Scales in financial data

Chapter 5

Mantegna, Stanley: An Introduction to Econophysics

Introduction

- Nowadays every transaction of every financial market is recorded.
  - on a daily basis since the 19th century
  - with a sampling rate of 1 min or less since 1984
  - transaction-by-transaction ('tick-by-tick') since 1993
- Main point of this chapter: the role of scales and reference units in finance and physics is rather different.

Price scales in financial markets

- *Physics:* metrology is devoted to find the optimal reference unit ⟹ accuracy.
- *Finance:* the scales used are often given in units (currencies) that are themselves fluctuating in time and transactions occur at random times with random intensities.

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Selection of the most appropriate variable is a difficult task.

- The price unit of financial goods is usually the currency of the country in which the particular financial market is located.
- The value of the currency is not constant in time.

Price scales in financial markets

- A currency can change its value because of
  - inflation,
  - economic growth or economic recession, and
  - fluctuations in the global currency market.
- What is the proper unit for a data table?

Price scales in financial markets

- Data tables, examples:

GDP at Constant Prices

1978	1979	1980	1981	1982	1983	1984	1985	1986	1987		99bp x
4.1	3.7	2.2	1.7	1.2	2.1	4.1	2.9	....	....	World	001
4.0	3.2	1.4	1.5	1.1	2.6	4.5	3.1	3.0	3.0	Industrial Countries	110
5.2	2.1	-.2	2.0	-2.5	3.7	6.6	3.0	3.8	3.1	United States	111
4.2	3.7	1.5	3.0	-3.4	3.7	6.1	4.3	3.0	....	Canada*	156
.8	3.4	3.4	2.0	3.1	.4	6.7	5.5	1.8	4.4	Australia	193
5.2	5.3	4.3	3.7	3.1	3.2	5.1	4.7	2.5	4.4	Japan*	158
—	2.7	.7	1.4	3.1	.1	6.6	1.5	....	....	New Zealand	196
.5	4.7	3.0	-.1	1.1	2.2	1.4	2.8	1.7	1.3	Austria	122
2.9	2.2	4.1	-1.3	1.5	.1	2.0	1.4	2.4	1.7	Belgium	124
1.5	3.5	-.4	-.9	3.0	2.5	4.4	4.2	3.3	-1.0	Denmark	128
2.2	7.3	5.4	1.6	3.6	3.0	3.3	3.5	2.4	....	Finland	172
3.3	3.2	1.6	1.2	2.5	.7	1.3	1.7	2.1	2.2	France	132
3.0	4.1	1.7	.2	11.7	1.5	2.8	2.1	2.6	1.8	Germany	134
5.9	5.0	-4.2	1.6	-1.5	-5.5	2.7	10.1	6.3	5.5	Iceland*	176
7.2	3.1	3.1	3.3	2.3	-1.1	3.8	1.1	-.3	....	Ireland	178
2.7	4.9	3.9	1.1	.2	1.0	3.2	2.8	2.9	3.1	Italy	136
4.7	4.0	2.9	.5	1.5	2.4	5.7	3.9	3.4	....	Luxembourg	137
2.5	2.4	.9	-.7	-1.4	1.4	3.2	2.3	2.4	2.2	Netherlands	138
4.5	5.1	I4.2	.9	.3	4.6	I5.7	5.3	I4.2	1.3	Norway	142
1.8	.2	1.5	-.2	1.2	1.8	1.9	2.1	3.6	....	Spain	184
1.8	3.8	1.7	-.3	.8	2.4	3.9	2.1	1.2	2.8	Sweden	144
.4	2.5	4.6	1.5	-1.1	.7	2.1	3.7	2.8	....	Switzerland	146
3.9	2.1	-2.1	-.9	1.1	3.5	2.1	3.9	2.9	3.6	United Kingdom	112

Fig. 5.4. Annual percent change of the gross domestic product of several countries over a 10-year period; data are obtained from International Financial Statistics (International Monetary Fund, 1988), page 165.

