藤堂研究室 ToDoGroup

東京大学大学院理学系研究科物理学専攻

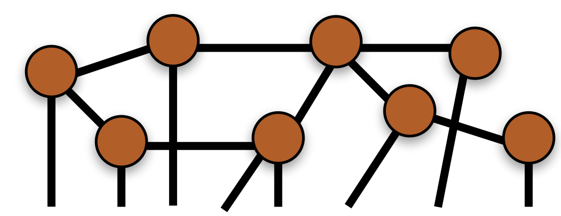
Department of Physics, The University of Tokyo

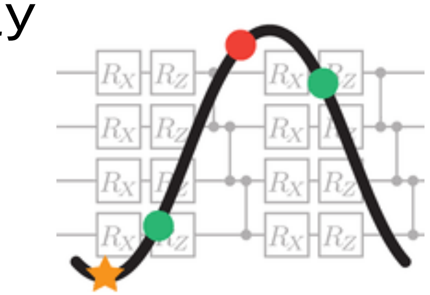
SQAI

Center of Innovation for Sustainable Quantum AI

Tensor Network and Quantum Computation

- Members
  - S. Todo (Prof.), H. Suwa (Assist. Prof.), T. Okubo (Proj. Assoc. Prof.), M. Yamada (Proj. Lecturer), S. Mukherjee/A. Iwaki/T. Sakashita (Proj. Assist. Prof.), 4 Proj. Researchers, 3 Visiting Researchers, 10 Graduate Students
- Research topics
  - Development of simulation methods for strongly correlated systems
  - Novel states, phase transition phenomena, and dynamics in strongly correlated many-body systems
  - Quantum computation algorithms and software
  - Data science and machine learning
  - Open-source software development for next generation parallel simulations





