Quantum Chebyshev generative modeling for Fragmentation functions

based on:

Jorge J. Martinez de Lejarza[1], Hsin-Yu Wu[3,5], Andrea Gentile[5], Germán Rodrigo[1], Oleksandr Kyriienko[3,4,5], Michele Grossi[2],
In preparation

1 VNIVERSITAT DE VALÈNCIA, IFIC INSTITUT DE FÍSICA CORPUSCULAR, EXCELENCIA SEVERO OCHOA, CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

2 CERN, QUANTUM TECHNOLOGY INITIATIVE

3 University of Exeter

4 University of Sheffield

5 PASQAL

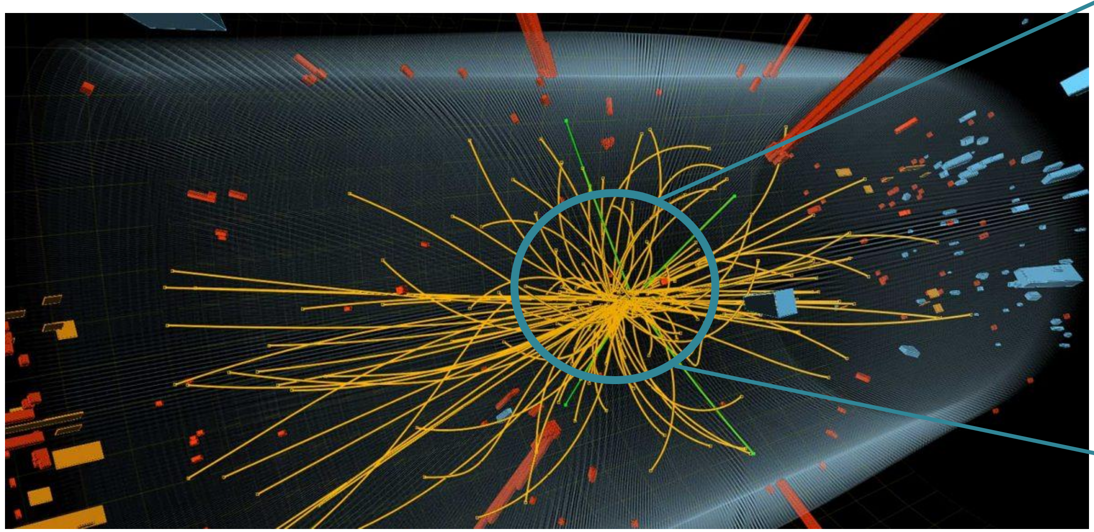
Speaker:

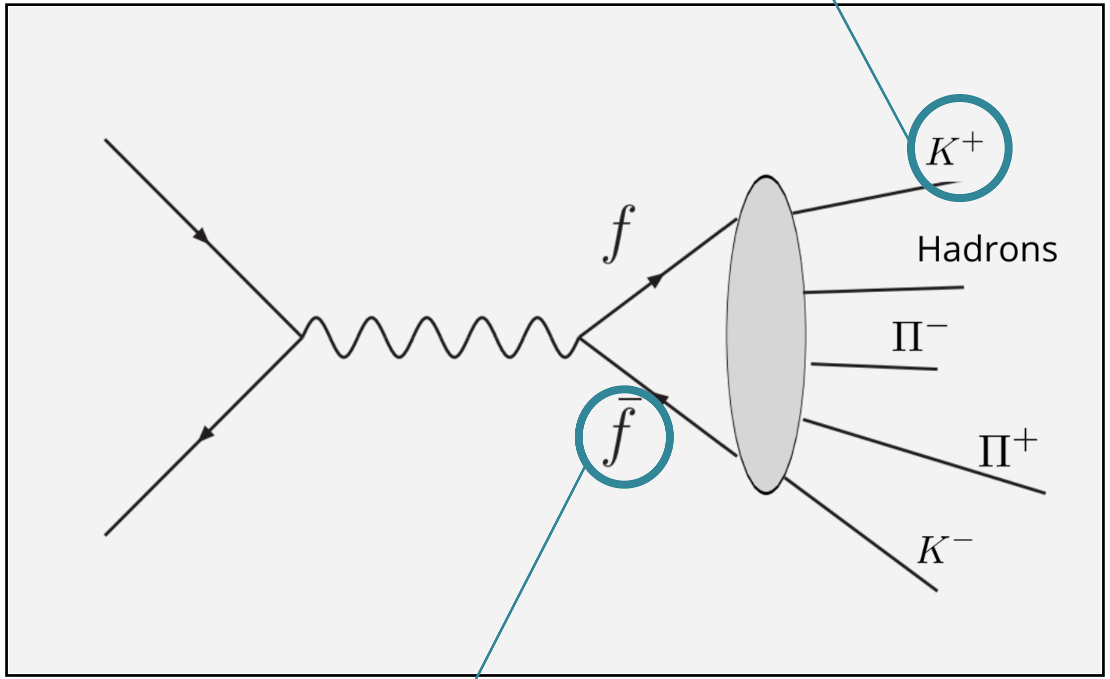
Jorge Martínez de Lejarza

QT4HEP 2025 – 23/01/2025

Motivation: Hadronization

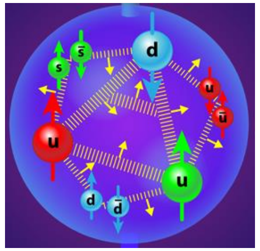
Hadronization: Partons → Hadrons





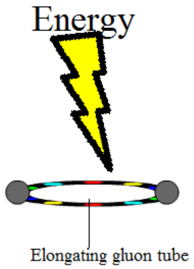
Particle Colliders (LHC)

Proton content

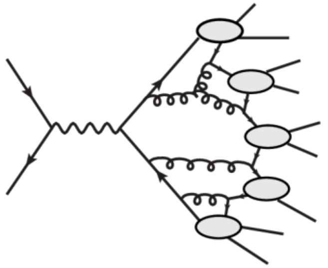


Partons: quarks and gluons

color particles QCD



Motivation: Fragmentation functions (FFs)



- **What** are FFs?

$$D_i^h(z,Q) \propto Prob(parton(i) \rightarrow hadron(h))$$

$z \equiv$ momentum fraction $Q \equiv$ energy scale

- **Why** are important?

