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Engineering periodic boundary conditions with circuit cutting for high-energy physics

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Diego Riste
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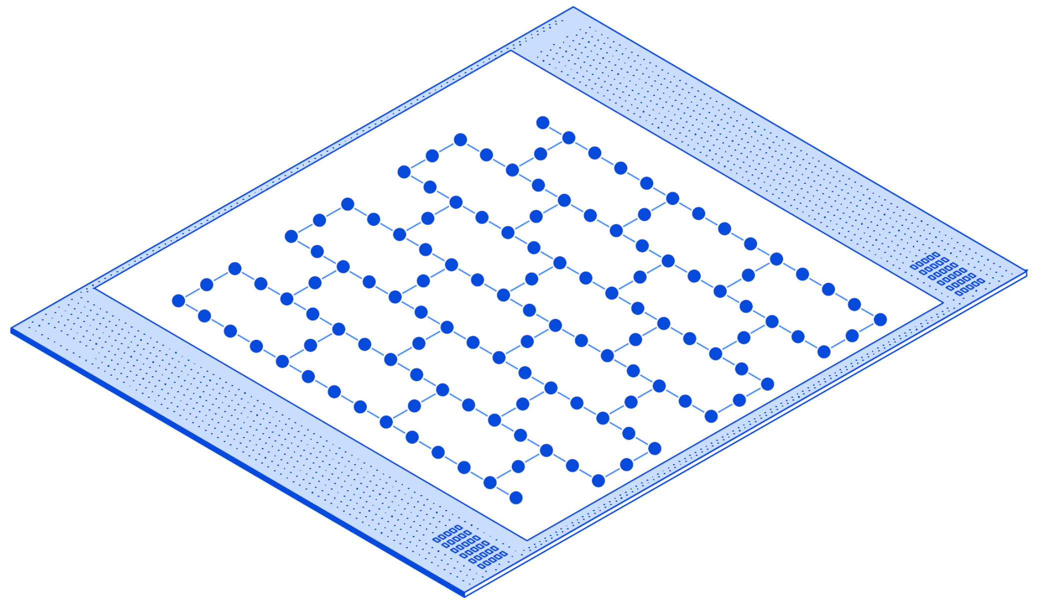
Stefan Woerner
Maika Takita
Henrike Cornelia Christ



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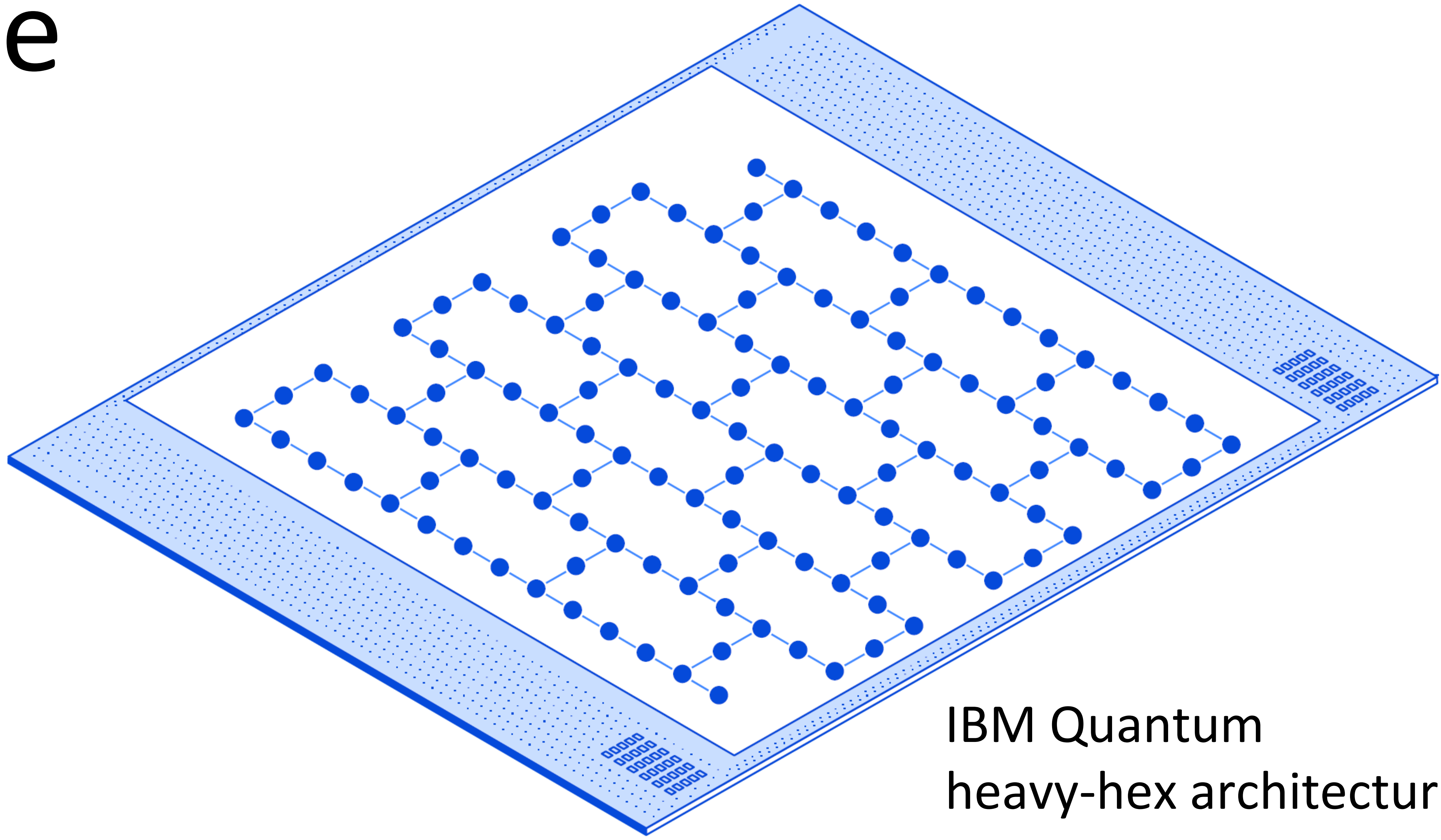
Superconducting processors have limited connectivity.



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heavy-hex architecture

Many computations require long-range connectivity.
Example: periodic boundary conditions.

How do we overcome this limited connectivity?

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Circuit knitting: theory

A virtual gate is implemented by a sum over several circuits

$$\mathcal{E}(\cdot) = \sum_i a_i \, \mathcal{E}_i(\cdot)$$

⇓

$$\mathcal{E}(\rho) = \gamma \sum_i \frac{|a_i|}{\gamma} sign(a_i) \mathcal{E}_i(\cdot)$$

Allows us to cut gates.

- A quantum channel $\mathcal{E}$ can be decomposed into a linear combination of several channels $\mathcal{E}_i$ with coefficients a_i.
- The a_i do not form a valid probability decomposition since some $a_i < 0 \Longrightarrow$ cannot sample.
- We transform to a valid probability distribution via
$$\gamma = \sum |a_i|$$
- Now $\sum \frac{|a_i|}{\gamma} = 1$ and $|a_i|/\gamma$ form a valid probability distribution.
- Many different implementations: w/wo classical communication, parallel gates, etc.

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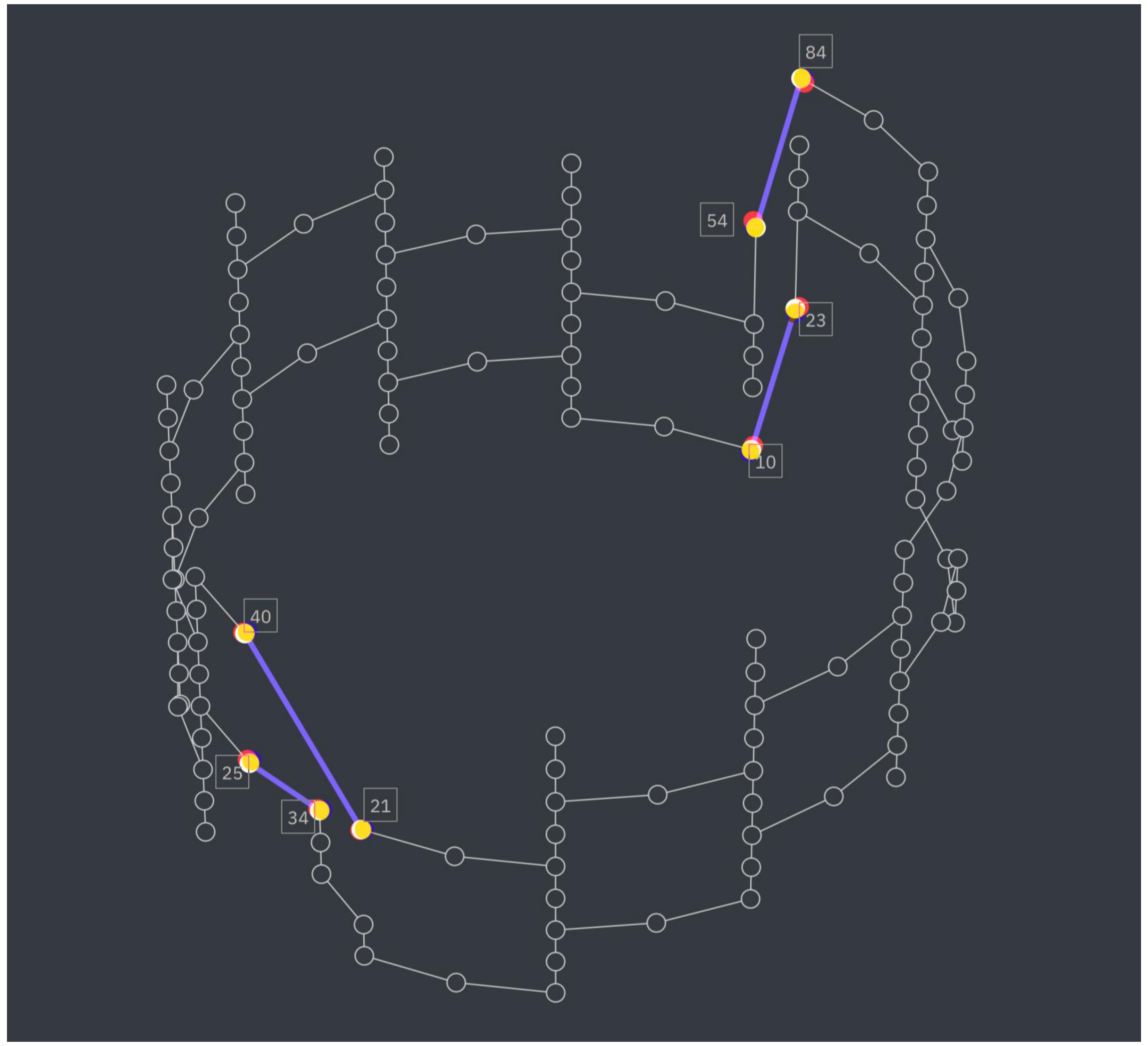
Circuit knitting: demonstration