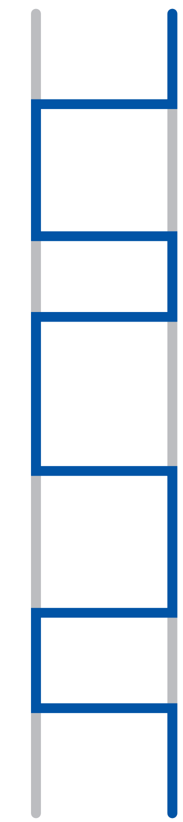
ALPS

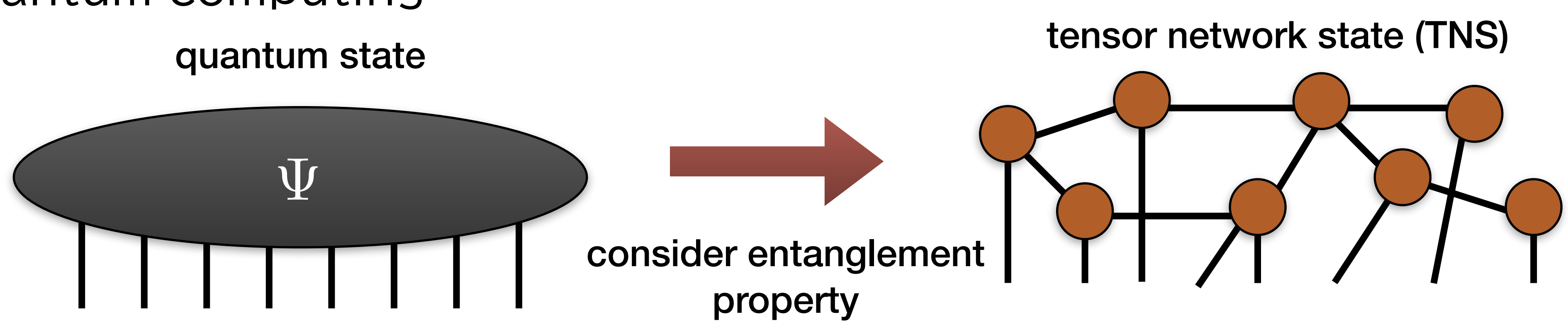
TeNeS

MateriApps

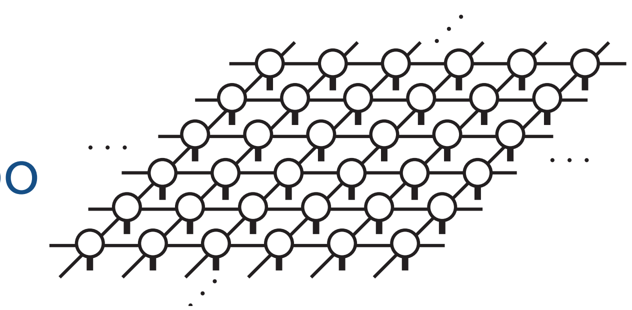
Development of Simulation Methods for Strongly Correlated Systems

- Hilbert space dimension ~ exponentially increasing with system size
  ⇒ Represented in a form that facilitates simulation without losing important physical properties
  - Randomized algorithms - classical and quantum Monte Carlo sampling
    - Representation of quantum fluctuations by imaginary-time path integrals
    - Detailed balance-breaking Markov chain Monte Carlo, event chain Monte Carlo, order-N methods for long-range interacting systems
    - Negative sign problems
      → Keisuke Murota, Sora Shiratani, Hidemaro Suwa
  - Information compression by tensor networks
    - Decompose information into networks by considering "entanglement" of information
    - Tensor network renormalization group method, application to quantum computing

