Large-volume Microwave Cavity Design for the Taiwan Axion Search Experiment with Haloscope

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Introduction & Outline

In this thesis, I introduce a large volume, tunable cavity design for the axion haloscope experiment, which is expected to improve the sensitivity to the axion-photon-photon coupling by a factor of 2.6 times compare to the previous result from TASEH.

Outline:

• Axion and its detection
• Conic-shell cavity
• 1st Aluminum prototype cavity
• 2nd Aluminum prototype cavity
• Summary and Conclusion
Axion & its detection

- Solution of strong-CP problem
- Cold dark matter candidate
- Detection through axion-photon-photon coupling
Axion

The QCD Lagrangian includes a CP-violating term

\[ \mathcal{L}_\theta = -\bar{\theta} \frac{g^2}{32\pi^2} G^\mu_\nu \tilde{G}^a_\mu_\nu \]

However, the evidence from the neutrino electrical dipole moment implies an extremely small \( \bar{\theta} \) value in strong interaction. This is called the Strong-CP problem.

Axion is a pseudo particle introduced in 1977, which was a results from Peccei-Quinn solution to the strong CP problem in QCD.

The electromagnetic interaction of axion is described by following Lagrangian:

\[ \mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a \vec{E} \cdot \vec{B} \]

In a strong magnetic field, axions have the potential to convert into photons. Therefore, current axion detection experiments primarily utilize this property.
Axion experiments

• Helioscope: Detect the solar axion flux produced in the solar core by a X-ray detector.

• Haloscope: Detect galactic halo axions by placing a cavity in a strong magnetic field on Earth.

• light-shining-through-walls: By shooting a light beam at one side of the wall under a strong magnetic field, one can pick up the photon signal on the other side once the photons in the light beam have been converted to axions and re-converted back to photons.
The Taiwan Axion Search Experiment with Haloscope (TASEH) is building a small scale local haloscope with a compatible sensitivity compared to others. During 2021, TASEH have carried out an experiment (CD102) which excluded the coupling constant above 11 times the KSVZ Axion model in a frequency range of 4.7075 to 4.7980 GHz.
The expected axion signal power is given by the function

\[ P_s = \left( g_{a\gamma\gamma}^2 \frac{\hbar^3 c^3 \rho_a}{m_a^2} \right) \times \left( \omega_c \frac{1}{\mu_0} B_0^2 V C_{mnl} Q_L \frac{\beta}{1 + \beta} \right) \]

and the signal to noise ratio is

\[ SNR = \frac{P_s}{\sigma_n} = \frac{P_s}{k_B T_{sys}} \sqrt{\frac{t}{\Delta f}} \]

\[ \propto g_{a\gamma\gamma}^2 V C_{mnl} Q_0 \frac{\beta}{(1 + \beta)^2} \sqrt{\frac{f_i T}{2 Q_L \Delta W}} \]

If we have a fixed target SNR, the sensitivity to the \( g_{a\gamma\gamma} \) of the axion haloscope is proportional to \( \sqrt{VCQ_0} \), which is the Figure of merit for our cavity design.

The quality factor is a measure of the loss of a resonant system.

\[ Q = \frac{\text{average energy stored}}{\text{energy loss rate}} = \frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e} \]
To maximize the form factor $C$:

$$C_{mnl} = \frac{\left( \int \vec{E}_{mnl} \cdot \vec{B} \, dV \right)^2}{B_0^2 V \int |\vec{E}_n|^2 \, dV}$$

Most axion haloscope experiments are targeting TM$_{010}$ mode with a cylindrical cavity.

The resonance frequency of the TM mode in a cylindrical cavity is:

$$f_{nml} = \frac{c}{2\pi \sqrt{\varepsilon_r \mu_r}} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$
Previous Cavity Detector

- OFHC copper, split cavity
- Volume $V$: $\sim 0.234$ L
- High quality $Q$: $\sim 18000$ (room temp.)
- Large form factor $C_{010}$: $\sim 0.62$
- Tunable frequency $f_c$: 4.65 - 4.95GHz
- Tunable coupling $\beta_2$: 0.5 - 3
Conic-shell cavity

- Idea proposed in 2020
- Large volume with good quality
- Limited by the bore size of the DR
For a rectangular cavity, if the external magnetic field is applied in the z direction (along h), the TE\textsubscript{110} mode is most suitable for axion searches because of its uniform polarization.

