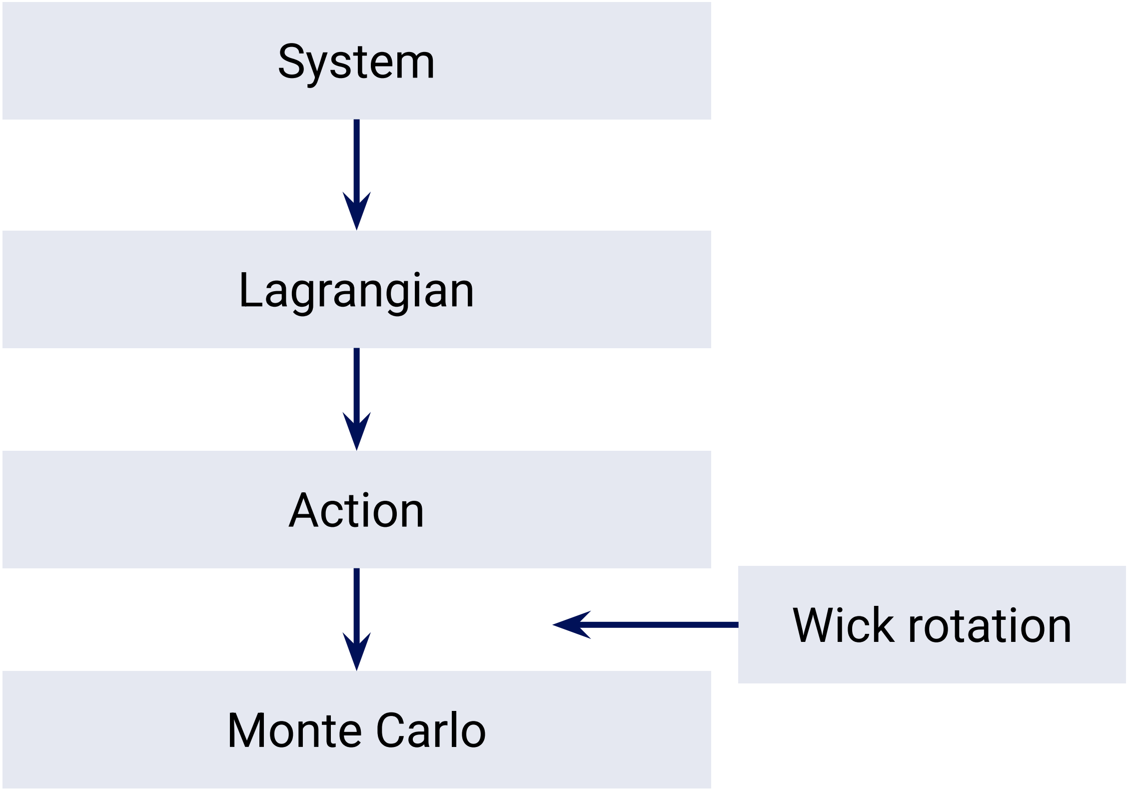
Projected Entangled Pair States for Lattice Gauge Theories with Dynamical Fermions

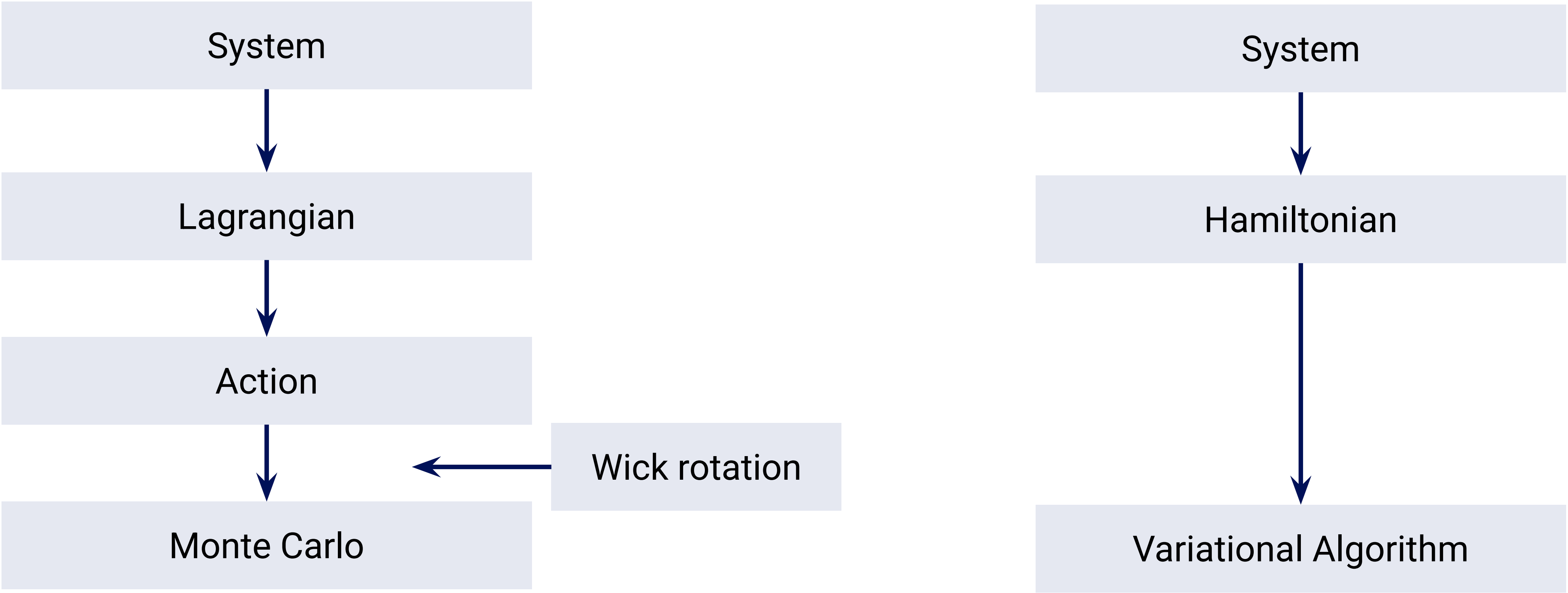
Patrick Emonts | QT4HEP 2025 | 23.01.2025 | CERN