Price scales in financial markets

- Data tables, examples:

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
1972	49.9	50.2	50.2	50.4	50.5	50.6	50.9	50.9	51.1	51.3	51.4	51.6
1973	51.7	52.1	52.6	53.0	53.3	53.6	53.8	54.7	54.9	55.3	55.8	56.1
1974	56.6	57.3	58.0	58.3	59.0	59.5	60.0	60.7	61.5	62.0	62.5	63.0
1975	63.2	63.7	63.9	64.3	64.5	65.1	65.8	66.0	66.3	66.7	67.1	67.4
1976	67.5	67.7	67.9	68.2	68.6	68.9	69.3	69.7	69.9	70.2	70.4	70.6
1977	71.0	71.8	72.2	72.8	73.2	73.7	74.0	74.3	74.6	74.8	75.1	75.4
1978	75.9	76.3	76.9	77.6	78.3	79.1	79.7	80.1	80.8	81.4	81.8	82.2
1979	82.9	83.9	84.7	85.7	86.8	87.8	88.7	89.6	90.5	91.3	92.2	93.2
1980	94.5	95.8	97.2	98.3	99.2	100.3	100.4	101.1	102.0	102.9	103.8	104.7
1981	105.6	106.6	107.4	108.1	109.0	109.9	111.2	112.0	113.2	113.4	113.7	114.1
1982	114.5	114.8	114.7	115.2	116.3	117.7	118.4	118.6	118.8	119.2	119.0	118.5
1983	118.8	118.8	118.9	119.7	120.4	120.8	121.3	121.7	122.3	122.6	122.8	123.0
1984	123.7	124.2	124.5	125.1	125.5	125.9	126.3	126.8	127.4	127.8	127.8	127.8
1985	128.1	128.6	129.2	129.7	130.2	130.6	130.8	131.1	131.5	131.9	132.3	132.7
1986	133.1	132.7	132.1	131.8	132.2	132.9	132.9	133.1	133.8	133.9	134.0	134.2

Fig. 5.5. Monthly consumer price index in the United States during the 15-year period 1972 to 1986, normalized to the value of 100 USD for the year 1980. Data from International Financial Statistics, Supplement on Price Statistics (International Monetary Fund, 1986), page 70.

Price scales in financial markets

- Let us define Y(t) as the price of a financial asset at time t.
- Which is the appropriate stochastic variable for us to investigate?

Price changes:

$$Z(t) \equiv Y(t + \Delta t) - Y(t)$$

- Nonlinear and stochastic approaches are not needed.
- It is seriously affected by changes in scale.

Price scales in financial markets

- Let us define Y(t) as the price of a financial asset at time t.
- Which is the appropriate stochastic variable for us to investigate?

Deflated or discounted price changes:

$$Z_{\mathrm{D}}(t) \equiv [Y(t+\Delta t) - Y(t)]D(t)$$

- Nonlinear transformations are not needed.
- Prices are given in terms of "constant" money.
- Deflators and discounting factors are unpredictable over the long term: there is no unique choice for D(t)

Price scales in financial markets

- Let us define Y(t) as the price of a financial asset at time t.
- Which is the appropriate stochastic variable for us to investigate?

Returns:

$$R(t) \equiv \frac{Y(t+\Delta t) - Y(t)}{Y(t)} = \frac{Z(t)}{Y(t)}$$

- Provide a direct percentage of gain or loss in a given time period.
- Sensitive to scale changes for long time horizons.

Price scales in financial markets

- Let us define Y(t) as the price of a financial asset at time t.
- Which is the appropriate stochastic variable for us to investigate?

Successive differences of the logarithm:

$$S(t) \equiv \ln Y(t + \Delta t) - \ln Y(t)$$

- The average correction of scale changes is incorporated without requiring deflators or discounting factors.
- The correction of scale change would be correct only if the growth rate of the economy were constant, but the growth rate generally fluctuates, and these fluctuations are not incorporated.
- Nonlinear transformation is used, and nonlinearity strongly affects the statistical properties of a stochastic process.

Price scales in financial markets

- The analysis of high-frequency financial data has become widespread in research institutes and financial institutions.
- How the above definitions are interrelated in the high-frequency regime?
- *For high-frequency data* and for investigations limited to a short time period in a time of low inflation:

$$S(t) \approx R(t) \approx C_1 Z(t) \approx Z_{\mathrm{D}}(t)$$

where the time dependence of C_1 is negligible.

