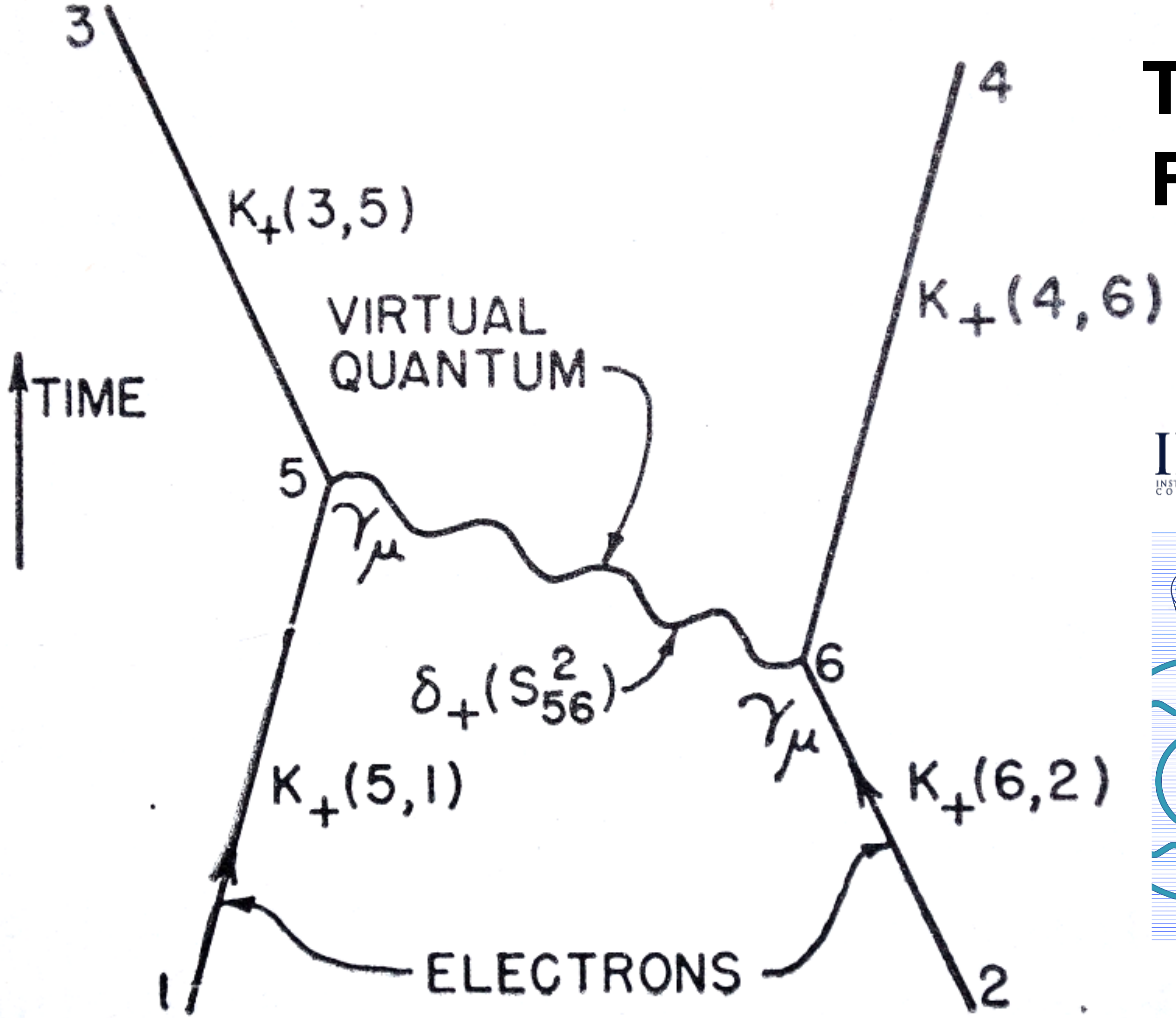

The quantum life of a Feynman propagator as a qubit

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Quantum Field Theory (QFT) is quantum


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High-energy colliders (LHC)
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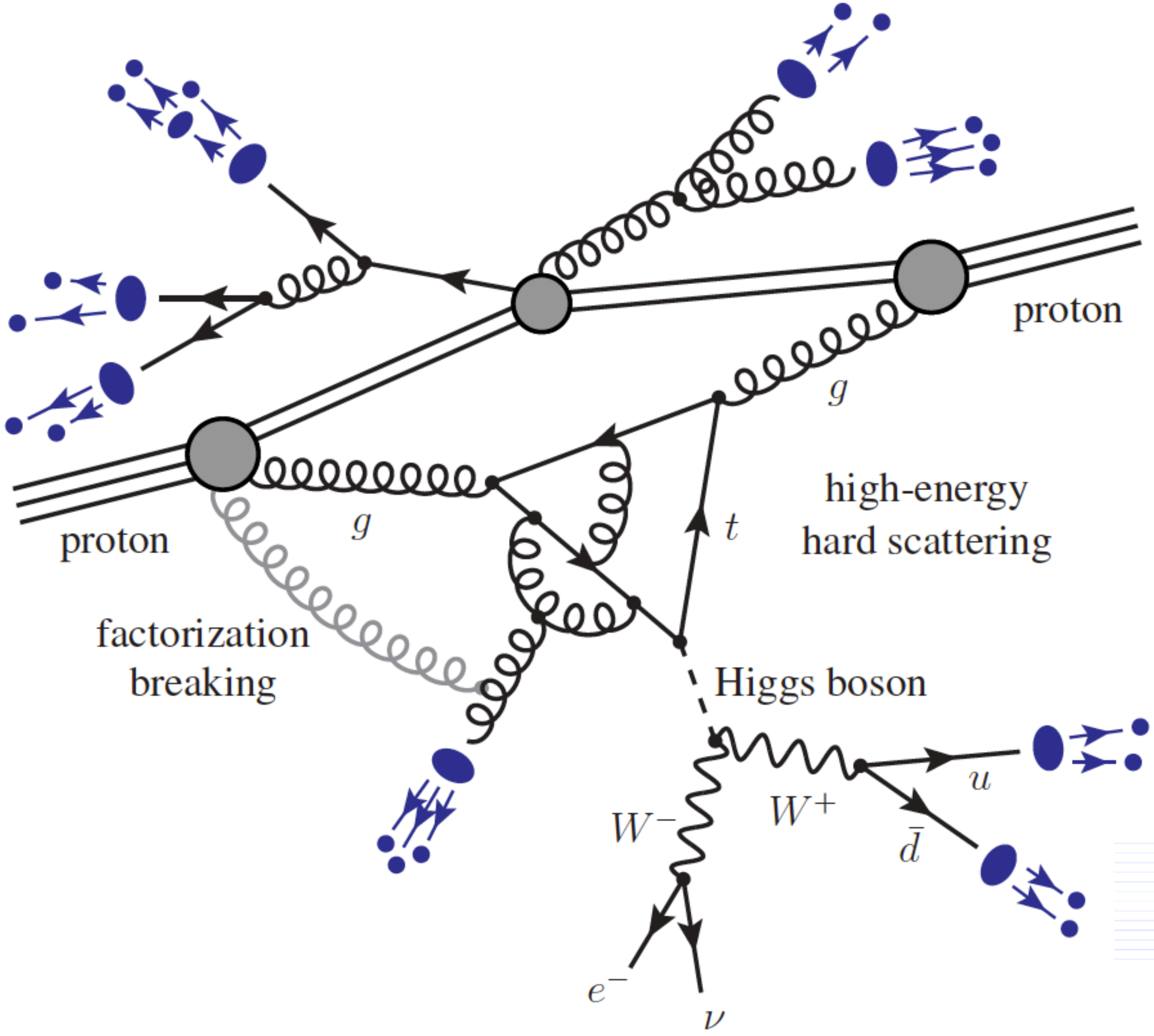
Article | Open access | Published: 18 September 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

Nature **633**, 542–547 (2024) | Cite this article


"Nature isn't classical, dammit, and if you want to make a simulation of nature, you better make it quantum"
- Richard P. Feynman



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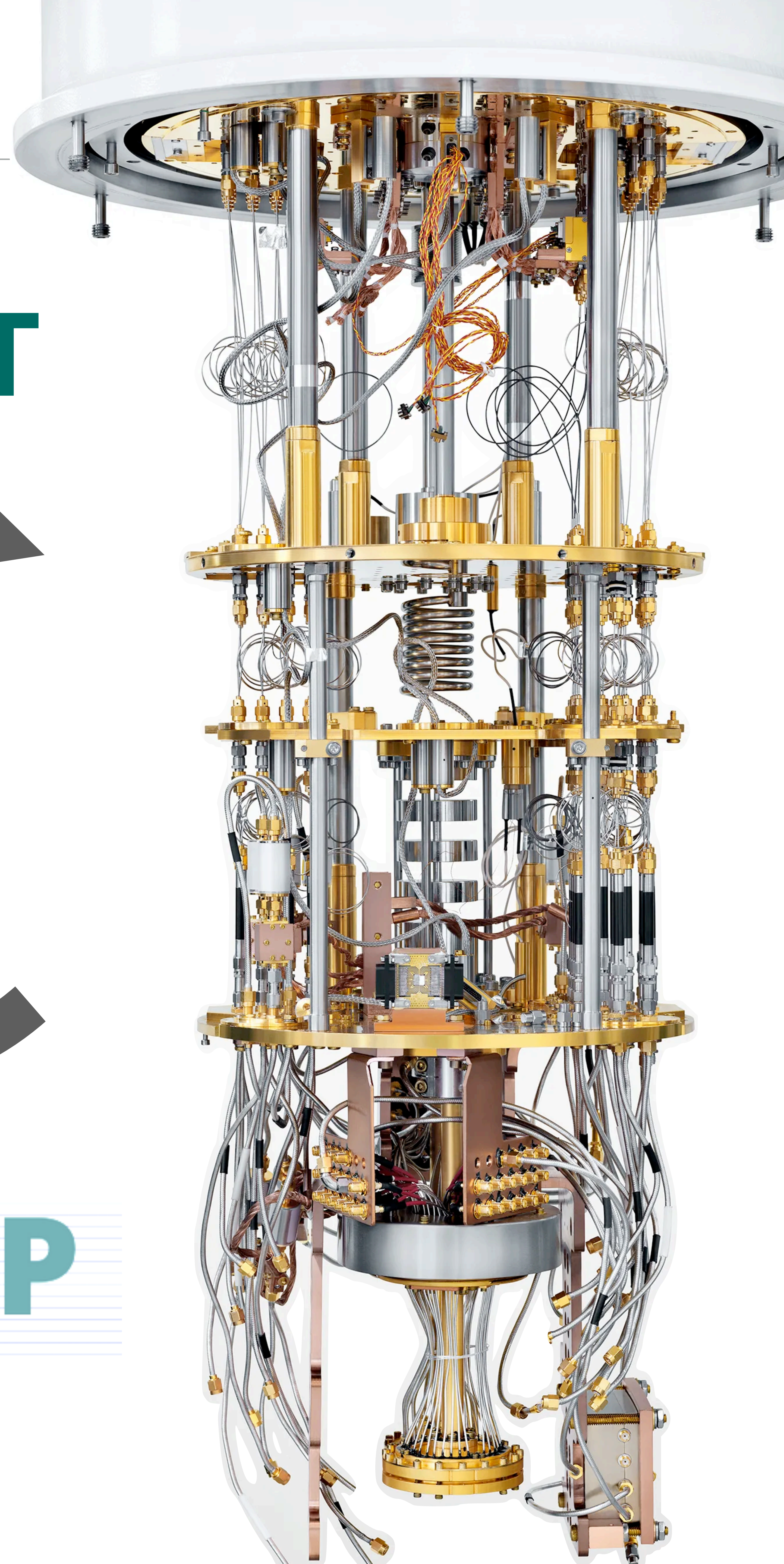
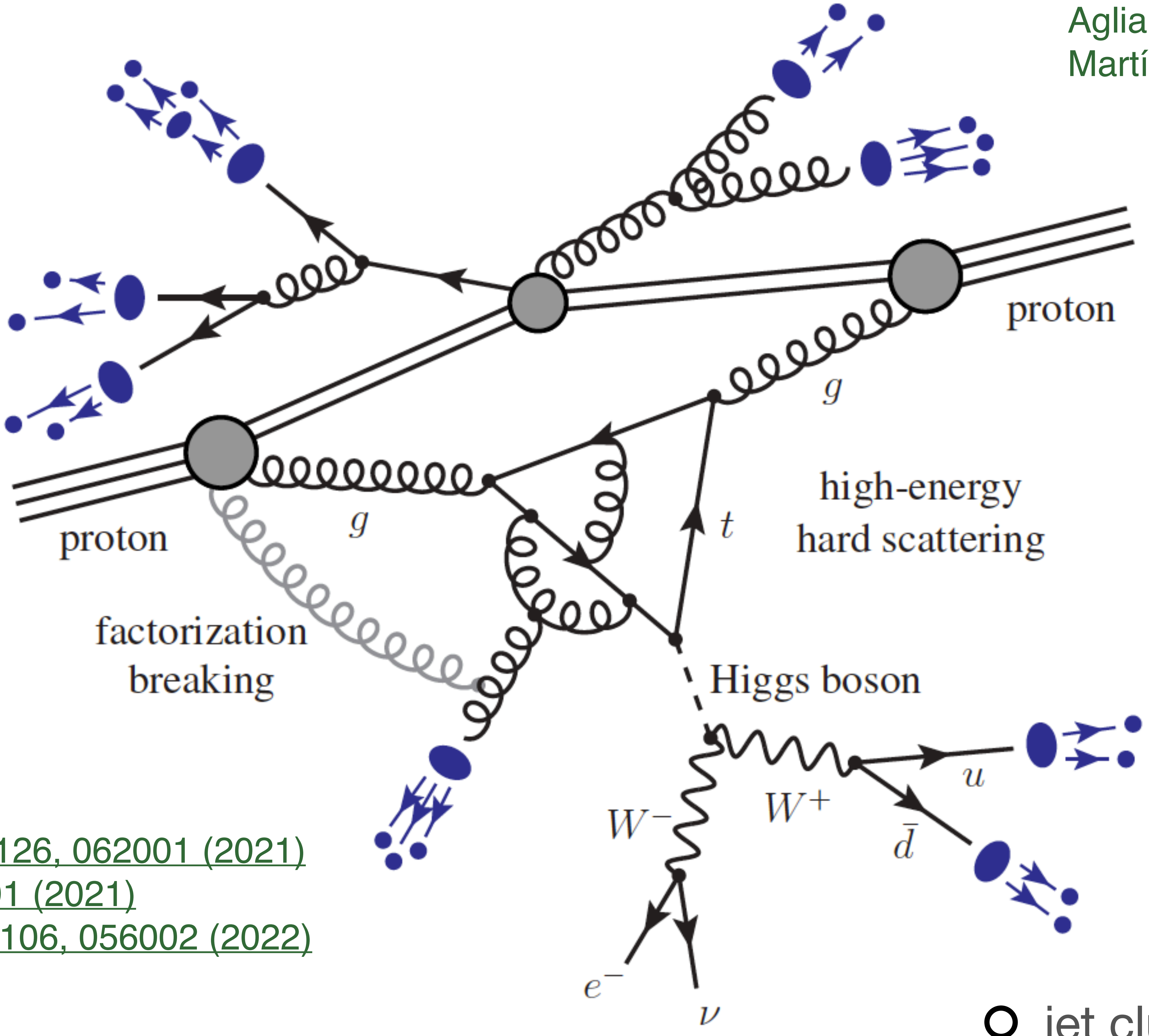