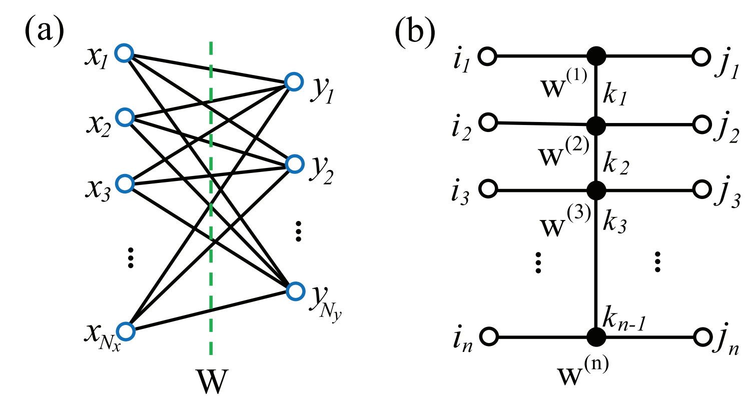




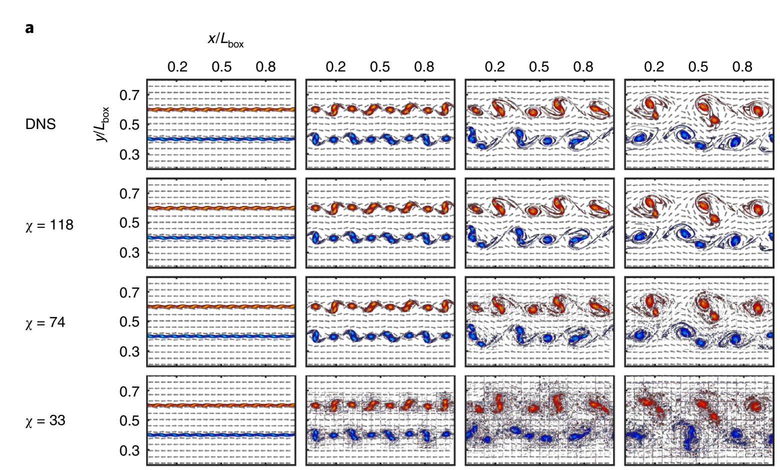
Tensor Network Representation



- Quantum state of quantum many-body systems / 量子多体系の量子状態
  - MPS, Tree TN, MERA, PEPS → Tohru Mashiko, Tsuyoshi Okubo
  - Sampling Complexity of MPS at finite temperature
    → Atsushi Iwaki (poster presentation) PRB 109, 224410 (2024)
- Partition function in statistical physics / 統計力学模型の分配関数
  - TN renormalization / TN繰り込み群
  - Application to lattice field theories
    → Ho Pai Kwok, Shinichiro Akiyama
- Machine learning using TN / テンソルネットワークによる機械学習
  - Compression of neural networks/generative models / ニューラルネットワーク・生成モデルの圧縮
    → collaboration with private companies
- Compression in hierarchical structure / 階層構造の情報圧縮
  - Quantics representation / 同次多項式表現
  - TN simulation of PDE / 偏微分方程式のTNシミュレーション
    → Rihito Sakurai



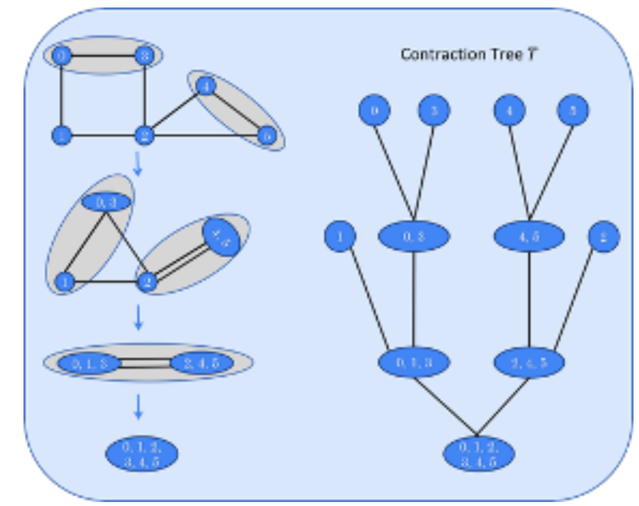
Gao et al (2020)

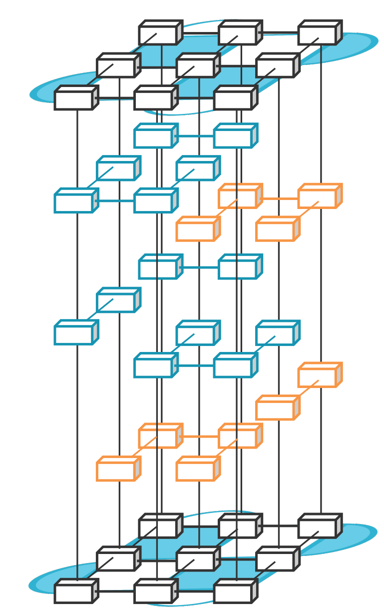


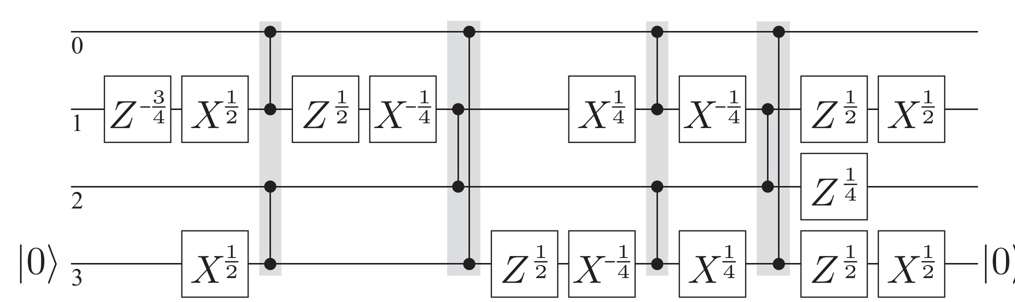
Gourianov et al (2022)

