Cosmological correlators in slow-roll violating inflation

Jason Kristiano (RESCEU, The University of Tokyo)
Work with Jun’ichi Yokoyama

References:
- Phys. Rev. Lett 132, 221003 (2024) [2211.03395]
- Phys. Rev. D 109, 103541 (2024) [2303.00341]
- JCAP 10 (2024) 036 [2405.12145]
- Springer textbook on PBH (invited chapter) [2405.12149]

Looping in the Primordial Universe (CERN-TH Workshop)

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Canonical inflation

The most minimal model of inflation.

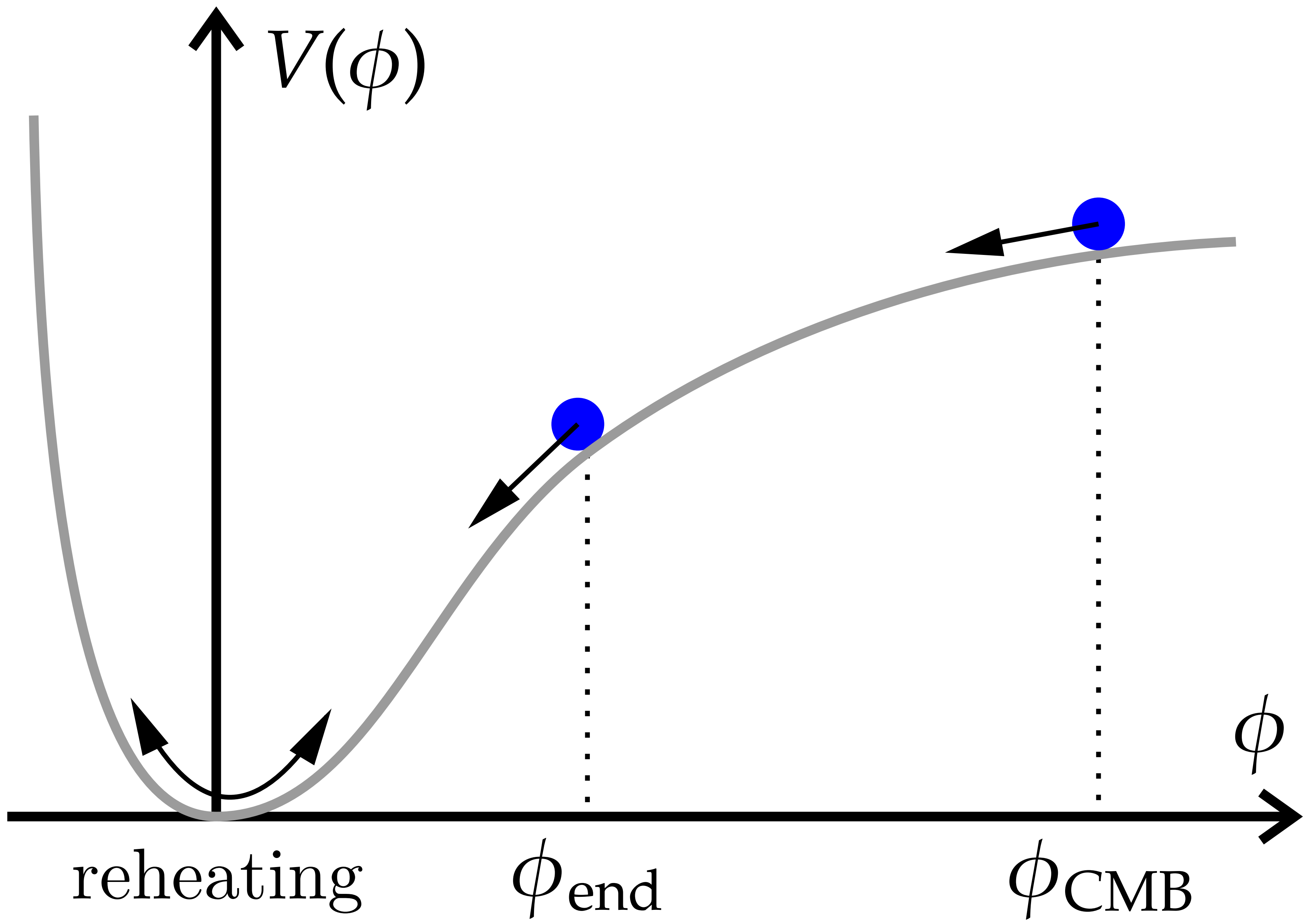
Action: $S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_\mathrm{P}^2 R - (\partial_\mu\phi)^2 - 2V(\phi)\right].$

Background: $\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\ \mathrm{d}\mathbf{x}^2 = a^2(\tau)(-\mathrm{d}\tau^2 + \mathrm{d}\mathbf{x}^2).$

Equation of motion:

Friedmann equation: $\dot{H} = -\frac{\dot{\phi}^2}{2M_\mathrm{P}^2}$ and $H^2 = \frac{1}{3M_\mathrm{P}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right).$

Klein-Gordon equation: $\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0.$



Slow-roll inflation

SR approximation: $\left|\frac{\ddot{\phi}}{\dot{\phi}H}\right| \ll 1$ and $\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2M_{\mathrm{P}}^2H^2} \ll 1.$

Performing SR approximation, the equations of motion become

$$H^2 \approx \frac{V(\phi)}{3M_{\mathrm{P}}^2} \approx \mathrm{const} \;\text{ and }\; \dot{\phi} \approx -\frac{V_{,\phi}}{3HM_{\mathrm{P}}} \longrightarrow \epsilon \approx \frac{M_{\mathrm{P}}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2.$$

Solution: $a(t) \simeq -\frac{1}{H\tau} \propto e^{Ht}$ (quasi-dS), with domain of conformal time $\tau < 0$.

Slow-roll inflation

More systematically, define n-th SR parameter: $\epsilon_{n+1} = \frac{\dot{\epsilon}_n}{\epsilon_n H}$ and $\epsilon_1 = -\frac{\dot{H}}{H^2}$.