Quantum at colliders



- Monte Carlo integration:
  Herbert, Q6, 823 (2022)
  Agliardi, Grossi, Pellen, Prati, PLB 832, 137228 (2022)
  Martínez de Lejarza, Grossi, Cieri, GR, 2305.01686

- track reconstruction:
  Mangano et al., PRD 105, 076012 (2022)
  Duckett, Facini, Jastrzebski, Malik, Scanlon, Rettie, PRD 109, 052002 (2024)
  Schwägerl, Issever, Jansen, Khoo, Kühn, Tüysüz, Weber, 2303.13249

- tree-level helicity amplitudes:
  Bepari, Malik, Spannowsky, Williams, PRD 103, 076020 (2021)

- multiloop scattering amplitudes:
  Ramírez, Rentería, GR, Sborlini, Vale Silva, JHEP 2205, 100 (2022)
  Clemente, Crippa, Jansen, Ramírez, Rentería, GR, Sborlini, Vale Silva, PRD 108, 096035 (2023)

- parton densities:
  Pérez-Salinas, Cruz-Martínez, Alhajri, Carrazza, PRD 103, 034027 (2021)

- parton showers:
  Bauer, de Jong, Nachman, Provasoli, PRL 126, 062001 (2021)
  Bauer, Freytsis, Nachman, PRL 127, 212001 (2021)
  Bepari, Malik, Spannowsky, Williams, PRD 106, 056002 (2022)

- jets in a medium:
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  Barata, Salgado, EPJC 81, 862 (2021)

- jet clustering:
  Wei, Naik, Harrow, Thaler, PRD 101, 094015 (2020)
  Pires, Bargassa, Seixas, Omar, 2101.05618
  Pires, Omar, Seixas, 2012.14514
  Martinez de Lejarza, Cieri, GR, PRD 106, 036021 (2022)

- quantum machine learning:
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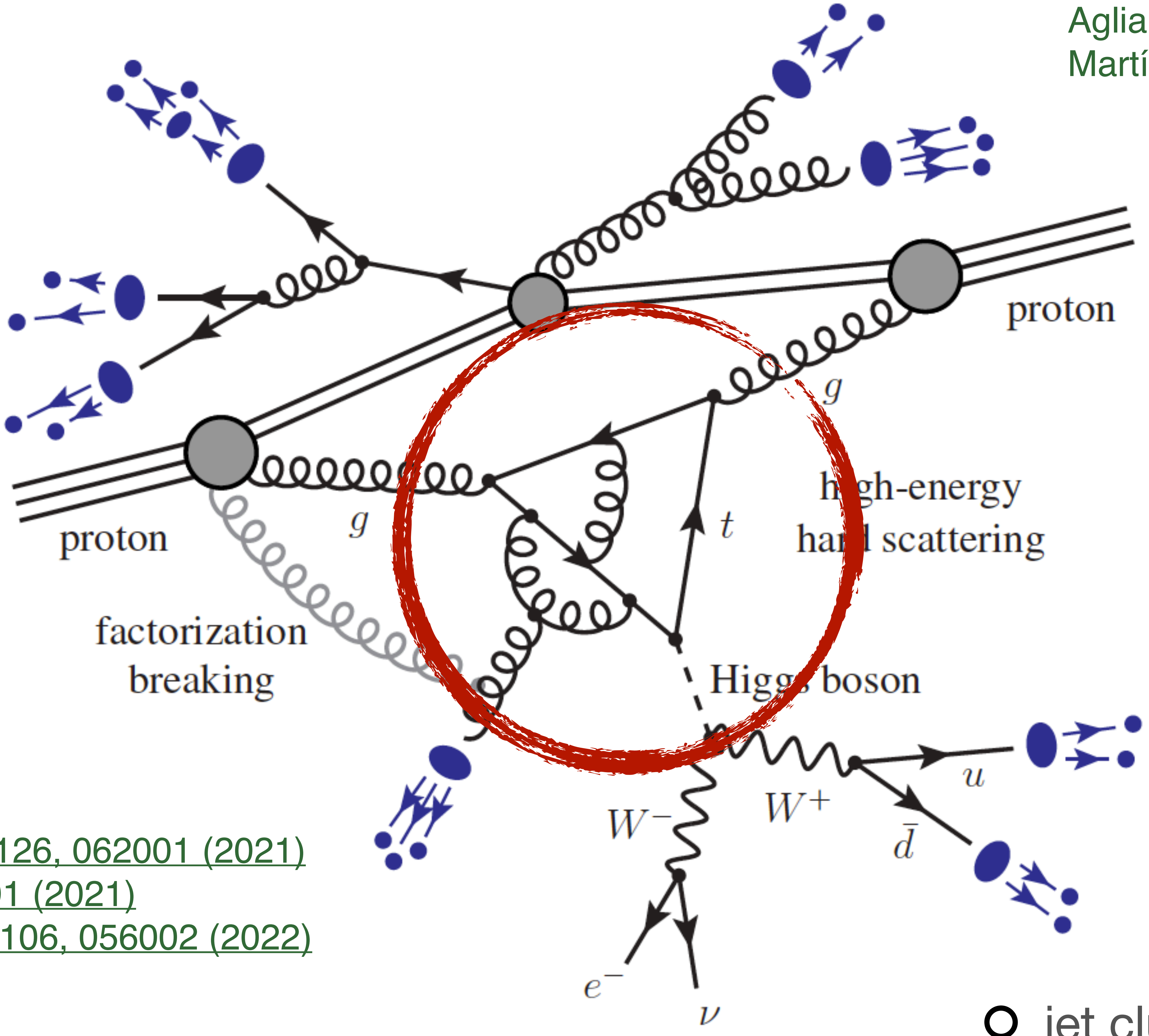
  Wu et al., JPG 48, 125003 (2021)
  Felser, Trenti, Sestini, Gianelle, Zuliani, Lucchesi, Montangero, npjQI 7, 111 (2021)

Quantum at colliders



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A Feynman propagator is a qubit

- A Feynman propagator describes a **quantum superposition** of propagation in both directions

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + \imath 0} \equiv \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right)$$

- A Feynman diagram is a superposition of 2^n states

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- A Feynman diagram is a superposition of 2^n states
- If a particle returns to the point of emission: it **travels back in time** and thus **breaks causality** ≡ cyclic configurations are nonphysical
- Causal configurations of Feynman diagrams are **directed acyclic graphs (DAG)** in graph theory

BC: Ramírez, Rentería, GR, Sborlini, Vale Silva, JHEP 2205, 100 (2022)
MCX: Ramírez, Rentería, GR, 2404.03544, European Patent EP24382213

