Spin polarization at pA system, new spin polarization effects and late time attractors

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Outline

- Introduction to spin polarization
- Spin polarization at pA system
  Also see talk by Cong Yi in the same section.
- New spin polarization effects
- Late time attractors in spin hydrodynamics
- Summary

Introduction to spin polarization

Spin in high energy physics

Striking spin effects have been observed in high energy reactions since 1970s

“Proton spin crisis” 质子自旋危机

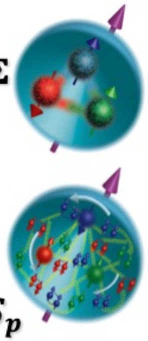
夸克模型：

夸克自旋之和Σ =质子自旋S_p

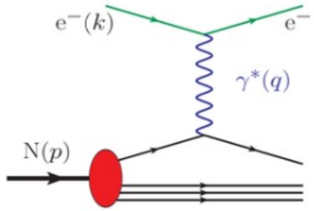
DIS实验：

89年: Σ~0

目前: Σ~20%S_p



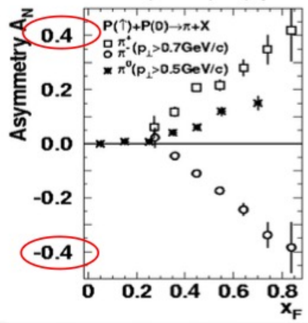
$e + p \to e + X$

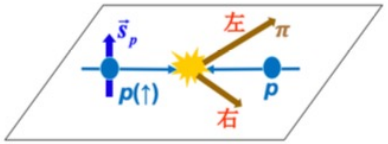


EMC, PLB 206.364 (1988)

“Single spin left-right asymmetry (SSA)”

$p(\uparrow) + p \to \pi + X$



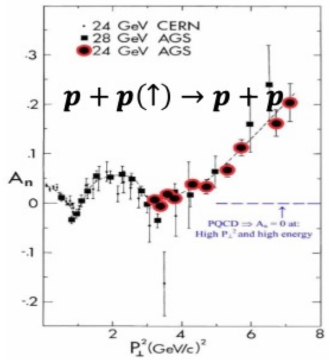


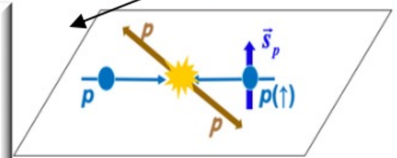
$$A_N \equiv \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

e.g. FNAL E704, PLB264, 462 (1991)

Predictions of pQCD ～ 0

“Spin analyzing power in $pp \to pp$”



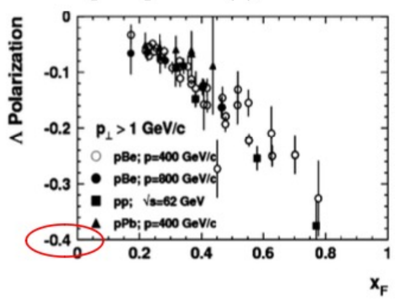


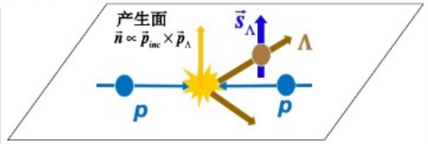
$$A_N \equiv -\frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

e.g. D. Grab *et al.*, PRL41, 1257 (1978)

“Transverse polarization of hyperon in $pp \to \Lambda X$”

$p + p \to \Lambda(\uparrow) + X$



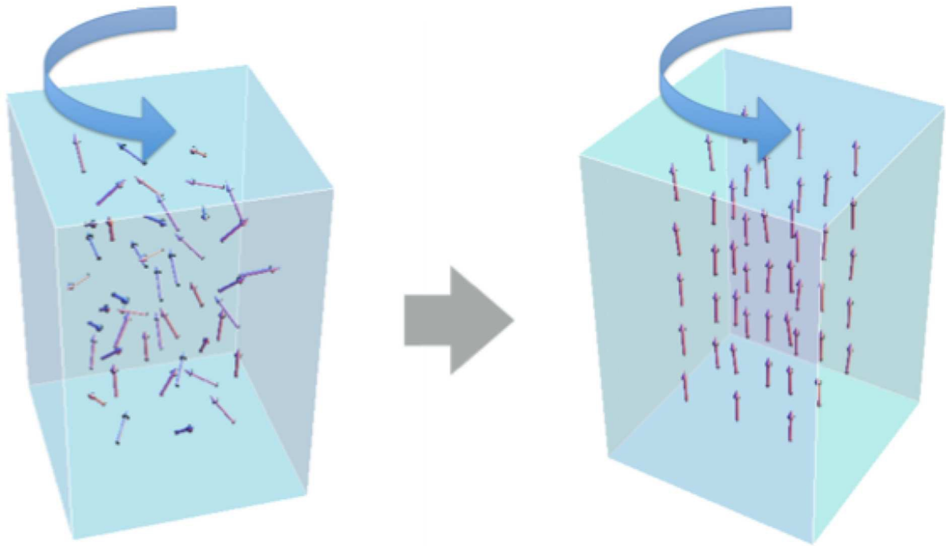


$$P_\Lambda \equiv \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$

e.g. S.A. Gourlay *et al.*, PRL56, 2244 (1986)

Slides copy from Prof. Zuo-tang Liang’s review talk

Barnet and Einstein-de Hass effects



Barnett effect:

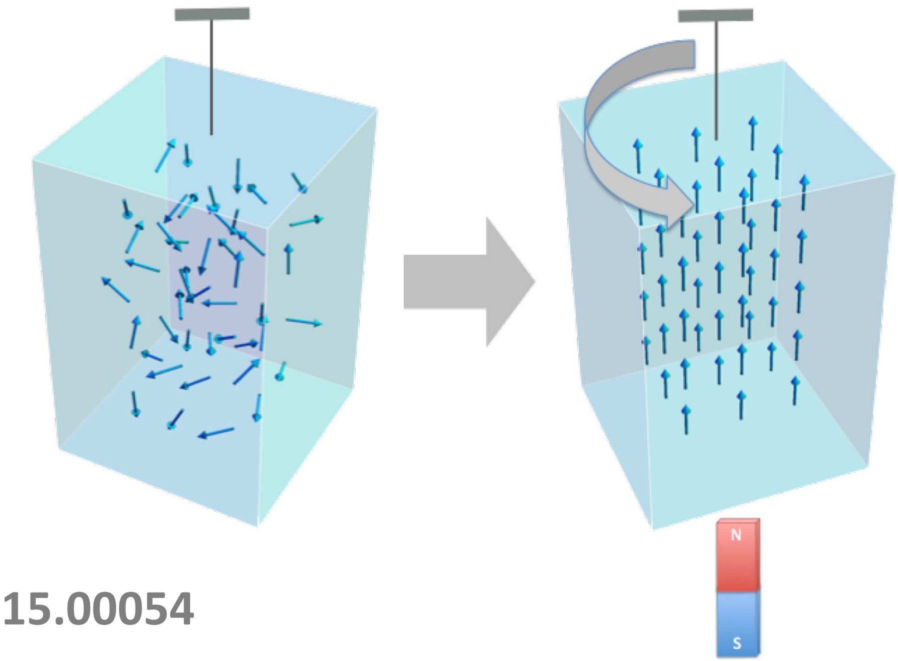
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

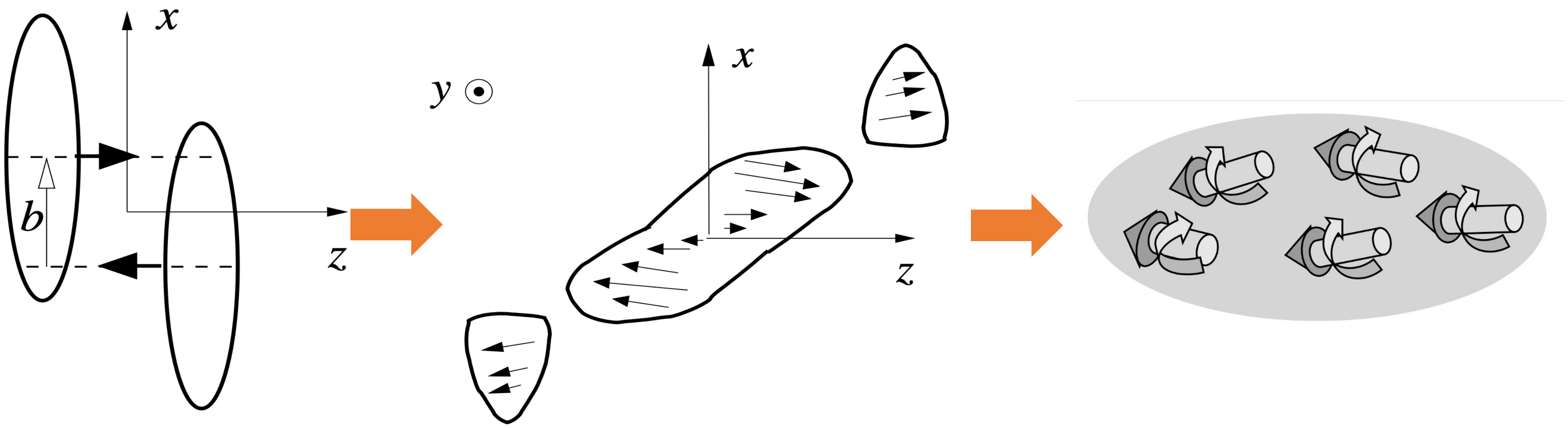
Magnetization $\Longrightarrow$ Rotation

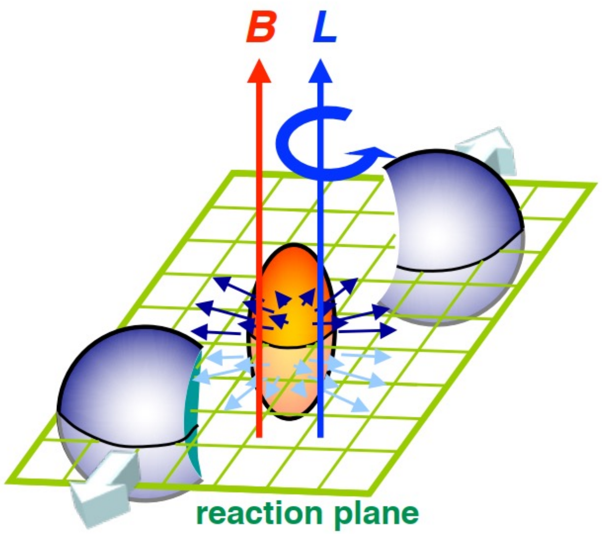
Einstein, de Haas, Experimental proof of the existence of Ampere’s molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.



Figures: copy from paper doi: 10.3389/fphy.2015.00054

OAM to spin polarization in HIC



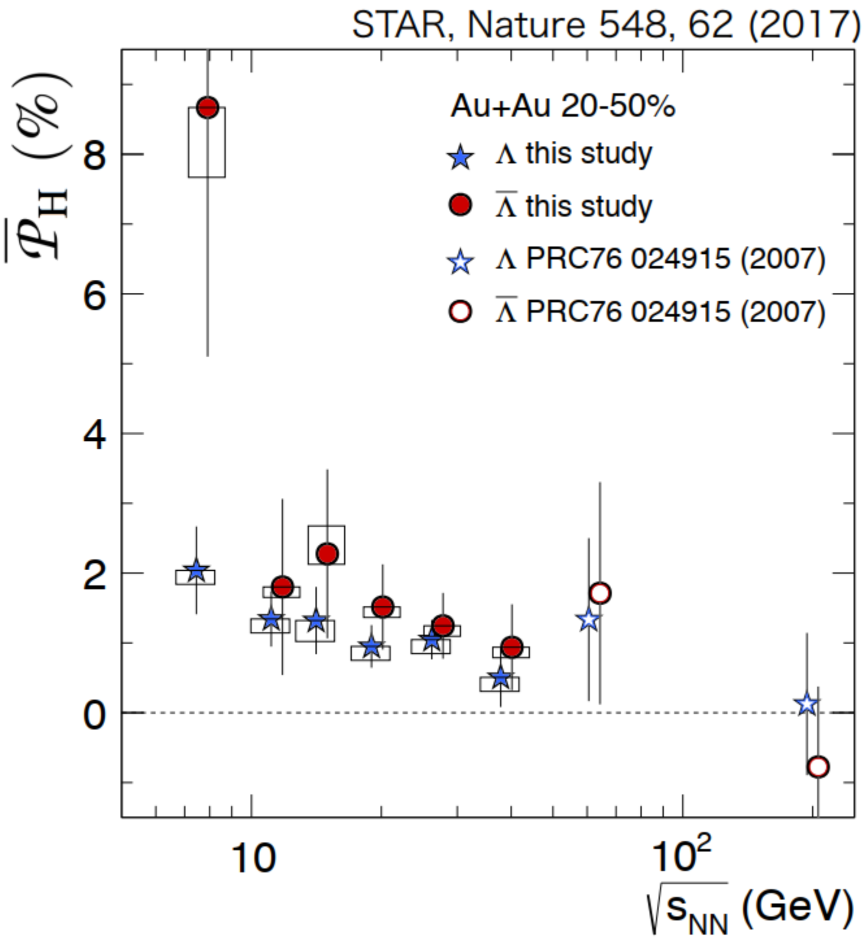


- **Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.**

- **Global orbital angular momentum leads to the polarizations of Λ hyperons and spin alignment of vector mesons through spin-orbital coupling.**

Liang, Wang, PRL (2005); PLB (2005);
Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization for Λ and $\bar{\Lambda}$ hyperons

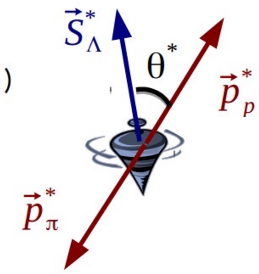


parity-violating decay of hyperons

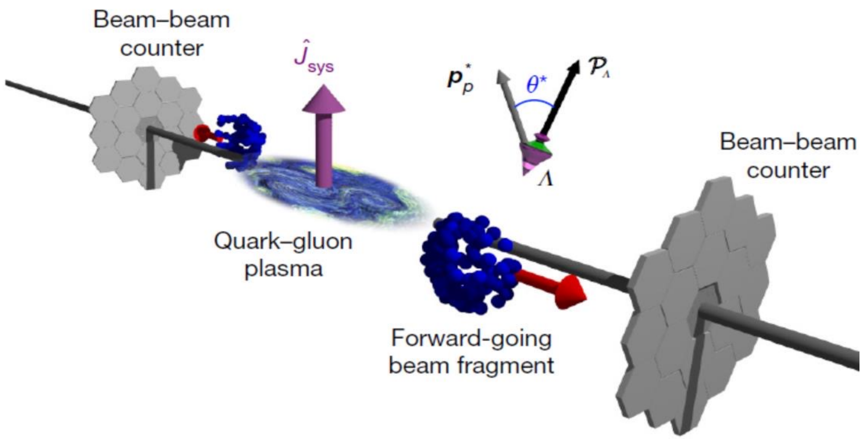
In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$\Lambda \rightarrow p + \pi^+$
(BR: 63.9%, cτ ~7.9 cm)



Most vortical fluid

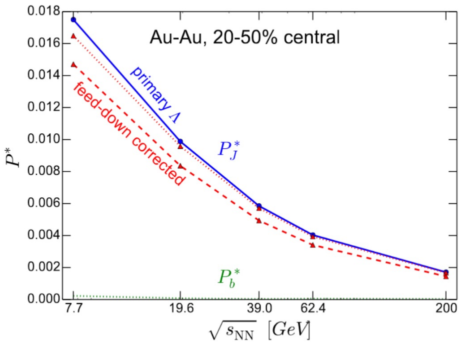
- Estimation given by Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC95, 054902 (2017)

$$\mathbf{P}_{\Lambda} \simeq \frac{\omega}{2T} + \frac{\mu_{\Lambda}\mathbf{B}}{T}$$
$$\mathbf{P}_{\overline{\Lambda}} \simeq \frac{\omega}{2T} - \frac{\mu_{\Lambda}\mathbf{B}}{T}$$

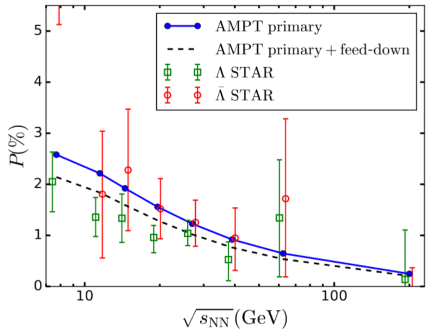
- $\omega = (9 \pm 1) \mathrm{x} 10^{21}/\mathrm{s}$, greater than previously observed in any system.
- QGP is most vortical fluid so far.