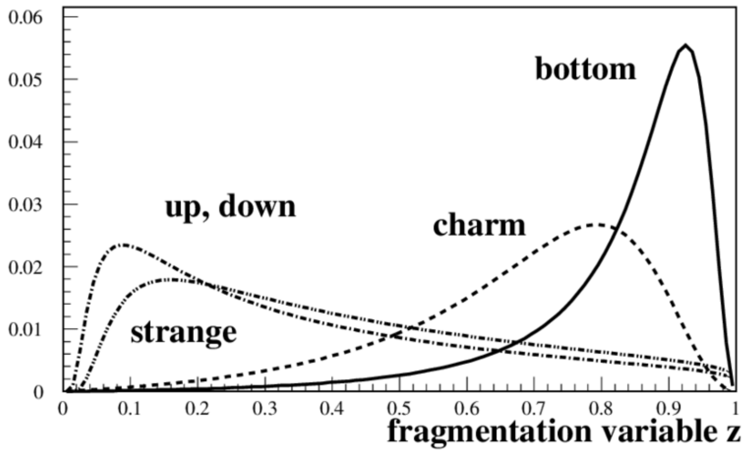
Compute integrals → predictions of observables (cross-sections)

$$\frac{d\sigma}{dz}(z,Q) \propto D_i^h(z,Q)$$

- **How** are determined?
  - Statistical and ML methods to **learn** FFs from data

$$D_i^h(z,Q_0) \propto z^\alpha(1-z)^\beta$$

  - **Caveat:** Interpolation for Q solving DGLAP evolution equations



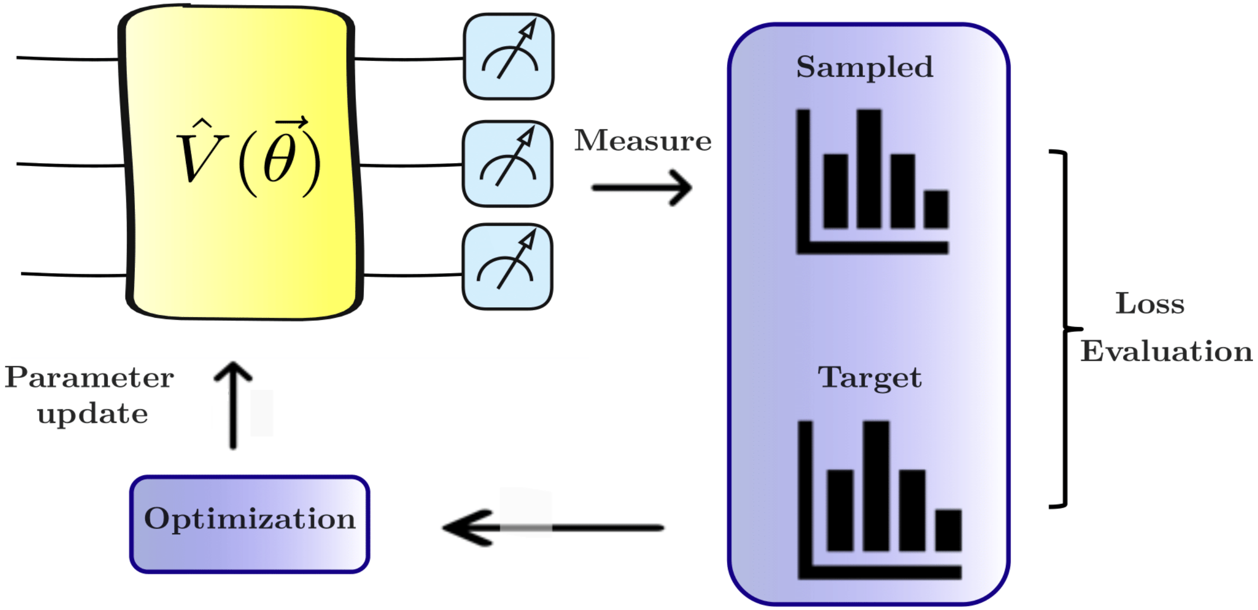
Motivation: Quantum Generative Models (QGM)

