FAULT-TOLERANT SIMULATION OF LATTICE GAUGE THEORIES WITH GAUGE COVARIANT CODES

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MOdeling Nuclear STructure and REaction

GOALS AND OBJECTIVES

Quantum error correction

Quantum computers can undergo errors.

We can define symmetries and conserved quantities.

If something is violated, we know an error occurred.

arXiv:2405.19293

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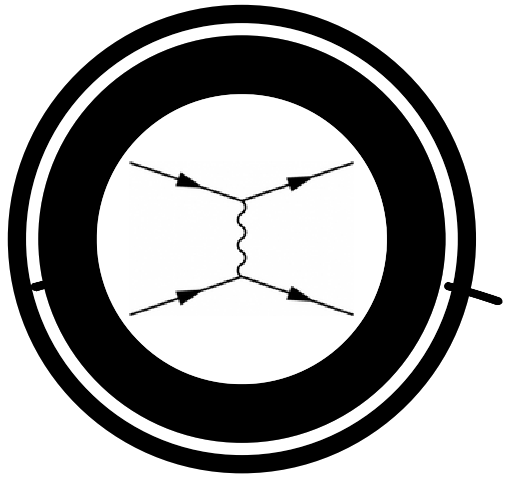
Gauge Theories

Gauge Theories are physical theories with a gauge symmetry, which is a local symmetry.



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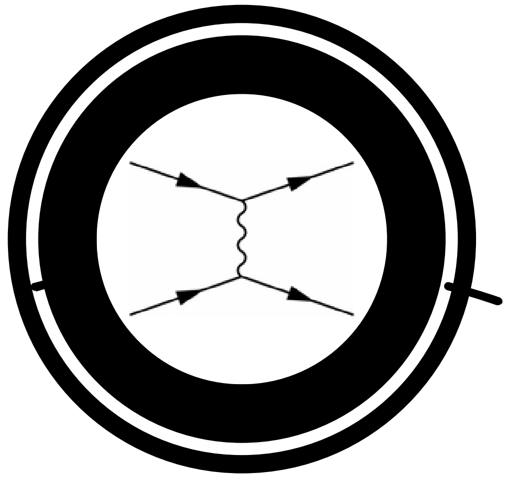

Linking two fields

We can use the Gauge symmetry as a symmetry to do error correction.

If it is violated, an error occurred.

QUESTIONS: what type and how many errors can we correct with the gauge symmetry?

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QUANTUM ERROR CORRECTION

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Classical computes:

bit: $0, 1$

Possible errors:

bit-flip: $0 \to 1$, $1 \to 0$

We can correct errors adding redundancy:

$$0_L = 000$$

$$010 \to 000$$

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Quantum computes:

qbit: $|\psi\rangle = a|0\rangle + b|1\rangle$

Possible errors:

bit-flip: $|0\rangle \rightarrow |1\rangle$
$|1\rangle \rightarrow |0\rangle$

phase-flip: $|0\rangle \rightarrow |0\rangle$
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$$|0\rangle_{LL} \rightarrow |0_L 0_L 0_L\rangle$$

$$|0\rangle_L \rightarrow |000\rangle$$

Since we have 2 possible errors, we need more redundancy

MEASUREMENT

Which operators are we alloed to measure without making the wavefunction collapse?

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Remember:

$$Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$

We can measure the parity between 2 qubits:

$$ZZ|00\rangle = |00\rangle$$

$$ZZ|01\rangle = -|01\rangle$$

$$ZZ|10\rangle = -|10\rangle$$

$$ZZ|11\rangle = |11\rangle$$

Without destroying superpositions:

$$ZZ(|00\rangle + |11\rangle) = (|00\rangle + |11\rangle)$$

$$ZZ(|01\rangle + |10\rangle) = -(|01\rangle + |10\rangle)$$

MEASUREMENT

Which operators are we alloed to measure without making the wavefunction collapse?