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)
…

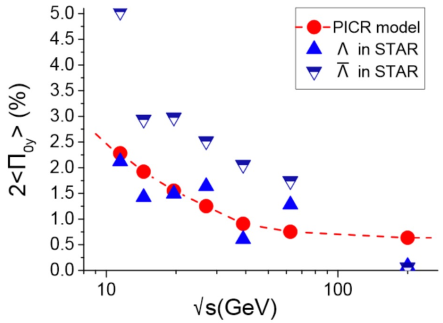
Phenomenological models for global polarization



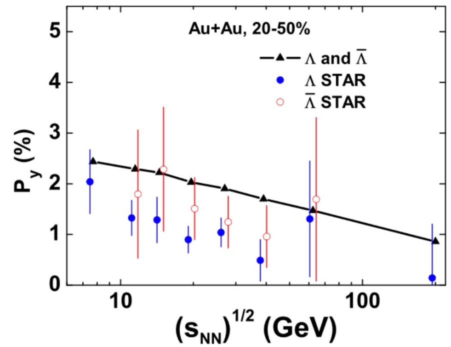
Karpenko, Becattini, EPJC(2017)



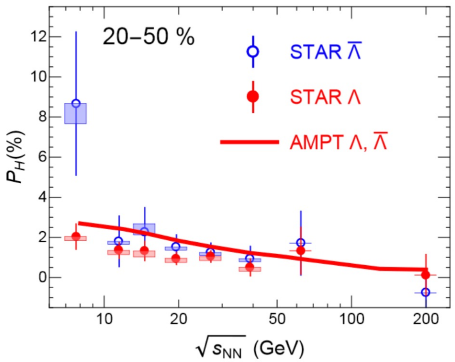
Li, Pang, Wang, Xia PRC(2017)



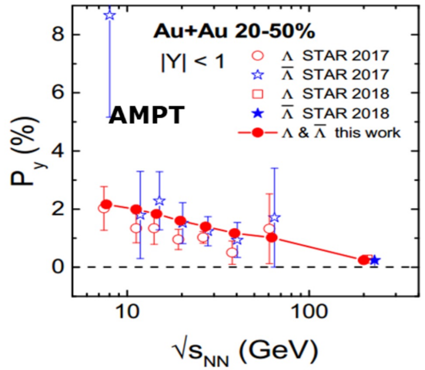
Xie, Wang, Csernai, PRC(2017)



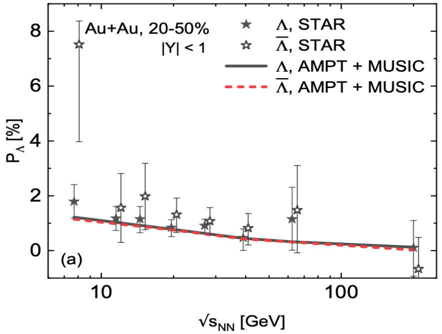
Sun, Ko, PRC(2017)



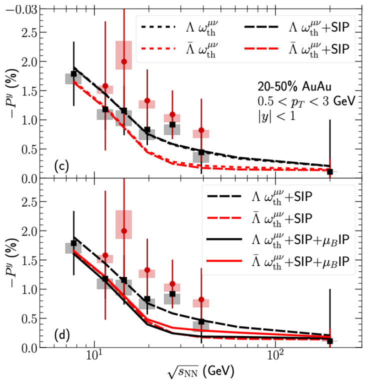
Shi, Li, Liao, PLB(2018)



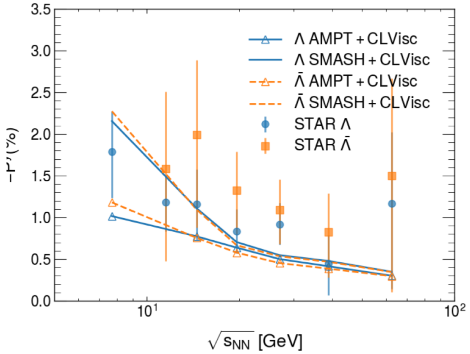
Wei, Deng, Huang, PRC(2019)



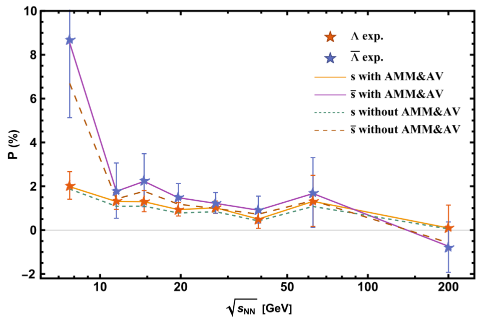
Fu, Xu, Huang, Song, PRC (2021)



S. Ryu, V. Jupic, C. Shen, PRC (2021)



Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

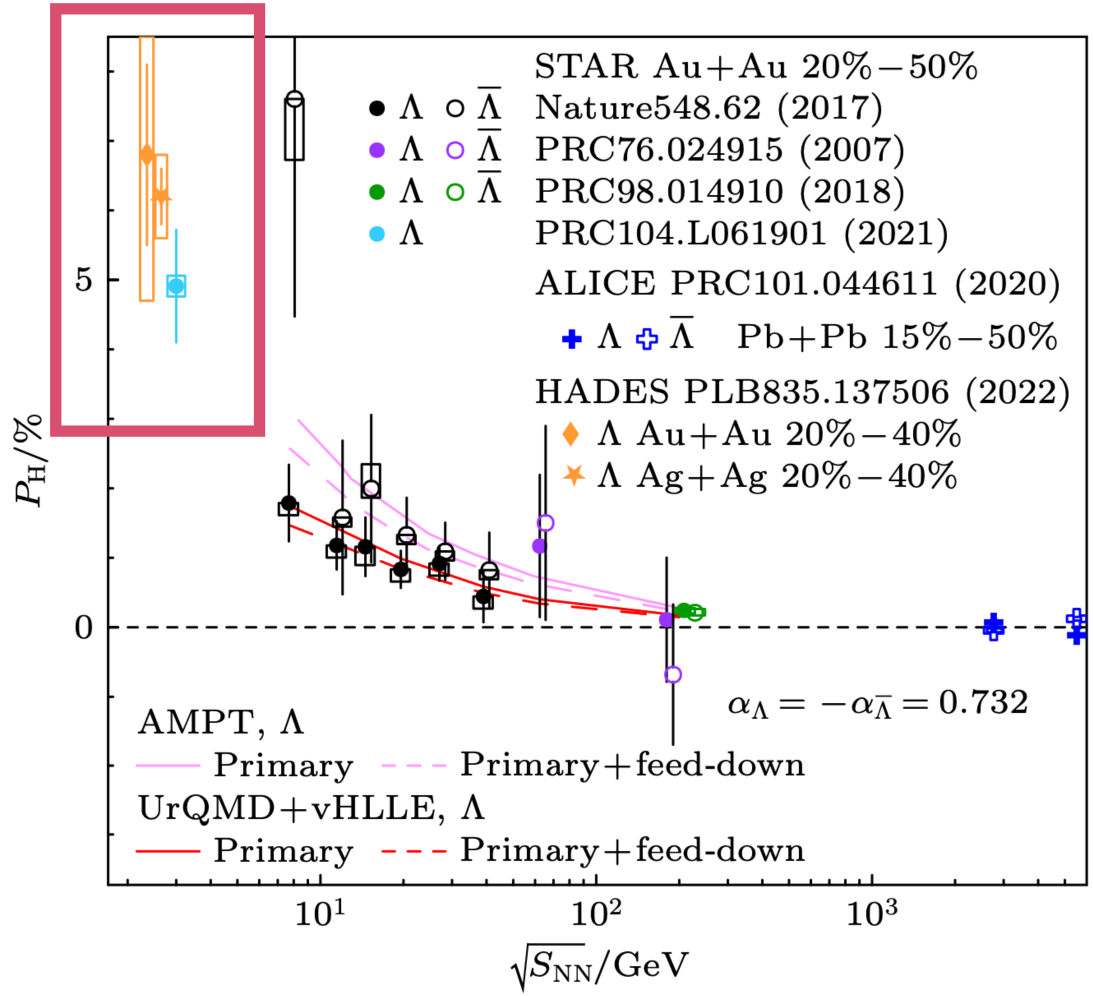


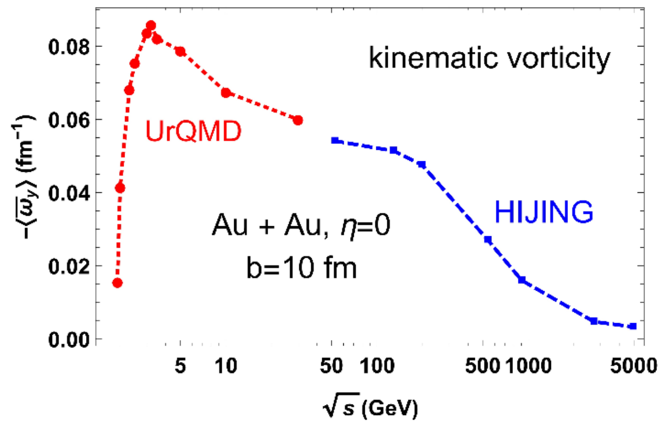
Xu, Lin, Huang, Huang, PRDL (2022)

Polarization at low energies

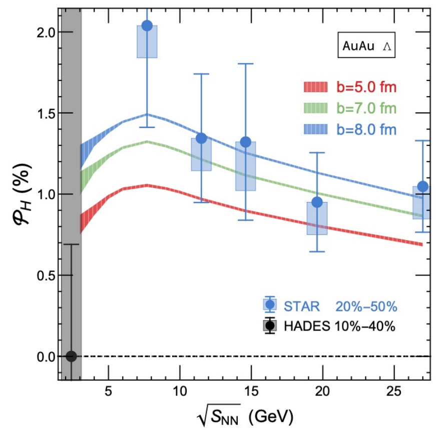
Will the polarization of Lambda be nonzero when $\sqrt{s_{NN}} \to 0$?

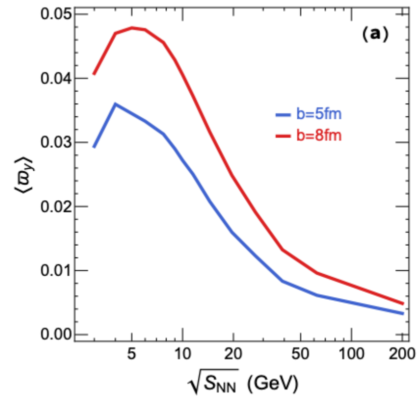
If not, how large the "critical $\sqrt{s_{NN}}$" will be?





Deng, Huang 2016; Deng, Huang, et. al 2020



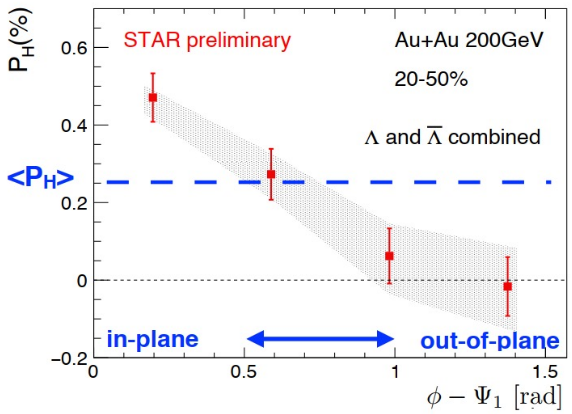


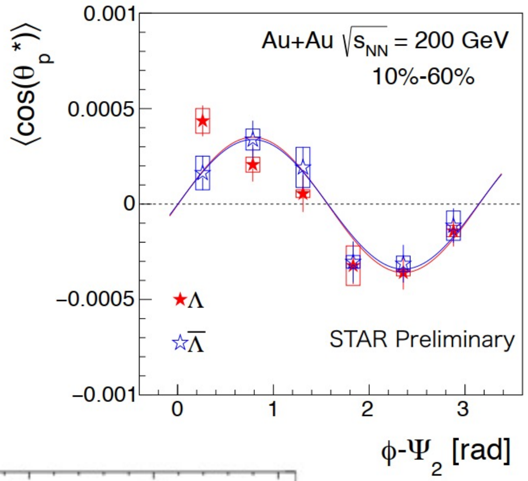
Sun Xu et al., Acta Phys. Sin. 72(7), 072401 (2023)

Guo, Liao, et. al, PRC 2021

Local polarization

$$\mathcal{S}^{\mu}_{\rm shear}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta}{(u\cdot p)T}\frac{1}{2}\{p^{\sigma}\boxed{(\partial_\sigma u_\nu+\partial_\nu u_\sigma)-Du_\nu}\}$$





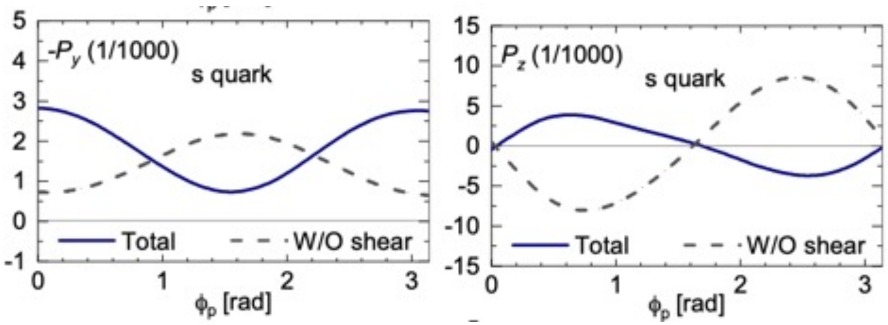
Early works:
(thermal vorticity only)

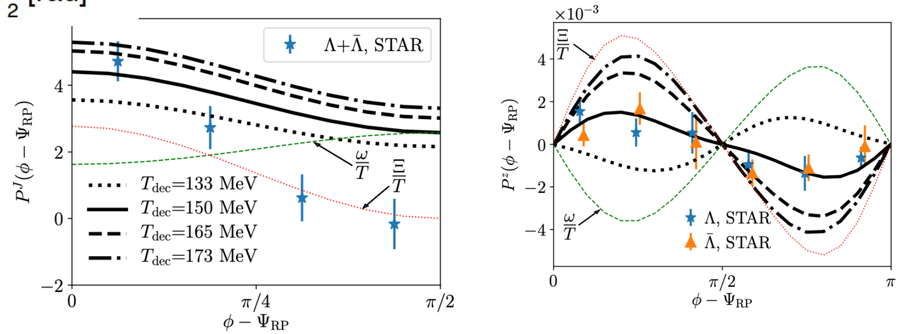
- **UrQMD :**

Becattini, Karpenko, PRL (2018)

- **AMPT:**

Xia, Li, Tang, Wang, PRC (2018)





s quark scenarios (Thermal vorticity + shear)

Fu, Liu, Pang, Song, Yin, PRL 2021

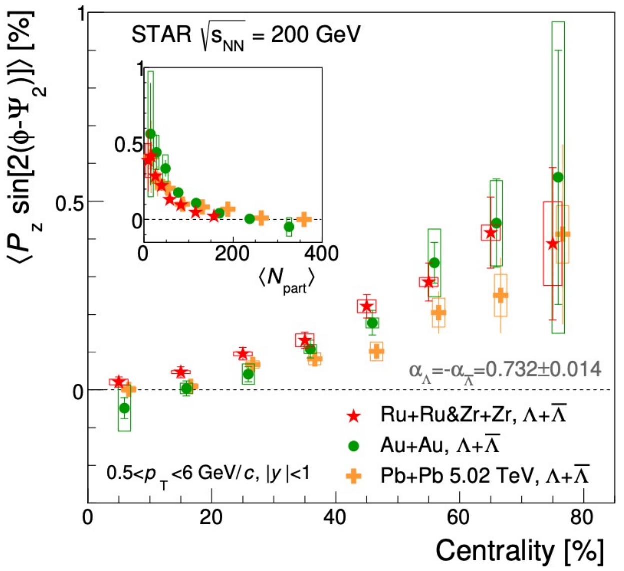
Isothermal equilibrium
(Thermal vorticity + shear)

