Classical description of quark interactions

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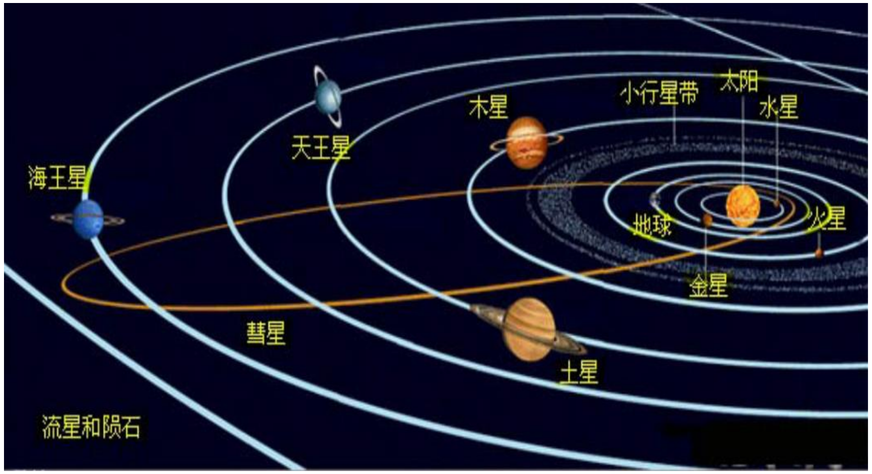
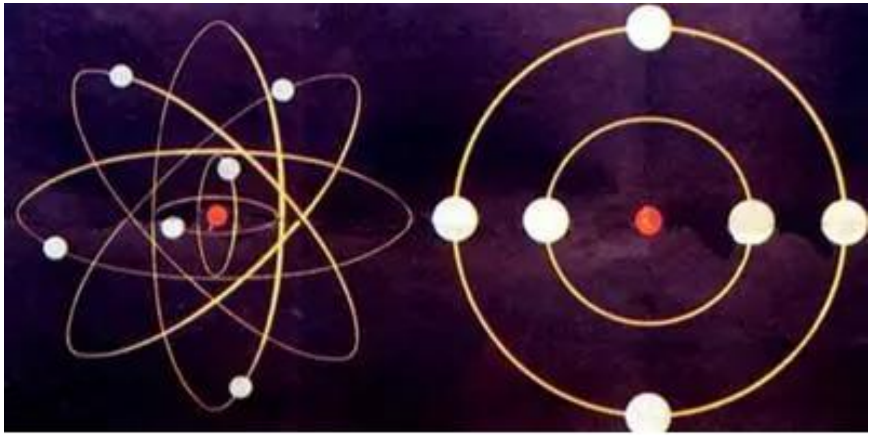
Classical interaction