We implement long range gates to engineer graph states with periodic boundary conditions on >100 qubits.

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Combining quantum processors with real-time classical communication

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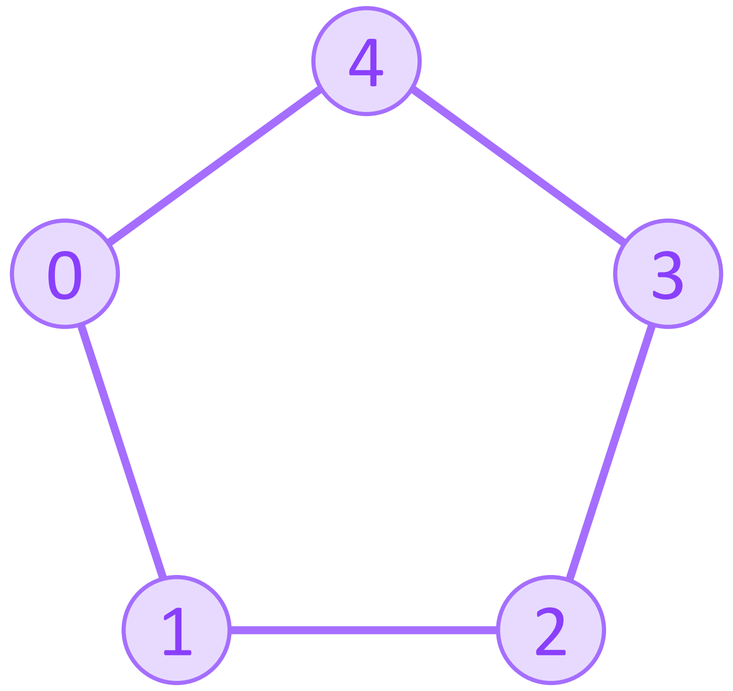
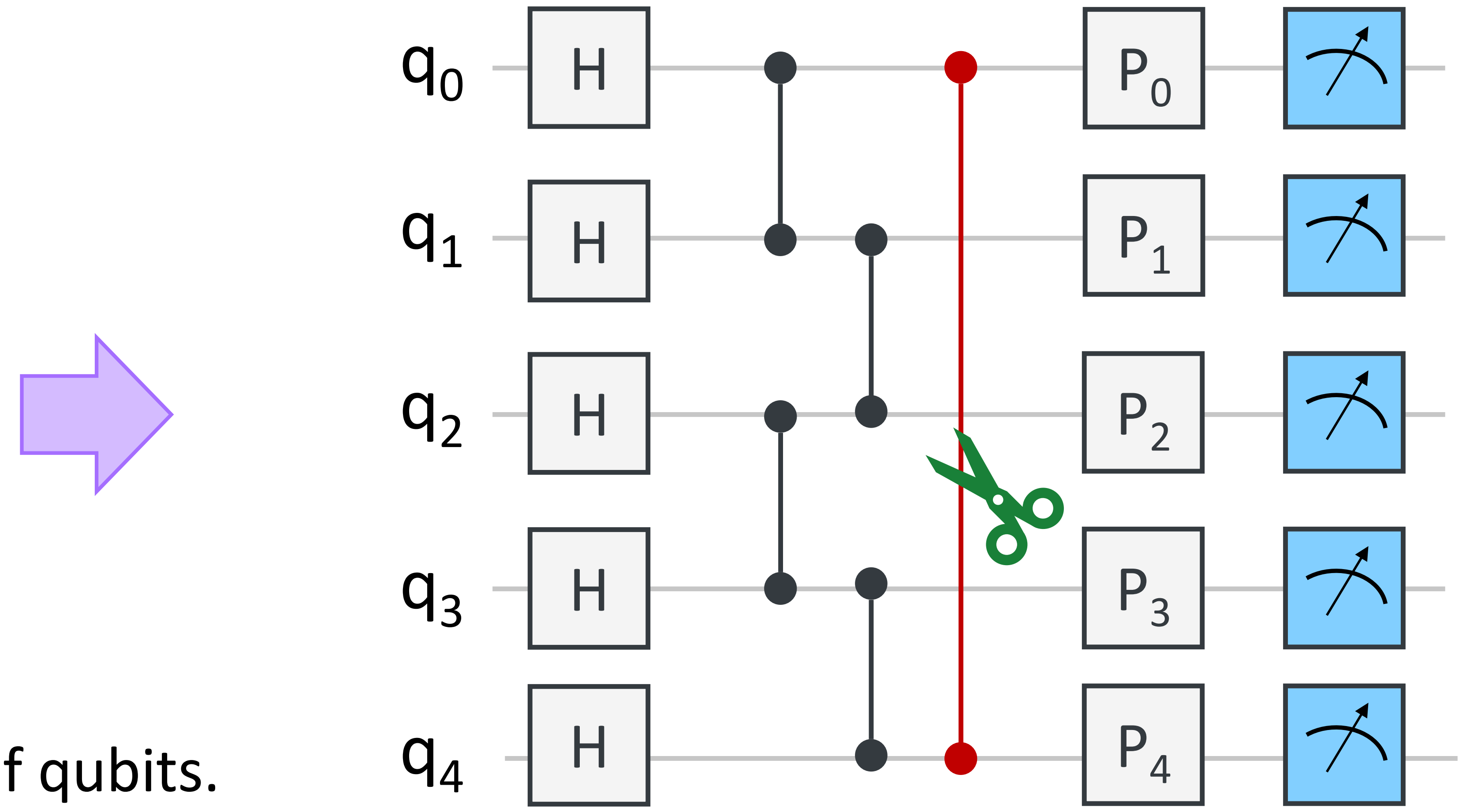


Periodic graph state on a line of qubits.



