QCD analysis of $\Delta S = 0$ Hadronic Parity Violation Through Next-to-Leading Order

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- G. Muralidhara, S.Gardner, Phys. Lett. B. Volume 849 (2024)

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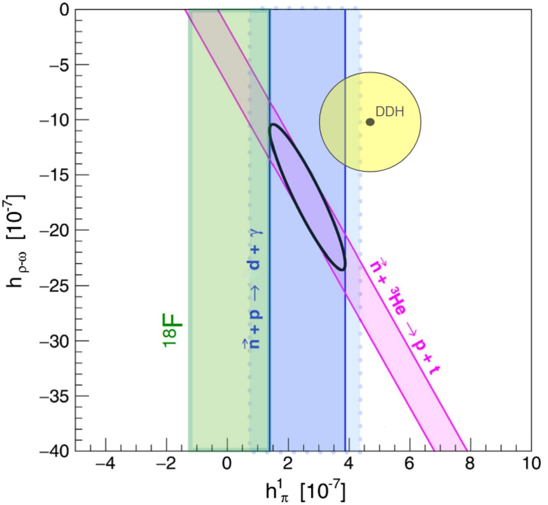
Need for $\Delta F = 0$ Effective Weak Hamiltonian

- *Weak processes* at low-energies are complex since interactions between quarks receive corrections due to QCD gluon loops.
- Flavor physics, $\Delta F = 1$ decay studies, have been extensively conducted[1], and NLO has been the standard since 1990s.
- But the status of $\Delta F = 0$ physics is *completely different*. Such rigorous studies have been limited.
- These studies become important in the context of Hadronic Parity Violation (HPV), especially in the light of new experimental results [2] [3] which demand better theoretical estimations and comparison of different isosectors.
- Additionally, $\Delta F = 0$ effective Hamiltonian is important in the studies of flavor neutral mesons like η, η' decays.

[1] Buras et.al., Rev. Mod. Phys., Vol. 68, 1996
[2] Blyth et al., 2018
[3] M. T. Gericke et al., 2020

Hadronic Parity Violation

Combined analysis of the $\vec{n}p \to d\gamma$ and $\vec{n}\,^3\mathrm{He} \to p\,^3\mathrm{H}$ experiments[1] [2] and DDH [3]



[1] Blyth et al., 2018

[2] M. T. Gericke et al., 2020

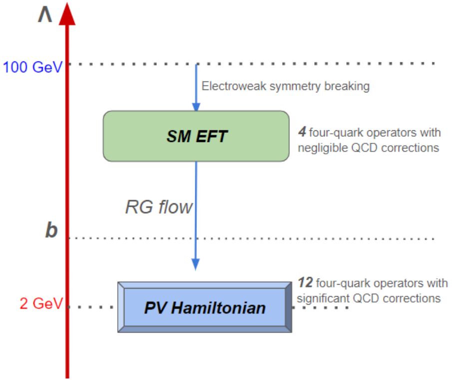
[3] Desplanques, Donoghue, and Holstein (DDH), 1980

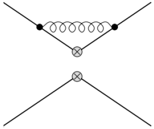
Our Work

- Performing RG analysis, we construct the the effective $\Delta F = 0$ PV Hamiltonian at $2\,\mathrm{GeV}$

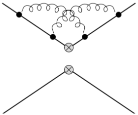
- We include both LO and NLO QCD corrections to weak interactions of quarks.

- Our analysis is comprehensive with all the three isosectors of HPV.