A Grover's based quantum algorithm

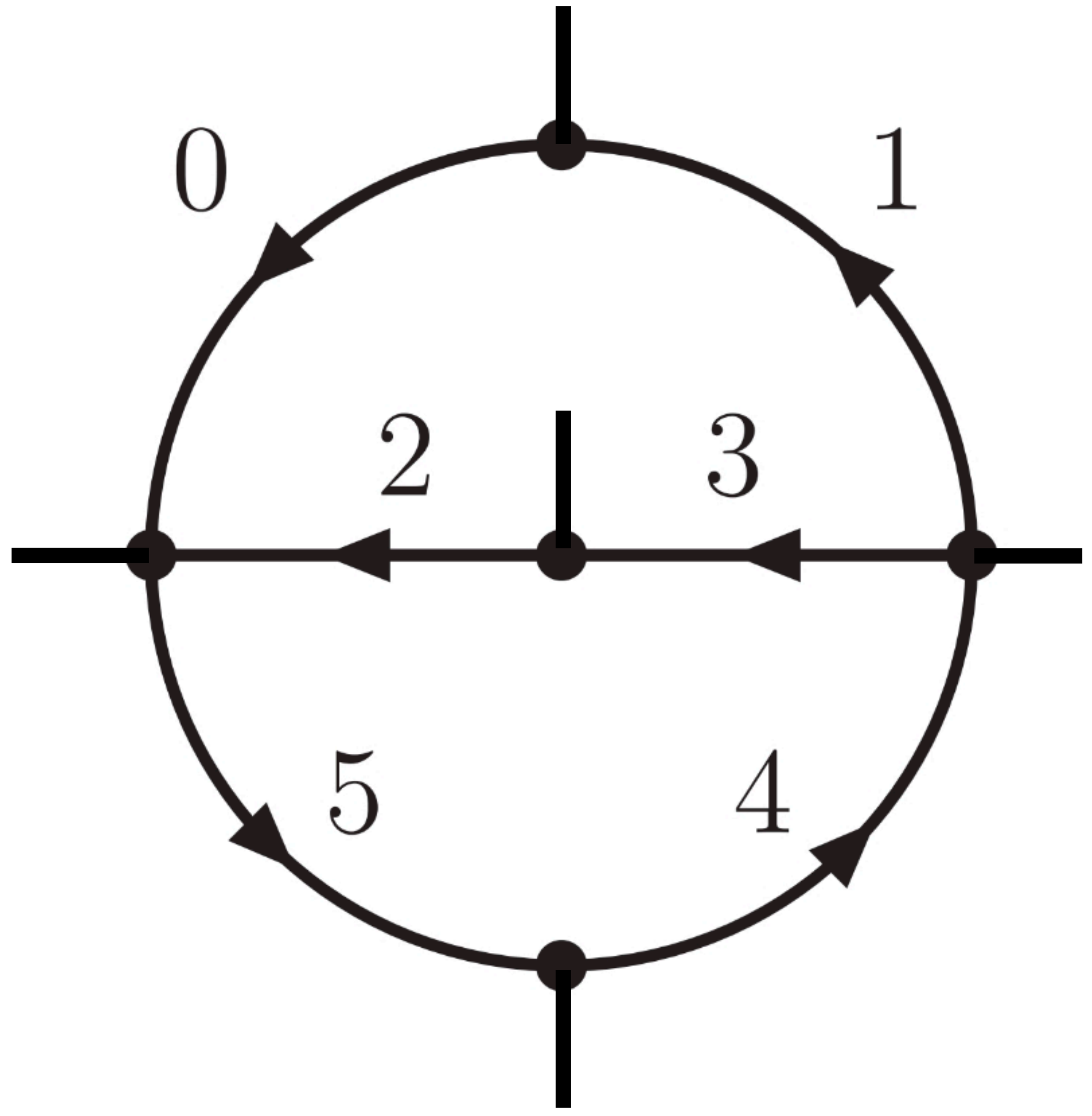
- **The $|e\rangle$ register** encodes the states of the edges/internal propagators: the qubit e_i is in the state $|1\rangle$ if the momentum flow of the corresponding edge is oriented in the direction of the original assignment, and $|0\rangle$ if it is in the opposite direction
- **The $|a\rangle$ register** stores the loop clauses that probe if all the qubits (edges) in each subloop form a cyclic circuit: constructed with **multi-controlled Toffoli gates** and NOT (Pauli-X) gates.
- **The Grover's marker** initialized to the Bell state $|out\rangle = |-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$
- **The oracle operator**

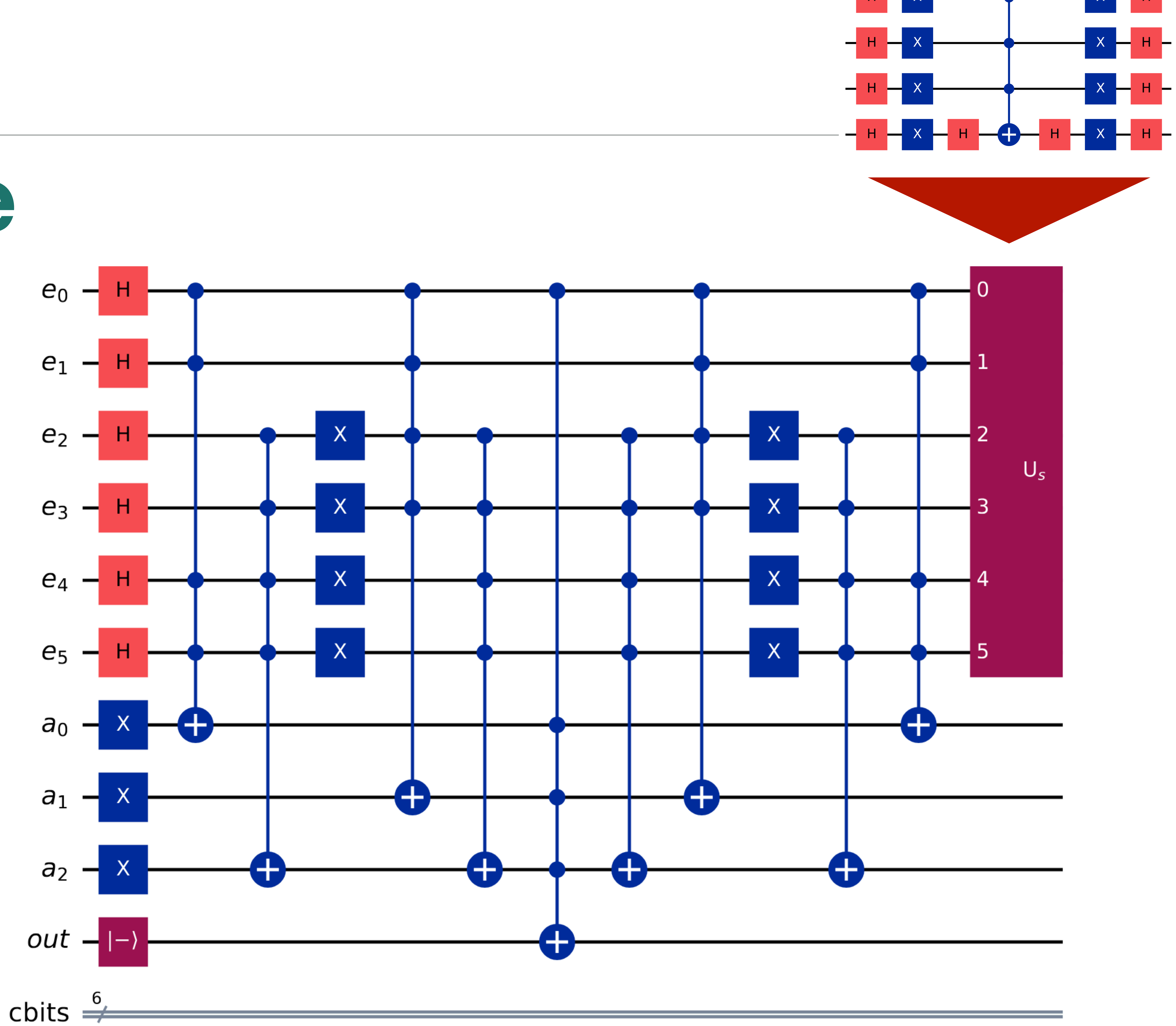
$$U_w|e\rangle|a\rangle|out\rangle = |e\rangle|a\rangle|out \otimes f(a,q)\rangle$$