Quantum Embedding: from QC to TN

- Conversion from QC to TN is straightforward / 量子回路からTNへの変換は容易
  - High-performance TN simulator / 高性能なTNシミュレーター
    - Optimization of TN contraction order based on graph theory / グラフ理論によるTN縮約順序の最適化
      → Sayan Mukherjee
    - Combination with MCMC sampling / MCMCサンプリングとの組み合わせ → ST, Hidemaro Suwa, Sora Shiratani
  - Quantum error correction / 量子誤り訂正
    - Noise model prediction using TN decoders and Bayesian estimation / TNデコーダーとベイズ推定によるノイズモデル予測
      → Takumi Kobori (arXiv:2406.08981)
  - Decomposition of multi-controlled gates / マルチコントロールゲートの分解 → Ken Nakanishi (poster presentation, PRA 110, 012604 (2024))



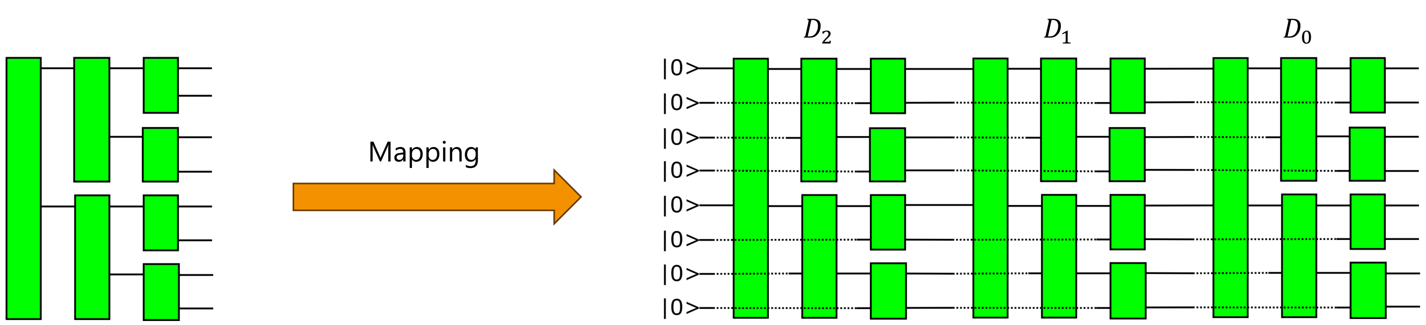


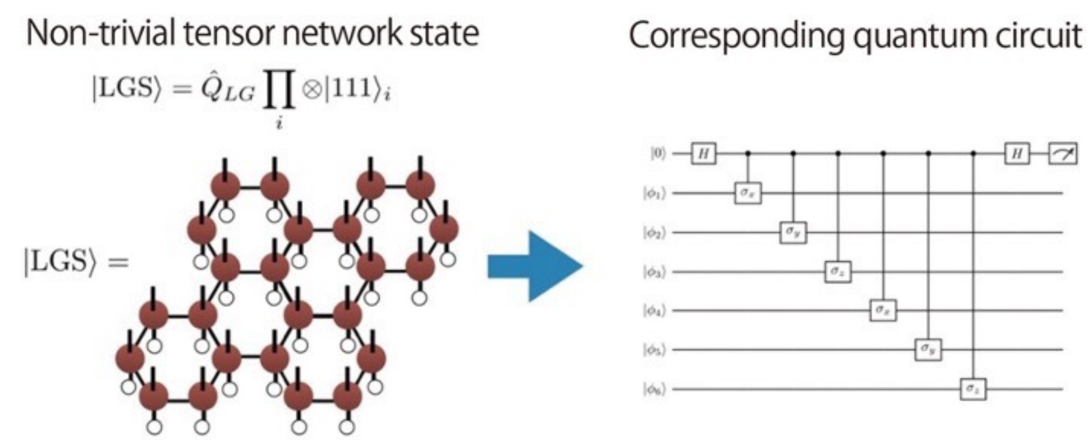


- Conversion from “Quantum State” to TN is not straightforward / 「量子状態」からTNへの変換は非自明