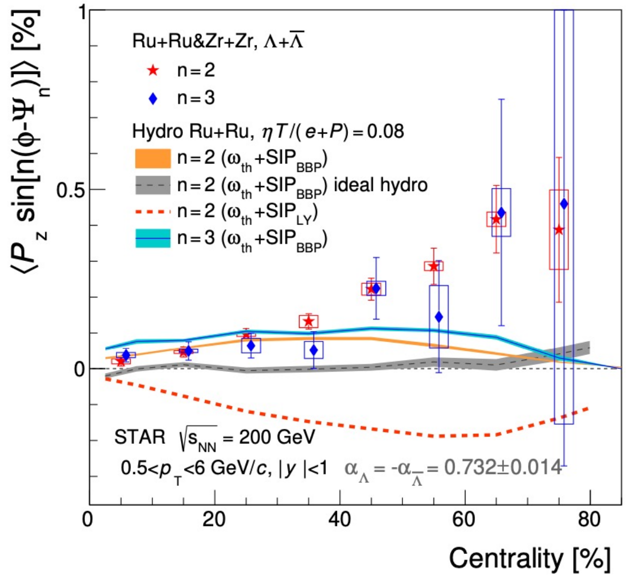
Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021

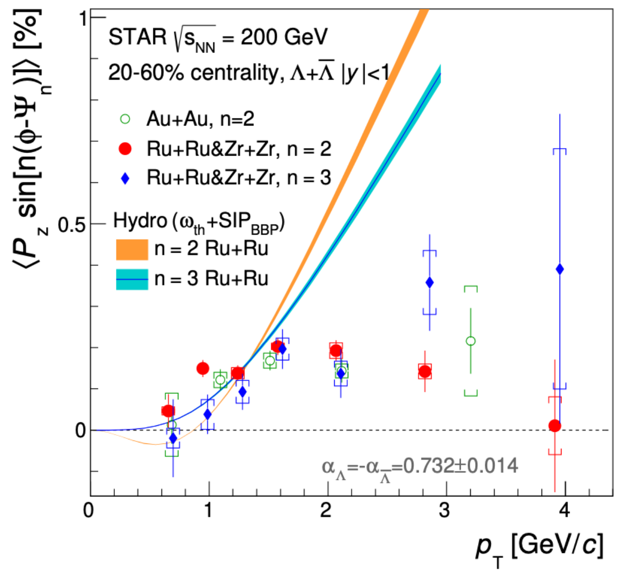
Also see:

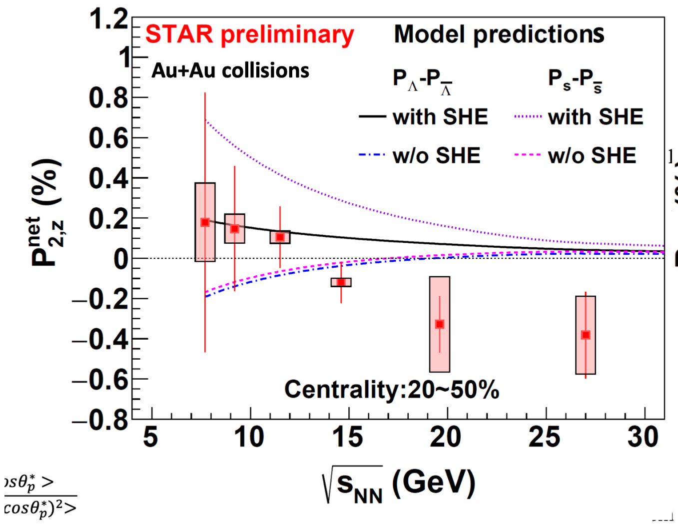
Yi, Pu, Yang, PRC (2021); Yi, Wu, Qin, Pu, PRC (2022)
Ryu, Jupic, Shen, PRC (2021)

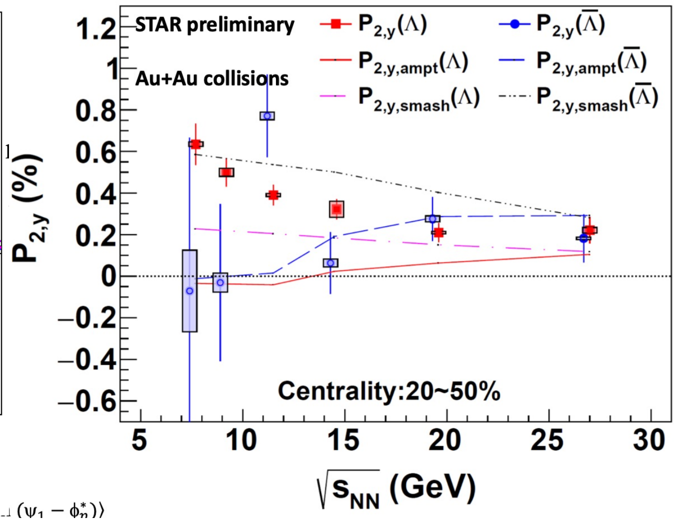
Puzzles in local polarization at AA system

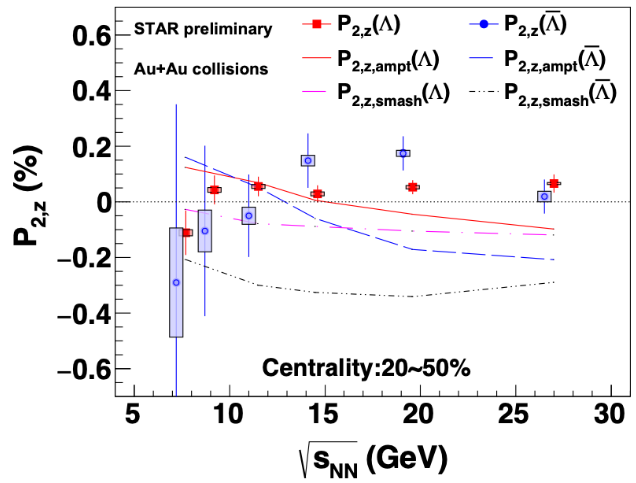












Spin polarization at pA system

C. Yi, X.Y. Wu, J. Zhu, SP, G.Y. Qin, arXiv: 2408.04296

Also see talk by Cong Yi in the same section.

Setup (I)

- **We follow the modified Cooper-Frye formula to compute the polarization pseudo-vector including the contribution from thermal vorticity and thermal shear tensor.**

$$
\begin{aligned}
\mathcal{S}^{\mu}(\mathbf{p}) &= \mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) + \mathcal{S}^{\mu}_{\text{th-shear}}(\mathbf{p}) \\
\mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) &= \hbar \int d\Sigma \cdot \mathcal{N}_p \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \varpi_{\alpha\beta}, \\
\mathcal{S}^{\mu}_{\text{th-shear}}(\mathbf{p}) &= \hbar \int d\Sigma \cdot \mathcal{N}_p \frac{\epsilon^{\mu\nu\alpha\beta} p_\nu n_\beta}{(n \cdot p)} p^\sigma \xi_{\sigma\alpha}
\end{aligned}
$$

thermal vorticity $$\varpi_{\alpha\beta} = \frac{1}{2}\left[\partial_\alpha\left(\frac{u_\beta}{T}\right) - \partial_\beta\left(\frac{u_\alpha}{T}\right)\right],$$

thermal shear tensor $$\xi_{\alpha\beta} = \frac{1}{2}\left[\partial_\alpha\left(\frac{u_\beta}{T}\right) + \partial_\beta\left(\frac{u_\alpha}{T}\right)\right]$$

Setup (II)

- **We consider three different scenarios:**
  - **Λ equilibrium:**

    It is assumed that Λ hyperons reach the local (thermal) equilibrium at the freeze-out hyper-surface.

  - **s quark equilibrium:**

    The spin of Λ hyperons is assumed to be carried by the constituent s quark. We take the s quark's mass instead of Λ's mass in the simulation.

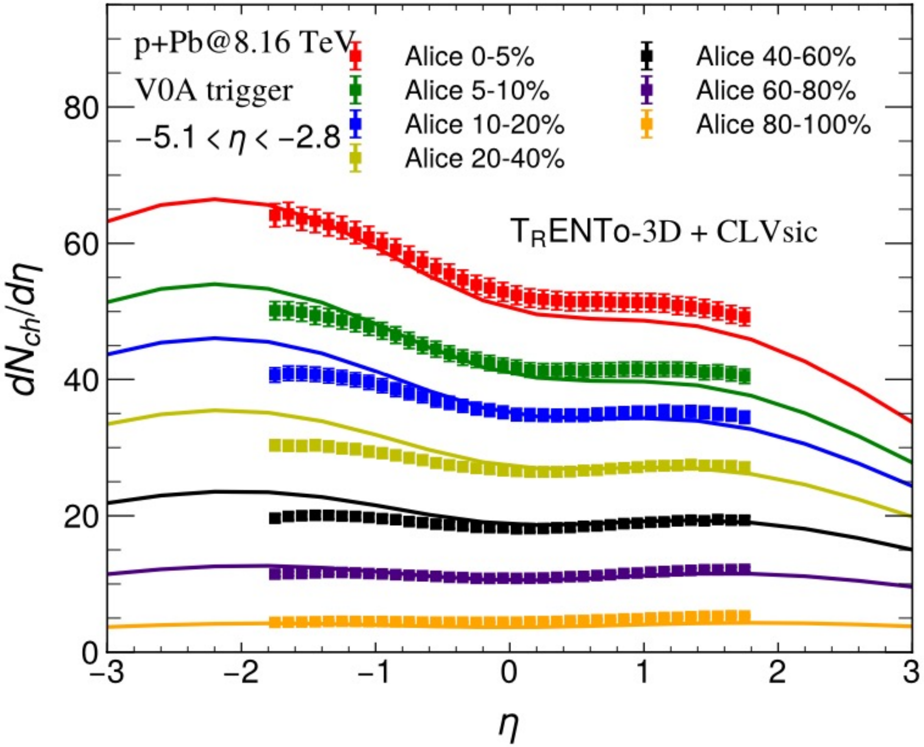
  - **Isothermal equilibrium:**

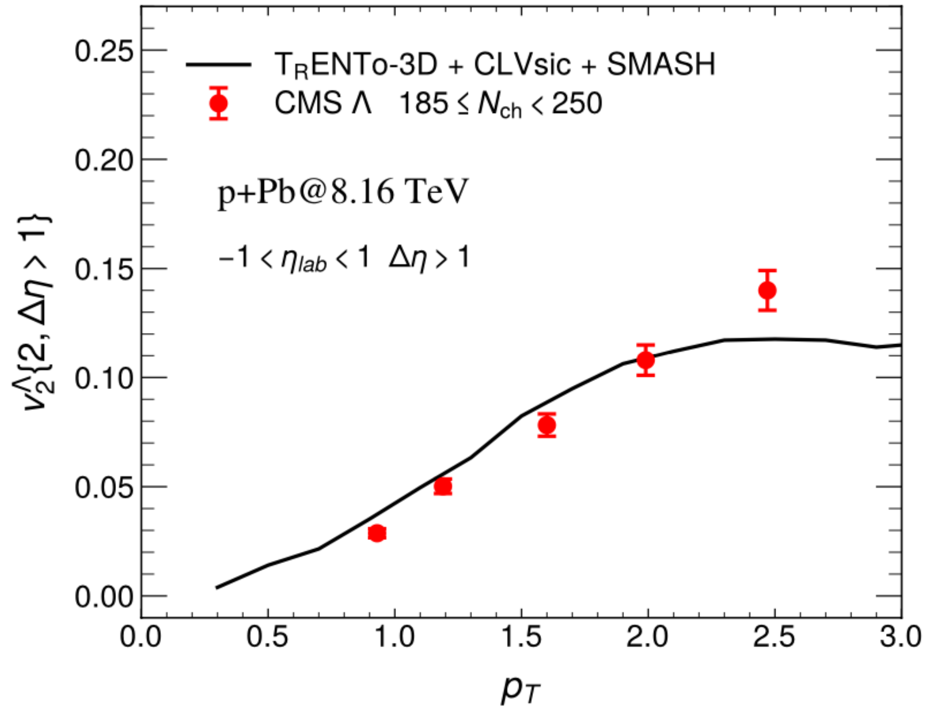
    The temperature of the system at the freeze-out hyper-surface is assumed to be constant. The time unit vector is taken as fluid velocity for simplicity.

Setup (III)

- **We implement the 3+1D CLVisc hydrodynamics model**
  - Pang, Wang, Wang, PRC (2012)
  - Wu, Qin, Pang, Wang, PRC (2022)
- **Initial condition: TRENTo-3D model**
  - Soeder, Ke, Paquet, Bass, 2306.08665
  - Moreland, Bernhard, Bass, PRC (2015); PRC (2020)
  - Ke, Moreland, Bernhard, Bass, PRC (2017)
- **EoS: HotQCD**
  - HotQCD, A. Bazavov et al., Phys. Rev. D 90, 094503 (2014)
- **Temperature dependent shear and bulk viscosity**
  - J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu, and U. Heinz, Phys. Rev. C 94, 024907 (2016)
- **p+Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV**

Fit parameters and test v2 of Λ

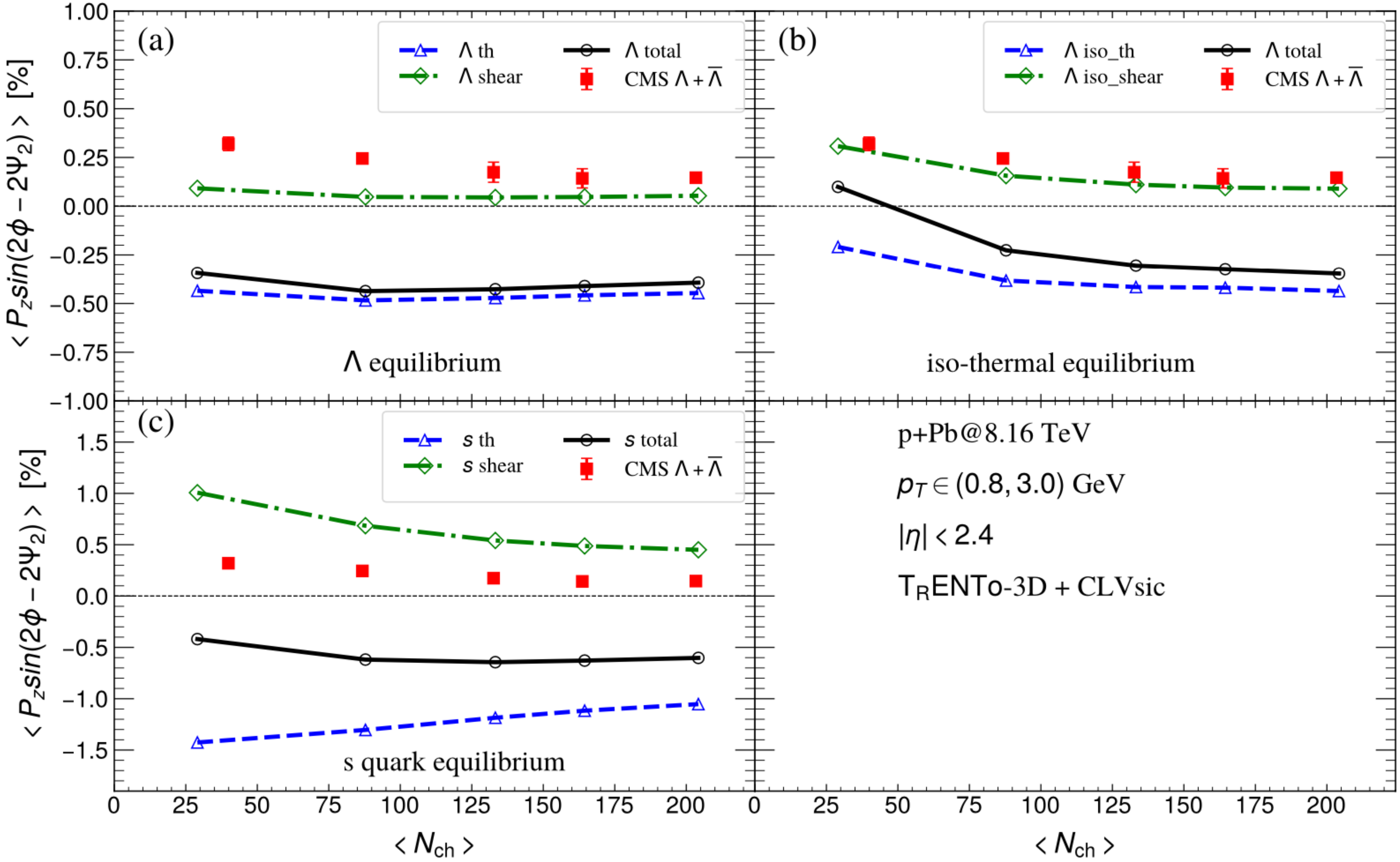




Multiplicity intervals	$\langle N_{\mathrm{ch}}\rangle_{\mathrm{exp}}$	$\langle N_{\mathrm{ch}}\rangle_{\mathrm{CLVisc}}$
[185,250)	203.3	204.2
[150,185)	163.6	164.5
[120,150)	132.7	133.57
[60,120)	86.7	87.7
[3,60)	40	29.3