The logical states:

$$|0\rangle_L \rightarrow |000\rangle$$
$$|1\rangle_L \rightarrow |111\rangle$$

Stabilizers:

$$S_1 = Z_1 Z_2$$
$$S_2 = Z_2 Z_3$$

Remember:

$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$

S_1	S_2	**error**
-	+	1
-	-	2
+	-	3

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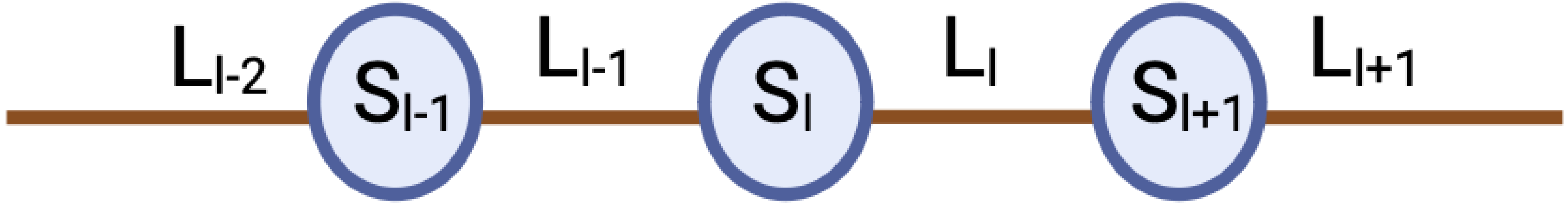
LATTICE GAUGE THEORIES



Sites can be:

empty $|0\rangle$

full $|1\rangle$

On links there is the Gauge field with 2 possible values:

zero $|0\rangle$

one $|1\rangle$

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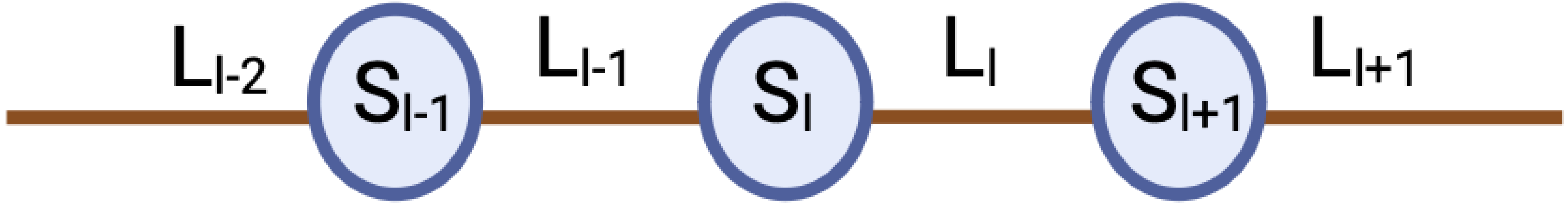

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The symmetry:

$$G_l = Z_{L_{l-1}} Z_{S_l} Z_{L_l}$$

$$G_l|\psi\rangle = |\psi\rangle$$

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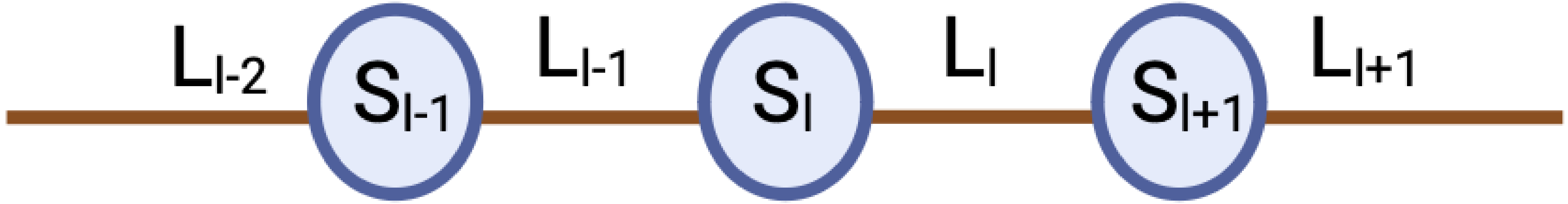

