Workshop on Advances, Innovations, and Future Perspectives in High-Energy Nuclear Physics
CCNU-Wuhan, October 19-24, 2024

Heavy quark transport in perturbative and non-perturbative QCD matter

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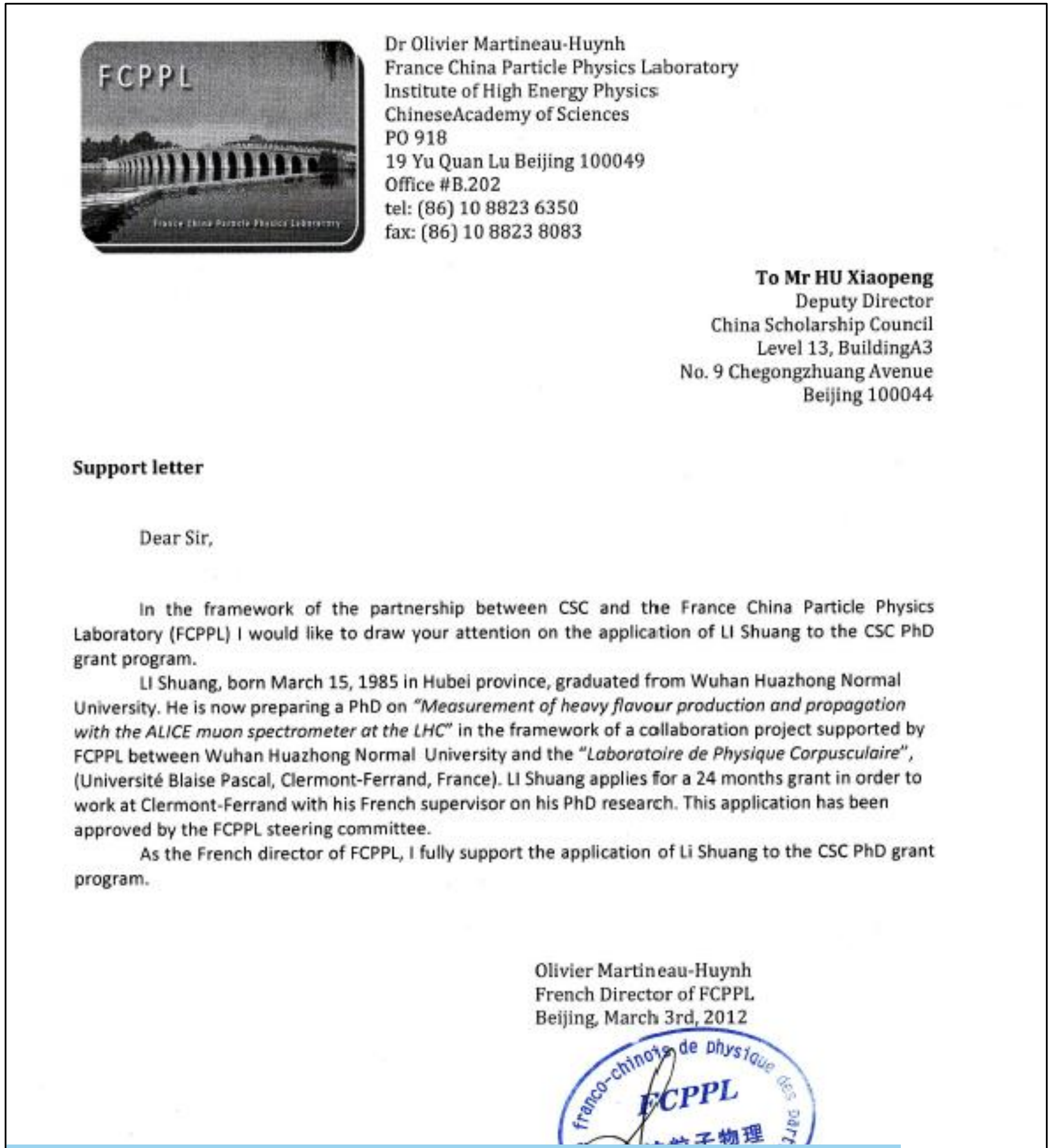

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Support letter

Dear Sir,

In the framework of the partnership between CSC and the France China Particle Physics Laboratory (FCPPL) I would like to draw your attention on the application of LI Shuang to the CSC PhD grant program.

LI Shuang, born March 15, 1985 in Hubei province, graduated from Wuhan Huazhong Normal University. He is now preparing a PhD on *"Measurement of heavy flavour production and propagation with the ALICE muon spectrometer at the LHC"* in the framework of a collaboration project supported by FCPPL between Wuhan Huazhong Normal University and the *"Laboratoire de Physique Corpusculaire"*, (Université Blaise Pascal, Clermont-Ferrand, France). LI Shuang applies for a 24 months grant in order to work at Clermont-Ferrand with his French supervisor on his PhD research. This application has been approved by the FCPPL steering committee.

As the French director of FCPPL, I fully support the application of Li Shuang to the CSC PhD grant program.

Olivier Martineau-Huynh
French Director of FCPPL
Beijing, March 3rd, 2012

Letter of support (FCPPL), 2012

CENTRAL CHINA NORMAL UNIVERSITY
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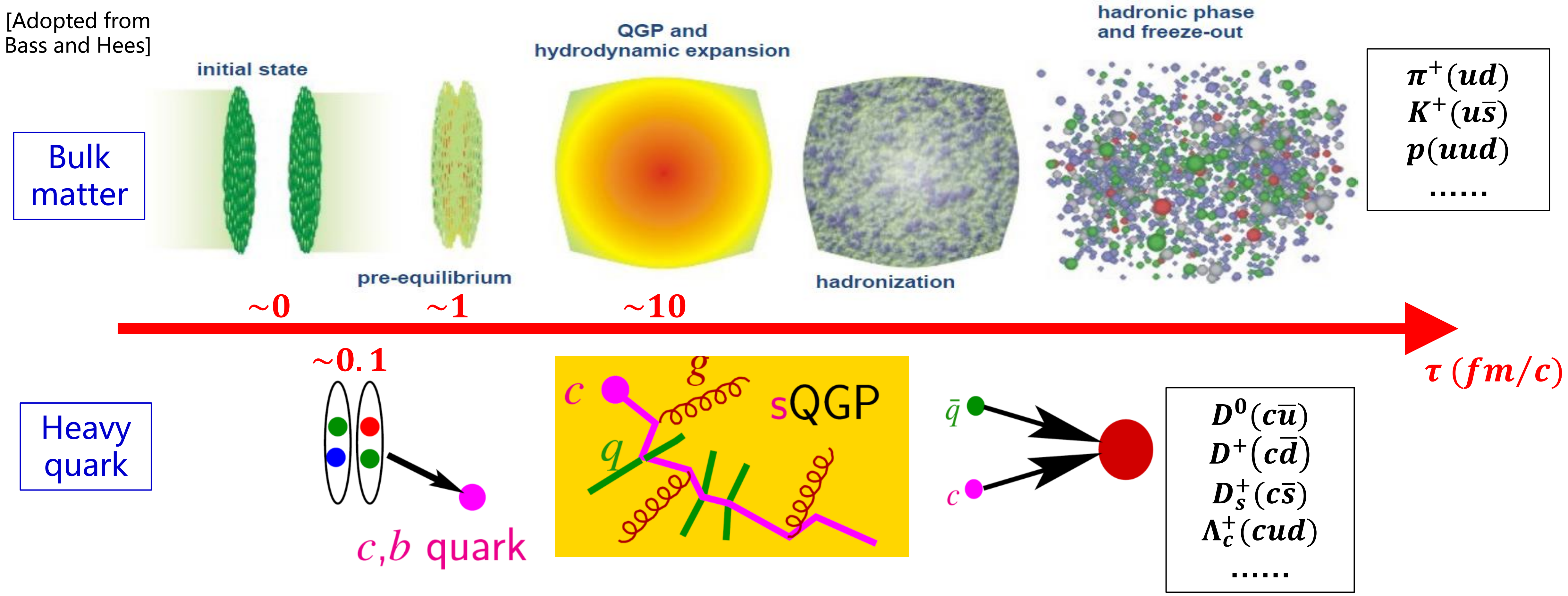
PHD THESIS

Muon production from heavy-flavour hadron decays in p–Pb and pp collisions with ALICE at the CERN-LHC

PhD thesis defense, 2015

Heavy quarks (HQ) as probes of QGP



- $m_Q \gg \Lambda_{QCD}$: their initial production can be well described by pQCD
- $m_Q \gg T$: thermal abundance in QGP is negligible ~ final multiplicity set by the initial hard production
- $m_Q \gg gT$: many soft scatterings necessary to change significantly the momentum of HQ ~ Brownian motion

Langevin dynamics with Gluon Radiation (LGR)

- Hybrid modeling

Bulk matter	Glauber model (SuperMC)	(3+1)D viscous hydro (vHLLE)	Cooper-Frye (iSS)
Heavy quark	Glauber for spatial and FONLL+EPS09 for momentum	Langevin transport equation modified for collisional+radiation	"Dual" hadronization model: fragmentation + coalescence

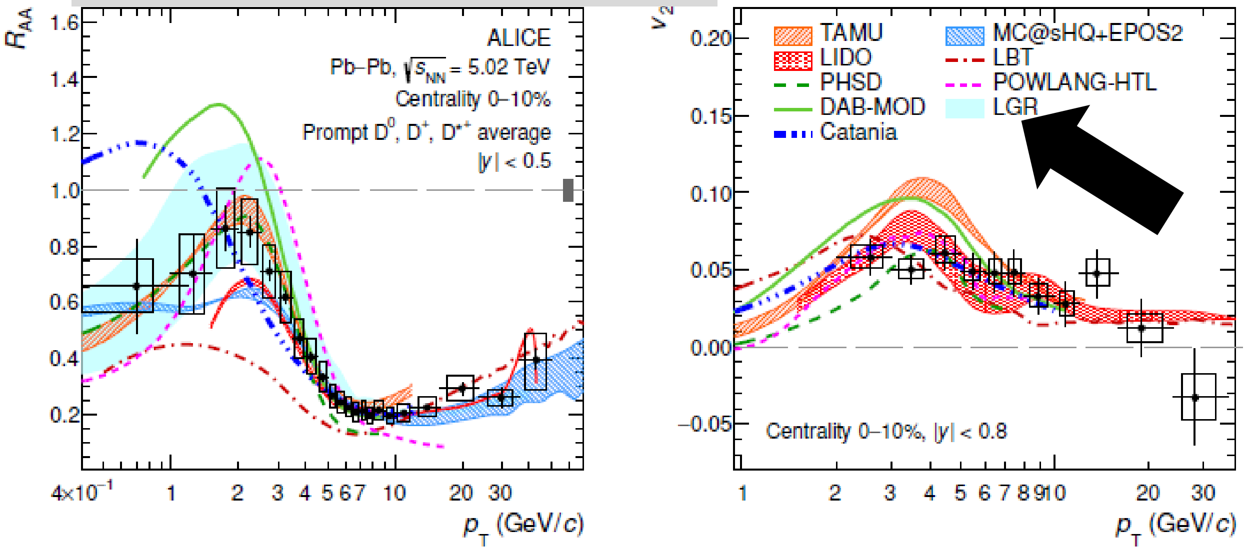
employed modules

SuperMC, iSS: Comput. Phys. Commun. 199, 61 (2016). vHLLE: Comput. Phys. Commun. 185, 3016 (2014) . FONLL: JHEP 05, 007 (1998); 03, 006 (2001); 10, 137 (2012). EPS09: JHEP 04, 065 (2009)

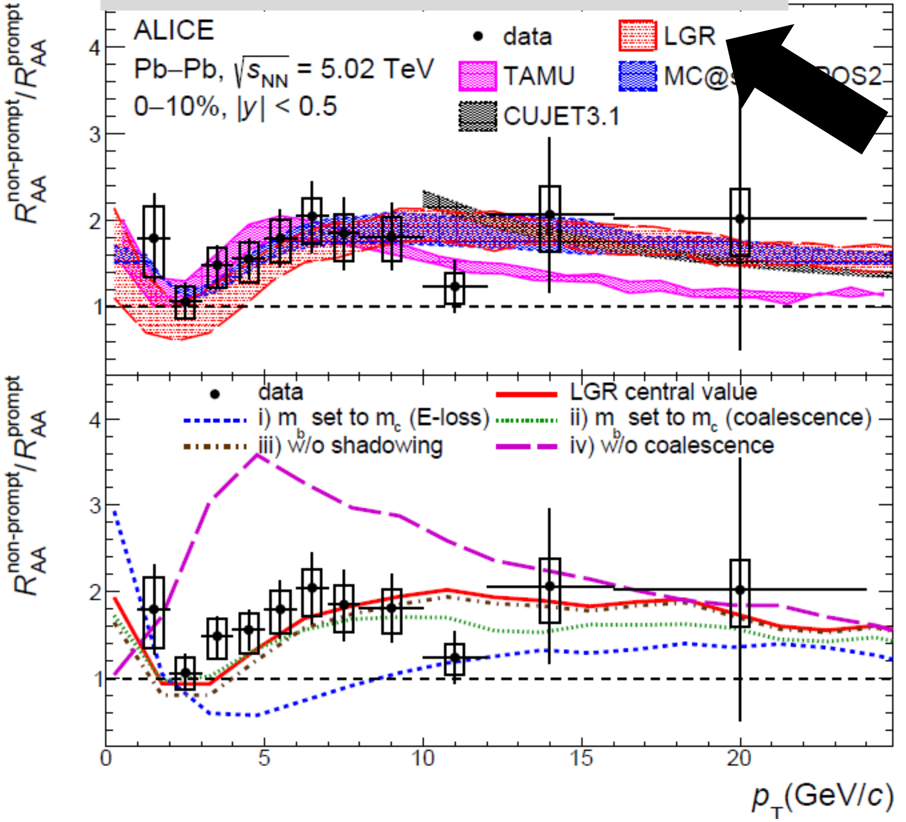
(selected) publications

PRC 98, 014909 (2018); PRC 98, 034914 (2018)
PRC 99, 054909 (2019); EPJC 80, 671 (2020)
EPJC 80, 1113 (2020); EPJC 81, 536 (2021)
PRC 109, 064907 (2024)