Circuit knitting: demonstration

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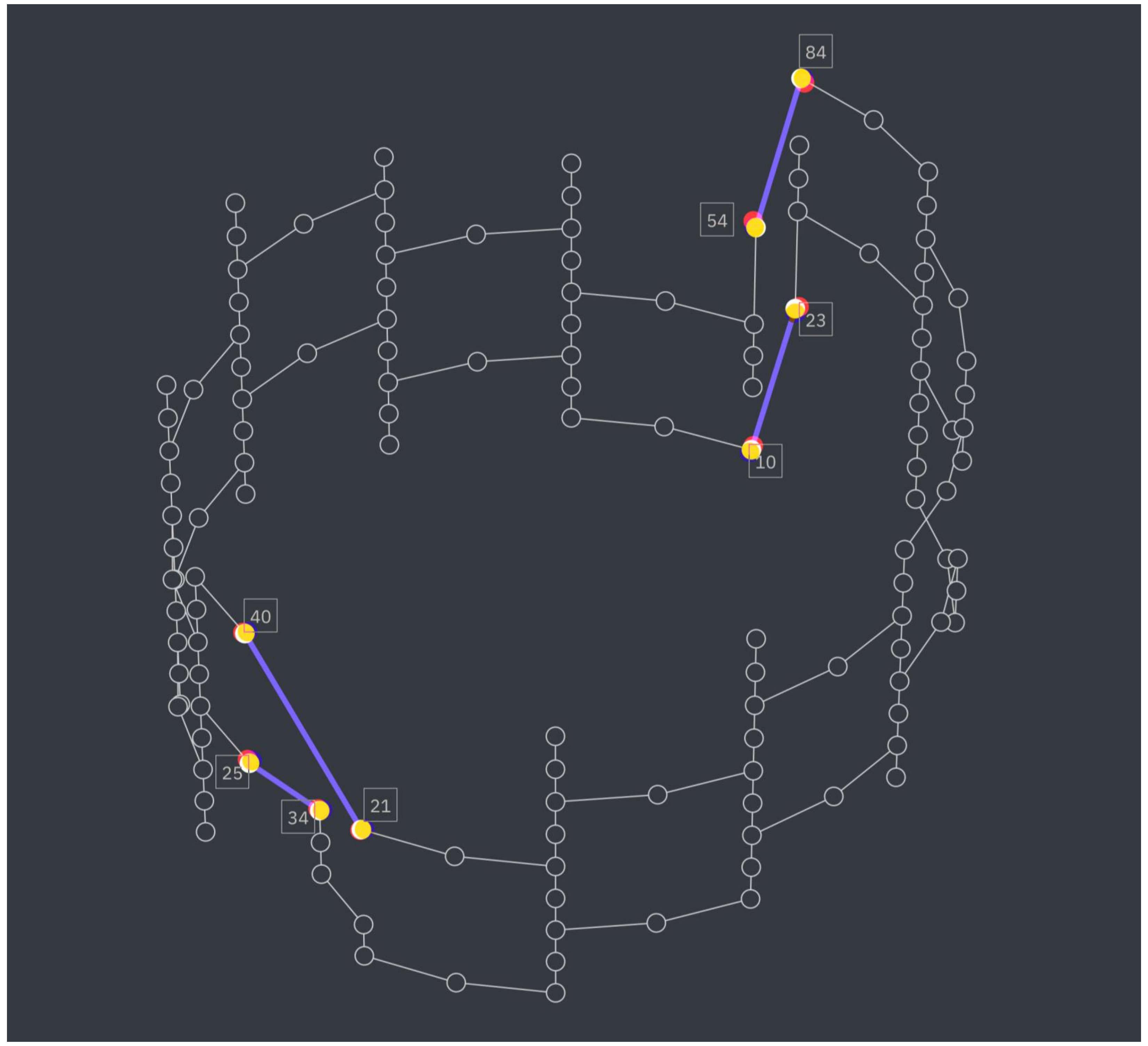
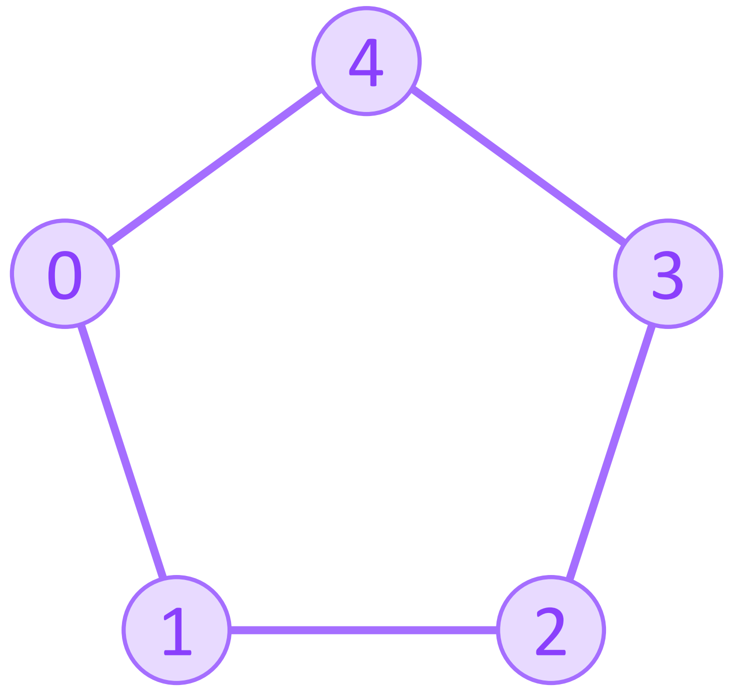


We implement long range gates to engineer graph states with periodic boundary conditions on >100 qubits.



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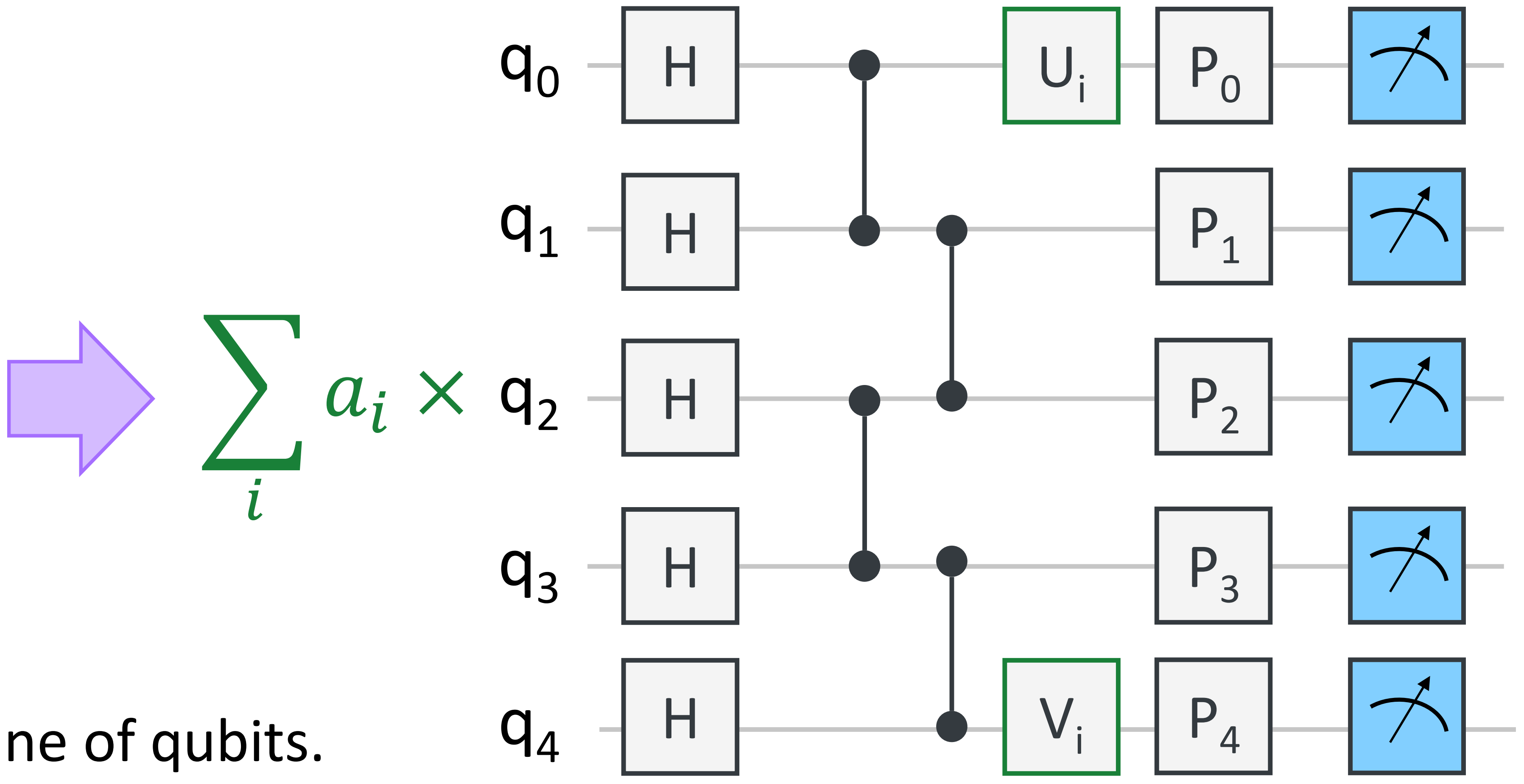
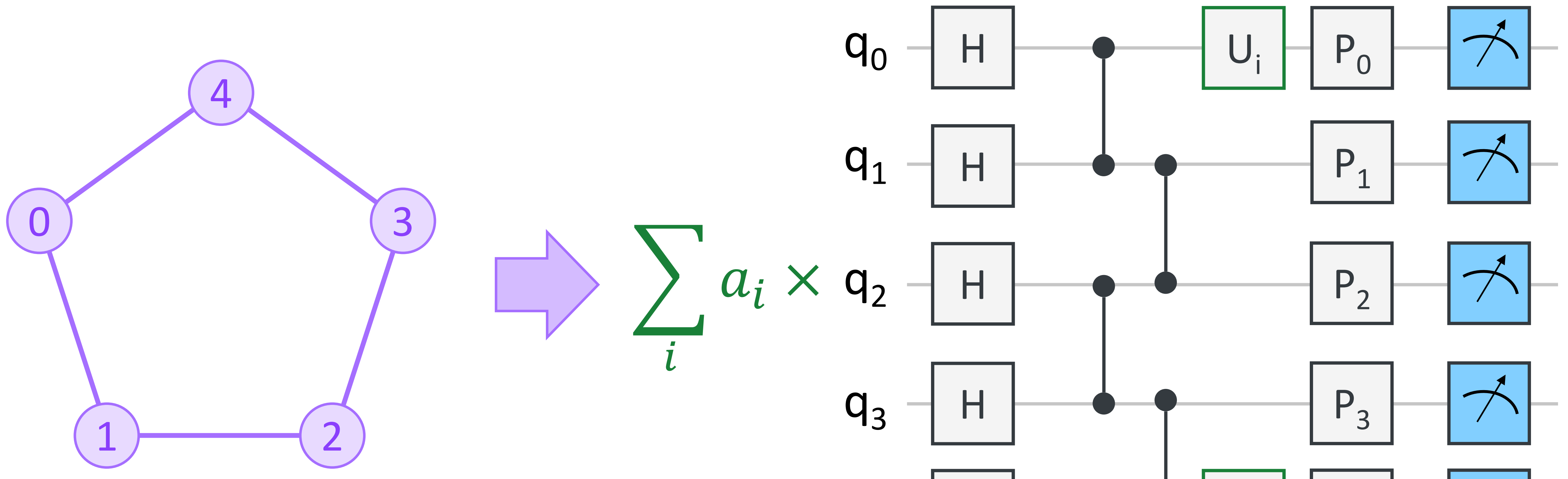