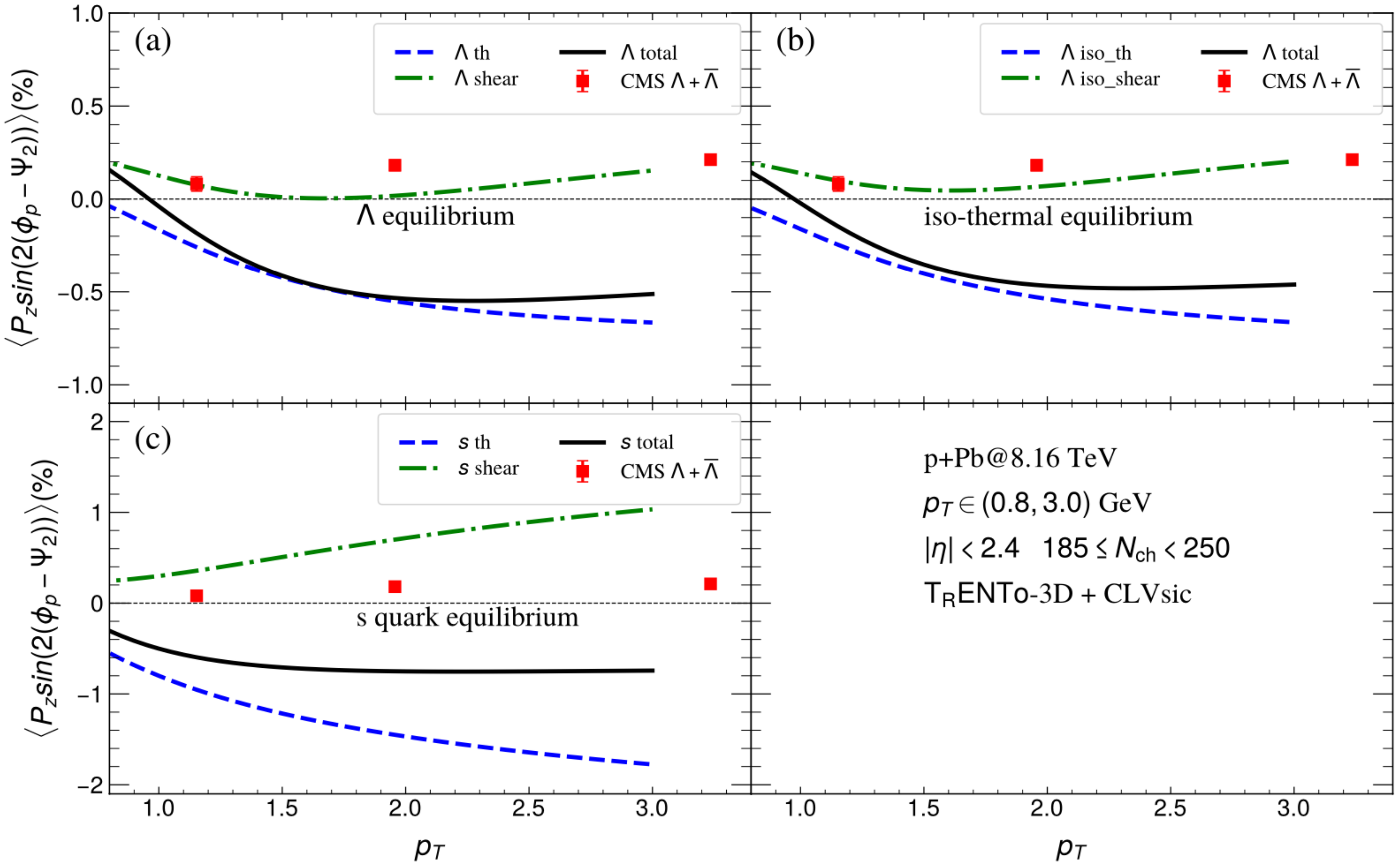
We have run 10^5 minimum bias events to divide the centrality. The centrality-dependent pseudo-rapidity distributions of charged hadrons and elliptic flow for Λ hyperons computed by our model are consistent with the experimental measurements.

Multiplicity (centrality) dependence

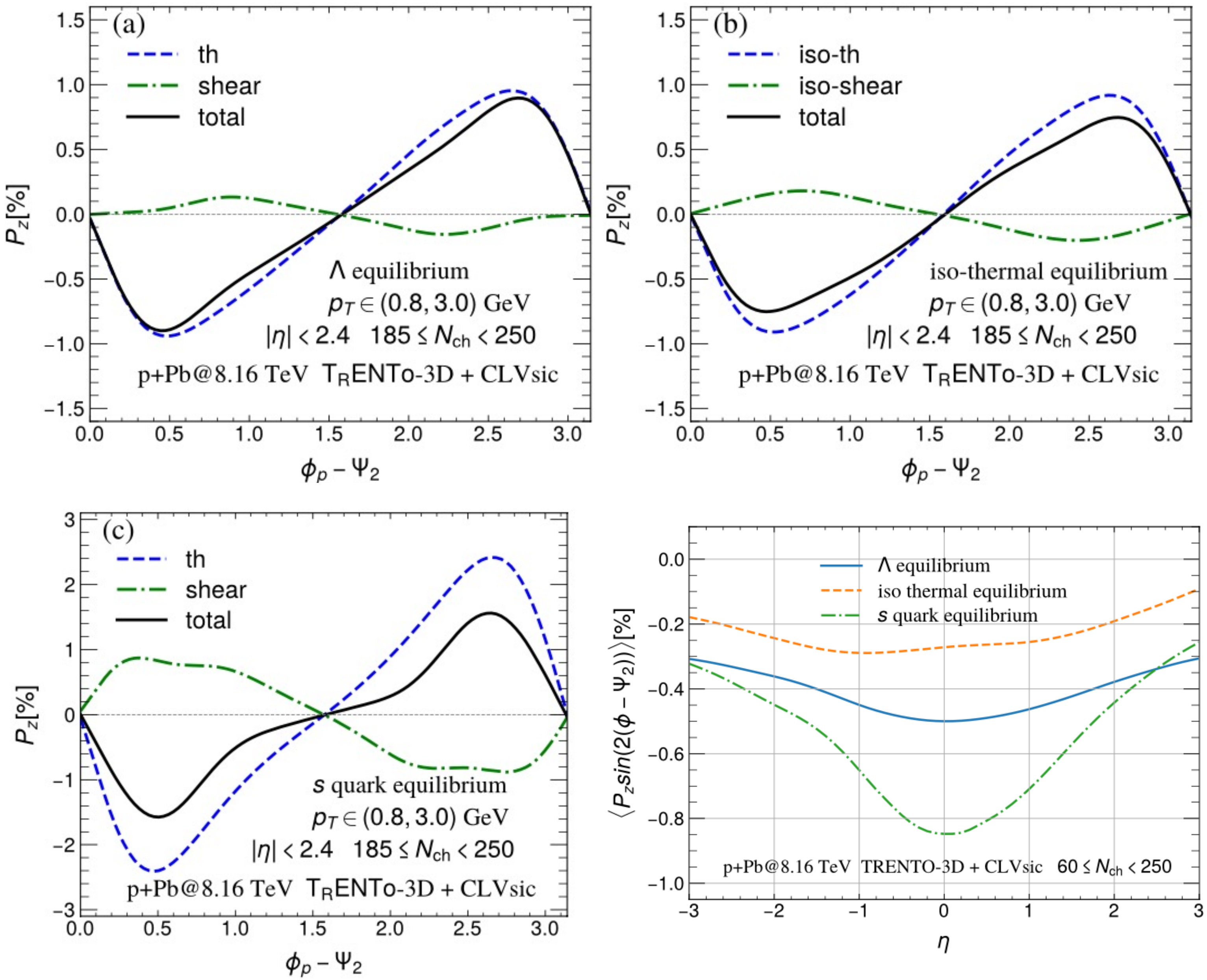


Multiplicity interval

p_T dependence



Azimuthal angle and pseudo-rapidity dependence



Why?

- **We implement the 3+1D CLVisc hydrodynamics model**

  Pang, Wang, Wang, PRC (2012)

  Wu, Qin, Pang, Wang, PRC (2022)

- **Initial condition: TRENTo-3D model**

  Soeder, Ke, Paquet, Bass, 2306.08665

  Moreland, Bernhard, Bass, PRC (2015); PRC (2020)

  Ke, Moreland, Bernhard, Bass, PRC (2017)

- **EoS: HotQCD**

  HotQCD, A. Bazavov et al., Phys. Rev. D 90, 094503 (2014)

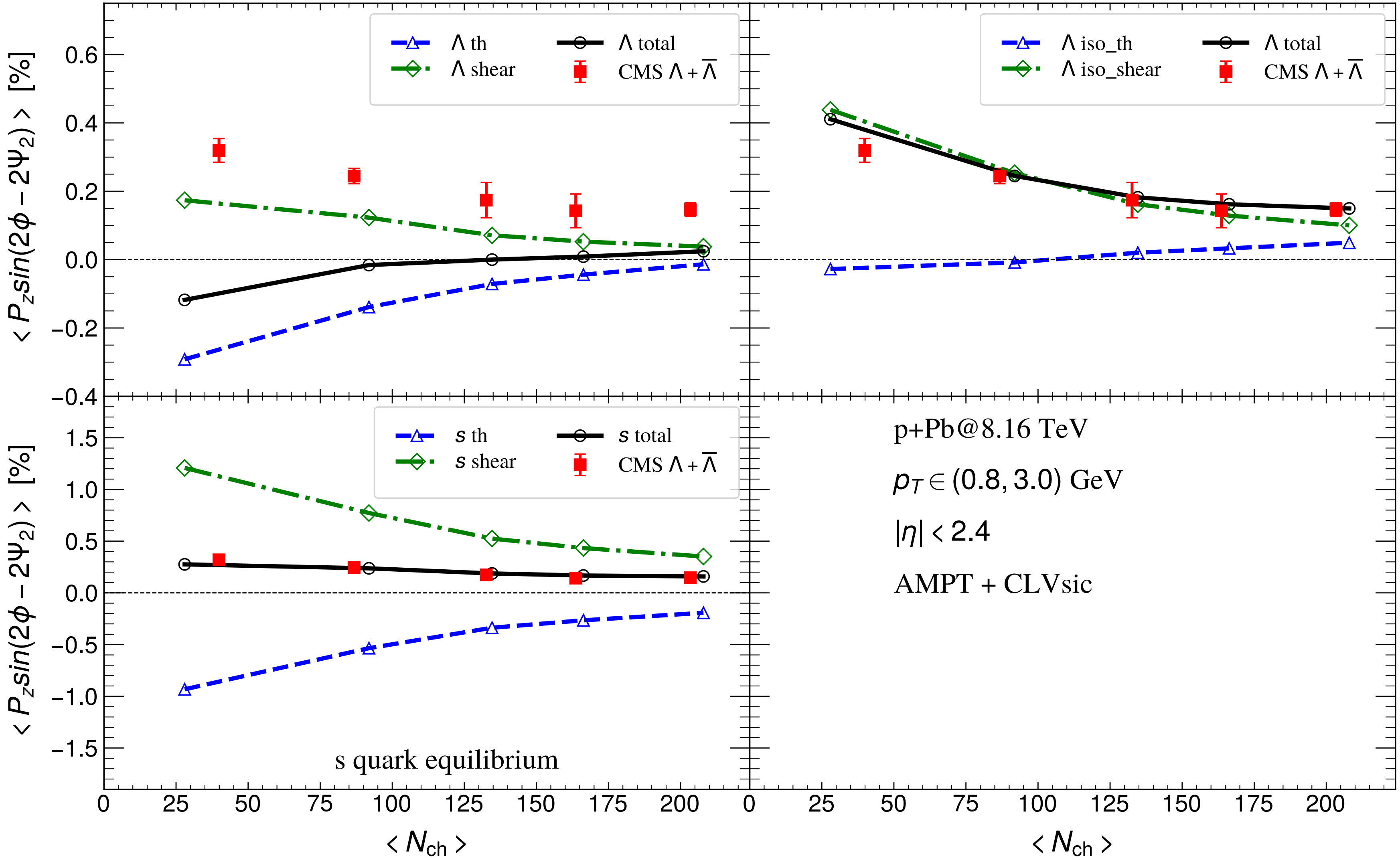
- **Temperature dependent shear and bulk viscosity**

  J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu, and U. Heinz, Phys. Rev. C 94, 024907 (2016)

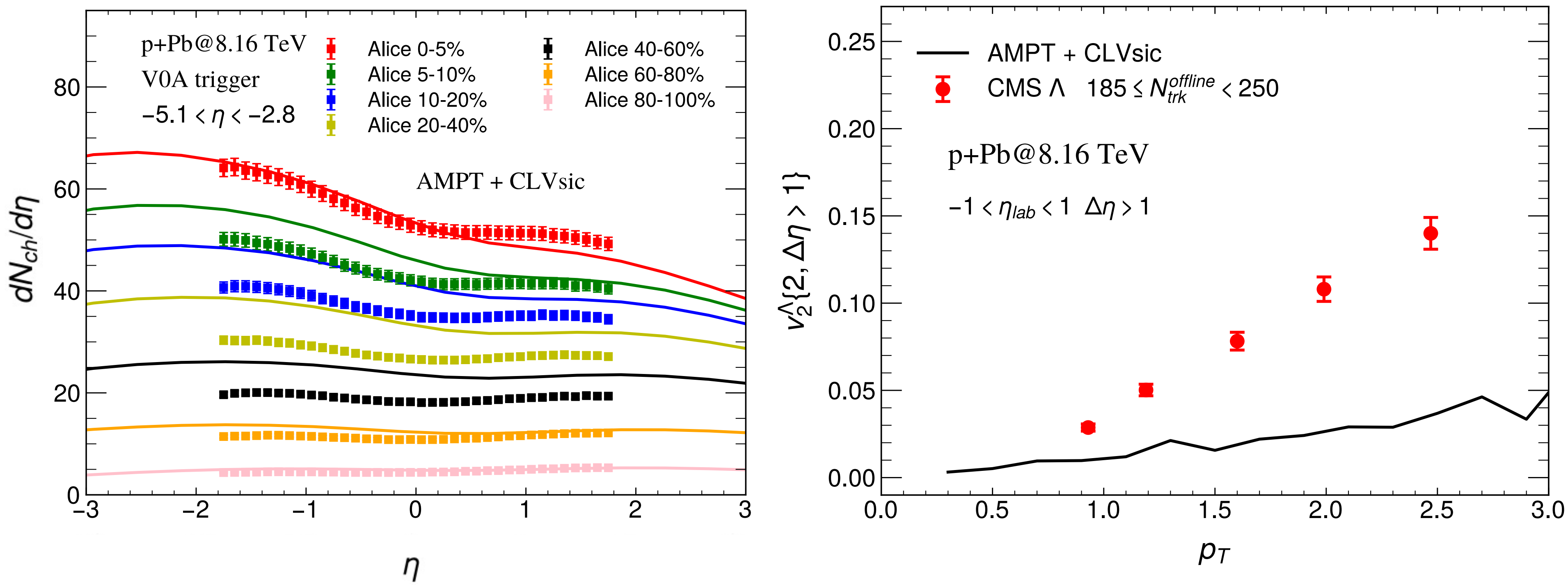
- **p+Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV**

Test for AMPT initial condtions

It describes data well in s quark and isothermal equilbirum scenarios?