- However, for investigations over longer time periods, a choice must be made.
- *The most commonly studied functions are S(t) and R(t).*

Time scales in financial markets

- Possible candidates for the 'correct' time scale include:
  - the physical time,
  - the trading (or market) time, or
  - the number of transactions.

- All the definitions have merits and all have problems.
- When examining price changes that take place when transactions occur, it is worth noting that each transaction occurring at a random time involves a random variable of the traded financial good: *the volume*.

Time scales in financial markets

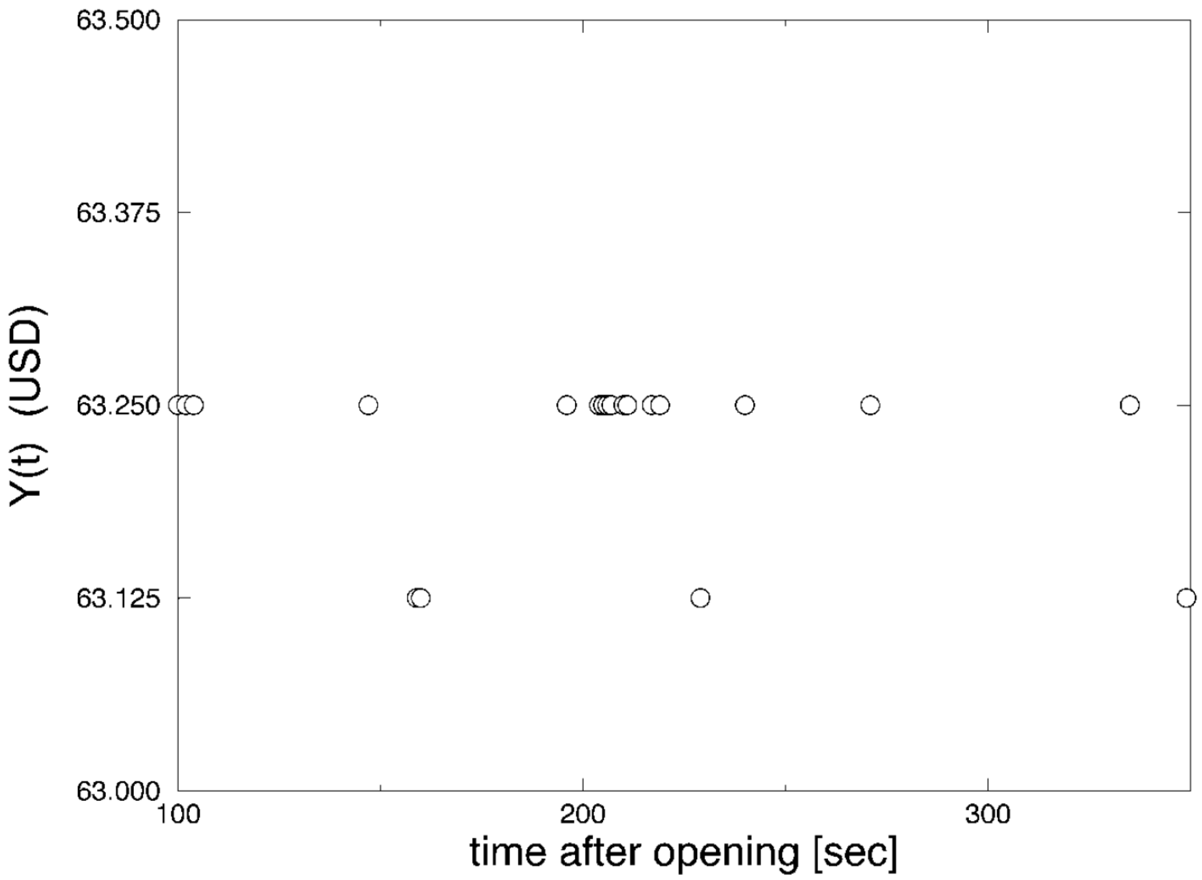

Fig. 5.6. Price change during the day of 3 January 1994 of an Exxon stock traded in the New York Stock Exchange. The price is recorded when a transaction occurs, and transactions occur randomly in time.