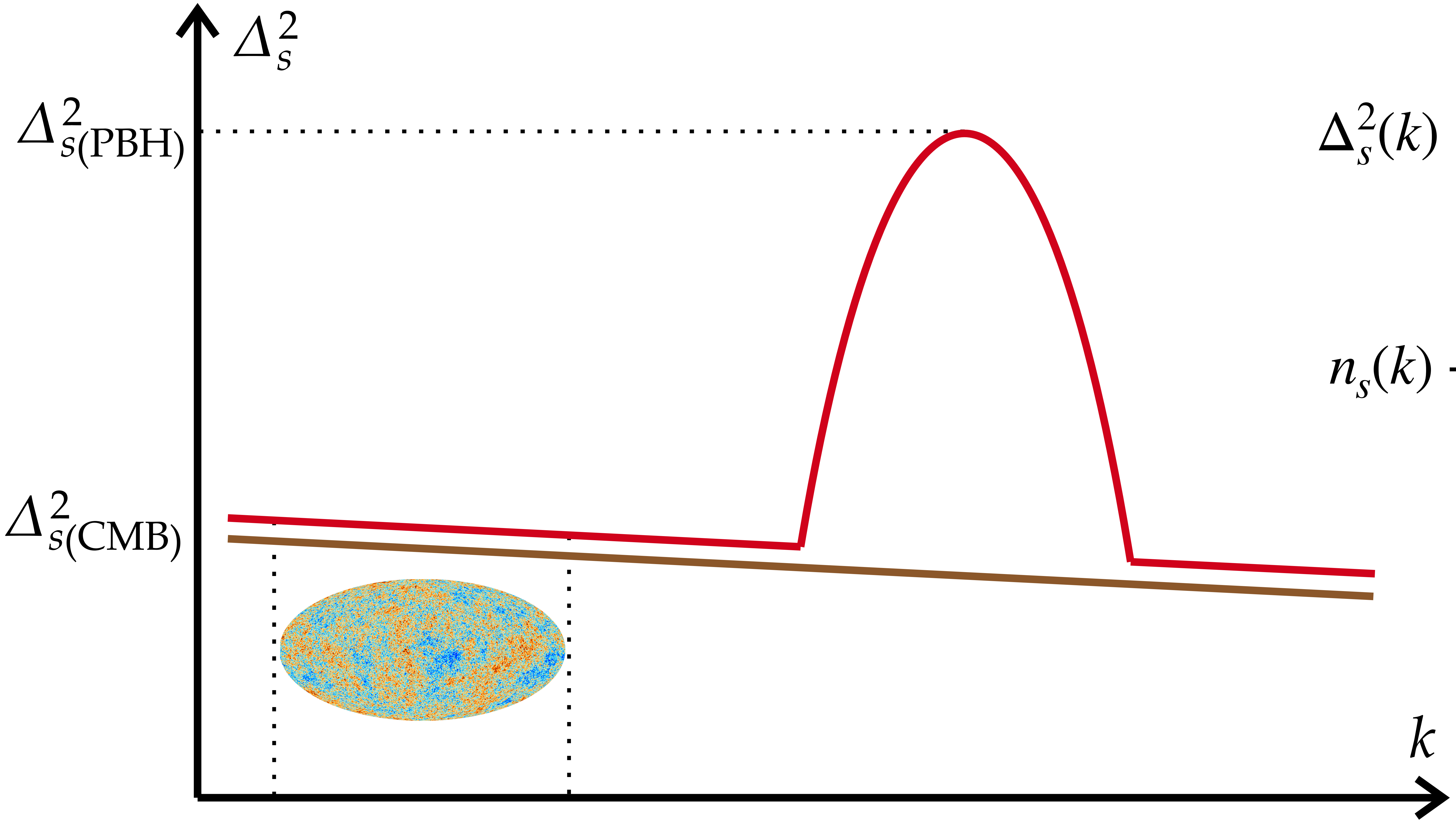
Substituting equation of motions:

$$\epsilon_1 = \frac{\dot{\phi}^2}{2M_{\rm P}^2 H^2} \, , \epsilon_2 = 2\epsilon_1 + 2\frac{\ddot{\phi}}{\dot{\phi}H} \, , \ldots$$

SR approximation: $|\epsilon_n| \ll 1.$

SR approximation implies quasi-dS, however converse statement is not true.

Power spectrum



$$\Delta_s^2(k) = \left[\frac{H^2(\tau)}{8\pi^2 M_{\mathrm{P}}^2 \epsilon_1(\tau)}\right]_{\tau=-1/k}$$

$$n_s(k) - 1 = \frac{\mathrm{d}\log\Delta_s^2}{\mathrm{d}\log k} \sim \mathcal{O}(\epsilon)$$

Decrease $\epsilon_1(\tau)$ at late time to amplify the power spectrum on small scales. How to achieve that?

Violation of SR approximation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

SR approximation: $3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} \approx 0$

$$\epsilon_1 = \frac{\dot{\phi}^2}{2M_\mathrm{P}^2H^2} \approx \frac{M_\mathrm{P}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1$$

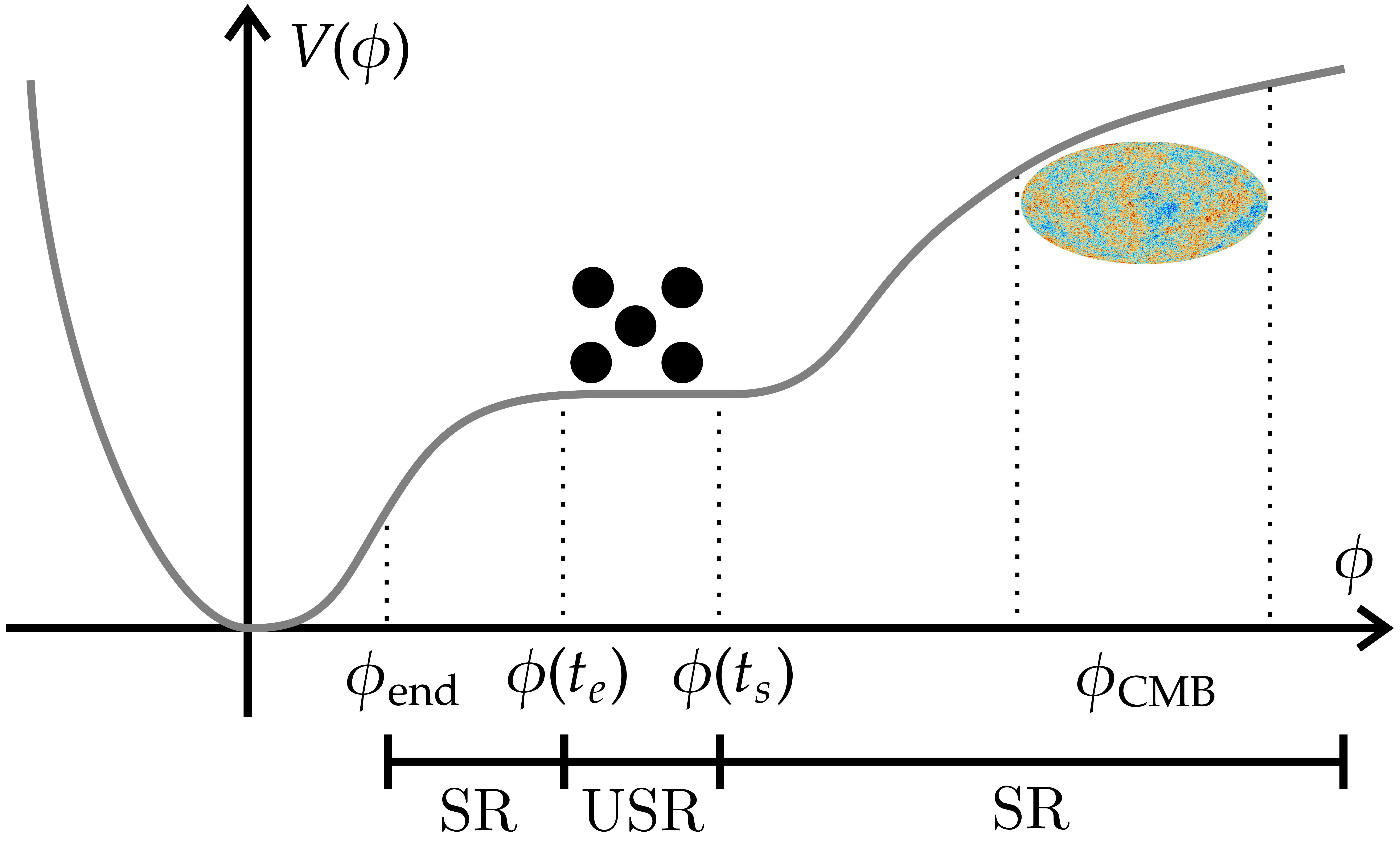
$$\epsilon_2 = 2\epsilon_1 + 2\frac{\ddot{\phi}}{\dot{\phi}H} \ll 1$$