- Universal gravitation

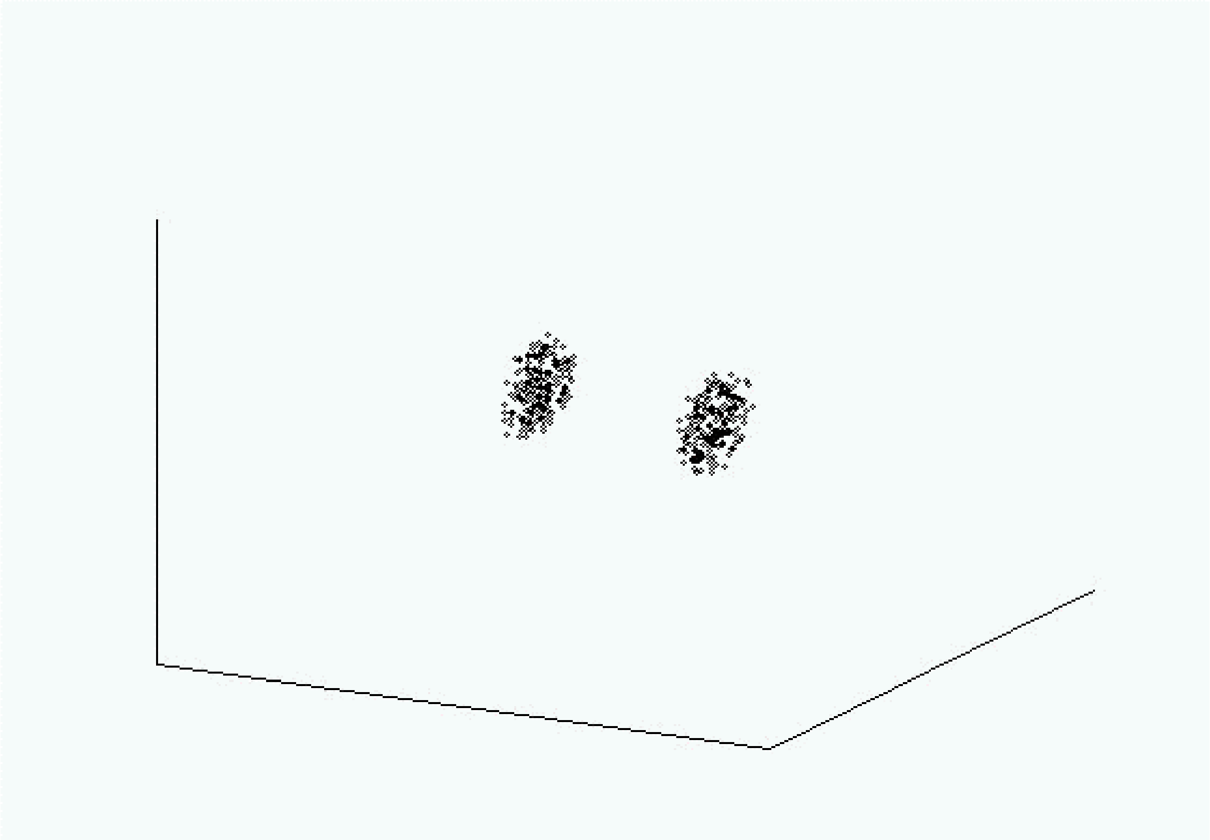
$$F = G\frac{Mm}{R^2} = m\frac{v^2}{R}$$

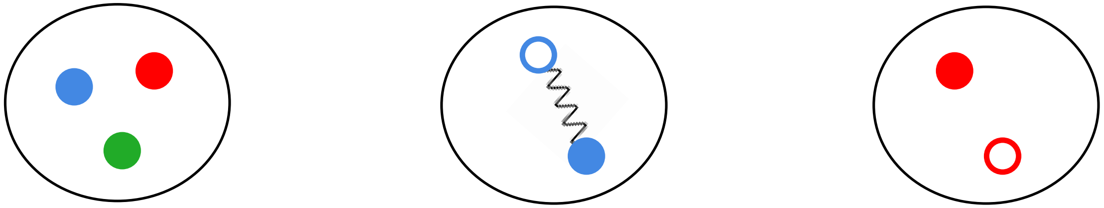


- Electrostatic attraction

$$F = k\frac{Qq}{R^2} = m\frac{v^2}{R}$$

How about quarks





Outline

1 Potential model

2 Classical description

3 Parameters and results

1. Potential model

- $V(r) = -\frac{a}{r} + br$ **Physical review D, 1980,21(1): 203.**

- $V(r) = k_0 e^{-\alpha^2 r^2/2} + cr - \frac{a}{r} - \frac{b}{r^2}$

  To Phys. J, 2019, 3: 197–215

- $V(r) = ar^2 + br - \frac{c}{r} + \frac{d}{r^2} + e$

  Modern Physics Letters A, 2022, 37(02): 2250010.

$$\nabla^2\psi + \frac{2\mu}{\hbar^2}[E - V(r)]\psi(r,\theta,\phi) = 0$$

$$M_{nl} = m_{q1} + m_{q2} + E_{nl}$$

Color charge

➢ **Three element color charges and their anti-charges**

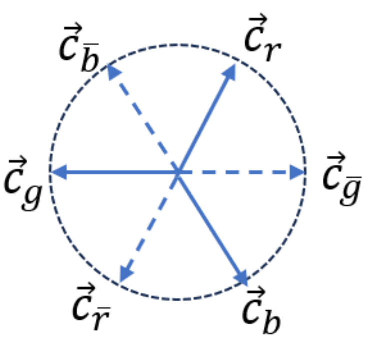
$$c_r \equiv e^{\theta i} \qquad c_{\bar r} \equiv e^{(\theta+\pi)i} = -c_r$$

$$c_g \equiv e^{\left(\theta+\frac{2\pi}{3}\right)i} \qquad c_{\bar g} \equiv e^{\left(\theta-\frac{\pi}{3}\right)i} = -c_g$$

$$c_b \equiv e^{\left(\theta-\frac{2\pi}{3}\right)i} \qquad c_{\bar b} \equiv e^{\left(\theta+\frac{\pi}{3}\right)i} = -c_b$$