Quantum Embedding: from TN to QC

- **Conversion from TN to QC is not straightforward / TNから量子回路への変換は非自明**
  - Matrix product state to QC / 行列積状態から量子回路
    - Quantum state preparation / 量子状態の準備
    - Generalization for Tree Tensor Network / ツリーテンソルネットワークへの一般化
      → Shota Sugawara (poster presentation)





  - Transformation from tensor networks including loops / ループを含むテンソルネットワークからの変換 → Tsuyoshi Okubo

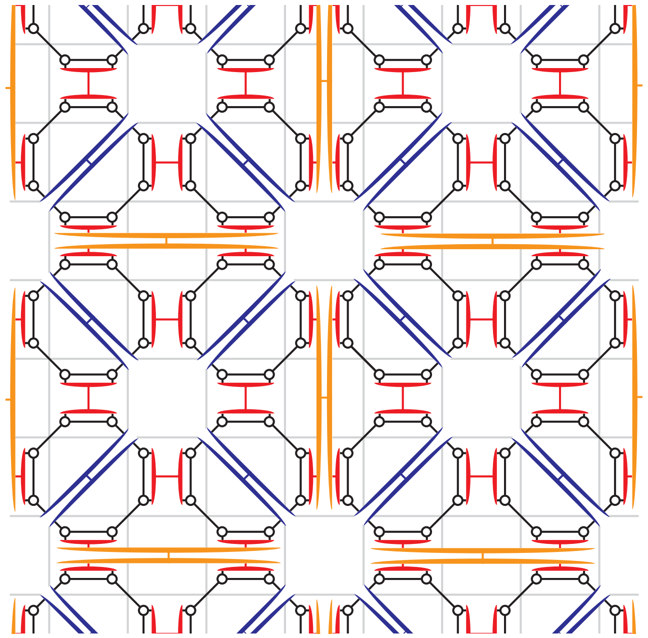
- **Embedding of real/imaginary-time evolution / 実時間・虚時間発展の埋め込み**
  - Quantum Simulation by Quantum Singular Value Transformation
    → Tokinori Oe
  - Tensor network quantum simulation on QC / QC上でのTN量子シミュレーション

