



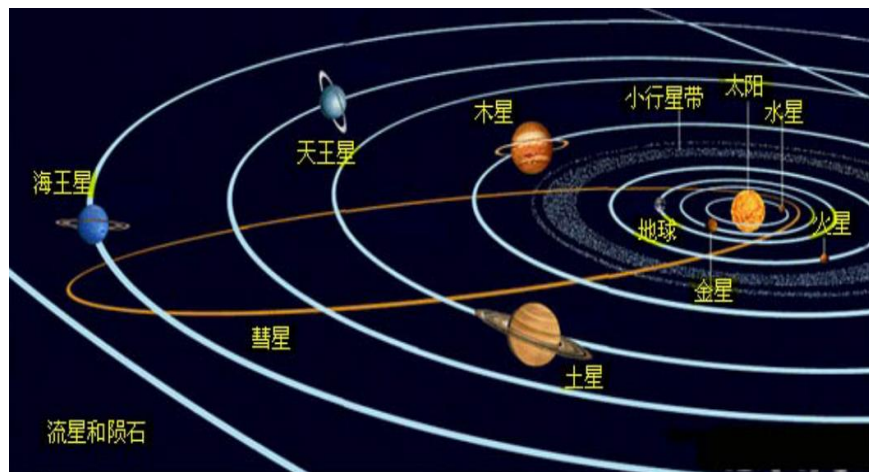
Classical description of quark interactions

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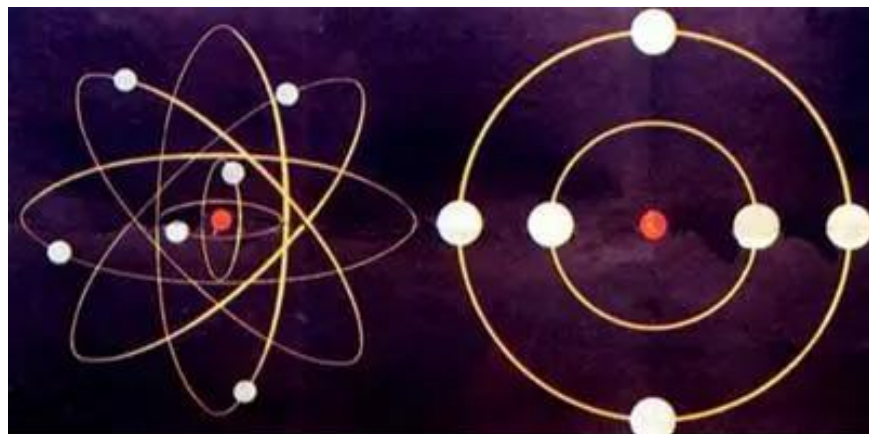
Classical interaction