USR condition: $\ddot{\phi} + 3H\dot{\phi} = 0 \longrightarrow \dot{\phi} \propto a^{-3}$

$$\epsilon_1 = \frac{\dot{\phi}^2}{2M_\mathrm{P}^2H^2} \propto a^{-6} \ll 1$$

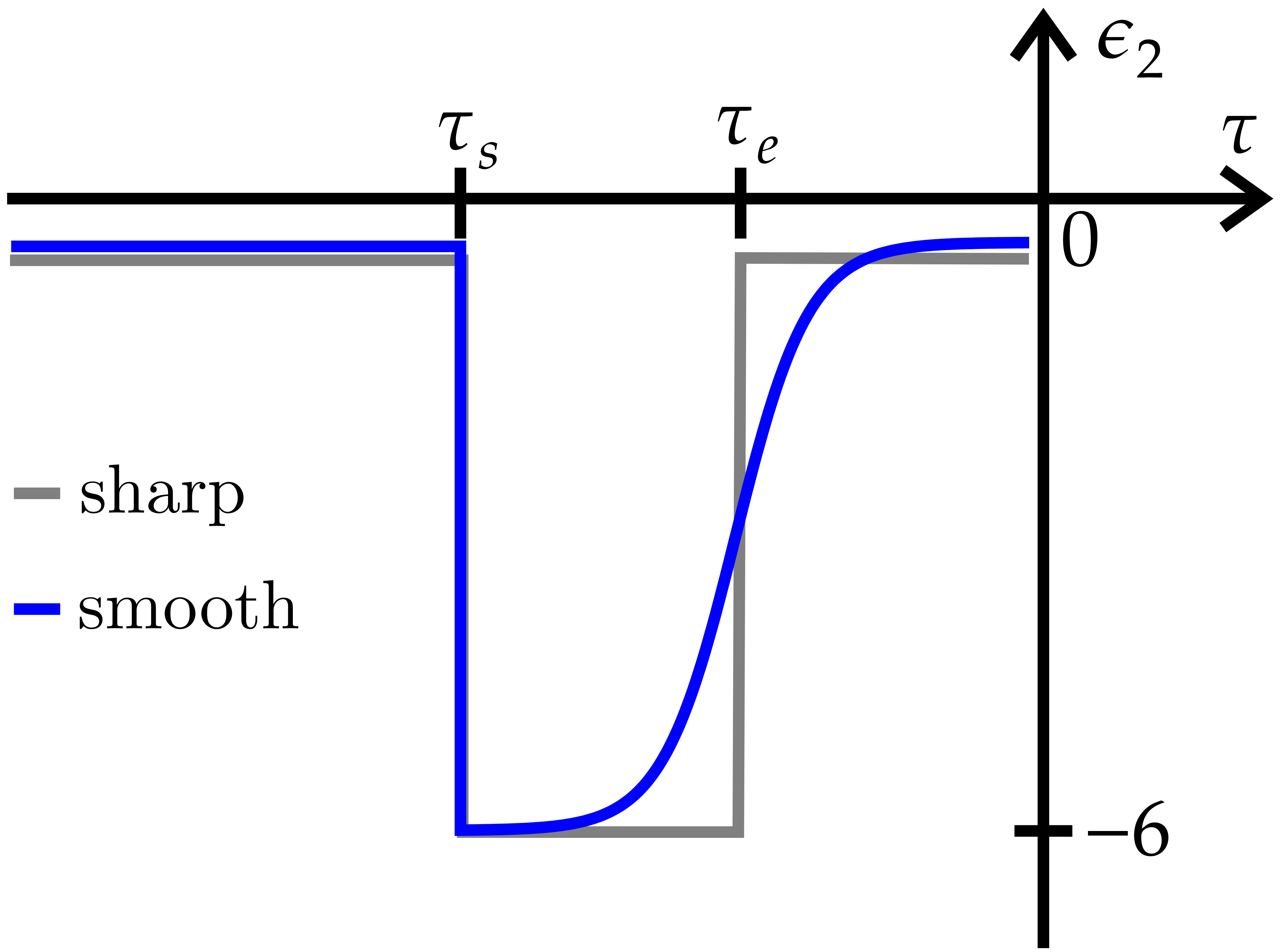
$$\epsilon_2 = 2\epsilon_1 + 2\frac{\ddot{\phi}}{\dot{\phi}H} \simeq -6$$

Potential of the inflaton



Evolution of the second SR parameter

- Sharp: step function at both $\tau = \tau_s$ and $\tau = \tau_e$.
- Smooth: continuous function at $\tau > \tau_s$.



Cosmological perturbations

Small perturbations:

- Inflaton: $\phi(\mathbf{x}, t) = \bar{\phi}(t) + \delta\phi(\mathbf{x}, t)$
- Spacetime: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$

Gauge fixing condition:

- Comoving: $\delta\phi = 0$ and $\gamma_{ij} = a^2 e^{2\zeta}\delta_{ij}$
- Flat-slicing: $\delta\phi \neq 0$ and $\gamma_{ij} = a^2\delta_{ij}$

(Non-linear) gauge transformation: $\zeta = \zeta_n + \frac{1}{4}\epsilon_2\zeta_n^2 + \frac{1}{H}\dot{\zeta}_n\zeta_n + \mathcal{O}(\zeta_n^3)$, $\zeta_n = -\frac{\delta\phi}{M_\mathrm{P}\sqrt{2\epsilon_1}}$

Compute correlation function of ζ_n , then obtain correlation function of ζ.

