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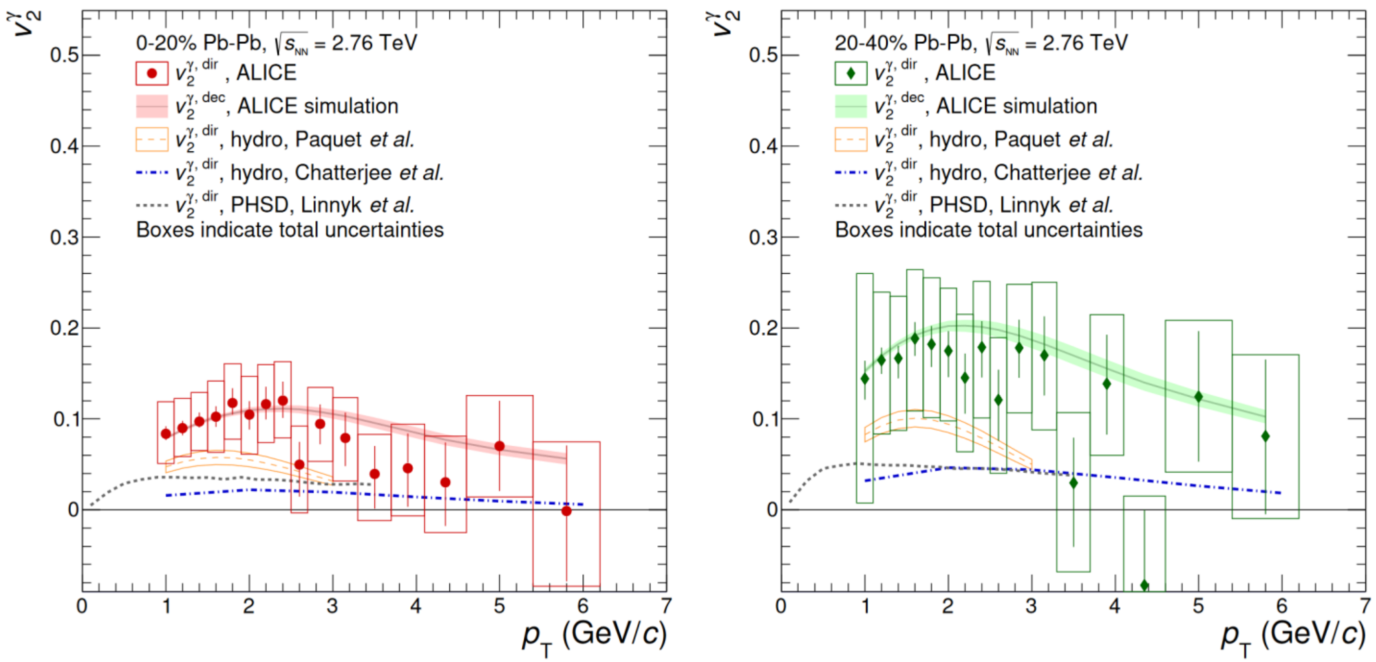
Hydrodynamic description of direct photon spectra and elliptic flow

Motivation

- Direct photon puzzle: the measured v_2 of direct photons is of the same order of magnitude as for hadrons.
- v_2 cannot be described simultaneously with direct photon spectra using the theoretical models known so far.