➢ **Mesons and baryons are color neutral**

$$c_i + c_{\bar i} = 0 \qquad c_r + c_g + c_b = 0$$



➢ **Multicolored charged particles**

$$C = \sum_{i=r,g,b} \left(n_i c_i + n_{\bar i} c_{\bar i}\right)$$

e.g. di-quark

2. Classical description

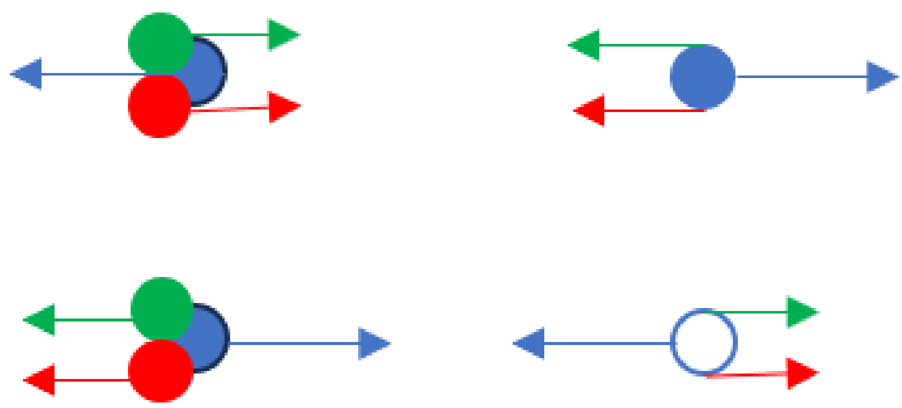
Interaction between two color charge

$$\vec{F}_{C_1C_2} = Z\frac{C_1\cdot C_2}{r^3}\vec{r} \Longrightarrow E_p = -\frac{Z}{r}$$

- **The dot product between two unit color charges**

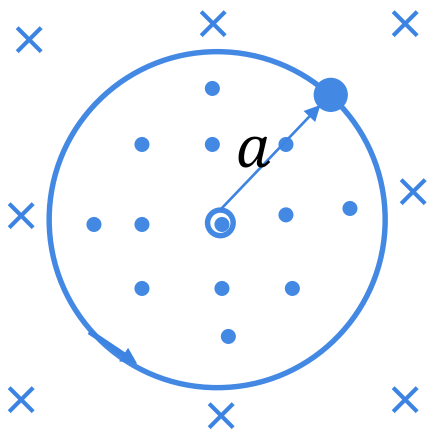
$$c_i\cdot(\pm c_j) = \begin{cases} \pm 1 & i = j \\ \mp\frac{1}{2} & i \neq j \end{cases}$$

- **Interaction between a quark and a Neutral particle**



2. Classical description

The color magnetic field



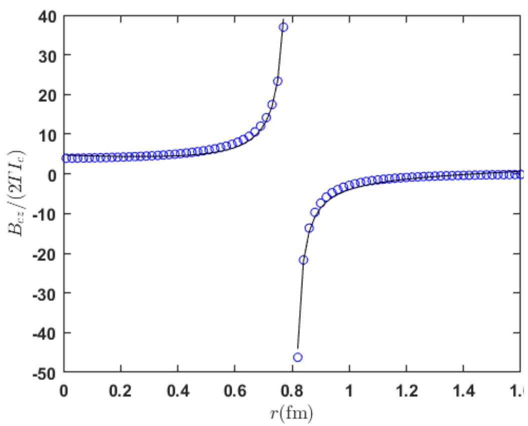
✓ Circular motion of color charge forms color flow

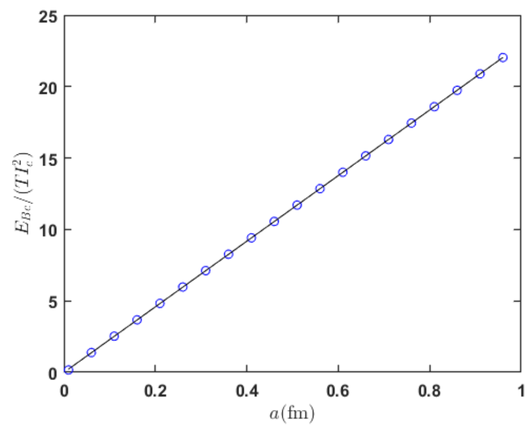
$$I_c = |c|\frac{v}{2\pi a} \qquad B_c = \int_l T\,\frac{I_c d\vec{l} \times \vec{r}}{r^3}$$