- 1 -



➤ Universal gravitation

$$F = G \frac{Mm}{R^2} = m \frac{v^2}{R}$$

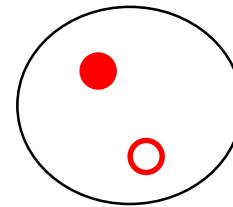
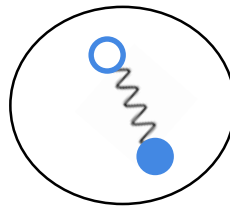
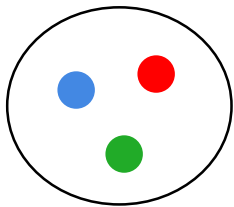
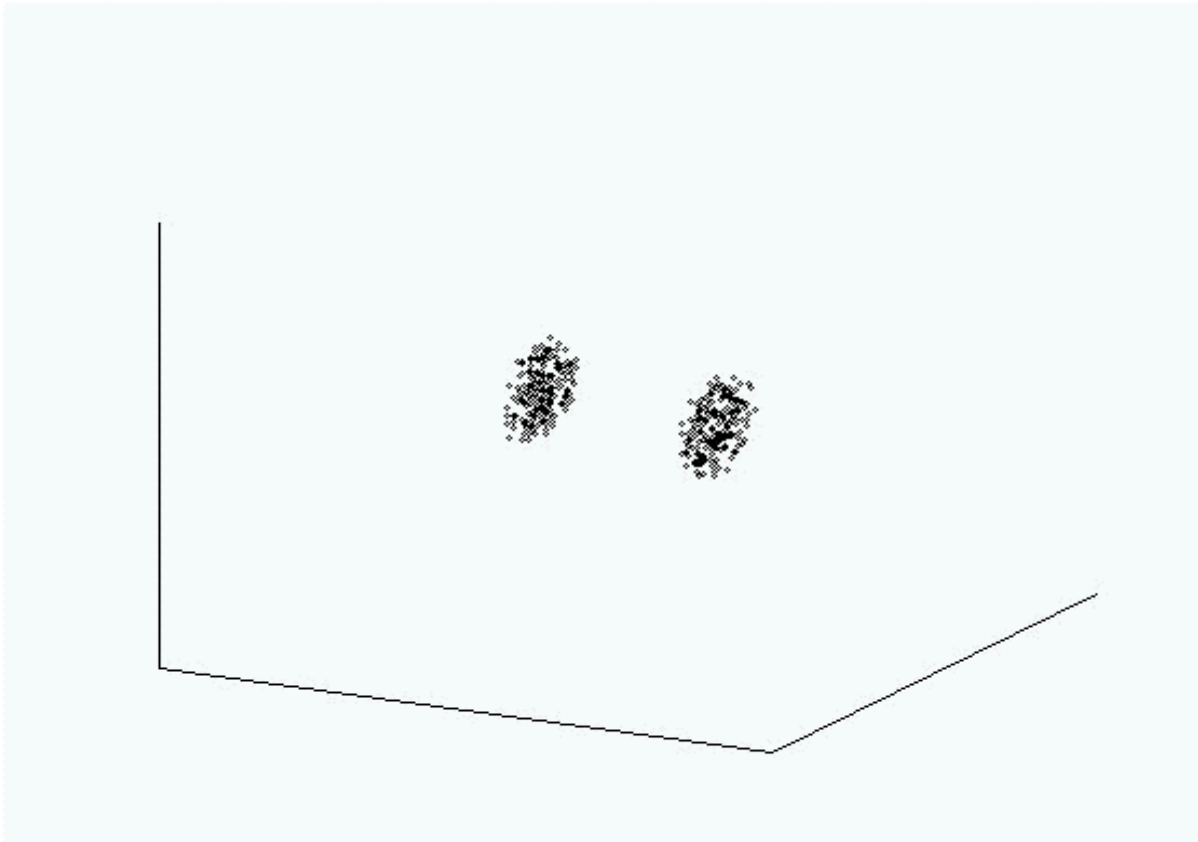


➤ Electrostatic attraction

$$F = k \frac{Qq}{R^2} = m \frac{v^2}{R}$$



How about quarks





Potential model



Classical description



Parameters and results



1. Potential model

➤ $V(r) = -\frac{a}{r} + br$ Physical review D, 1980,21(1): 203.

➤ $V(r) = k_0 e^{-\alpha^2 r^2 / 2} + cr - \frac{a}{r} - \frac{b}{r^2}$

To Phys. J, 2019, 3: 197–215

➤ $V(r) = ar^2 + br - \frac{c}{r} + \frac{d}{r^2} + e$

Modern Physics Letters A, 2022, 37(02): 2250010.