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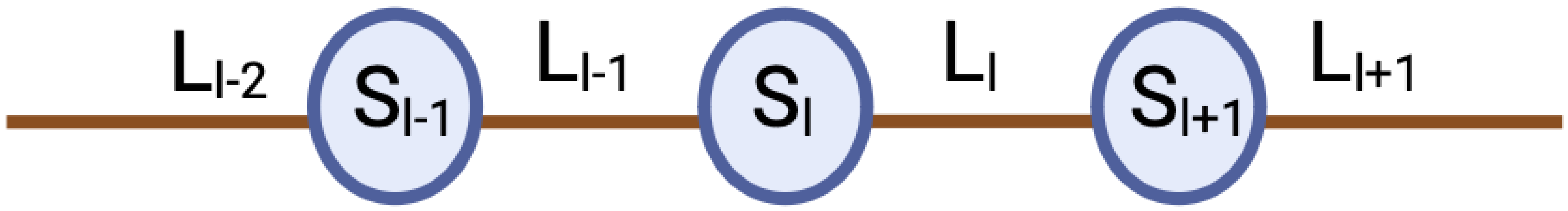

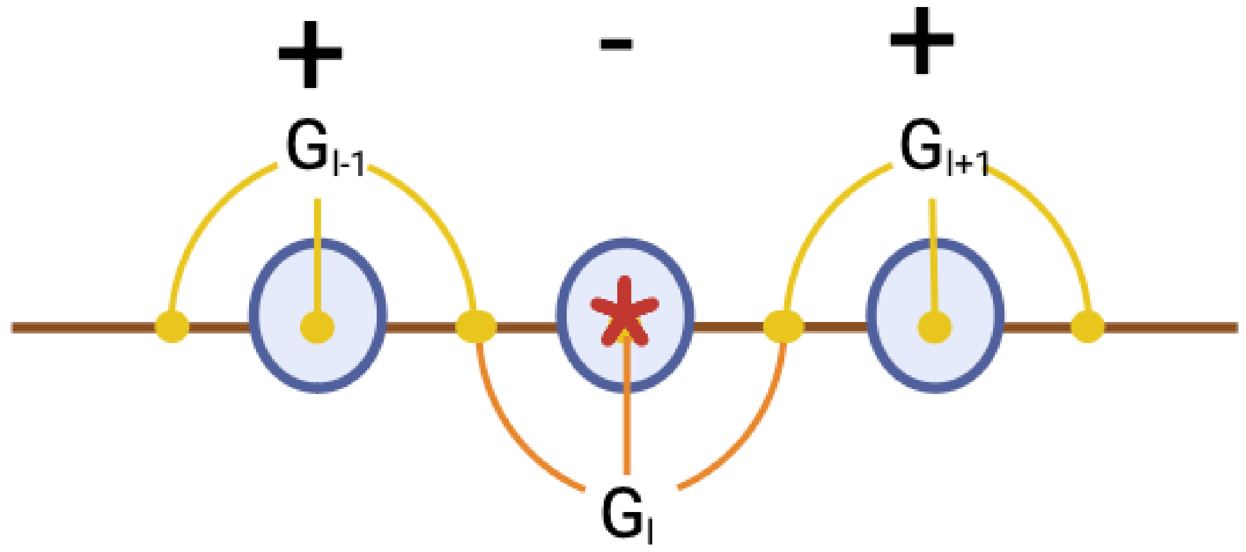

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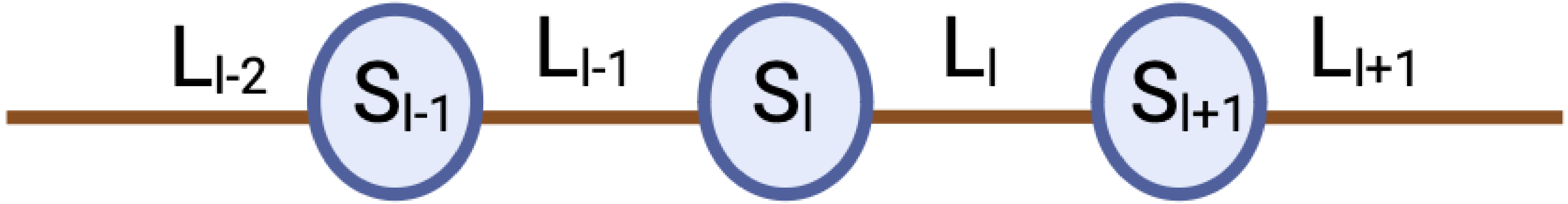