Second-order action

Second-order action: $S^{(2)} = M_{\rm P}^2 \int {\rm d}t\ {\rm d}^3x\ \epsilon_1 a^3 \left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i \zeta)^2\right]$

Mukhanov-Sasaki (MS) variable: $v = z\zeta M_{\rm P}$, $z = a\sqrt{2\epsilon_1}$

$$S^{(2)} = \frac{1}{2}\int {\rm d}\tau\ {\rm d}^3x \left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z}v^2\right]$$

Equation of motion: $v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0, \frac{z''}{z} = (aH)^2\left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{1}{2}\epsilon_2\epsilon_3\right)$

- SR ($\epsilon_1, |\epsilon_2|, |\epsilon_3| \ll 1$)
- USR ($\epsilon_1, |\epsilon_3| \ll 1$, $\epsilon_2 = -6$)

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right)v_k = 0$$

Curvature perturbation

Pure USR inflation ($V(\phi) = \text{constant}$) corresponds to $\mathscr{A}_k = 1$ and $\mathscr{B}_k = 0$

$$\lim_{k\to 0} \zeta_k(\tau) = \frac{iH}{2M_\text{P}\sqrt{k^3\epsilon_1(\tau)}} \longrightarrow \Delta_s^2(k \to 0, \tau) = \frac{H^2}{8\pi^2 M_\text{P}^2 \epsilon_1(\tau)} \propto a^3(\tau)$$

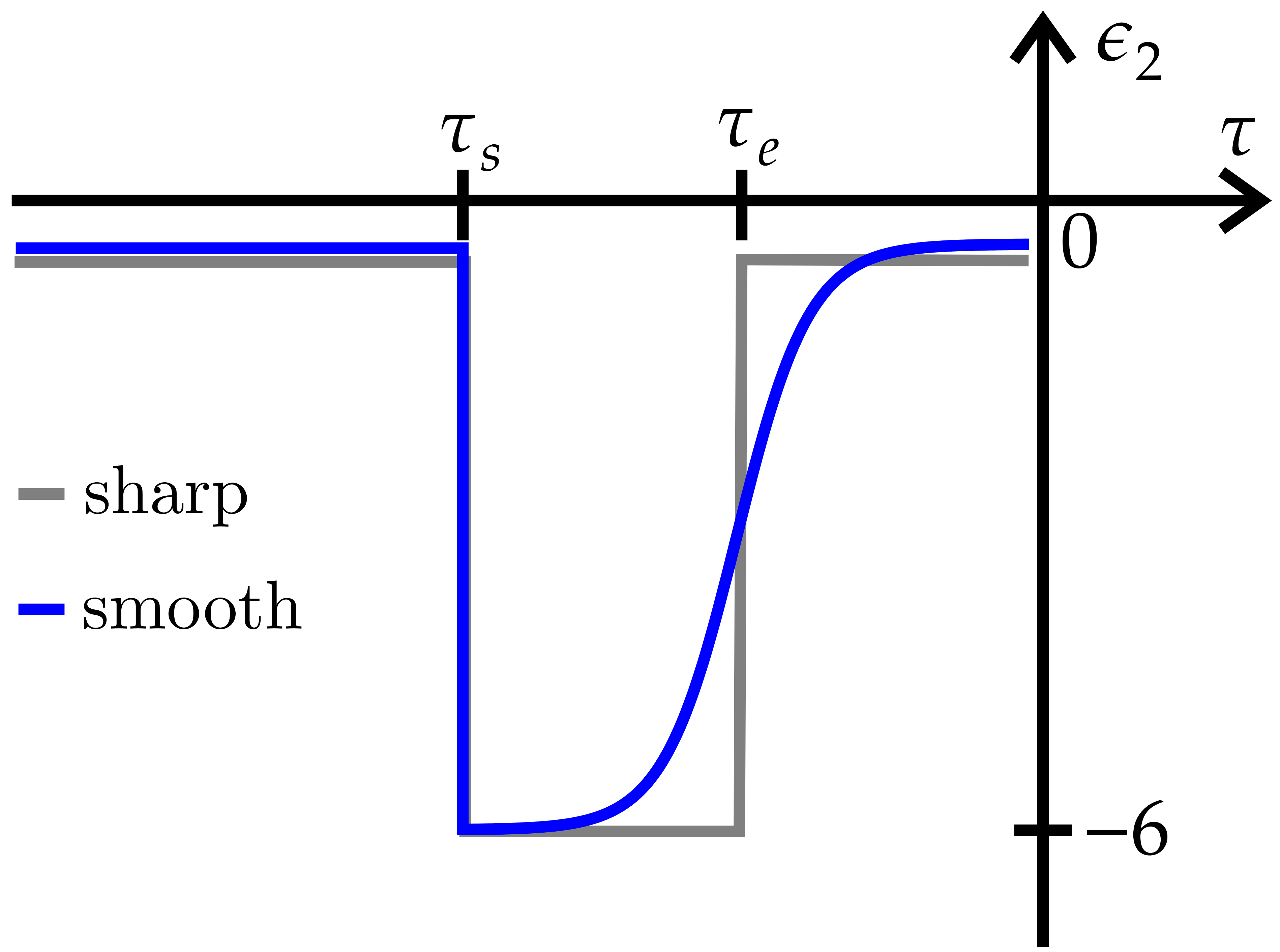
Superhorizon evolution of scale-invariant perturbation even at tree-level.

Transition makes initial condition of the USR period deviates from Bunch-Davies.

$$\zeta_k(\tau) = \frac{iH}{2M_\text{P}\sqrt{k^3\epsilon_1(\tau)}} \left[\mathscr{A}_k e^{-ik\tau}(1+ik\tau) - \mathscr{B}_k e^{ik\tau}(1-ik\tau)\right]$$

Coefficients $\mathscr{A}_k$ and $\mathscr{B}_k$ are obtained by requiring continuity of $\zeta_k(\tau)$ and $\zeta_k'(\tau)$ at the transition.