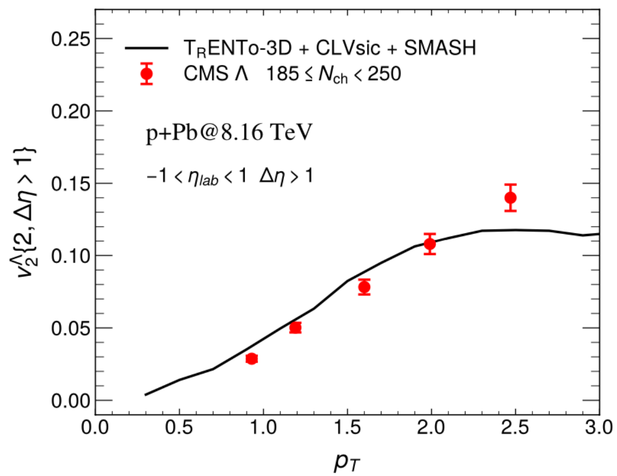


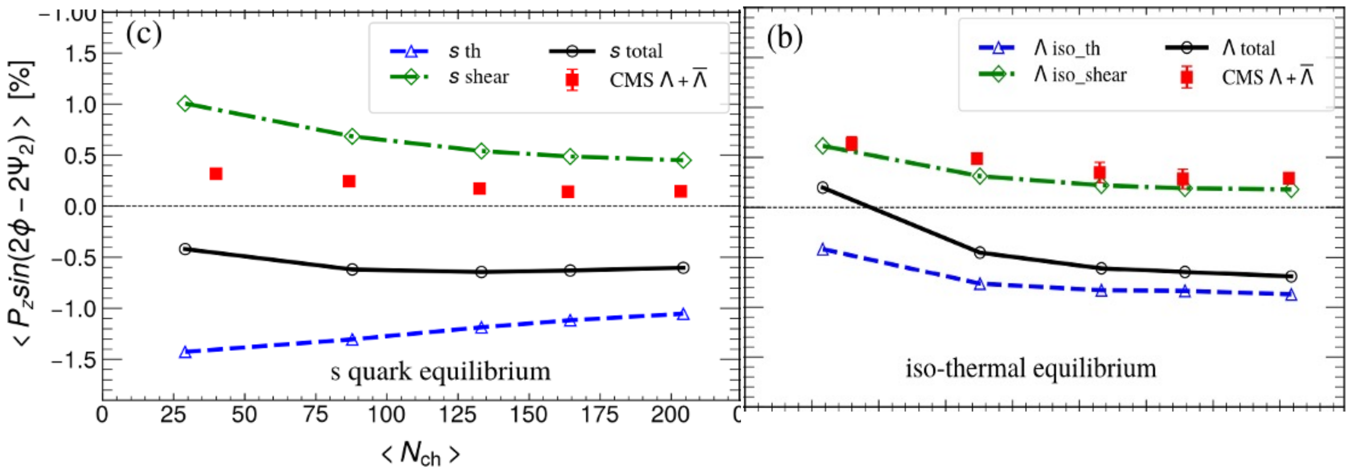
Test for AMPT initial conditions



We fix the parameters in 3+1D CLVisc hydrodynamic model with AMPT initial conditions by the spectrum of charged hadrons. But, it cannot describe v2 well.

However ...

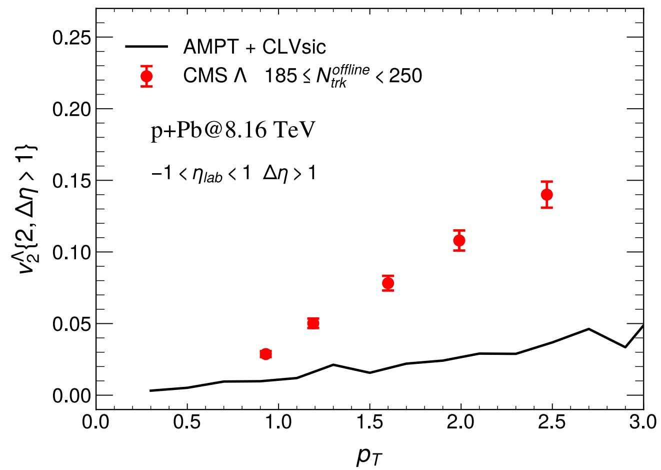


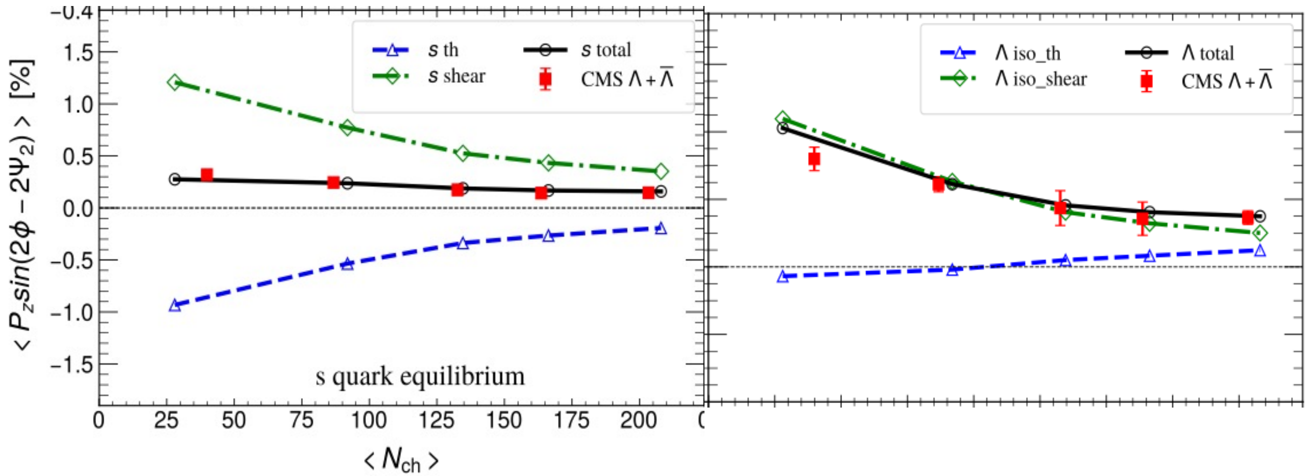


Smaller v2 gives a larger polarization along beam direction ?

Smaller v2, larger shear induced polarization, smaller thermal vorticial induced polarization

Sensitive to initial conditions?





Connection between P_z and v_2

- **Assuming we consider a Bjorken-like flow**

$$\mathcal{S}^{z}_{\text{thermal}} = -\frac{1}{4m_\Lambda N}\frac{1}{T}\frac{dT}{d\tau}\bigg|_{\Sigma}\partial_\phi \int d\Sigma_\alpha p^\alpha f_V^{(0)} \cosh\eta$$

since

$$\int d\Sigma_\lambda p^\lambda f_V^{(0)} = \frac{dN}{2\pi E_p p_T dp_T dY}\left[1+\sum_{n=1}^{\infty} 2v_n(p_T, Y)\cos n\phi\right]$$

one can get

$$\mathcal{S}^{z}_{\text{thermal}} \approx \frac{1}{m_\Lambda}\frac{1}{T}\frac{dT}{d\tau}\bigg|_{\Sigma} v_2(p_T, 0)\sin 2\phi$$

Becattini, Karpenko, PRL (2018); C. Yi, SP, J.H. Gao, D.L. Yang, PRC (2022)

What is the relation between flow and polarization along beam direction?

New spin polarization effects

S. Fang, SP, arXiv:2408.09877

Spin Boltzmann equations

- **We derive the spin Boltzmann equation incorporating Møller scattering process using hard thermal loop approximations.**

$$p^{\mu}\partial_{\mu}f_{\mathrm{A}}^{<}(p)+\hbar\partial_{\mu}S^{(u),\mu\alpha}(p)\partial_{\alpha}f_{\mathrm{V}}^{<}(p) \quad = \quad \mathcal{C}_{\mathrm{A}}+\hbar\partial_{\mu}\left(S_{(u)}^{\mu\alpha}C_{\mathrm{V},\alpha}[f_{\mathrm{V}}^{<}]\right)$$

S. Fang, SP, D.L. Yang, PRD (2022); S. Fang, SP, arXiv:2408.09877

- **Scenario (I): particle distribution function is off-equilibirum**

$$\partial\sim\lambda^{-1}\mathrm{Kn}\ll\lambda^{-1}$$

Kn: Knudsen number
λ: **mean free path**

- **Scenario (II): particle distribution function is at local equilibrium**

  Similar to standard kinetic theory, e.g. AMY

New corrections from scattering

Let us start from the kinetic theory for massless fermions.

$$p \cdot \partial f_0 = C_{pp' \to kk'}[\delta f],$$

We consider the system close to the global equilibrium,

$$f = f_0 + \delta f,$$

We can estimate

$$\delta f \sim A p_\mu p_\nu \pi^{\mu\nu}, \quad A \sim 1/C_{pp' \to kk'}[f] \sim 1/e^4,$$

Recalling the spin current in phase space,

$$j^\mu(p) = p^\mu f + S^{\mu\nu}\partial_\nu f + \int_{p',k,k'} C_{pp' \to kk'}[f]\Delta^\mu,$$

$$j^\mu \sim \int_{p',k,k'} C_{pp' \to kk'}[\delta f]\Delta^\mu \sim \int_{p',k,k'} C_{pp' \to kk'}[A p_\mu p_\nu \pi^{\mu\nu}]\Delta^\mu$$

$$\sim \cancel{e}^4 \frac{1}{\cancel{e}^4} \pi^{\mu\nu} p_\mu p_\nu.$$

- **Leading order in gradient expansion!**
- **Corrections from scatterings but do not depend on coupling constant**

It can also be derived by Kubo formula.

Connection to condensed matter physics

- **After we finish this work, we find that the same discoveries have been derived in condensed matter physics in their quantum kinetic theory.**

T. Valet, R. Raimondi, arXiv:2410.08975

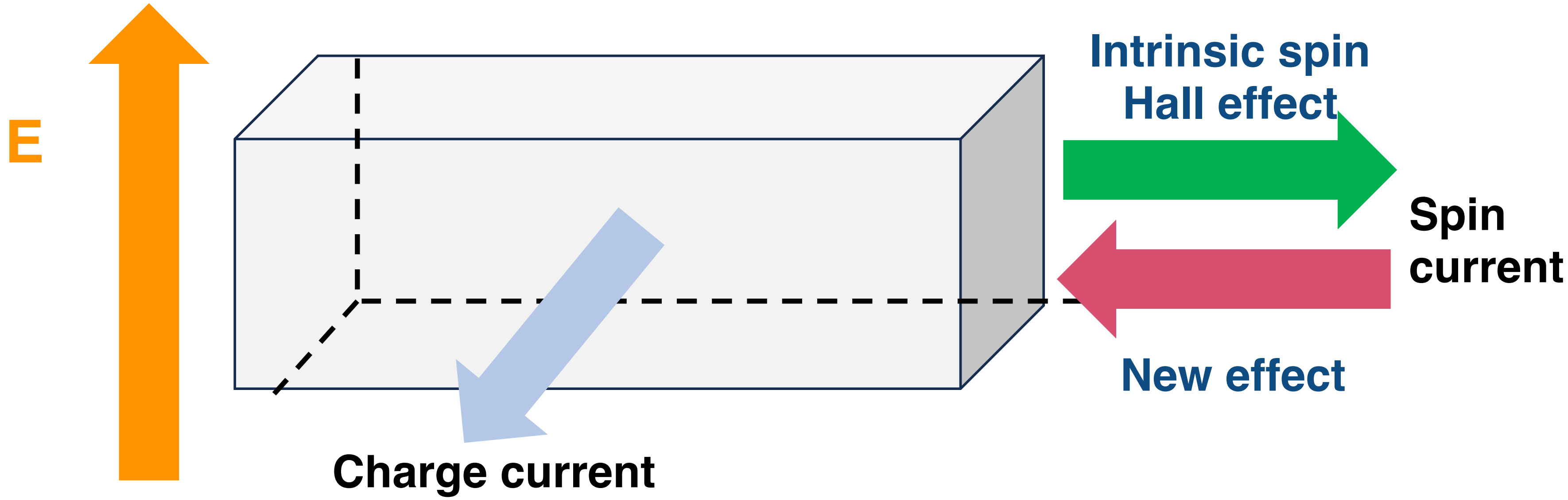
$$\hat{\rho}^{(1)}_{nm} = -\hbar\frac{f^{(0)}_n - f^{(0)}_m}{\varepsilon_n - \varepsilon_m} e\boldsymbol{E}\cdot\hat{\mathcal{A}}_{nm} - \hbar\partial_{\boldsymbol{x}}\frac{f^{(1)}_n + f^{(1)}_m}{2}\cdot\hat{\mathcal{A}}_{nm}$$

$$\boxed{\begin{aligned} &- \frac{\mathrm{i}\hbar}{2(\varepsilon_n - \varepsilon_m)}\frac{2\pi n_i v_0^2}{\hbar}\sum_q\int\frac{d^d \underset{\sim}{p}}{(2\pi\hbar)^d}\hat{P}_n\hat{P}_{\underset{\sim}{q}}\hat{P}_m \times \cdots \qquad (1) \\ &\cdots\times\left[\delta\left(\underset{\sim}{\varepsilon}_q - \varepsilon_n\right)\left(f^{(1)}_{\underset{\sim}{q}} - f^{(1)}_n\right) + \delta\left(\underset{\sim}{\varepsilon}_q - \varepsilon_m\right)\left(f^{(1)}_{\underset{\sim}{q}} - f^{(1)}_m\right)\right], \end{aligned}}$$

Replacing the electric force by shear force, the results are consistent with what we found.

Private communication and check with T. Valet in workshop at Hangzhou.

Physical meaning of new effects



$$\hat{\rho}^{(1)}_{nm} = \boxed{-\hbar \frac{f_n^{(0)} - f_m^{(0)}}{\varepsilon_n - \varepsilon_m} e\boldsymbol{E} \cdot \hat{\mathcal{A}}_{nm}} - \hbar \partial_{\boldsymbol{x}} \frac{f_n^{(1)} + f_m^{(1)}}{2} \cdot \hat{\mathcal{A}}_{nm}$$

$$\boxed{\begin{aligned} & - \frac{i\hbar}{2(\varepsilon_n - \varepsilon_m)} \frac{2\pi n_i v_0^2}{\hbar} \sum_q \int \frac{d^d \underset{\sim}{p}}{(2\pi\hbar)^d} \hat{P}_n \hat{P}_{\underset{\sim}{q}} \hat{P}_m \times \cdots \qquad (1) \\ & \cdots \times \left[\delta\left(\varepsilon_{\underset{\sim}{q}} - \varepsilon_n\right) \left(f_{\underset{\sim}{q}}^{(1)} - f_n^{(1)} \right) + \delta\left(\varepsilon_{\underset{\sim}{q}} - \varepsilon_m\right) \left(f_{\underset{\sim}{q}}^{(1)} - f_m^{(1)} \right) \right], \end{aligned}}$$

New corrections to shear induced polarization

- **Scenario (I):**

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I})}(\mathbf{p}) = \delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{shear}} + \delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{chem}} + \mathcal{O}(\hbar^2\partial^2)$$

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{shear}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 g_2(E_{\mathbf{p}})\epsilon^{\mu\nu\rho\sigma}p_{\rho}u_{\sigma}\sigma_{\nu\alpha}p^{\alpha},$$

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{chem}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 g_1(E_{\mathbf{p}})\epsilon^{\mu\nu\rho\sigma}p_{\rho}u_{\sigma}\nabla_{\nu}\alpha_0.$$

They come from scatterings but do not depend on coupling constant explicitly. They correspond to anomalous spin Hall conductivity in condensed matter.

S. Fang, SP, arXiv:2408.09877

Also see the similar findings: S. Lin and Z. Wang, arXiv:2406.10003.

Furthermore

What can we learn?

- Spin current (in phase space) is quite different with the charge current.

- What is the complete leading order in gradient expansion in general for spin current? Are there other similar effects?

What is next?

- What is the underlying physics?

- How large will the correction be? How to measure it?

- Dissipative or non-dissipative? Topological invariant?

