Building quantum event generators through sparse Fock representations

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QT4HEP
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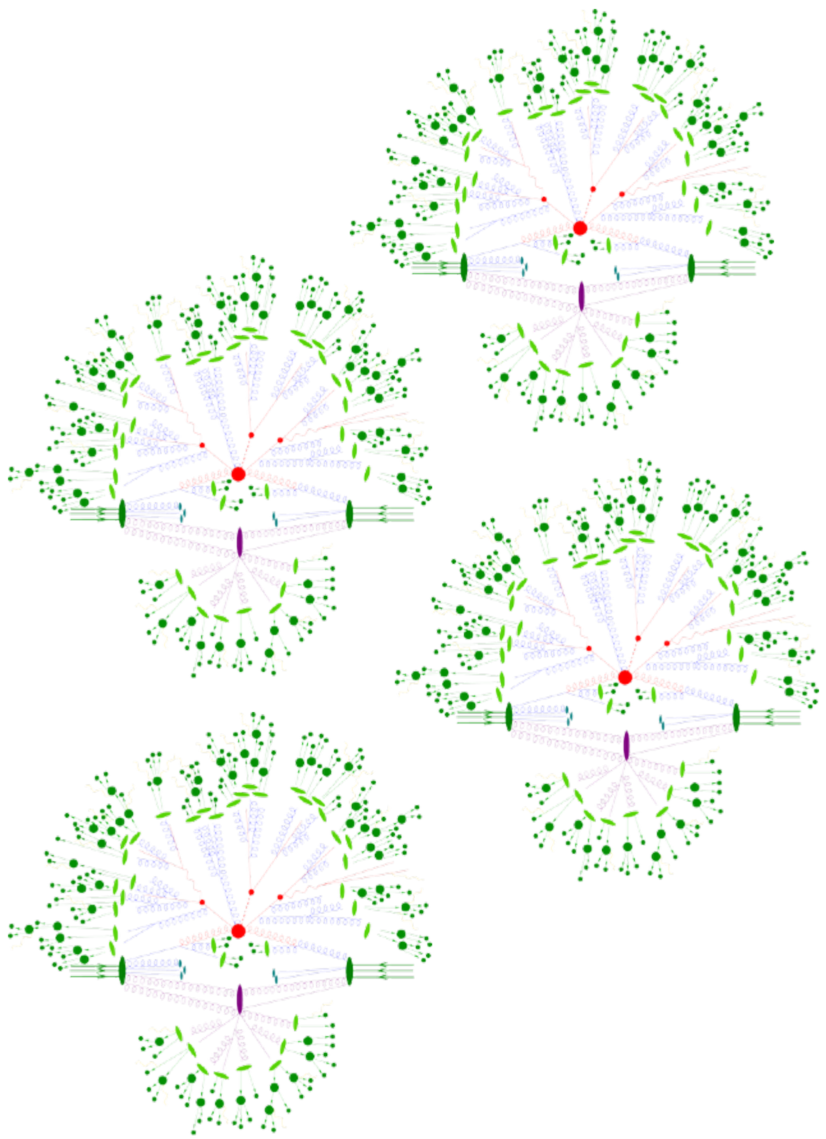
Event generator in a nutshell

Process description

- Incoming particles
- Primary outgoing particles
- Kinematic constraints
- Algorithm
- Precision
- etc.

Event generator

- Knows how to calculate the $d\sigma/d\Omega$ of the desired process
- Computes $\sigma=\int(d\sigma/d\Omega)d\Omega$
- Samples four-vectors according to $dP(\Omega) = (d\sigma/d\Omega)d\Omega/\sigma$



Many, many events

= Lists of stable particles and their four-vectors

"one of the computational pillars of any HEP experiment"

(HEP Software Foundation review)

A practical case for something different

Current event generators suffer from fundamental scaling problems:

- Event complexity scales ~factorially with perturbation order
- Integration time scales ~exponentially with final-state multiplicity

Timing and memory usage (Sherpa 3.x.y + HDF5)

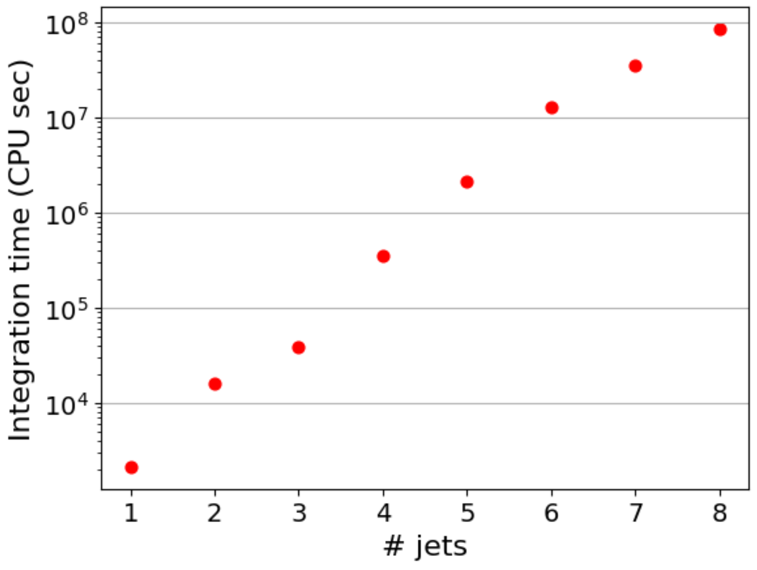
LO ME level event generation only (Comix; $\gamma, Z, h, \mu, \nu_\mu, \tau, \nu_\tau$ off)

Process $W^+ +$	1j	2j	3j	4j
RAM Usage	21 MB	43 MB	48 MB	85 MB
Init/startup time	<1s / <1s	<1s / <1s	2s / <1s	32s / <1s
Integration time	8×4m26s	16×16m42s	32×20m26s	64×1h32m
MC uncertainty	0.22%	0.46%	0.89%	0.97%
Unweighting eff	$6.59 \cdot 10^{-3}$	$7.50 \cdot 10^{-4}$	$2.71 \cdot 10^{-4}$	$1.47 \cdot 10^{-4}$
10k evts	1m 2s	15m 5s	1h 3m	5h 56m

Numbers generated on dual 8-core Intel® Xeon® E5-2660 @ 2.20GHz

Process $W^+ +$	5j	6j*	7j*	8j†
RAM Usage	189 MB	484 MB	1.32 GB	1.32 GB
Init/startup time	3m5s / 1s	24m52s / 5s	3h0m / 18s	5h55m / 29s
Integration time	128×4h38m	256×13h53m	512×19h0m	1024×23h8m
MC uncertainty	1.0%	0.99%	2.38%	4.68%
Unweighting eff	$9.56 \cdot 10^{-5}$	$7.66 \cdot 10^{-5}$	$7.20 \cdot 10^{-5}$	$7.51 \cdot 10^{-5}$
10k evts	24h 40m	2d 11h	10d 15h	78d 1h