Universiteit Leiden
The Netherlands

The usual pipeline



Our Approach



Mari Carmen Bañuls and Krzysztof Cichy (2020) Rep. Prog. Phys. 83 p. 024401;
John Kogut and Leonard Susskind (1975) Phys. Rev. D 11 pp. 395–408;
Kenneth G. Wilson (1974) Phys. Rev. D 10 pp. 2445–2459

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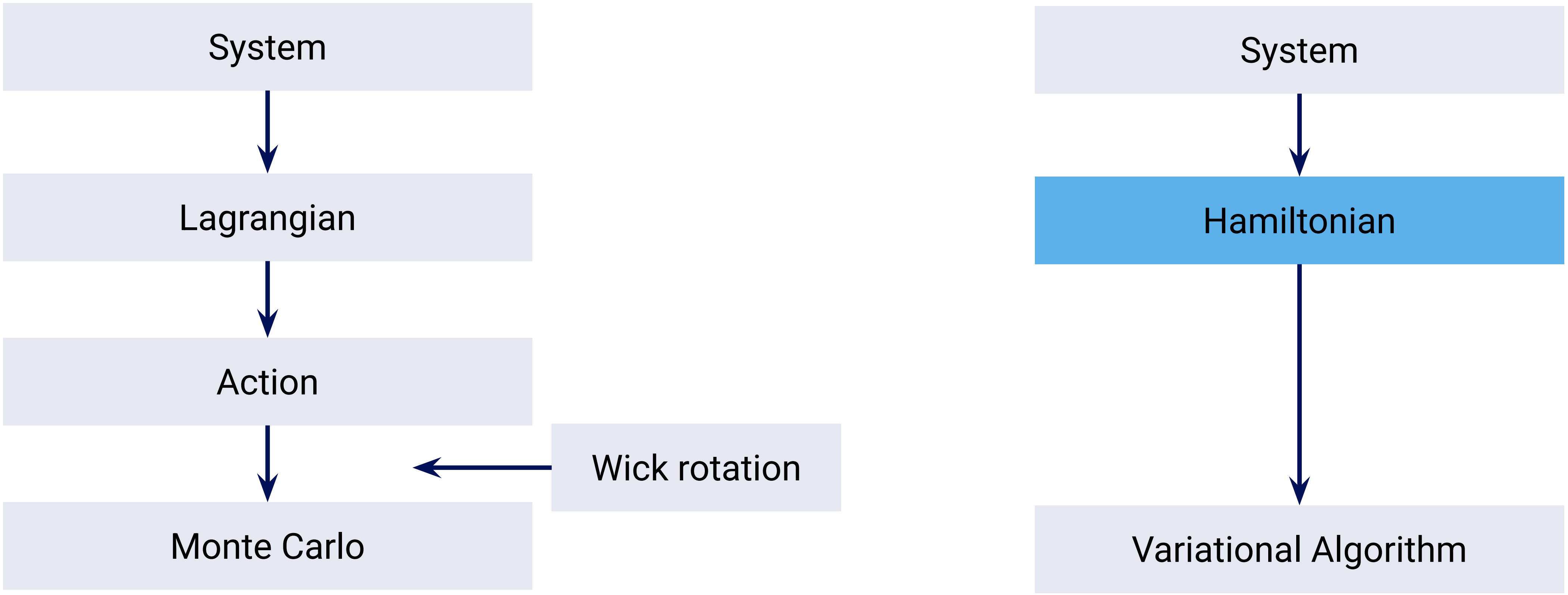

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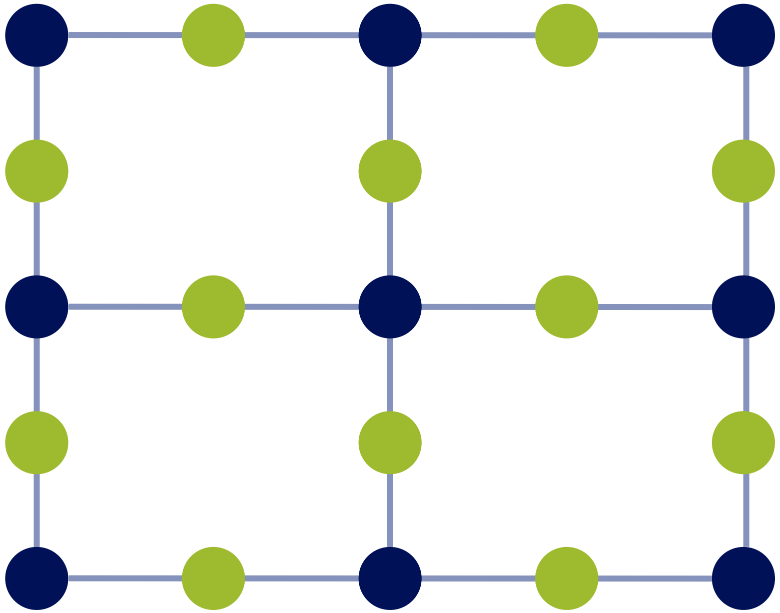
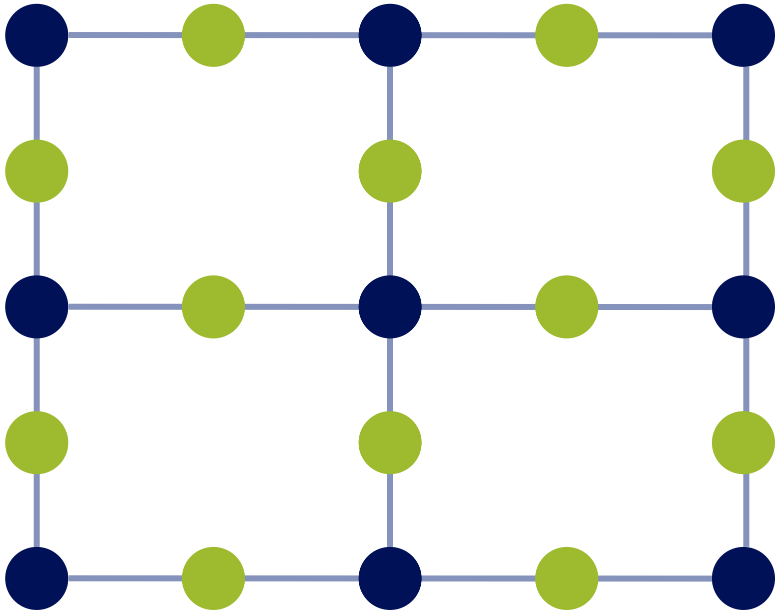
Hilbert spaces and Lattices