The resonance frequency of the TE mode in a cylindrical cavity is:

$$f_{lmn} = \frac{c}{2\pi \sqrt{\varepsilon_r \mu_r}} \sqrt{\left(\frac{l\pi}{L}\right)^2 + \left(\frac{m\pi}{w}\right)^2 + \left(\frac{n\pi}{h}\right)^2}$$
Conic Shell Cavity

Design model (assisted by HFSS simulation)

- TM$_{010}$ mode exists in cavity between two cones
- Maximize available space in magnet bore
- $f_{TM_{010}} \sim 4.75$ GHz, $V \sim 1.66$ L
- Frequency tuning via inner cone position
- $\lambda/4$ RF choke to reduce radiation loss from gap

E-field pattern of TM$_{010}$

<table>
<thead>
<tr>
<th>Material</th>
<th>R</th>
<th>w</th>
<th>h</th>
<th>$f_{TM_{010}}$</th>
<th>$\theta_{in}$</th>
<th>$\theta_{out}$</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>64 (mm)</td>
<td>31.35 (mm)</td>
<td>200 (mm)</td>
<td>4.785 (GHz)</td>
<td>3.5 (deg)</td>
<td>3.57 (deg)</td>
<td>1.5 (mm)</td>
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</tbody>
</table>

Outer cone

- Outer cone half-angle $\theta_{out}$

Inner cone

- Inner cone half-angle $\theta_{in}$

Cavity space

- $THK_{out}$
- $THK_{in}$
- $gap_{out}$
- $gap_{in}$

Tuning vertically

$\lambda/4$ Choke

$f_{010} \sim c/2w$
Choke Design

Field leak out through the gap

\[ Q_0 = 508 \]

\[ Q_0 = 10602 \]

Choke design greatly reduces the radiation loss through the gap.
By optimizing the outer cone half-angle, the E-field maximal distribution can be located at the center.
- The form factor can be slightly increased due to alignment of E- and B-fields.
- Less radiation loss through the gap.
Mode Map

The diagram illustrates the mode map with a frequency range from 4.65 GHz to 5.1 GHz on the y-axis and an inner cone position range from -15 mm to 15 mm on the x-axis. The graph shows a tuning range of 223 MHz and a 24 mm tuning range. The mode identified is $\text{TM}_{010}$. The plot includes several data points indicating the frequency variation with the inner cone position.
Mode Crossing

Mode map

Frequency (Hz)

$10^9$

Inner cone position (mm)

$TM_{010}$
Form Factor and Expected Cavity Performance

- Volume $V \sim 1.66$ L, form factor $C_{010} \sim 0.73$
- $\sim 2.6$ sensitivity improvement refer to the 1st results
Inner & outer cone misalignment

Inner cone shift of 0.2 mm along y axis

Inner cone rotation of 0.2 degree around x-axis (anti-clockwise, about 0.7 mm displacement on top)
Fabrication of 1st Test Cavity

- The 1st aluminum prototype cavity was made in late 2022.
- The alignment mechanism was designed with a long rod.
- The inner cone position was tuned by a moving table.
Fabrication of 1st AL prototype Cavity

- Whole weight ~ 6 kg
- Inner cone weight ~ 1 kg

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<tr>
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<th>$f_{TM010}$</th>
<th>$\theta_{in}$</th>
<th>$\theta_{out}$</th>
<th>d</th>
<th>gap$_{in}$</th>
<th>gap$_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6061 Al alloy</td>
<td>64</td>
<td>31.35</td>
<td>200</td>
<td>4.855</td>
<td>3.63</td>
<td>3.68</td>
<td>1.5</td>
<td>1.5</td>
<td>1.0</td>
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</table>
Scattering Parameters Measurement

Measured $S_{22}$ at different inner cone tuning position

- Frequency tuning mechanism is confirmed.
- Frequency tuning has nearly identical tuning range as design
- ~ 70 MHz off from design
E-Field Distribution Measurement via Bottom Slots

- $f_c \sim 4.75$ GHz (at −9 mm tuning position)
- Angular dependence of field strength possibly due to inner cone offset
E-Field Distribution Measurement via Side Slots

Probe depth: 8mm

top view

• Measurement results are abnormal, but repeatable
• Not much similar to field distribution in angular position of bottom slot measurements
Bead-pull Measurement

Bead-pull measurement is an application of the cavity material perturbation technique.