Numbers generated on dual 8-core Intel® Xeon® E5-2660 @ 2.20GHz
*,† Number of quarks limited to ≤6/4



Source: Schultz 2018

3 billions CPU hours/year
15% is MC integration

phase-space factor — integrand

$$\sigma = \frac{1}{F}\int d\Phi |M|^2 \Theta(\Phi - \Phi_c)$$

phase-space cuts

probability distributions/ matrix element

Agliardi, Grossi, Pellen, Prati "**Quantum integration of elementary particle processes.**" https://doi.org/10.1016/j.physletb.2022.137228

Michele Grossi (CHEP24)

QUANTUM TECHNOLOGY INITIATIVE — M. Grossi

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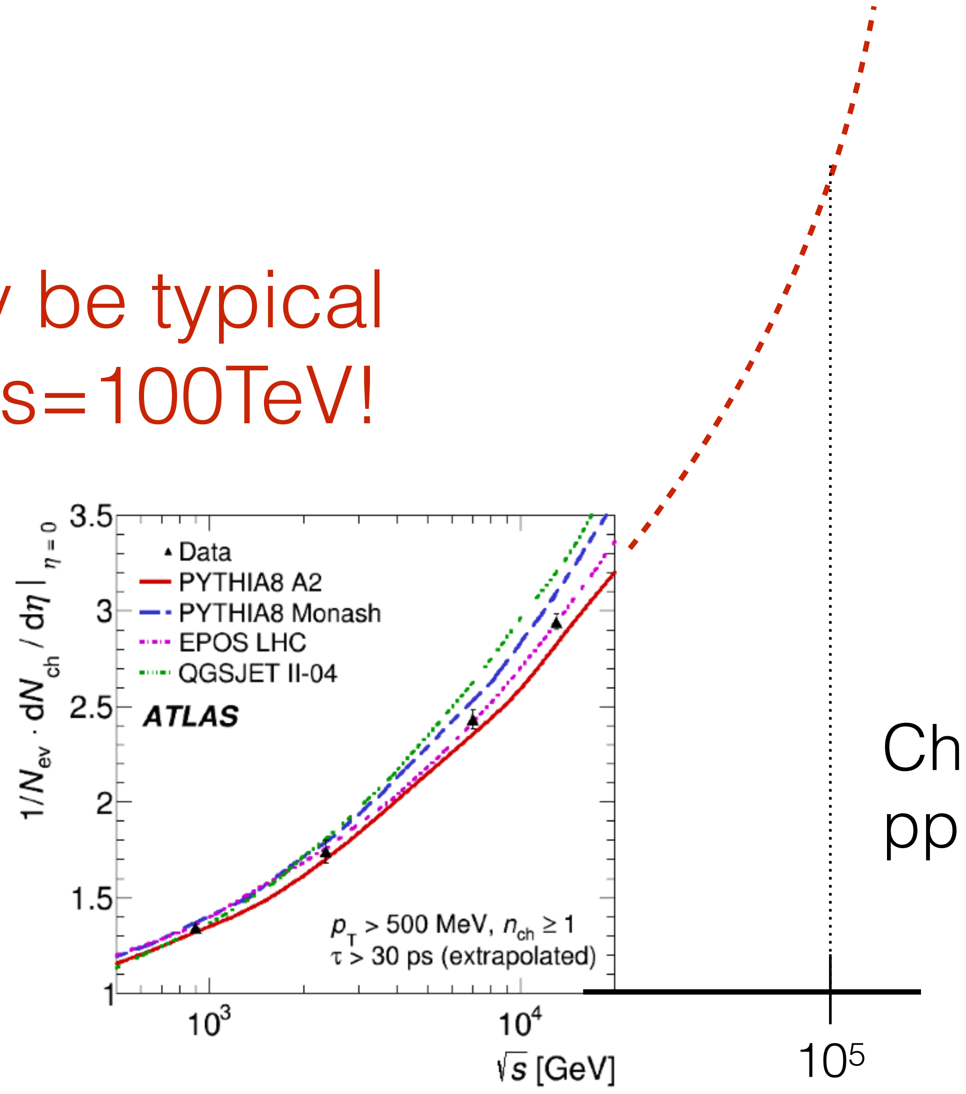
May be typical @ √s=100TeV!



Charged particle multiplicity in pp collisions (PLB 758 67)

Source: Schultz 2018

A practical case for something different

Current event generators suffer from fundamental scaling problems:

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Inefficiencies are also problems:

- Sampling = variant of hit-and-miss. $d\sigma/d\Omega$ variance ↑⇒ hit efficiency ↓

  Ref: Eff(W+4jets @ $\sqrt{s}$=13TeV) = 0.1%
- "Sign problem": Events at ≥NLO in QCD can have negative weights

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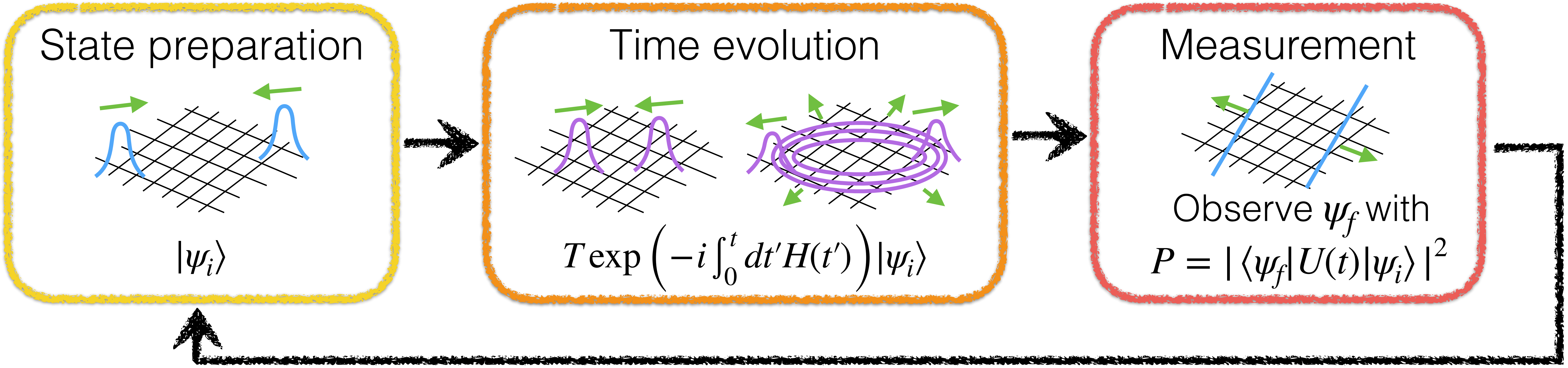
All difficulties are consequences of simulating a quantum system with classical computers

What a quantum event generator would look like

Real-time dynamics simulation + shot-by-shot sampling:



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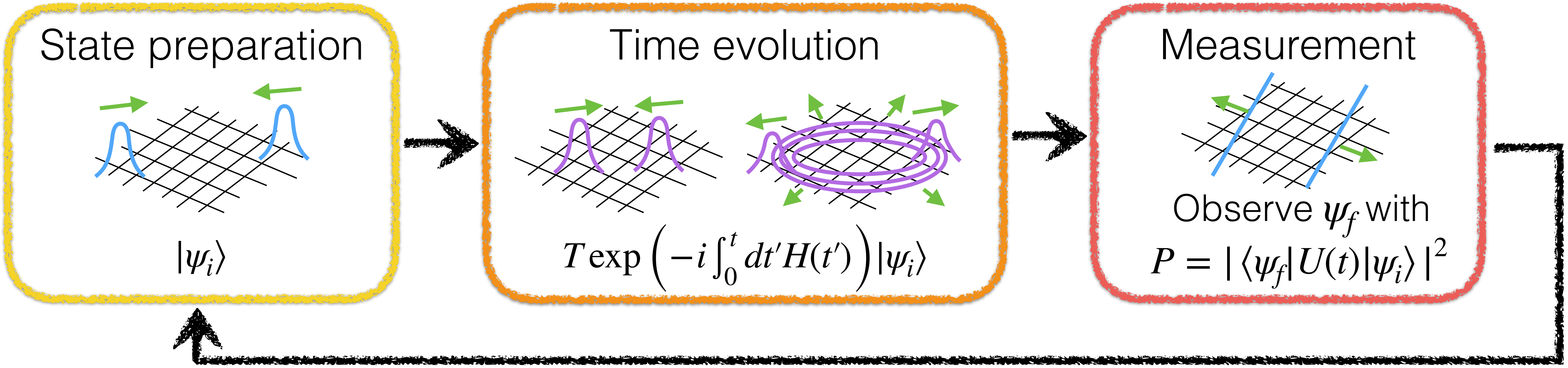
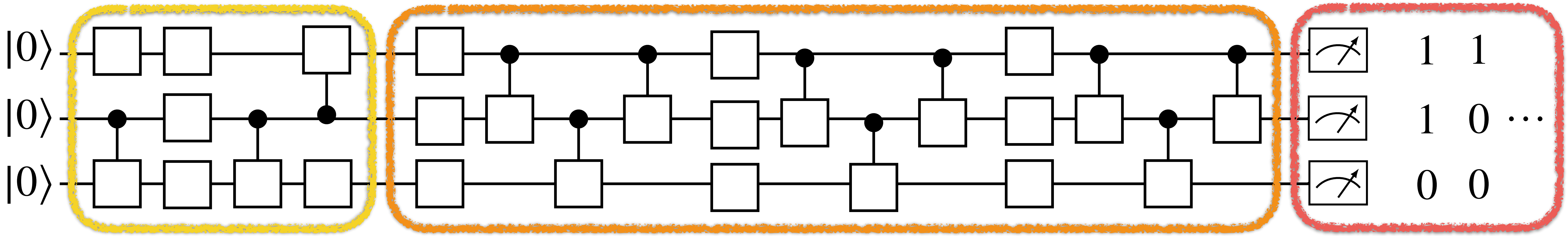


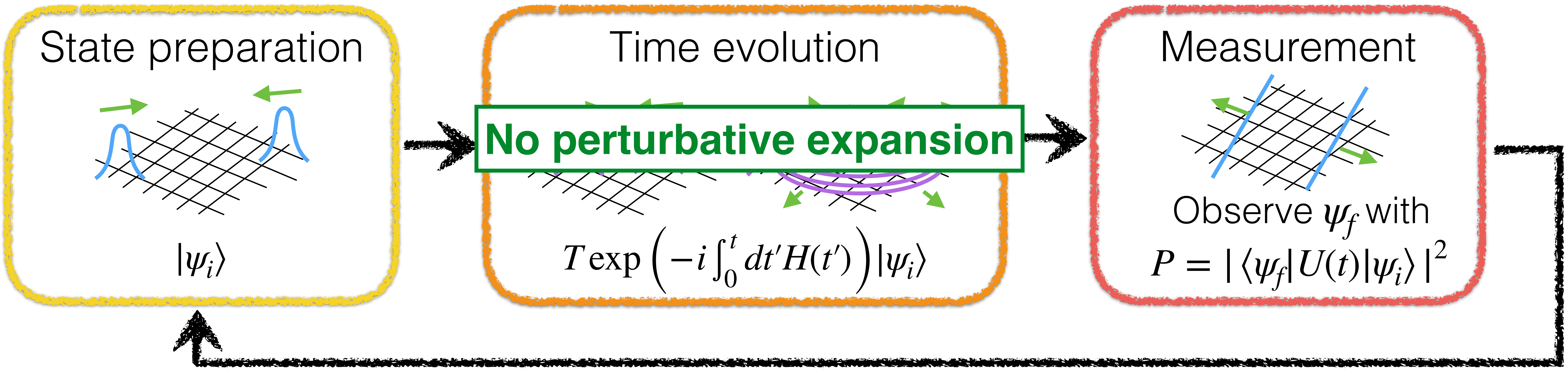
Which is, incidentally, how quantum computation works:



* If based on the quantum circuit model of quantum computing

What a quantum event generator would look like

Real-time dynamics simulation + shot-by-shot sampling:



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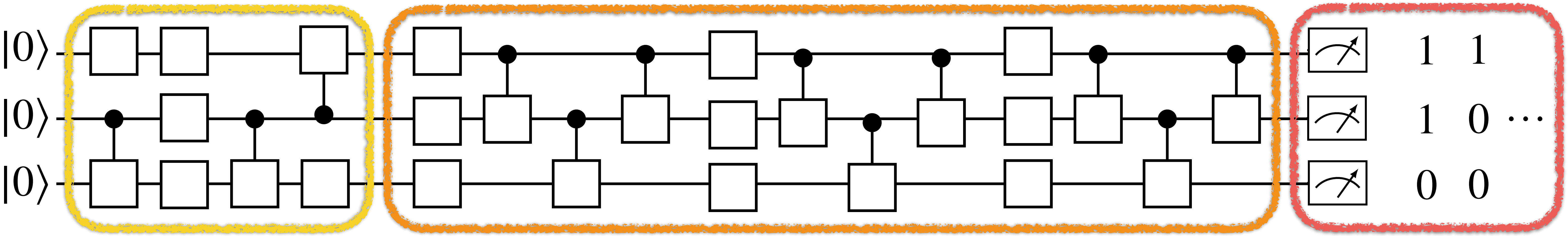

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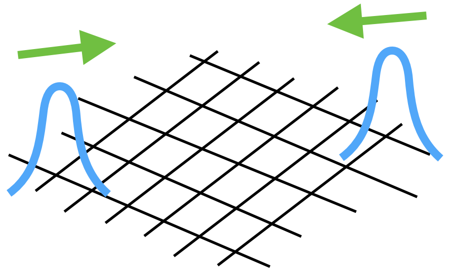
What a quantum event generator would look like

Real-time dynamics simulation + shot-by-shot sampling:

State preparation



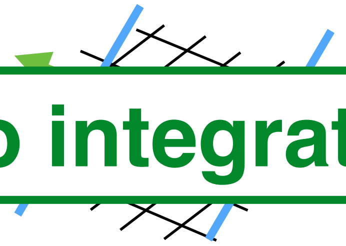
$|\psi_i\rangle$

Time evolution

No perturbative expansion

$T\exp\left(-i\int_0^t dt' H(t')\right)|\psi_i\rangle$

Measurement



No integration

Observe ψ_f with

$P = |\langle\psi_f|U(t)|\psi_i\rangle|^2$

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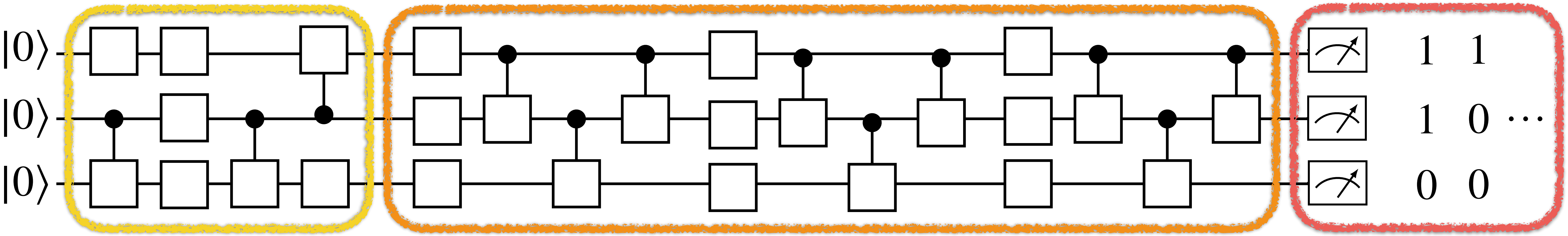

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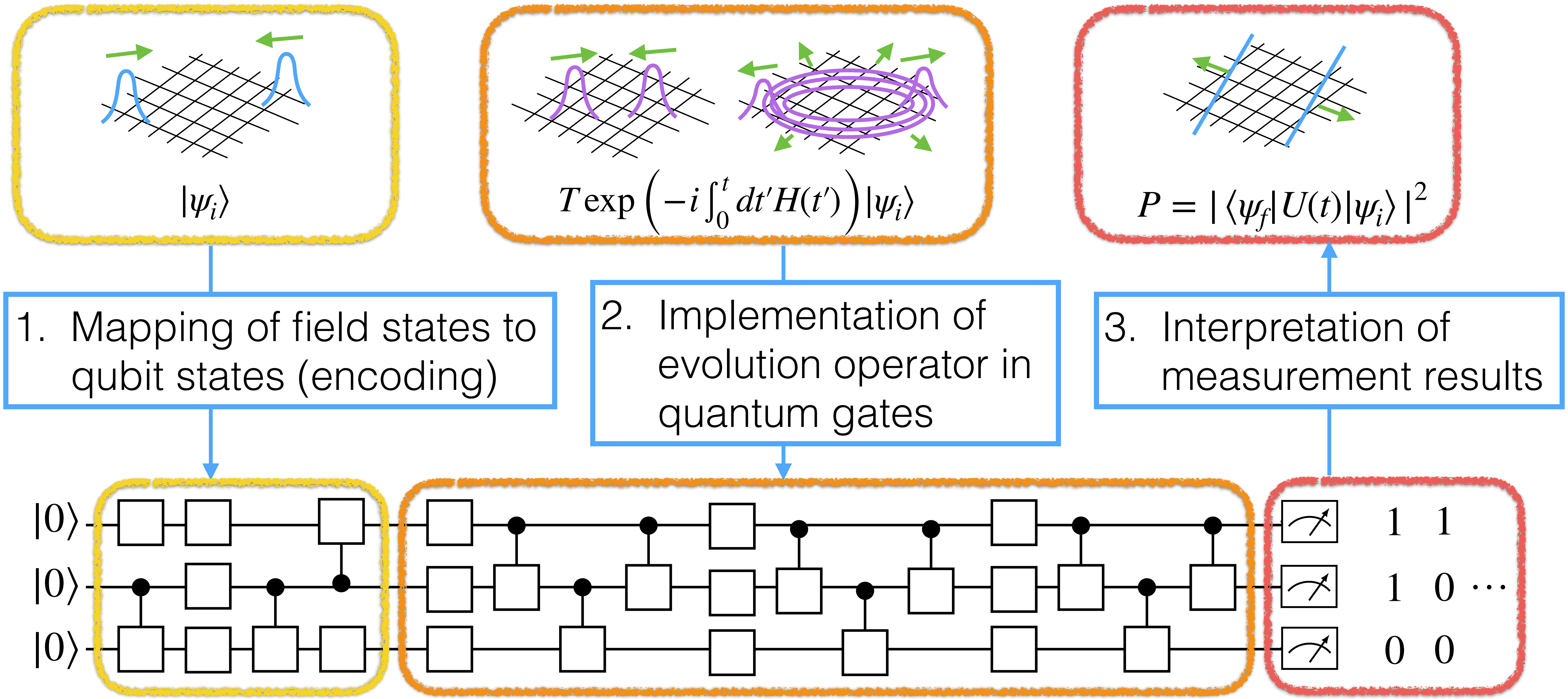
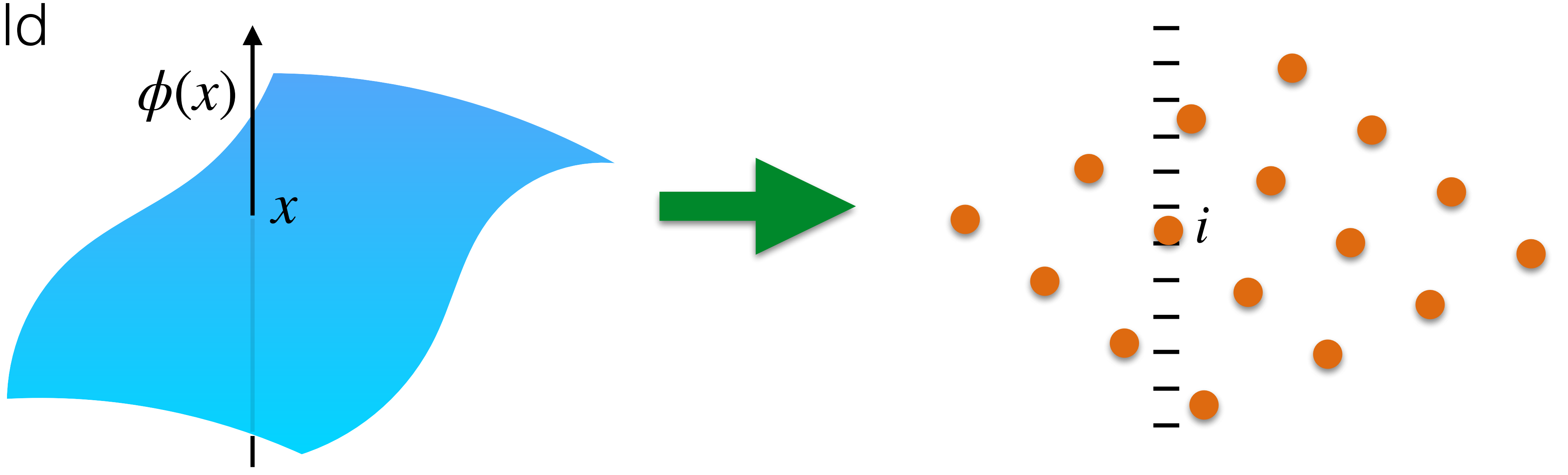
Ingredients of a quantum event generator