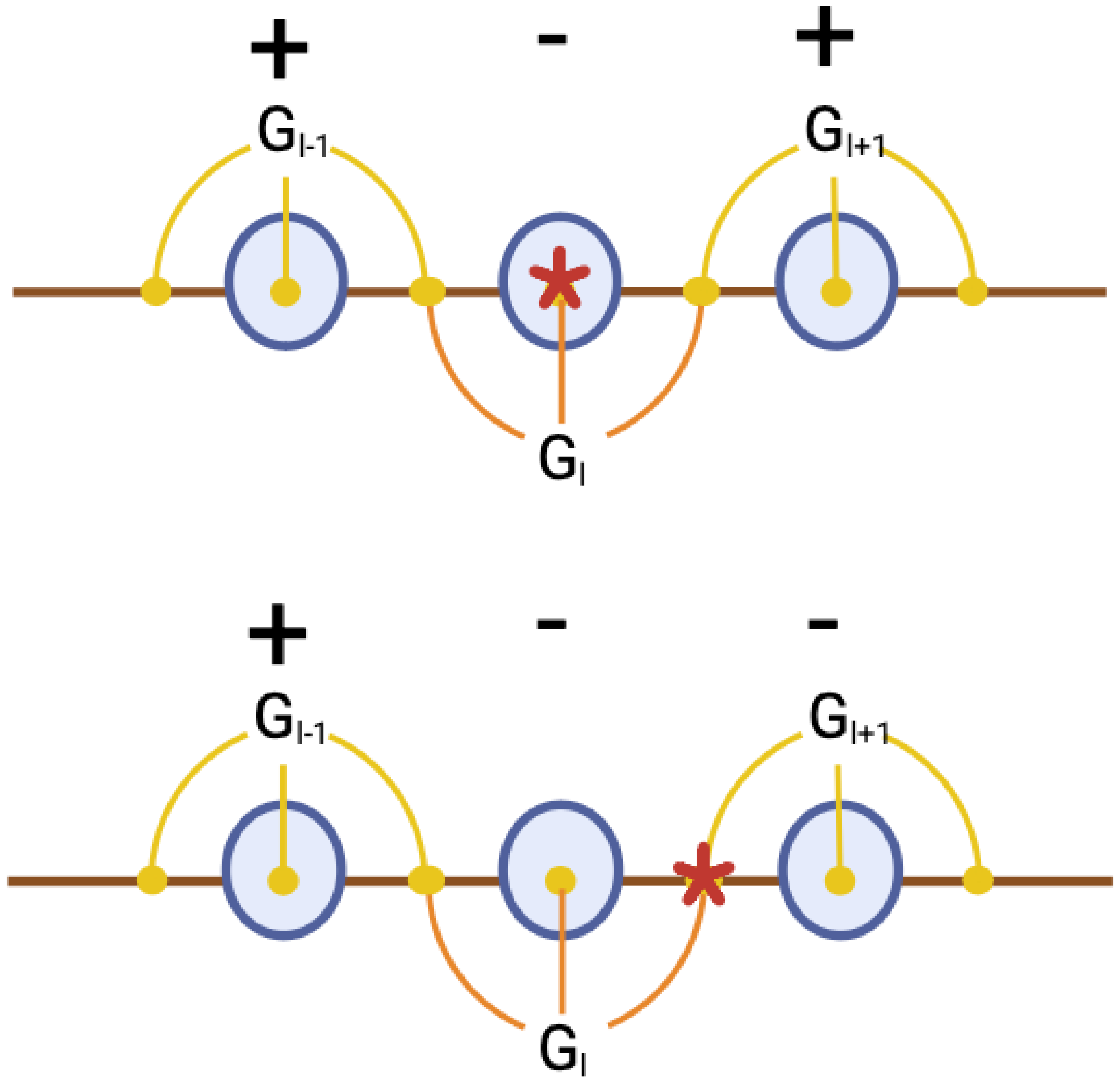

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FUTURE WORK

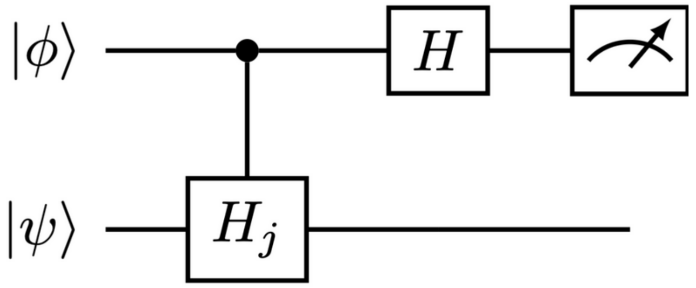
arXiv:2405.19293

- ✓ more spatial dimensions
- ✓ larger electric cutoff
- ? non-abelian theories

✓ Fault-tolerant quantum simulation (Trotter, QSP)