$$|out \otimes 0\rangle = |out\rangle$$
$$|out \otimes 1\rangle = -|out\rangle$$

- **The diffuser operator U_s**

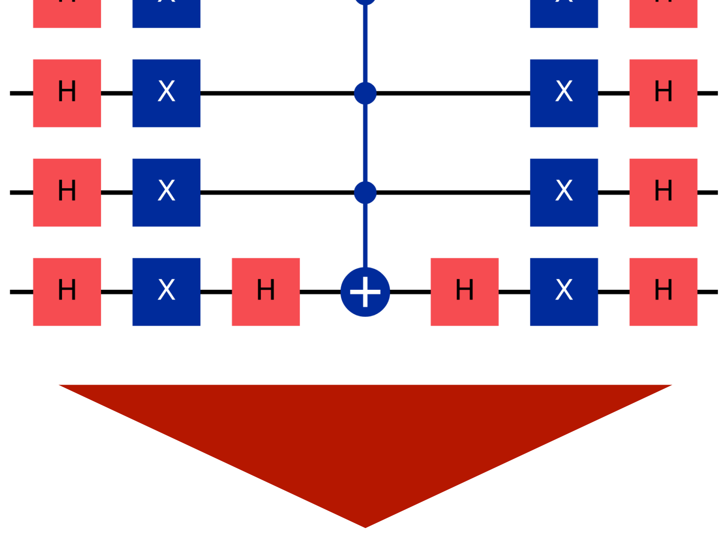
A two-loop example

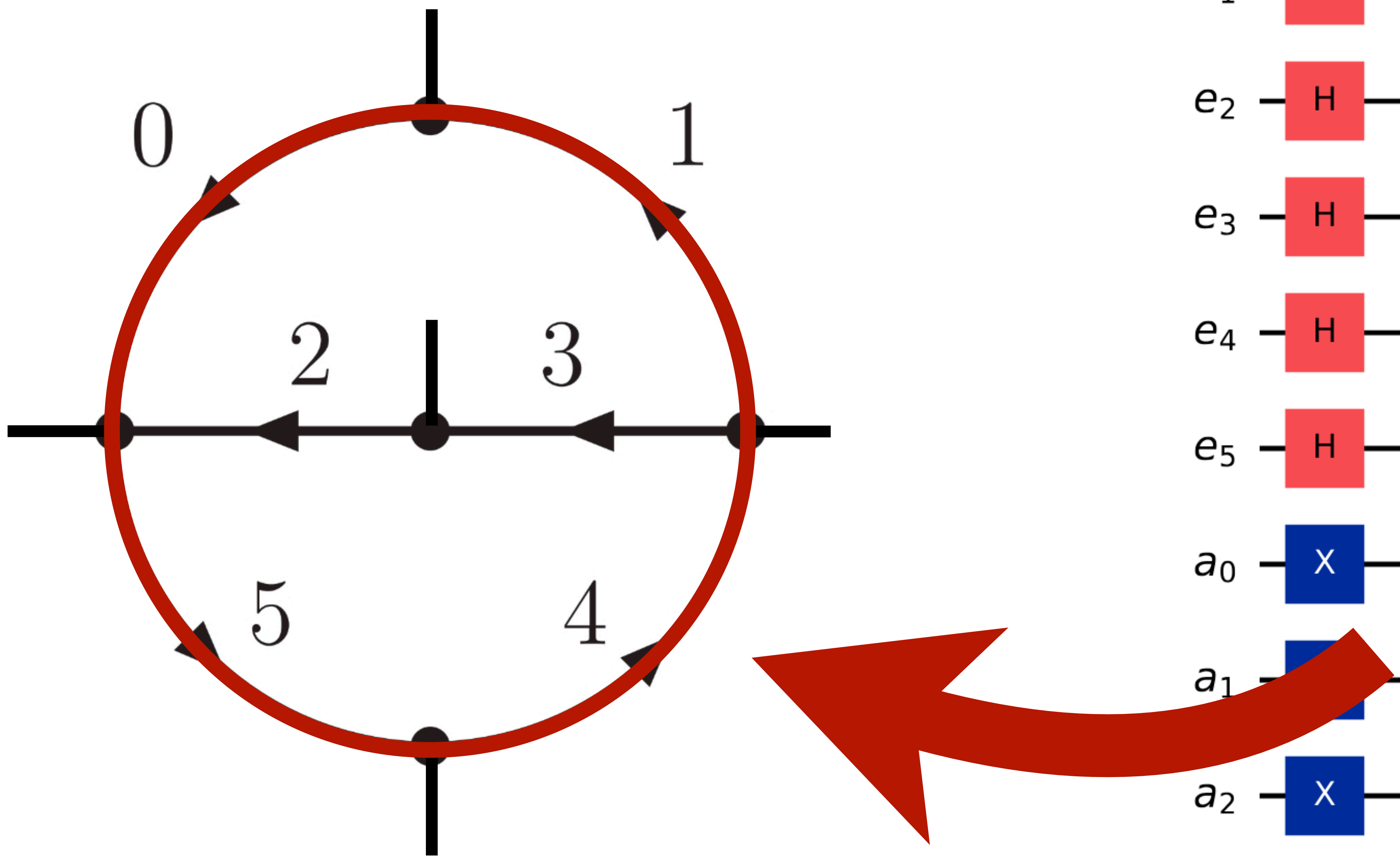


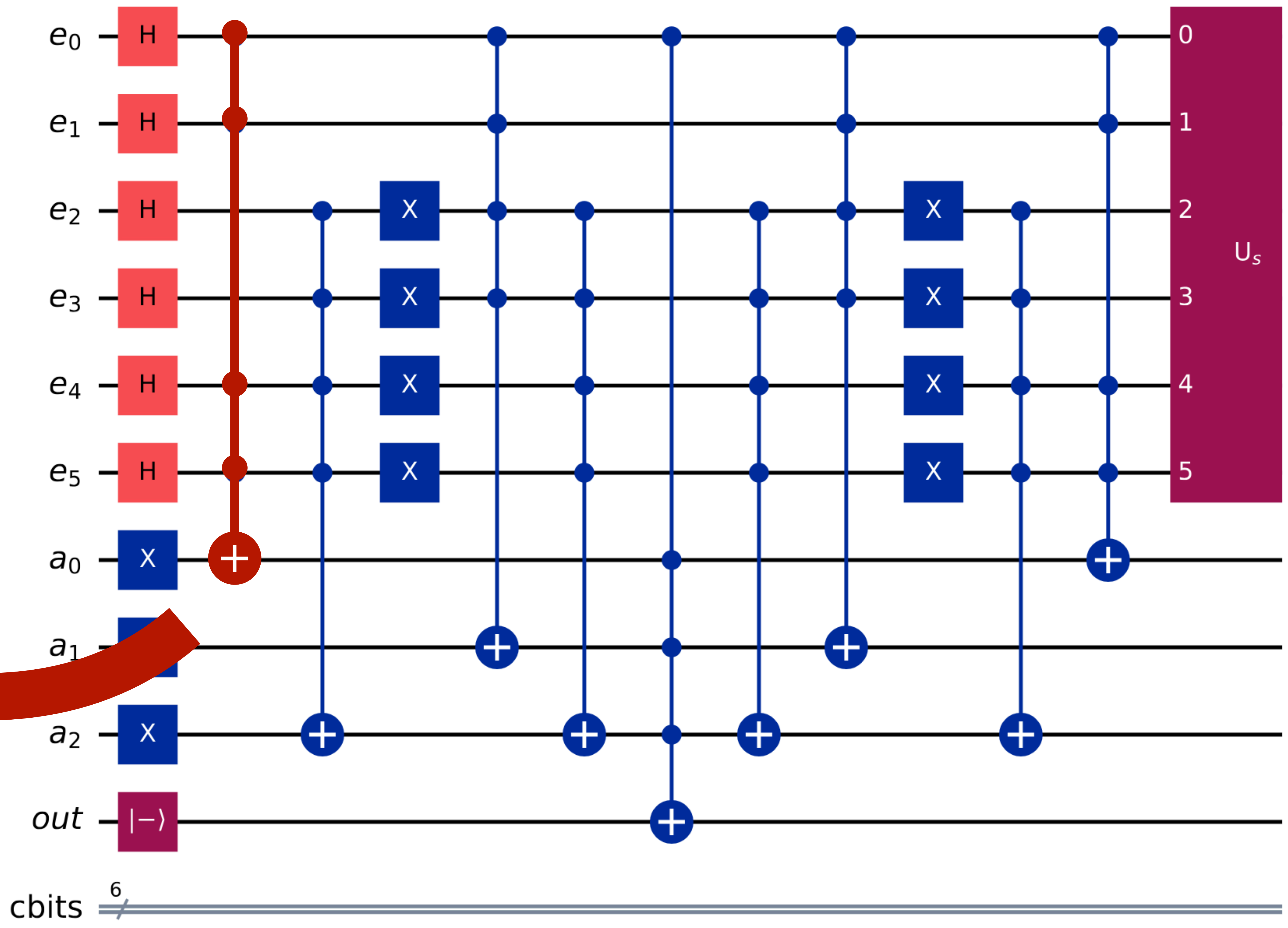


eloops (edges)	Qubits	Quantum Depth	θ	$\sin^2(\theta_t)$	Causal states	Total states
two (6)	10 | 16	14 | 22	36.8°	0.87	23	64

A two-loop example







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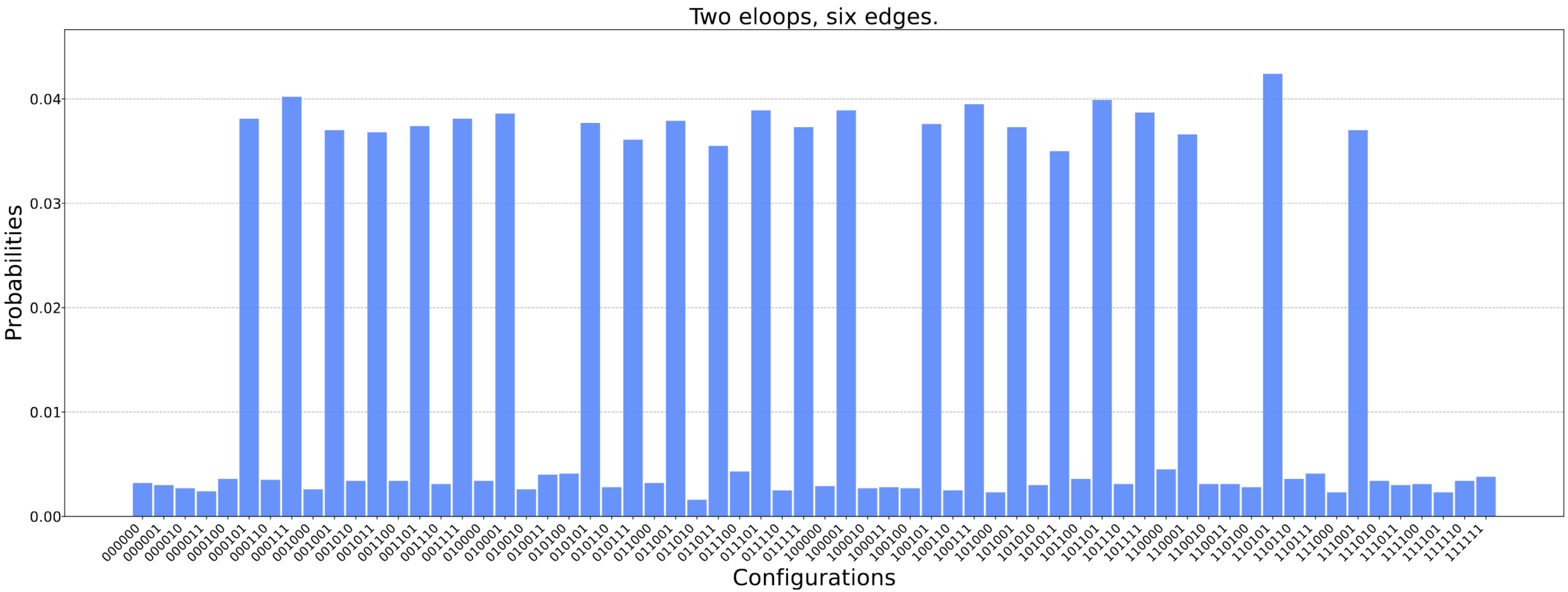
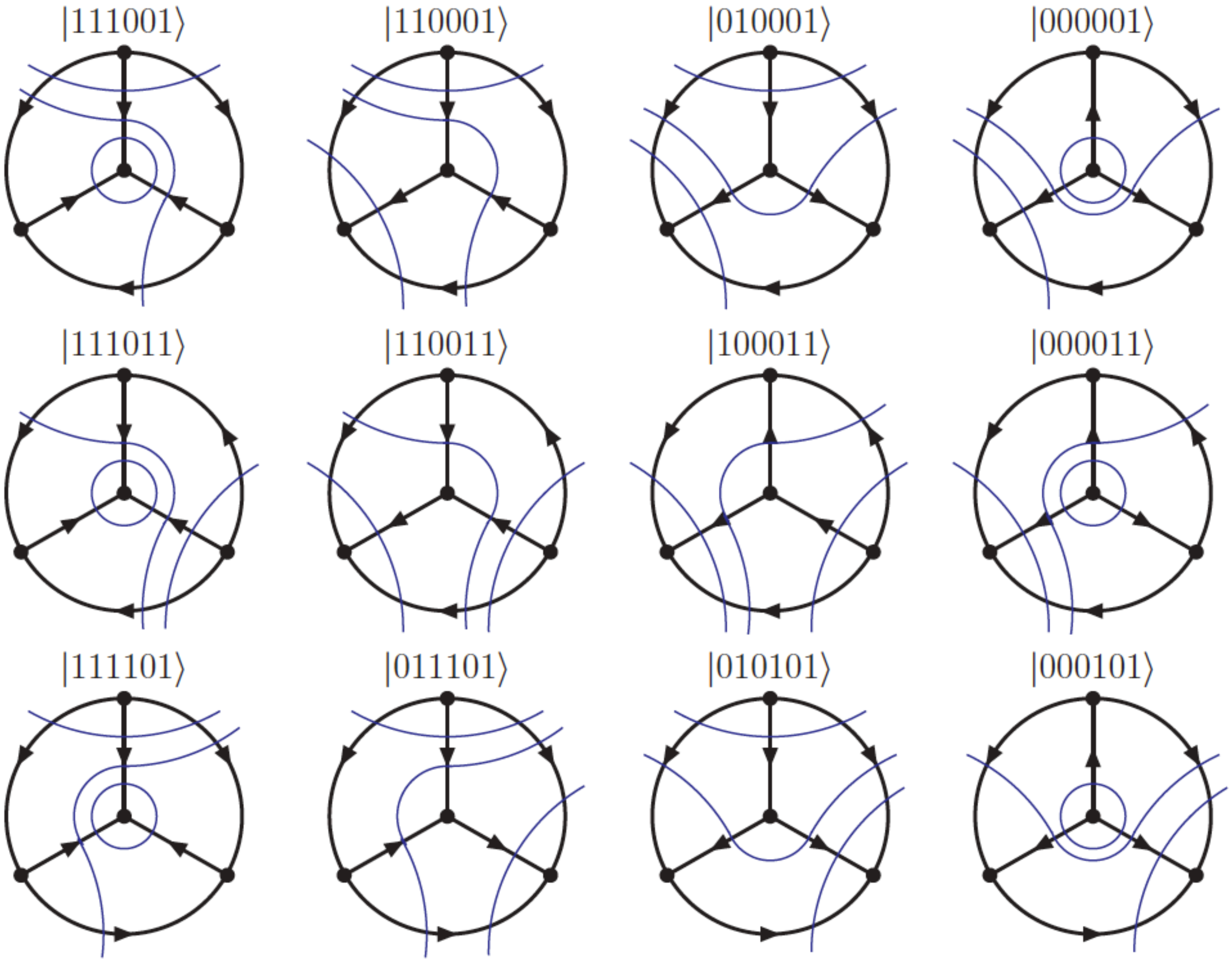
MCX | BC (binary clauses)

Probability distribution



Causal interpretation



BC: Ramírez, Rentería, GR, Sborlini, Vale Silva, JHEP 2205, 100 (2022)
MCX: Ramírez, Rentería, GR, 2404.03544, European Patent EP24382213