Hilbert space

$$\mathcal{H} \subset \mathcal{H}_{\text{gauge fields}} \otimes \mathcal{H}_{\text{fermions}}$$

Hilbert spaces and Lattices



Hilbert space

$$\mathcal{H} \subset \mathcal{H}_{\text{gauge fields}} \otimes \mathcal{H}_{\text{fermions}}$$

A general state

$$|\Psi\rangle = \int D\mathcal{G} |\mathcal{G}\rangle |\psi_F(\mathcal{G})\rangle$$

with $$D\mathcal{G} = \prod_{x,k} dg(x,k)$$

Which way to go?

Quantum Computation/
Quantum Simulation

Classical Simulation

M. C. Bañuls and K. Cichy, Rep. Prog. Phys. **83**, 024401 (2020).

M. C. Bañuls et al., Eur. Phys. J. D 74, 165 (2020).

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A wishlist

General State Formulation

$$|\Psi\rangle = \int D\mathcal{G}|\mathcal{G}\rangle|\psi_F(\mathcal{G})\rangle$$

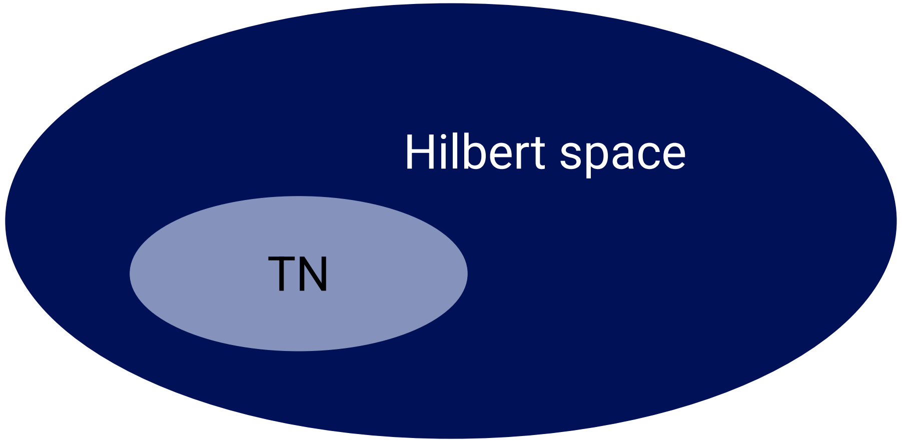
ToDo List

1. How do we construct $|\psi_F(\mathcal{G})\rangle$?
2. How do we efficiently calculate the expectation values?
3. Are those states useful?

Finding an Ansatz

Idea

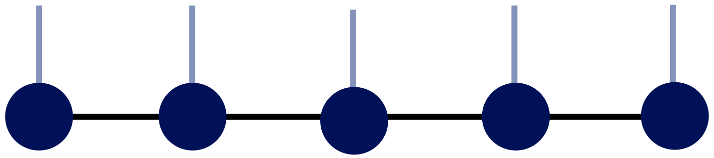
Use an Ansatz with polynomially many parameters although the Hilbert space has exponentially many states



We explore only a small part of the Hilbert space

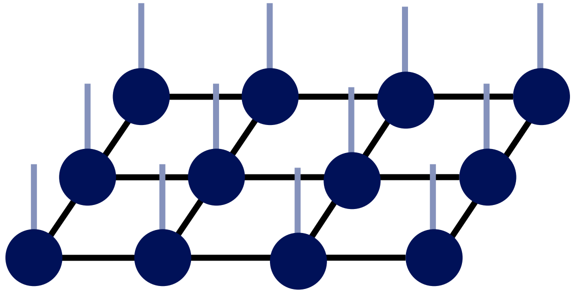
M. Fannes, B. Nachtergaele, and R. F. Werner (1992) Commun.Math. Phys. 144 pp. 443–490
J. I. Cirac, D. Pérez-García, N. Schuch, and F. Verstraete, Rev. Mod. Phys. 93, 045003 (2021).

Different Families of Tensor Networks



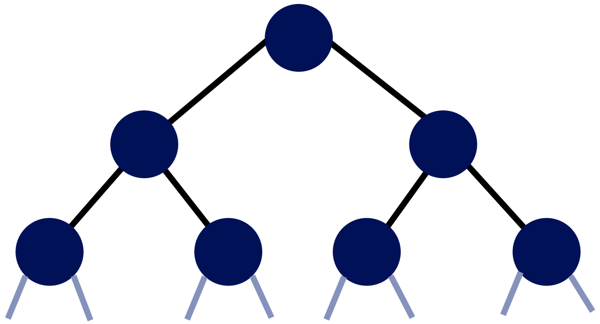
Matrix Product States (MPS)

M. Fannes, B. Nachtergaele, and R. F. Werner (1992)
Commun.Math. Phys. 144 pp. 443–490



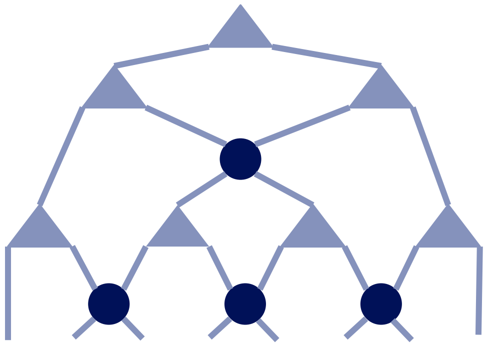
Projected Entangled Pair States (PEPS)