Time scales in financial markets

Physical time

- Well-defined, but...
- ...stock-exchanges closed during night, weekends and holidays.
- Similar limitations in foreign exchange market: active 24h/day but temporal constraints (varies by location) are present due to the biological cycles and social organizations of business.
- Not known how to model the stochastic dynamics of prices and the arrival of information during hours in which the market is closed.

Time scales in financial markets

Trading time

- Well-defined in stock-exchanges: it is the time that elapses during open market hours.
- *In the foreign markets: physical time = trading time.*
- The variance determined by considering closure values of successive days is only approximately 20% lower than the variance determined by considering closure values across weekends.

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- *Trading time is used in the modeling of price dynamics.*

Time scales in financial markets

Problems with Trading time:

- Information affecting the dynamics of the price of a financial asset can be released while the market is closed.
- In high-frequency analyses overnight price changes are treated as shorttime price changes.
- The market activity is implicitly assumed to be uniform during market hours.

Time scales in financial markets

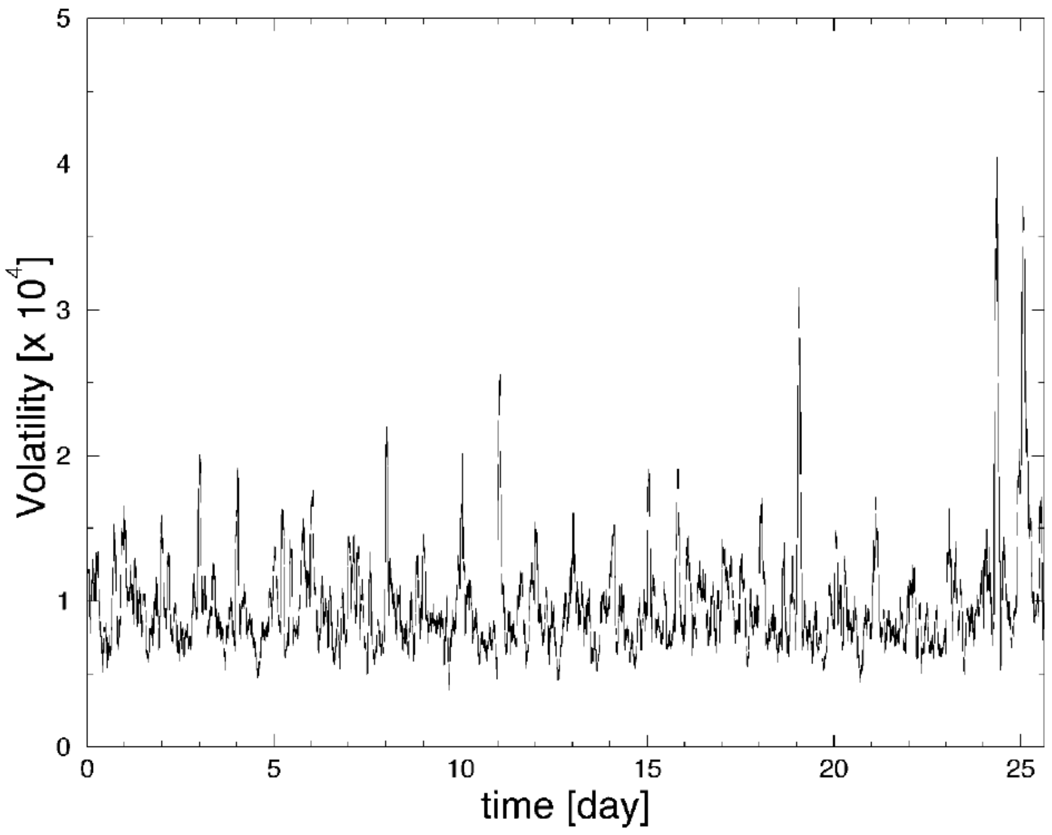


Fig. 5.7. Volatility (to be discussed in Chapter 7) of the S&P 500 high-frequency data. A daily cycle with a period of 6.5 trading hours is clearly observed in the time evolution.