1 loop, LO



2 loop, NLO

HPV Operators

The Z^0-channel operator set that is closed under mixing,

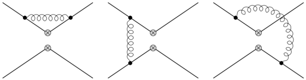
$$\Theta_1 = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$
$$\Theta_2 = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\alpha}$$
$$\Theta_3 = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\beta\beta}$$
$$\Theta_4 = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\beta\alpha}$$
$$\Theta_5 = [(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$
$$\Theta_6 = [(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\alpha}$$
$$\Theta_7 = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\beta}$$
$$\Theta_8 = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\alpha}$$

The operators from the W-channel form a separate closed group,

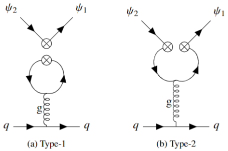
$$\Theta_9 = (\bar{u}d)_V^{\alpha\alpha}(\bar{d}u)_A^{\beta\beta} + (\bar{d}u)_V^{\alpha\alpha}(\bar{u}d)_A^{\beta\beta}$$
$$\Theta_{10} = (\bar{u}d)_V^{\alpha\beta}(\bar{d}u)_A^{\beta\alpha} + (\bar{d}u)_V^{\alpha\beta}(\bar{u}d)_A^{\beta\alpha}$$
$$\Theta_{11} = (\bar{u}s)_V^{\alpha\alpha}(\bar{s}u)_A^{\beta\beta} + (\bar{s}u)_V^{\alpha\alpha}(\bar{u}s)_A^{\beta\beta}$$
$$\Theta_{12} = (\bar{u}s)_V^{\alpha\beta}(\bar{s}u)_A^{\beta\alpha} + (\bar{s}u)_V^{\alpha\beta}(\bar{u}s)_A^{\beta\alpha}$$

HPV Operator Mixing and ADM

Operators renormalize and mix even under LO QCD corrections. Inserting
$\Theta_1 = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$



LO current-current corrections



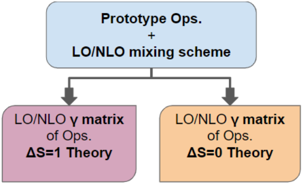
LO Penguin corrections

$$\Theta_1 \to \Theta_1 + \frac{g^2\Gamma(\frac{\varepsilon}{2})}{(4\pi)^2\mu^{\varepsilon}}\left(\frac{2}{9}\Theta_1 - \frac{2}{3}\Theta_2 + 1\Theta_3 - 3\Theta_4\right)$$

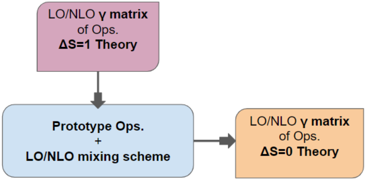
The mixing can be characterized by ***Anomalous Dimension Matrix (ADM)***

A Rule-book for constructing γ^{HPV}

- QCD corrections are flavor blind up to quark mass effects, a general *prototype operator set* and corresponding *ADM* should exist[4].



- With adequate information on mixing in a theory, can the ADM for prototype operators be *re*-constructed? *Yes!*



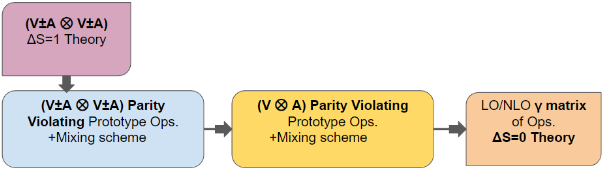
[4] Miller, McKellar, Phys. Rev. D 28 (4) (1983)

One small detail...

- NLO corrections to the *6 operators* ($\mathbf{O}_1 \ldots \mathbf{O}_6$) of $\Delta S = 1$ *Kaon-physics* have been calculated in detail [5].

- The flavor physics $\Delta S = 1$ operators are *chiral* in nature. i.e. bilinears with $(V \pm A) \otimes (V \pm A)$ and $(V \pm A) \otimes (V \mp A)$ structures.

- We need to translate this information, suitable to our explicitly parity violating operators i.e. bilinears with $(V \otimes A)$ like structures.