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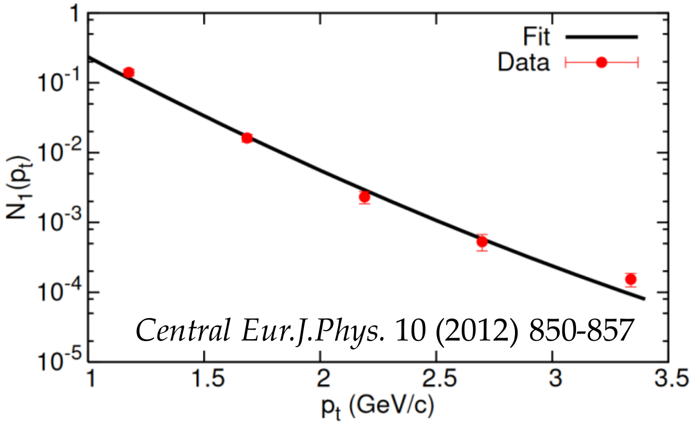
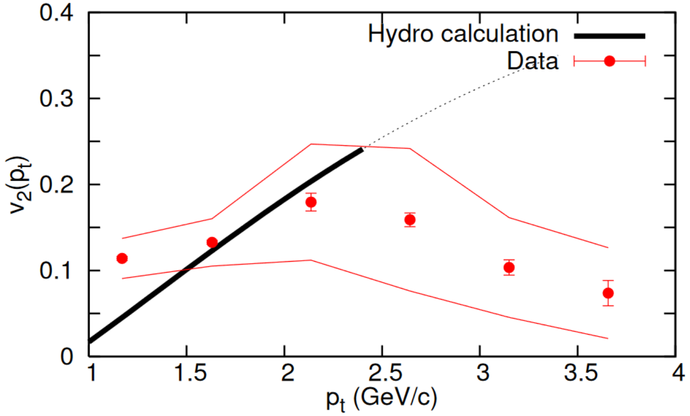
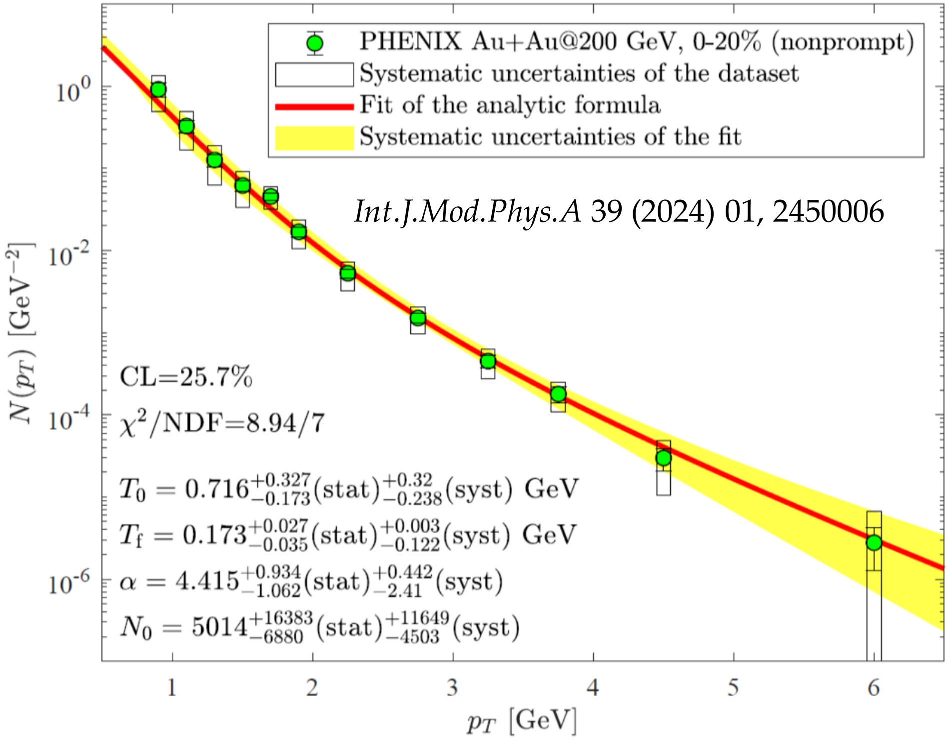
Earlier success of analytic hydro

- Non-prompt component of the direct photon spectra and v_2 are dominated by hydrodynamic evolution.







Simple 1+3d model

- Based on the Csörgő-Csernai-Hama-Kodama solution of relativistic hydro. *Acta Phys.Hung.A* 21 (2004) 73-84
- No acceleration, Gaussian temperature (so density becomes infinite), but 1+3 dimensional.
- Csanád and Májer: analytic calculation of spectra and v_2 at y=0 using second-order saddle-point approximation. *Central Eur.J.Phys.* 10 (2012) 850-857
- Lökös and Kasza (L&K): numerical calculation of spectra and v_2 at y=0, no appromixations applied. e-Print: 2403.13599 [hep-ph]

Simple 1+3d model (L&K)

- Source function:

$$S(x,p)d^4x = \frac{g}{(2\pi\hbar)^3}\frac{\Theta(\tau-\tau_0)-\Theta(\tau-\tau_{\rm f})}{\tau_{\rm R}} p^\mu u_\mu \exp\left(\frac{p^\mu u_\mu}{T}\right) dt\, d^3x$$