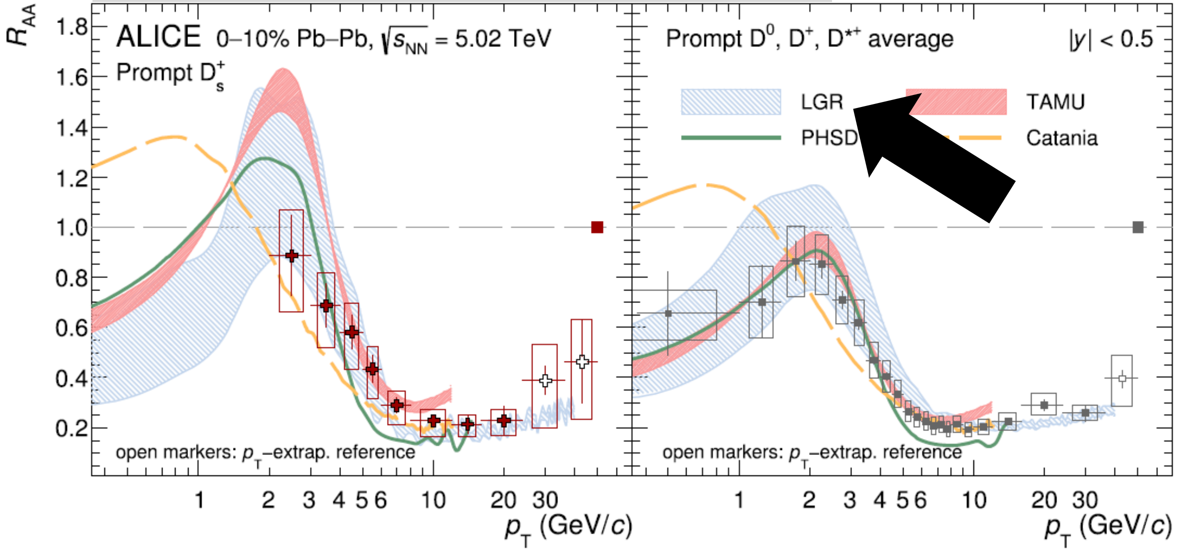
ALICE, JHEP 01, 174 (2022)



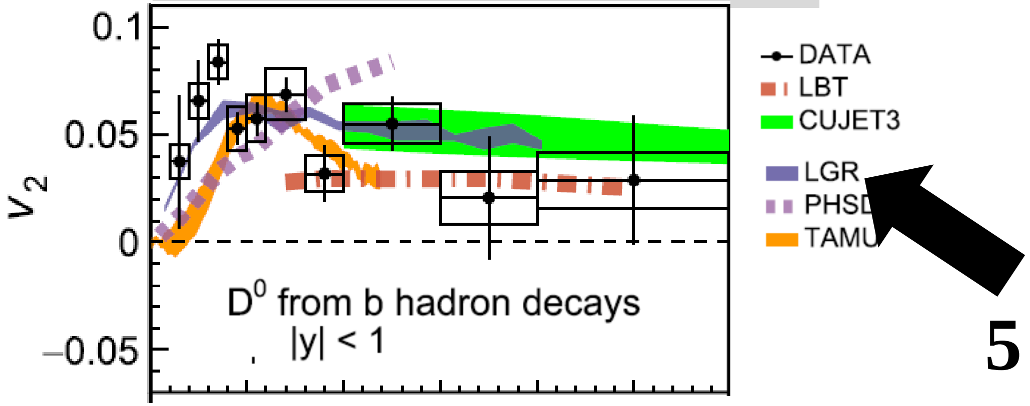
ALICE, JHEP 12, 126 (2022)



ALICE, PLB 827, 136986 (2022)



CMS, PLB 850, 138389 (2024)



Can we even go deeper, in particular at low and moderate p_{T}?

+ based on: PRD 109, 096028 (2024)
EPJC 81, 536 (2021)

Outline

- **HQ transport in perturbation theory: the soft-hard factorized approach**
- **HQ transport in non-perturbation theory: the background field effective theory**
- **Numerical results for HQ energy loss and transport coefficients**
- **Summary and outlooks**

Soft-hard factorization model

- Divergence from t-channel contribution $\frac{d\sigma}{dt} \propto \overline{|\mathcal{M}^2|} \propto \frac{1}{t^2}$

  ~ infrared divergence when $|t| \to 0$

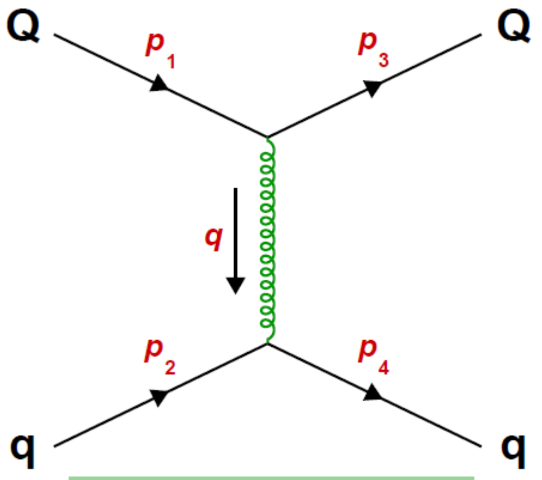
- Infrared regulator can be well determined on first principles: soft-hard factorization approach [Braaten and Yuan, PRL PRL 66, 2183 (1991)]
  - ✓ hard collision: $|t| > |t^*|$, where the pQCD Born approx. is valid
  - ✓ soft collision: $|t| < |t^*|$, where the t-channel long wavelength gluons are screened by the mediums ~ they feel the presence of the medium and require the resummation ~ Hard Thermal Loop (HTL) approximation

- The intermediate scale t^* is formally chosen as

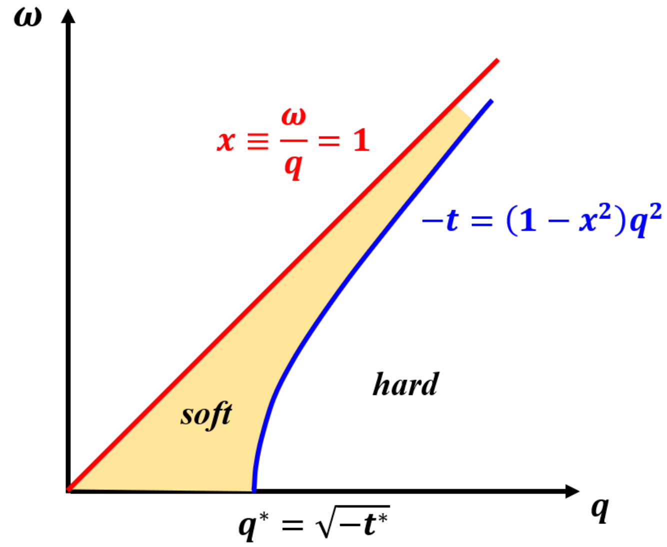
$$m_D^2 \ll -t^* \ll T^2$$

implying weak-coupling or high-temperature limit

[Braaten and Yuan, PRL 66, 2183 (1991)]



$$p_1^\mu - p_3^\mu = (\omega, \vec{q})$$



Collisional energy loss: strategy

- The energy loss per traveling distance

$$-\frac{dE}{dz} = \int d^3\vec{q}\,\frac{d\Gamma}{d^3\vec{q}}\frac{\omega}{v_1}$$

where, Γ is the interaction rate between HQ and medium partons,

$$\Gamma = \Gamma_{(t)}^{soft} + \Gamma_{(t)}^{hard} + \Gamma_{(s+u)}$$

so the total energy loss

$$-\frac{dE}{dz} = \left[-\frac{dE}{dz}\right]_{(t)}^{soft} + \left[-\frac{dE}{dz}\right]_{(t)}^{hard} + \left[-\frac{dE}{dz}\right]_{(s+u)}$$

Collisional energy loss: hard component

- The interaction rate for a given elastic process $Q+i \to Q+i \quad (i=q,g)$

$$\Gamma^{Qi}(E_1,T)=\frac{1}{2E_1}\int_{p_2}\frac{n(E_2)}{2E_2}\int_{p_3}\frac{1}{2E_3}\int_{p_4}\frac{\bar{n}(E_4)}{2E_4}\overline{|\mathcal{M}^2|}^{Qi}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

and the total energy loss for the hard collisions in ***t*-channel**

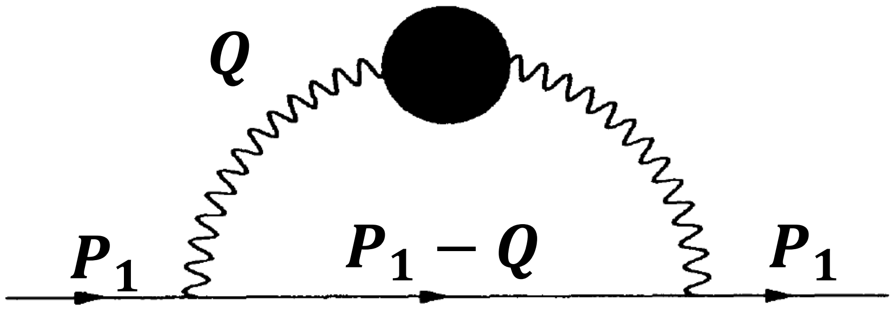
$$\boxed{\left[-\frac{dE}{dz}\right]_{(t)}^{hard}=\frac{1}{256\pi^3\vec{p}_1^{\,2}}\sum_{i=q,g}\int_{|\vec{p}_2|_{min}}^{\infty}d|\vec{p}_2|E_2 n(E_2)\int_{-1}^{cos\psi|_{max}}d(cos\psi)\int_{t_{min}}^{t^*}dt\,\frac{b}{a^3}\overline{|\mathcal{M}^2|}_{Qi(t)}}$$

- The contributions from ***s*- and *u*-channels** are not divergent for small momentum transfers → no need to introduce the intermediate cutoff

$$\boxed{(i=g) \qquad (|\vec{p}_2|_{min}\Rightarrow 0) \qquad (cos\psi|_{max}\Rightarrow 1) \qquad (t^*\Rightarrow 0)}$$

Collisional energy loss: soft component

- Basic formulas [Weldon, PRD 28, 2007 (1983)]



$$\Gamma(E_1,T) = -\frac{1}{2E_1}\bar{n}_F(E_1)Tr[(P_1\cdot\gamma + m_1)Im\Sigma(P_1)]$$

with the HQ self-energy in Minkowski space

$$\Sigma(P_1) = iC_F g^2 \int \frac{d^4Q}{(2\pi)^4}\Delta^{\mu\nu}(Q)\gamma_\mu \frac{1}{(P_1 - Q)\cdot\gamma - m_1}\gamma_\nu$$

and the HTL gluon propagator in Coulomb gauge

$$\Delta^{\mu\nu}(Q) = -(\delta^{\mu 0}\delta^{\nu 0})\Delta_L - (\delta^{ij} - \hat{q}^i\hat{q}^j)\Delta_T$$

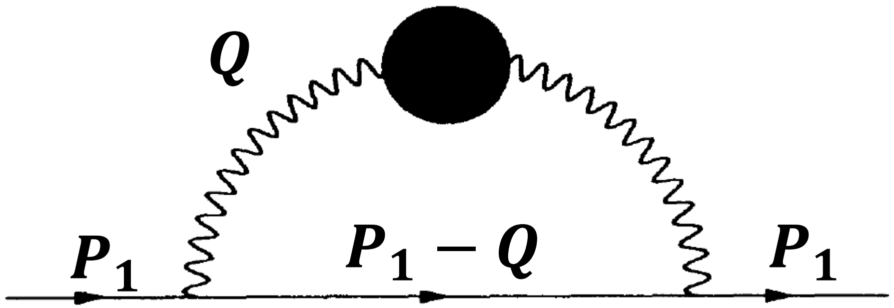
The longitudinal and transverse effective propagators are

$$(\Delta_L)^{-1} = \vec{q}^2 + \Pi_L \qquad (\Delta_T)^{-1} = (q^0)^2 - \vec{q}^2 - \Pi_T$$