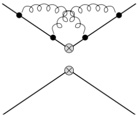
[5] Buras et. al. Nucl.Phys.B 400 (1993)

Current-Current Basis

Renormalization of the $\Delta S = 1$ physics operators:

$$\mathbf{O_1} = (\bar{s}u)^{\alpha\alpha}_{V-A}(\bar{u}d)^{\beta\beta}_{V-A} \quad \mathbf{O_2} = (\bar{s}u)^{\alpha\beta}_{V-A}(\bar{u}d)^{\beta\alpha}_{V-A}$$
$$\mathbf{O_5} = (\bar{s}d)^{\alpha\alpha}_{V-A}\Sigma^f_q(\bar{q}q)^{\beta\beta}_{V+A} \quad \mathbf{O_6} = (\bar{s}d)^{\alpha\beta}_{V-A}\Sigma^f_q(\bar{q}q)^{\beta\alpha}_{V+A}$$

due to insertions into diagrams such as:



give the information of the mixing of prototype basis:

$$\Phi_1 = (\psi_1\psi_2)^{\alpha\alpha}_V(\psi_3\psi_4)^{\beta\beta}_A$$
$$\Phi_2 = (\psi_1\psi_2)^{\alpha\beta}_V(\psi_3\psi_4)^{\beta\alpha}_A$$
$$\Phi_3 = (\psi_1\psi_2)^{\alpha\alpha}_A(\psi_3\psi_4)^{\beta\beta}_V$$
$$\Phi_4 = (\psi_1\psi_2)^{\alpha\beta}_A(\psi_3\psi_4)^{\beta\alpha}_V$$

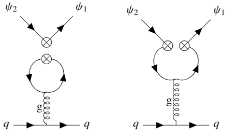
$$C_{NLO} = \left(\frac{\alpha_s}{4\pi}\right)^2 \begin{pmatrix} \frac{1279}{12} - \frac{20f}{3} & \frac{17}{4} - \frac{4f}{3} & \frac{2f}{9} - \frac{173}{12} & \frac{173}{4} - \frac{2f}{3} \\ \frac{95}{2} - \frac{5f}{3} & \frac{149}{6} - \frac{17f}{3} & -\frac{f}{3} & \frac{202}{3} - \frac{7f}{9} \\ \frac{2f}{9} - \frac{173}{12} & \frac{173}{4} - \frac{2f}{3} & \frac{1279}{12} - \frac{20f}{3} & \frac{17}{4} - \frac{4f}{3} \\ -\frac{f}{3} & \frac{202}{3} - \frac{7f}{9} & \frac{95}{2} - \frac{5f}{3} & \frac{149}{6} - \frac{17f}{3} \end{pmatrix}$$

Penguin Basis

Renormalization of the $\Delta S = 1$ operators:

$$\mathbf{O_3}=(\bar{s}d)^{\alpha\alpha}_{V-A}\Sigma^f_q(\bar{q}q)^{\beta\beta}_{V-A} \quad \mathbf{O_4}=(\bar{s}d)^{\alpha\beta}_{V-A}\Sigma^f_q(\bar{q}q)^{\beta\alpha}_{V-A}$$
$$\mathbf{O_5}=(\bar{s}d)^{\alpha\alpha}_{V-A}\Sigma^f_q(\bar{q}q)^{\beta\beta}_{V+A} \quad \mathbf{O_6}=(\bar{s}d)^{\alpha\beta}_{V-A}\Sigma^f_q(\bar{q}q)^{\beta\alpha}_{V+A}$$

From insertions in 2-loop versions of P1 and P2 such as:



(a) Type-1 (b) Type-2

give the information of the mixing of prototype basis:

$$\Phi_5=(\bar{\psi}_1\psi_2)^{\alpha\alpha}_A\Sigma^f_q(\bar{q}q)^{\beta\beta}_V \quad \Phi_6=(\bar{\psi}_1\psi_2)^{\alpha\beta}_A\Sigma^f_q(\bar{q}q)^{\beta\alpha}_V$$
$$\Phi_7=(\bar{\psi}_1\psi_2)^{\alpha\alpha}_V\Sigma^f_q(\bar{q}q)^{\beta\beta}_A \quad \Phi_8=(\bar{\psi}_1\psi_2)^{\alpha\beta}_V\Sigma^f_q(\bar{q}q)^{\beta\alpha}_A$$

$$\mathscr{P}_1=\left(\frac{\alpha_s}{4\pi}\right)^2\begin{pmatrix} -\frac{4}{9} & \frac{4}{3} & 0 & 0 \\ -\frac{1565}{243} & \frac{1277}{81} & -\frac{11}{27} & -\frac{5}{9} \\ -\frac{2}{9} & \frac{2}{3} & 16 & 0 \\ -\frac{7}{27} & \frac{7}{9} & \frac{11}{3} & 5 \end{pmatrix}; \qquad \mathscr{P}_2=\left(\frac{\alpha_s}{4\pi}\right)^2\begin{pmatrix} -\frac{814}{243} & \frac{670}{81} & \frac{44}{27} & \frac{20}{9} \\ -\frac{1}{3} & 1 & 8 & 0 \\ -\frac{814}{243} & \frac{670}{81} & \frac{44}{27} & \frac{20}{9} \\ -\frac{1}{3} & 1 & 8 & 0 \end{pmatrix}$$

Running through energy thresholds

So far, we have been working with only **u, d** and **s** quarks. But, the number of active quarks(f) varies at different energy thresholds.

$\mu \leq$	f	quarks
M_w	5	u,d,s,c,b
M_b	4	u,d,s,c
M_c	3	u,d,s

The extension to include heavier quarks is made possible by the structure shared by *u-like* quarks and *d-like quarks*. For eg.

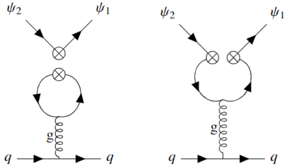
$$\Theta_1(2\,\mathrm{GeV}) = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$

$$\Theta_1(\mathrm{M_w}) = [(\bar{u}u)_V + (\bar{c}c)_V + (\bar{d}d)_V + (\bar{s}s)_V + (\bar{b}b)_V]^{\alpha\alpha}[(\bar{u}u)_A + (\bar{c}c)_A - (\bar{d}d)_A - (\bar{s}s)_A - (\bar{b}b)_A]^{\beta\beta}$$

HPV operator corrections

- Consider the penguin corrections to $\Theta_1 = [(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\beta}$.
- Type-1 penguins with possible $\bar{\psi}_1\psi_2(\bar{\psi}_3\psi_3)$ fermion operator insertions versus type-2 penguins with possible $\bar{\psi}_1(\psi_3\bar{\psi}_3)\psi_2$ insertions. E.g. $(\bar{u}u)_A(\bar{d}d)_V$

$$(\bar{u}u)_A(\bar{d}d)_V \xrightarrow{Type-1} (\bar{u}u)_A\sum_q^f(\bar{q}q)_V \implies (\bar{u}u)_A\sum_q^f(\bar{q}q)_V \xrightarrow{Type-1} f(\mathscr{P}_1)_{11}(\bar{u}u)_A\sum_q^f(\bar{q}q)_V$$



(a) Type-1 (b) Type-2

$$[\gamma(\Theta_1)]_{1j} \xrightarrow[corrections]{penguin} (f(\mathscr{P}_1)_{11} + 2(\mathscr{P}_2)_{11})[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\alpha\alpha}\sum_q^{N_f}(\bar{q}q)_V^{\beta\beta}$$

$$+ \ldots\ldots$$

$$+(-q)((\mathscr{P}_1)_{32} + (\mathscr{P}_1)_{34})[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta}\sum_q^{N_f}(\bar{q}q)_A^{\beta\alpha}$$

$$+ \ldots\ldots$$

$$= \left(\sum_{i=1}^{4}(f(\mathscr{P}_1)_{1i} + 2(\mathscr{P}_2)_{1i})\Theta_i\right) - q((\mathscr{P}_1)_{31} + (\mathscr{P}_1)_{33})\Theta_7 - q((\mathscr{P}_1)_{32} + (\mathscr{P}_1)_{34})\Theta_8$$

where $q = \#d - \#u$, difference in the no. of d-like and u-like dynamical quarks.