Quantum resources and depth

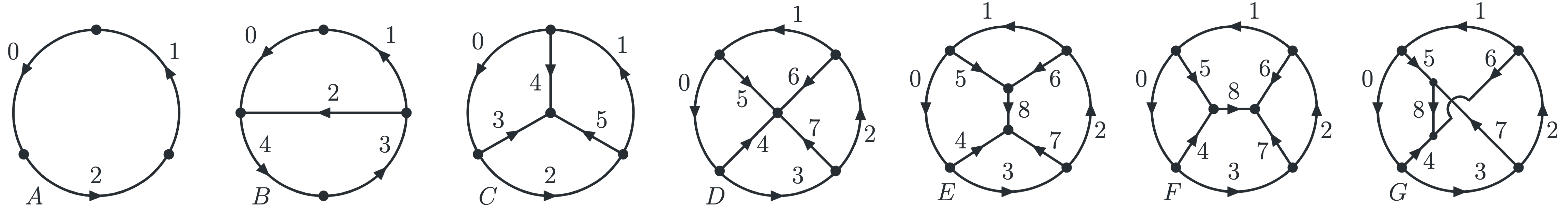
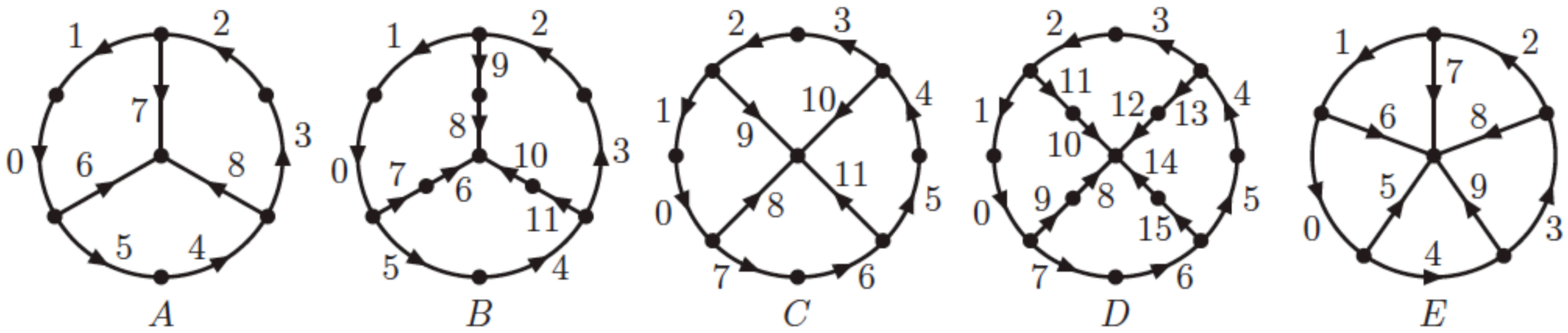


Fig.	eloops (edges)	Qubits		Quantum Depth		θ	$\sin^2(\theta_t)$	Causal states	Total states
1A	one (3)	5	7	6	18	37.7°	0.84	3	8
1B	two (5)	9	14	12	21	32.0°	0.99	9	32
1C	three (6)	11	19	18	24	25.7°	0.95	12	64
1D	four$^{(c)}$ (8)	14	25	16	23	33.5°	0.97	39	256
1E, 1F	four$^{(t,s)}$ (9)	15	28	20	25	26.5°	0.97	102	512
1G	four$^{(u)}$ (9)	19	33	32	28	28.3°	0.99	115	512

MCX | BC

BC: Ramírez, Rentería, GR, Sborlini, Vale Silva, JHEP 2205, 100 (2022)
MCX: Ramírez, Rentería, GR, 2404.03544, European Patent EP24382213

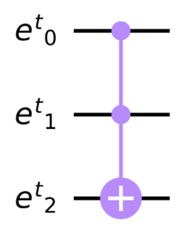
Quantum resources and depth



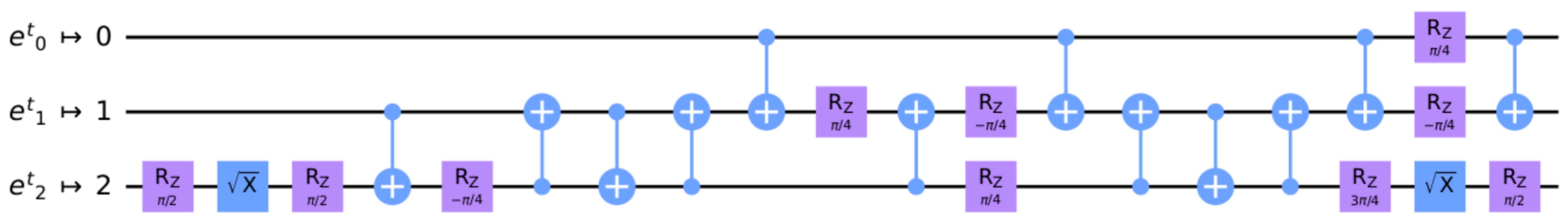
eloops(edges)	Total Qubits	Quantum Depth	θ	$\sin^2(\theta_t)$	$\|e\rangle$	$\|a\rangle$	Toffoli Gates	NOT Gates	Causal States	Total states
three (9)	14 | 25	18 | 27	35.2°	0.93	9	4	14	45	170	512
three(12)	21 | 36	32 | 33	28.0°	0.99	13	7	21	48	1804	8192
four$^{(c)}$(12)	18 | 33	16 | 31	32.8°	0.98	12	5	17	66	1199	4096
four$^{(c)}$(16)	31 | 50	46 | 53	27.7°	0.98	17	13	39	85	28343	131072
five$^{(c)}$(10)	17 | 31	24 | 25	28.9°	0.99	10	6	25	37	240	1024

MCX | BC

The quantum area



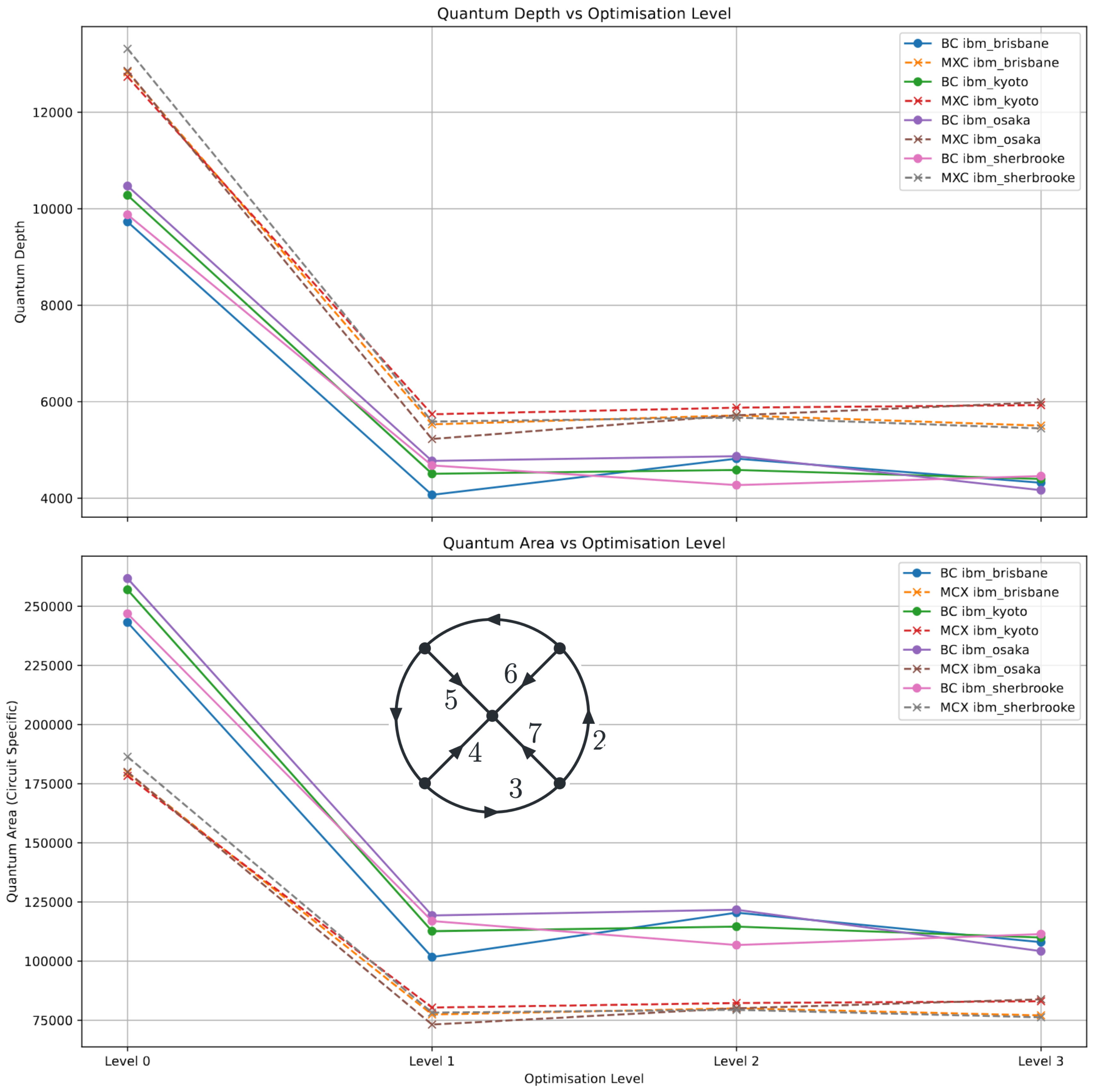
a) Toffoli gate

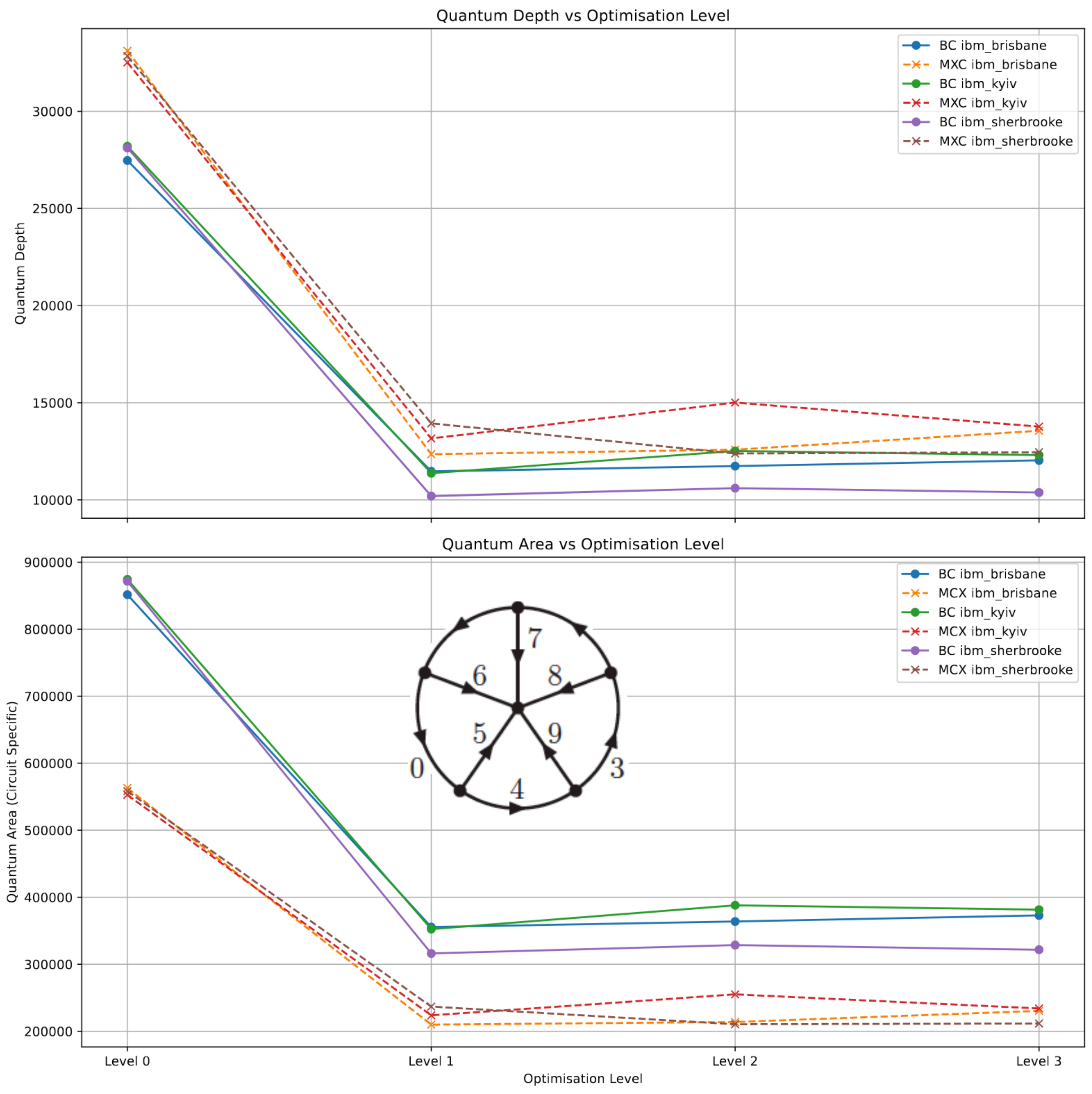


b) Transpiled Toffoli gate.