F. Verstraete and J. I. Cirac, arXiv:cond-mat/0407066.



Tree Tensor Network

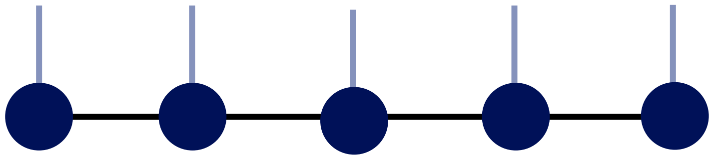
Y.-Y. Shi, L.-M. Duan, and G. Vidal, Phys. Rev. A 74, 022320 (2006).



Multiscale Entanglement Renormalization Ansatz (MERA)

G. Vidal, Phys. Rev. Lett. 101, 110501 (2008).

Different Families of Tensor Networks



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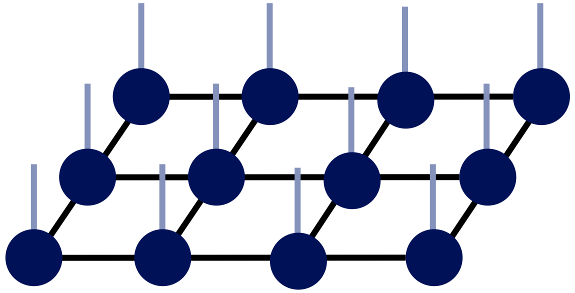

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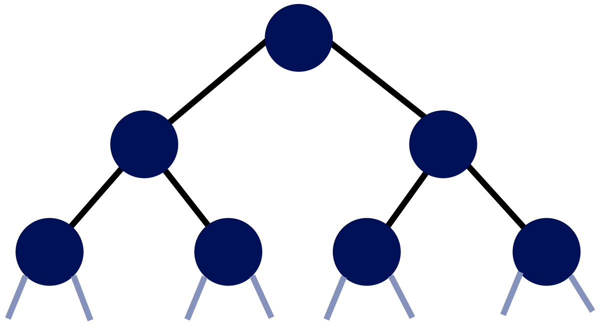

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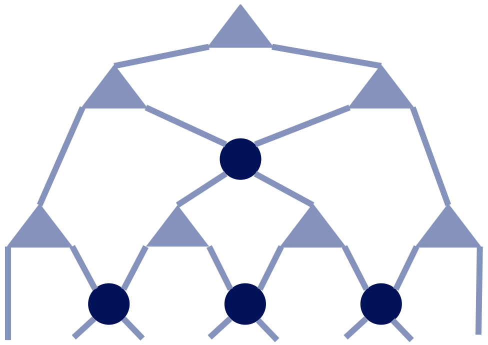

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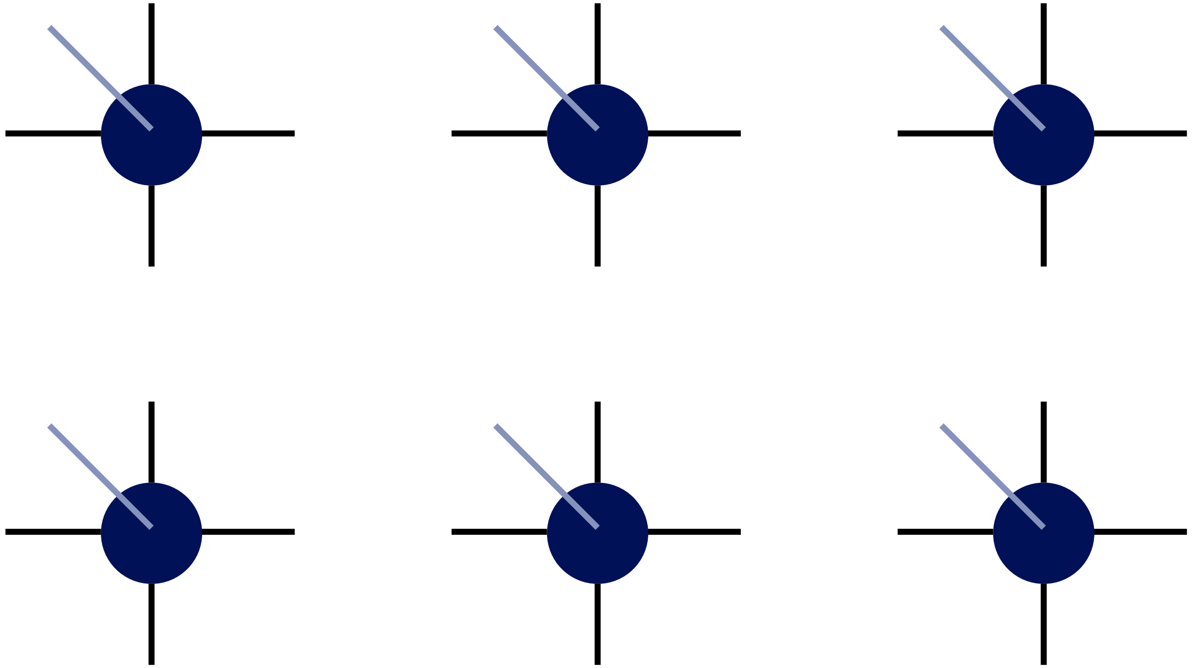
G. Vidal, Phys. Rev. Lett. 101, 110501 (2008).