By inserting a foreign object into the cavity, the fractional change of the resonance frequency will be

$$\frac{\Delta \omega}{\omega_1} = - \frac{\int (\Delta \varepsilon |E_1|^2 + \Delta \mu |H_1|^2) \, d\Delta V}{\int (\varepsilon |E_0|^2 + \mu |H_0|^2) \, dV}$$

And with specific materials,

$$\frac{\Delta \omega}{\omega_0} \approx - \frac{\int (\Delta \varepsilon |E_0|^2) \, d\Delta V}{\int (\varepsilon |E_0|^2 + \mu |H_0|^2) \, dV} \propto |E_0|^2$$
Bead-pull measurement setup
E-Field Distribution Measurement via Bead-pull

Wire: 0.5mm
Bead: $r_{\text{bead}} = 3.8\,\text{mm}$, dielectric

- TM$_{010}$-like but with peaks and fluctuation
Quality Factor

**No slot**

- Data
- Fit function

Fit result:
- $X^2/\text{ndf} = 0.0010$
- $f_r = 4.70203$
- $Q_1 = 10913$
- $Q_{np} = 9476$
- Scale = -0.0130

**With Horizontal slot**

- Data
- Fit function

Fit result:
- $X^2/\text{ndf} = 0.0027$
- $f_r = 4.75881$
- $Q_1 = 6361$
- $Q_{np} = 6096$
- Scale = -0.0203
Fabrication of 2\textsuperscript{nd} Test Cavity

- The 2\textsuperscript{nd} aluminum prototype cavity was made in early 2024.
- The alignment mechanism was designed with guiding rails.
- The inner cone position was tuned by aluminum sheets. (gearbox is still under development)
Fabrication of 2nd AL prototype Cavity

- Whole weight ~ 6 kg
- Inner cone weight ~ 1 kg

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<th>d</th>
<th>gap$_{in}$</th>
<th>gap$_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6061 Al alloy</td>
<td>64 (mm)</td>
<td>31.35 (mm)</td>
<td>200 (mm)</td>
<td>4.805 (GHz)</td>
<td>3.5 (deg)</td>
<td>3.353 (deg)</td>
<td>1.5 (mm)</td>
<td>1.5 (mm)</td>
<td>0.1 (mm)</td>
</tr>
</tbody>
</table>
E-Field Distribution Measurement via Bead-pull

Wire: 0.09mm
Bead: $r_{\text{bead}} = 4.6\text{mm}$, $h_{\text{bead}} = 6\text{mm}$, Teflon

- Field concentrate at higher location
- Agree with simulation result
• Frequency tuning has nearly identical tuning range as design
• ~70 MHz off from design
- Rough frequency tuning method -> unstable
Summary

• To enhance the sensitivity to $g_{\alpha\gamma\gamma}$, big volume cavity is needed
• Conic-shell cavity design
  • Alignment
• 1st Aluminum prototype cavity
  • Capability of frequency tuning
  • Low Q (cause by slots)
  • Abnormal E-field distribution
• 2nd Aluminum prototype cavity
  • Higher Q (No slots)
  • E-field distribution agree with simulation

The design is confirmed and can be used in next phase experiment
Thanks for your attention
\[
SNR = \frac{P_s}{\sigma_n} = \frac{P_s}{k_B T_{sys}} \frac{t}{\Delta f}
\]

\[
\propto g_{\alpha \gamma \gamma} V C_{mnl} Q_0 \frac{\beta}{(1 + \beta)^2} \frac{f_r T}{2 Q_L \Delta W}
\]

\[
\Delta f = \frac{1}{2 Q_L} = \frac{1}{2 Q_0} \frac{f_r}{1 + \beta}
\]

\[
t = \frac{T}{N_{step}} - s
\]

\[
N_{step} = \frac{\Delta W}{\Delta f}
\]

- \(\sigma_n\): noise fluctuation
- \(T_{sys}\): noise temperature
- \(\Delta f\): frequency resolution bandwidth
- \(t\): data integration time
- \(N_{step}\): step number
- \(T\): data taking time
- \(s\): the operation time for each step
- \(\Delta W\): signal bandwidth
E-Field Distribution Measurement via Bead-pull

$10^5$ bead-pull measurement of a TE mode

Bead-pull measurement of hole1

Frequency change df (Hz)

Bead position (mm)
E-Field Distribution Measurement via Side Slots

- Deeper cavity position in radius, stronger field
- Oscillating field distribution (possibly from TE mode mixing)
- Affect form factor calculation