Other second order corrections

- **Scenario (II):**

$$\begin{aligned}\delta\mathcal{P}^{\mu}_{(\mathrm{II})}(\mathbf{p}) \;=\;& \mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\nabla T}+\mathcal{P}^{\mu}_{(\mathrm{II}),\nabla\omega}+\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\mathrm{chem}}+\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\mathrm{shear}}\\ &+\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{chem}-\nabla\mathrm{T}}+\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{shear}-\nabla\mathrm{T}}+\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{shear}-\mathrm{chem}}+\mathcal{O}(\hbar^{2}\partial^{3})\end{aligned}$$

$$\begin{aligned}
\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\nabla T} &= -\hbar^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot pa^{\mu}_{(\mathrm{II})}\left[d_{2}\left(E_{\mathbf{p}}-d_{1}\frac{1}{\beta_{0}}\right)\omega^{\alpha}\nabla_{\alpha}\beta_{0}-d_{6}\beta_{0}p_{\langle\alpha}p_{\rho\rangle}\omega^{\alpha}\nabla^{\rho}\beta_{0}\right],\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\nabla\omega} &= \hbar^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot pa^{\mu}_{(\mathrm{II})}\left[\left(E_{\mathbf{p}}-d_{1}\frac{1}{\beta_{0}}\right)d_{2}\beta_{0}\nabla^{\alpha}\omega_{\alpha}+d_{6}\frac{1}{2}\beta_{0}^{2}\nabla^{\alpha}\omega^{\rho}p_{\langle\alpha}p_{\rho\rangle}\right],\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\mathrm{chem}} &= \hbar^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot pa^{\mu}_{(\mathrm{II})}\left[\left(E_{\mathbf{p}}-d_{1}\frac{1}{\beta_{0}}\right)d_{3}\beta_{0}\omega^{\alpha}\nabla_{\alpha}\alpha_{0}-d_{8}\beta_{0}^{2}\omega^{\alpha}\nabla^{\rho}\alpha_{0}p_{\langle\alpha}p_{\rho\rangle}\right],\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\mathrm{shear}} &= -\hbar^{2}\beta_{0}\int_{\Sigma}\mathrm{d}\Sigma\cdot pa^{\mu}_{(\mathrm{II})}\left[-d_{4}\omega^{\rho}\sigma^{\alpha}_{\rho}p_{\langle\alpha\rangle}+d_{9}\beta_{0}^{2}\omega^{\beta}\sigma^{\alpha\lambda}p_{\langle\beta}p_{\alpha}p_{\lambda\rangle}\right]\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{chem}-\nabla\mathrm{T}} &= \hbar^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot pa^{\mu}_{(\mathrm{II})}d_{5}\epsilon^{\rho\nu\alpha\beta}u_{\beta}\nabla_{\nu}\alpha_{0}\nabla_{\rho}\beta_{0}p_{\langle\alpha\rangle},\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{shear}-\nabla\mathrm{T}} &= \hbar^{2}\beta_{0}\int_{\Sigma}\mathrm{d}\Sigma\cdot pa^{\mu}_{(\mathrm{II})}d_{6}\epsilon^{\beta\nu\sigma\rho}\sigma^{\alpha}_{\beta}u_{\sigma}\nabla_{\nu}\beta_{0}p_{\langle\alpha}p_{\rho\rangle},\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{shear}-\mathrm{chem}} &= -\hbar^{2}\beta_{0}^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot pa^{\mu}_{(\mathrm{II})}d_{7}\epsilon^{\mu\nu\sigma\rho}\sigma^{\alpha}_{\mu}u_{\sigma}\nabla_{\nu}\alpha_{0}p_{\langle\alpha}p_{\rho\rangle},
\end{aligned}$$

Self energy correction to Wigner function

$$\left[\frac{i\hbar}{2}\gamma^\mu\nabla_\mu + \gamma^\mu\Pi_\mu - m + \underline{\overline{\Sigma}_{\mathrm{g}}\star}\right] S^<(q,X) = -\frac{i\hbar}{2}(\Sigma_{\mathrm{g}}^> \star S^< - \Sigma_{\mathrm{g}}^< \star S^>),$$

$$S^<\left(-\frac{i\hbar}{2}\gamma^\mu\overleftarrow{\nabla}_\mu + \gamma^\mu\overleftarrow{\Pi}_\mu - m\right) + S^< \underline{\star\overline{\Sigma}_{\mathrm{g}}} = -\frac{i\hbar}{2}(S^> \star \Sigma_{\mathrm{g}}^< - S^< \star \Sigma_{\mathrm{g}}^>),$$

$\star$ denotes the Moyal product

$S^<$: Wigner function

$$\overline{\Sigma}_{\mathrm{g}}(q,X) = \Sigma^\delta(X) + \mathrm{Re}\Sigma_{\mathrm{g}}^{\mathrm{r}}$$

For a long time, we always neglect the self-energy terms for simplicity. Now, we consider the contributions from them carefully.

New corrections from self-energies

- **We consider effects from the thermal QCD background. After a heavy calculation, we get the corrections to polarization vectors from self-energies:**

Corrections to polarization induced by:

$$\delta\mathcal{P}^{\mu}_{\mathrm{therm}}(t,\mathbf{q}) = -\frac{\hbar^2}{2mN}\int_{\Sigma} q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{m_f^2 T}{E_q^3}\epsilon^{\mu\nu\alpha\beta}q_\nu\underline{\partial_\alpha\left(\frac{u_\beta}{T}\right)},$$

⟷ **Thermal vorticity**

$$\delta\mathcal{P}^{\mu}_{\mathrm{shear}}(t,\mathbf{q}) = -\frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot d\sigma G_{\omega_1}(E_q,\mathbf{q})\frac{m_f^2}{E_q^3}\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{E_q}\underline{q^\gamma\sigma_{\nu\gamma}},$$

⟷ **Shear tensor**

$$\delta\mathcal{P}^{\mu}_{\mathrm{chem}}(t,\mathbf{q}) = -\frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{C_{\mathrm{F}}g^2\mu T}{4\pi^2 E_q^2}\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{E_q}\underline{\nabla_\nu\left(\frac{\mu}{T}\right)},$$

⟷ **Gradient of chemical Potential over temperature**

$$\delta\mathcal{P}^{\mu}_{\mathrm{acc}}(t,\mathbf{q}) = \frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{3m_f^2}{E_q^3}\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma \underline{Du_\nu},$$

⟷ **Fluid acceleration**

$$\delta\mathcal{P}^{\mu}_{\mathrm{vor}}(t,\mathbf{q}) = \frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot d\sigma\frac{m_f^2}{E_q^2}\left[\underline{\omega^\mu}\left(4G_{\mathrm{T}}(E_q,\mathbf{q})-\frac{|q_\perp|^2}{E_q^2}G_{\omega_1}(E_q,\mathbf{q})+2G_{\omega_2}(E_q,\mathbf{q})\right)\right.$$
$$\left.-\frac{(\underline{\omega}\cdot q)}{E_q}\left(6u^\mu G_{\mathrm{T}}(E_q,\mathbf{q})+\frac{q_\perp^\mu}{E_q}G_{\omega_1}(E_q,\mathbf{q})\right)\right],$$

⟷ **Kinetic vorticity**

Shuo Fang, Shi Pu, Di-Lun Yang, PRD (2024), arXiv: 2311.15197

Corrections from space-time dependent EM fields

- **We derived the corrections to Wigner function and polarization from varying EM fields.**

$$\mathcal{S}^{\mu}_{(2)} = \frac{1}{8mN}\sum_{m=1,2,3}\int d\Sigma^{\sigma}p_{\sigma}X^{\mu}_{(m)}f_5^{(m)},$$

$$\begin{aligned}
X^{\mu}_{(0)} &= \frac{1}{p_u^3}\left(u^{\mu}u^{\nu}u_{\lambda}-\frac{1}{p_u}p^{\mu}u^{\nu}u_{\lambda}-\frac{2}{p_u}u^{\mu}u^{\nu}p_{\lambda}-\frac{1}{2p_u^2}u^{\mu}p^{\nu}p_{\lambda}+\frac{1}{p_u^2}p^{\mu}u^{\nu}p_{\lambda}+u_{\lambda}g^{\mu\nu}\right)F_{\nu\rho}F^{\lambda\rho},\\
X^{\mu}_{(1)} &= \frac{1}{p_u^2}\left(\frac{1}{2}p^{\mu}u^{\nu}u_{\lambda}+u^{\mu}u^{\nu}p_{\lambda}-\frac{1}{4p_u}u^{\mu}p^{\nu}p_{\lambda}-\frac{1}{2p_u}p^{\mu}u^{\nu}p_{\lambda}-p_u g^{\mu\nu}u_{\lambda}\right)(F_{\nu\gamma}\Omega^{\lambda\gamma}+\Omega_{\nu\gamma}F^{\lambda\gamma})\\
&\quad+\frac{\beta}{p_u^3}\left(p^{\mu}u^{\nu}u_{\lambda}+2u^{\mu}u^{\nu}p_{\lambda}-\frac{1}{p_u}u^{\mu}p_{\mu}p_{\lambda}-\frac{2}{p_u}p^{\mu}u^{\nu}p_{\lambda}+\frac{1}{2p_u^2}p^{\mu}p^{\nu}p_{\lambda}-2p_u g^{\mu\nu}u_{\lambda}+g^{\mu\nu}p_{\lambda}\right)F_{\nu\rho}F^{\lambda\rho}
\end{aligned}$$

Corrections for 2nd order constant EM fields

$$\begin{aligned}
X^{\mu}_{(2)} &= \frac{1}{p_u}\left(\frac{1}{2}u^{\mu}p^{\nu}p_{\lambda}+p^{\mu}u^{\nu}p_{\lambda}-p_u g^{\mu\nu}p_{\lambda}\right)\Omega_{\nu\rho}\Omega^{\lambda\rho}\\
&\quad+\frac{\beta}{p_u^2}\left(p^{\mu}u^{\nu}p_{\lambda}+\frac{1}{2}u^{\mu}p^{\nu}p_{\lambda}-\frac{1}{4p_u}p^{\mu}p^{\nu}p_{\lambda}-p_u g^{\mu\nu}p_{\lambda}\right)(F_{\nu\gamma}\Omega^{\lambda\gamma}+\Omega_{\nu\gamma}F^{\lambda\gamma})\\
&\quad+\frac{\beta^2}{2p_u^3}\left(u^{\mu}p_{\mu}p_{\lambda}+2p^{\mu}u^{\nu}p_{\lambda}-\frac{1}{p_u}p^{\mu}p_{\mu}p_{\lambda}-2p_u g^{\mu\nu}p_{\lambda}\right)F_{\nu\rho}F^{\lambda\rho},\\
X^{\mu}_{(3)} &= \frac{\beta}{2p_u}p^{\mu}p^{\nu}p_{\lambda}\Omega_{\nu\rho}\Omega^{\lambda\rho}+\frac{\beta^2}{4p_u^2}p^{\mu}p^{\nu}p_{\lambda}(F_{\nu\gamma}\Omega^{\lambda\gamma}+\Omega_{\nu\gamma}F^{\lambda\gamma})+\frac{\beta^3}{3p_u^3}p^{\mu}p^{\nu}p_{\lambda}F_{\nu\rho}F^{\lambda\rho}.
\end{aligned}$$

$$\mathcal{S}^{\mu}_{\partial,\mathrm{EM}} = \frac{1}{8mN}\sum_{m=0,1,2,3}\int d\Sigma^{\sigma}p_{\sigma}Y^{\mu}_{(m)}f_5^{(m)},$$

Corrections for varying EM fields

$$\begin{aligned}
Y^{\mu}_{(0)} &= -\frac{2}{3p_u^2}\left(u_{\lambda}u_{\nu}-\frac{1}{2p_u}u_{\lambda}p_{\nu}-\frac{1}{2p_u}p_{\lambda}u_{\nu}\right)\partial^{\lambda}F^{\mu\nu}\\
&\quad+\frac{1}{3p_u^2}\left(2u^{\mu}u^{\nu}-\frac{1}{p_u}p^{\mu}u^{\nu}-\frac{1}{p_u}u^{\mu}p^{\nu}\right)\partial^{\lambda}F_{\lambda\nu},\\
Y^{\mu}_{(1)} &= +\frac{2\beta}{3p_u}\left(u_{\lambda}u_{\nu}-\frac{1}{p_u}u_{\lambda}p_{\nu}-\frac{1}{p_u}p_{\lambda}u_{\nu}+\frac{1}{2p_u^2}p_{\lambda}p_{\nu}\right)\partial^{\lambda}F^{\mu\nu}\\
&\quad+\frac{\beta}{6p_u}\left(u^{\mu}u^{\nu}+\frac{4}{p_u}p^{\mu}u^{\nu}+\frac{4}{p_u}u^{\mu}p^{\nu}-\frac{2}{p_u^2}p^{\mu}p^{\nu}\right)\partial^{\lambda}F_{\lambda\nu}\\
&\quad-\frac{\beta}{3p_u^2}\left(u^{\mu}u^{\lambda}p^{\nu}-\frac{1}{2p_u}p^{\mu}u^{\lambda}p^{\nu}-\frac{1}{2p_u}u^{\mu}p^{\lambda}p^{\nu}\right)u^{\rho}\partial_{\lambda}F_{\nu\rho}\\
&\quad\frac{4\beta}{3p_u}\partial_{\lambda}F^{\mu\lambda}+\frac{\beta}{3p_u}u^{\nu}u^{\rho}\partial^{\mu}F_{\nu\rho},
\end{aligned}$$

$$\begin{aligned}
Y^{\mu}_{(2)} &= -\frac{\beta^2}{3}\left(u_{\lambda}u_{\nu}-\frac{2}{p_u}u_{\lambda}p_{\nu}+\frac{1}{p_u^2}p_{\lambda}p_{\nu}\right)\partial^{\lambda}F^{\mu\nu}\\
&\quad+\frac{\beta^2}{6}\left(\frac{1}{p_u^2}p^{\mu}u^{\nu}+\frac{2}{p_u^2}p^{\mu}p^{\nu}\right)\partial^{\lambda}F_{\lambda\nu}+\frac{\beta^2}{3p_u}p^{\nu}u^{\rho}\partial^{\mu}F_{\nu\rho}\\
&\quad+\frac{\beta^2}{3p_u}\left(u^{\mu}u^{\lambda}p^{\nu}-\frac{1}{p_u}p^{\mu}u^{\lambda}p^{\nu}-\frac{1}{p_u}u^{\mu}p^{\lambda}p^{\nu}+\frac{1}{2p_u^2}p^{\mu}p^{\lambda}p^{\nu}\right)u^{\rho}\partial_{\lambda}F_{\nu\rho},\\
Y^{\mu}_{(3)} &= \frac{\beta^3}{3p_u}\left(p^{\mu}u^{\lambda}p^{\nu}-\frac{1}{2p_u}p^{\mu}p^{\lambda}p^{\nu}\right)u^{\rho}\partial_{\lambda}F_{\nu\rho}.
\end{aligned}$$

S. Z. Yang, J.H. Gao, SP, arXiv: 2409.00456

Attractors and focusing behavior in spin hydrodynamics

D.L. Wang, Y. Li, SP, arXiv: 2408.03781

Basic conservation equations in canonical form

- **Total angular momentum conservation**

$$\partial_\alpha J^{\alpha\mu\nu}_{\text{can}} = 0 \qquad J^{\lambda\mu\nu} = \underbrace{x^\mu\Theta^{\lambda\nu} - x^\nu\Theta^{\lambda\mu}}_{\text{Orbital part}} + \underbrace{\Sigma^{\lambda\mu\nu}}_{\text{Spin tensor}},$$

$$\Rightarrow \boxed{\partial_\lambda\Sigma^{\lambda\mu\nu} = -2\Theta^{[\mu\nu]},}$$

- **Energy-momentum conservation**

$$\partial_\mu\Theta^{\mu\nu} = 0,$$

- **Currents conservation**

$$\partial_\mu j^\mu = 0,$$

Spin tensor, spin density and chemical potential

Spin density:
has 6 independent components
S^{ij} 3 rotating; S^{0i} 3 boosting

μ, ν is anti-symmetic

$$\Sigma^{\alpha\boxed{\mu\nu}} = u^\alpha S^{\mu\nu} + \Sigma^{\alpha\mu\nu}_{(1)}$$

spin tensor

Parallel to fluid velocity u^μ;
Leading order

Perpendicular to fluid velocity u^μ;
Higher order

Thermodynamic relations

$$e + p = Ts + \mu n + \omega_{\mu\nu} S^{\mu\nu}$$

energy density

pressure

temperature X entropy density

spin chemical potential

spin density

6-d.o.f Spin hydrodynamics

- **By using entropy principle, one can get**

$$
\begin{aligned}
\Theta^{\mu\nu} &= (e+p)u^\mu u^\nu - pg^{\mu\nu} + 2h^{(\mu}u^{\nu)} + 2q^{[\mu}u^{\nu]} + \pi^{\mu\nu} + \phi^{\mu\nu}, \\
q^\mu &= \lambda[(u\cdot\partial)u^\mu + \frac{1}{T}\Delta^{\mu\nu}\partial_\nu T - 4\omega^{\mu\nu}u_\nu], \\
\phi^{\mu\nu} &= 2\gamma[T\omega_{th}^{\mu\nu} + 2(g^{\mu\alpha} - u^\mu u^\alpha)(g^{\nu\beta} - u^\nu u^\beta)\omega_{\alpha\beta}]/T.
\end{aligned}
$$

Spin hydrodynamics:

Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
Montenegro, Tinti, Torrieri (2017-2019);
Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022)
S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060Weickgenannt, Wanger, Speranze, Rischke, PRD 2022; PRD 2022; Weickgennatt, Wanger, Speranza, PRD 2022; arXiv:2306.05936

Recent review:

SP, X.G. Huang, "Relativistic spin hydrodynamics", Acta Phys.Sin. 72 (2023) 7, 071202

Analytic solutions for spin hydrodynamics

- **Solution for Bjorken type spin hydrodynamics:**

$$\omega^{xy}(\tau) = \omega_0^{xy}\left(\frac{\tau_0}{\tau}\right)^{1/3}\exp\left[-\frac{2\gamma\tau_0}{a_1 T_0^3}\left(\frac{\tau^2}{\tau_0^2}-1\right)\right]\left\{1+\left(\frac{2}{3}\frac{\eta_s}{s}+\frac{\zeta}{s}\right)\frac{1}{T_0^4}\right.$$
$$\left.\times\left[\frac{T_0^3}{\tau_0}\left(\left(\frac{\tau_0}{\tau}\right)^{2/3}-1\right)+\frac{\gamma}{a_1}\left(3\left(\frac{\tau}{\tau_0}\right)^2-\frac{9}{2}\left(\frac{\tau}{\tau_0}\right)^{4/3}+\frac{3}{2}\right)\right]\right\}$$
$$+\mathcal{O}\left((\omega_0^{xy}/T_0)^2,(\eta_s/s)^2,(\zeta/s)^2,(\eta_s\zeta/s^2)\right),$$

D.L. Wang, S. Fang, SP, Phys.Rev.D 104 (2021) 11, 114043

- **Solution for Gubser type spin hydrodynamics:**

$$S^{0x} = \frac{4L^2}{\tau}C_+G(L,\tau,x_\perp)^{-1}, \qquad S^{xz} = \frac{4L^2}{\tau}D_+G(L,\tau,x_\perp)^{-1},$$
$$S^{0y} = \frac{4L^2}{\tau}C_-G(L,\tau,x_\perp)^{-1}, \qquad S^{yz} = \frac{4L^2}{\tau}D_-G(L,\tau,x_\perp)^{-1}.$$

D.L. Wang, X.Q. Xie, S.Fang, SP, Phys.Rev.D 105 (2022) 11, 114050

- **Spin density: Power law X exponential decay**

  Ordinary hydro variables: power law decay

No spin effects at late time?