[Blaizot and Iancu, Phys. Rep. 359, 355 (2002); Alberico et. al., EPJC 71, 1666 (2011)]

Collisional energy loss: soft component

- Basic formulas [Weldon, PRD 28, 2007 (1983)]



$$\Gamma(E_1, T) = -\frac{1}{2E_1}\bar{n}_F(E_1)Tr[(P_1 \cdot \gamma + m_1)Im\Sigma(P_1)]$$

$$\Gamma(E_1, T) = C_F g^2 \int_q \int d\omega \bar{n}_B(\omega)\delta(\omega - \vec{v}_1 \cdot \vec{q})\{\rho_L(\omega, q) + \vec{v}_1^2[1 - (\hat{v}_1 \cdot \hat{q})^2]\rho_T(\omega, q)\}$$

with the longitudinal and transverse spectral functions

$$\rho_{L/T}(\omega, q) \equiv 2 \cdot Im\Delta_{L/T}(\omega + i\eta, \vec{q}) \quad (\eta \to 0_+)$$

[Blaizot and Iancu, Phys. Rep. 359, 355 (2002)]

The total energy loss in soft collisions reads

$$\boxed{\left[-\frac{dE}{dz}\right]_{(t)}^{soft} = \frac{C_F g^2}{8\pi^2 v_1^2}\int_{t^*}^{0} dt(-t)\int_0^{v_1} dx \frac{x}{(1-x^2)^2}[\rho_L(t, x) + (v_1^2 - x^2)\rho_T(t, x)]}$$

Collisional energy loss: hard+soft

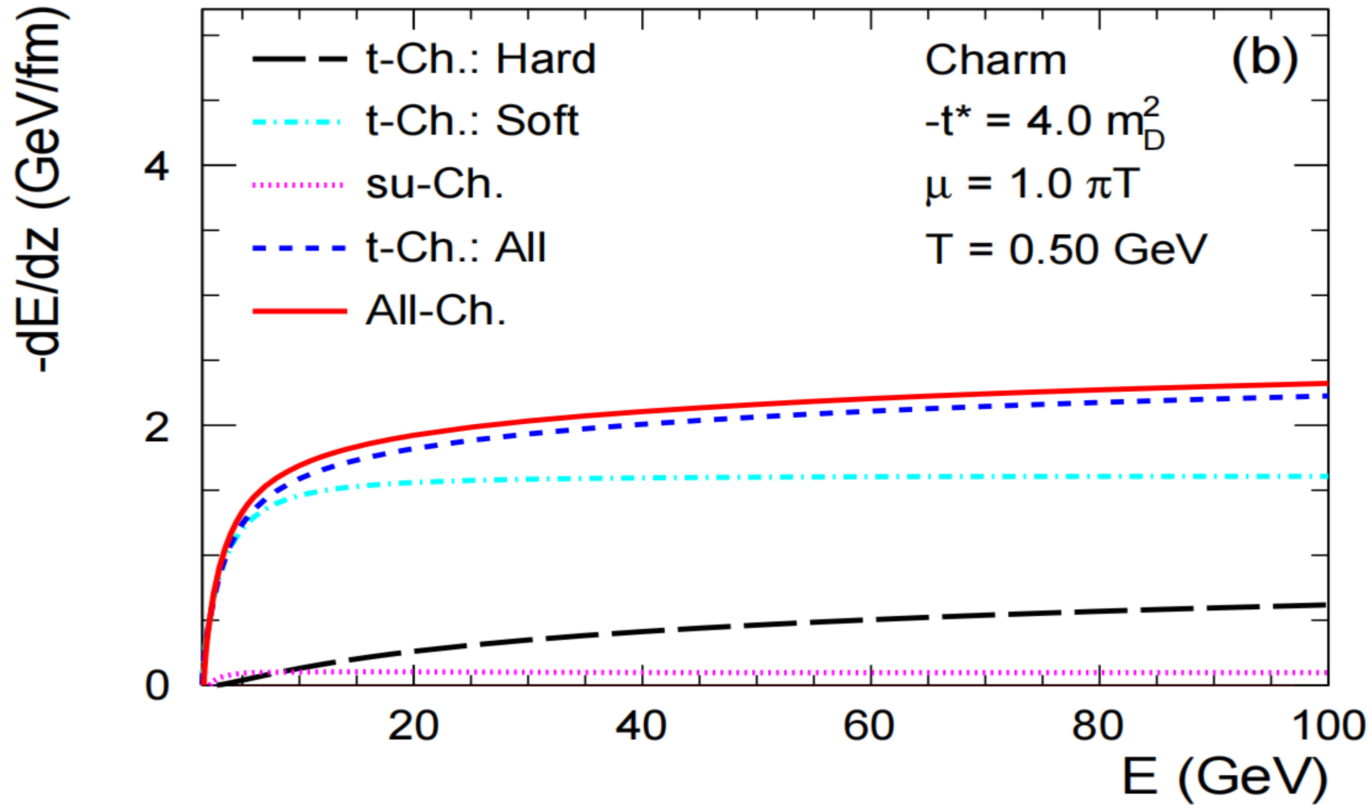
$$-\frac{dE}{dz} = \left[-\frac{dE}{dz}\right]_{(t)}^{soft} + \left[-\frac{dE}{dz}\right]_{(t)}^{hard} + \left[-\frac{dE}{dz}\right]_{(s+u)}$$

$$\left[-\frac{dE}{dz}\right]_{(t)}^{soft} = \frac{C_F g^2}{8\pi^2 v_1^2}\int_{t^*}^{0} dt(-t)\int_0^{v_1} dx\,\frac{x}{(1-x^2)^2}\left[\rho_L(t,x) + (v_1^2 - x^2)\rho_T(t,x)\right]$$

$$\left[-\frac{dE}{dz}\right]_{(t)}^{hard} = \frac{1}{256\pi^3 \vec{p}_1^{\,2}}\sum_{i=q,g}\int_{|\vec{p}_2|_{min}}^{\infty} d|\vec{p}_2|E_2 n(E_2)\int_{-1}^{cos\psi|_{max}} d(cos\psi)\int_{t_{min}}^{t^*} dt\,\frac{b}{a^3}\,\overline{|\mathcal{M}^2|}_{Qi(t)}$$

$$\left[-\frac{dE}{dz}\right]_{(s+u)} = \frac{1}{256\pi^3 \vec{p}_1^{\,2}}\int_0^{\infty} d|\vec{p}_2|E_2 n(E_2)\int_{-1}^{1} d(cos\psi)\int_{t_{min}}^{0} dt\,\frac{b}{a^3}\,\overline{|\mathcal{M}^2|}_{Qg(s+u)}$$

Collisional energy loss



[PRD 109, 096028 (2024)]

- s + u channel contribution is negligible since the relevant interaction rate is much smaller than the others
- The soft contribution dominates in the considered energy region

Toward an analytical form

① High-energy approach (HEA): $E \gg m_Q^2/T$ ② Weak-coupling approximation: $m_D^2 \ll -t^* \ll T^2$

③ Large momentum transfer: $-t \sim s \gg m_1^2$ ④ Backward scattering: $\theta = \pi$

$$\left[-\frac{dE}{dz}\right]^{soft}_{(t)} \Rightarrow \left[-\frac{dE}{dz}\right]^{soft-HEA}_{(t)} \approx \frac{C_F}{16\pi}\left(\frac{N_c}{3}+\frac{N_f}{6}\right)g^4T^2 ln\frac{-2t^*}{m_D^2}$$

$$\left[-\frac{dE}{dz}\right]^{hard}_{(t)} \Rightarrow \left[-\frac{dE}{dz}\right]^{hard-HEA}_{Qq(t)} \approx \frac{N_f N_c}{216\pi}g^4T^2\left(ln\frac{8E_1T}{-t^*}-\frac{3}{4}+c\right)$$

$$\left[-\frac{dE}{dz}\right]^{hard-HEA}_{Qg(t)} = \frac{N_c^2-1}{96\pi}g^4T^2\left(ln\frac{4E_1T}{-t^*}-\frac{3}{4}+c\right)$$

$$\left[-\frac{dE}{dz}\right]_{(s+u)} \Rightarrow \left[-\frac{dE}{dz}\right]^{hard-HEA}_{Qg(s+u)} = \frac{N_c^2-1}{432\pi}g^4T^2\left(ln\frac{4E_1T}{m_1^2}-\frac{5}{6}+c\right)$$

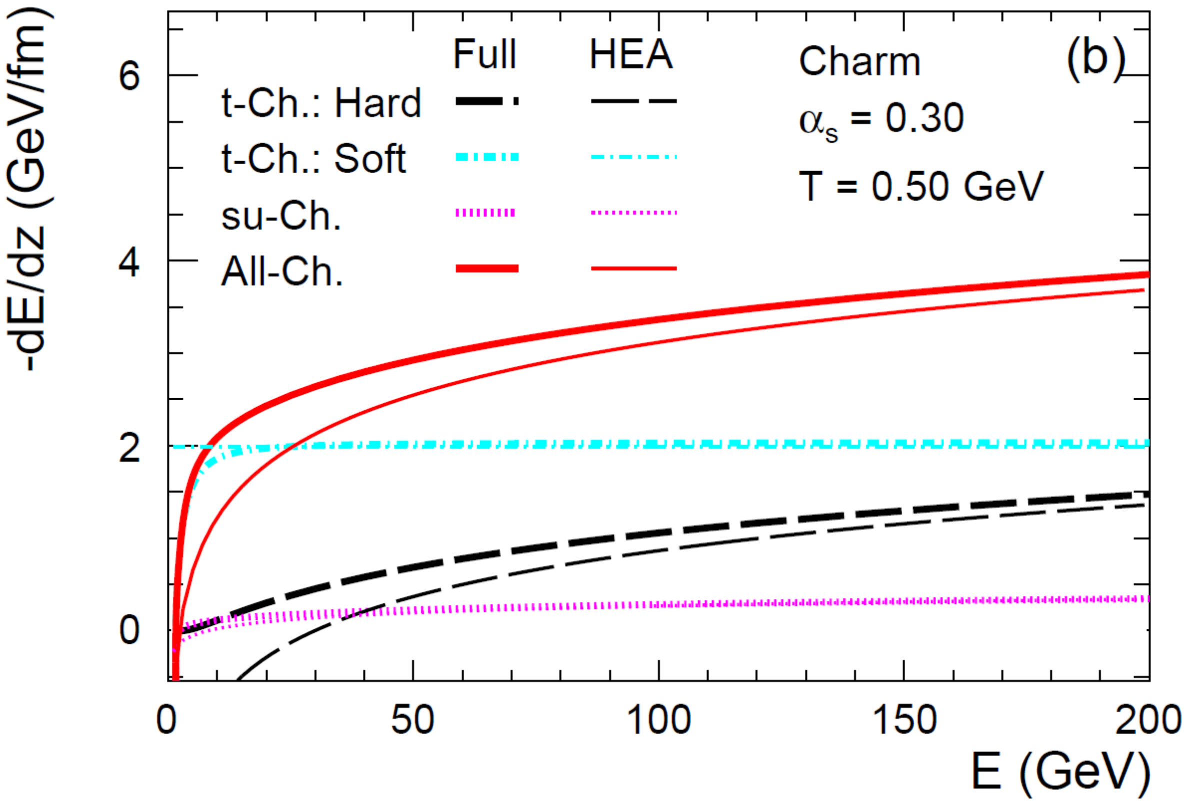
$$\left[-\frac{dE}{dz}\right]^{HEA}_{Qq+Qg} = \frac{4}{3}\pi\alpha_s^2T^2\left[\left(1+\frac{N_f}{6}\right)ln\frac{E_1T}{m_D^2} + \frac{2}{9}ln\frac{E_1T}{m_1^2}+d(N_f)\right],$$

The sum of these contributions cancels the t^*-dependence

[Peigne and Peshier, PRD 77, 114017 (2008)]

- The T- and E-dependencies are similar to the results for the scattering of a light hard parton off a light soft parton [Qin et. al., PRL 100, 072301 (2008)]