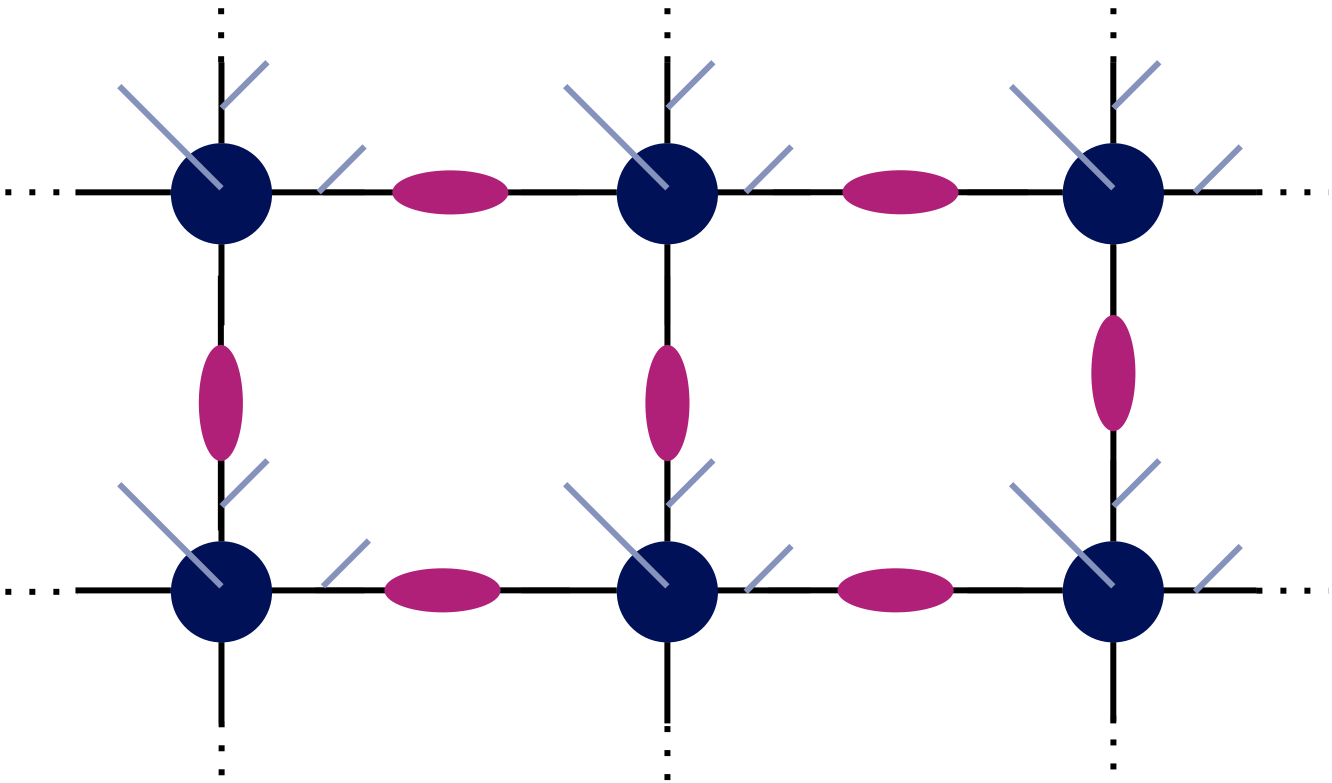
Building a State – GGPEPS



Construction

$$\prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x})|\Omega\rangle$$

Building a State – GGPEPS



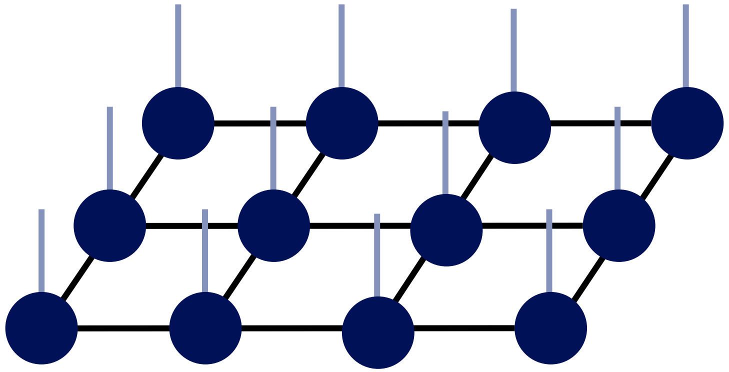
Construction

$$|\psi_F(\mathcal{G})\rangle = \langle\Omega_v| \prod_{\ell} \omega_\ell \prod_{\ell} U_\ell(\mathcal{G}) \prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x})|\Omega\rangle$$

How to compute an expectation value?

ToDo List

1. How do we construct $|\psi_F(\mathcal{G})\rangle$? ✓
2. How do we efficiently calculate the observables?
3. Are those states useful?



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How to compute an expectation value?

ToDo List

1. How do we construct $|\psi_F(\mathcal{G})\rangle$? ✓
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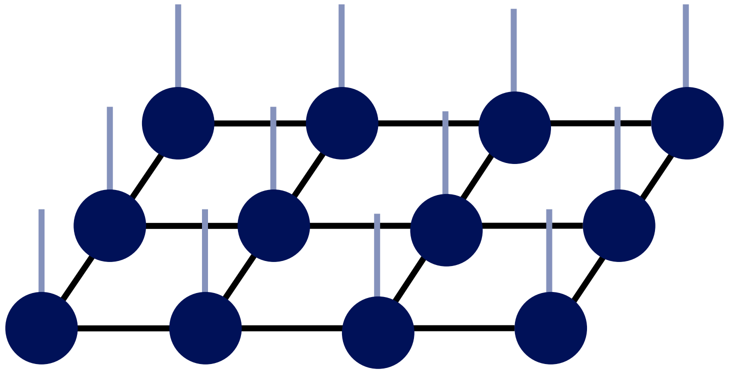
$$p(\mathcal{G}) = \frac{\langle\psi_F(\mathcal{G})|\psi_F(\mathcal{G})\rangle}{\int D\mathcal{G}'\langle\psi_F(\mathcal{G}')|\psi_F(\mathcal{G}')\rangle}$$