- Trading activity is not uniform during trading hours, either in terms of volume or in number of contracts.
- In the figure, clearly seen is a daily cycle with a period of 6.5 trading hours.

Time scales in financial markets

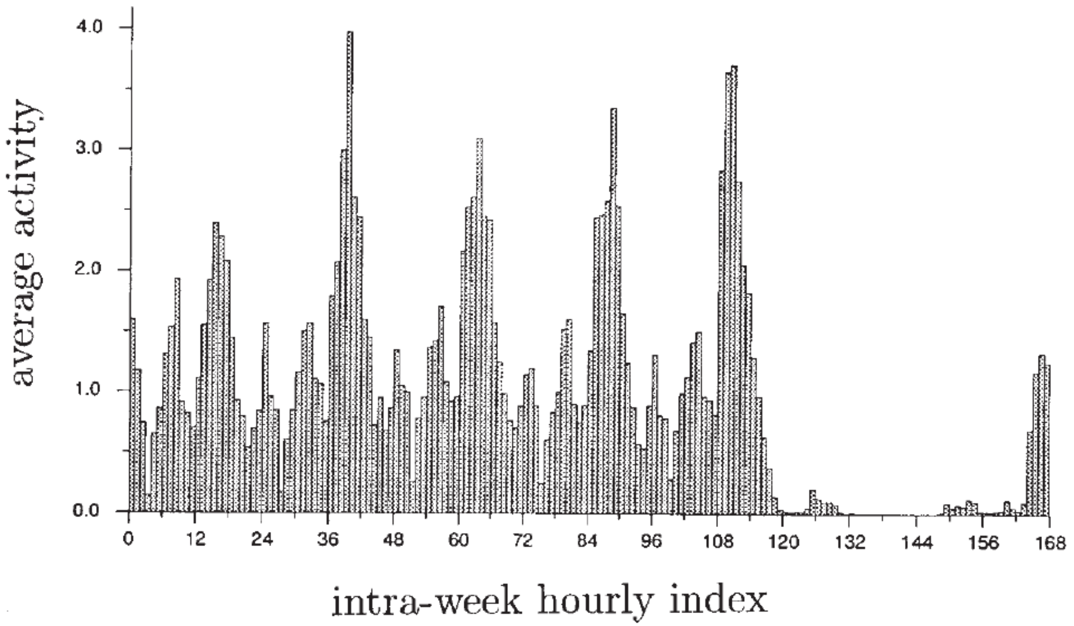


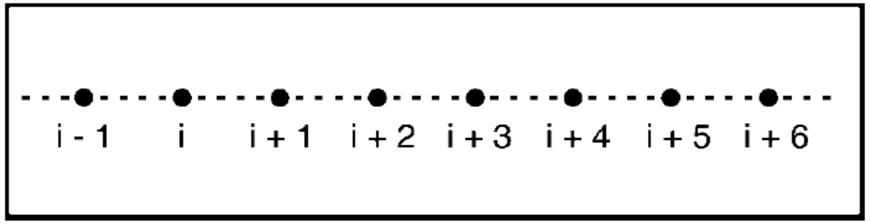
Fig. 5.8. Average hourly activity in the foreign exchange global market. Intraday cycles are also observed. Note that the three peaks are related to the maximal activity in each of the three main geographic areas, America, Asia, and Europe. Adapted from [41].

- Almost periodic behavior is observed in the average activity of the foreign exchange market.

Time scales in financial markets

Time index of the number of effective transactions

- Time is defined in terms of number of transactions.



- Tick-by-tick data are necessary to perform statistical analysis.
- Nowadays it is possible (at least for some financial markets).
- One source of randomness is eliminated: the time elapsing between transactions.
- The second source of randomness, the volume of the transaction, still remains.

Summary

- It is not straightforward to select the price function and the time reference frame to be used in the analysis and modeling of the stochastic dynamics of a price.
- Empirical analyses are often performed with slightly different
  - definitions of variables,
  - periods of time analyzed,
  - frequency of recorded data.
- Results are really sensitive to these choices.

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- *Statistical properties of price changes is still lacking, despite a large number of empirical analyses.*