Encoding field states: discretization

Bosonic field



- Continuous (infinite) space $V = \int dx$
- Continuous unbounded field value ϕ

$$\Rightarrow \mathscr{H} = \text{span}\left(\left\{ |\phi\rangle \middle| \phi \in \mathbb{R} \right\}\right)^{\otimes \int dx}$$

$(\text{span}(\{|0\rangle, |1\rangle\})$ for fermions)

- Discrete finite lattice $N = L^d$
- Discrete truncated field values $0, 1, \ldots, K-1$

$$\Rightarrow \mathscr{H} = \text{span}\left(\{|0\rangle, |1\rangle, \ldots |K-1\rangle\}\right)^{\otimes N}$$

Discretization parameters determine the expressible dynamic range:

- $p_{\max}/p_{\min} \sim L$
- $\phi_{\max}/\phi_{\min} \sim K$

Field-based encoding is infeasible

Use an n-bit quantum register per lattice point per field:

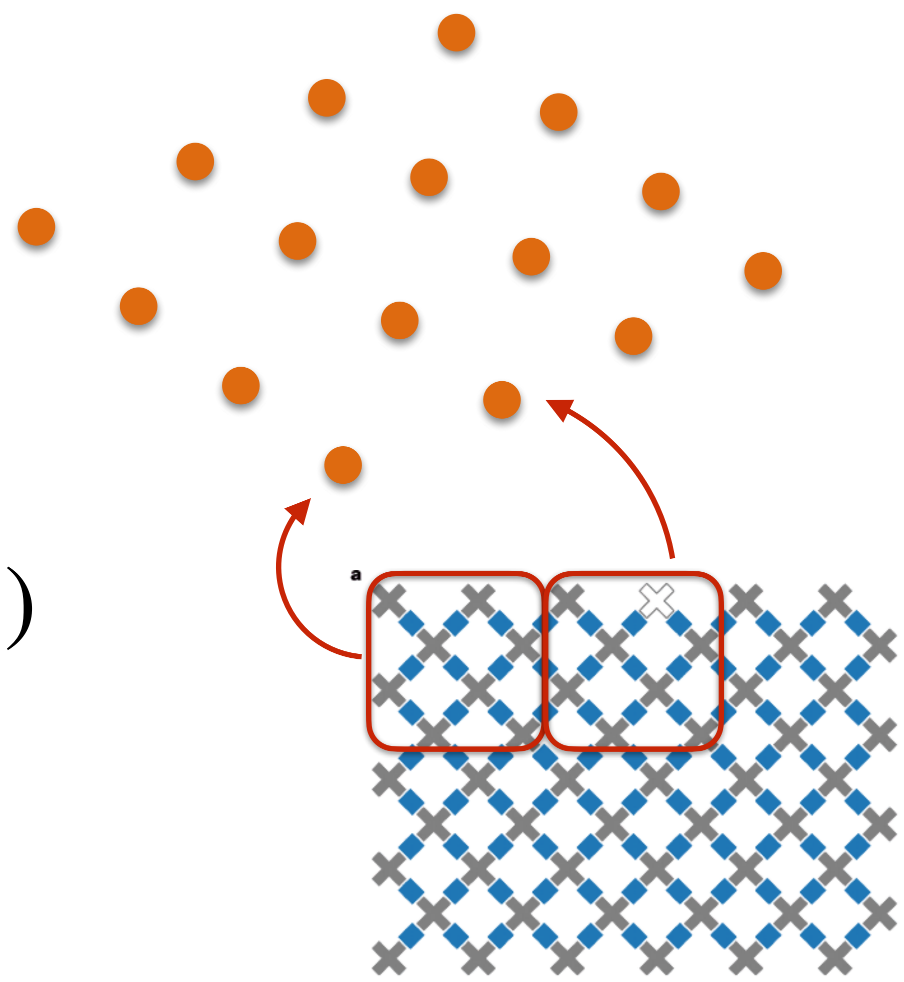
$$|\text{system}\rangle = |j_1\rangle \otimes |j_2\rangle \otimes \cdots \otimes |j_N\rangle \;\; (j_i = 0,\ldots,2^n - 1)$$

Field value at site 1

Can also encode a Fock representation:

$$|\text{system}\rangle = |k_{p_1}\rangle \otimes |k_{p_2}\rangle \otimes \cdots \otimes |k_{p_N}\rangle \;\; (k_{p_i} = 0,\ldots,2^n - 1)$$

Number of excitations of mode p_1



⇒ **Qubit count**: nL^d

For $p_{\max}/p_{\min}$ = 10 TeV / 100 MeV = 10^5 and d = 3 we need ~$10^{15}n$ qubits

Alternative: Particle-based encoding

Assign a quantum register to each particle, maximum M particles

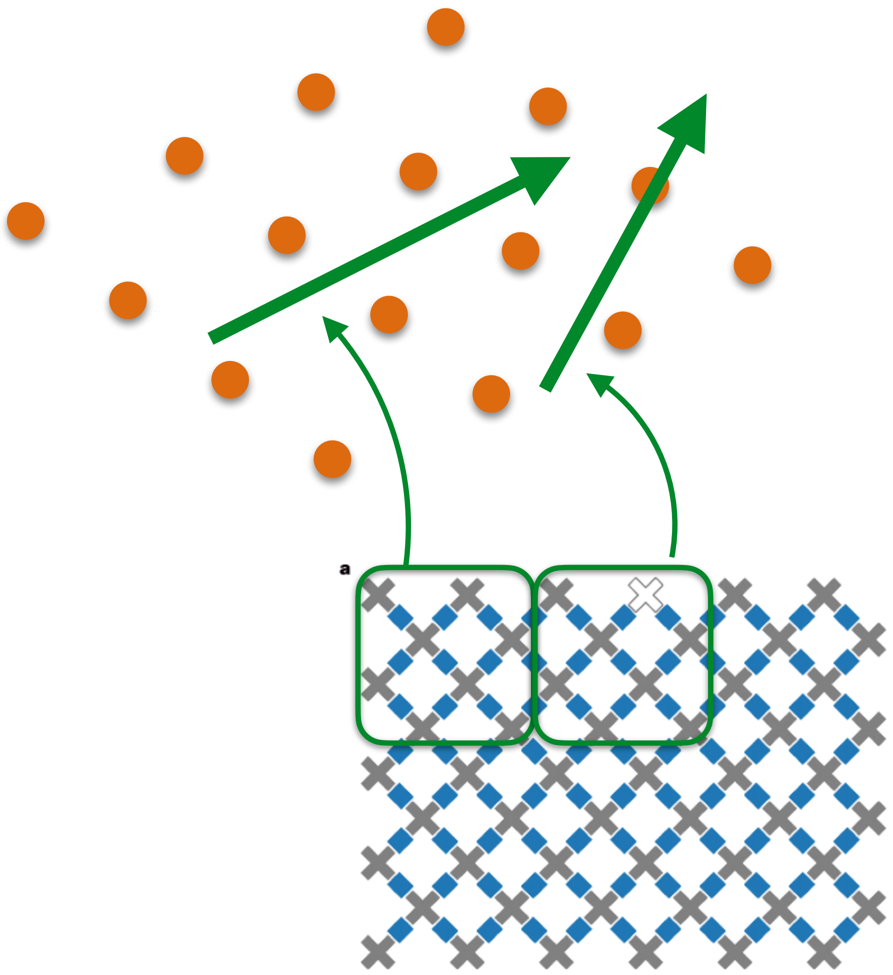
→ Field theory as multi-body quantum mechanics

