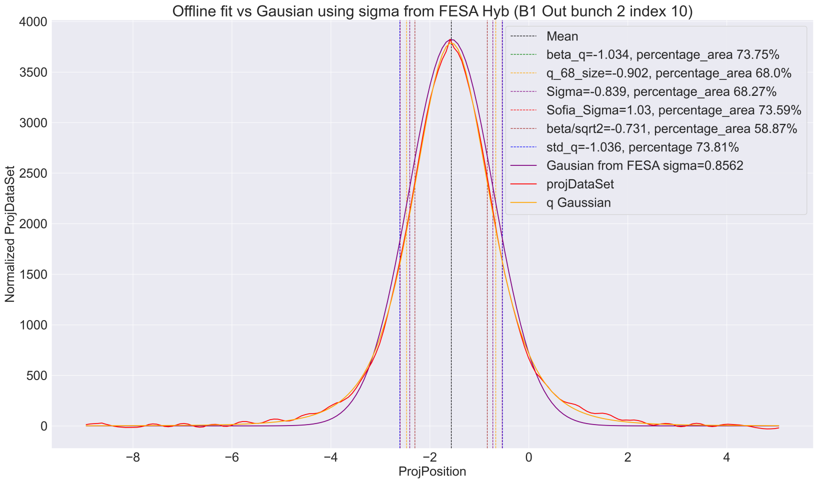
Percentage Analysis - FT



q Parameter Analysis



Offline Fit vs Gaussian



Conclusion

The equivalent interval σ_{qg} for a q-Gaussian distribution that contains 68% of the probability mass is analogous to the standard deviation σ for a Gaussian distribution. This interval can be found by solving the integral equation numerically.

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