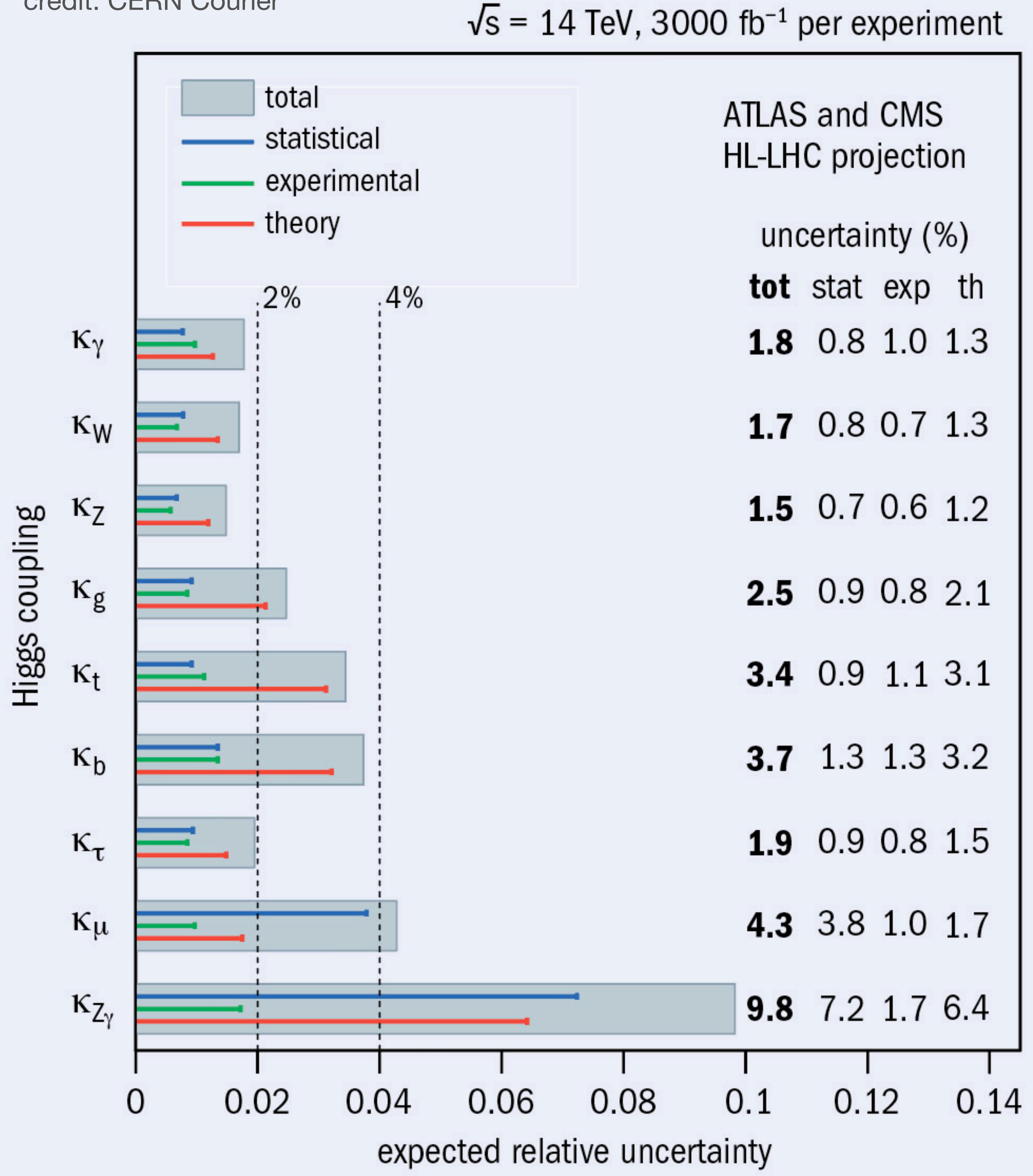
- Huge difference between the theoretical quantum depth and the quantum depth of the **transpiled** quantum circuit
- **Transpilation** may involve more qubits than initially thought

- **Quantum area:** the product of the quantum depth and number of qubits required for the transpiled quantum circuit
- Hardware dependent
- Good measure of the dramatic difference in running times (**from minutes to seconds**)





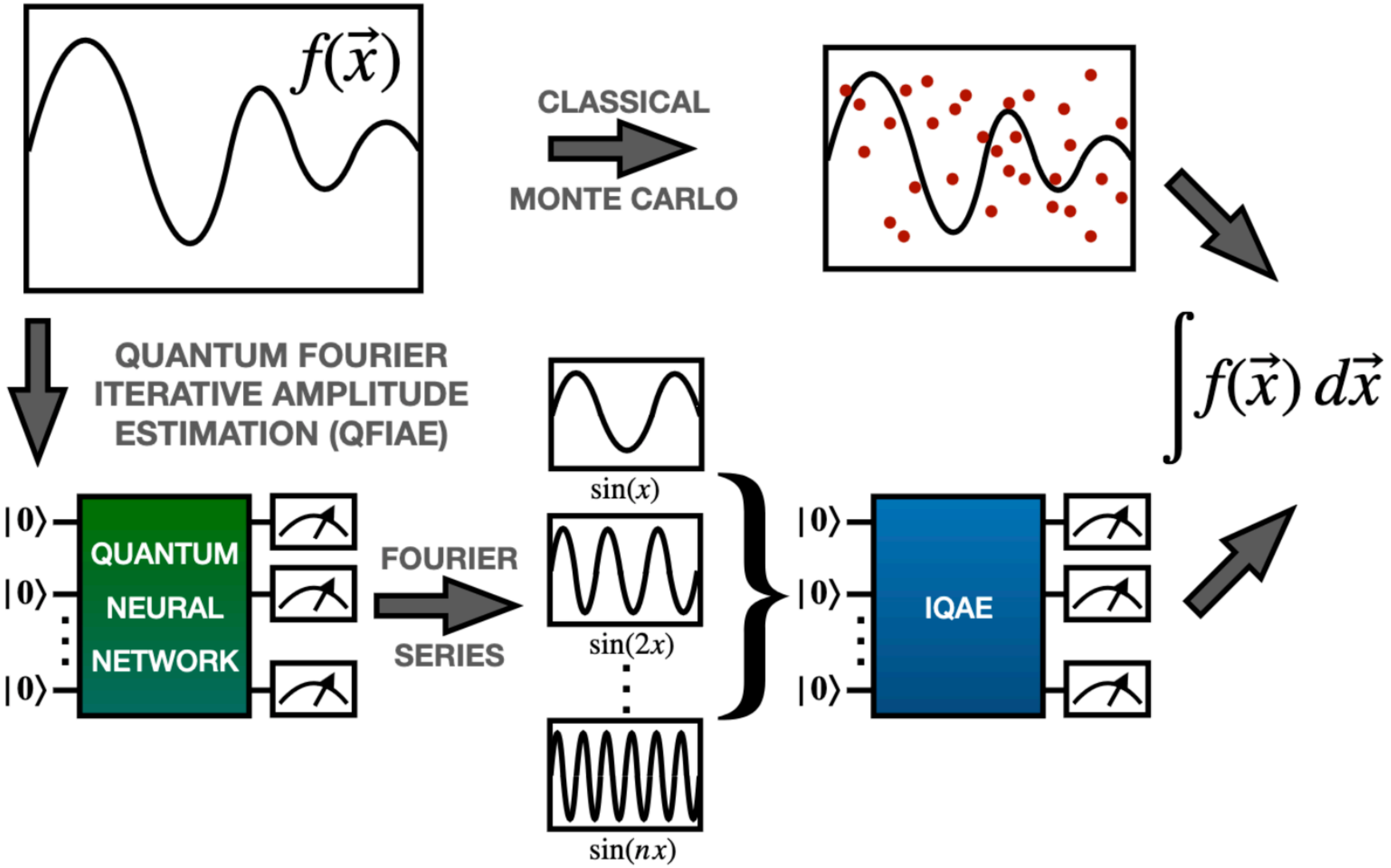
credit: CERN Courier



- Need to go beyond the current state of the art in **theory predictions** to match precision measurements and unveil potential discoveries

Martínez de Lejarza, Grossi, Cieri, GR, IEEE QCE2023 [2305.01686]
Martínez de Lejarza, Cieri, Grossi, Vallecorsa, GR, PRD 110, 074031 (2024)

Quantum Fourier Iterative Amplitude Estimation (QFIAE)



- Integration of multidimensional functions
- Quantum Machine Learning + Grover's amplification
- Fourier series using a **Quantum Neural Network (QNN)**
- Integrates each trigonometric component using Iterative Quantum Amplitude Estimation (IQAE) [Grinko, Gacon, Zoufal, Woerner, npj QI 7, 52 (2021)], a variant of Grover's algorithm
- Long-term dream: a **quantum event generator**