1. **Learn** target distributions

2. Generate new **samples**

3. Potentially more **efficient**

QGM



4. Examples: QGANs, QCBM, QAE, **QChGM**

Quantum Chebyshev Generative Models

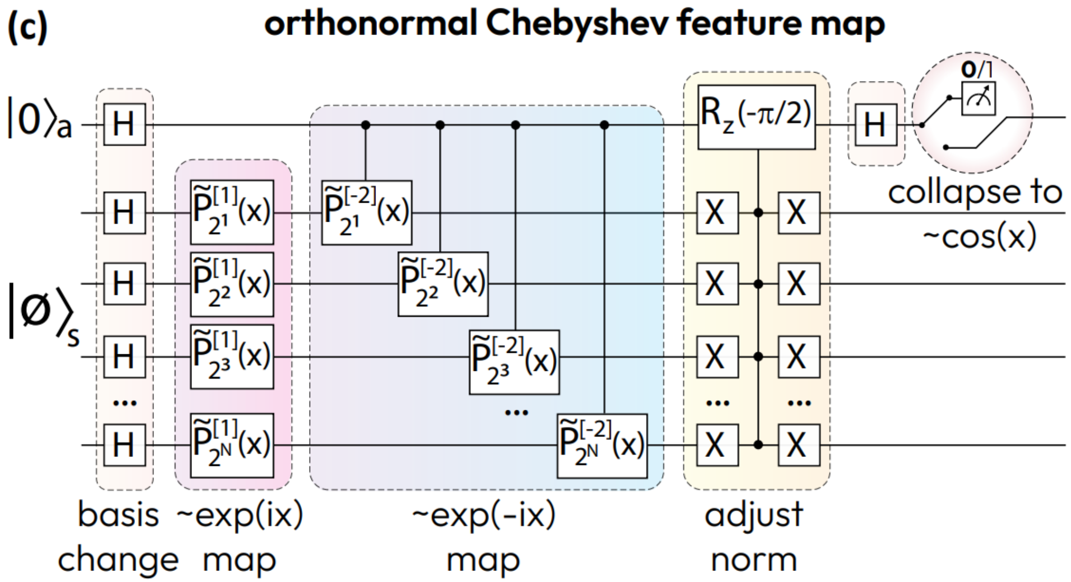
➢ Chebyshev Feature map:

- ➢ Chebyshev polynomials:

$$T_k(x) \equiv \cos\big(k \arccos(x)\big)$$

- ➢ Quantum state with Cheb. poly. as **amplitudes**

$$|\tau(x)\rangle = \frac{1}{2^{N/2}} T_0(x)|0\rangle + \frac{1}{2^{(N-1)/2}} \sum_{k=1}^{2^N-1} T_k(x)|k\rangle$$



$P_{meas} = \dfrac{I - Z}{2}$

$|\tau(x)\rangle$

- ➢ The feature map such $\hat{\mathcal{U}}_\tau(x)|0\rangle = |\tau(x)\rangle$

Orthogonal at the nodes $x_j^{\mathrm{Ch}} := \cos(\pi/2(2j+1)/2^N)$

Quantum Chebyshev Generative Models

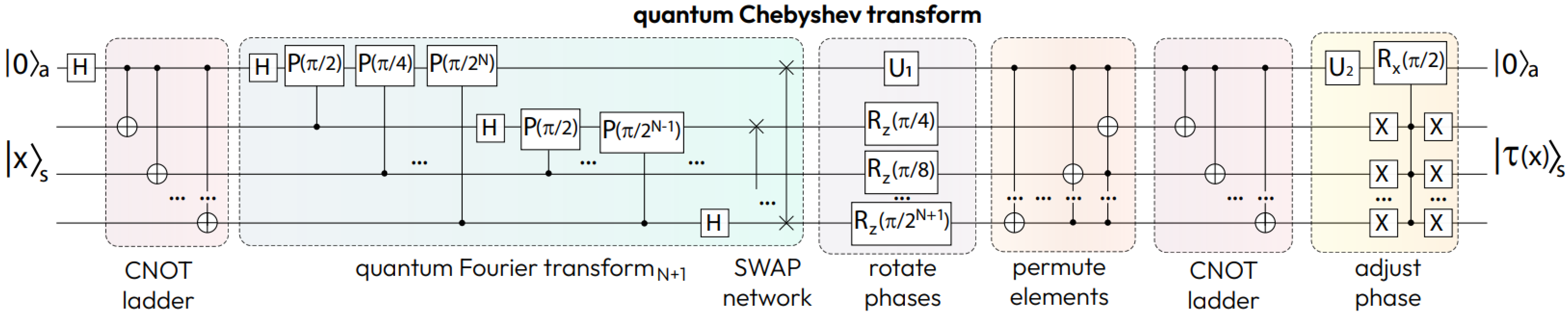
➢ Quantum Chebyshev Transform:

- ➢ Need a **map** between Cheb ↔ Computational basis

$$\hat{\mathcal{U}}_{\mathrm{QChT}} = \sum_{j=0}^{2^N-1} |\tau(x_j^{\mathrm{Ch}})\rangle\langle x_j|$$

- ➢ One can **go back** to computational basis (f):

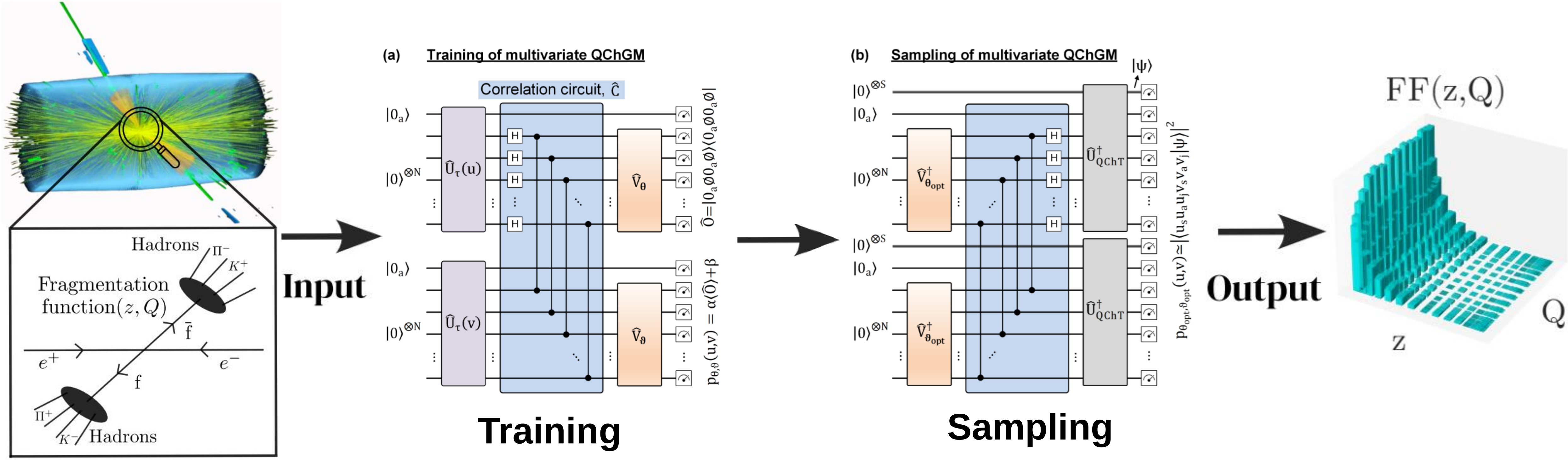
$$\hat{\mathcal{U}}_f = \hat{\mathcal{U}}^\dagger{}_{\mathrm{QChT}}\, \hat{\mathcal{U}}_\tau(x)$$