- Using the CCHK solution:

$$u_\mu = \gamma\,(1,\mathbf{v}) = \gamma\left(1,\frac{\mathbf{r}}{t}\right)$$ *Hubble-type velocity field*

$$T(\tau,s) = T_{\rm f}\left(\frac{\tau_{\rm f}}{\tau}\right)^{3/\kappa}\mathcal{T}(s)$$ *Inhomogeneous temperature profile*

$$\mathcal{T}(s) = \exp(-bs/2)$$ *Scale function is chosen to be Gaussian*

- Scale variable:

$$s = \frac{r^2}{R^2}\left(1+\epsilon_2\cos(2\alpha)\right) + \frac{r_z^2}{Z^2}$$

$$\epsilon_2 = \frac{Y-X}{Y+X}$$

$$\frac{1}{R^2} = \left(\frac{1}{X^2}+\frac{1}{Y^2}\right)$$

Simple 1+3d model: observables

- Invariant transverse momentum spectrum:

$$N_1(p_{\mathrm{T}},\phi) = E\left.\frac{d^3N}{dp^3}\right|_{p_z=0} = \left.\frac{d^3N}{d\phi dp_{\mathrm{T}} dy}\right|_{y=0} = \int S(t,r,\alpha,r_z,p_{\mathrm{T}},\phi)dt\, r d\alpha\, dr\, dr_z$$

$$\left.\frac{d^2N}{p_{\mathrm{T}} dp_{\mathrm{T}} dy}\right|_{y=0} = \int_0^{2\pi} d\phi N_1(p_{\mathrm{T}},\phi)$$

- Elliptic flow:

$$v_2(p_{\mathrm{T}}) = \frac{\int_0^{2\pi} d\phi \cos(2\phi) N_1(p_{\mathrm{T}},\phi)}{\int_0^{2\pi} d\phi N_1(p_{\mathrm{T}},\phi)}$$

Simultaneous fit (current status)