Sharp transition

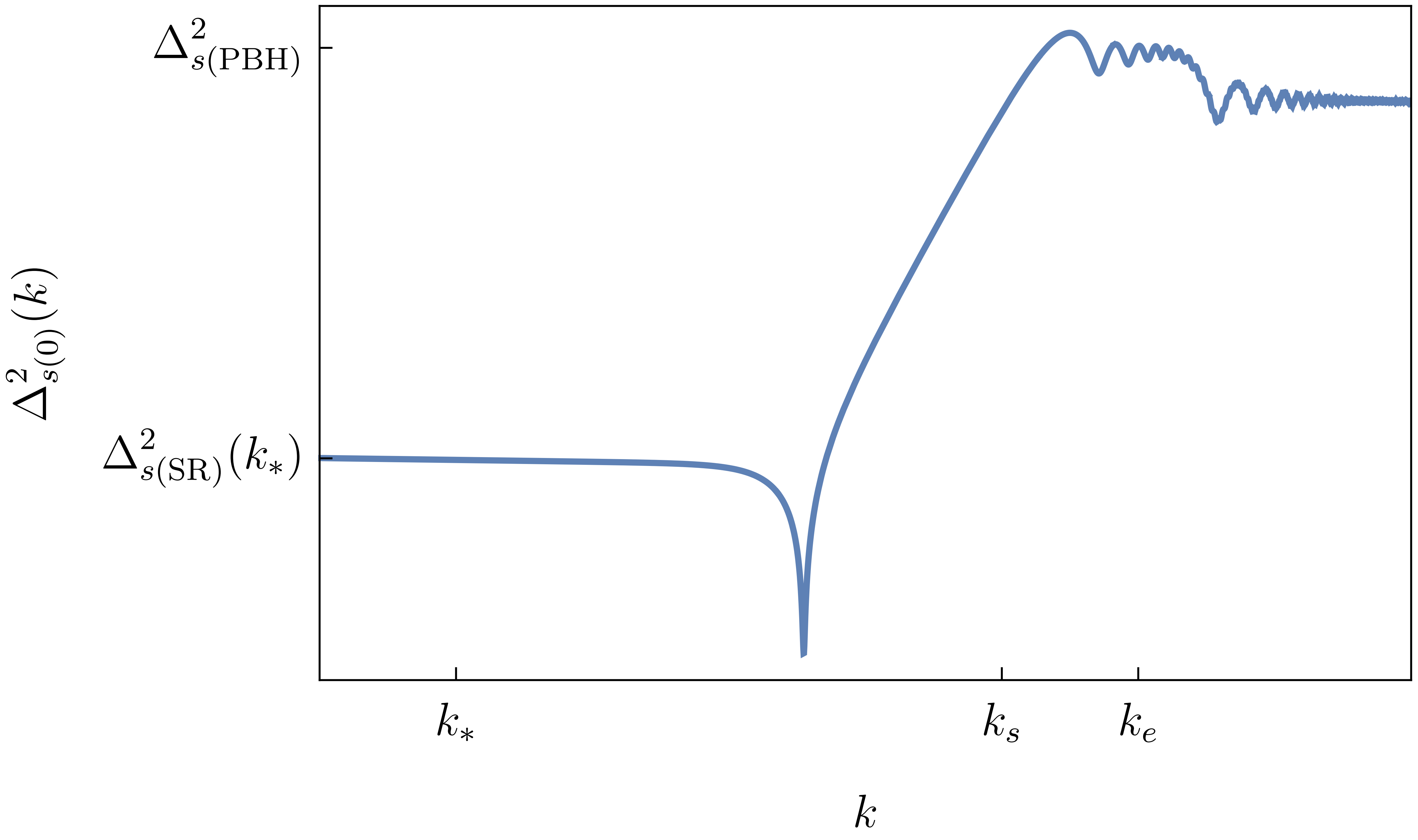


Two-point functions

Power spectrum at the end of inflation: $\Delta^2_{s(0)}(k) = \frac{k^3}{2\pi^2}\,|\,\zeta_k(\tau \to 0)\,|^2$.

Large scale: $\Delta^2_{s(\mathrm{SR})}(k) \equiv \Delta^2_{s(0)}(k \ll k_s) = \frac{H^2}{8\pi^2 M_\mathrm{P}^2 \epsilon_1(\tau_s)}$.

Small scale: $\Delta^2_{s(\mathrm{PBH})} \approx \Delta^2_{s(\mathrm{SR})}(k_s)\left(\frac{k_e}{k_s}\right)^6$.



Cubic self-interaction

Derivative of the potential: $V_n \equiv \dfrac{\mathrm{d}^n V}{\mathrm{d}\phi^n}$

Cubic action: $S^{(3)}_{\delta\phi} = -\dfrac{1}{6}\int \mathrm{d}t\ \mathrm{d}^3x\ a^3 V_3 \delta\phi^3 = -\dfrac{1}{6}M_\mathrm{P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \left(a^2\epsilon_1\epsilon_2'\right)' \zeta_n^3$

As an alternative, performing IBP on the cubic action:

$$S^{(3)}_{\delta\phi} = -\frac{1}{6}M_\mathrm{P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \left(a^2\epsilon_1\epsilon_2'\right)' \zeta_n^3 = \frac{1}{2}M_\mathrm{P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x\ a^2\epsilon_1\epsilon_2'\zeta_n'\zeta_n^2\,,$$

that is justified because $(f(\tau)\zeta_n^3)'$ does not contribute to correlation of ζ_n.

No subtlety for computing tree-level bispectrum (1st order perturbation), but we have to be careful when computing tree-level trispectrum and one-loop power spectrum (2nd order perturbation).

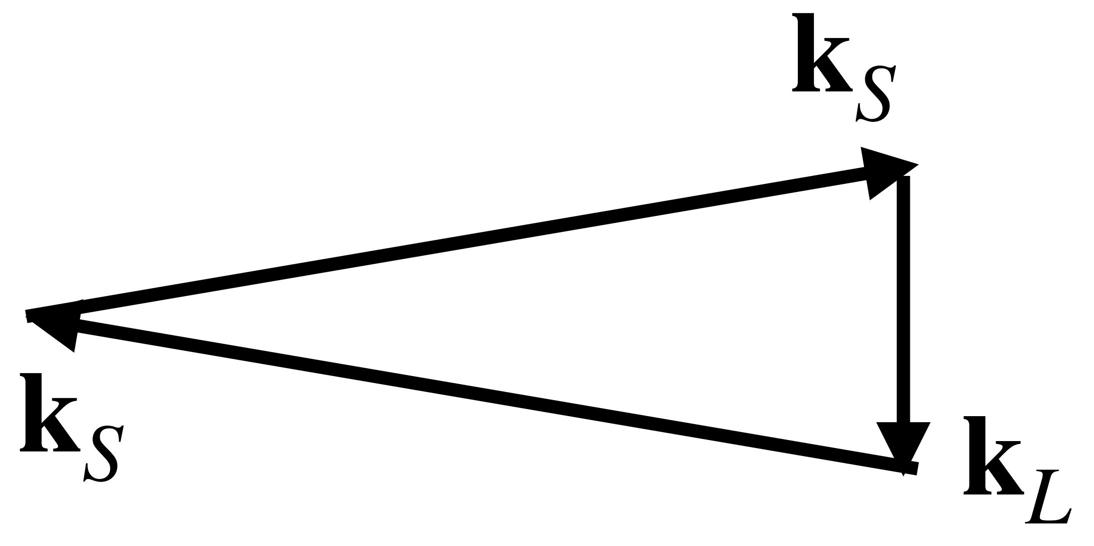
Three-point functions

In-in formalism: $\langle \mathcal{O}(\tau) \rangle = 2\int_{-\infty}^{\tau} \mathrm{d}\tau_1 \ \mathrm{Im} \left\langle \hat{\mathcal{O}}(\tau) H_{\rm int}(\tau_1) \right\rangle$ with $\mathcal{O}(\tau) = \zeta_{\mathbf{k}_1}(\tau)\zeta_{\mathbf{k}_2}(\tau)\zeta_{\mathbf{k}_3}(\tau)$.