$$U_1 = P(-\pi/2^{N+1})R_z(-\pi(2^N-1)/2^{N+1}), \quad U_2 = P(-\pi/2)R_Y(-\pi/2)$$

Quantum Chebyshev Generative Models

➢ **Workflow of the model:**



Results: QChGM for FF

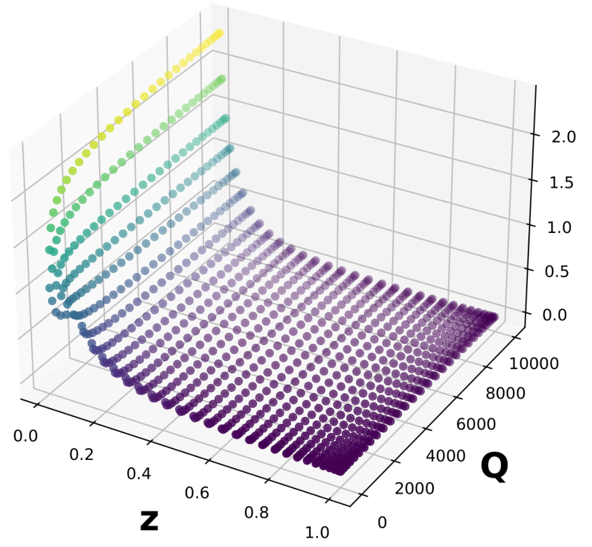
- **Simulation setup:**
  - Analyze FF data: $D^h_{u^+}, D^h_{d^++s^+}, D^h_{c^+}, D^h_{b^+}, D^h_g$ $\left\{ \begin{array}{l} \text{with } h = \pi^\pm, K^\pm \\ \text{for } z \in [0.01,1], Q \in [1, 10000] \end{array} \right.$
  - Quantum simulation: → per **variable**:
    4 qubits, 3 ansatz layers=16 parameters
  - Optimizer: ADAM, iterations=10000, learning rate ∈ $[0.1,1]$
  - Data from: http://lhapdf.hepforge.org/
    - NNF10_PIsum_nnlo
    - NNF10_KAsum_nnlo

LHAPDF6: parton density access in the LHC precision era: Andy Buckley, James Ferrando, Stephen Lloyd, Karl Nordstrom, Ben Page, Martin Ruefenacht, Marek Schoenherr, Graeme Watt, *Eur.Phys.J.C* 75 (2015) 132

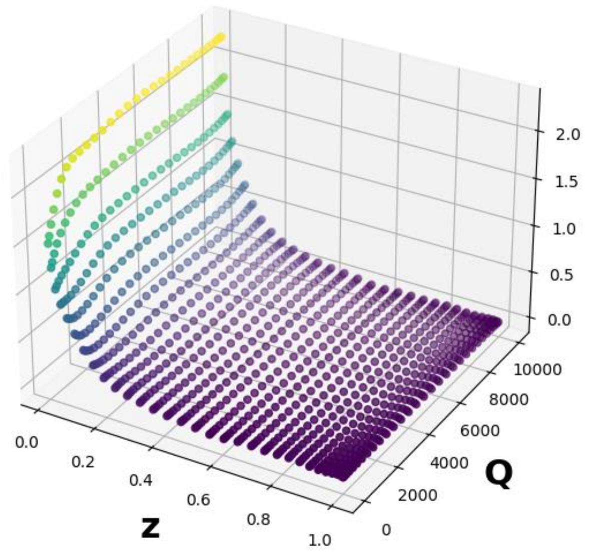
Results: QChGM for FF

- **Training and sampling:**
  - Example $D_g^{K^{\pm}}$:

Target:



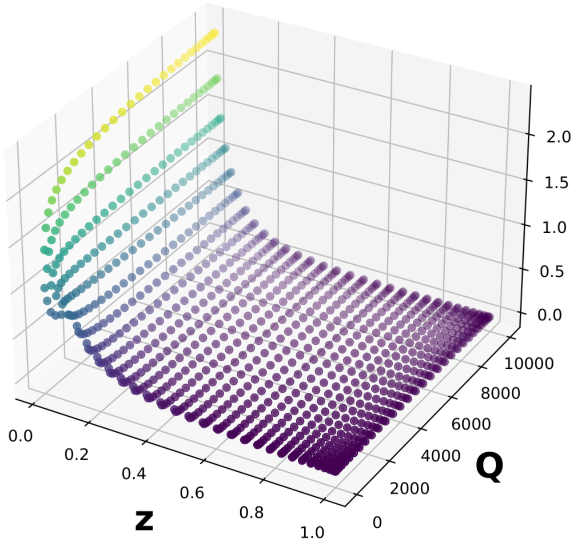
Predictions: $R^2 = 0.99$



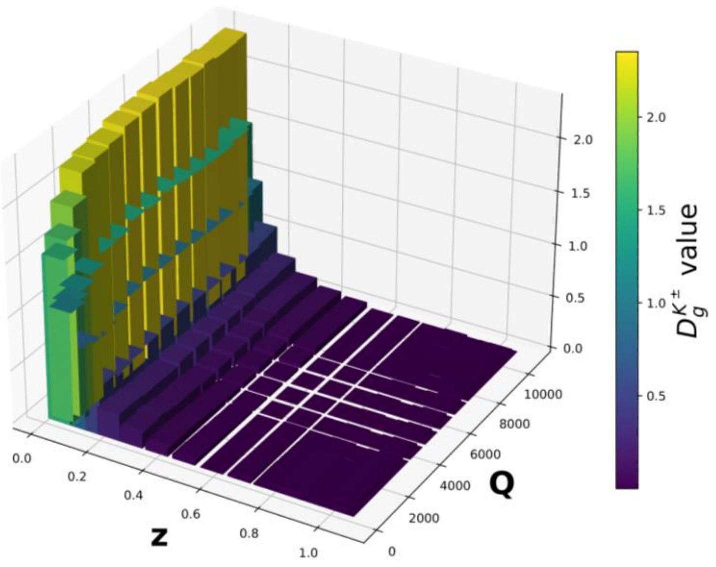
Results: QChGM for FF

- **Training and sampling:**
  - Example $D_g^{K^\pm}$:

Target:



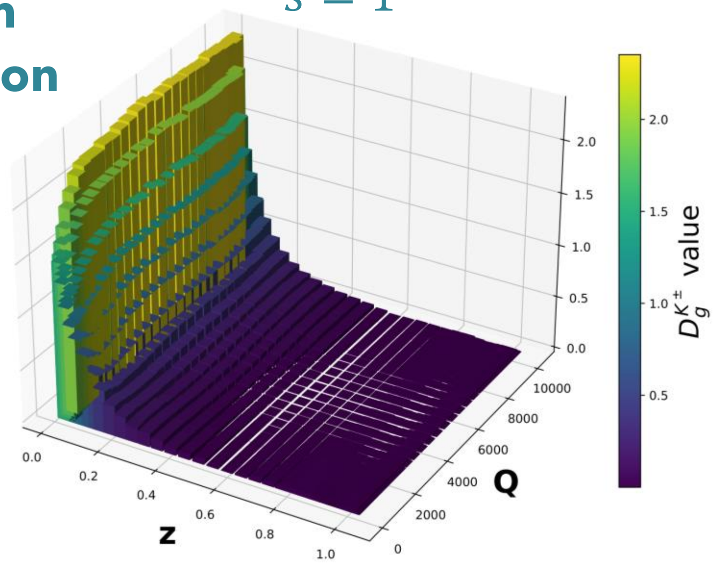
Sampling:



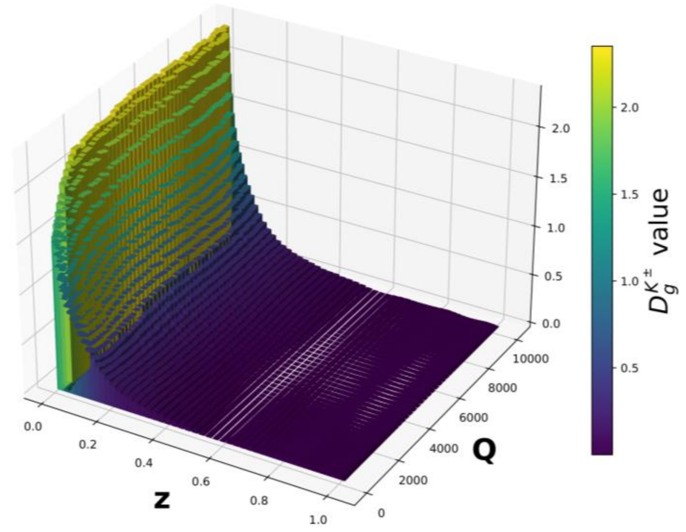
Extended sampling (more qubits)