✓ Circular color flow excited color magnetic field

$$B_{cz} = 2TI_c\left[\frac{1}{a-r}E(k) + \frac{1}{a+r}K(k)\right]$$

✓ The color magnetic field energy



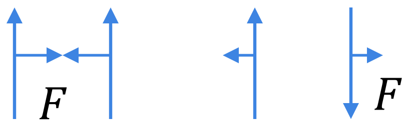


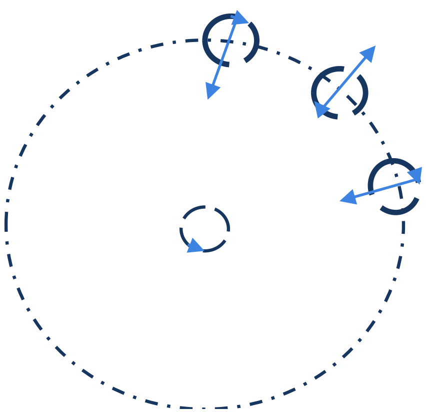
$$E_{Bc} = 22.97TI_c^2 r$$

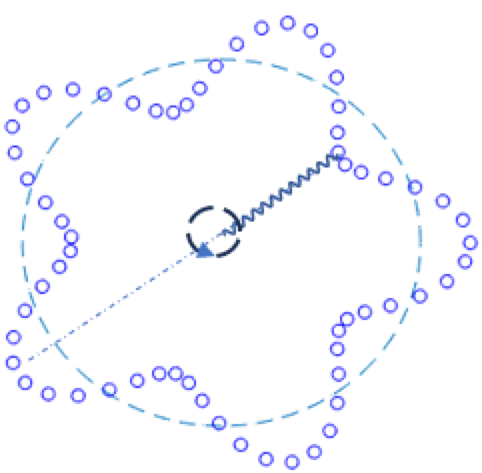
$$= 0.5818T\frac{Z}{mr^2}$$

2. Classical description

Interaction from spin







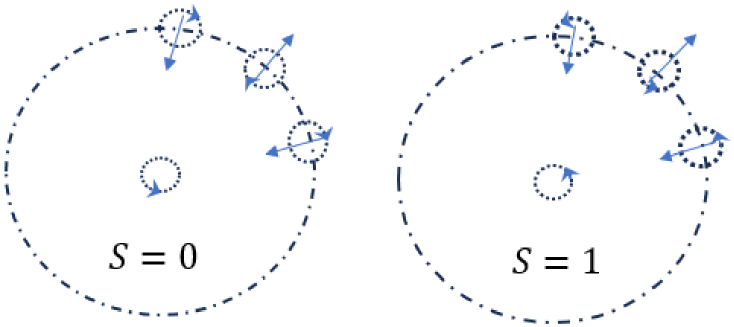
- ✓ Color flow attracts in the same direction and repels in the opposite direction
- ✓ Treating spin as the rotational motion of color charge
- ✓ the harmonic oscillator potential caused by spin

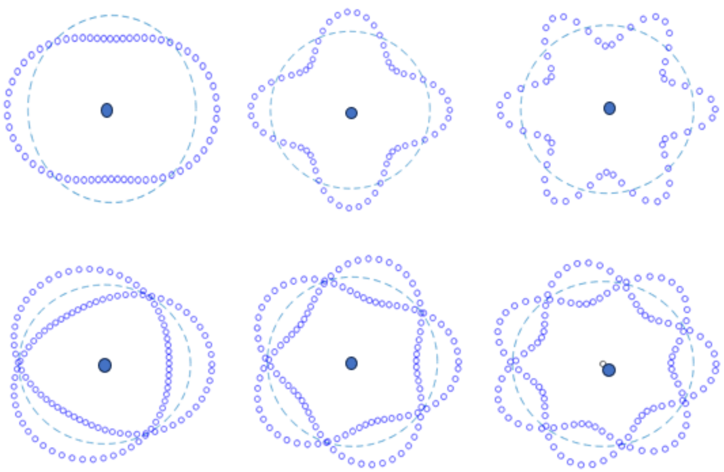
$$F_{S_1S_2} = -k(r - r_0) = -\frac{1}{2}m\omega^2(r - r_0)$$