Circuit knitting: CZ example

- A CZ gate can be implemented by a QPD with 6 circuits.
- Cost of this QPD γ=3.
  [Mitarai, New. J. Phys. (2023)]
- In practice, better LO decompositions exist that reach the same γ as LOCC.
  [Ufrecht, arXiv:2312.09679 (2023), Schmitt, arXiv:2312.11638 (2023)]



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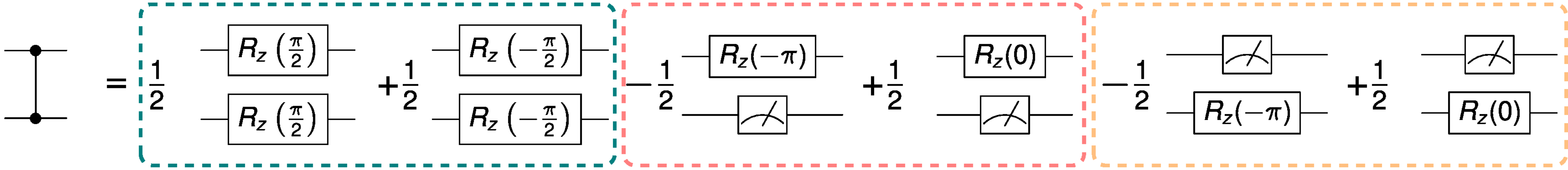

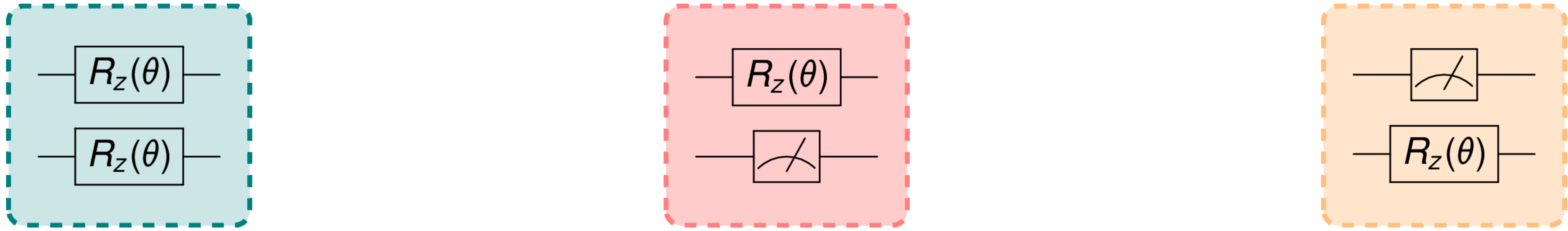

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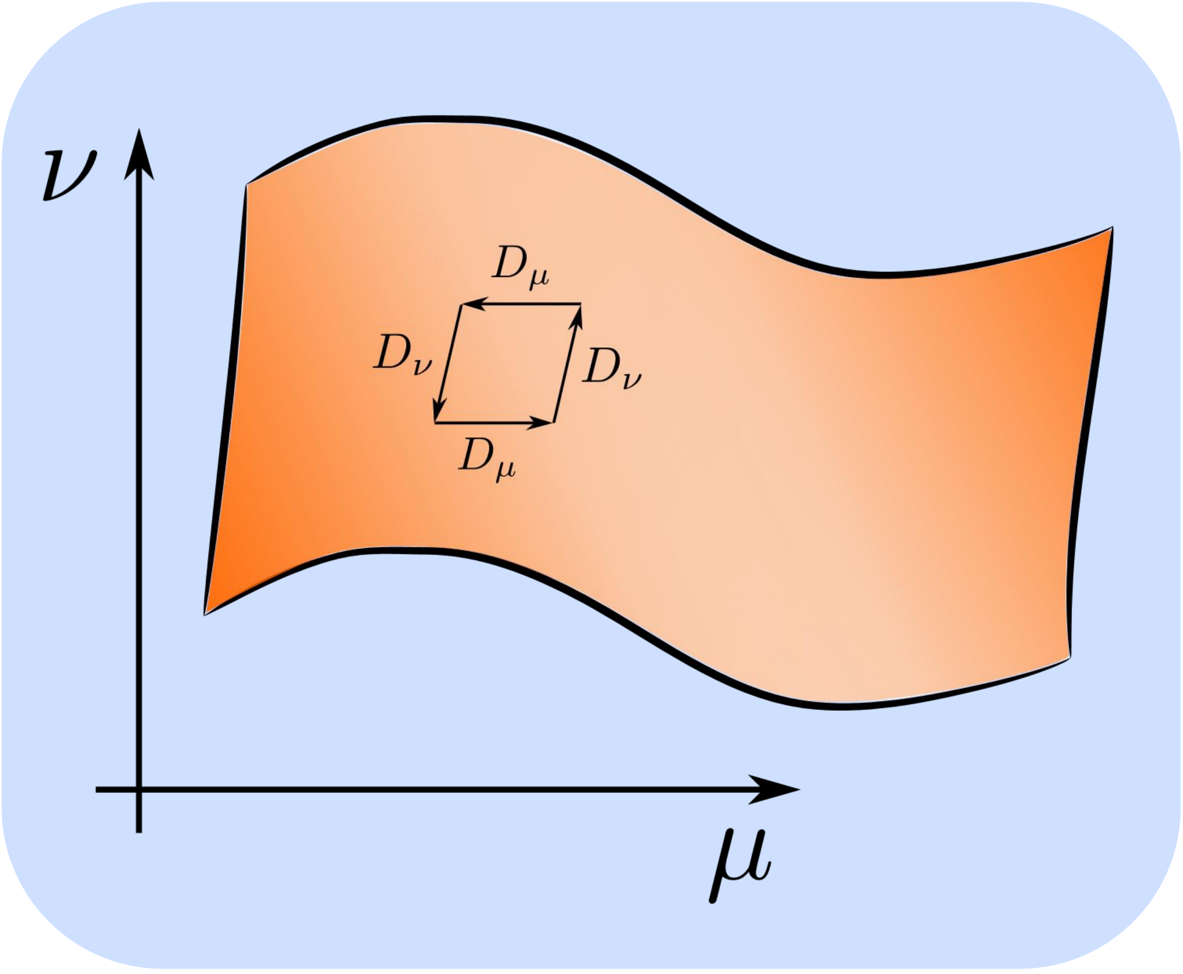
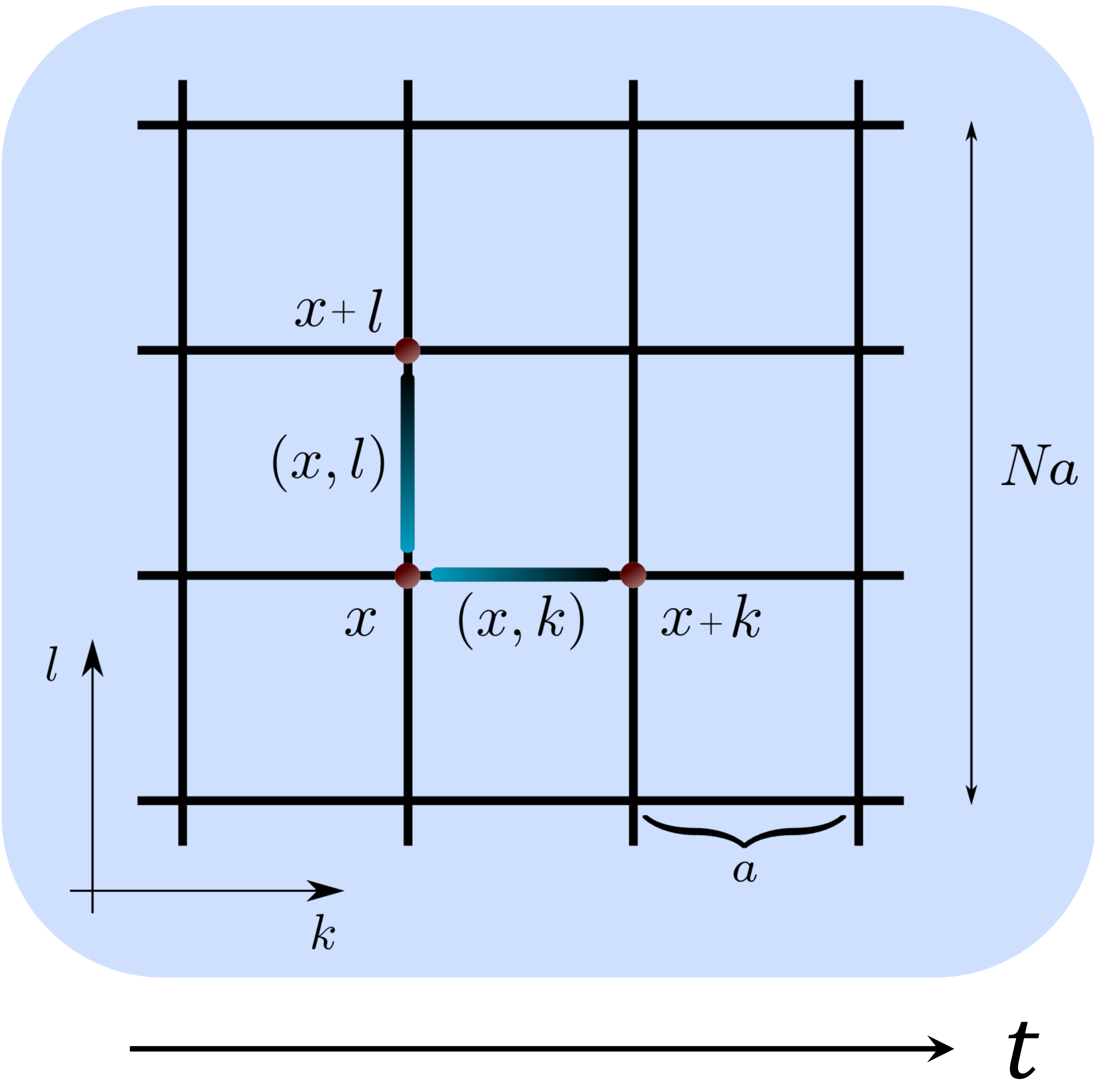
High-energy physics simulation strategy on quantum computers