Quantum interpolation

1 extra qubit for variable $s = 1$

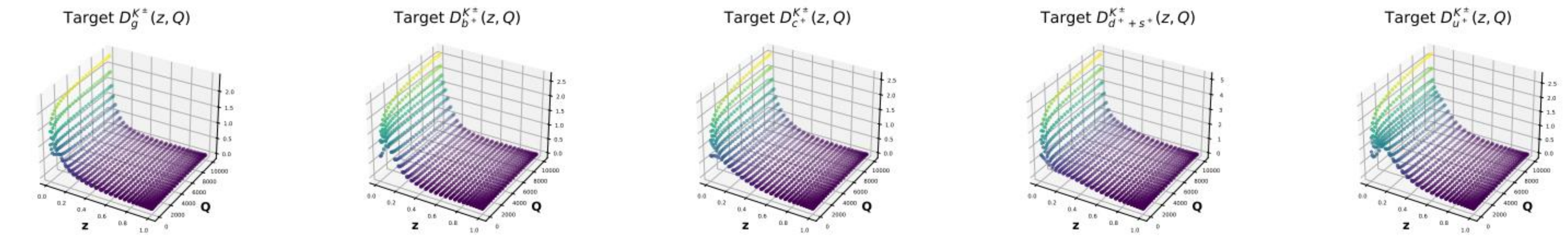


2 extra qubits for variable $s = 2$

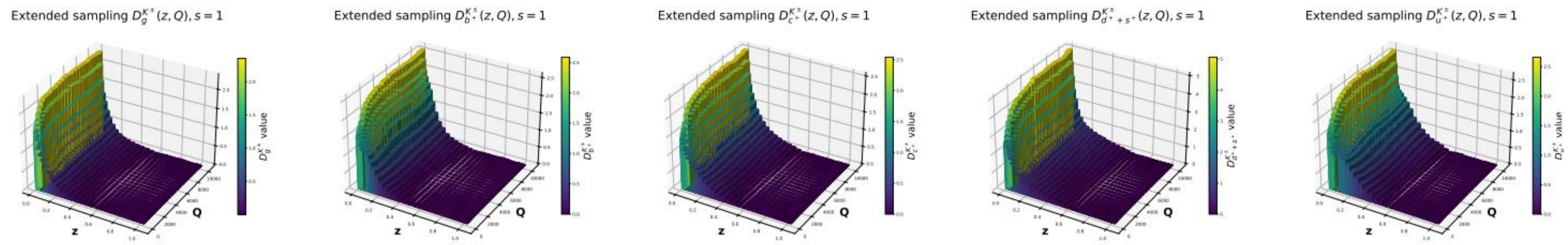


Results: QChGM for FF

➢ **All the results:** Fragmentation Functions of $K^{\pm}$



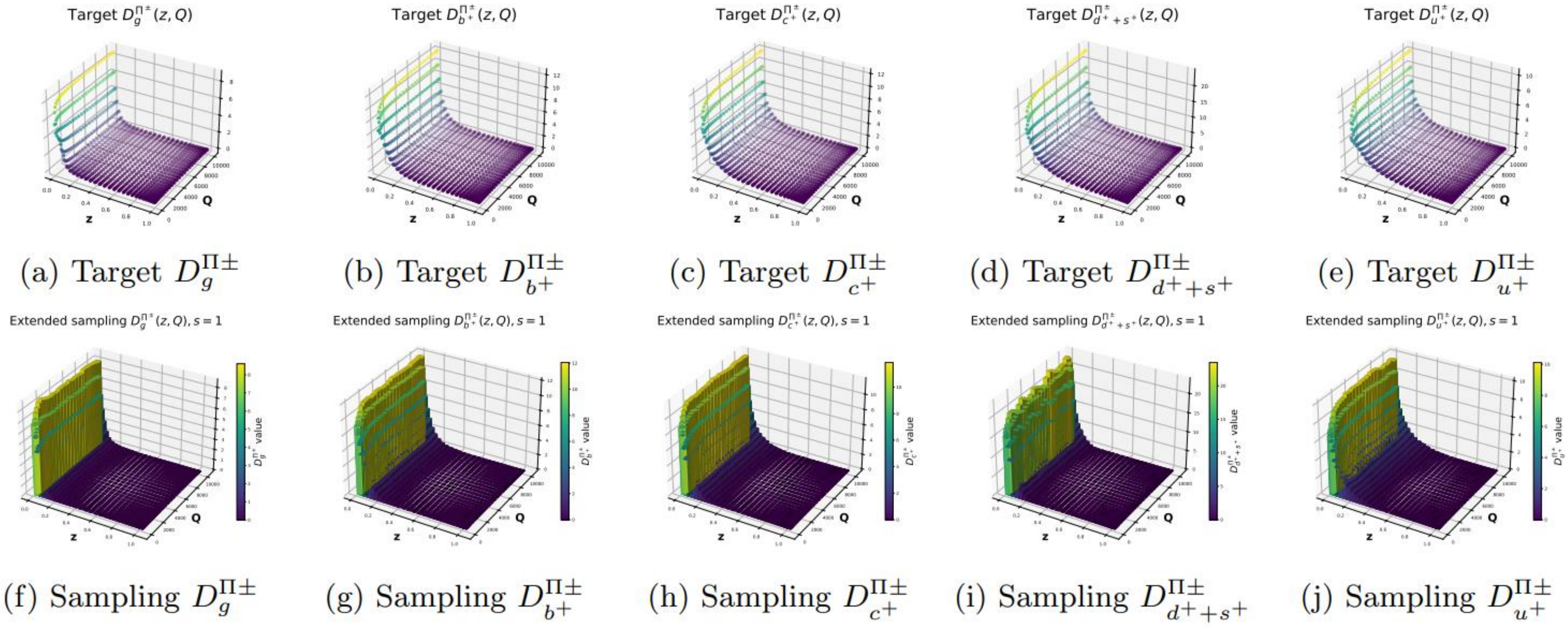
(a) Target $D_g^{K^\pm}$ (b) Target $D_{b^+}^{K^\pm}$ (c) Target $D_{c^+}^{K^\pm}$ (d) Target $D_{d^+ + s^+}^{K^\pm}$ (e) Target $D_{u^+}^{K^\pm}$



(f) Sampling $D_g^{K^\pm}$ (g) Sampling $D_{b^+}^{K^\pm}$ (h) Sampling $D_{c^+}^{K^\pm}$ (i) Sampling $D_{d^+ + s^+}^{K^\pm}$ (j) Sampling $D_{u^+}^{K^\pm}$

Results: QChGM for FF

- **All the results:** Fragmentation Functions of $\pi^{\pm}$



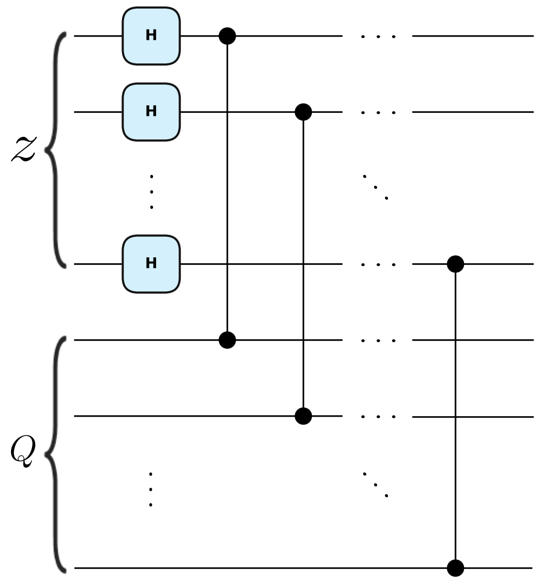
(a) Target $D_g^{\Pi\pm}$ (b) Target $D_{b+}^{\Pi\pm}$ (c) Target $D_{c+}^{\Pi\pm}$ (d) Target $D_{d^+ + s^+}^{\Pi\pm}$ (e) Target $D_{u^+}^{\Pi\pm}$