$$\nabla^2 \psi + \frac{2\mu}{\hbar^2} [E - V(r)] \psi(r, \theta, \phi) = 0$$

$$M_{nl} = m_{q1} + m_{q2} + E_{nl}$$



2. Classical description

Color charge

➤ Three element color charges and their anti-charges

● $c_r \equiv e^{\theta i}$

○ $c_{\bar{r}} \equiv e^{(\theta+\pi)i} = -c_r$

● $c_g \equiv e^{(\theta+\frac{2\pi}{3})i}$

○ $c_{\bar{g}} \equiv e^{(\theta-\frac{\pi}{3})i} = -c_g$

● $c_b \equiv e^{(\theta-\frac{2\pi}{3})i}$

○ $c_{\bar{b}} \equiv e^{(\theta+\frac{\pi}{3})i} = -c_b$

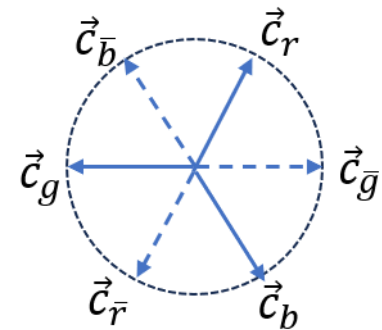
➤ Mesons and baryons are color neutral

$$c_i + c_{\bar{i}} = 0$$

$$c_r + c_g + c_b = 0$$

➤ Multicolored charged particles

$$C = \sum_{i=r,g,b} (n_i c_i + \bar{n}_i c_{\bar{i}}) \text{ quark}$$



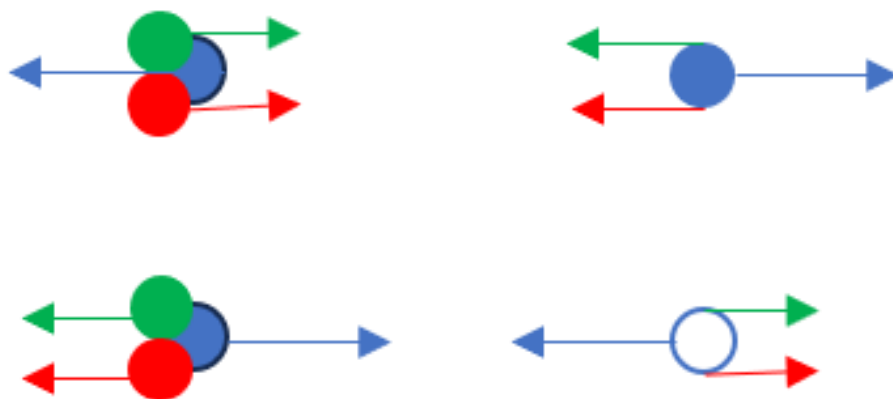
Interaction between two color charge

$$\vec{F}_{c_1 c_2} = Z \frac{c_1 \cdot c_2}{r^3} \vec{r} \quad \Rightarrow \quad E_p = -\frac{Z}{r}$$

- The dot product between two unit color charges

$$c_i \cdot (\pm c_j) = \begin{cases} \pm 1 & i = j \\ \mp \frac{1}{2} & i \neq j \end{cases}$$

- Interaction between a quark and a Neutral particle





2. Classical description

The color magnetic field

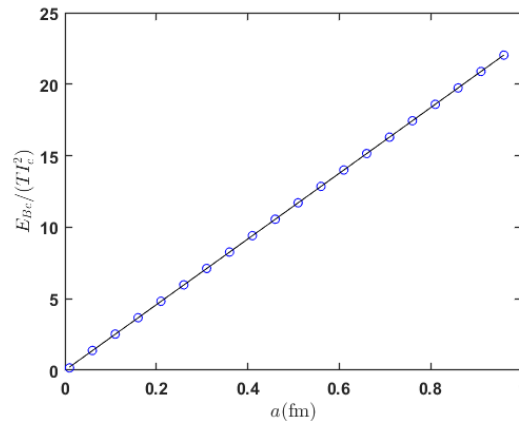
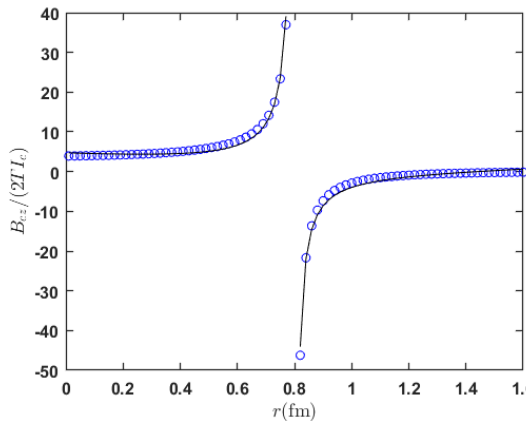
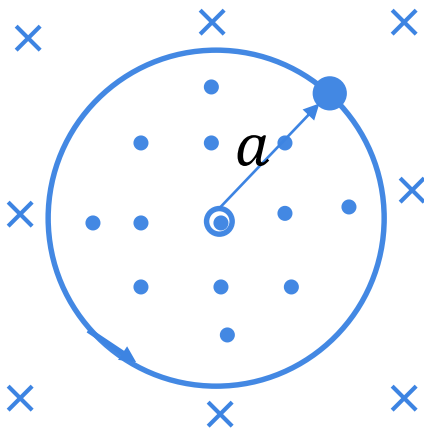
✓ Circular motion of color charge forms color flow

$$I_c = |c| \frac{v}{2\pi a} \quad B_c = \int_l T \frac{I_c d\vec{l} \times \vec{r}}{r^3}$$