Continuous space and time

- Principles of Gauge invariance
- Basic elements of a Yang-Mills Theory

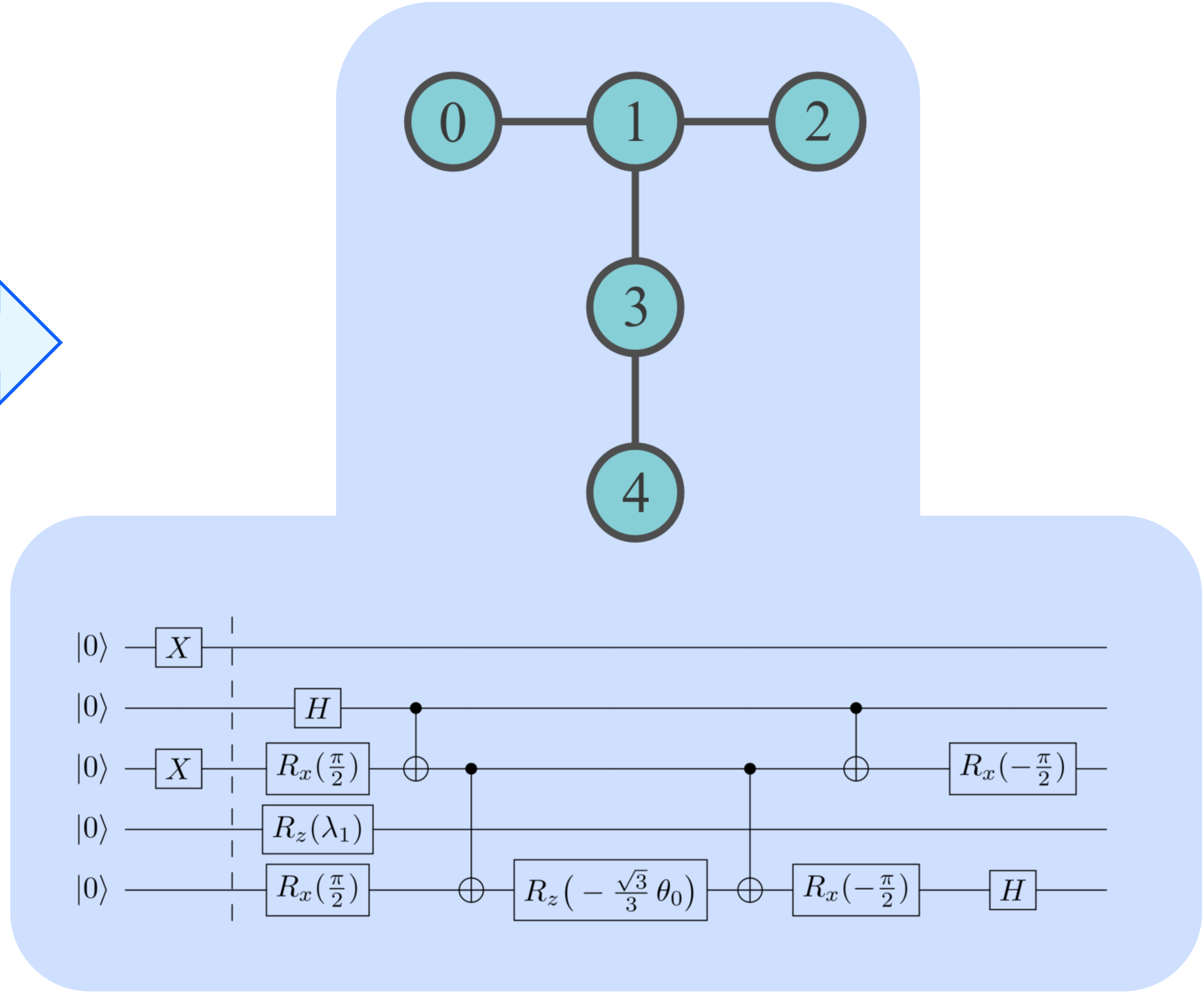


Discrete space continuous time

- Hamiltonian formulation
- Quantum link model
- Gauge- and matter-field operator relations

Quantum simulation

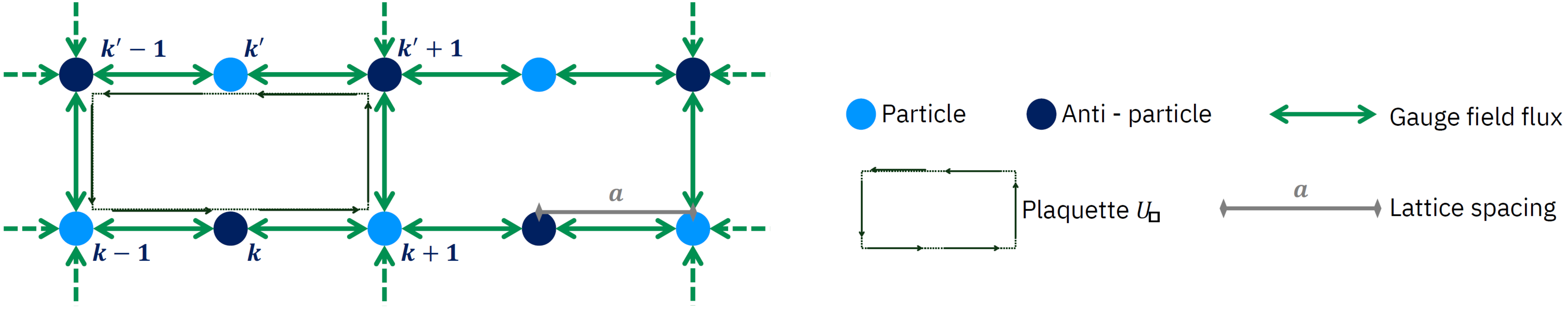
- Real variational quantum simulation
- Trotterized time evolution



High-energy physics simulation strategy on quantum computers

Discretized model of fermions ψ interacting with a gauge field E/U

$$H_{LGT} = \underbrace{\sum_k m(-1)^k\,\psi_k^\dagger\,\psi_k}_{\substack{\text{Mass term}\\ \text{1-local}}} - \underbrace{\sum_k 2\sigma(-1)^k\,E_k}_{\substack{\text{Field term}\\ \text{1-local}}} - \underbrace{\sum_k c\left(\psi_k^\dagger\,U_{k,k+1}\psi_{k+1} + h.c.\right)}_{\substack{\text{Interaction term}\\ \text{3-local \& }\textbf{potentially long-range}\text{ on}\\ \text{periodic BC}}}$$