- Dataset: ALICE Pb+Pb@2.76 TeV, 0-20%

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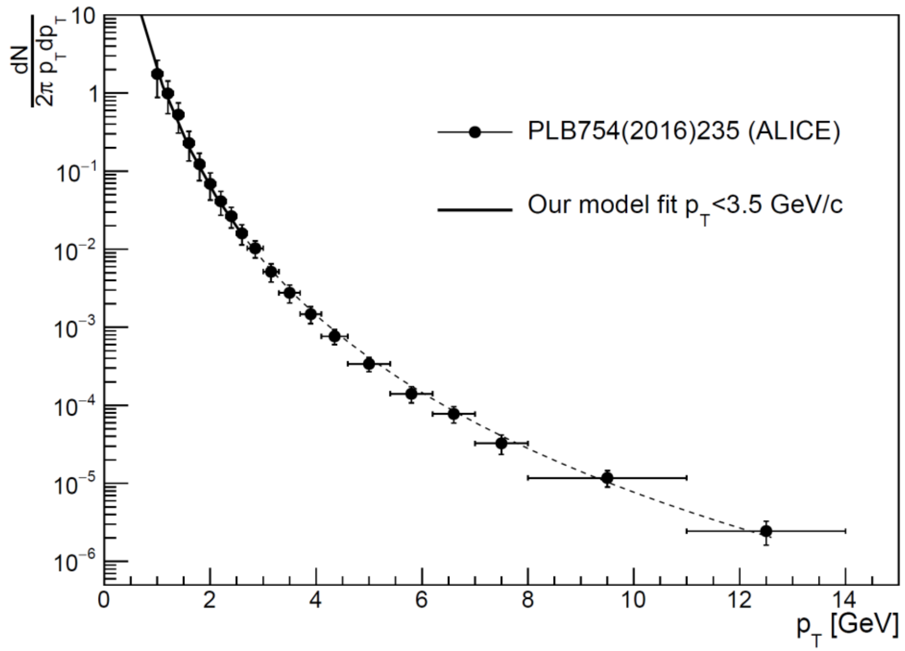
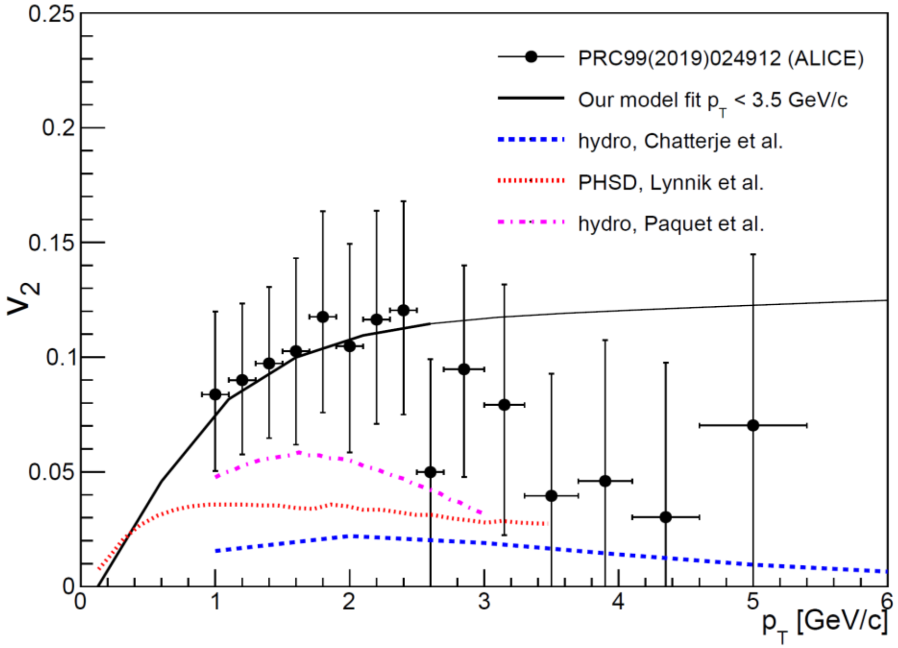
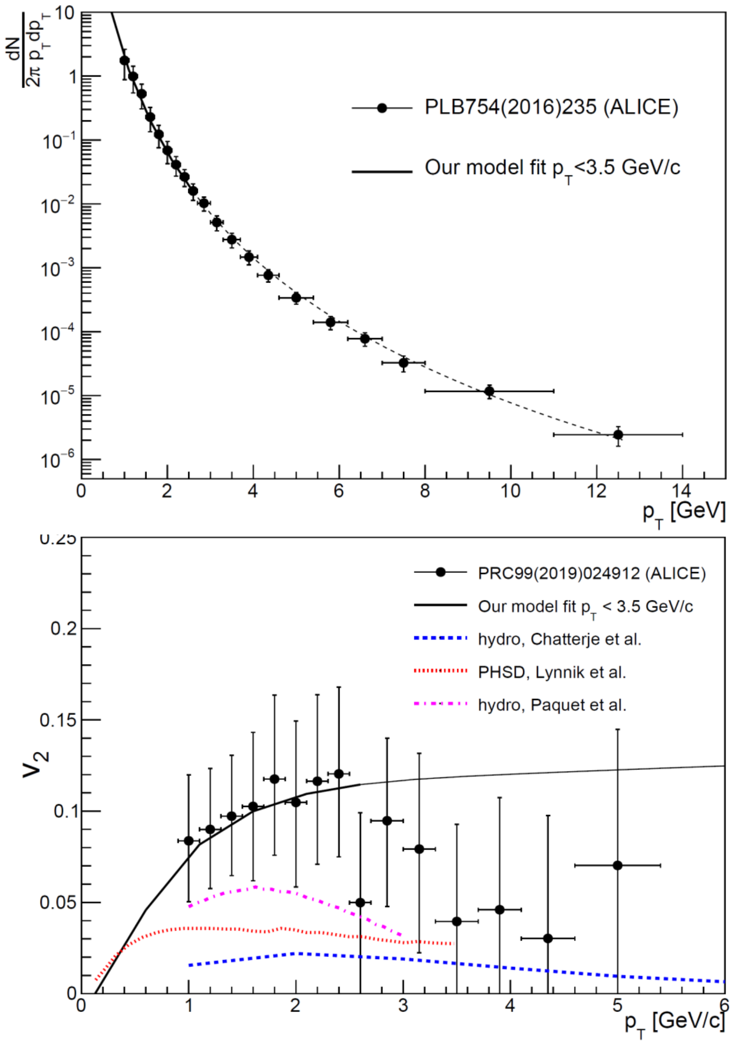




- Our toy-model works better than the advanced state-of-art models.
- Problem: why our model works on the whole p_T-range?

Simultaneous fit (current status)



- T_f = 0.123081 ± 0.00766099 GeV (limited)
- τ_f = 5.51272 ± 0.484373 fm/c (limited)
- $(dR/dt)/\sqrt{b}$ = 1.9 (fixed)
- $(dZ/dt)/\sqrt{b}$ = 1.2 (fixed)
- ϵ_2 = 0.271011 ± 0.0120792 (limited)
- κ = 4.16755 ± 0.074314 (limited)
- normalization = 0.0642342 ± 0.0180748

1+1d model with 2nd order PT

- Based on the Csörgő-Kasza-Csanád-Jiang solution of relativistic hydro with the addition of describing a 2nd order PT. *Universe* 4 (2018) 6, 69
- Accelerating, inhomogeneous solution, but only 1+1 dimensional.
- Thermal component of direct photon spectrum at y=0 is derived analytically with saddle-point approximation.
- The spectrum is embedded to the 1+3d space, but v_2 cannot be calculated.
- The spectrum has a hadronic and a QGP component.