✓ Circular color flow excited color magnetic field

$$B_{cz} = 2TI_c \left[\frac{1}{a-r} E(k) + \frac{1}{a+r} K(k) \right]$$

✓ The color magnetic field energy



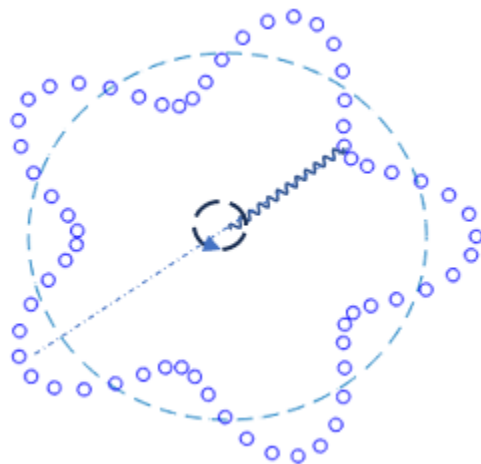
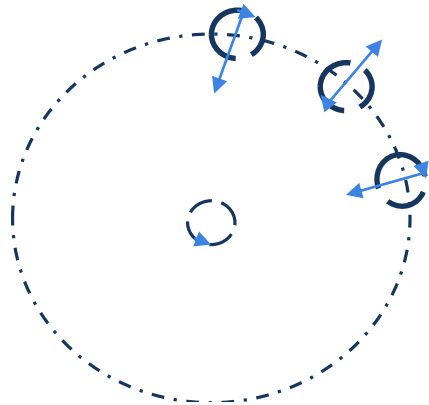
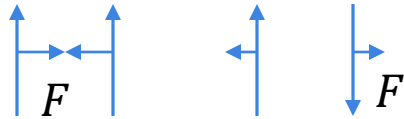
$$E_{BC} = 22.97 T I_c^2 r$$

$$= 0.5818 T \frac{Z}{mr^2}$$



2. Classical description

Interaction from spin



- ✓ Color flow attracts in the same direction and repels in the opposite direction
- ✓ Treating spin as the rotational motion of color charge
- ✓ the harmonic oscillator potential caused by spin

$$F_{S_1 S_2} = -k(r - r_0) = -\frac{1}{2} m \omega^2 (r - r_0)$$

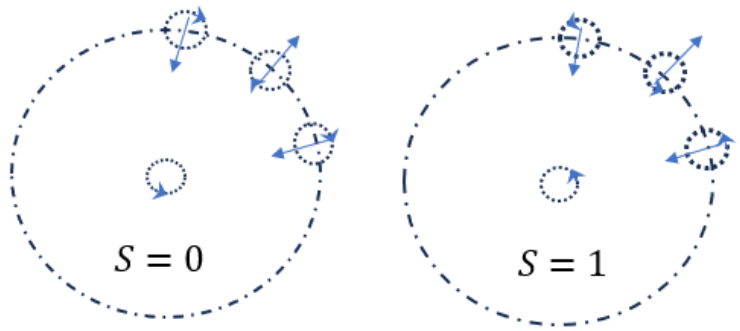
$$E_S = \frac{1}{2} k A^2 = \frac{1}{2} k (r_M - r)^2$$

$$= \frac{1}{2} k r_M^2 + \frac{1}{2} k r^2 - k r_M r$$



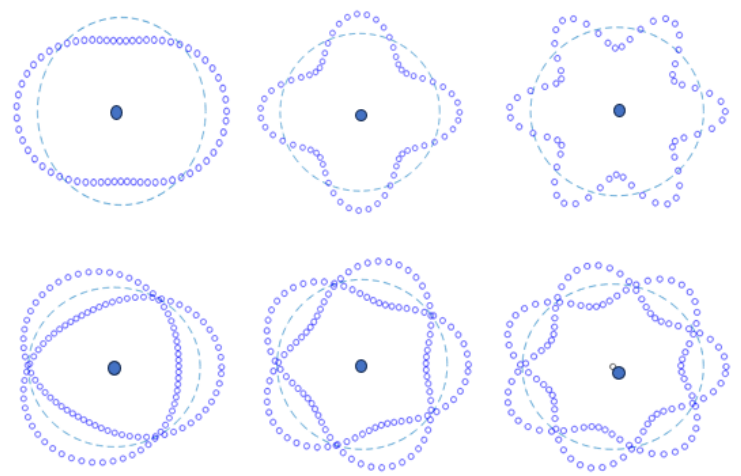
2. Classical description

Quantization hypothesis



$$E_S = \left(L + \frac{1}{2} \right) \hbar \omega \quad L = 0, 1, 2, \dots$$

$$\frac{2\pi r_n}{v_n} = \begin{cases} \left(n + \frac{1}{2} \right) \frac{2\pi}{2\omega} & (S = 0) \\ (n + 1) \frac{2\pi}{2\omega} & (S = 1) \end{cases} \quad n = 1, 2, \dots$$