GGPEPS can be contracted via covariance matrices

N. Schuch, M. M. Wolf, F. Verstraete, and J. I. Cirac, Phys. Rev. Lett. **98**, 140506 (2007).

Let's consider a Z2 LGT with Fermions

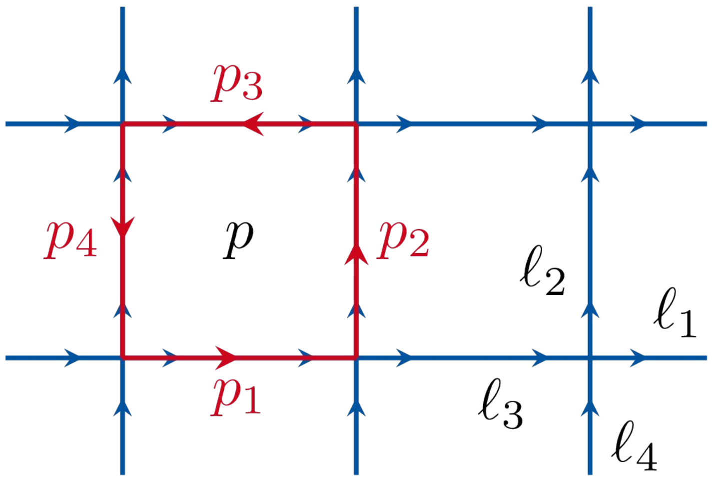
Kogut Susskind Hamiltonian

$$H = g_{\mathrm{E}} H_{\mathrm{E}} + g_{\mathrm{B}} H_{\mathrm{B}} + g_{\mathrm{I}} H_{\mathrm{I}} + g_{\mathrm{M}} H_{\mathrm{M}}$$

$$H_{\mathrm{E}} = \sum_{\ell} 2\left[1 - \sigma^z_{\ell}\right]$$

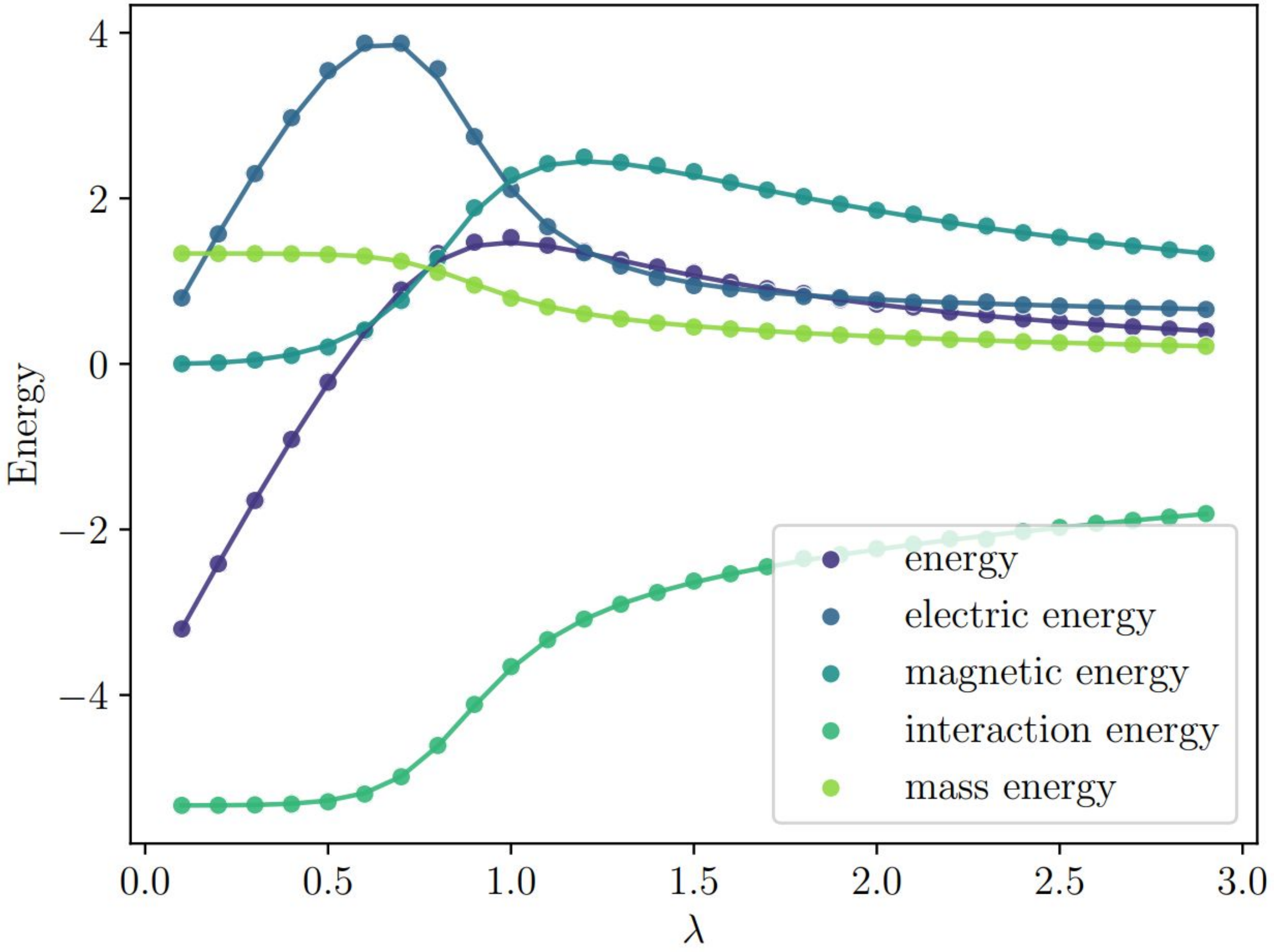
$$H_{\mathrm{B}} = \sum_{p} \left[1 - \sigma^x_{p_1}\sigma^x_{p_2}\sigma^x_{p_3}\sigma^x_{p_4}\right]$$

$$H_{\mathrm{M}} = \sum_{x} \left(\frac{1}{2} + (-1)^x \psi^\dagger(x)\psi(x)\right)$$