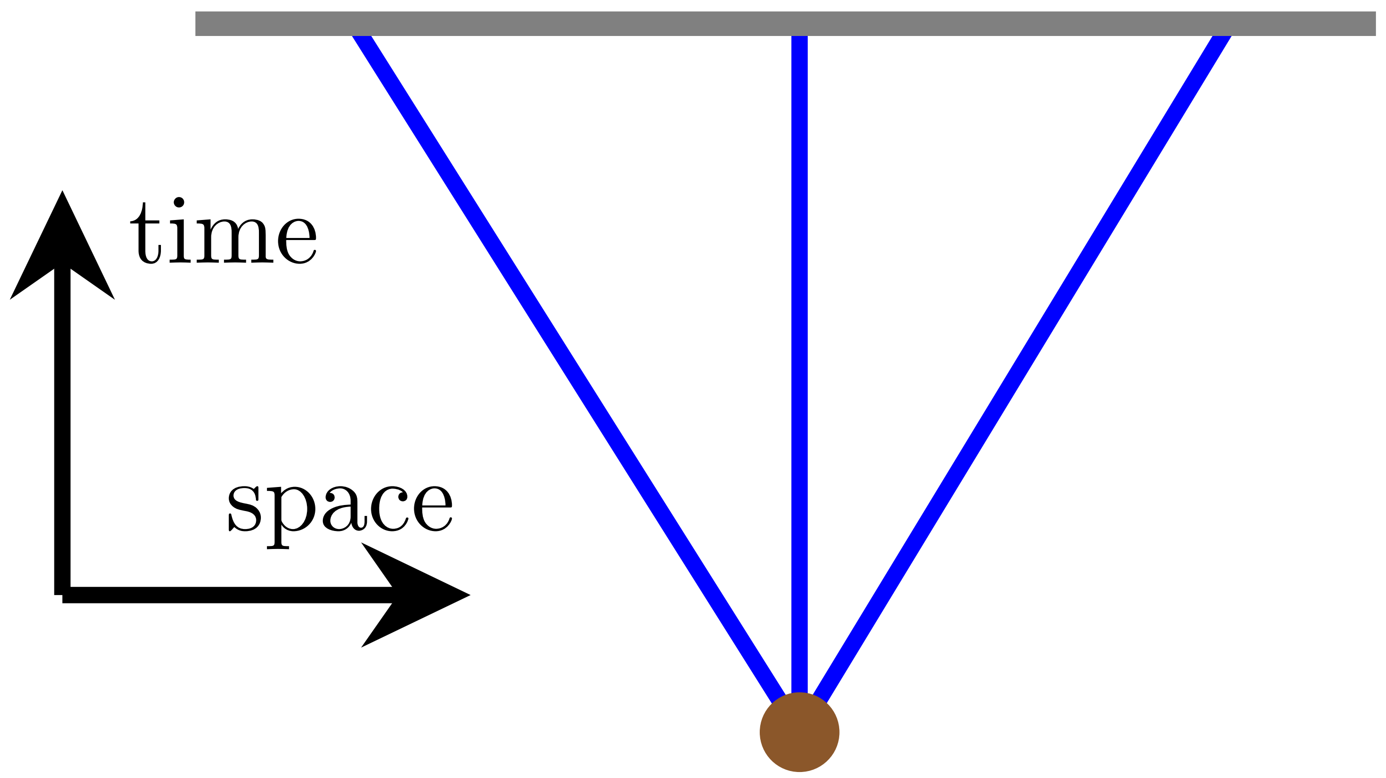
Bispectrum: $\langle \zeta_{\mathbf{k}_1}(\tau)\zeta_{\mathbf{k}_2}(\tau)\zeta_{\mathbf{k}_3}(\tau) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \langle\langle \zeta_{\mathbf{k}_1}(\tau)\zeta_{\mathbf{k}_2}(\tau)\zeta_{\mathbf{k}_3}(\tau) \rangle\rangle$.

Maldacena's theorem on squeezed limit of the bispectrum:

$$\lim_{k_L \to 0} \langle\langle \zeta_{\mathbf{k}_L}(\tau)\zeta_{\mathbf{k}_S}(\tau)\zeta_{-\mathbf{k}_S}(\tau) \rangle\rangle = -\left(n_s(k_S, \tau) - 1\right) |\, \zeta_{k_S}(\tau)\, |^2 \,|\, \zeta_{k_L}(\tau)\, |^2, \; n_s(k, \tau) - 1 = \frac{\mathrm{d} \log \Delta_s^2(k, \tau)}{\mathrm{d} \log k} \, .$$



Bispectrum



Leading interaction: $H^{(3)}_{\delta\phi} = -\frac{1}{2}M_P^2 \int \mathrm{d}^3x\ \epsilon_1 \epsilon_2' a^2 \zeta_n' \zeta_n^2$

Time integral: $\int_{-\infty}^{0} \mathrm{d}\tau\ \epsilon_2'(\tau) f(\tau) = \Delta\epsilon_2 [f(\tau_e) - f(\tau_s)]$