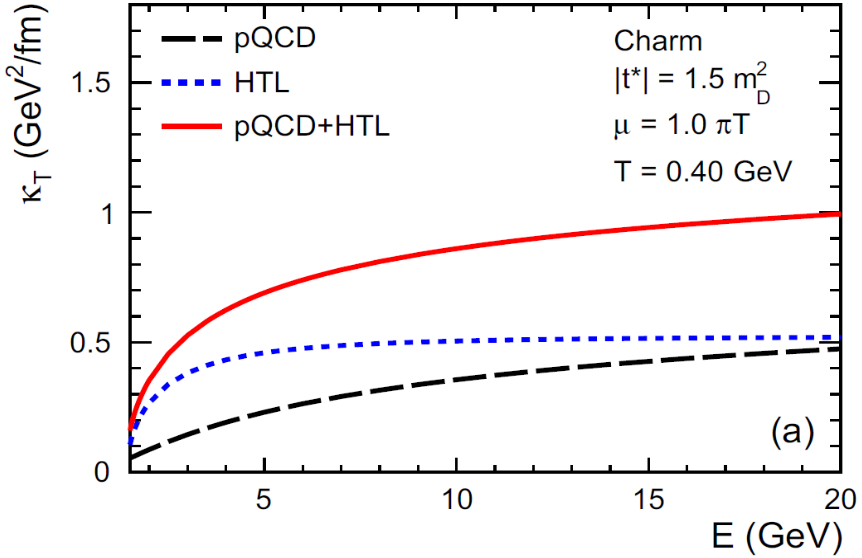
“Full” vs. “HEA”

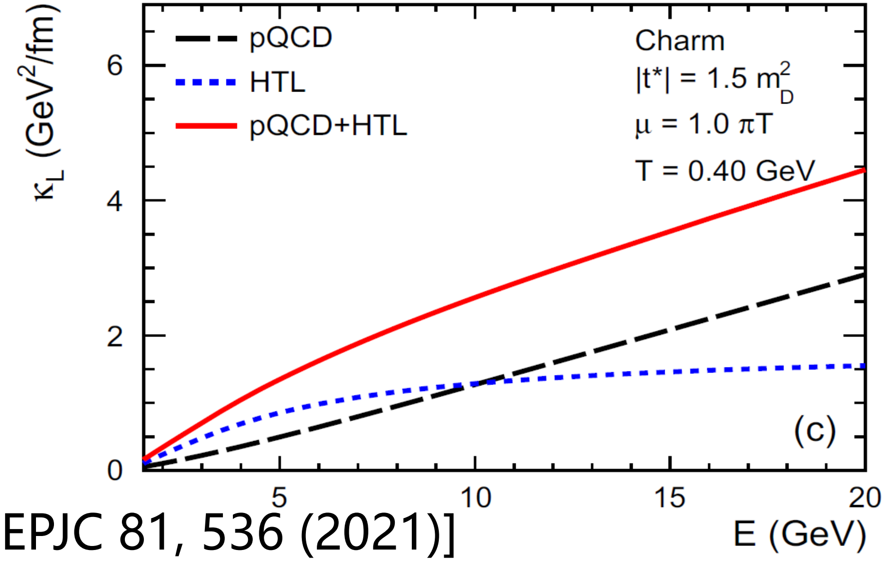


[PRD 109, 096028 (2024)]

- As expected, the asymptotic behavior is presented toward high energy, while a considerable variation is found at low and moderate energy for each channel

Transport coefficients





[EPJC 81, 536 (2021)]

$$\kappa_T = \frac{1}{2}\int d\Gamma\, \mathbf{q}_T^2 = \frac{1}{2}\int d\Gamma \left[\omega^2 - t - \left(\frac{2\omega E_1 - t}{2|\mathbf{p}_1|}\right)^2\right]$$

$$\kappa_L = \int d\Gamma\, q_L^2 = \frac{1}{4\mathbf{p}_1^2}\int d\Gamma\, (2\omega E_1 - t)^2$$

$$p_1^\mu - p_3^\mu = (\omega, \vec{q}) = (\omega, \vec{q}_T, q_L)$$

- $\kappa_{L/T}$ describes the momentum fluctuations in the direction that parallel / perpendicular (i.e., longitudinal / transverse) to the HQ propagation
- It is found that the soft components are significant at low energy, while they are compatible at larger values

How about the non-perturbative contributions ?

+ based on: arXiv: 2410.*****

Semi-QGP near T_c

$T \ll \Lambda_{QCD}$
Vacuum: confined

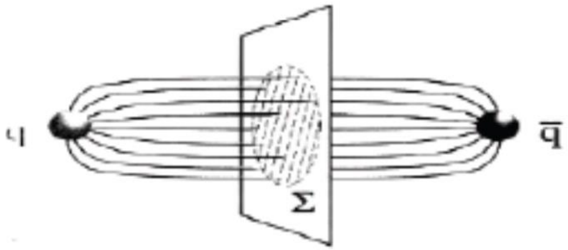
T_C

$T \sim \Lambda_{QCD}$
sQGP

few T_C

$T \gg \Lambda_{QCD}$
wQGP: screening

T



weak-coupling approach
(high-temperature limit)

$$m_D^2 \ll -t^* \ll T^2$$

- Strong-coupling behavior in the "semi-QGP", $T_C < T \lesssim 3 - 4\, T_C$, may have an important influence on the HQ energy loss which should be reconsidered in an effective theory
- Non-perturbative contribution included

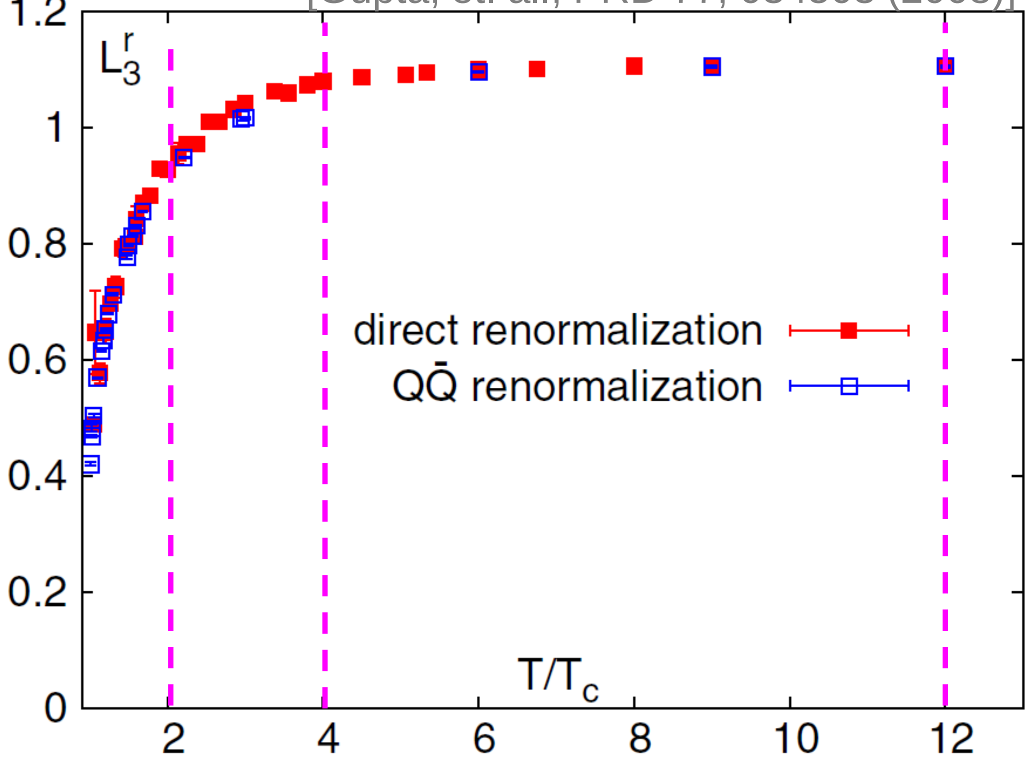
[Hidaka and Pisarski, PRD 78, 071501 (2008)]

How to describe the semi-QGP ?

- For pure gauge theory without quarks, the order parameter of the deconfining phase transition is the Polyakov loop which has a nontrivial dependence on the temperature

$T < T_c$: $\ell \approx 0$	$T \sim T_c^+$: $\ell \approx 0.5$
$T \approx 2T_c$: $\ell \approx 0.9$	$T \approx 4T_c$: $\ell \approx 1.1$
$4T_c \lesssim T \lesssim 12T_c$: $\ell \approx const.$	

[Gupta, et. al., PRD 77, 034503 (2008)]



Phase	Temperature	ℓ from LQCD	Method
QGP	$T \gtrsim 3-4T_c$	$\ell \approx 1$	Perturbation theory (pQCD + HTL)
semi-QGP	$T_c < T \lesssim 3-4T_c$	$0 < \ell < 1$	Background field effective theory
Hadronic	$T < T_c$	$\ell \approx 0$	Effective theory (HRG)

The background field effective theory

- Introducing a classical background field to describe the nontrivial Polyakov loop in the deconfining phase transition for $SU(N)$

$$(A_0^{cl})_{ab} = \frac{1}{g} \mathcal{Q}^a \delta_{ab}; \quad \boldsymbol{L} = \mathcal{P} \exp(ig \int_0^\beta A_0^{cl}\, d\tau); \quad \ell = \frac{1}{N}\, Tr\boldsymbol{L}$$

$q^a \equiv \mathcal{Q}^a/(2\pi T) = (q, 0, -q)$

$q^{ab} \equiv q^a - q^b$

- The effective potential reads

$$\mathcal{V} = \mathcal{V}_{pt} + \mathcal{V}_{npt} = \frac{2\pi^2 T^4}{3} \sum_{ab} \mathcal{P}^{ab,ba}\, B_4(|q^{ab}|) + \frac{M^2 T^2}{2} \sum_{ab} \mathcal{P}^{ab,ba}\, B_2(|q^{ab}|)$$

- The T-dependent background field obtained from the relevant EoM for the background field ($N = 3$)

$$q_{conf} = \frac{1}{3} \qquad\qquad q_{deconf} = \frac{1}{36}\left(9 - \sqrt{81 - 80\frac{T_c^2}{T^2}}\right)$$

[Hidaka and Pisarski, PRD 80, 036004 (2008); Guo and Kuang, PRD 104, 014015 (2021)]

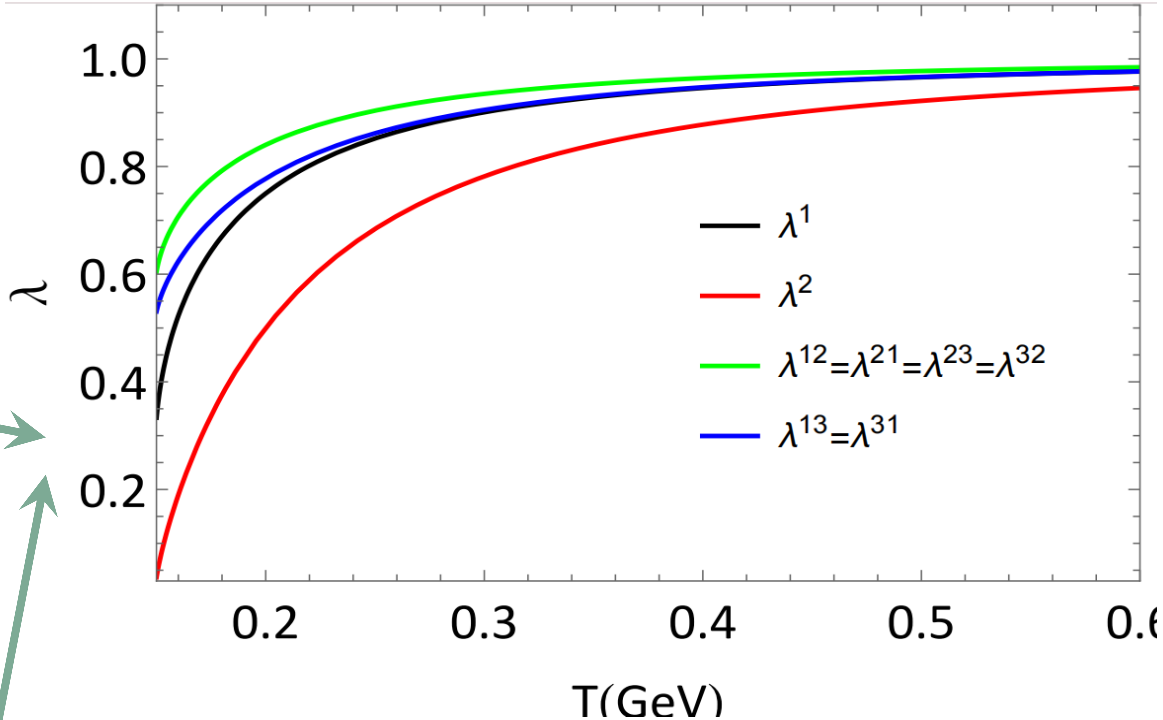
Resummed gluon propagator

- The off-diagonal components

$$D_{\mu\nu}^{de.fg}(Q^{de}) = \Delta_L^{de,fg} B_{ij} + \Delta_T^{de,fg} A_{ij}$$

$$\Delta_T^{de,fg} = \delta^{dg}\delta^{ef}\frac{1}{\omega^2 - q^2 - \lambda^{de}\Pi_T}$$

$$\Delta_L^{de,fg} = \delta^{dg}\delta^{ef}\frac{1}{q^2 + \lambda^{de}\Pi_L}$$



- The diagonal components

$$\sum_{color} P^{dd.ff} D_{\mu\nu}^{dd.ff}(Q) = \sum_{\alpha=1}^{N-1}\left[\frac{1}{q^{o2} - q^2 - \lambda^\alpha \Pi_T} A_{ij} + \frac{1}{q^2 + \lambda^\alpha \Pi_L} B_{ij}\right]$$