John Kogut and Leonard Susskind (1975) Phys. Rev. D 11 pp. 395–408
D. Horn, M. Weinstein, and S. Yankielowicz (1979) Phys. Rev. D 19 pp. 3715–3731

Results – 2x2 Benchmark



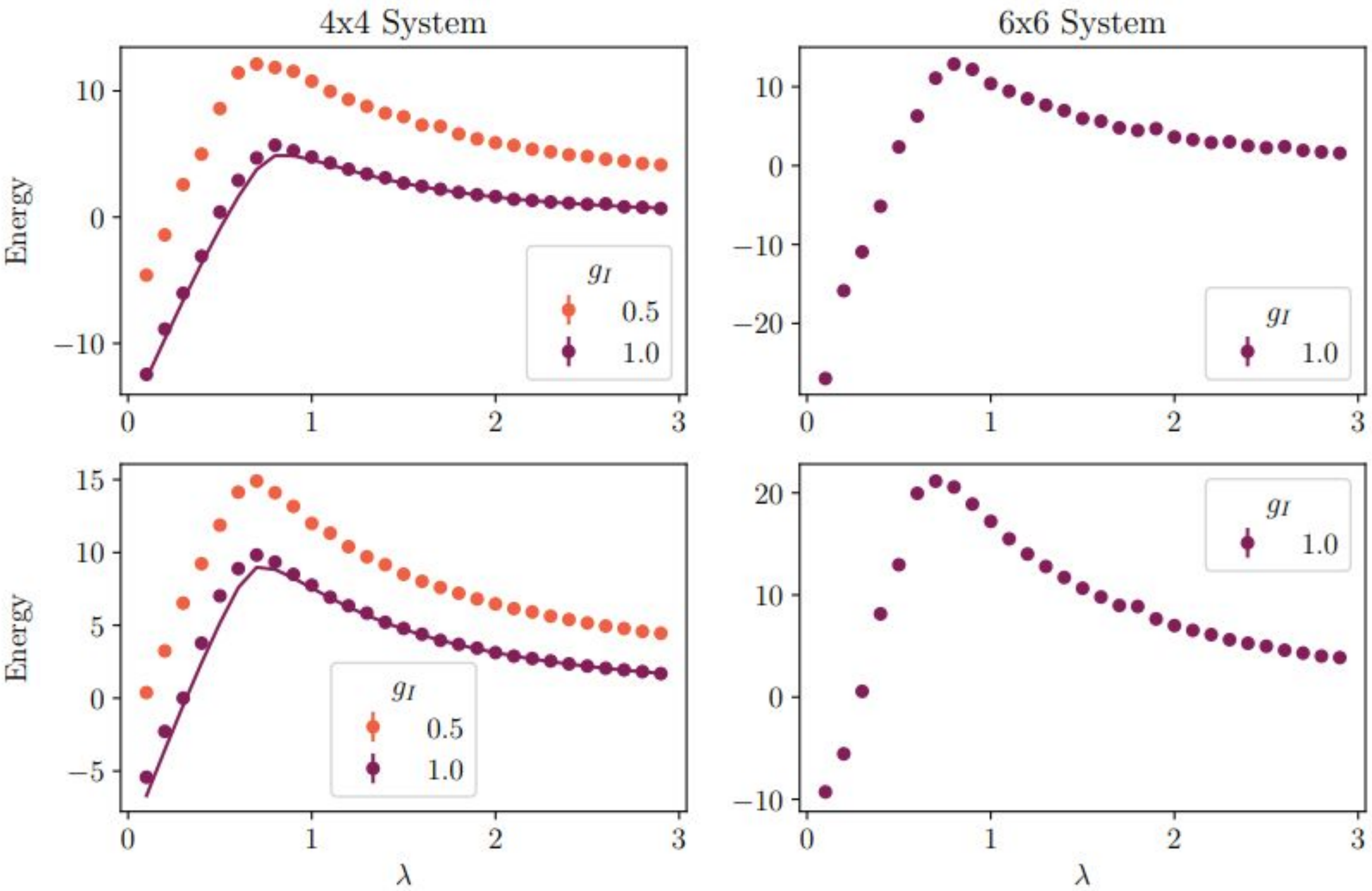
$$g_{\mathrm{E}} = \lambda$$
$$g_{\mathrm{I}} = 1.0$$
$$g_{\mathrm{M}} = 1.0$$
$$g_{\mathrm{B}} = 1/\lambda$$

A. Kelman, U. Borla, P. Emonts, and E. Zohar, arXiv:2412.16951.

Results – Monte Carlo Sampling

$g_{\mathrm{M}} = 0.0$

$g_{\mathrm{M}} = 1.0$



$g_{\mathrm{E}} = \lambda$

$g_{\mathrm{B}} = 1/\lambda$

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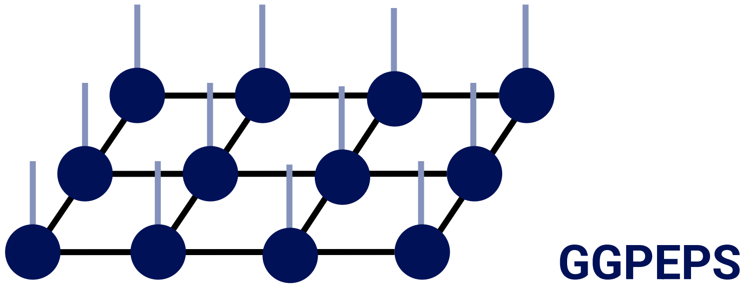
Summary