HPV NLO anomalous dimension matrix

- Use prototype basis $\vec{\Phi}_{cc}$ and corresponding ADM C_{NLO} on operators of HPV: $\gamma^{HPV}_{cc,NLO}$

- Use prototype basis $\vec{\Phi}_{p}$ and mixing schemes $\mathscr{P}_1$ and $\mathscr{P}_2$ on operators of HPV: $\gamma^{HPV}_{penguin,NLO}$

- The NLO mixing matrix for the Z-sector: $\gamma^{HPV}_{cc,NLO} + \gamma^{HPV}_{penguin,NLO} = \gamma^{HPV}_{NLO} =$

$$\frac{\alpha_s^2}{(4\pi)^2}\begin{pmatrix}
\frac{97087}{972}-\frac{64f}{9} & \frac{6737}{324} & \frac{2f}{9}-\frac{1205}{108} & \frac{1717}{36}-\frac{2f}{3} & 0 & 0 & -\frac{142q}{9} & -\frac{2q}{3} \\
\frac{281}{6}-\frac{1970f}{243} & \frac{818f}{81}+\frac{161}{6} & 16-\frac{20f}{27} & \frac{202}{3}-\frac{4f}{3} & 0 & 0 & -\frac{92q}{27} & -\frac{52q}{9} \\
-\frac{20525}{972} & \frac{19373}{324} & \frac{28f}{3}+\frac{11863}{108} & \frac{313}{36}-\frac{4f}{3} & 0 & 0 & \frac{4q}{9} & -\frac{4q}{3} \\
-\frac{16f+18}{27} & \frac{208}{3} & 2f+\frac{127}{2} & \frac{149-4f}{6} & 0 & 0 & \frac{1664q}{243} & -\frac{1232q}{81} \\
\frac{2q}{3} & -2q & -16q & 0 & \frac{553}{6}-\frac{58f}{9} & \frac{95}{2}-2f & -\frac{836}{243} & \frac{1700}{81} \\
\frac{1628q}{243} & -\frac{1340q}{81} & -\frac{88q}{27} & -\frac{40q}{9} & \frac{95}{2}-2f & \frac{553}{6}-\frac{58f}{9} & \frac{46}{3} & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{80f}{9}+\frac{43121}{486} & \frac{11095}{162} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{377}{6}-\frac{1322f}{243} & \frac{1178f}{81}+\frac{565}{6}
\end{pmatrix}$$

HPV Isosectors

- HPV operators are a blend of isospin $I = 0, 1, 2$ four-quark structures and can be separated into *isovector* and *isoscalar* $\oplus$ *isotensor* sub-operators.
- This means, the study of the HPV can be divided into the study of Hamiltonians: $\mathscr{H}^{I=1}$ and $\mathscr{H}^{I=0\oplus 2}$. A useful distinction in estimating the different meson-nucleon couplings later.

$$\mathscr{H}^{I=1}_{\rm eff}(\mu) = \frac{G_F s_W^2}{3\sqrt{2}} \sum_{i=1}^{10} C_i^{I=1}(\mu)\Theta_i^{I=1} \qquad \mathscr{H}^{I=0\oplus 2}_{\rm eff}(\mu) = \frac{G_F s_W^2}{3\sqrt{2}} \sum_{i=1}^{10} C_i^{I=0\oplus 2}(\mu)\Theta_i^{I=0\oplus 2}$$