$$e^{iHt} = e^{it\sum_j c_j H_j} \approx \prod_j e^{itc_j H_j}$$

$$e^{itc_j H_j} =$$



FUTURE WORK

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- ? non-abelian theories

✓ “bosonization” of the Hamiltonian

✓ Fault-tolerant quantum simulation (Trotter, QSP)

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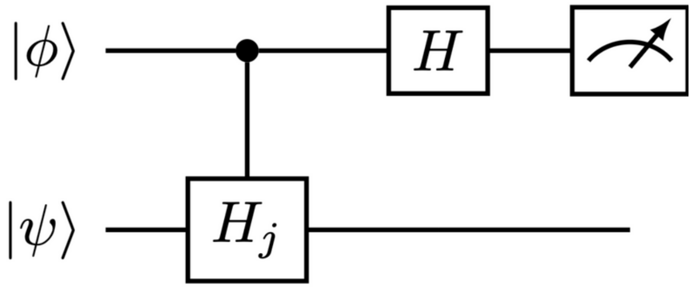

CONCLUSIONS

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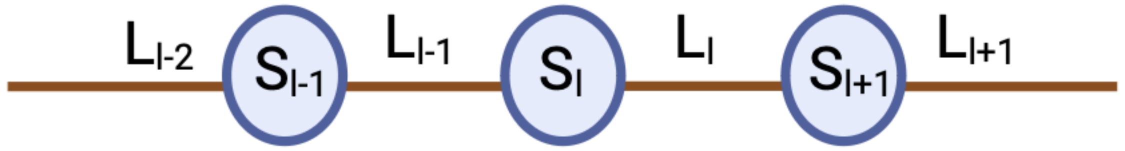

We want to simulate a system with a gauge symmetry

$$G_l = Z_{L_{l-1}} Z_{S_l} Z_{L_l}$$

We can use the gauge symmetry to detect and correct every X error

In this way we can save memory, and easily perform quantum simulations

$$e^{iHt} = e^{it\sum_j c_j H_j} \approx \prod_j e^{itc_j H_j}$$

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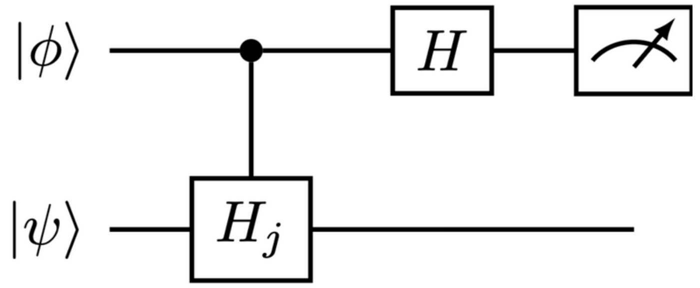
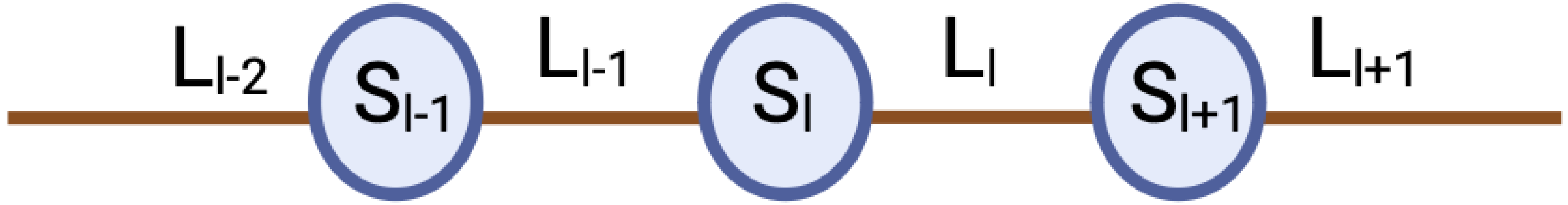