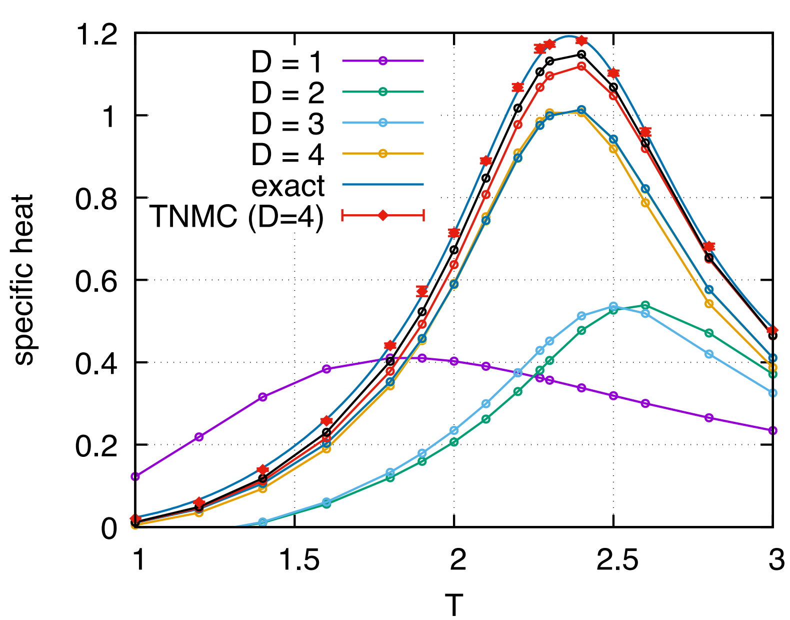
Tensor Network Monte Carlo

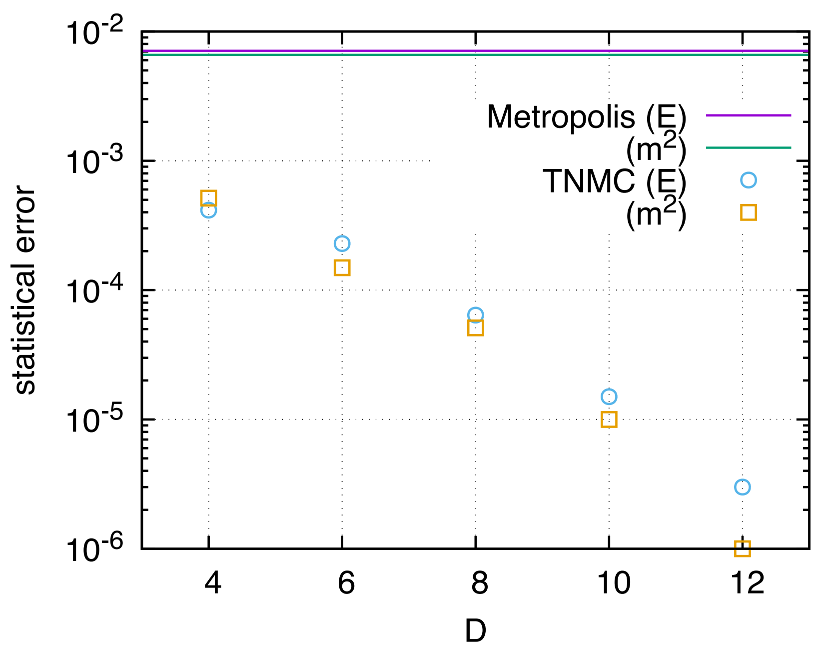
ST in preparation

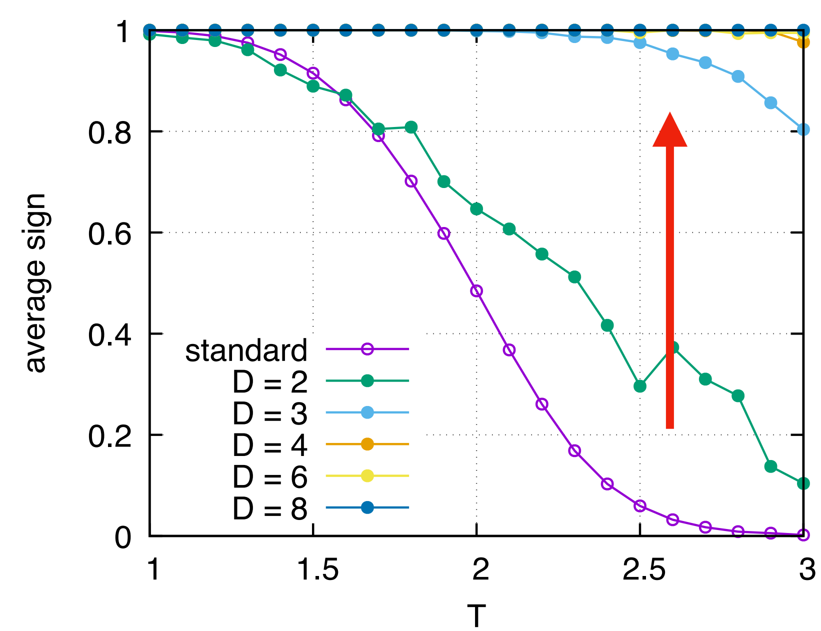
- Many physical problems can be formulated in terms of tensor network
  - classical lattice models (Ising model, etc)
  - quantum lattice models (via Suzuki-Trotter decomposition, etc)
  - quantum circuits, real-time unitary evolution
- Exact contraction is not possible in higher dimensions
  - low-rank approximation based on SVD
- Markov-chain Monte Carlo sampling of “projectors”
  - can remove systematic bias of low-rank approximation
  - can reduce statistical error by MCMC
    - exponential speed up of MCMC
  - can solve “negative sign problem”