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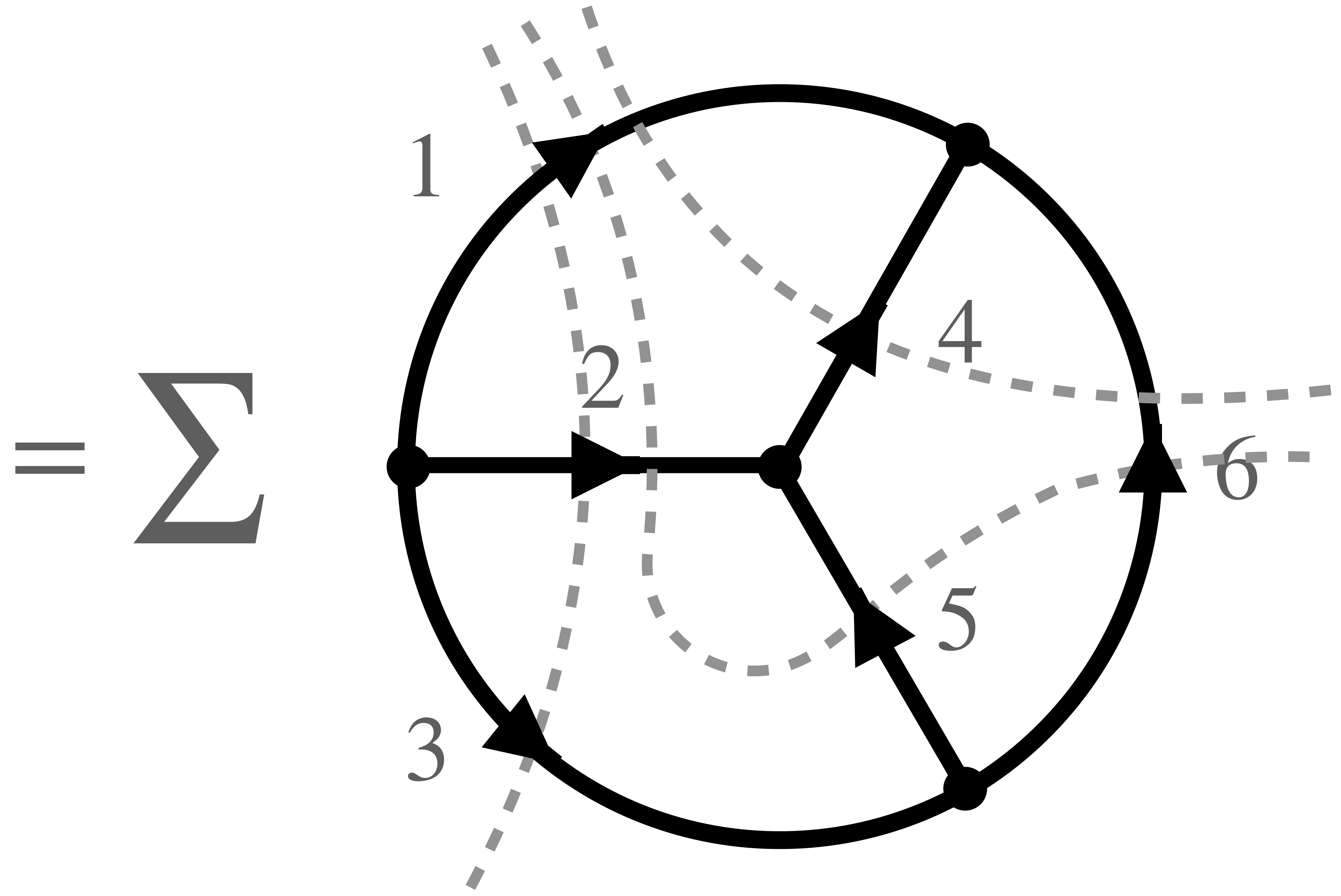
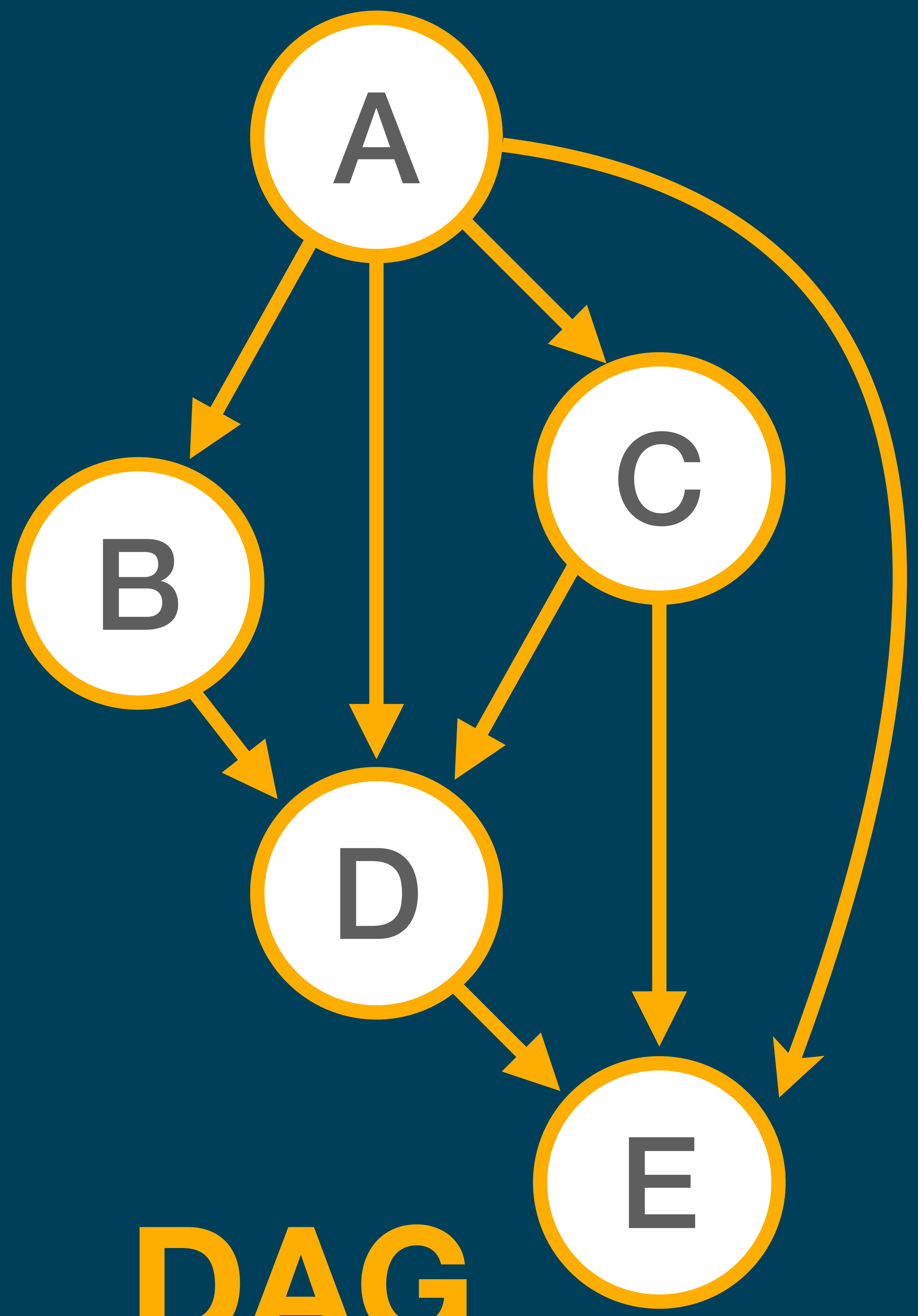
- **Vacuum amplitudes** (scattering amplitudes without external particles) as the optimal building blocks in the loop-tree duality (LTD), because it is **manifestly causal**



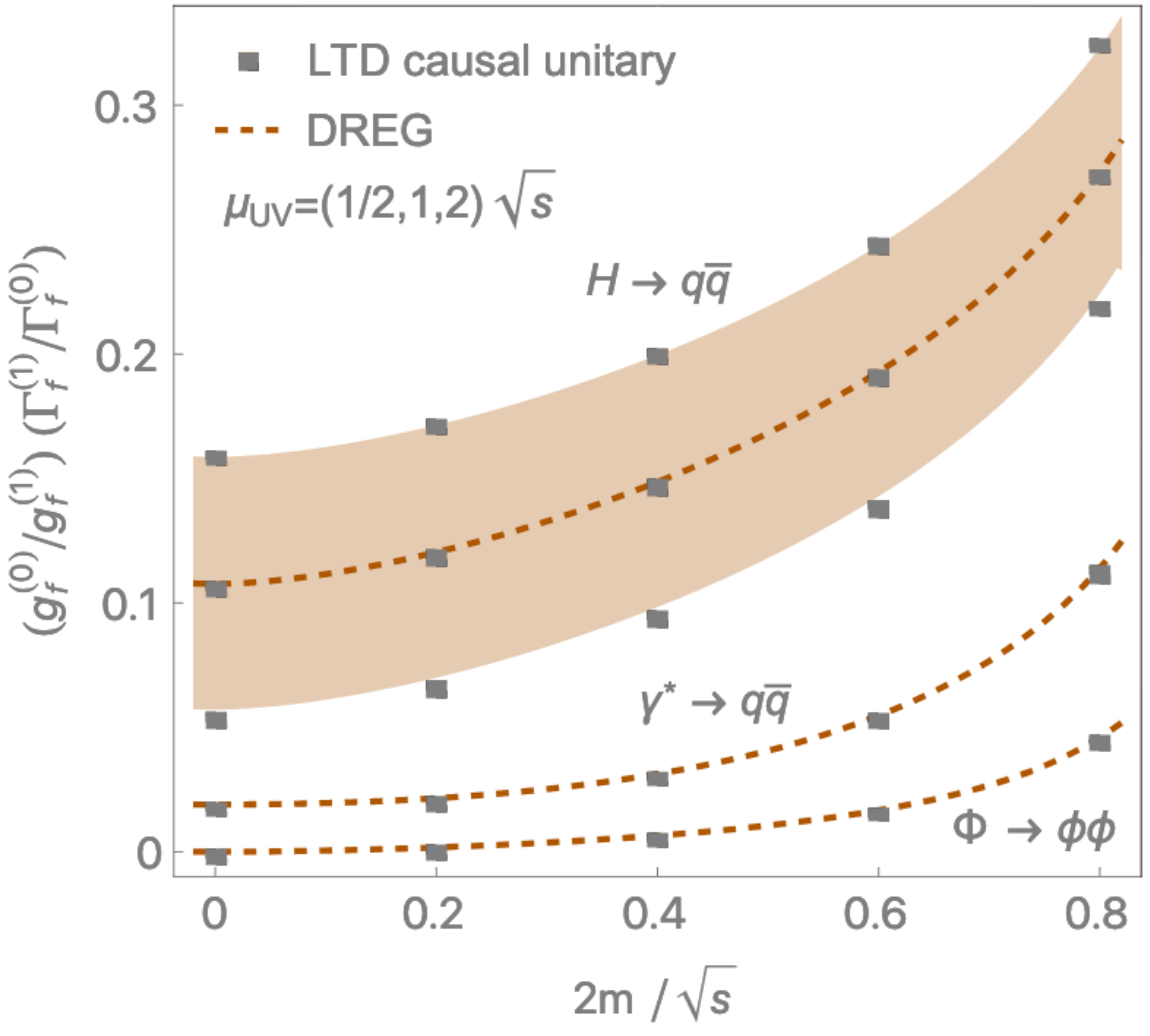
- S. Ramírez Uribe, P.K. Dhani, G.F.R. Sborlini, GR, "***Rewording theoretical predictions at colliders with vacuum amplitudes***," PRL133, 211901 (2024)