$$\begin{aligned}
\Theta_1{}^{I=1} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta} \\
\Theta_2{}^{I=1} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha} \\
\Theta_3{}^{I=1} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\beta} \\
\Theta_4{}^{I=1} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\alpha} \\
\Theta_5{}^{I=1} &= (\bar{s}s)_V^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta} \\
\Theta_6{}^{I=1} &= (\bar{s}s)_V^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha} \\
\Theta_7{}^{I=1} &= (\bar{s}s)_A^{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\beta} \\
\Theta_8{}^{I=1} &= (\bar{s}s)_A^{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\alpha} \\
\Theta_9{}^{I=1} &= (\bar{u}s)_V^{\alpha\alpha}(\bar{s}u)_A^{\beta\beta} + (\bar{s}u)_V^{\alpha\alpha}(\bar{u}s)_A^{\beta\beta} \\
\Theta_{10}{}^{I=1} &= (\bar{u}s)_V^{\alpha\beta}(\bar{s}u)_A^{\beta\alpha} + (\bar{s}u)_V^{\alpha\beta}(\bar{u}s)_A^{\beta\alpha}
\end{aligned}$$

$$\begin{aligned}
\Theta_1{}^{I=0\oplus 2} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{s}s)_A]^{\beta\beta} \\
\Theta_2{}^{I=0\oplus 2} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta}[(\bar{s}s)_A]^{\beta\alpha} \\
\Theta_3{}^{I=0\oplus 2} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{s}s)_V]^{\beta\beta} \\
\Theta_4{}^{I=0\oplus 2} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{s}s)_V]^{\beta\alpha} \\
\Theta_5{}^{I=0\oplus 2} &= [(\bar{u}u)_V - (\bar{d}d)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta} + (\bar{s}s)_V^{\alpha\alpha}(\bar{s}s)_A^{\beta\beta} \\
\Theta_6{}^{I=0\oplus 2} &= [(\bar{u}u)_V - (\bar{d}d)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha} + (\bar{s}s)_V^{\alpha\beta}(\bar{s}s)_A^{\beta\alpha} \\
\Theta_7{}^{I=0\oplus 2} &= [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\beta\beta} \\
\Theta_8{}^{I=0\oplus 2} &= [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\alpha} \\
\Theta_9{}^{I=0\oplus 2} &= (\bar{u}d)_V^{\alpha\alpha}(\bar{d}u)_A^{\beta\beta} + (\bar{d}u)_V^{\alpha\alpha}(\bar{u}d)_A^{\beta\beta} \\
\Theta_{10}{}^{I=0\oplus 2} &= (\bar{u}d)_V^{\alpha\beta}(\bar{d}u)_A^{\beta\alpha} + (\bar{d}u)_V^{\alpha\beta}(\bar{u}d)_A^{\beta\alpha}
\end{aligned}$$

- $\gamma_{NLO}^{I=1}$ and $\gamma_{NLO}^{I=0\oplus 2}$, 10×10 matrices, are calculated following the procedures discussed before.

RG Flow

- The WCs, C_i flow between energies, $\vec{C}^I(\mu) = U(\mu, m, \gamma^I_{LO}, \gamma^I_{NLO})\vec{C}^I(m)$. For example, LO flow of WCs between $\mathrm{M_w}$ scale and 2 GeV,

$$\vec{C}^I(2\,\mathrm{GeV}) = \exp\left[\int_{\mathrm{g_s(M_w)}}^{\mathrm{g_s}(\mu)} \mathrm{dg_s} \frac{\gamma^{I\ T}_{LO}(\mu)}{\beta(\mathrm{g_s})}\right] \tilde{C}^I(\mathrm{M_w})$$

- The form of NLO-evolution operator involving both γ^I_{LO} and γ^I_{NLO} is a bit more complicated.
- *A caution*! strong coupling parameter α_s is discontinuous with changing number of dynamical quarks and needs to be accounted for.