[Guo and Kuang, PRD 104, 014015 (2021)]

Interaction rate: hard component

BF effective theory

$$\Gamma^{hard-BF}_{Qi(t)}(E_1,T) = \frac{1}{2E_1}\int_{p_2} \boxed{\frac{n_B^+(E_2,q^{de})+n_B^-(E_2,q^{de})}{4E_2}} \int_{p_3}\frac{1}{2E_3}\int_{p_4}\frac{\bar{n}(E_4)}{2E_4}$$
$$\boxed{\overline{|\mathcal{M}^2|}^{BF}_{Qi(t)}}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4),$$

$$n_B^+(k_0,q^{de}) = \frac{1}{e^{|k_0|/T-2\pi i q^{de}}-1}$$
$$n_B^-(k_0,q^{de}) = \frac{1}{e^{|k_0|/T+2\pi i q^{de}}-1}$$

$$\overline{|\mathcal{M}^2|}^{BF} = \frac{1}{8}\sum_{de}(1-\frac{1}{3}\delta^{de})\overline{|\mathcal{M}^2|}$$

(BF enters like an imaginary chemical potential)

$$n_B(k) = \frac{1}{e^{k/T}-1}$$

Perturbative

$$\Gamma^{hard}_{Qi}(E_1,T) = \frac{1}{2E_1}\int_{p_2}\boxed{\frac{n(E_2)}{2E_2}}\int_{p_3}\frac{1}{2E_3}\int_{p_4}\frac{\bar{n}(E_4)}{2E_4}$$
$$\times\boxed{\overline{|\mathcal{M}^2|}^{Qi}}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

Interaction rate: soft component

- The off-diagonal components

$$\Gamma^{soft}_{offdiag}(E_1,T)=\frac{1}{2N}g^2\sum_{d,e=1}^{N}\int_q\int d\omega\,\bar{n}_B(\omega)\delta(\omega-\vec{v}_1\cdot\vec{q}\,)\Big\{\rho^{de}_L(\omega,q)+\vec{v}_1^{\,2}\big[1-(\hat{v}_1\cdot\hat{q})^2\big]\rho^{de}_T(\omega,q)\Big\}$$

- The diagonal components

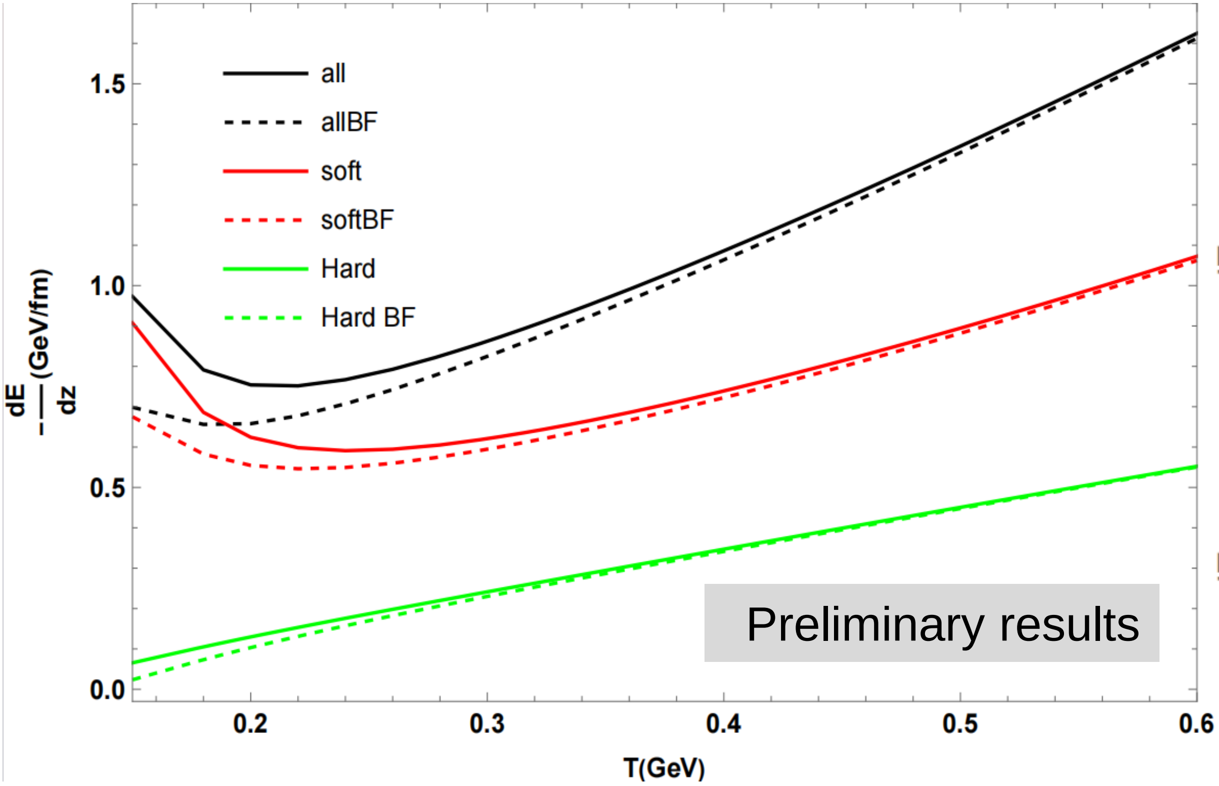
$$\Gamma^{soft}_{diag}(E_1,T)=\frac{1}{2N}g^2\sum_{\alpha=1}^{N-1}\int_q\int d\omega\,\bar{n}_B(\omega)\delta(\omega-\vec{v}_1\cdot\vec{q}\,)\Big\{\rho^{\alpha}_L(\omega,q)+\vec{v}_1^{\,2}\big[1-(\hat{v}_1\cdot\hat{q})^2\big]\rho^{\alpha}_T(\omega,q)\Big\}$$

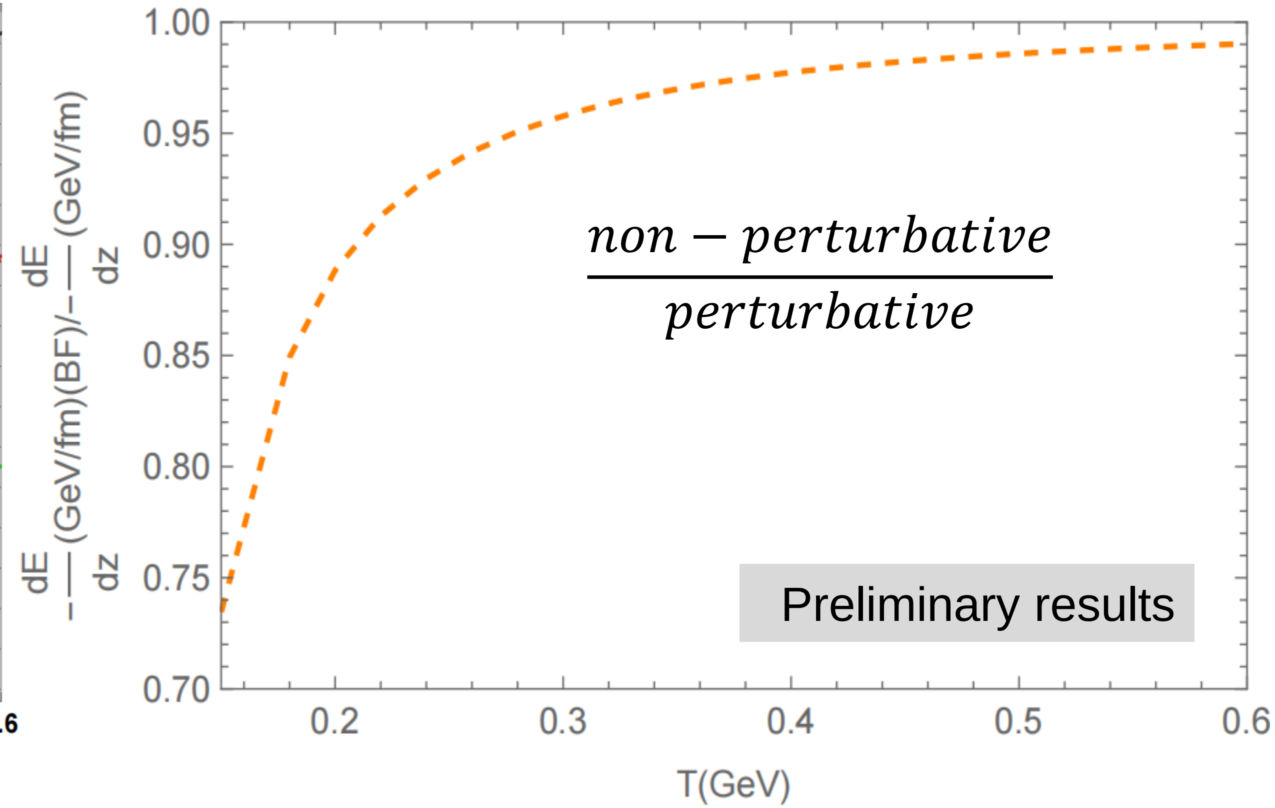
cross check

$$\Gamma^{soft}_{offdiag}(E_1,T)+\Gamma^{soft}_{diag}(E_1,T)\xrightarrow[\text{high temperature}]{\text{sum over color}}\Gamma^{soft}(E_1,T)$$

BF effective theory | **Perturbative**

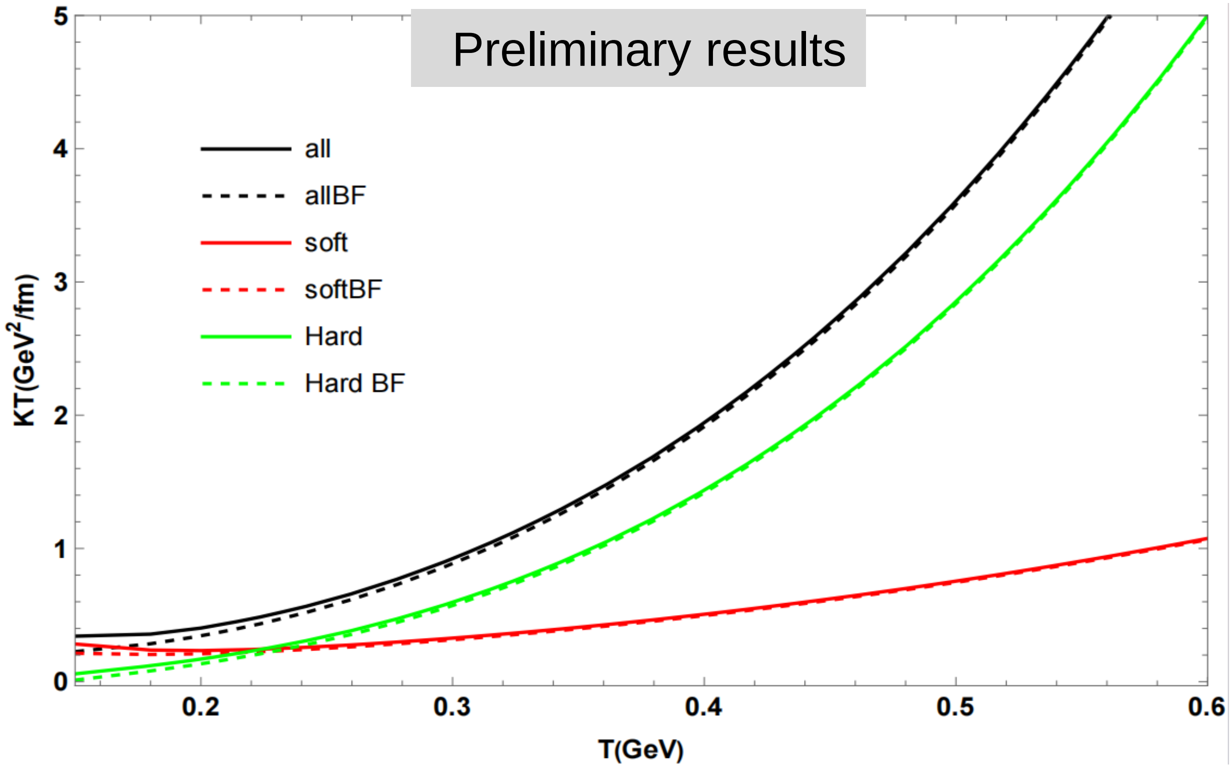
dE/dx: non-pert. vs pert.