$$|\text{system}\rangle = \mathcal{S}|p_1 \ldots p_J \Omega \ldots \Omega\rangle$$

J occupied slots M-J unoccupied slots

Symmetrization (bosons) or antisymmetrization (fermions)

→ Slater determinant



Essentially, a sparse Fock representation

⇒ **Qubit count**: $M(d \log_2 L)$

For $p_{\max}/p_{\min}$ = 10^5 and d = 3 we need ~ $50M$ qubits

Constructing field operators

$$a_p \mathcal{S} |p_1 \ldots p \ldots p_J \, \Omega \ldots \Omega\rangle = \sqrt{n_p} \mathcal{S} |p_1 \ldots p_J \, \Omega\Omega \ldots \Omega\rangle$$

Annihilation operator de-occupies one slot..

$$a_q \mathcal{S} |p_1 \ldots p_J \, \Omega \ldots \Omega\rangle = 0 \quad (q \notin \{p_j\}_j)$$

or annihilates the ket if no matching occupied slot exists.

$$a_q^\dagger \mathcal{S} |p_1 \ldots p_J \, \Omega\Omega \ldots \Omega\rangle = \sqrt{n_q + 1} \mathcal{S} |p_1 \ldots p_J q \, \Omega \ldots \Omega\rangle$$

Creation operator fills one slot..

$$a_q^\dagger \mathcal{S} |p_1 \ldots p_M\rangle = 0$$

or annihilates the ket if it is maximally filled.

All operators can be expressed with combinations of a and $a^\dagger$

⇒ Figure out the implementation of $\mathcal{S}$, a, and $a^\dagger$!

Proposed implementations

- Barata et al. (PRA 103, 2021)

Only for bosons

$$\mathcal{S}|p_1 \ldots p_J \,\Omega \ldots \Omega\rangle = \frac{1}{\sqrt{\mathcal{N}}} \sum_{P \in \mathrm{perm}(M)} |P(p_1 \ldots p_J \,\Omega \ldots \Omega)\rangle$$

$a_p^\dagger = \frac{1}{\sqrt{M}} \sum_j a_p^{\dagger(j)}$ where $a_p^{\dagger(j)}$ creates a particle in register j

- Gálves-Viruet and Llanes-Estrada (arXiv 2406.03147)

$$\mathcal{S}|p_1 \ldots p_J \,\Omega \ldots \Omega\rangle = \frac{1}{\sqrt{\mathcal{N}}} \sum_{P \in \mathrm{perm}(J)} \sigma_P |P(p_1 \ldots p_J)\, \Omega \ldots \Omega\rangle$$

Sign of P

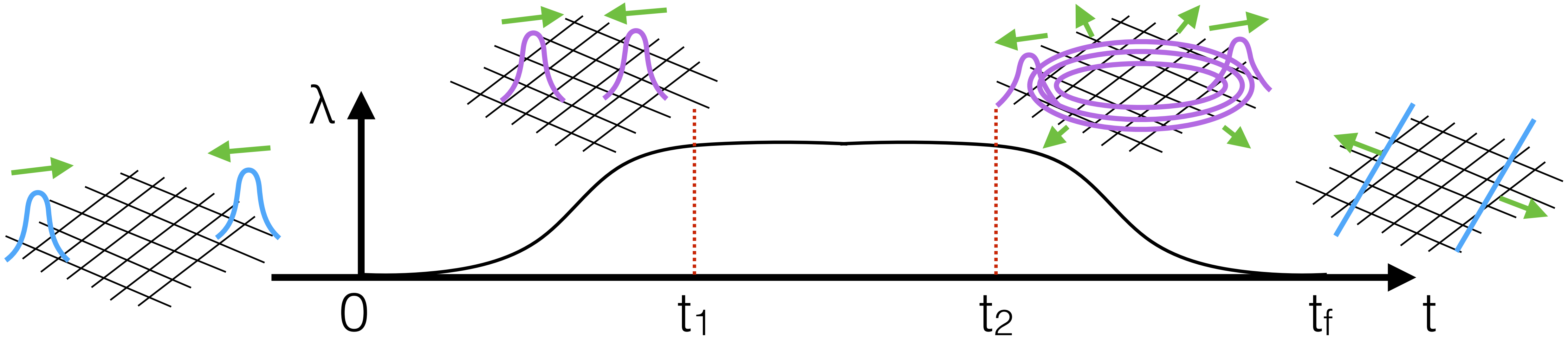
$a_p^\dagger = \sum_j \mathcal{T}_{j \leftarrow (j-1)} \, a_p^{\dagger(j)}$ where $a_p^{\dagger(j)}$ creates a particle in register j

and $\mathcal{T}_{j \leftarrow (j-1)}$ is a “step (anti)symmetrizer”

Event synopsis

Details in Barata et al.

- State preparation = Create wave packets $\sum_{\mathbf{p}_0,\mathbf{p}_1} \Psi_0(\mathbf{p}_0)\Psi_1(\mathbf{p}_1)\mathcal{S}|\mathbf{p}_0\mathbf{p}_1\,\Omega\ldots\Omega\rangle$
- Evolution in three time windows
  - $0 < t < t_1$: Adiabatic transition to physical single-particle states
    $H(t) = H_0 + f(t)\,H_I$ with $f(0) = 0, f(t_1) = 1$
  - $t_1 < t < t_2$: Evolution with full Hamiltonian e^{-iHt} (scattering)
  - $t_2 < t < t_f$: Adiabatic transition to Fock final states
- Measurement → Each bit string corresponds to a Fock state



Is sparse Fock simulation accurate?