$$E_S = \frac{1}{2}kA^2 = \frac{1}{2}k(r_M - r)^2$$

$$= \frac{1}{2}kr_M^2 + \frac{1}{2}kr^2 - kr_Mr$$

Quantization hypothesis





$$E_S = \left(L+\frac{1}{2}\right)\hbar\omega \qquad L=0,1,2,\cdots$$

$$\frac{2\pi r_n}{v_n} = \begin{cases} \left(n+\frac{1}{2}\right)\frac{2\pi}{2\omega} & (S=0) \\ (n+1)\frac{2\pi}{2\omega} & (S=1) \end{cases} \qquad n=1,2,\cdots$$

$$r_n^3 = \begin{cases} \left(n+\frac{1}{2}\right)^2 \frac{Z}{m\omega^2} = \left(\frac{2n+1}{3}\right)^2 r_1^3 & (S=0) \\ \left(\frac{2n+2}{2}\right)^2 \frac{Z}{m\omega^2} = \left(\frac{n+1}{2}\right)^2 r_1'^3 & (S=1) \end{cases}$$

3. Parameters and results

$$\vec{F}_{C_1C_2} = Z\frac{C_1 \cdot C_2}{r^3}\vec{r} \qquad \vec{B}_c = \int_l T\frac{I_c d\vec{l} \times \vec{r}}{r^3}$$

- $$M_{n,L} = M - \frac{Z}{2r_n} + 0.5818T\frac{Z}{mr_n^2} + \left(L + \frac{1}{2}\right)\hbar\omega$$

with $M = m_Q + m_q, m = \frac{m_q m_q}{M}$

- Using some experimental data on mass and radius of π & ρ from PDG to estimate Z and T

$n^{2S+1}L_j$	Name	$q\bar{q}'$	$\sqrt{\langle r_1\rangle^2}$ (fm)	M_1(GeV)
1^1S_0	$\pi^{\pm}$	ud	0.6626	0.1400
1^1S_1	$\rho^{\pm}$	ud	0.7483	0.7750

$Z \approx 1.33 \times 10^{-26} Nm^2,$ $\qquad T \approx 4.43 \times 10^{-44} Ns^2$

3. Parameters and results

➢ Calculation on some other mesons' radii and compare with literatures

$n^{2S+1}L_j$	Name	$q\bar{q}'$	M(GeV)	$\sqrt{r^2}$(fm)[Ref.]	this work
1^1S_0	$\pi^{\pm}$	ud	0.140	0.6626	0.6621
2^1S_0	$\pi(1300)^{\pm}$	ud	1.300	0.59∼ 0.92[16]	1.038
3^1S_0	$\pi(1800)^{\pm}$	ud	1.812		1.256
1^3S_0	$\rho^{\pm}$	ud	0.775	0.7483	0.7475
2^3S_1	$\rho(1450)^{\pm}$	ud	1.465		1.310
3^3S_1	$\rho(1900)^{\pm}$	ud	1.900		1.568
1^1S_0	$K^{\pm}$	us	0.494	0.58[17]	0.554
1^1S_0	h^0	ss	0.701	0.36[18]	0.417
1^1S_0	$D^{\pm}$	cd	1.87	0.426[19]	0.557
1^1S_0	D^0	cu	1.865?		0.692
1^1P_1	$D_1(2430)^0$	cu	2.412		1.573
1^1D_2	$D_2(2740)^0$	cu	2.740		1.992
1^1S_0	D_s^+	cs	1.945	0.26[18]	0.277
1^1P_1	$D_{s1}(2460)^{\pm}$	cs	2.46		0.435
1^1S_0	$B^{\pm}$	ub	5.279?	0.62[19]	0.690
1^1S_0	η_c	cc	2.984	0.20[20]	0.174
1^3S_1	J/Psi	cc	3.097	0.212[20]	0.199
1^1P_1	h_c	cc	3.525	0.265[20]	0.267
1^1S_0	η_b	bb	9.399	0.07[19]	0.064

Reference data source

[15] C. Patrignani, K. Agashe, G. Aielli et al., (Particle Data Group),Review of Particle Physics. Chin. Phys. C 40, 100001 (2016).doi:10.1088/1674-1137/40/10/100001

[16] Höll A, Krassnigg A, Maris P, et al. Electromagnetic properties of ground-state and excited-state pseudoscalar mesons[J]. Physical Review C - Nuclear Physics, 2005, 71(6): 065204.