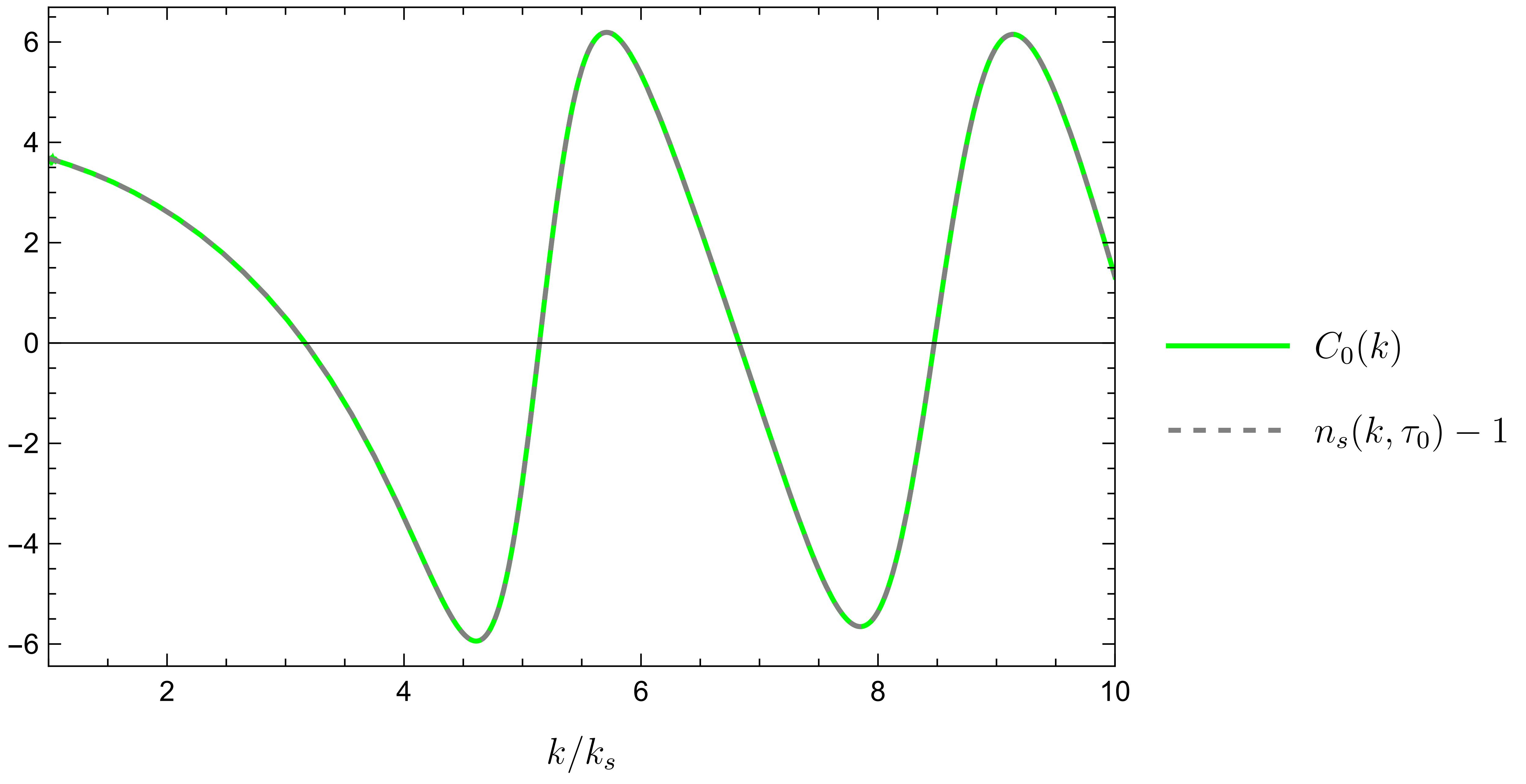
Bispectrum:

$$\langle\langle \zeta_{\mathbf{k}_1}(\tau_0)\zeta_{\mathbf{k}_2}(\tau_0)\zeta_{\mathbf{k}_3}(\tau_0)\rangle\rangle = -2M_P^2 \int_{-\infty}^{\tau_0} \mathrm{d}\tau\ \epsilon_1(\tau)\epsilon_2'(\tau)a^2(\tau)\mathrm{Im}\left[\zeta_{k_1}(\tau_0)\zeta_{k_2}(\tau_0)\zeta_{k_3}(\tau_0)\zeta^*_{k_1}(\tau)\zeta^*_{k_2}(\tau)\zeta'^*_{k_3}(\tau)\right] + \text{perm.}$$

Bispectrum

Squezeed limit: $\langle\langle \zeta_{\mathbf{k}_1}(\tau_0)\zeta_{\mathbf{k}_2}(\tau_0)\zeta_{-\mathbf{k}_2}(\tau_0)\rangle\rangle = -C_0(k_2)\,|\,\zeta_{k_1}(\tau_0)\,|^2\,|\,\zeta_{k_2}(\tau_0)\,|^2,$

$$C_0(k) = 4M_{\rm P}^2\Delta\epsilon_2 {\rm Im}\left\{ \frac{\zeta_k^2(\tau_0)}{|\,\zeta_k(\tau_0)\,|^2}\left[\epsilon_1(\tau_e)a^2(\tau_e)\zeta_k^*(\tau_e)\zeta_k'^*(\tau_e) - \epsilon_1(\tau_s)a^2(\tau_s)\zeta_k^*(\tau_s)\zeta_k'^*(\tau_s)\right]\right\}.$$



One-loop correction

One-loop correction generated by cubic-order action is computed using in-in perturbation theory:

$$\langle \mathcal{O}(\tau) \rangle = \langle \mathcal{O}(\tau) \rangle^{\dagger}_{(0,2)} + \langle \mathcal{O}(\tau) \rangle_{(1,1)} + \langle \mathcal{O}(\tau) \rangle_{(0,2)} \,,$$

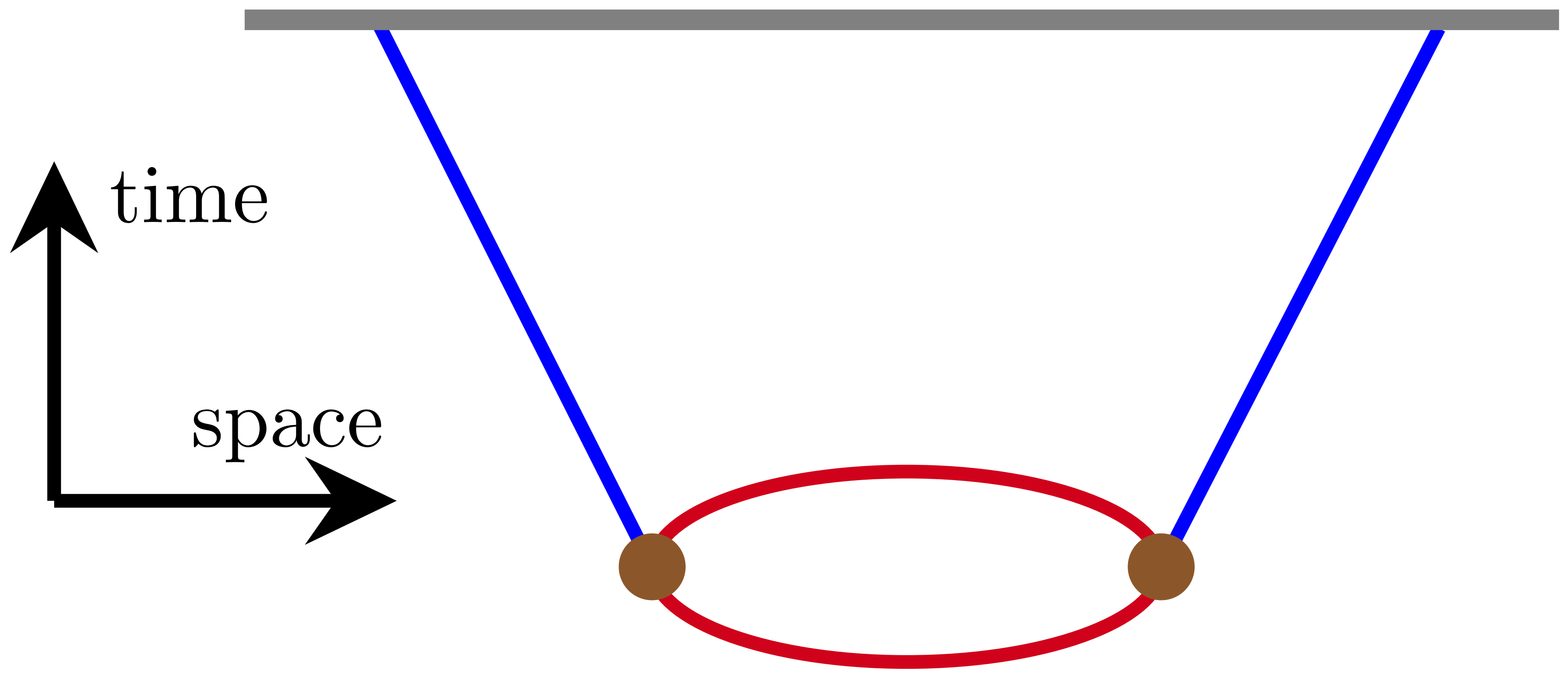
$$\langle \mathcal{O}(\tau) \rangle_{(1,1)} = \int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau} \mathrm{d}\tau_2 \left\langle H_{\mathrm{int}}(\tau_1) \hat{\mathcal{O}}(\tau) H_{\mathrm{int}}(\tau_2) \right\rangle \,,$$