Demonstration: Scattering in Schwinger model

Compare time evolution by full and truncated Hamiltonians

$$H = \sum_{n=0}^{N-1} \left[-\frac{i}{2\alpha} \left(e^{i\theta_n} \Phi_n^\dagger \Phi_{n+1} - \mathrm{h.c.} \right) + (-1)^n m \Phi_n^\dagger \Phi_n + J L_n^2 \right]$$

Periodic boundary condition → translationally invariant
→ momentum eigenstates!

with

$$\Phi_{n,\mathrm{even}} = \sqrt{\frac{2}{N}} \sum_k \frac{1}{\sqrt{\cosh w_k}} \left(e^{\frac{2\pi i}{N} kn} \cosh \frac{w_k}{2} a_k + e^{-\frac{2\pi i}{N} kn} \sinh \frac{w_k}{2} b_k^\dagger \right)$$

$$\Phi_{n,\mathrm{odd}} = \sqrt{\frac{2}{N}} \sum_k \frac{1}{\sqrt{\cosh w_k}} \left(e^{\frac{2\pi i}{N} kn} \sinh \frac{w_k}{2} a_k + e^{-\frac{2\pi i}{N} kn} \cosh \frac{w_k}{2} b_k^\dagger \right)$$

rapidity (w_k)

Incorporating Gauss' law constraint →

$$H = \sum_k \mathcal{E}_k \left(a_k^\dagger a_k + b_k^\dagger b_k \right) + J \sum_n L_n^2$$

Sparse Fock simulation lite

After solving the Gauss' law, all dynamics are encoded in $\sum_n L_n^2$.

In the Fock basis, No excitations 1 e⁻ with k₁, 1 e⁺ with k₂ ...

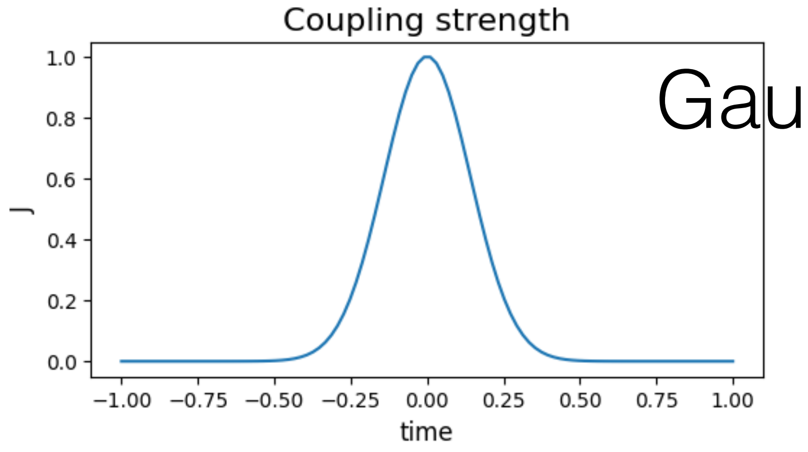
$$\sum_n L_n^2 = \begin{pmatrix} * & * & \cdots \\ * & * & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

⇒ Evolution in M-sparse Fock system simulated by nullifying rows and columns with $n = n^- + n^+ > M$

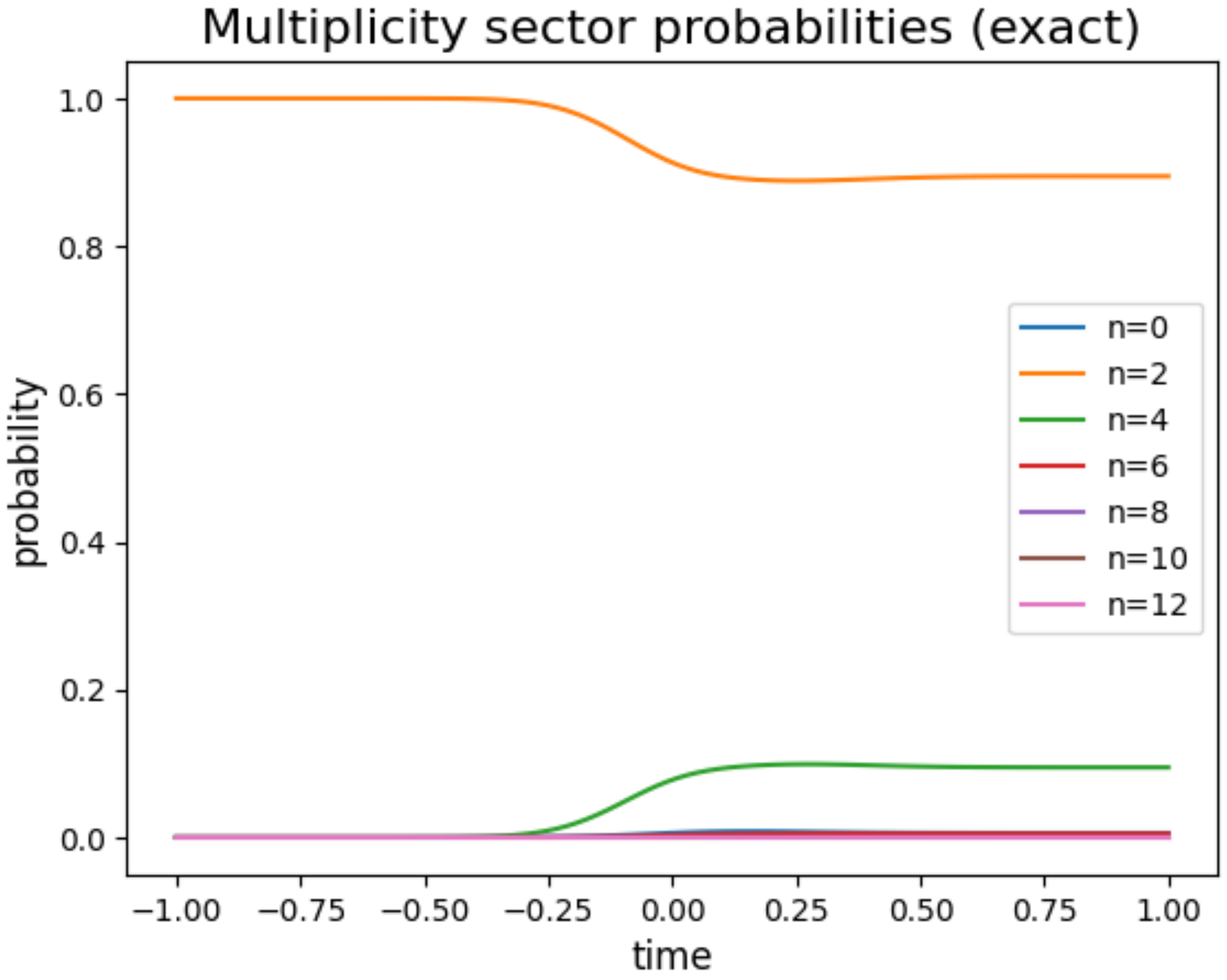
Schwinger model scattering (12 sites)