(f) Sampling $D_g^{\Pi\pm}$ (g) Sampling $D_{b+}^{\Pi\pm}$ (h) Sampling $D_{c+}^{\Pi\pm}$ (i) Sampling $D_{d^+ + s^+}^{\Pi\pm}$ (j) Sampling $D_{u^+}^{\Pi\pm}$

Results: QChGM for FF

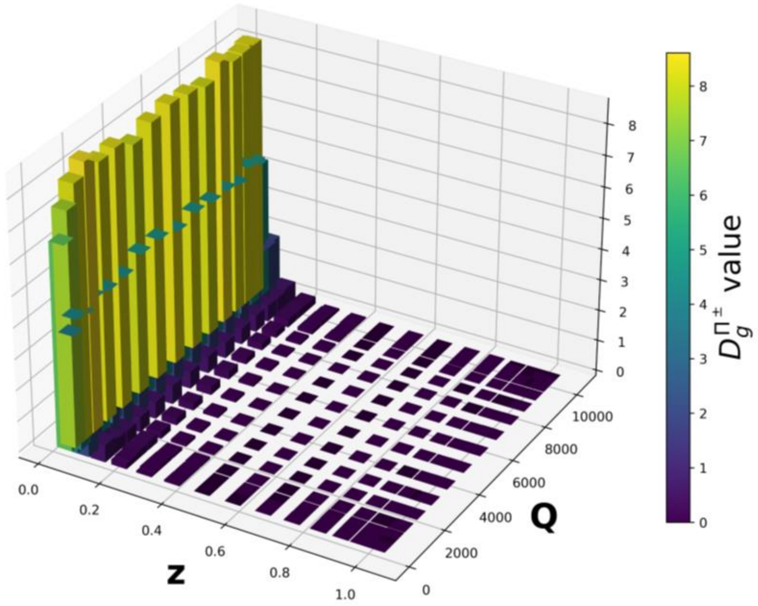
- **Study of correlations:**

Correlations circuit

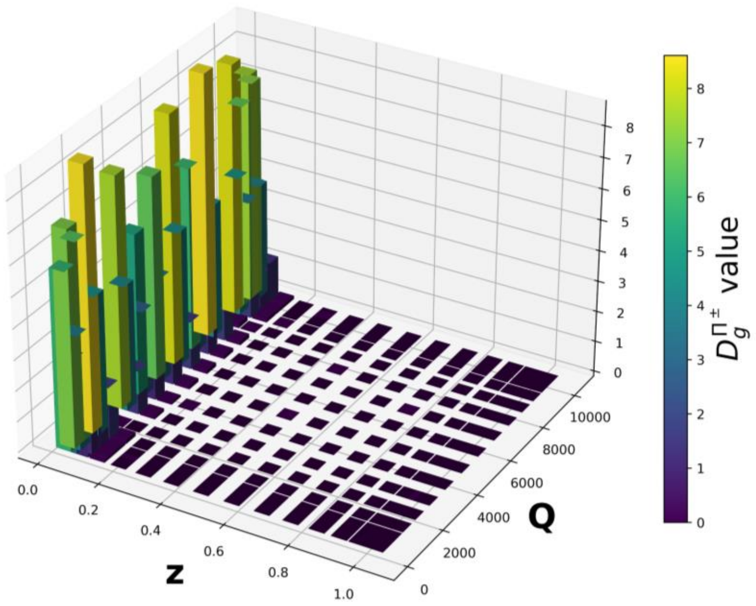


Example $D_g^{\pi^\pm}$: 4 qubits, 3 layers, 10000 iter., learning rate ∈ $[0.1,1]$

W/ CC: $R^2 = 0.99$



W/o CC: $R^2 = 0.85$



Results: QChGM for FF

Extending findings of:

Better than classical? The subtle art of benchmarking quantum machine learning models:
Joseph Bowles, Shahnawaz Ahmed, Maria Schuld, arXiv:2403.07059

They: classification

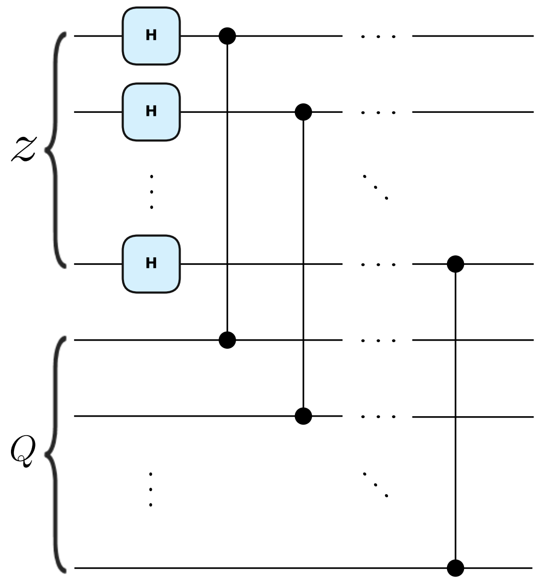
We: multivariate prob. distr.