[17] Krutov A F, Polezhaev R G, Troitsky V E. Radius of the meson determined from its decay constant[J]. Physical Review D, 2016, 93(3): 036007.

[18] Hernández-Pinto R J, Gutiérrez-Guerrero L X, Bashir A, et al. Electromagnetic form factors and charge radi-i of pseudoscalar and scalar mesons: A comprehensive contact interaction analysis[J]. Physical Review D, 2023, 107(5): 054002.

[19] Hwang C W. Charge radii of light and heavy mesons[J]. The European Physical Journal C-Particles and Fields, 2002, 23: 585-590.

[20] R. J. Hernandez-Pinto, L. X. Gutierrez-Guerrero, A. Bashir, M. A. Bedolla, and I. M. Higuera-Angulo, Phys. Rev. D 107 5, 054002, (2023).

Summary

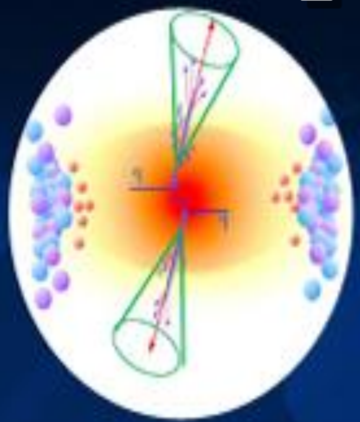
- we propose a classical expression based on color charge interactions between a pair of quarks.

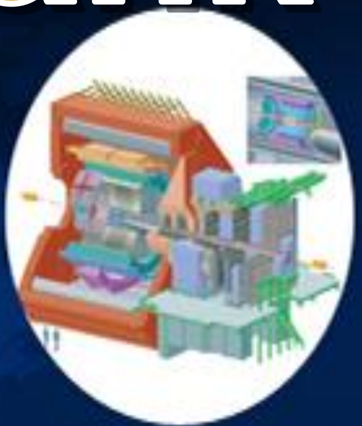
- Especially, the design of the superposition and dot multiplication of quark color charges predicts the existence of non unit color charge elementary particles.

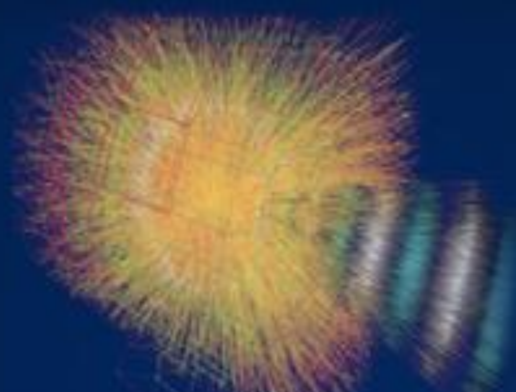
- The estimated vacuum static color gravitational constant Z and color magnetic field constant T presented in this talk are quite rough. However, we are eager to obtain more accurate values as soon as possible.

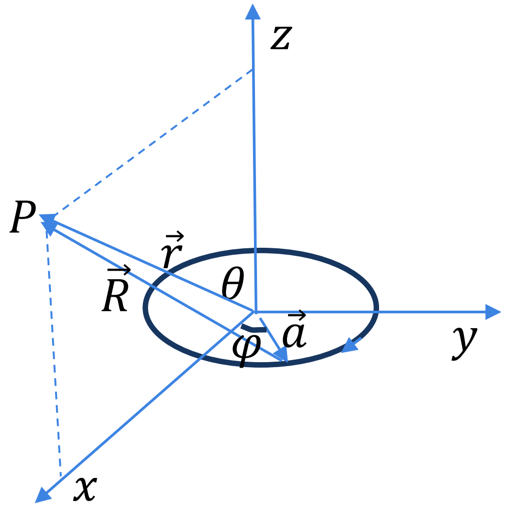
- we hope that the classic approach in this report can provide a simple and visualizable approach to study the interior of microscopic particles.

Thank you !