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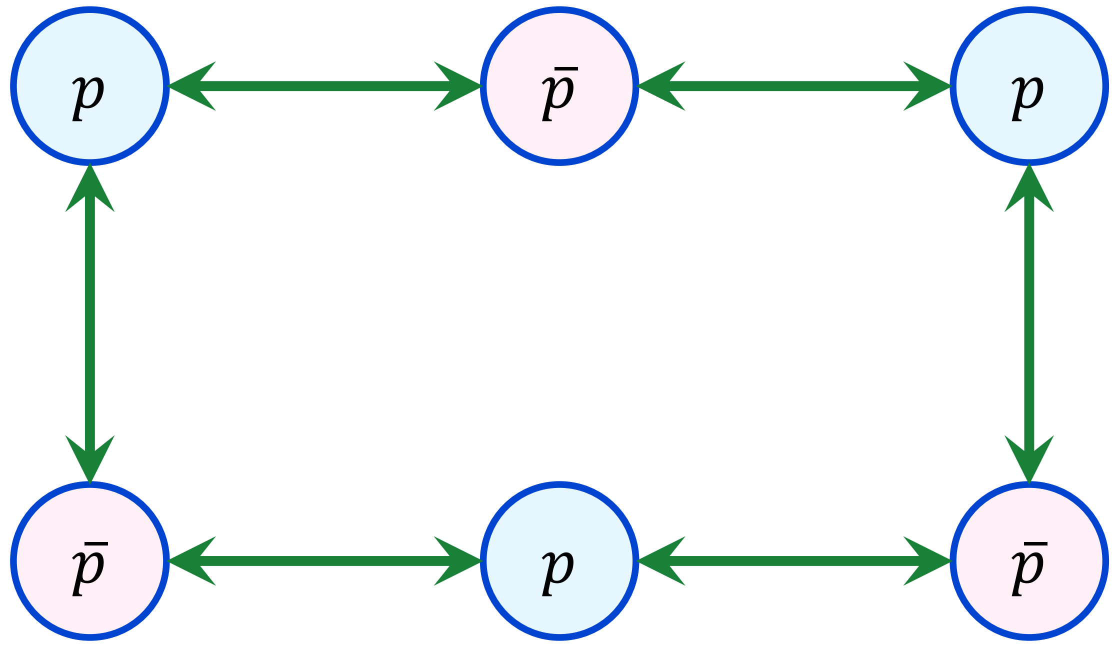
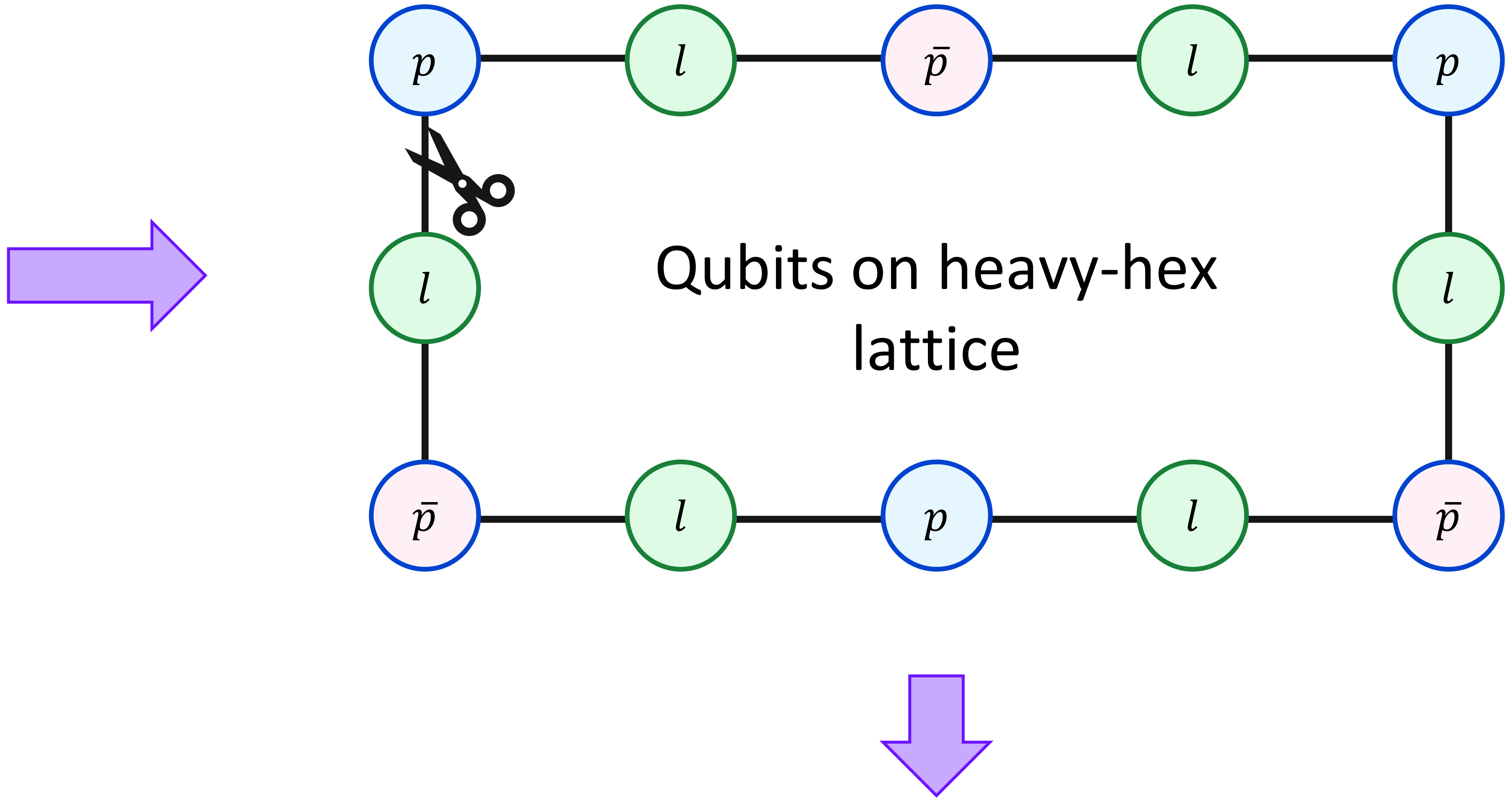
High-energy physics simulation strategy on quantum computers

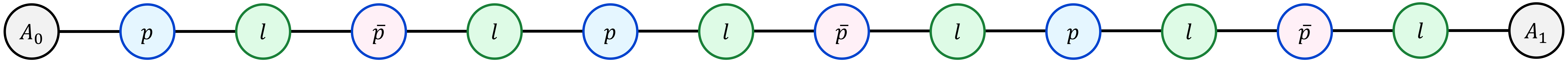
Experiment 1: ring of 3 particles and anti-particles $\Longrightarrow$ 12 qubits



Lattice model



Qubits on heavy-hex lattice



Ancilla qubit for circuit cutting

Line of qubits

Ancilla qubit for circuit cutting

Both the hex ring and the line are native to IBM hardware
$\Longrightarrow$ Compare periodic BC with circuit cutting and without.

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High-energy physics simulation strategy on quantum computers

Experiment 1: ring of 3 particles and anti-particles ⟹ 12 qubits

Run 1: Open BC on a line

Run 2: Periodic BC on the ring

Run 3: Periodic BC on a line with standard circuit cutting
[Schmitt et al. arXiv:2312.11638, Harrow & Lowe arXiv:2403.01018]

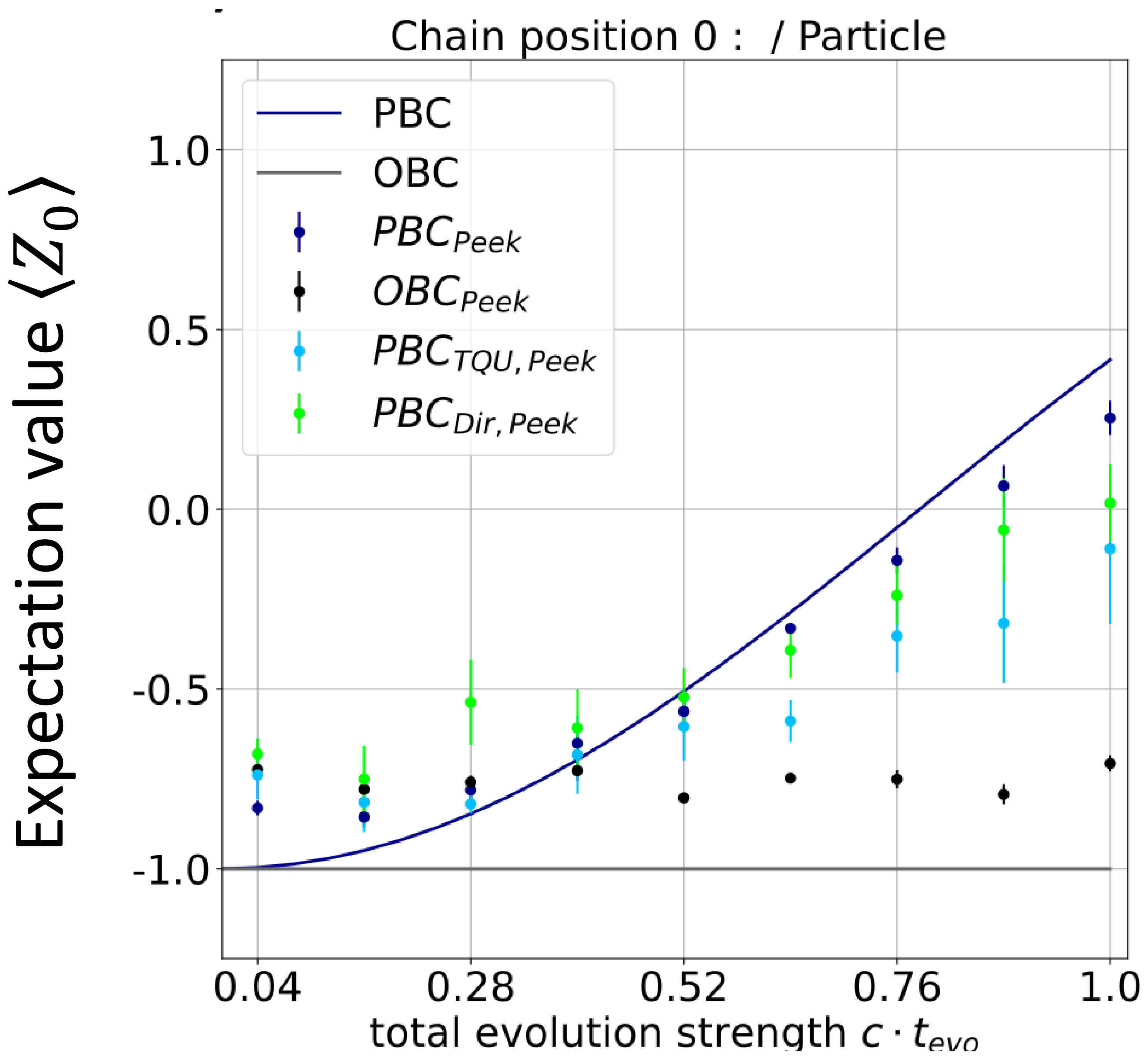
Run 4: Periodic BC on a line with circuit cutting tailored to the model

$$U_{int}(\theta) = \exp[-i\,\theta\,(XXX\ - XYY + YYX + YXY)]$$

$$= \left(1 + \frac{\cos(\theta) - 1}{4}\right) III + \frac{\cos(\theta) - 1}{4}(IZZ\ - ZIZ\ - ZZI) + \frac{-i\,\sin(\theta)}{4}(\mathrm{XXX}\ - \mathrm{XYY} + \mathrm{YXY} + \mathrm{YYX})$$