$$r_n^3 = \begin{cases} \left(n + \frac{1}{2} \right)^2 \frac{Z}{m\omega^2} = \left(\frac{2n + 1}{3} \right)^2 r_1^3 & (S = 0) \\ \left(\frac{2n + 2}{2} \right)^2 \frac{Z}{m\omega^2} = \left(\frac{n + 1}{2} \right)^2 r_1'^3 & (S = 1) \end{cases}$$



3. Parameters and results

$$\vec{F}_{C_1 C_2} = Z \frac{C_1 \cdot C_2}{r^3} \vec{r} \qquad \vec{B}_c = \int_l T \frac{I_c d\vec{l} \times \vec{r}}{r^3}$$

➤ $M_{n,L} = M - \frac{Z}{2r_n} + 0.5818T \frac{Z}{mr_n^2} + \left(L + \frac{1}{2}\right) \hbar\omega$

with $M = m_Q + m_q, m = \frac{m_q m_q}{M}$

➤ Using some experimental data on mass and radius of π & ρ from PDG to estimate Z and T

$n^{2S+1}L_j$	Name	$q\bar{q}'$	$\sqrt{\langle r_1 \rangle^2}$ (fm)	M_1 (GeV)
1^1S_0	π^\pm	ud	0.6626	0.1400
1^1S_1	ρ^\pm	ud	0.7483	0.7750

$$Z \approx 1.33 \times 10^{-26} Nm^2,$$

$$T \approx 4.43 \times 10^{-44} Ns^2$$



3. Parameters and results

- Calculation on some other mesons' radii and compare with literatures

$n^{2S+1}L_j$	Name	qq'	$M(\text{GeV})$	$\sqrt{r^2}(\text{fm})$ [Ref.]	this work
1^1S_0	π^\pm	ud	0.140	0.6626	0.6621
2^1S_0	$\pi(1300)^\pm$	ud	1.300	0.59~ 0.92 [16]	1.038
3^1S_0	$\pi(1800)^\pm$	ud	1.812		1.256
1^3S_0	ρ^\pm	ud	0.775	0.7483	0.7475
2^3S_1	$\rho(1450)^\pm$	ud	1.465		1.310
3^3S_1	$\rho(1900)^\pm$	ud	1.900		1.568
1^1S_0	K^\pm	us	0.494	0.58 [17]	0.554
1^1S_0	h^0	ss	0.701	0.36 [18]	0.417
1^1S_0	D^\pm	cd	1.87	0.426 [19]	0.557
1^1S_0	D^0	cu	1.865?		0.692
1^1P_1	$D_1(2430)^0$	cu	2.412		1.573
1^1D_2	$D_2(2740)^0$	cu	2.740		1.992
1^1S_0	D_s^+	cs	1.945	0.26 [18]	0.277
1^1P_1	$D_{s1}(2460)^\pm$	cs	2.46		0.435
1^1S_0	B^\pm	ub	5.279?	0.62 [19]	0.690
1^1S_0	η_c	cc	2.984	0.20 [20]	0.174
1^3S_1	J/Ψ	cc	3.097	0.212 [20]	0.199
1^1P_1	h_c	cc	3.525	0.265 [20]	0.267
1^1S_0	η_b	bb	9.399	0.07 [19]	0.064