Revisited Bjorken type spin hydro (I)

- **For Bjorken type spin hydro, we have**

$$\frac{d^2 S^{xy}}{dw^2} + (\Delta_1^{-1} + w^{-1})\frac{dS^{xy}}{dw} + \Delta_1^{-2}(w^{-1} - w^{-2} + 8\alpha w^{\Delta_2})S^{xy} = 0.$$

$$\mathrm{Kn}^{-1} \approx w \equiv \frac{\tau}{\tau_\phi}. \qquad w \equiv \frac{\tau}{\tau_\phi} = \left(\frac{\tau}{\tau_1}\right)^{\Delta_1}, \quad \frac{\tau_\phi \gamma}{\chi} = \alpha w^{\Delta_2},$$

Δ_1, Δ_2 are constant

γ: transport coefficient
τ_ϕ: relaxation time
χ: spin susceptibility

$$f(w) \equiv \Delta_1 \frac{w}{S^{xy}}\frac{dS^{xy}}{dw} = \frac{\tau}{S^{xy}}\frac{dS^{xy}}{d\tau}$$

Here, we assume the γ is proper time dependent different with our previous work.

$$\Delta_1 w f' + f^2 + wf + w - 1 + 8\alpha w^{2+\Delta_2} = 0,$$

Revisited Bjorken type spin hydro (II)

$$\Delta_1 w f' + f^2 + wf + w - 1 + 8\alpha w^{2+\Delta_2} = 0,$$

$$w \equiv \frac{\tau}{\tau_\phi} = \left(\frac{\tau}{\tau_1}\right)^{\Delta_1}, \quad \frac{\tau_\phi \gamma}{\chi} = \alpha w^{\Delta_2}, \qquad f(w) \equiv \Delta_1 \frac{w}{S^{xy}} \frac{dS^{xy}}{dw} = \frac{\tau}{S^{xy}} \frac{dS^{xy}}{d\tau}$$

If $\alpha w^{2+\Delta_2} \to 0$, then the late time beahivor reads

$$\cancel{\Delta_1 w f'} + f^2 + \cancel{wf} + \cancel{w} - 1 + \cancel{8\alpha w^{2+\Delta_2}} = 0,$$

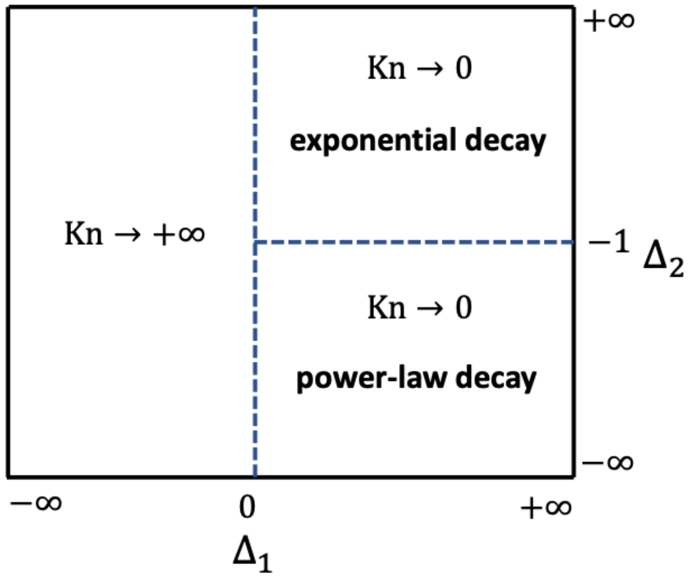
$$\Rightarrow \quad f \sim \pm 1$$

Trivial solution? But one kind of attactors!

Spin density: power law decay

Asymptotic solutions for $S = S^{xy}$

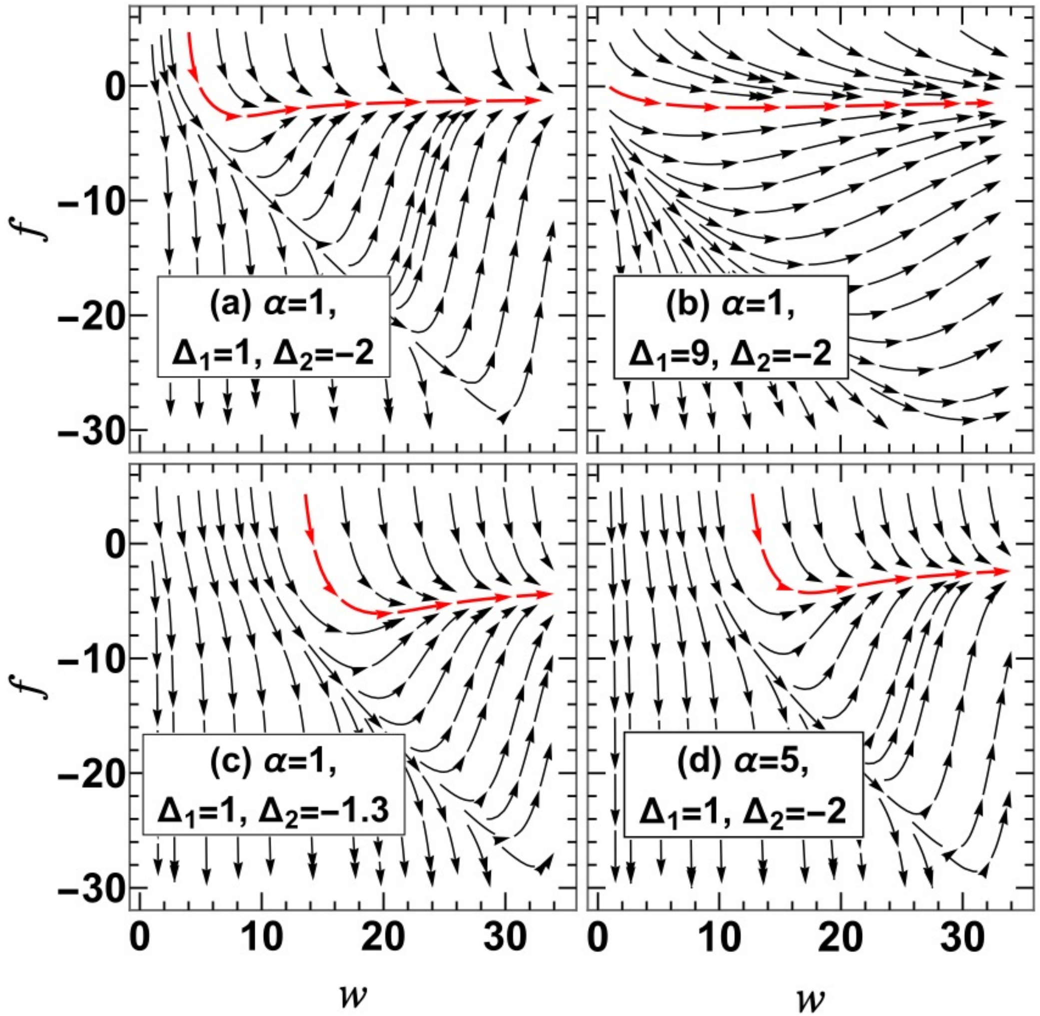
	$w \to +\infty$
$\Delta_2 > 0$	$S_{(1),(2)} \propto e^{-w/(2\Delta_1)}$
$\Delta_2 = 0$	$S_{(1),(2)} \propto e^{-w(1\pm\sqrt{1-32\alpha})/(2\Delta_1)}$
$-1 < \Delta_2 < 0$	$S_{(1)} \propto e^{-w/\Delta_1}$, $S_{(2)} \propto \exp\left[-\frac{8\alpha w^{1+\Delta_2}}{\Delta_1(1+\Delta_2)}\right]$
$\Delta_2 = -1$	$S_{(1)} \propto e^{-w/\Delta_1}$, $\boxed{S_{(2)} \sim w^{-(1+8\alpha)/\Delta_1}}$
$\Delta_2 < -1$	$S_{(1)} \propto e^{-w/\Delta_1}$, $\boxed{S_{(2)} \sim w^{-1/\Delta_1}}$



We focus on the region:

$$\Delta_1 > 0, \quad \Delta_2 \leq -1.$$

Late time attactors



$$f_{(1)} \rightarrow -w,$$

$$f_{(2)} \rightarrow \begin{cases} -1-8\alpha, & \Delta_2 = -1, \\ -1, & \Delta_2 < -1, \end{cases}$$

Why late time attactors exist?

- **Assuming spin susceptibility is a constant for simplicity.**

$$\gamma \sim \tau^{1+\Delta_2-1/\Delta_1}$$

When γ is small (or $\Delta_1 > 0, \Delta_2 \leq -1$),

$$\tau_\phi \Delta^{\mu\alpha}\Delta^{\nu\beta}u^\rho\nabla_\rho\phi_{\alpha\beta} + \phi^{\mu\nu} = 2\gamma\Delta^{\mu\alpha}\Delta^{\nu\beta}(\cancel{\nabla_{[\alpha}u_{\beta]}} + 2\omega_{\alpha\beta}),$$

$$\Rightarrow \quad \phi^{xy} \approx \phi_0 \exp\left(-\frac{w}{\Delta_1}\right) + \mathcal{O}(\gamma),$$

While ϕ is the source generating spin density

$$\partial_\lambda \Sigma^{\lambda xy} \approx 0$$

$$\frac{dS^{xy}}{d\tau} + \frac{1}{\tau}S^{xy} \approx 0, \quad \Rightarrow \quad S^{xy} \approx S_0\frac{\tau_1}{\tau} = S_0 w^{-1/\Delta_1}$$

In this case, spin density decays due to expanding only, just like energy or number density in a Bjorken flow.
Beyond the non-hydro modes?

New discovery: focusing behavior

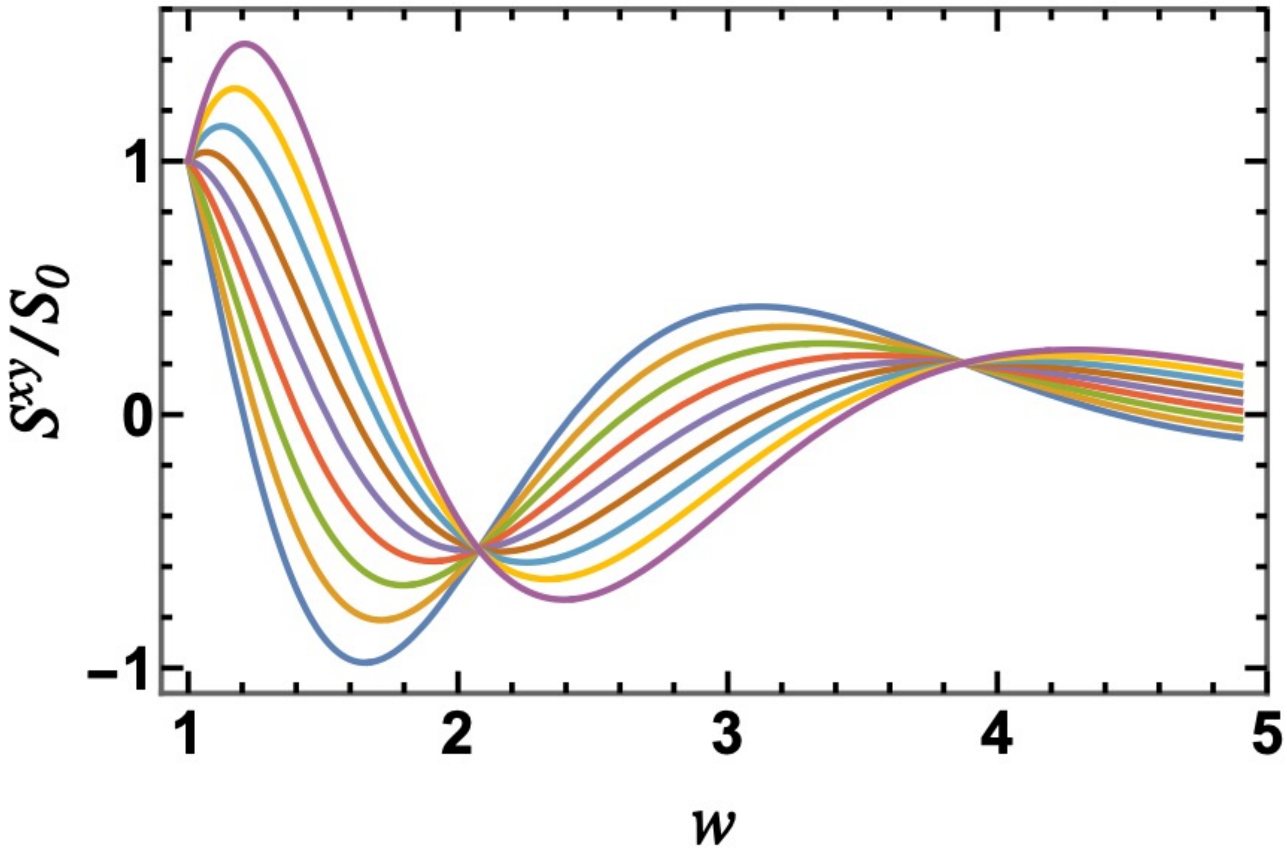
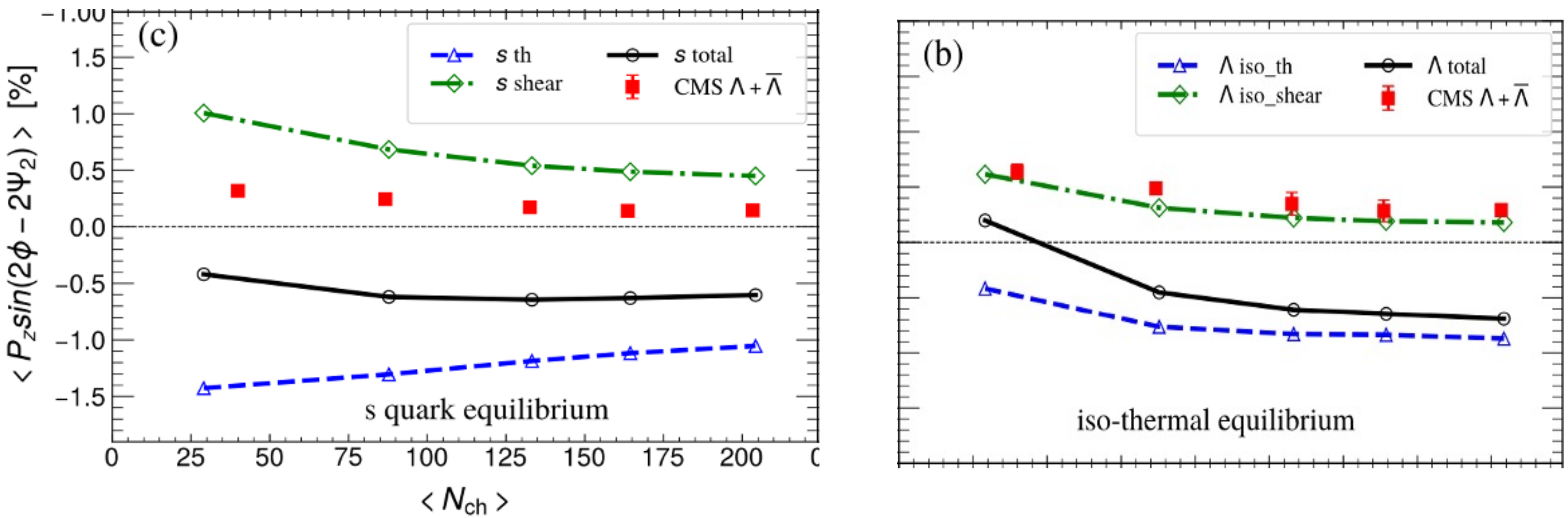


FIG. 5. The focusing behavior for $S^{xy}(w)/S_0$ with different S_0'. The parameters are set to be $\Delta_1 = 1$, $\Delta_2 = -1.5$, and $\alpha = 2$. The initial conditions are chosen as $w_0 = 1$ and $S_0' = -4.9$, -3.7, -2.5, -1.3, -0.1, 1.1, 2.3, 3.5, and 4.7. All solutions $S^{xy}(w)/S_0$ pass through the same point at $w = 2.077$, 3.876, 6.804, and 11.974 (the last two are not shown in this figure).