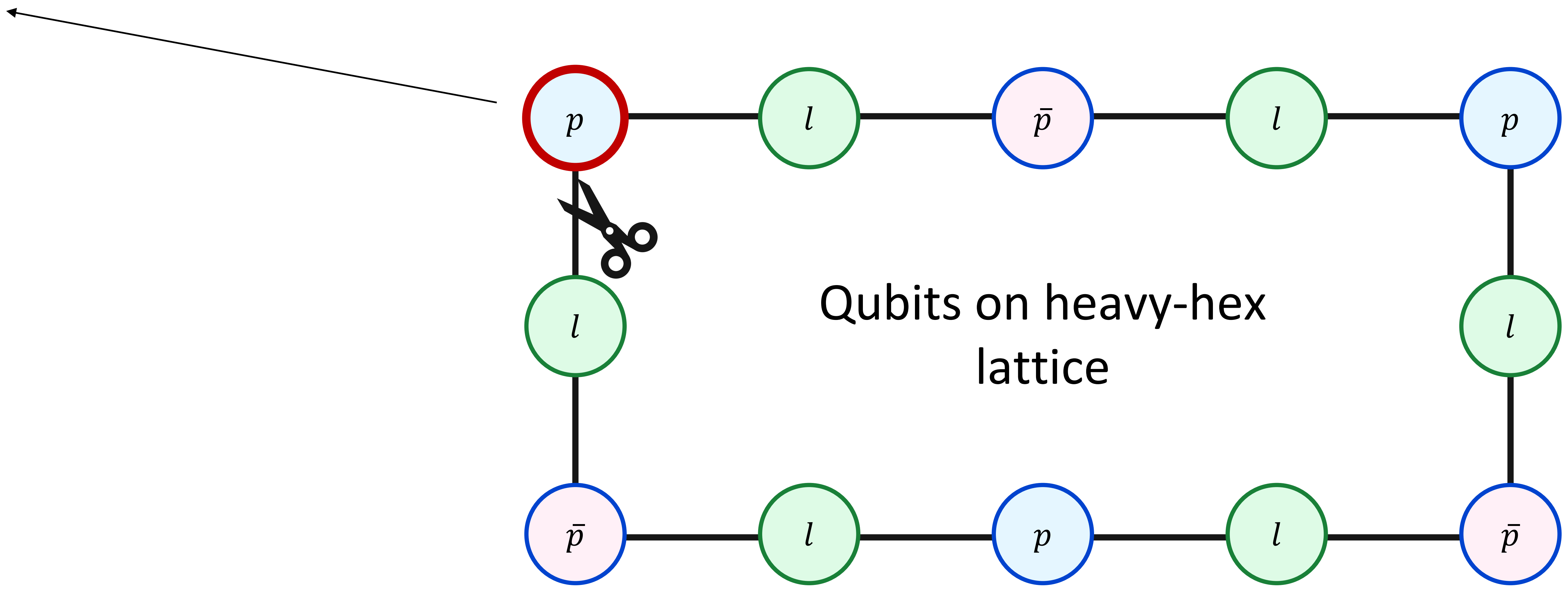
High-energy physics simulation strategy on quantum computers

Experiment 1: ring of 3 particles and anti-particles $\Longrightarrow$ 12 qubits



Mean-squared error with the ideal noiseless simulations

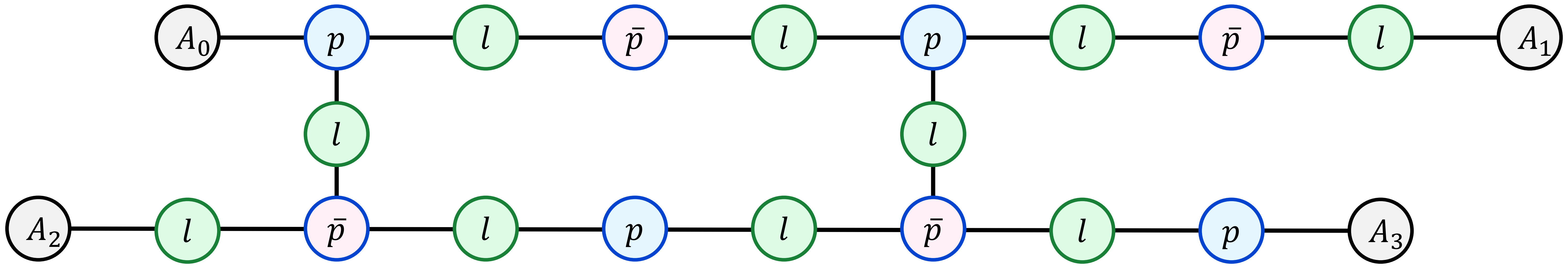
Reference	OBC Run 1	PBC Run 2	PBC Run 3	PBC Run 4
Ideal PBC	10.6%	2.4%	4.5%	3.2%
Ideal OBC	3.7%	13.9%	9.9%	12.8%