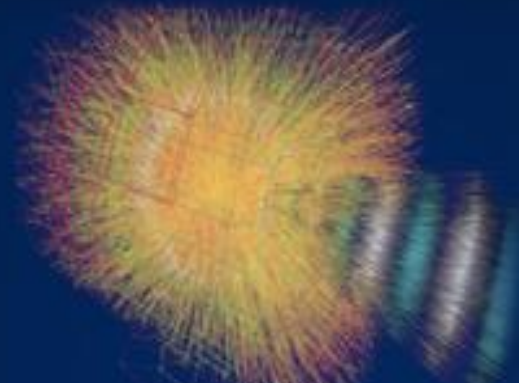
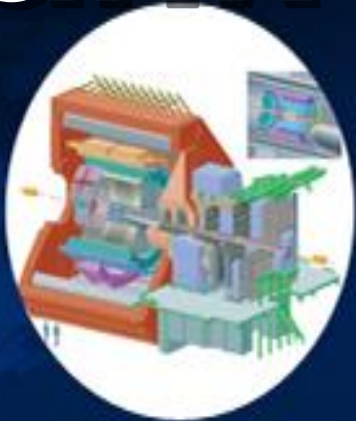
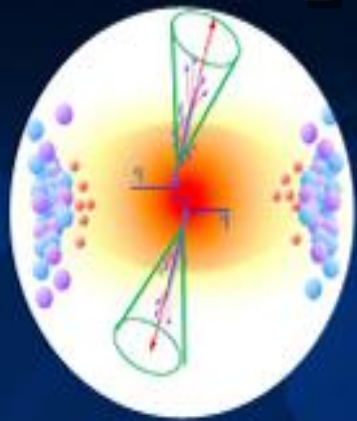
- [15] C. Patrignani, K. Agashe, G. Aielli et al., (Particle Data Group), Review of Particle Physics. Chin. Phys. C 40, 100001 (2016).doi:10.1088/1674-1137/40/10/100001
- [16] Höll A, Krassnigg A, Maris P, et al. Electromagnetic properties of ground-state and excited-state pseudoscalar mesons[J]. Physical Review C - Nuclear Physics, 2005, 71(6): 065204.
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- [18] Hernández-Pinto R J, Gutiérrez-Guerrero L X, Bashir A, et al. Electromagnetic form factors and charge radii of pseudoscalar and scalar mesons: A comprehensive contact interaction analysis[J]. Physical Review D, 2023, 107(5): 054002.
- [19] Hwang C W. Charge radii of light and heavy mesons[J]. The European Physical Journal C-Particles and Fields, 2002, 23: 585-590.
- [20] R. J. Hernandez-Pinto, L. X. Gutierrez-Guerrero, A. Bashir, M. A. Bedolla, and I. M. Higuera-Angulo, Phys. Rev. D 107 5, 054002, (2023).

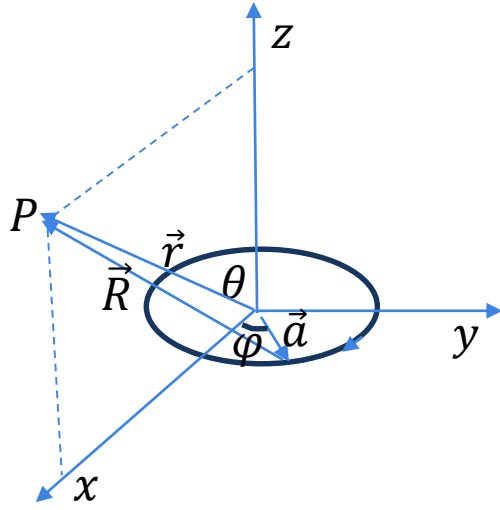


- we propose a classical expression based on color charge interactions between a pair of quarks.
- Especially, the design of the superposition and dot multiplication of quark color charges predicts the existence of non unit color charge elementary particles.
- The estimated vacuum static color gravitational constant Z and color magnetic field constant T presented in this talk are quite rough. However, we are eager to obtain more accurate values as soon as possible.
- we hope that the classic approach in this report can provide a simple and visualizable approach to study the interior of microscopic particles.



Thank you !



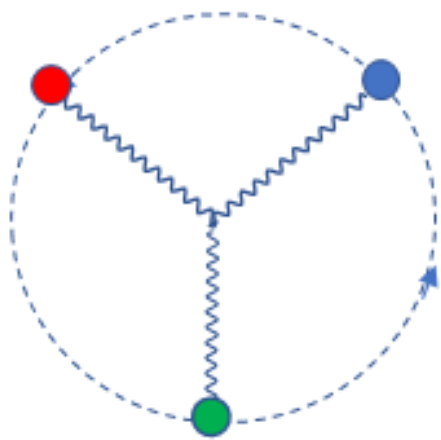


$$\begin{aligned}
 B_{cx} &= T I_c a r \cos \theta \int_0^{2\pi} \frac{\cos \varphi d\varphi}{(r^2 + a^2 - 2ra \sin \theta \cos \varphi)^{3/2}}, \\
 B_{cy} &= 0, \\
 B_{cz} &= T I_c \int_0^{2\pi} \frac{a^2 - ar \sin \theta \cos \varphi d\varphi}{(r^2 + a^2 - 2ra \sin \theta \cos \varphi)^{3/2}}. \quad (14)
 \end{aligned}$$