- ✓ more spatial dimensions
- ✓ larger electric cutoff
- ? non-abelian theories
- ✓ fault-tolerant time evolution
- ✓ bosonization

THANK YOU

MEASUREMENT AND CORRECTION



$$G_l = Z_{L_{l-1}} Z_{S_l} Z_{L_l}$$

$$G_l|\psi\rangle = |\psi\rangle$$

G_{l-1}	G_l	G_{l+1}	error location
$+$	$+$	$+$	none
$-$	$-$	$+$	L_{l-1}
$+$	$-$	$+$	S_l
$+$	$-$	$-$	L_l

We can correct every X error in the system, but we cannot detect Z errors

To correct Z errors we can use more layers of redundancy

TIME EVOLUTION

The hamiltonian, can be written as

$$H = \sum_j c_j H_j$$

The time evolution operator we want to apply is

$$e^{iHt} = e^{it\sum_j c_j H_j}$$

To simplify the implementation, we can break up the operator, approximating it:

$$e^{it\sum_j c_j H_j} \approx \prod_j e^{itc_j H_j}$$

This is the first-order Trotter formula, and the error is:

$$\left\| e^{itH} - \prod_j e^{itc_j H_j} \right\| \leq t^2 \sum_j \left\| \sum_k [H_j, H_k] \right\|$$

So we need a way to implement the single exponentials

But we can apply easily on the system only the logical operations

They correspond to Pauli matrices on the logical qubits

TIME EVOLUTION

How do we implement

$$e^{itc_jH_j}$$

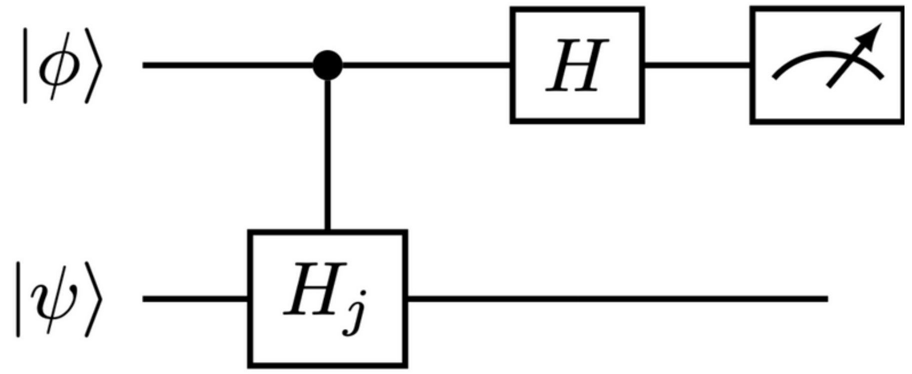
Assuming the Hamiltonian is
a sum of Pauli matrices:

$$e^{itc_jH_j} = \cos(tc_j) + i\sin(tc_j)H_j$$

To do this, let us assume to have an ancilla qubit on which we can do arbitrary rotations, prepared in the following state:

$$|\phi\rangle = \cos(tc_j)|0\rangle + i\sin(tc_j)|1\rangle$$

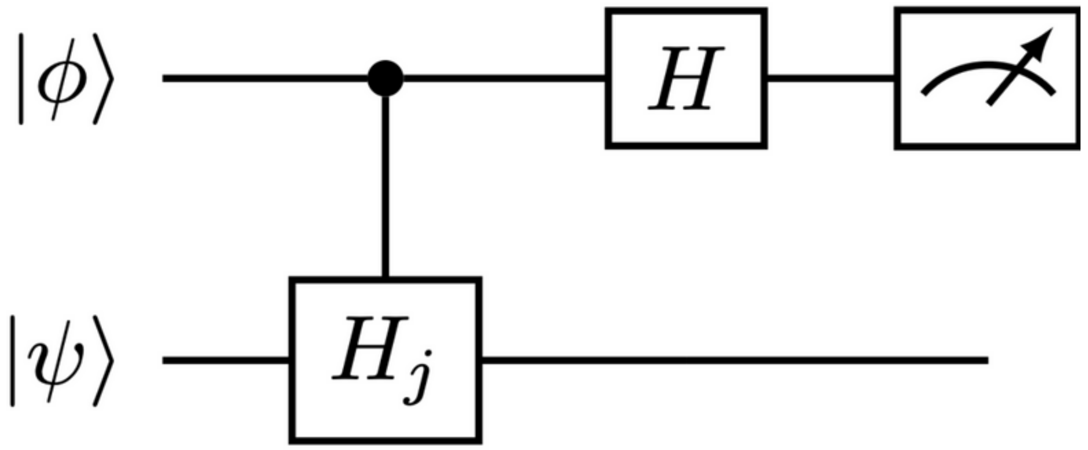
Then, the following circuit applies the right exponential:



In this way we move the problem of applying the exponential, to the problem of preparing an ancilla qubit state

IMPLEMENTATION OF TROTTER

Why the following circuit implements the right exponential?



$$|\phi\rangle = \cos(tc_j)|0\rangle + i\sin(tc_j)|1\rangle$$

$$|\phi\rangle|\psi\rangle = \cos(tc_j)|0\rangle|\psi\rangle + i\sin(tc_j)|1\rangle|\psi\rangle$$

$$CH_j \to \cos(tc_j)|0\rangle|\psi\rangle + i\sin(tc_j)|1\rangle H_j|\psi\rangle$$

$$\begin{aligned} H &\to \cos(tc_j)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|\psi\rangle + i\sin(tc_j)\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)H_j|\psi\rangle \\ &= \frac{1}{\sqrt{2}}(\cos(tc_j) + i\sin(tc_j)H_j)|0\rangle|\psi\rangle + \frac{1}{\sqrt{2}}(\cos(tc_j) - i\sin(tc_j)H_j)|1\rangle|\psi\rangle \\ &= \frac{1}{\sqrt{2}}|0\rangle e^{itc_jH_j}|\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle e^{-itc_jH_j}|\psi\rangle \end{aligned}$$

The circuit applies with probability 1/2 the right exponential, with probability 1/2 its hermitian conjugate

We can apply always the right exponential with a cycle of oblivious amplitude amplification

HAMILTONIAN

The starting Hamiltonian:

$$H = m\sum_l (-1)^l \psi_l^\dagger \psi_l + \epsilon \sum_l (\psi_l^\dagger Q_l \psi_{l+1} + \psi_{l+1}^\dagger Q_l^\dagger \psi_l) + 2\lambda_E \sum_l P_l$$

In terms of Pauli matrices:

$$H = \frac{m}{2}\sum_l (-1)^l (1-(-1)^l Z_{S_l})$$
$$+ \frac{\epsilon}{2}\sum_l (1 + Z_{S_l} Z_{S_{l+1}}) X_{S_l} X_{L_l} X_{S_{l+1}} + 2\lambda_E \sum_l Z_{L_l}$$

In terms of logical operations:

$$H = \frac{m}{2}\sum_l (-1)^l (1-\overline{Z}_{l-1}\overline{Z}_l) + \frac{\epsilon}{2}\sum_l \left(1-\overline{Z}_{l-1}\overline{Z}_{l+1}\right)\overline{X}_l + 2\lambda_E \sum_l \overline{Z}_l$$

Fermionic operators $\psi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(1+Z)X$