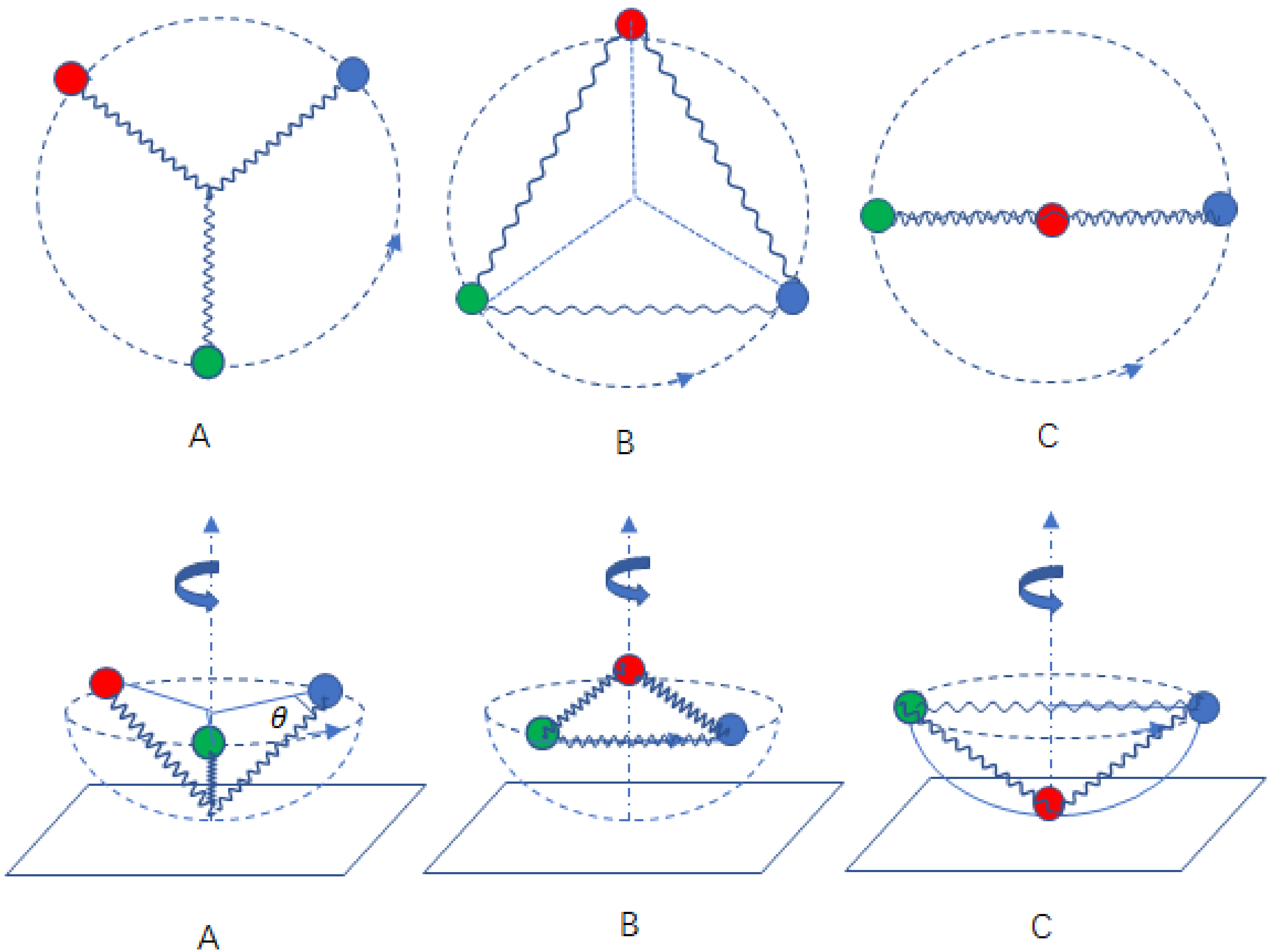


$$
\begin{aligned}
B_{cx} &= TI_c ar\cos\theta\int_0^{2\pi}\frac{\cos\varphi d\varphi}{(r^2+a^2-2ra\sin\theta\cos\varphi)^{3/2}},\\
B_{cy} &= 0,\\
B_{cz} &= TI_c\int_0^{2\pi}\frac{a^2-ar\sin\theta\cos\varphi d\varphi}{(r^2+a^2-2ra\sin\theta\cos\varphi)^{3/2}}. \qquad (14)
\end{aligned}
$$

Points on the color flow plane, $\theta=\pi/2, \sin\theta=1, \cos\theta=0$, therefore, $B_{cx}=B_{cy}=0$.