$$\sigma^2 = \frac{\sigma_0^2(1+2\tau_{\rm int})}{M}$$





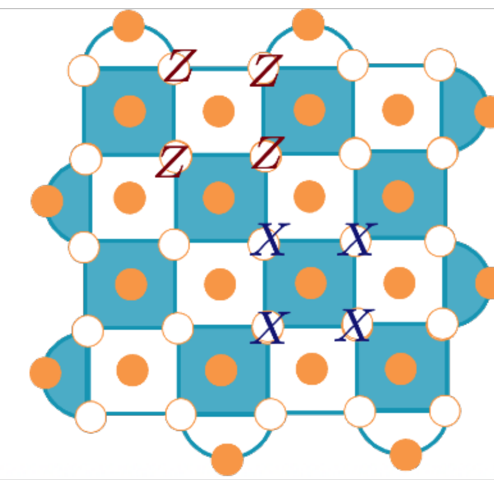


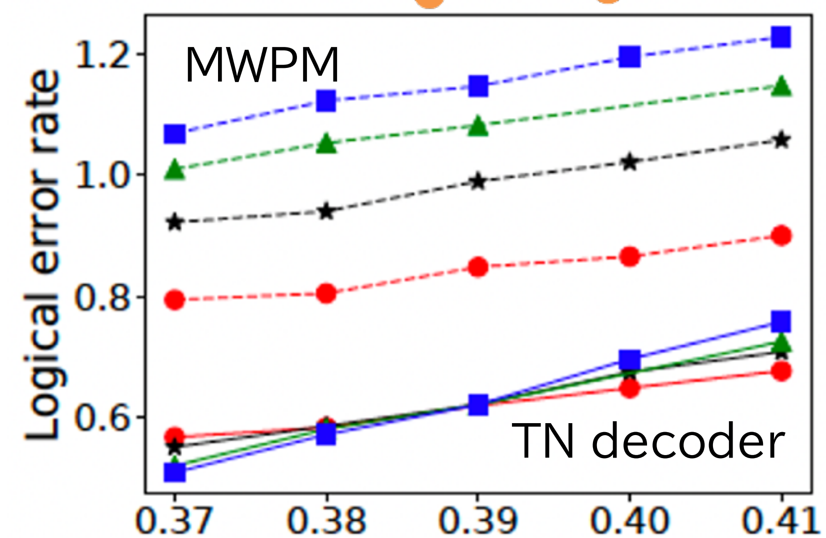


Tensor Network Decoder for Error Correction

- Decoding in Stabilizer Code (e.g., surface code)
  - Using information from noise models can improves performance of TN decoder
- How to determine / infer the noise model?
  - Is it possible to extract from syndrome measurement results?