Field operators

$$Q = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = X$$

$$P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$$

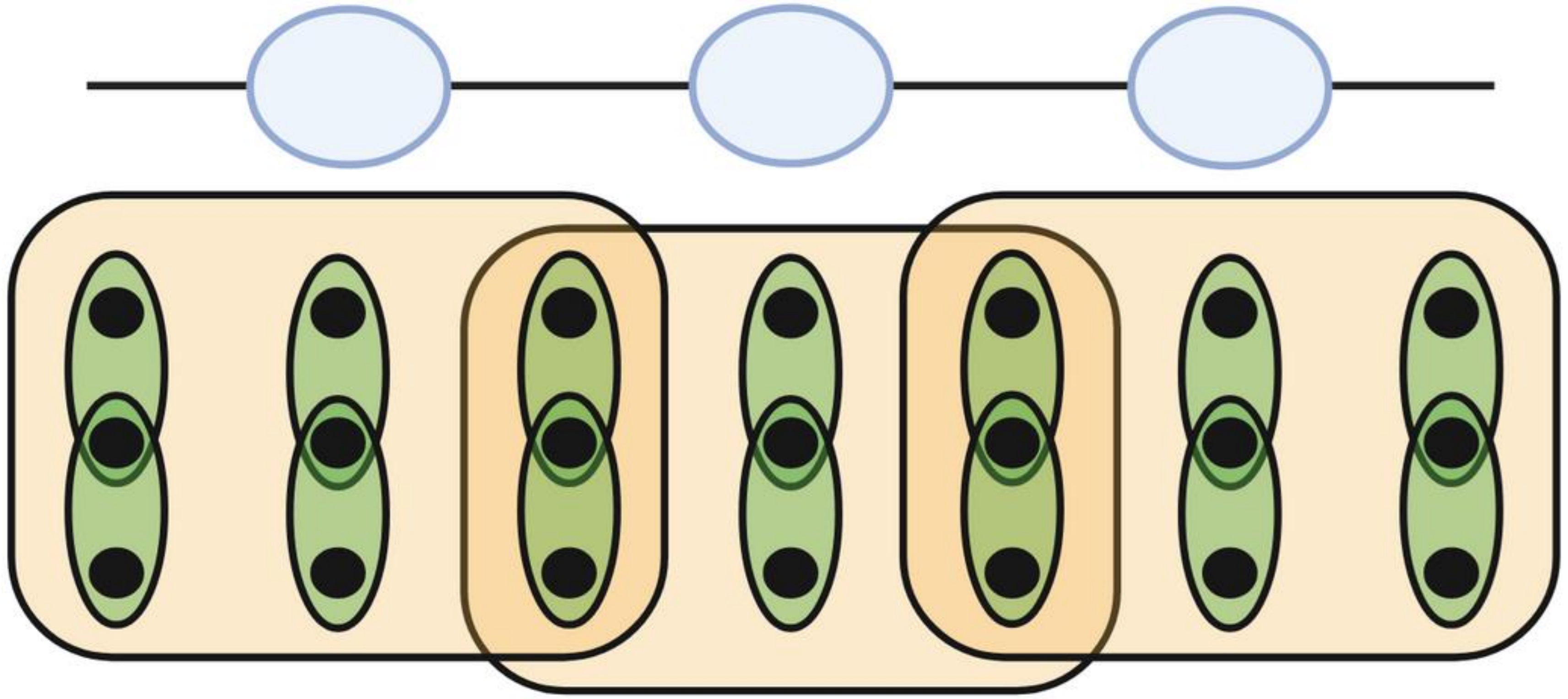
Logical operations:

$$\bar{Z}_l = Z_{L_l}$$

$$\bar{X}_l = X_{S_l} X_{L_l} X_{S_{l+1}}$$

$$\bar{X}|0\rangle_L = |1\rangle_L$$
$$\bar{X}|1\rangle_L = |0\rangle_L$$
$$\bar{Z}|0\rangle_L = |0\rangle_L$$
$$\bar{Z}|1\rangle_L = -|1\rangle_L$$

FULL ENCODING



3 qubits per site
3 qubits per link

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

codewords

$$|0\rangle_L \to |+++\rangle$$
$$|1\rangle_L \to |---\rangle$$

stabilizers

$$S_1 = X_1 X_2$$
$$S_1 = X_2 X_3$$

$$X|+\rangle = |+\rangle$$
$$X|-\rangle = -|-\rangle$$

STABILIZER CODES

Let “P” be the n-qubits Pauli group

Define “S” the stabilizer group as an abelian subgroup of P

A codword is a state such that, for every element of S

$$S_i|x\rangle = |x\rangle$$

The element of S are traceless, with eigenvalues +1 or -1

By adding an element to S, we half the Hilbert space of codewords

If we start with n physical qubits

we define n-k stabiliser operators

we will have a number of codewords equal to

$$2^n/2^{n-k} = 2^k$$

So we will have k logical qubits

Logical operators are elements of P that commute with S

They are 2k operators. Every operator commute with all other operators but one that has to anti commute