$$
\begin{aligned}
B_{cz} &= TI_c\int_0^{2\pi}\frac{a^2-ar\cos\varphi d\varphi}{(r^2+a^2-2ra\cos\varphi)^{3/2}}\\
&= 2TI_c\left[\frac{1}{a-r}E(k)+\frac{1}{a+r}K(k)\right]\\
&= 2TI_c X(a,r). \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
a=0.5,\quad X_i &= 1.2215\frac{1}{a-r}+9.9248(a-r),\\
X_o &= -0.8433\frac{1}{r-a}+5.0440(r-a);\\
a=0.6,\quad X_i &= 1.2001\frac{1}{a-r}+7.0576(a-r),\\
X_o &= -0.8593\frac{1}{r-a}+3.6412(r-a);\\
a=0.7,\quad X_i &= 1.1833\frac{1}{a-r}+5.2816(a-r),\\
X_o &= -0.8717\frac{1}{r-a}+2.7549(r-a);\\
a=0.8,\quad X_i &= 1.1696\frac{1}{a-r}+4.1042(a-r),\\
X_o &= -0.8817\frac{1}{r-a}+2.1587(r-a);\\
a=0.9,\quad X_i &= 1.1583\frac{1}{a-r}+3.2828(a-r),\\
X_o &= -0.8900\frac{1}{r-a}+1.7381(r-a);\\
a=1.0,\quad X_i &= 1.1487\frac{1}{a-r}+2.6866(a-r),\\
X_o &= -0.8970\frac{1}{r-a}+1.4301(r-a).
\end{aligned}
$$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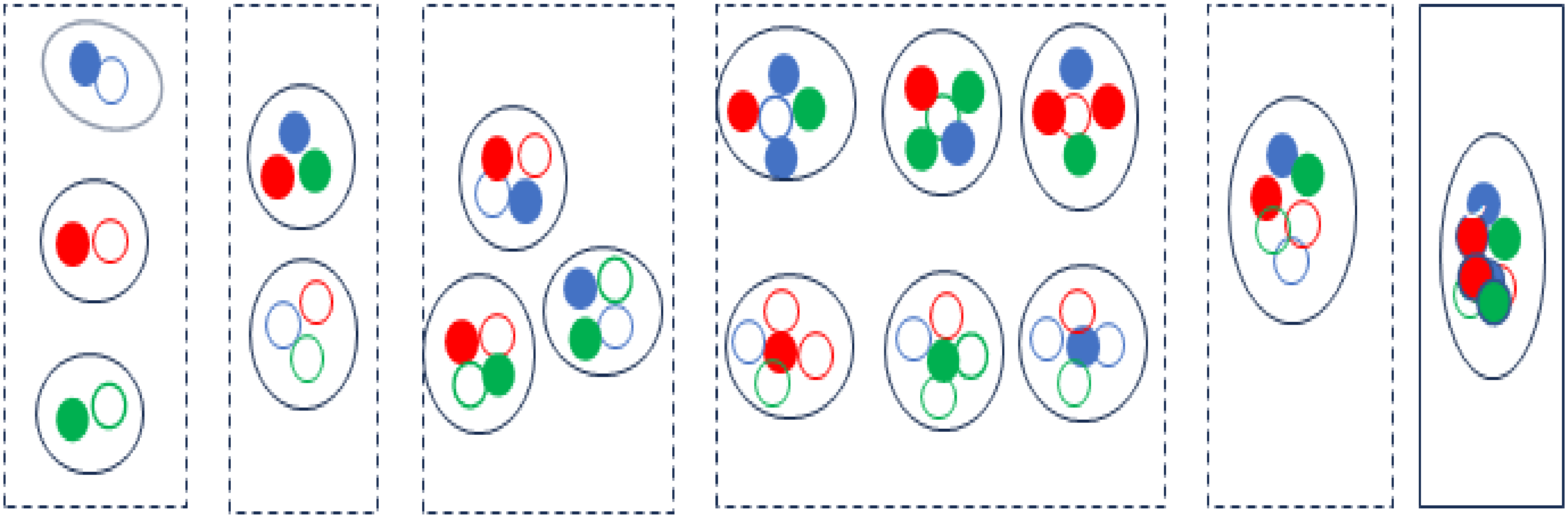

mesons → baryons → tetraquarks → pentaquarks → multiquarks ……