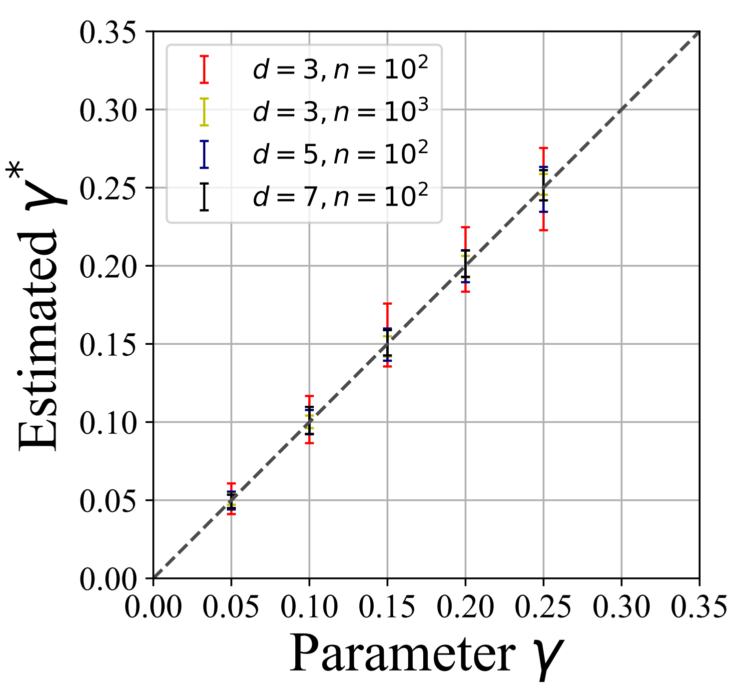
- Syndrome measurement + TN likelihood calculation
  + Bayesian inference + Monte Carlo sampling
  → Quantum Noise Estimation → Decoder



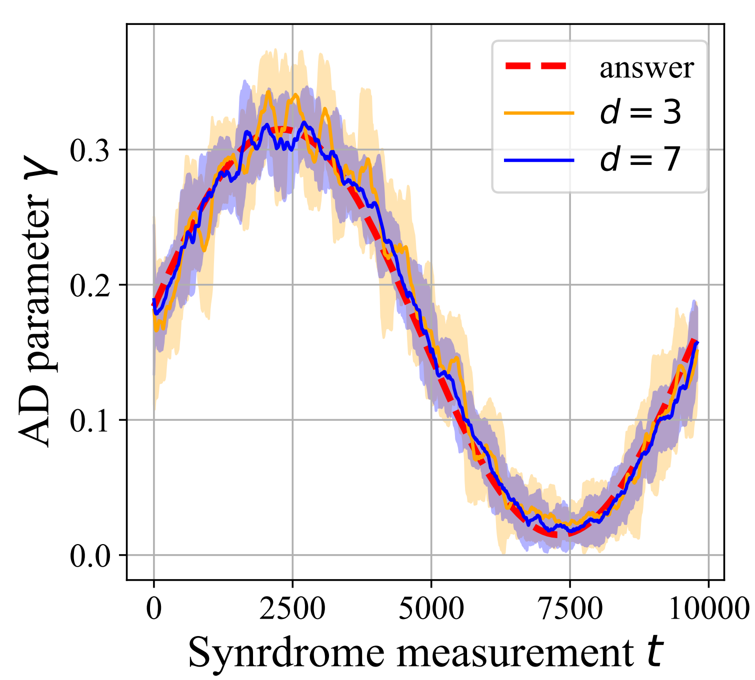


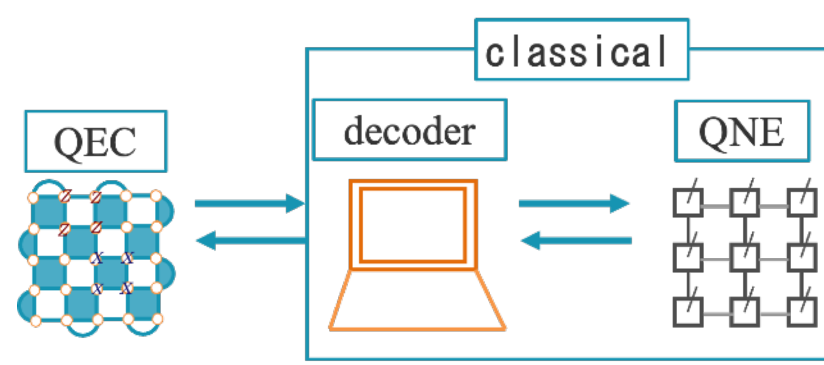
Darmawan, Poulin (2018)

static noise model case



time-dependent case





Takumi Kobori and ST, arXiv:2406.08981