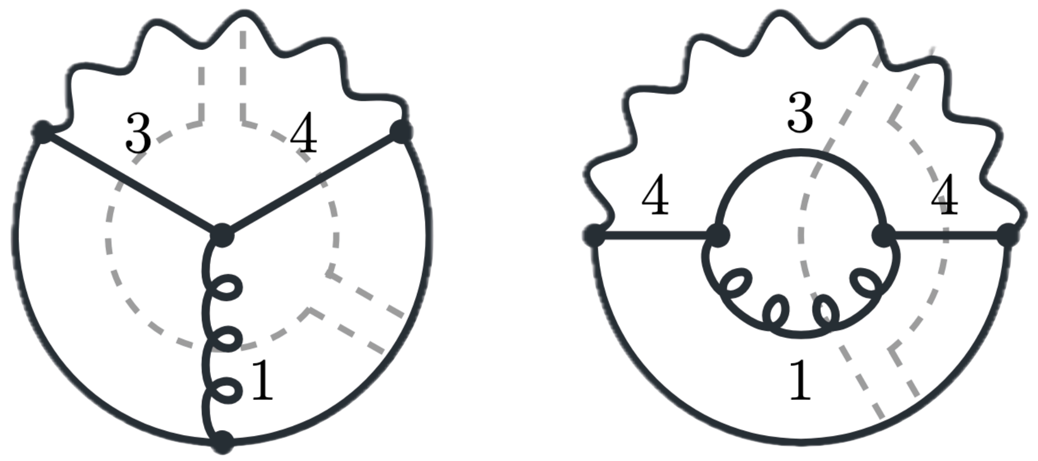


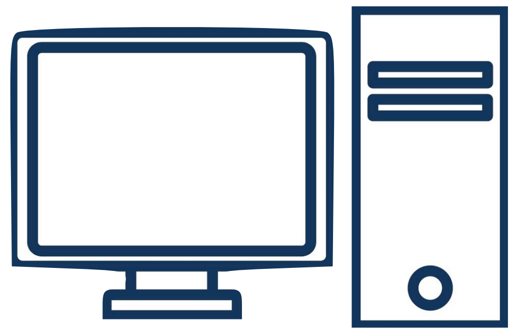
DAG

Directed Acyclic Graph

Decay rate of the Higgs/photon/toy-scalar at NLO



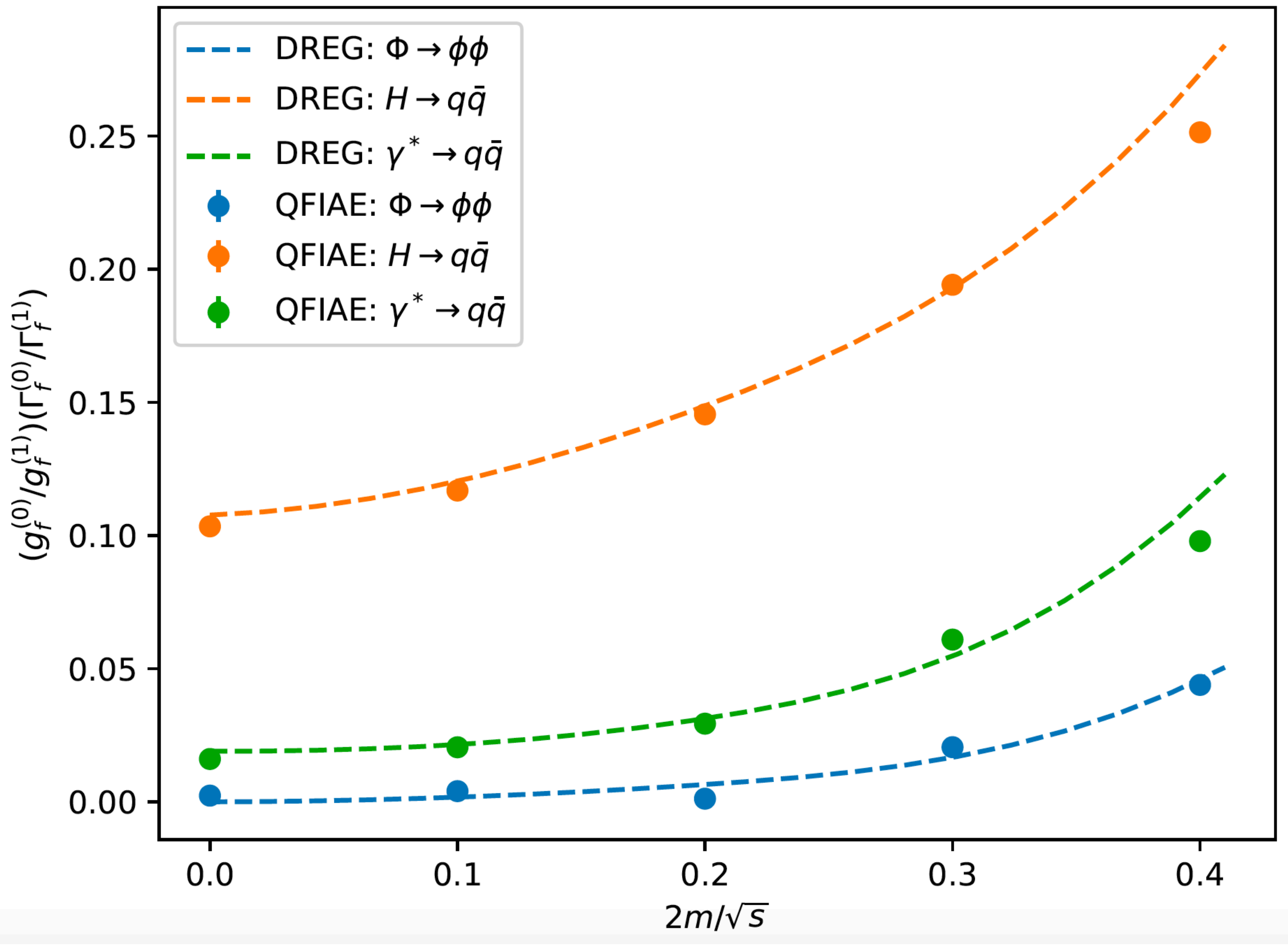




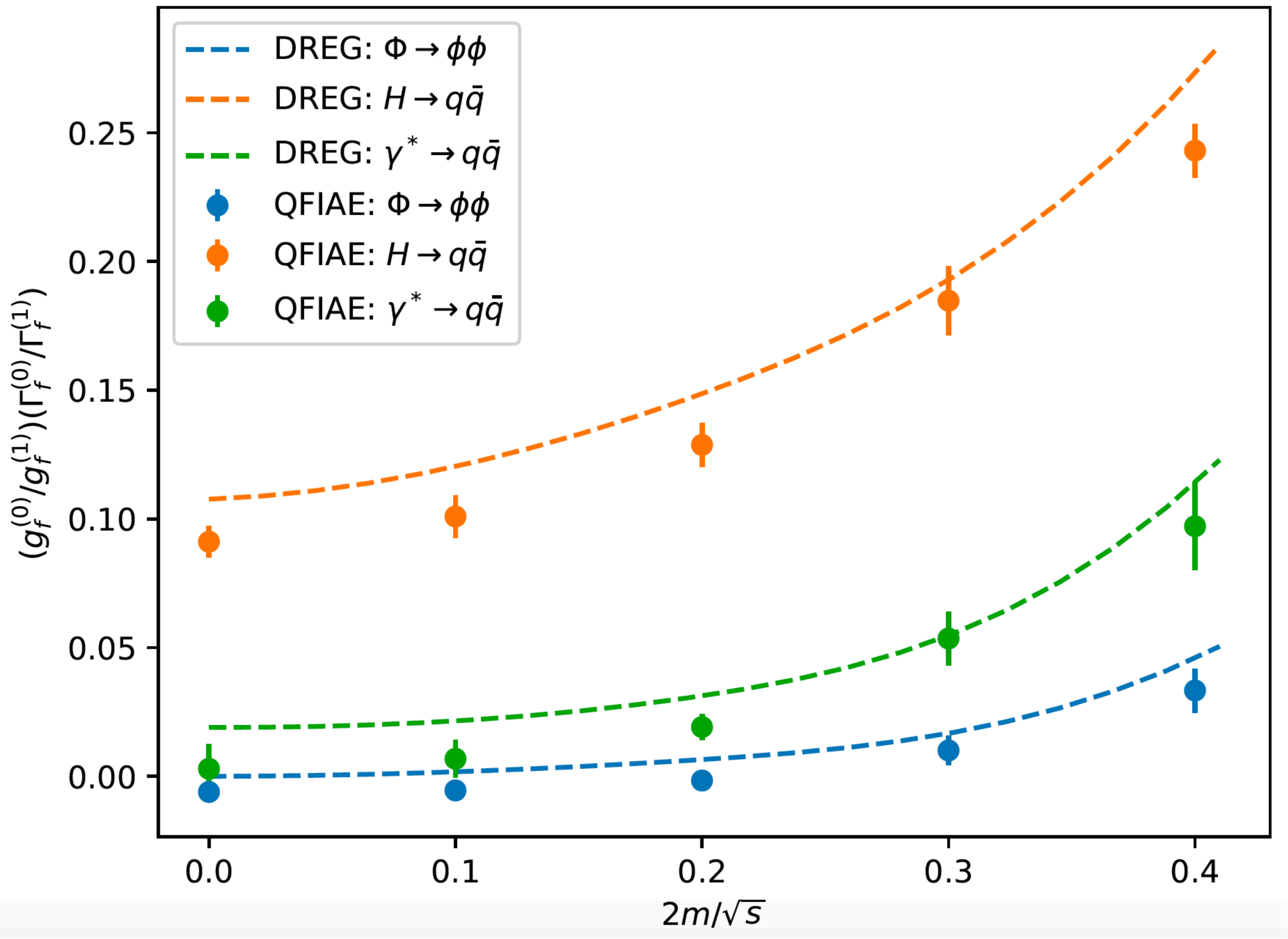
- Classical Monte Carlo

Vacuum amplitudes and time-like causal unitary in the loop-tree duality, LTD Collaboration, to appear in JHEP [2404.05492]

Quantum integration of decay rates at second order in perturbation theory
J.J. Martínez de Lejarza, D.F. Rentería Estrada, M. Grossi, GR, to appear in QST [2409.12236]

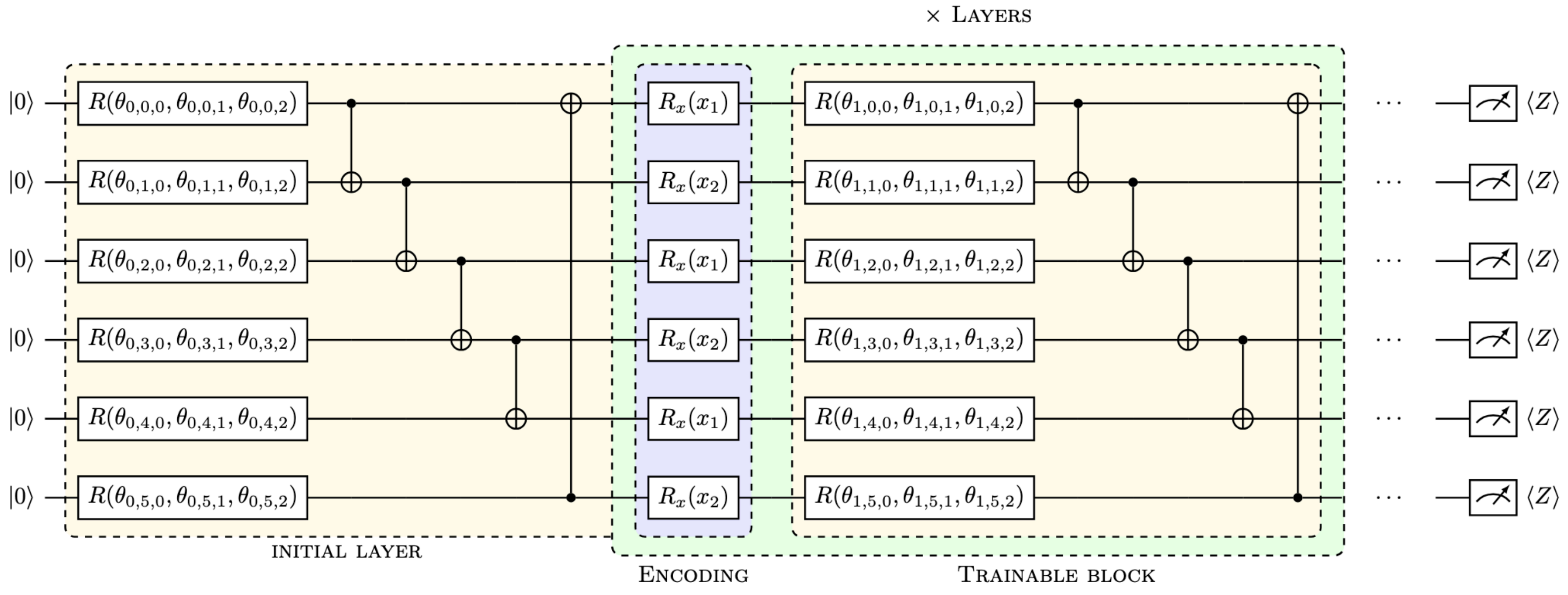


- Quantum simulator



- Partially in quantum hardware

Technical details



Decay	$2m/\sqrt{s}$	Hardware	Simulator	DREG
$\Phi \to \phi\phi(\phi)$	0.0	−0.0061(28)	0.0023(5)	0.0000
	0.1	−0.0055(31)	0.0040(6)	0.0018
	0.2	−0.0016(30)	0.0011(6)	0.0065
	0.3	0.0101(56)	0.0205(11)	0.0167
	0.4	0.0333(85)	0.0439(15)	0.0459
$H \to q\bar{q}(g)$	0.0	0.0911(61)	0.1034(13)	0.1077
	0.1	0.1009(83)	0.1169(14)	0.1204
	0.2	0.1288(85)	0.1455(14)	0.1486
	0.3	0.1847(135)	0.1941(20)	0.1928
	0.4	0.2431(104)	0.2513(30)	0.2730
$\gamma^* \to q\bar{q}(g)$	0.0	0.0029(96)	0.0161(14)	0.0190
	0.1	0.0068(74)	0.0205(13)	0.0215
	0.2	0.0191(50)	0.0293(13)	0.0313
	0.3	0.0535(103)	0.0609(20)	0.0547
	0.4	0.0971(171)	0.0979(30)	0.1140

- Pennylane to construct and train the QNN (6-qubit Ansatz), 20 layers
- IQAE module implemented with Qibo on quantum simulators, and with Qiskit on a real hardware (5 qubits), executed on the 27-qubit IBMQ superconducting device *ibmq_mumbai*
- error mitigation: pulse-efficient transpilation, error suppression Dynamical Decoupling (DD) within the circuit execution and error mitigation Zero Noise Extrapolation (ZNE) to the output

Conclusions

- Departing from the **starting hypothesis** that a Feynman propagator is a qubit, in the sense that it represents the quantum superposition of propagation in both directions

- Stress test of quantum algorithms for querying of causal states or DAG configurations, and a new indicator of algorithmic complexity, the **transpiled quantum area**

- To the quantum integration of multidimensional integrals, in particular, loop and phase-space integrals

- And a **quantum event generator** (with NNLO, …, $N^kLO + N^kLL$ accuracy), as a challenging long-term goal.