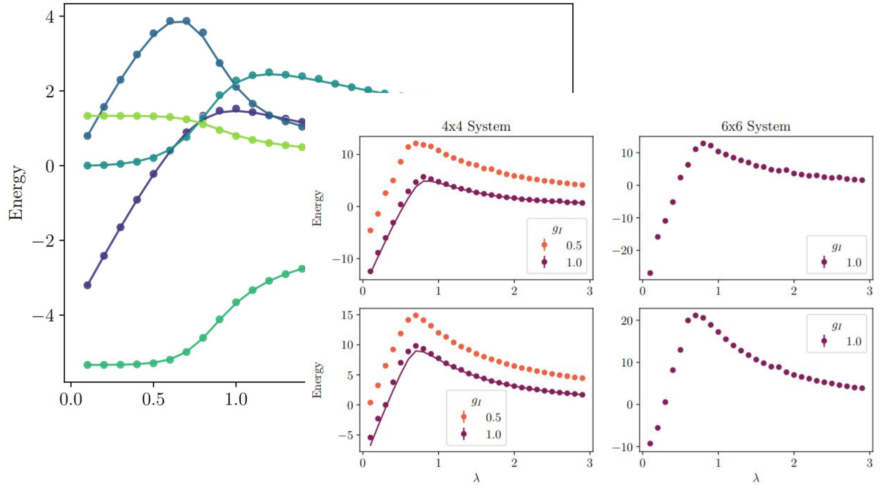
How do we construct $|\psi_F(\mathcal{G})\rangle$?



How do we efficiently calculate the expectation values?

Covariance Matrices and Variational Monte Carlo Sampling

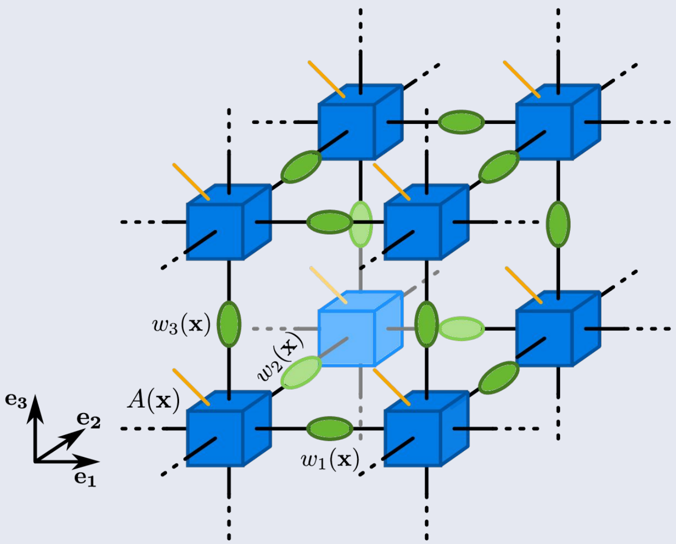
Are those states useful?



A. Kelman, U. Borla, P. Emonts, and E. Zohar, arXiv:2412.16951.

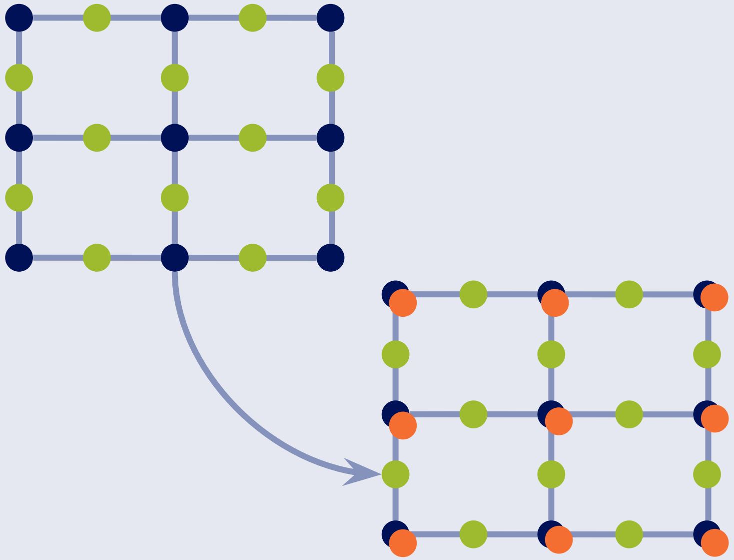
What are the next steps?

Three spatial dimensions?



P. Emonts and E. Zohar,. Phys. Rev. D 108, 014514 (2023).

Sign Problem affected cases?



Non-Abelian Gauge Groups?

$$SU(2)$$

$$SU(3)$$

E. Zohar and J. I. Cirac, Phys. Rev. D 97, 034510 (2018).

Science is a team effort

Ariel Kelman

Umberto Borla

Sergej Moroz

Snir Gazit

Ignacio Cirac

Erez Zohar

Mari Carmen Banuls

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arXiv:2412.16951

Variational Approaches

early works with DMRG/TNS

Bymes PRD (2002)
Suihara NPB (2004)
Tagliacozzo PRB (2011)
Sugihara JHEP (2005)
Meurice PRB (2013)

Schwinger Model U(1) in 1+1 d

Banuls et al., JHEP 11 158 (2013)
Rico et al., PRL (2014)
Buyens et al., PRL (2014)
Kühn et al., PRA 90 (2014)
Banuls et al., PRD (2015)
Buyens et al., PRD (2016)
Picher alt al., PRX (2016)

Non-Abelian in 1D
string breaking dynamics

Kühn et al., JHEP 07 (2015)
Silvi et al., Quantum (2017)
Kühn et al., PRX (2017)

3+1 d

Magnifico et al., Nat. Comm. 12 (2021)

2+1 d

Felser et al., PRX 10 (2020)
Robaina et al,. PRL 126 (2021)
Emonts et al., PRD 102 (2020)
Kelman et al., PRD 110 (2024)

SU(3) Quantum Link Model

Sivli et al. PRD (2019)

and more …

Photo by Jack Anstey on Unsplash

The Algorithm

Draw a new gauge field configuration

Build the state $|\psi_F(\mathcal{G})\rangle$

Calculate the acceptance probability by computing $\langle\psi_F(\mathcal{G})|\psi_F(\mathcal{G})\rangle$

Accept or Decline