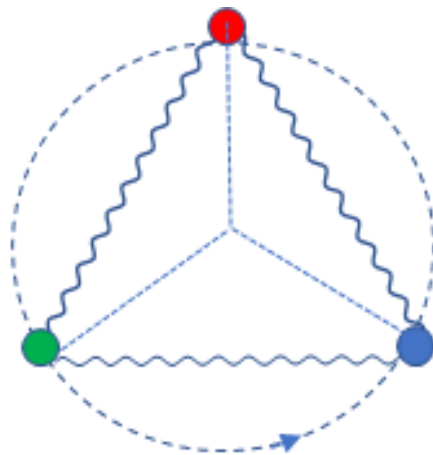
Points on the color flow plane, $\theta = \pi/2, \sin \theta = 1, \cos \theta = 0$, therefore, $B_{cx} = B_{cy} = 0$.

$$\begin{aligned}
 B_{cz} &= T I_c \int_0^{2\pi} \frac{a^2 - ar \cos \varphi d\varphi}{(r^2 + a^2 - 2ra \cos \varphi)^{3/2}} \\
 &= 2T I_c \left[\frac{1}{a-r} E(k) + \frac{1}{a+r} K(k) \right] \\
 &= 2T I_c X(a, r). \quad (15)
 \end{aligned}$$

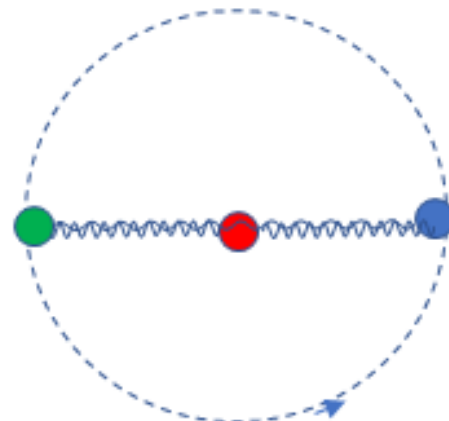
$$\begin{aligned}
 a = 0.5, \quad X_i &= 1.2215 \frac{1}{a-r} + 9.9248(a-r), \\
 X_o &= -0.8433 \frac{1}{r-a} + 5.0440(r-a); \\
 a = 0.6, \quad X_i &= 1.2001 \frac{1}{a-r} + 7.0576(a-r), \\
 X_o &= -0.8593 \frac{1}{r-a} + 3.6412(r-a); \\
 a = 0.7, \quad X_i &= 1.1833 \frac{1}{a-r} + 5.2816(a-r), \\
 X_o &= -0.8717 \frac{1}{r-a} + 2.7549(r-a); \\
 a = 0.8, \quad X_i &= 1.1696 \frac{1}{a-r} + 4.1042(a-r), \\
 X_o &= -0.8817 \frac{1}{r-a} + 2.1587(r-a); \\
 a = 0.9, \quad X_i &= 1.1583 \frac{1}{a-r} + 3.2828(a-r), \\
 X_o &= -0.8900 \frac{1}{r-a} + 1.7381(r-a); \\
 a = 1.0, \quad X_i &= 1.1487 \frac{1}{a-r} + 2.6866(a-r), \\
 X_o &= -0.8970 \frac{1}{r-a} + 1.4301(r-a).
 \end{aligned}$$