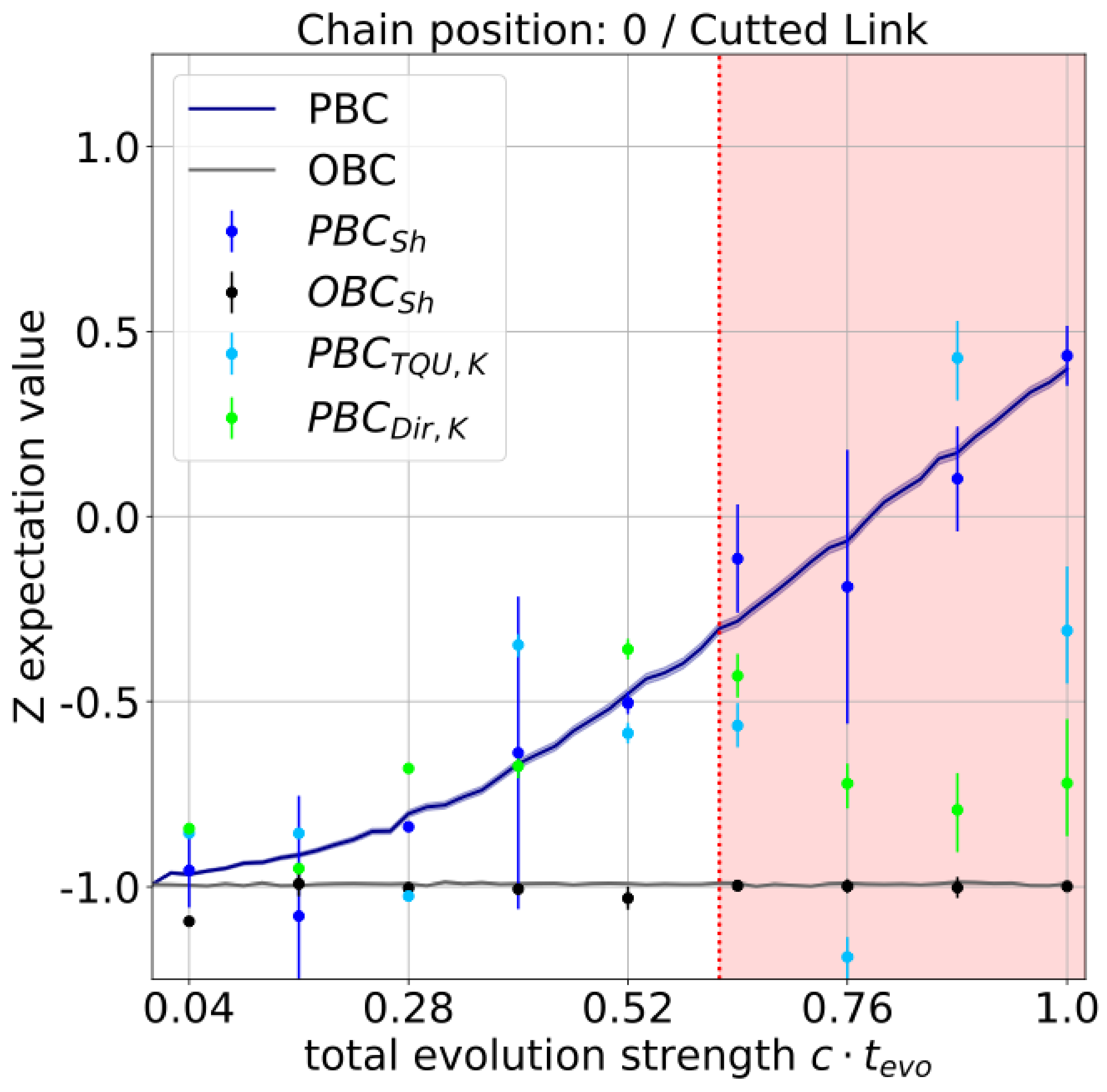


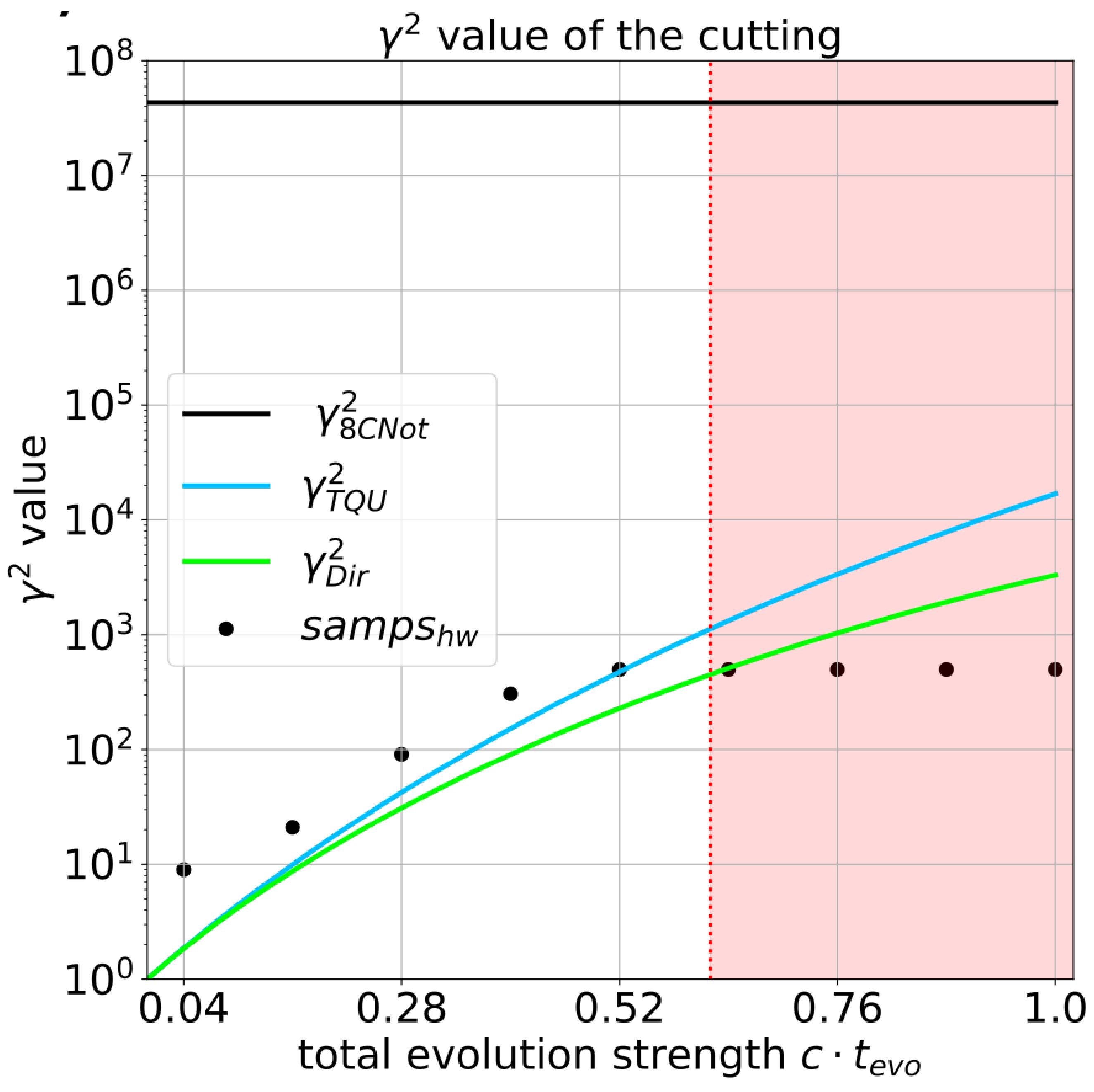
Qubits on heavy-hex lattice

High-energy physics simulation strategy on quantum computers

Experiment 2: Two rings of 8 particles and anti-particles ⟹ 18 qubits







- MSE with PBC of 4.3% and 1.1% for the circuit cutting compared to 57.8% with OBC.
- Results are good so long we increase the sampling overhead with γ.

Conclusion & Outlook

We can go beyond planar topologies with circuit cutting

- Graph states at utility scale $O(100)$ qubits
- First circuit cutting experiments with Trotter simulations of LGT on $O(10)$ qubits
- Problem tailored circuit cutting reduces γ

Circuit cutting is exponentially expensive

- Can we use it sparingly to engineer classically hard problems?
- Can we scale-up the LGT simulations within a reasonable γ-budget?

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High-energy physics simulation strategy on quantum computers

Goal: compute the time dynamics under H_{LGT}, implement $e^{-itH_{LGT}}$ with quantum gates

$$H_{LGT} = \underbrace{\sum_k m(-1)^k \, \psi_k^\dagger \, \psi_k}_{\text{Mass term}} - \underbrace{\sum_k 2\sigma(-1)^k \, E_k}_{\text{Field term}} - \underbrace{\sum_k c\left(\psi_k^\dagger \, U_{k,k+1}\psi_{k+1} + h.c.\right)}_{\text{Interaction term}}$$

$$\Downarrow$$

- Mapping from fermion & fields to qubits
- Each particle and anti-particle is mapped to a qubit.
- The field is modeled as a single qubit with 2 levels

$$H_{LGT} = \underbrace{\sum_{j \in ferm} \frac{m}{2}(-1)^j \, Z_j}_{\text{Mass term}} - \underbrace{\sum_{j \in link} 2\sigma(-1)^j \, Z_k}_{\text{Field term}} - \underbrace{\sum_{j \in link} \frac{c}{4}\left(X_{j-1}X_jX_{j+1} - X_{j-1}Y_jY_{j+1} + Y_{j-1}Y_jX_{j+1} + Y_{j-1}X_jY_{j+1}\right)}_{\text{Interaction term}}$$

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High-energy physics simulation strategy on quantum computers

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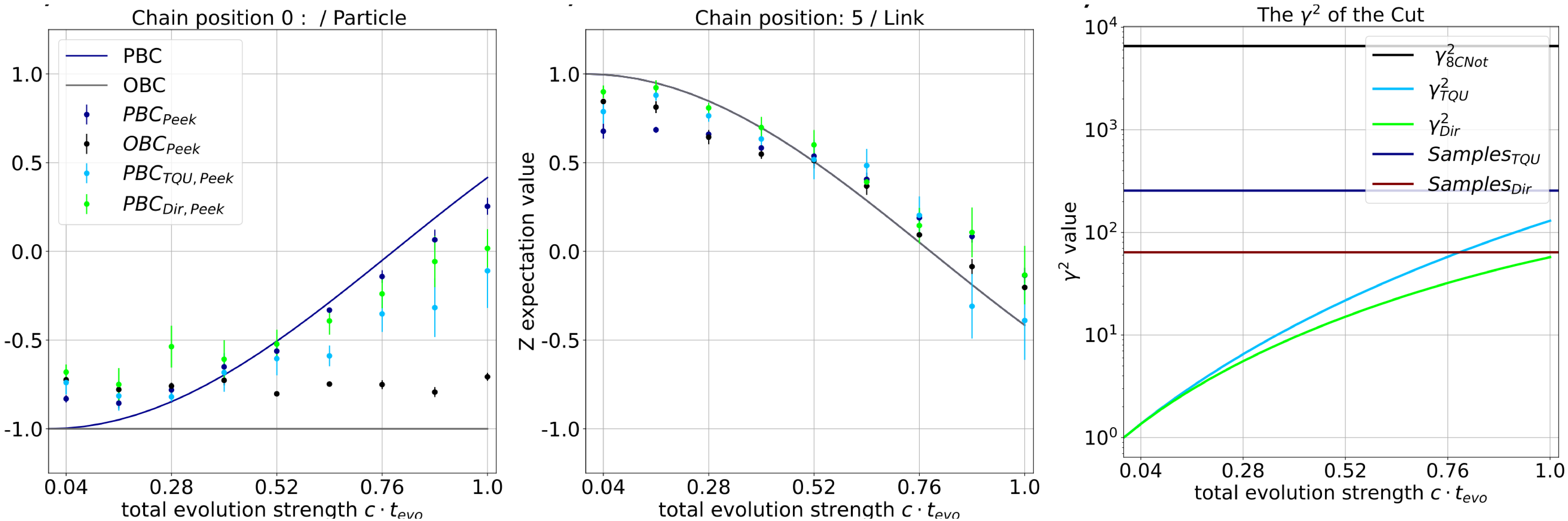

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