"**Quantumness**" **help** the models to learn

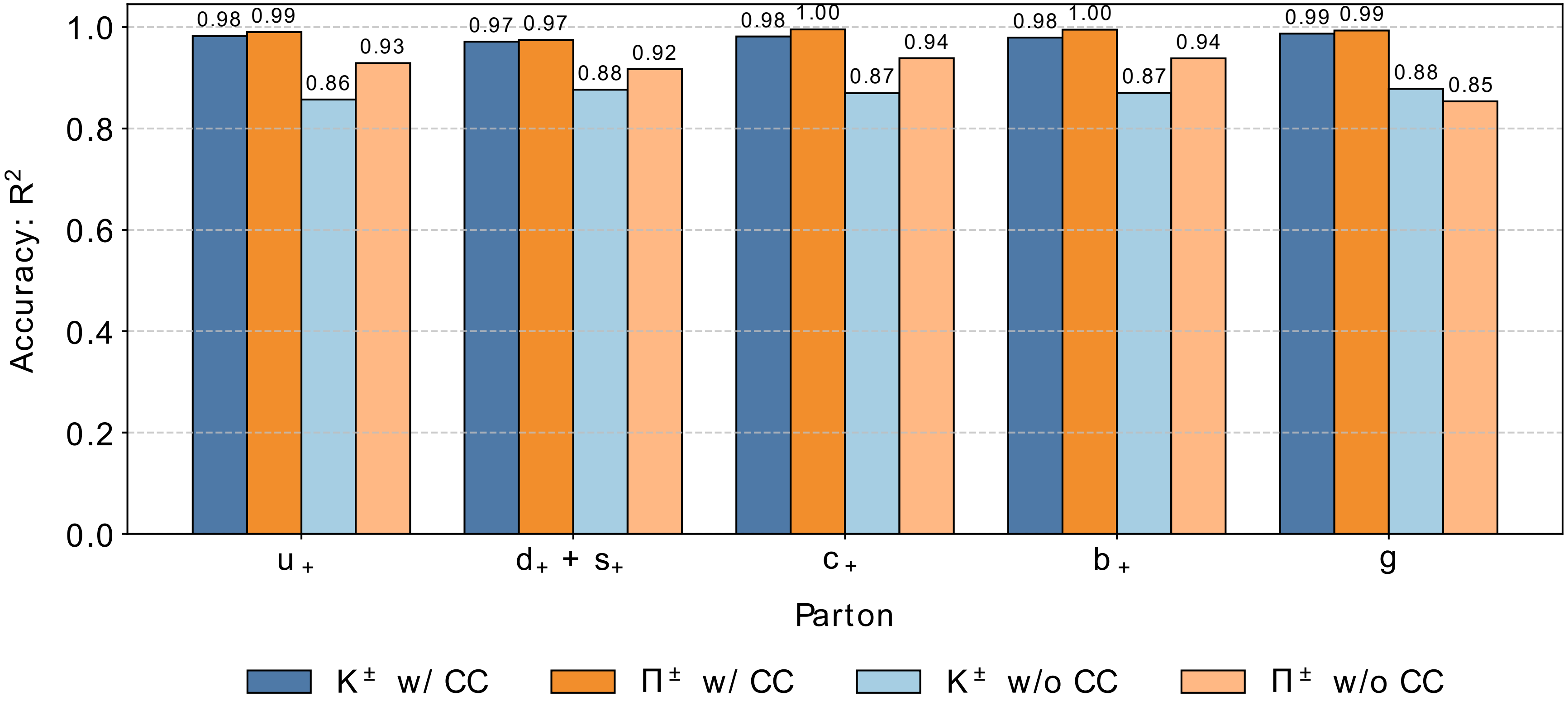
Similar to: Yacine Haddad's poster

➢ Study of correlations:

Correlations circuit

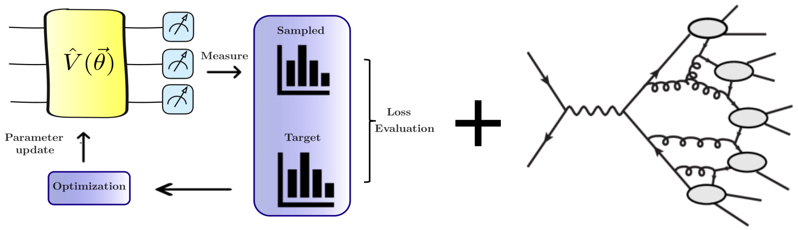


Comparison all FF

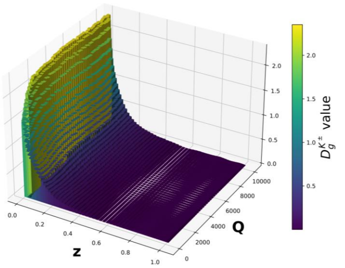


Outlook

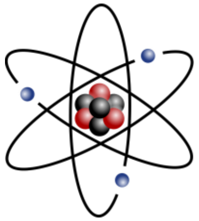
1. Succesfully extended **QChGM** to **learn FFs**

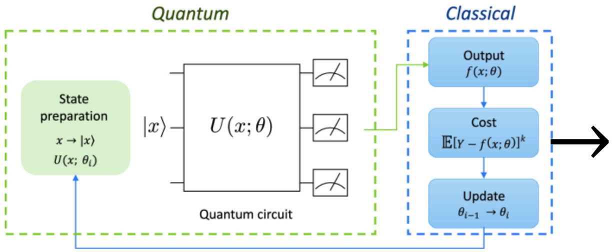


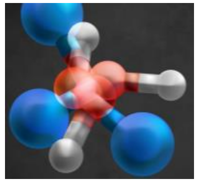
2. Efficient **sampling** with extended register
   - Natural **quantum interpolation**



3. **Correlations** help the quantum model to **learn**
   - Apply **QML** to analyze correlations in **physical processes**







Thank you for your attention!

If questions drop me an email:
jormard@ific.uv.es