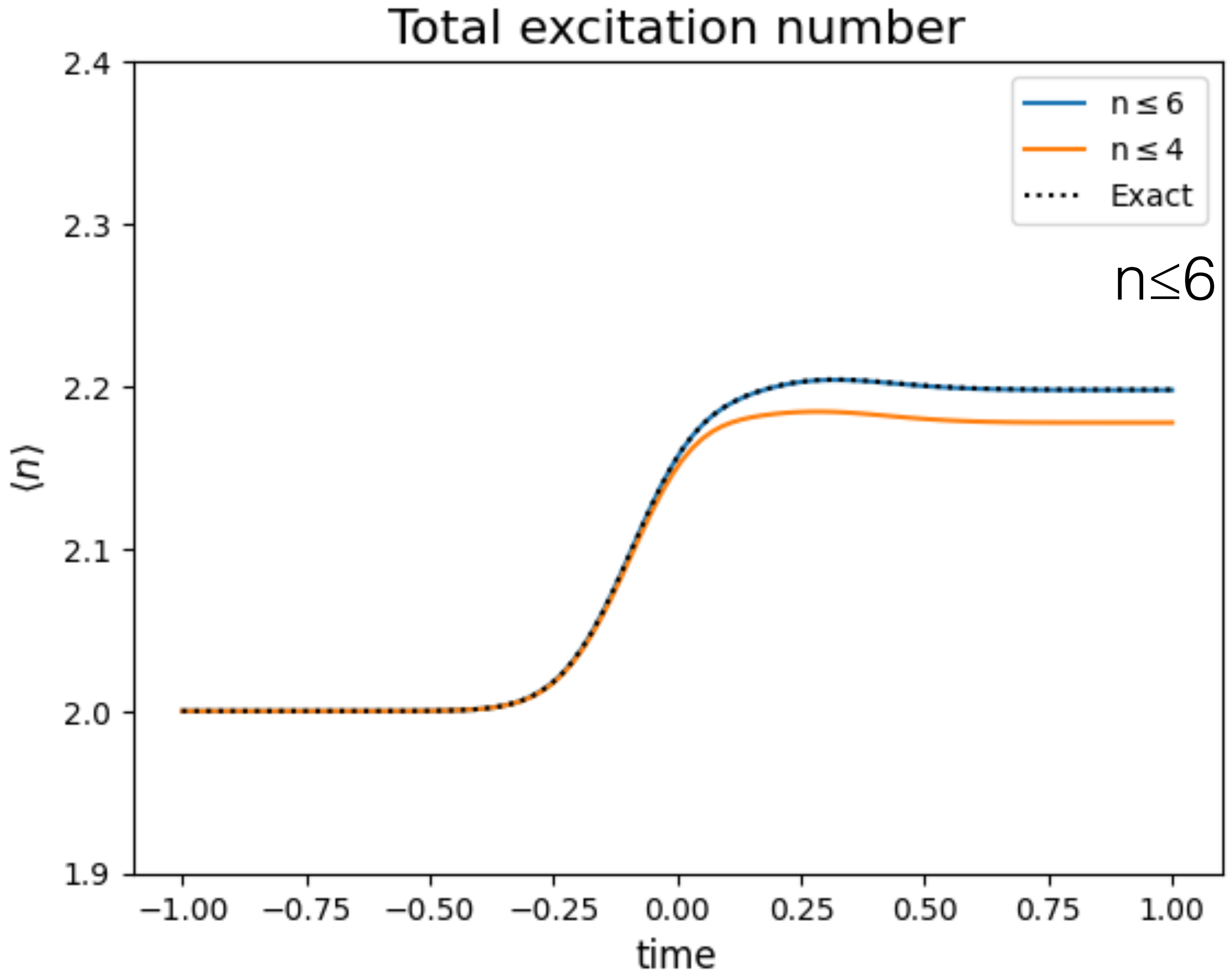
Initial state: $|e^{-}; -k_{\mathrm{max}}\rangle \otimes |e^{+}; +k_{\mathrm{max}}\rangle$

am=0.5
aJ=0.5



Gaussian interaction profile





n≤6 and no-trunc overlap

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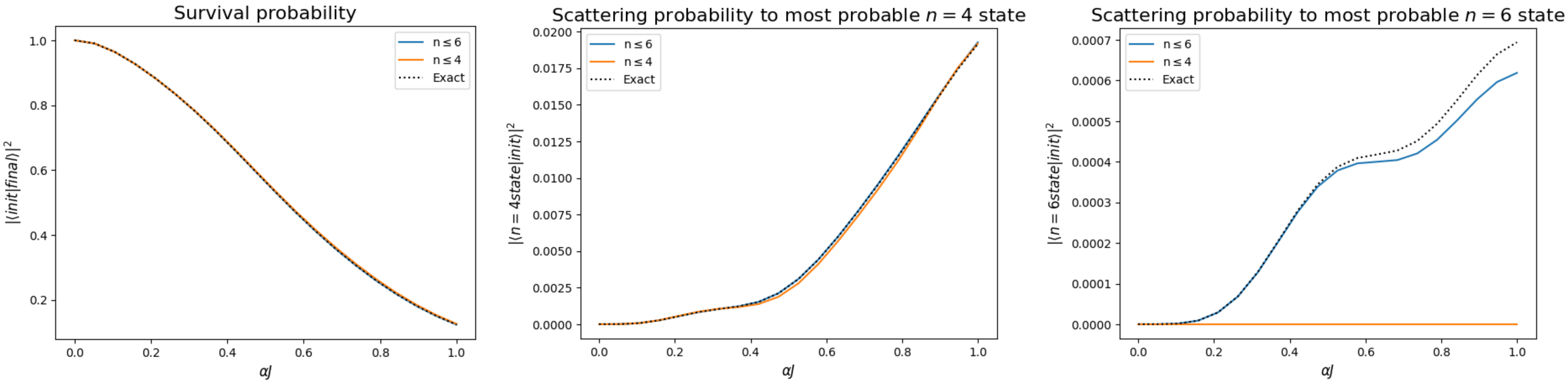

- Scattering probability difference is barely visible for n_{final}=4, even with M=4
- Obviously, probability of seeing n_{final}=6 is 0 for M=4

→ Sparse Fock representation viable, at least for this simple system

Particle-based simulation is viable.
What are the biggest questions?

- Optimality of the encoding?

  Symmetrizers are complex & non-unitary. Any way around?

- How do we select final states?

  A faithful LHC simulation will generate uninteresting events 99.999% of the time

- Circuit depth?

  Interaction Hamiltonian requires $O(L^d)$ gates per time step / poly degree

And a lot more!

Conclusion

- There is a practical case for quantum-simulating QFTs in the perturbative regime too
- Quantum computers operate like event generators already
- Field-based encoding is infeasible and unnecessary
- Encoding based on sparse Fock representation is promising
- For a scattering problem in the Schwinger model, sparsity does not kill the accuracy of simulation