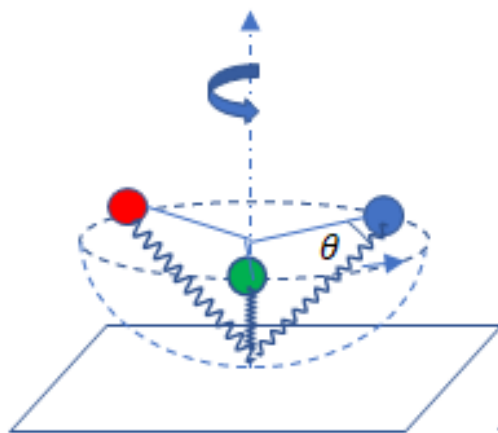
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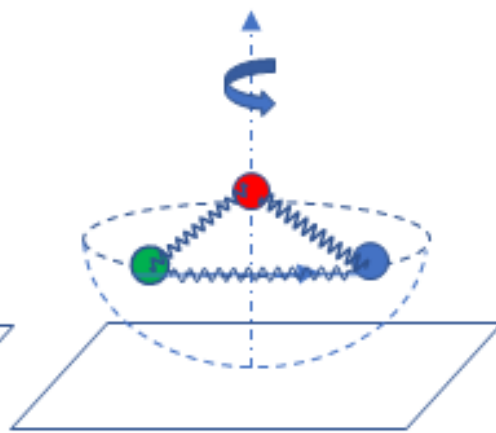
B



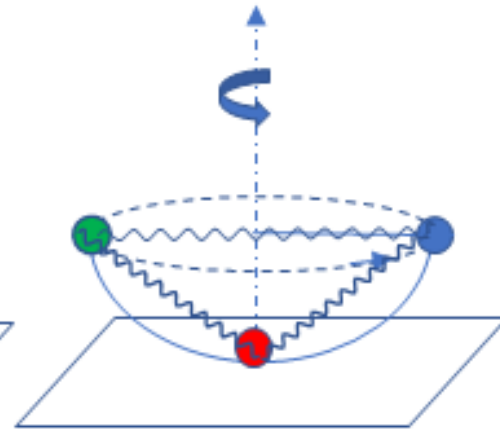
C



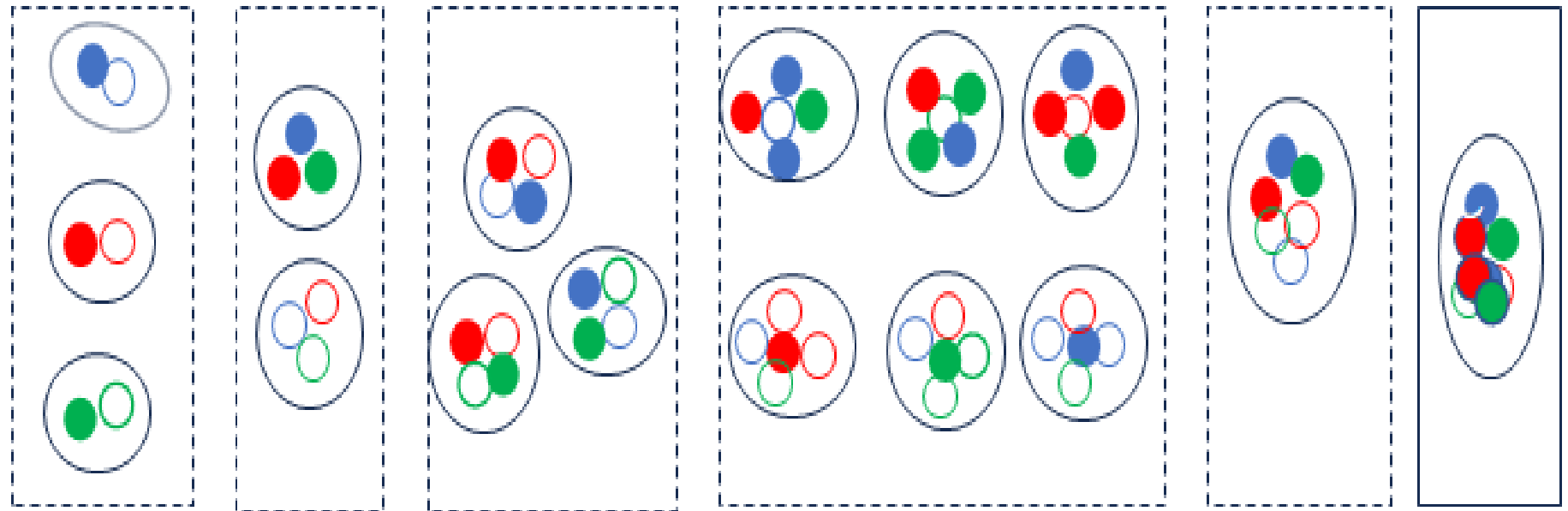
A



B



C



mesons → baryons → tetraquarks → pentaquarks → multiquarks