$$\vec{C}^{I=1} = \begin{pmatrix} \underline{M_W} \\ 1 \\ 0 \\ 0 \\ 0 \\ 3.49 \\ 0 \\ 3.49 \\ 0 \\ -13 \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} \underline{2\,GeV} & \underline{[1.5\,GeV - 4\,GeV]} \\ 0.955 & [0.947...0.964] \\ -0.009 & [-0.007\cdots-0.007] \\ 0.107 & [0.139...0.053] \\ -0.377 & [-0.446\cdots-0.253] \\ 3.83 & [3.98...3.72] \\ -1.28 & [-1.49\cdots-0.906] \\ 3.83 & [3.89...3.72] \\ -1.28 & [-1.49\cdots-0.906] \\ -16.2 & [-16.1\cdots-15.0] \\ 6.38 & [6.22...4.30] \end{pmatrix} \qquad \vec{C}^{I=0\oplus 2} = \begin{pmatrix} \underline{M_W} \\ -1 \\ 0 \\ 0 \\ 0 \\ -3.49 \\ 0 \\ 0 \\ 0 \\ -13 \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} \underline{2\,GeV} & \underline{[1.5\,GeV - 4\,GeV]} \\ -0.976 & [-0.966\cdots-0.978] \\ -0.029 & [0.011...0.007] \\ -0.091 & [-0.192\cdots-0.106] \\ 0.446 & [0.488...0.286] \\ -3.83 & [-3.97\cdots-3.75] \\ 1.28 & [1.36...0.864] \\ -0.129 & [-0.080\cdots-0.050] \\ 0.303 & [0.438...0.226] \\ -16.2 & [-16.1\cdots-15.0] \\ 6.38 & [6.22...4.30] \end{pmatrix}$$

- Last two entries of $\vec{C}^{I=1}$ should be multiplied by a factor $\sin^2\theta_c$ and that of $\vec{C}^{I=0\oplus 2}$ by $\cos^2\theta_c$.

Meson-Nucleon couplings: h_M^I

- DDH introduced HPV via meson nucleon coupling terms in their effective Hamiltonian ($\mathscr{H}_{\text{DDH}}$): h_π^1, $h_\rho^{0,1,2}$, $h_\omega^{0,1}$.

- $\langle MN'|\mathscr{H}_{\text{eff}}^I|N\rangle = \langle MN'|\mathscr{H}_{\text{DDH}}|N\rangle$

- As an eg., for pion : $\mathscr{H}_{\text{DDH}}^\pi = ih_\pi^1(\pi^+\bar{p}n - \pi^-\bar{n}p)$.

- Pion is a pseudoscalar meson. Fierz $V \otimes A$ operators to $S \otimes P$ and make use of *factorization approximation* to evaluate these matrix elements.

$$\langle \pi^+ n|\mathscr{H}_{eff}^{I=1}|p\rangle = -ih_\pi^1 \bar{u}_n u_p = \frac{G_F s_w^2}{3\sqrt{2}} \langle \pi^+|(\bar{u}\gamma_5 d)|0\rangle \left(\frac{2C_1^{I=1}}{3} + 2C_2^{I=1} - \frac{2C_3^{I=1}}{3} + 2C_4^{I=1}\right) \langle n|\bar{d}u|p\rangle$$

- LQCD result for quark scalar charge of the nucleon[1]

$$\langle n|\bar{d}u|p\rangle = g_s^{u-d}\bar{u}_n u_p; \quad g_s^{u-d} = 1.022(80)(60)$$

[1] g_s: ($N_f = 2+1+1$) [Gupta et al., Phys. Rev. D 98 (2018)]