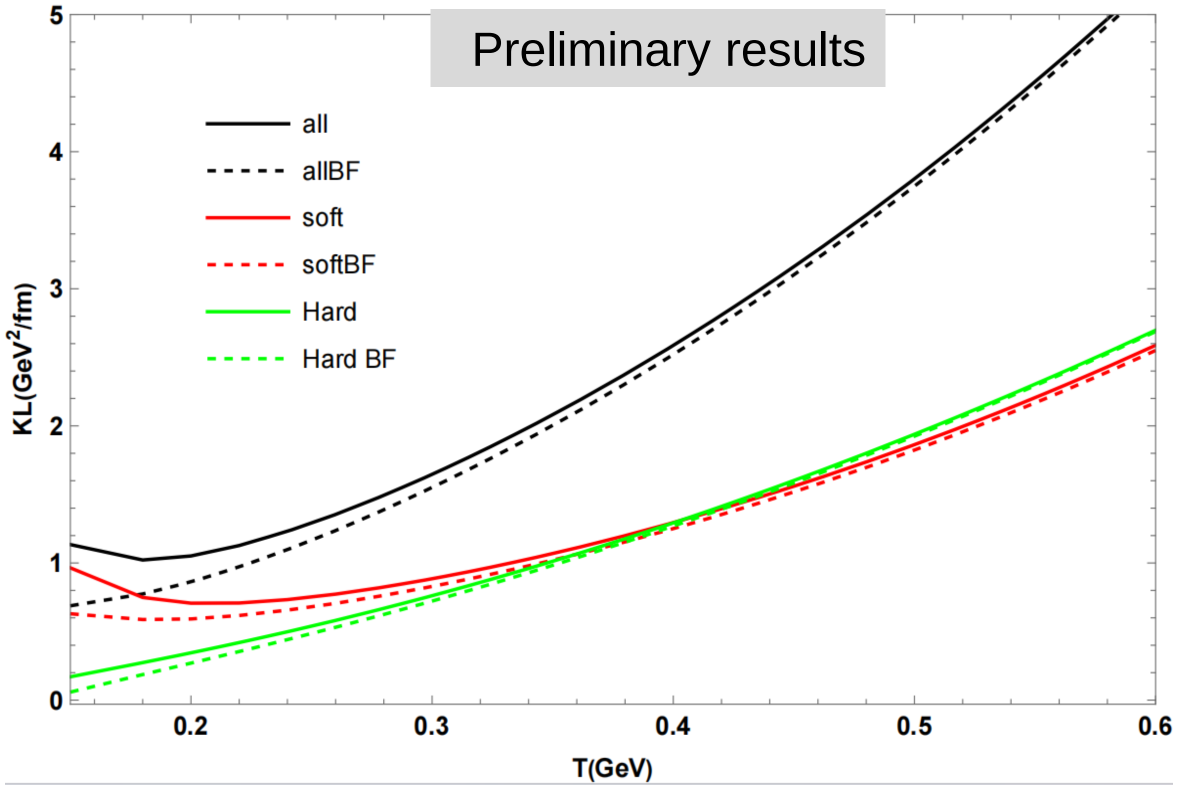




- The dependence on the medium temperature is similar
- As expected, the non-perturbative contribution is more significant in the entire semi-QGP region

$\kappa_{T/L}$: non-pert. vs pert.





- Similar trend observed

Summary and outlooks

- **Within the soft-hard factorization approach, we calculated the HQ collisional energy loss and transport coefficients at leading order in the QCD coupling constant.** Our results show
  - ✓ a better description of HQ transport in perturbative QCD medium, in particular at $E \lesssim 50\,GeV$, where the heavy-flavor probes are measured comprehensively at RHIC and LHC energies
  - ✓ the full dE/dz can be simplified to an analytical form (Peigne-Peshier formula) in the high energy approximation $E \gg m_Q^2/T$

- **To investigate the non-perturbative contributions, we updated the above framework by using the background field effective theory.** Our (preliminary) results show
  - ✓ dependence of dE/dx and $\kappa_{T/L}$ on the medium temperature is similar w.r.t. that obtained in perturbative QCD medium
  - ✓ comparing with the results from perturbative theory, BF suppress dE/dx, in particular in the entire semi-QGP region $T_c < T \lesssim 3 - 4T_c$

- **New paths forward for future work**
  - ✓ perform the model-data comparisons at RHIC and LHC energies

Three Gorges Dam

Backup

Langevin dynamics with Gluon Radiation (LGR)

S. Cao, G.Y. Qin, and S. A. Bass, Phys. Rev. C 88, 044907 (2013).

$$\frac{d\vec{p}}{dt} = \vec{F}_D + \vec{F}_T + \vec{F}_G$$

- Deterministic drag force: $\vec{F}_D = -\eta_D \cdot \vec{p}$
- Stochastic thermal force: $< F_T^i(t) \cdot F_T^j(t') >= [\kappa_T P_\perp^{ij} + \kappa_L P_\parallel^{ij}] \cdot \delta(t - t')$
- Recoil force from gluon radiation: $\vec{F}_G = -\sum_{j=1}^{N_g} \frac{d\vec{p}_G}{dt}$

where, the radiation probability taken from Higher Twist calculation

$$\frac{dN_g}{dz dk_\perp^2 dt} = \frac{2\alpha_s(k_\perp) C_A}{\pi k_\perp^4} \cdot P(z)\, \hat{q}_q \cdot \left(\frac{k_\perp^2}{k_\perp^2 + z^2 m_Q^2}\right)^4 \cdot sin^2\left(\frac{\Delta t}{2\tau_f}\right)$$

Guo and Wang, Phys. Rev. Lett. 85, 3591 (2000); Zhang *et.al.*, Phys. Rev. Lett. 93, 072301 (2004); Qin and Muller, Phys. Rev. Lett. 106, 162302 (2011)

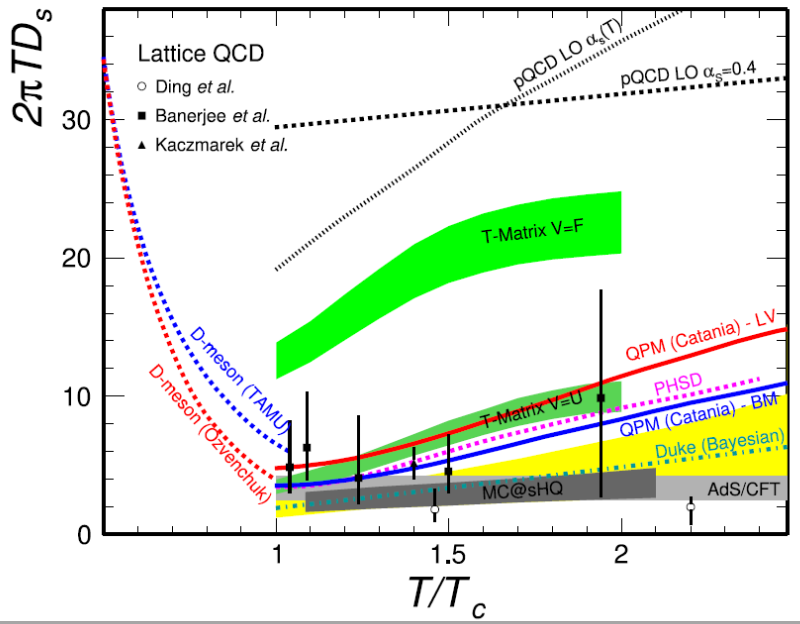
"Minimal model" as the first step

Assumptions

- Isotropic mom. diff.: $\kappa_T = \kappa_L \equiv \kappa$;
- momentum independent behavior of κ: $\partial\kappa / \partial p = 0$;
- Einstein relation $\eta_D = \kappa/(2TE)$.

$$\begin{cases} \eta_D = \dfrac{1}{2\pi T D_s}\dfrac{2\pi T^2}{E} \\ \kappa = \dfrac{1}{2\pi T D_s} 4\pi T^3 \\ \hat{q}_q \approx 2\kappa \end{cases}$$

Only one parameter left: (scaled) spatial diffusion coefficient $2\pi T \cdot D_s(T)$



- Modeling via polynomials: $2\pi T D_s\left(\frac{T}{T_c}\right) = d_0 + d_1\left(\frac{T}{T_c}\right) + d_2\left(\frac{T}{T_c}\right)^2 + \cdots$
  - ✓ without the need of assuming any theoretically-motivated temperature dependence
- Linear ansatz: $2\pi T D_s\left(\frac{T}{T_c}\right) \approx \alpha\left(\frac{T}{T_c}\right) + \beta$, where the slope $0 \le \alpha \le 9$ and the intercept $-8.5 \le \beta \le 4$, will be optimized without presuming any reasonable values → $\alpha = 6.5$ and $\beta = -5.5$ obtained via model-data comparison ($\chi^2 \sim 1.1$)

X. Dong and V. Greco, Prog. Part. Nucl. Phys. 104, 97-141 (2019)

Hadronization via heavy-light coalescence

K.C. Han *et.al.*, Phys. Rev. C 93, 045207 (2016)

- Instantaneous approach utilized
- Momentum spectrum of **mesons** ($\boldsymbol{M}$) formed from the coalescence of **heavy quark** ($\boldsymbol{Q}$) and **anti-light-quark** ($\bar{\boldsymbol{q}}$) is then given by

$$\frac{dN_M}{d^3\vec{p}_M} = g_M \int d^6\xi_Q\, d^6\xi_{\bar{q}} f_Q f_{\bar{q}} \cdot \sum_{n=0}^{1} \overline{W}_M^{(\boldsymbol{n})}\, \delta^3(\vec{p}_M - \vec{p}_Q - \vec{p}_{\bar{q}})$$

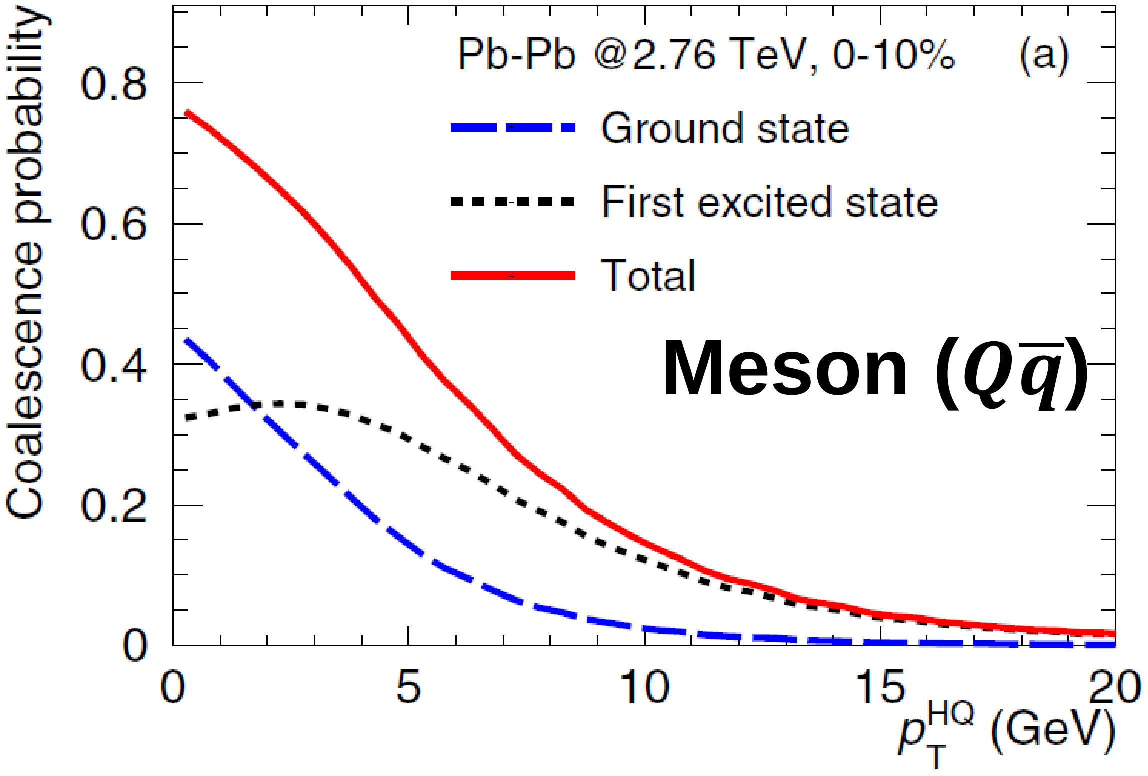
where the overlap integral of the Wigner function of the $Q\bar{q}$ pair and the **meson in *n* excited state**

