Hint to Supersymmetry from GR Vacuum †

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† with Gia Dvali and Archil Kobakhidze, [arXiv:2406.18402]

Classical landscape of GR vacua

General relativity is given by

$$S = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\,(R(g) - 2\Lambda)\,,$$

where g is a metric and Λ is a cosmological constant.
Classically we have three different possibilities,

- de Sitter $\Lambda > 0$
- anti-de Sitter (AdS) $\Lambda < 0$
- Minkowski $\Lambda = 0$

on top of them we have fluctuations which should be understood as an expectation value of the graviton field operator in a (coherent) quantum states,

$$\delta g_{\mu\nu}(x) = \frac{\langle \hat{h}_{\mu\nu}(x) \rangle}{M_{pl}}$$

Quantum landscape of GR vacua 1

- Eternal de Sitter is incompatible with Quantum gravity
  Dvali, Gomes '14,'16+Zell '17
- The ground state should not evolve in time, de Sitter does
  $T \propto H$
  Gibbons, Hawking '77
- Non eternal de Sitter can exist and should be understood as a BRST invariant state on a valid vacuum
  Berezhiani, Dvali, Sakhelashvili '21

To summarize,

$$t_Q \sim \frac{M_{pl}^2}{H^3}$$

Rigidity = double-scaling limit $M_{pl} \to \infty$, H fixed, but
$2 \to 2$ Graviton interaction

$$\alpha_{gr} = P^2 / M_{pl}^2 \to 0,$$

is trivial.

Quantum landscape of GR vacua 2

- AdS cosmology leads to big crunch and singular cosmology
- The only vacuum supported by cosmology = Minkowski
- S-matrix formulation singles out the Minkowski vacuum Dvali '20
- Isolated AdS are also part of quantum gravity landscape and supported by AdS/CFT duality Maldacena '98

Can we fix an unique Minkowski vacuum?

Let us imagine, we tuned cosmological constant to zero,

$$\Lambda = 0$$

Then we have Minkowski vacuum, and quantum gravity with cosmology.

We could ask if we are in a consistent theory.

The answer is no, if there multiple vacua with different energies. We can not pick one and discard others.

An example is QCD θ-vacua, $\mathcal{E} \propto \theta^2$.

If $\theta = 0$ is Minkowski, $\theta' \neq \theta$ is in de Sitter.

The above promotes the strong CP puzzle into the consistency problem Dvali '22

The QCD vacuum

The QCD vacuum has topological property,

$$\pi_3(SU(N_c)) = Z$$

and Instanton processes, with rate,

$$\mathcal{M} \sim e^{-\frac{8\pi^2}{g^2}}$$

This makes θ-angle physical

$$\mathcal{L}_\theta = \theta \frac{g^2}{16\pi^2} G\tilde{G}$$

and vacuum energy depends on,

$$\mathcal{E} \propto \theta^2$$

Callan, Dashen, Gross '76, Jackiw, Rebbi '76
$\theta = 0$ is a minimum of energy Vafa, Witten '84

The (traditional) Strong CP puzzle

$\theta \leq 10^{-10}$ From EDMN e.g. C. Abel, et al. '20

A quark with chiral symmetry

$$\psi \to e^{i\gamma_5 \alpha}\psi,$$

$$\theta \to \theta + 2\alpha$$

Or in the integral form of anomaly

$$Q_5(t=\infty) - Q_5(t=-\infty) = 2n,$$

The quark could be massive, with Peccei, Quinn '77 symmetry

$$|\Phi| e^{-i\frac{a(x)}{f_a}} \bar{\psi}\psi$$

implies an axion Wilczek '78, Weinberg '78 with

$$a(x) \to a(x) - 2\alpha f_a$$

How does the axion work?

Lets look at the following correlator,

$$\mathrm{FT}\langle G\tilde{G}(x)\ G\tilde{G}(0)\rangle_{p\to 0} \propto \left.\frac{p^2}{p^2-m^2}\right|_{p\to 0}$$

If $m=0$, θ is physical, and

$$\theta \propto \langle \tilde{G}G\rangle$$

Axion makes θ unphysical, with $m\neq 0$. This effect alternatively can be understood as the 3-form Higgs effect $\tilde{G}G =^* dC$ Dvali '05

Axion quality problem

$a \to a + c$ not exact means,

$$\mathrm{FT}\langle G\tilde{G}(x)\ G\tilde{G}(0)\rangle_{p\to 0} \neq 0$$

This is considered as a quality problem.

If we add gravity, we create de Sitter
In our context consistency problem

Alternatively 2-form axion can solve the problem, which can not be undone via continues deformations.

$$\mathcal{L} \;=\; \frac{1}{f_a^2}(C - f_a dB)^2$$

Gravitational Instantons 1

Egguchi and Hanson '78 (EH) found euclidean solution of GR,

$$ds^2 = \left(1 - \frac{a^4}{r^4}\right)^{-1} dr^2 + r^2\left(\sigma_x^2 + \sigma_y^2\right) + r^2\left(1 - \frac{a^4}{r^4}\right)\sigma_z^2$$

σ's are $SU(2)$ elements (We have 3-angles ϕ, θ, ψ).

$$d\sigma_x = 2\sigma_y \wedge \sigma_z$$

For example,

$$\sum_{i=1}^{4} dx_i^2 = dr^2 + r^2(\sigma_x^2 + \sigma_y^2 + \sigma_z^2)$$

$$\sigma_z \sim d\psi + \cos\theta d\phi$$

Gravitational Instantons 2

The EH instanton is locally flat at infinity, compatible with the $\mathcal{S}$-matrix, has zero action and non-trivial topology

The boundary at infinity S^3/Z_2 and the boundary at $r = a$ (coordinate singularity) is S^2