Couplings

$$h^1_\pi = 2.14{\pm}0.21{+}\binom{+0.17}{-0.34} \times 10^{-7} \text{ (LO result: } 3.06{\pm}0.34{+}\binom{+1.29}{-0.64}{\times}10^{-7}\text{)}$$

$$\text{(NPDGamma : } 2.6(1.2)_{stat}(0.2)_{sys} \times 10^{-7}\text{)}$$

$$h^0_\omega = 0.277{\pm}0.014{+}\binom{+0.008}{-0.34}{\times}10^{-7} \quad \text{(LO: } 0.270{\times}10^{-7}\text{)}$$

$$h^1_\omega = 1.58{\pm}0.10{+}\binom{+0.01}{-0.02}{\times}10^{-7} \quad \text{(LO: } 1.82{\times}10^{-7}\text{)}$$

$$h^1_\rho = -0.275{\pm}0.040{+}\binom{+0.006}{-0.002}{\times}10^{-7} \quad \text{(LO:} -0.294{\times}10^{-7}\text{)}$$

$$h^0_\rho = -10.6{\pm}0.6{+}\binom{+0.02}{+0.9}{\times}10^{-7} \quad \text{(LO: } -11.0{\times}10^{-7}\text{)}$$

$$h^2_\rho = 9.27{\pm}0.67{+}\binom{-0.41}{+0.85}{\times}10^{-7} \quad \text{(LO: } 8.57{\times}10^{-7}\text{)}$$

An experimentally measurable parameter in $\vec{n}\,{}^3\mathrm{He} \to p\,{}^3\mathrm{H}$:

$$h_{\rho-\omega} \equiv h^0_\rho + 0.605 h^0_\omega - 0.605 h^1_\rho - 1.316 h^1_\omega - 0.026 h^2_\rho$$

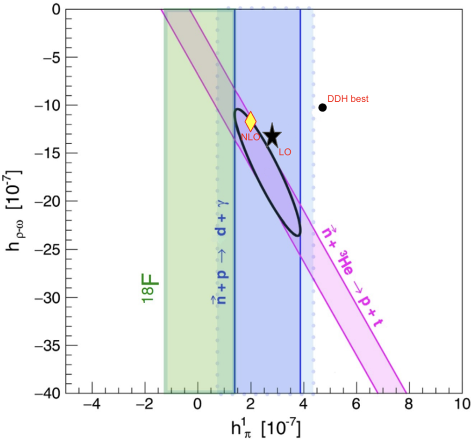
$$h_{\rho-\omega} = -12.2{\pm}0.62{+}\binom{0.07}{0.74}{\times}10^{-7} \quad \text{(LO:} -12.9{\pm}0.52{+}\binom{0.97}{-1.9}{\times}10^{-7}\text{)}$$

$$\vec{n}\,{}^3He : h_{\rho-\omega} = (-17.0 \pm 6.56) \times 10^{-7}$$

Observations: Both h_π and $h_{\rho-\omega}$ within $\pm 1\sigma$ of experiment. NLO values tend to be smaller compared to the LO results. The WCs come with scale dependence. The signature of scale ambiguities reflected in the variation of meson-nucleon couplings between 1.5 GeV – 4 GeV, is *reduced*, and more stable about 2 GeV.

A broader impact of our work

Combined analysis of the $\vec{n}p \to d\gamma$ and $\vec{n}\,^3\mathrm{He} \to p\,^3\mathrm{H}$ experiments[1] [2] and DDH [3]



[1] Blyth et al., 2018

[2] M. T. Gericke et al., 2020

[3] Desplanques, Donoghue, and Holstein (DDH), 1980

Summary

- We have performed a complete NLO RG analysis in QCD to get the effective PV Hamiltonian for hadronic processes.
- I have presented NLO + LQCD analysis of the parity-violating meson-nucleon coupling constants.
- Our results compare favorably to the couplings determined in the NPDGamma and n3He experiments
- The method developed here greatly eases the calculations of anomalous dimension matrices. The idea itself is not specific to HPV, and can be used to study other systems.
- The complete flavor conserving physics NLO effective Hamiltonian is available now . This can serve as the foundational ingredient for all such studies in $\Delta F = 0$, as in η decay cases.

Thank You!