Previously: 1+1d model with constant c_s

- The direct photon spectrum was derived earlier with constant speed of sound from the following source function:

$$S\left(x^\mu, p^\mu\right) d^4x = \frac{g}{(2\pi\hbar)^3} \frac{H(\tau)}{\tau_\mathrm{R}} \frac{p_\mu d\Sigma^\mu}{\exp\left(\frac{p^\mu u_\mu}{T}\right) - 1}$$

- Based on CKCJ solution:

$$u^\mu = \left(\cosh\left(\Omega\right), \sinh\left(\Omega\right)\right)$$

Accelerating flow: Ω=Ω(η)

$$T\left(\tau, \eta_z\right) = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{\kappa}} \left[1 + \frac{\kappa - 1}{\lambda - 1} \sinh^2\left(\Omega\left(\eta_z\right) - \eta_z\right)\right]^{-\frac{\lambda}{2\kappa}}$$

- The spectrum:

$$\left.\frac{d^2N}{2\pi p_\mathrm{T} dp_\mathrm{T} dy}\right|_{y=0} = N_0 \frac{2\alpha}{3\pi^{3/2}} \left[\frac{1}{T_\mathrm{f}^\alpha} - \frac{1}{T_0^\alpha}\right]^{-1} p_\mathrm{T}^{-\alpha-2}\, \Gamma\left(\alpha + \frac{5}{2}, \frac{p_\mathrm{T}}{T}\right)\Bigg|_{T=T_\mathrm{f}}^{T=T_0}$$

1+1d solution with T dependent c_s

- In the case of zero bariochemical potential, if κ(T) satisfies the following equations ...

$$\frac{d}{dT}\left[\frac{\kappa(T)T}{1+\kappa(T)}\right]=\frac{c_Q}{1+\kappa(T)}$$ $T > T_c$, $\kappa(T)=\kappa_Q(T)$

$$\frac{d}{dT}\left[\frac{\kappa(T)T}{1+\kappa(T)}\right]=\frac{c_H}{1+\kappa(T)}$$ $T < T_c$, $\kappa(T)=\kappa_H(T)$

- … then the solutions for the temperature are in the same class as in the case of constant c_s:

$$T(\tau,\eta_z)=T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{c_H}}\left[1+\frac{c_H-1}{\lambda-1}\sinh^2\left(\Omega-\eta_z\right)\right]^{-\frac{\lambda}{2c_H}}$$ $T < T_c$

$$T(\tau,\eta_z)=T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{c_Q}}\left[1+\frac{c_Q-1}{\lambda-1}\sinh^2\left(\Omega-\eta_z\right)\right]^{-\frac{\lambda}{2c_Q}}$$ $T > T_c$

1+1d model with 2nd order PT

- If $\kappa_Q(T)$ and $\kappa_H(T)$ are matched at T_c: hydro-based EoS describing 2nd order PT

$$\kappa(T) = \Theta(T - T_c)\kappa_H(T) + \Theta(T_c - T)\kappa_Q(T)$$

- Two-component direct photon spectrum:

$$N(p_T) = \frac{dN}{2\pi p_T dp_T} = \underbrace{\int_{\tau_f}^{\tau_c} N(p_T,\tau,c_H)d\tau}_{\text{Hadronic component}} + \underbrace{\int_{\tau_c}^{\tau_0} N(p_T,\tau,c_Q)d\tau}_{\text{QGP component}}$$

Hadronic component $(c_H, \tau_f, \tau_c, \lambda, N_H)$

QGP component $(c_Q, \tau_0, \tau_c, \lambda, N_Q)$

- "Preliminary results" from fit to PHENIX Au+Au@200 GeV data:
  - Realistic values were obtained for the fit parameters.
  - Values of c_H and c_Q: the hydro-based EoS is in agreement with the lQCD EoS qualitatively.

Open questions

- Why a simple 1+3d hydrodynamic model with pre-existing flow can describe the direct photon spectrum and the elliptic flow simultaneously while more complex models fail this problem?
- Will the 1+3d hydrodynamic model be successful in desribing the PHENIX data as well? *According to our preliminary results, yes.*
- Can the effect of a possible second-order phase transition be detected in the direct photon spectrum?
- Open problem: analytic solution of relativistic hydrodynamics with an EoS that desribing crossover type quark-hadron transition and with inhomogeneous temperature field.