We get two topological invariants,

$$\chi = \frac{1}{8\pi^2} \int d^4x \sqrt{g} \left(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\alpha\beta}^2\right) + \text{bound. terms} = 2$$

$$\tau = -\frac{1}{24\pi^2} \int d^4x \, R\tilde{R} = 1$$

Gravitational Instantons 3

Instantons must have finite action, we add,

$$\Delta S \; = \; c\,\frac{\chi}{2}$$

For large c, EFT works

$$\mathcal{M} \sim e^{-c}$$

c encodes the cut-off scale

$$c \sim \left(\frac{M_{pl}}{\Lambda_{gr}}\right)^2$$

The Gravity CP-problem

We could add the θ-term to the theory

$$S = \frac{\theta}{24\pi^2}\int d^4x R\tilde{R}$$

Since the theory has θ-vacuum structure (Instantons carry non-zero τ),

$$\mathrm{FT}\langle \tilde{R}R(x)\ \tilde{R}R(0)\rangle_{p\to 0} \neq 0$$

The vacuum angle is physical

In the $\mathcal{S}$-matrix framework, it can be thought as a consistency problem, or simply as a new CP puzzle.

Now we try to solve the Gravity-CP problem

Solving the problem

The fermions carry gravitational anomaly chiral anomaly Delbourgo, Salam '72

$$\partial_\mu j_5^\mu \propto R\tilde{R}$$

Naively, this should solve the problem. There is a caveat,

$$Q_5(t=\infty) - Q_5(t=-\infty) = 0$$

Helicity 1/2 fermion does not have zero modes

Fermion with helicity 3/2 has zero modes Egguchi, Hanson '78

$$|I_{3/2}| = 2$$

Chiral redefinition of gravitiono implies $\theta \to \theta + 2\alpha$

$$\psi_\mu \to \mathrm{e}^{i\gamma_5\alpha}\psi_\mu$$

A SUGRA?

Consistency of spin/helicity 3/2 particle requires supergravity.

The gauge transformation has form,

$$\psi_\mu \to \psi_\mu + \partial_\mu \xi$$

To remove ghosts in the interaction theory we need to promote it to a symmetry

This is a local (gauge) version SUSY, SUGRA
see e.g. Freedman, Proeyen, Supergravity (book)

So we get a powerful conclusion

The solution of Gravity CP requires SUGRA

Breaking of SUSY

After taking into account instanton effects, we get effective t'Hooft vertex,

$$\frac{W^*_{3/2}}{M^2_{pl}} \bar{\psi}^{\mu} \sigma_{\mu\nu} \psi^{\nu}$$

Breaks R symmetry and lowers theory in AdS, with vacuum energy $\propto -3|W_{3/2}|^2/M^2_{pl}$.
We uplift the theory to Minkowski, with extra Superfield X and superpotential,

$$W \;=\; X\Lambda_X^2 + W_{3/2}$$

We end up in the Polonyi model with broken SUSY

We predict an ALP (phase of X, $\langle X \rangle \sim M_{pl}$) with mass $\sim m_{3/2}$ and decay constant M_{pl} (maybe a good Dark matter)

The fate of 1/2 fermion anomalies

The 1/2 helicity fermion can not solve Gravity CP, still we have anomaly,

$$\partial_\mu j_5^\mu \propto R\tilde{R}$$

Consistency requires cancellation of it, or explicit breaking of it.

Also the R-symmetry should be exact (Up to helicity 3/2 anomaly)

This has ramification in the SUSY framework, let us add an extra Y-fields,

$$W = \hat{X}\Lambda_X^2 - g\hat{X}\hat{Y}_j^2 + W_{3/2}$$

which sets the theory in AdS, and going back to Minkowski requires, extra fields $\bar{Y}$'s

$$W = \hat{X}\Lambda_X^2 - g\hat{X}\hat{Y}_j^2 + M\hat{\bar{Y}}_j\hat{Y}_j + W_{3/2}$$

The 1/2-anomaly is cancelled.

An alternative approaches

We could ask, what happens if we rely all the physics on gravitino condesate,

$$\langle \bar{\psi}^{\mu} \sigma_{\mu\nu} \psi^{\nu} \rangle \neq 0$$

In this scenario, role of the axion is played by η_R, which has mass $m_{3/2}$ and decay constant M_{pl}. We still study the mechanism of the SUSY breaking.

Why we do not use the two-form $B_{\mu\nu}$, like in QCD?
There are potential consistency issues Duff, Nieuwenhuizen '80

Conclusions

- We argued that Quantum gravity works only on Minkowski and eternal AdS without cosmology
- We used existence of instantons in GR
- We studied topological structure of GR vacua
- We defined Gravity CP problem
- We found necessity of SUGRA and breaking of SUSY
- We predict existence of ALP with the mass of the order of gravitino mass
- We constrain representations of the $1/2$ fermion via requirement of perturbative gravitational anomaly cancellation.

Thank you

Backup slide (Instanton)

$$u^2 = r^2(1 - \frac{a^4}{r^4})$$

$r = a,\ u = 0$

$$ds^2 \simeq \frac{1}{4}du^2 + \frac{1}{4}u^2(d\psi + \cos\theta d\phi)^2 + \frac{1}{4}a^2 d\Omega^2$$

Backup slide (SUSY)

$$X_0 = \pm M_{pl}(\sqrt{3}+1)$$

$$W_{3/2} = \mp \Lambda_X^2 M_{pl}(\sqrt{3}+2)$$

$$m_{3/2} = W/M_{pl}^2 = \Lambda_X^2/M_{pl}$$

$$gXY_j^2 \simeq \Lambda_Y^2$$