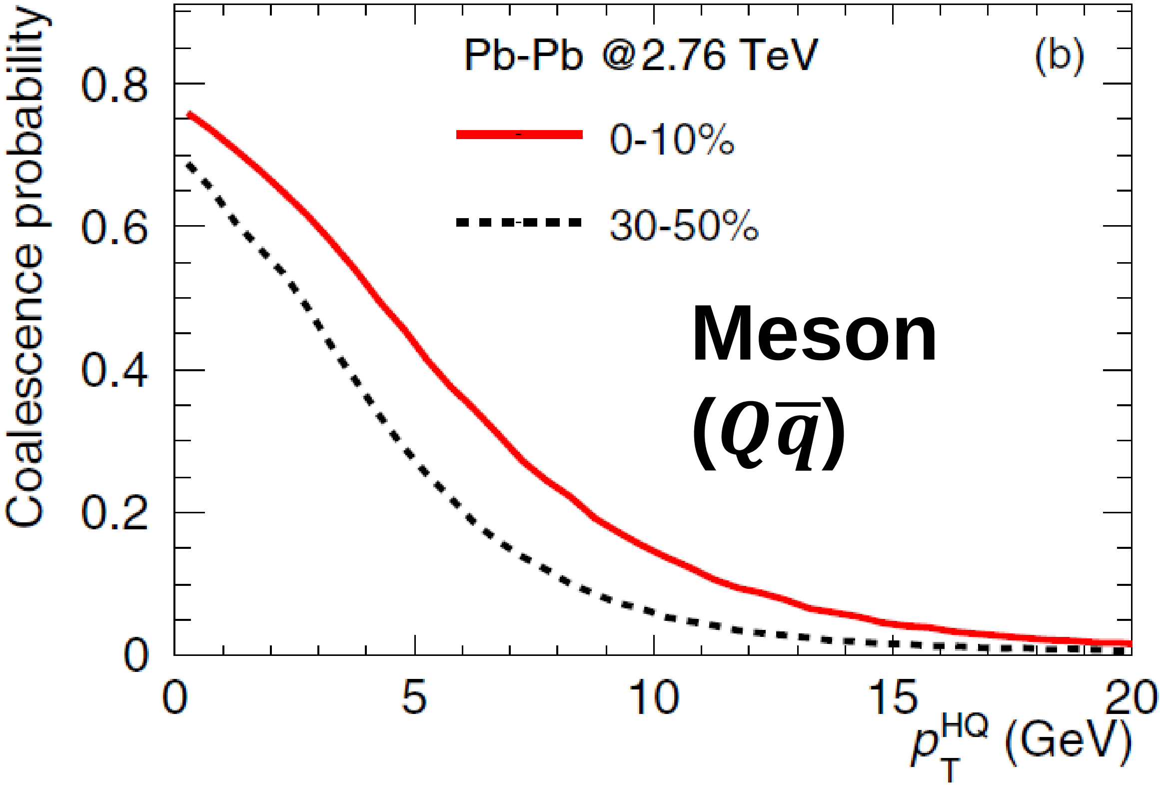
$$\overline{W}_M^{(\boldsymbol{n})}\left(\vec{y}_M, \vec{k}_M\right) = \int \frac{d^6\xi'_Q}{(2\pi)^3} \frac{d^6\xi'_{\bar{q}}}{(2\pi)^3} W_Q(\vec{x}'_Q, \vec{p}'_Q) W_{\bar{q}}(\vec{x}'_{\bar{q}}, \vec{p}'_{\bar{q}}) W_M^{(n)}(\vec{x}'_M, \vec{p}'_M)$$

$$= \frac{\lambda^n}{n!} e^{-\lambda} \qquad \lambda = \frac{1}{2}\left(\frac{\vec{y}_M^2}{\sigma_M^2} + \sigma_M^2 \vec{k}_M^2\right)$$

✓ parton (meson) wave function behaves the Gaussian wave packet (simple harmonic oscillator)

Hadronization via heavy-light coalescence



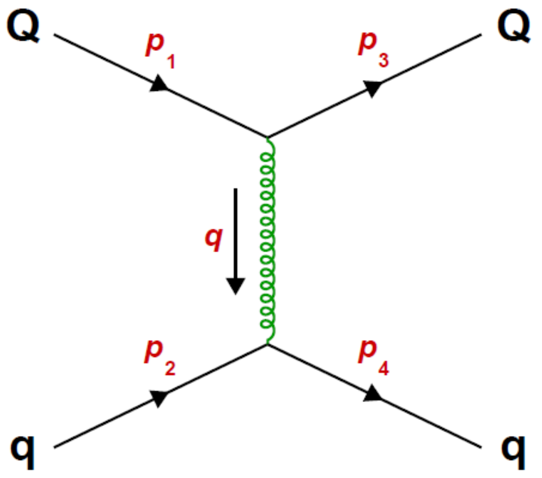


- Coalescence into a ground state has maximum probability at $p_T \sim 0$, and then decreases toward high p_T, due to the difficulty of finding a coalescence partner in this region
- Larger coalescence probability for more central collisions → the coalescence partner density is larger in **0-10%** than in **30-50%**, resulting in a larger probability to form heavy-light combinations

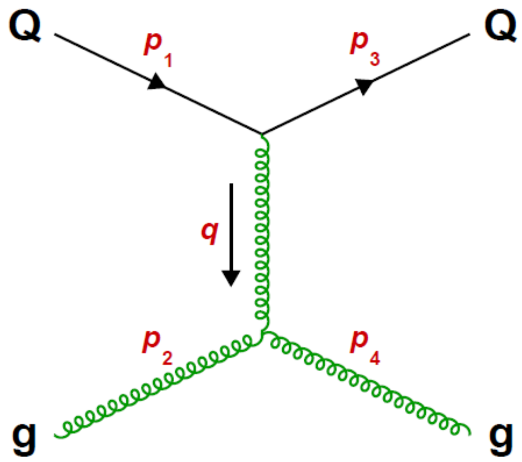
Tree-level Feynman diagrams in vacuum

- The elastic scattering processes between heavy quark (Q) and the quark-gluon plasma constituents ($i = q, g$)

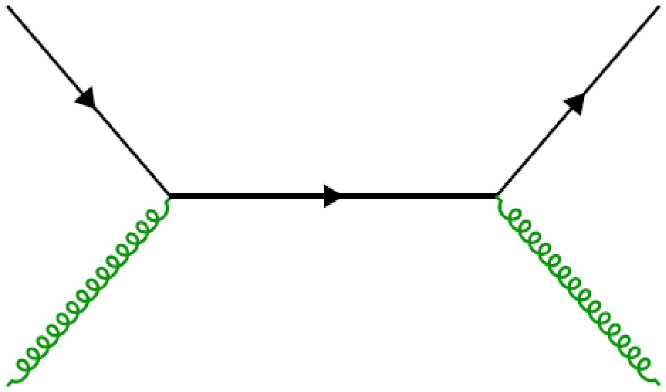
$$\boldsymbol{Q(P_1) + i(P_2) \rightarrow Q(P_3) + i(P_4)}$$



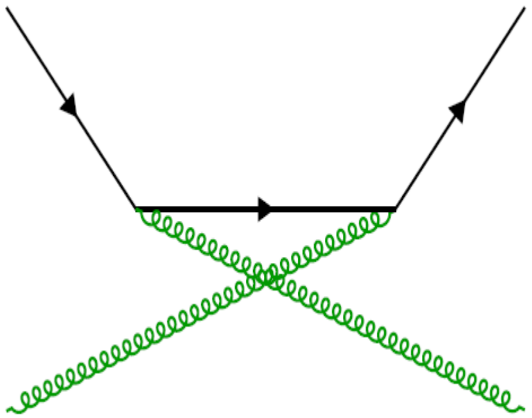
Qq: $t - channel$



Qg: $t - channel$



Qg: $s - channel$



Qg: $u - channel$

Spectral functions

$$\rho_T(\omega,q) = \frac{\pi\omega m_D^2}{2q^3}(q^2-\omega^2)\Bigg\{\Big[q^2-\omega^2 + \frac{\omega^2 m_D^2}{2q^2}\left(1+\frac{q^2-\omega^2}{2\omega q}ln\frac{q+\omega}{q-\omega}\right)\Big]^2 + \left[\frac{\pi\omega m_D^2}{4q^3}(q^2-\omega^2)\right]^2\Bigg\}^{-1} \quad \text{(A.15)}$$

$$\rho_L(\omega,q) = \frac{\pi\omega m_D^2}{q}\Bigg\{\Big[q^2+m_D^2\left(1-\frac{\omega}{2q}ln\frac{q+\omega}{q-\omega}\right)\Big]^2 + \left(\frac{\pi\omega m_D^2}{2q}\right)^2\Bigg\}^{-1}, \quad \text{(A.16)}$$

$$t=\omega^2-q^2<0 \qquad q^2=\frac{-t}{1-x^2}$$
$$x=\frac{\omega}{q}<v_1 \qquad \omega = x\sqrt{\frac{-t}{1-x^2}}, \quad \text{(A.19)}$$

resulting in

$$dtdx = \left|\frac{\partial(t,x)}{\partial(q,\omega)}\right| dqd\omega = 2(1-x^2)dqd\omega. \quad \text{(A.20)}$$

$$|\mathbf{p}_2|_{min} = \frac{|t^*|+\sqrt{(t^*)^2+4m_1^2|t^*|}}{4(E_1+|\mathbf{p}_1|)} \quad \text{(B.28)}$$

$$cos\psi|_{max} = min\left\{1, \frac{E_1}{|\mathbf{p}_1|} - \frac{|t^*|+\sqrt{(t^*)^2+4m_1^2|t^*|}}{4|\mathbf{p}_1|\cdot|\mathbf{p}_2|}\right\} \quad \text{(B.29)}$$

$$t_{min} = -\frac{(s-m_1^2)^2}{s} \quad \text{(B.30)}$$

$$\omega_{max/min} = \frac{b\pm\sqrt{D}}{2a^2} \; with \quad \text{(B.31)}$$

$$a = \frac{s-m_1^2}{|\mathbf{p}_1|} \quad \text{(B.32)}$$

$$b = -\frac{2t}{\mathbf{p}_1^2}\left[E_1(s-m_1^2)-E_2(s+m_1^2)\right] \quad \text{(B.33)}$$

$$c = -\frac{t}{\mathbf{p}_1^2}\left\{t\left[(E_1+E_2)^2-s\right]+4\mathbf{p}_1^2\mathbf{p}_2^2 sin^2\psi\right\} \quad \text{(B.34)}$$

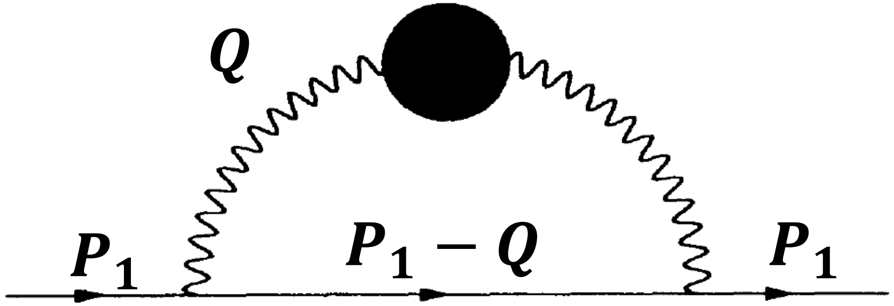
$$D = b^2+4a^2c = -t\left[ts+(s-m_1^2)^2\right]\cdot\left(\frac{4E_2 sin\psi}{|\mathbf{p}_1|}\right)^2 \quad \text{(B.35)}$$

$$G(\omega) = -a^2\omega + b\omega + c \quad \text{(B.36)}$$

Collisional energy loss: soft component

- Basic formulas [Weldon, PRD 1983]

$$\Gamma(E_1,T) = -\frac{1}{2E_1}\overline{n}_F(E_1)Tr[(P_1\cdot\gamma + m_1)Im\Sigma(P_1)]$$



with the HQ self-energy

$$\Sigma(P_1) = iC_F g^2 \int \frac{d^4Q}{(2\pi)^4}\Delta^{\mu\nu}(Q)\gamma_\mu \frac{1}{(P_1-Q)\cdot\gamma - m_1}\gamma_\nu$$

and the HTL gluon propagator in Coulomb gauge

$$\Delta^{\mu\nu}(Q) = -[\delta^{\mu 0}\delta^{\nu 0}]\Delta_L - \left[\delta^{ij} - \hat{q}^i\hat{q}^j\right]\Delta_T$$

The longitudinal and transverse effective propagators are

$$(\Delta_L)^{-1} = \vec{q}^2 + \Pi_L \qquad \Pi_L = m_D^2[1-Q(x)] \qquad x \equiv q^0/|\vec{q}|$$

$$(\Delta_T)^{-1} = (q^0)^2 - \vec{q}^2 - \Pi_T \qquad \Pi_T = m_D^2[x^2 + (1-x^2)Q(x)]/2 \qquad Q(x) \equiv \frac{x}{2}ln\frac{x+1}{x-1}$$

[Blaizot and Iancu, Phys.Rep. 2002; Alberico et. al., EPJC 2011]