Summary and outlook

Summary (I)

How to understand the local polarization at pA system?



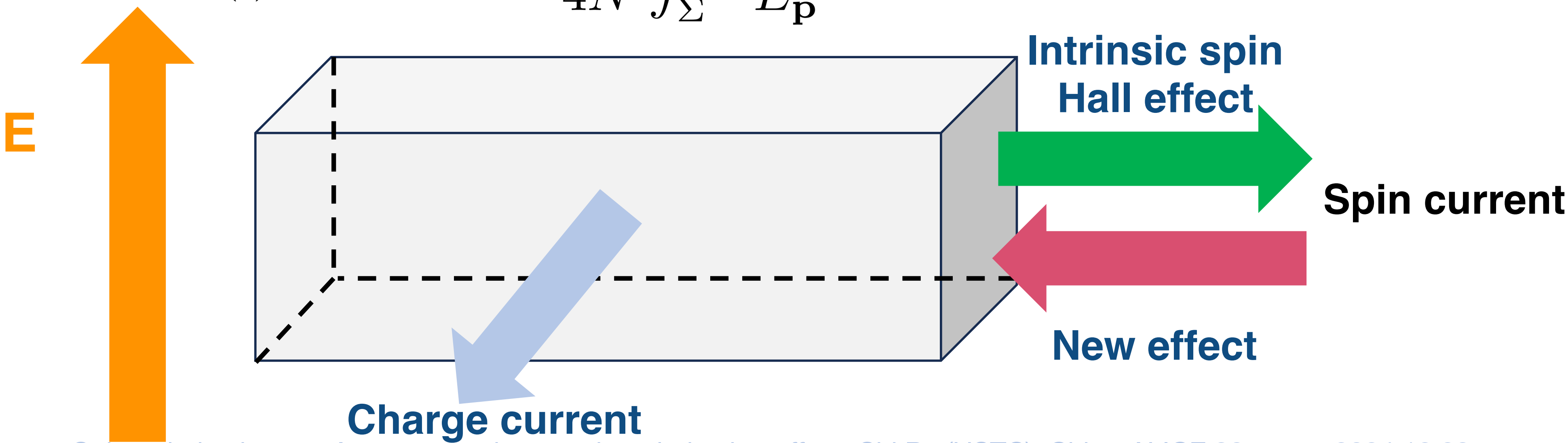
Summary (II)

New corrections to shear induced polarization come from scatterings but do not depend on coupling constant explicitly.

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I})}(\mathbf{p}) = \delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{shear}} + \delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{chem}} + \mathcal{O}(\hbar^2\partial^2)$$

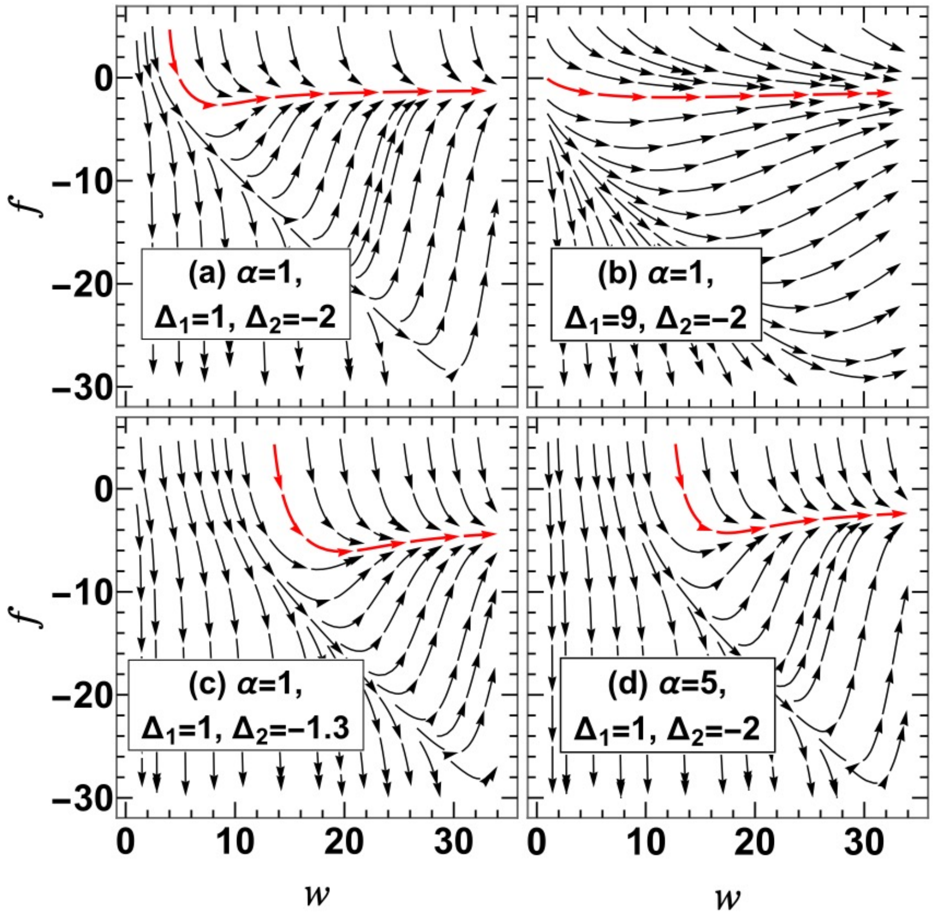
$$\delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{shear}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 g_2(E_{\mathbf{p}})\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \sigma_{\nu\alpha}p^{\alpha},$$

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{chem}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 g_1(E_{\mathbf{p}})\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \nabla_\nu \alpha_0.$$



Summary (III)

We derive the late time attractors and focusing behavior for spin hydrodynamics. It implies that spin density can be treated as other thermodynamic variables in certain region.

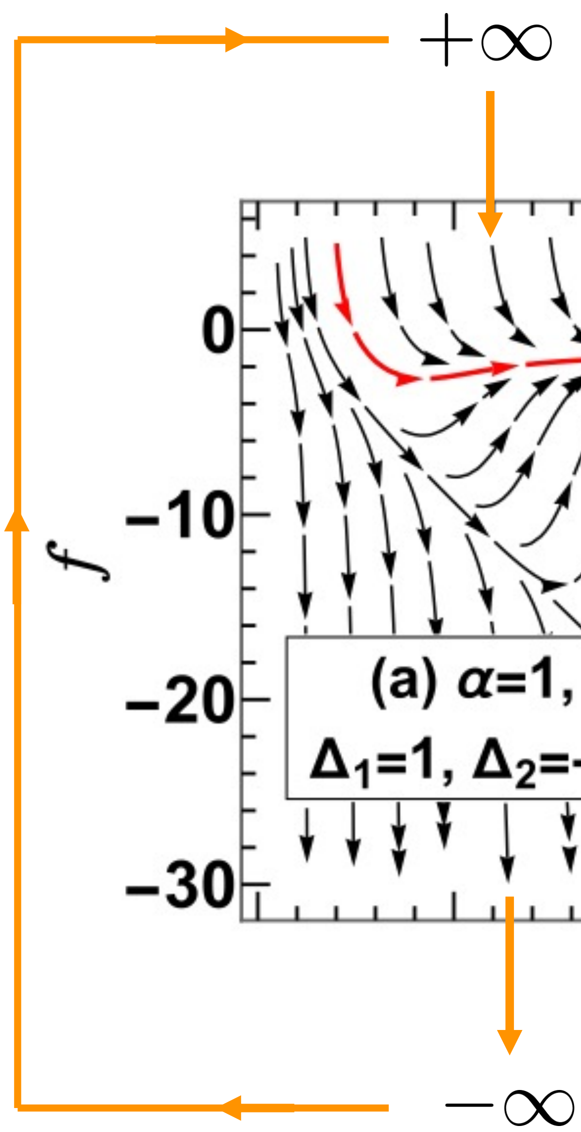


Thank you for your time!

Any comments and suggestions are welcome!

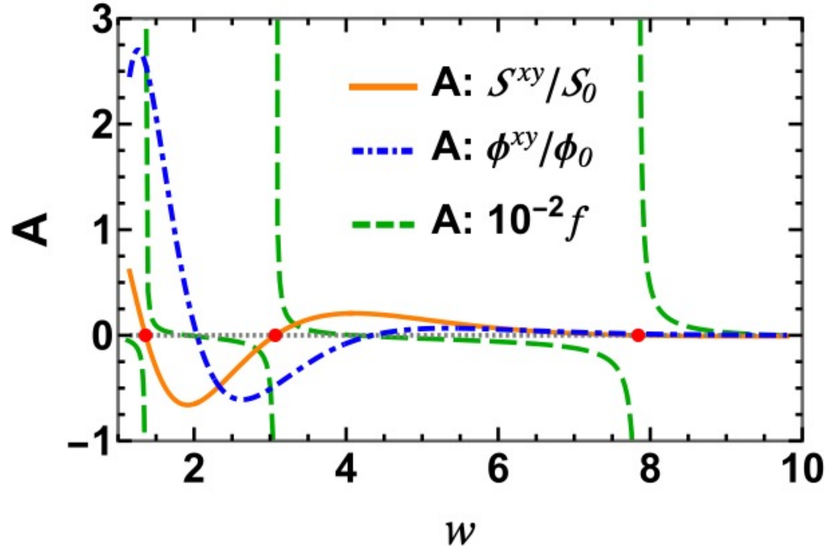
Backup

No singularity for spin density



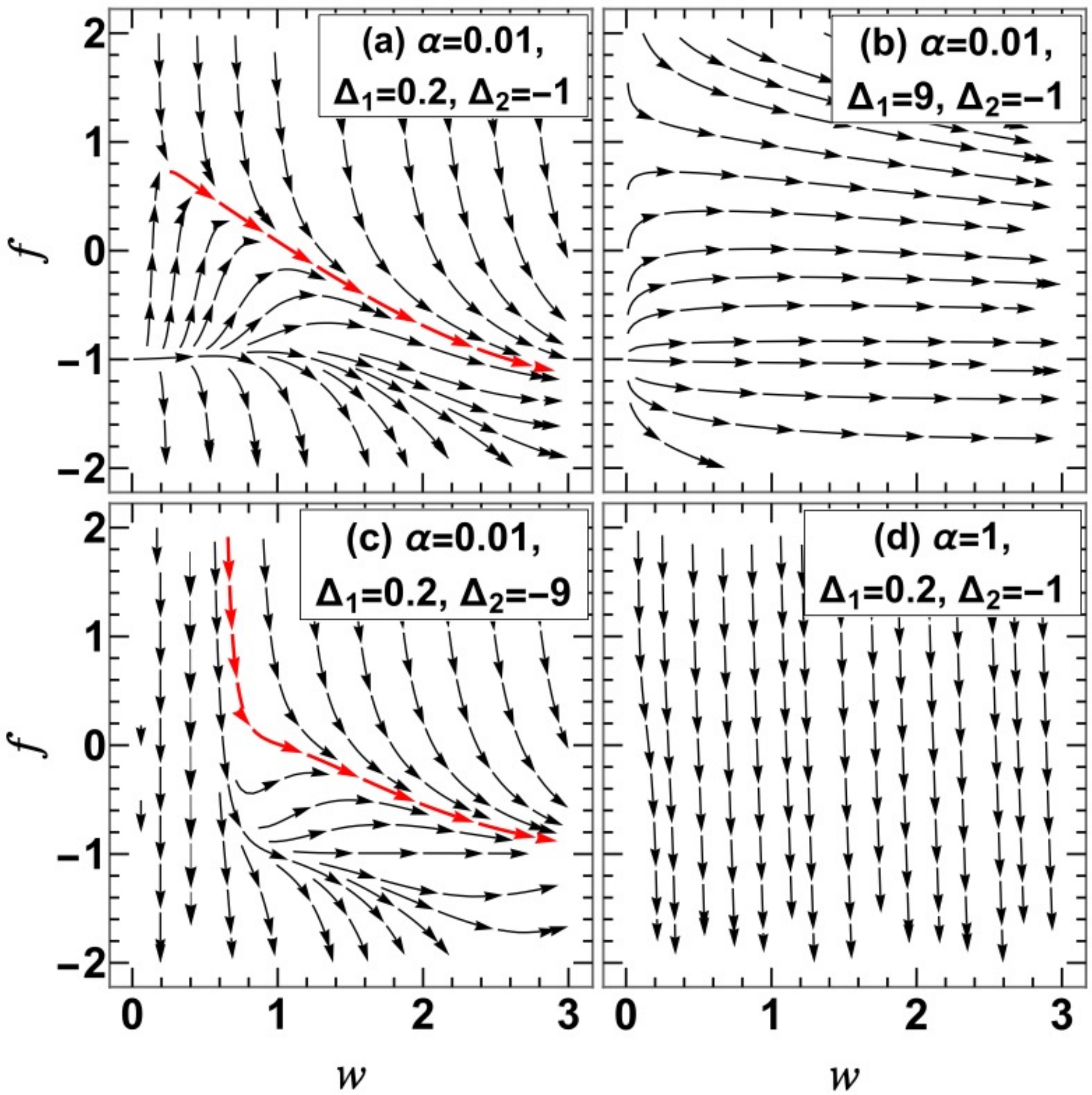
$$f(w) \equiv \Delta_1 \frac{w}{S^{xy}} \frac{dS^{xy}}{dw} = \frac{\tau}{S^{xy}} \frac{dS^{xy}}{d\tau}$$

$$S^{xy} \to 0, \ \ f \to \pm\infty$$



$\Delta_1 = 1, \ \Delta_2 = -2, \text{ and } \alpha = 2$

Early time attractors



Puzzle : T-odd/T-even VS dissipative/non-dissipative

Are shear induced polarization or spin alignment non-dissipative?

Q1: If the coefficient is T-even, it is non-dissipative.

CME	$\mathbf{j} \sim C\mathbf{B}$	Taking T transformation →	$\mathbf{j} \rightarrow -\mathbf{j}$ $\mathbf{B} \rightarrow -\mathbf{B}$	C: T-even ✓
Spin polarization vector	$\mathcal{S}^i \sim C^{ijk}(\partial_j u_k + \partial_k u_j)$	→	$\mathcal{S}^i \rightarrow -\mathcal{S}^i$ $\partial_j u_k \rightarrow -\partial_j u_k$	C: T-even Non-dissipative?
Spin alignment	$\epsilon^i(\lambda)\epsilon^{*j}(\lambda')\rho_{\lambda\lambda'} \sim a\pi^{ij}$	→	$\rho_{\lambda\lambda'} \rightarrow (-1)^{\lambda+\lambda'}\rho_{\lambda\lambda'}$ $\epsilon^{s*}_{\mu}(\mathbf{p}) \rightarrow -(-1)^s \tilde{\delta}^{\alpha}_{\mu}\epsilon^{-s*}_{\alpha}(-\mathbf{p})$ $\pi^{ij} \rightarrow -\pi^{ij}$	

For ρ_{00}, the coefficient "a" is T-odd. Dissipative?

In Zubarev approach, the non-dissipative means the results does NOT depend on hypersurface. But, shear tensor comes from local equilibrium operators and should always depends on hypersurface. So, shear induced something is always dissipative?