Coupling constant

- g is quantified by the two-loop QCD beta-function

[Kaczmarek and Zantow, PRD 2005]

$$g^{-2}(\mu) = 2\beta_0 ln\left(\frac{\mu}{\Lambda_{QCD}}\right) + \frac{\beta_1}{\beta_0} ln\left[2ln(\frac{\mu}{\Lambda_{QCD}})\right]$$

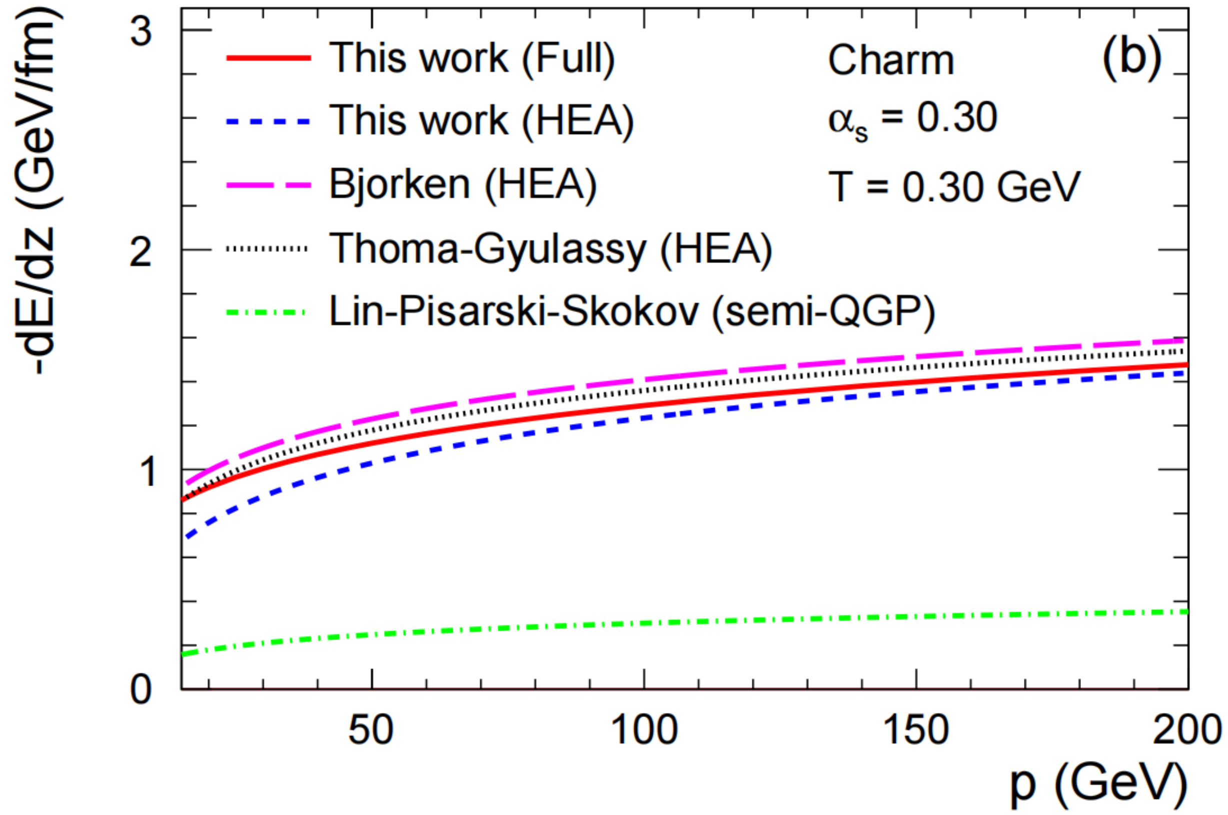
$$\beta_0 = \frac{1}{16\pi^2}\left(11 - \frac{2}{3}N_f\right)$$

$$\beta_1 = \frac{1}{(16\pi^2)^2}\left(102 - \frac{38}{3}N_f\right)$$

✓ for hard collision, $\mu = \sqrt{-t}$

✓ for soft collision, $\mu = K \cdot \pi T$

Comparison with other models



- ***Bjorken*** : keep only the logarithmically divergent integral over momentum transfer; imposing physically reasonable upper and lower limits to regulate the divergences

[Bjorken, FERMILAB-Pub-82/59-THY]

- ***Thoma-Gyulassy*** : update the Bjorken approach by including a more careful treatment of the infrared divergences

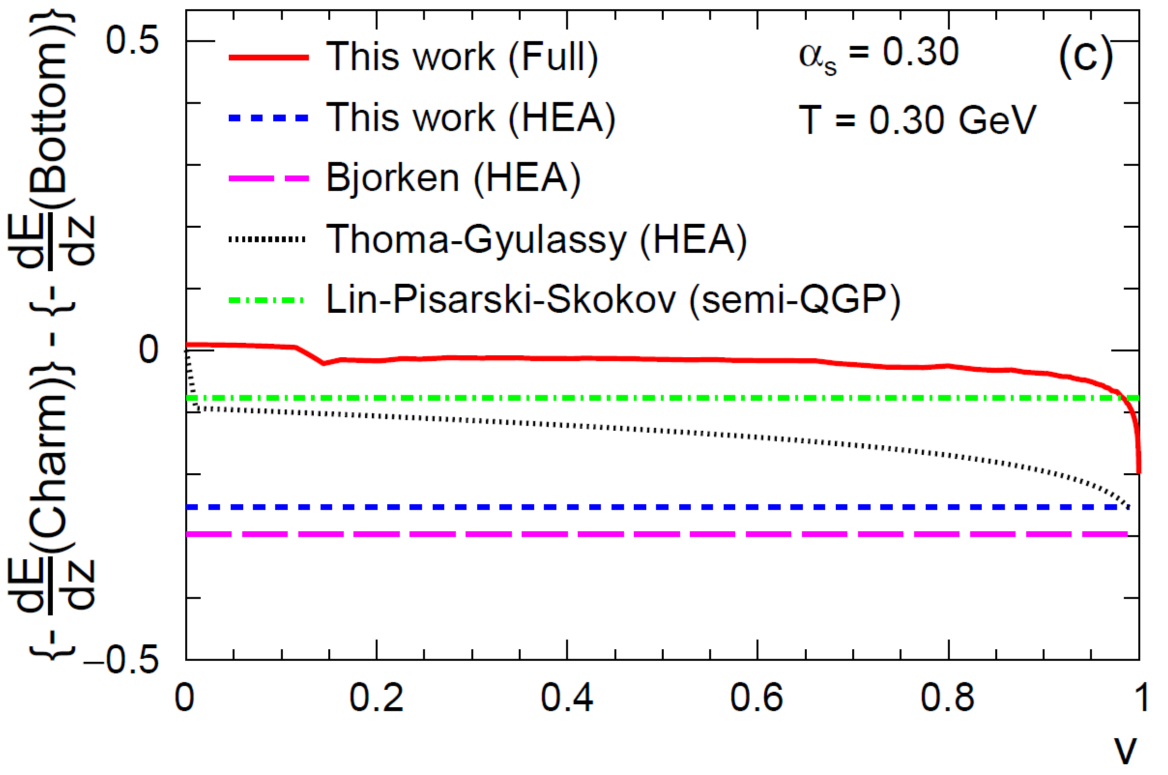
[Thoma and Gyulassy, NPB 1991]

- ***Lin-Pisarski-Skokov*** : incorporate partially confinement effect through purely imaginary background color charge determined by Polyakov loop from lattice studies, leading reduced quark and gluon degrees of freedom

[Lin, Pisarski and Skokov, PLB 2014]

- A common behavior is observed for all the models

Charm vs. Bottom



- For a give velocity, quark with larger mass will lose more its initial energy

Bjorken (HEA)

$$-\frac{dE}{dz} = \frac{4}{3}\pi\alpha_s^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6}\right) \ln \frac{q_{\max}^2}{q_{\min}^2}. \qquad (41)$$

[Bjorken, FERMILAB-Pub-82/59-THY (1982)]

This work (HEA)

$$\left[-\frac{dE}{dz}\right]^{\mathrm{HEA}}_{Qq+Qg} = \frac{4}{3}\pi\alpha_s^2 T^2 \left[\left(1+\frac{N_f}{6}\right)\ln\frac{E_1 T}{m_D^2} + \frac{2}{9}\ln\frac{E_1 T}{m_1^2} + d(N_f)\right],$$

Thoma-Gyulassy (HEA)

$$-\frac{dE}{dz} = \frac{4}{3}\pi\alpha_s^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6}\right) \ln \frac{4T|\vec{p}_1|}{(E_1 - |\vec{p}_1| + 4T)m_D^2},$$

[Thoma and Gyulassy, NPB 351, 491 (1991)]

Lin (HEA)

$$-\frac{dE}{dz} = \pi\alpha_s^2 T^2 \left\{ S^{qk}\frac{N_f(N_c^2-1)}{12N_c}\ln\left(\frac{E_1 T}{m_D^2}\right) + S^{gl}\left[\frac{N_c^2-1}{6}\ln\left(\frac{E_1 T}{m_D^2}\right) + \frac{C_F^2}{6}\ln\left(\frac{E_1 T}{m_1^2}\right)\right]\right\}$$

[Lin, et. al., PLB 730, 236 (2014)]

Transport coefficients

$$\kappa_T(E_1,T)=\frac{C_Fg^2}{16\pi^2v_1^3}\int_{t^*}^{0}dt\,(-t)^{3/2}\int_0^{v_1}dx\,\frac{v_1^2-x^2}{(1-x^2)^{5/2}}\times\left[\rho_L(t,x)+(v_1^2-x^2)\rho_T(t,x)\right]\times coth\left(\frac{x}{2T}\sqrt{\frac{-t}{1-x^2}}\right) \tag{9}$$

$$\kappa_L(E_1,T)=\frac{C_Fg^2}{8\pi^2v_1^3}\int_{t^*}^{0}dt\,(-t)^{3/2}\int_0^{v_1}dx\,\frac{x^2}{(1-x^2)^{5/2}}\times\left[\rho_L(t,x)+(v_1^2-x^2)\rho_T(t,x)\right]\times coth\left(\frac{x}{2T}\sqrt{\frac{-t}{1-x^2}}\right), \tag{10}$$

Soft collisions

$$\kappa_T^{Qi}(E_1,T)=\frac{1}{256\pi^3|\mathbf{p}_1|E_1}\int_{|\mathbf{p}_2|_{min}}^{\infty}d|\mathbf{p}_2|E_2n_2(E_2)\times\int_{-1}^{cos\psi|_{max}}d(cos\psi)\int_{t_{min}}^{t^*}dt\frac{1}{a}\times\left[-\frac{m_1^2(D+2b^2)}{8\mathbf{p}_1^2a^4}+\frac{E_1tb}{2\mathbf{p}_1^2a^2}-t\left(1+\frac{t}{4\mathbf{p}_1^2}\right)\right]\overline{|\mathcal{M}^2|}^{Qi} \tag{13}$$

$$\kappa_L^{Qi}(E_1,T)=\frac{1}{256\pi^3|\mathbf{p}_1|^3E_1}\int_{|\mathbf{p}_2|_{min}}^{\infty}d|\mathbf{p}_2|E_2n_2(E_2)\times\int_{-1}^{cos\psi|_{max}}d(cos\psi)\int_{t_{min}}^{t^*}dt\frac{1}{a}\times\left[\frac{E_1^2(D+2b^2)}{4a^4}-\frac{E_1tb}{a^2}+\frac{t^2}{2}\right]\overline{|\mathcal{M}^2|}^{Qi}. \tag{14}$$

Hard collisions

Notations

- Bernoulli polynomial

$$B_4(x) = x^2(1-x)^2 - 1/30$$

$$B_2(x) = x^2 - x + 1/6.$$

- The projection operator in the double line basis

$$\mathcal{P}^{ab,cd} = \mathcal{P}^{ab}_{dc} = \delta^a_d \delta^b_c - \frac{1}{N}\delta^{ab}\delta_{cd}.$$

Notations

$$\rho_T^{de(\alpha)}(\omega,q) = \frac{\pi\omega\lambda^{de(\alpha)}m_D^2}{2q^3}(q^2-\omega^2)\left\{\left[q^2-\omega^2+\frac{\omega^2\lambda^{de(\alpha)}m_D^2}{2q^2}\right.\right.$$
$$\left.(1+\frac{q^2-\omega^2}{2\omega q}ln\frac{q+\omega}{q-\omega})\right]^2$$
$$\left.+\left[\frac{\pi\omega\lambda^{de(\alpha)}m_D^2}{4q^3}(q^2-\omega^2)\right]^2\right\}^{-1},$$

$$\rho_L^{de(\alpha)}(\omega,q) = \frac{\pi\omega\lambda^{de(\alpha)}m_D^2}{q}\left\{\left[q^2+\lambda^{de(\alpha)}m_D^2(1-\frac{\omega}{2q}ln\frac{q+\omega}{q-\omega})\right]^2\right.$$
$$\left.+\left(\frac{\pi\omega\lambda^{de(\alpha)}m_D^2}{2q}\right)^2\right\}^{-1},$$