$$\langle \mathcal{O}(\tau) \rangle_{(0,2)} = -\int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau_1} \mathrm{d}\tau_2 \left\langle \hat{\mathcal{O}}(\tau) H_{\mathrm{int}}(\tau_1) H_{\mathrm{int}}(\tau_2) \right\rangle \,.$$

Operator: $\mathcal{O}(\tau_0) = \zeta_{\mathbf{p}}(\tau_0)\zeta_{-\mathbf{p}}(\tau_0)$ where $\tau_0 \to 0$

Leading interaction: $H^{(3)}_{\delta\phi} = -\dfrac{1}{2} M_{\mathrm{P}}^2 \int \mathrm{d}^3x \; \epsilon_1 \epsilon_2' a^2 \zeta_n' \zeta_n^2$

One-loop correction



One-loop correction to general scale p :

$$\langle \zeta_{\mathbf{p}}(\tau_0)\zeta_{-\mathbf{p}}(\tau_0)\rangle_{(1)} = \frac{1}{4} M_{\mathrm{P}}^4 \epsilon_1^2(\tau_e) a^4(\tau_e)(\Delta\epsilon_2)^2 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} |\zeta_{|\mathbf{k}-\mathbf{p}|}(\tau_e)|^2 f(p,k;\tau_e) \ ,$$

$$f(p,k;\tau_e) = 8|\zeta_k(\tau_e)|^2 \mathrm{Im}[\zeta_p(\tau_0)\zeta_p'^*(\tau_e)]^2 + 32|\zeta_k'(\tau_e)|^2 \mathrm{Im}[\zeta_p(\tau_0)\zeta_p^*(\tau_e)]^2$$

$$+32\mathrm{Im}[\zeta_p(\tau_0)\zeta_p^*(\tau_e)]\mathrm{Im}[\zeta_p(\tau_0)\zeta_p'^*(\tau_e)]\mathrm{Re}[\zeta_k'^*(\tau_e)\zeta_k(\tau_e)] + 16|\zeta_p(\tau_0)|^2 \mathrm{Im}[\zeta_p(\tau_e)\zeta_p'^*(\tau_e)]\mathrm{Im}[\zeta_k(\tau_0)\zeta_k'^*(\tau_e)].$$

One-loop correction

For large loop momentum $k \gg p$:

$$\langle \zeta_{\mathbf{p}}(\tau_0)\zeta_{-\mathbf{p}}(\tau_0)\rangle_{(1)} = \frac{1}{4} M_{\rm P}^4 \epsilon_1^2(\tau_e) a^4(\tau_e) (\Delta\epsilon_2)^2 |\zeta_p(\tau_0)|^2 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} 16 \left[|\zeta_k|^2 \ \mathrm{Im}(\zeta_p' \zeta_p^*) \ \mathrm{Im}(\zeta_k' \zeta_k^*) \right]_{\tau=\tau_e} .$$

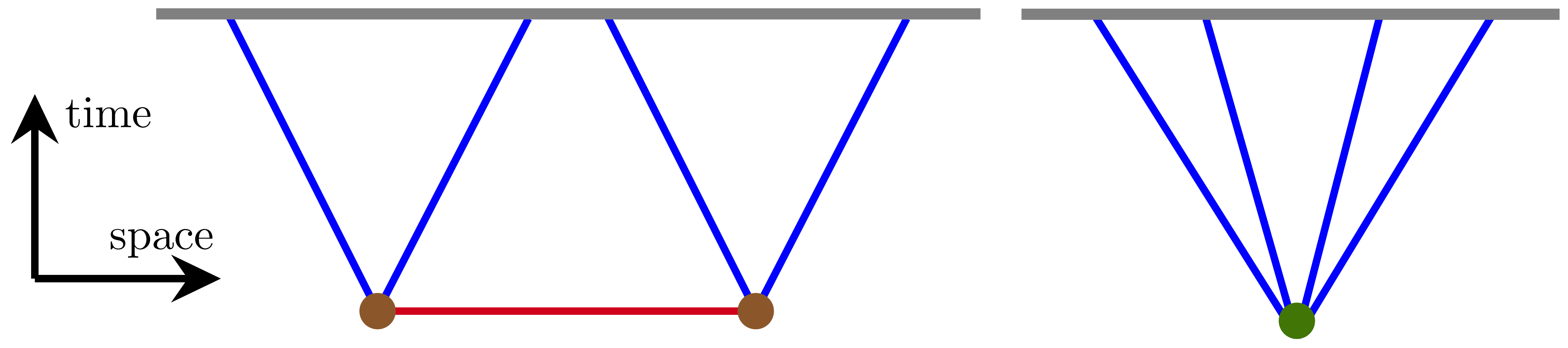
Substituting $[\mathrm{Im}(\zeta_k' \zeta_k^*)]_{\tau=\tau_e} = -\dfrac{1}{4M_{\rm P}^2 \epsilon_1(\tau_e) a^2(\tau_e)}$, we obtain

$$\Delta^2_{s(1)}(p, \tau_0) = \frac{1}{4} (\Delta\epsilon_2)^2 \Delta^2_{s(0)}(p, \tau_0) \int \frac{\mathrm{d}k}{k} \Delta^2_{s(0)}(k, \tau_e) \ .$$

$$\Delta^2_{s(\mathrm{PBH})} \ll \frac{1}{(\Delta\epsilon_2)^2}$$

Scale-invariant one-loop correction? Note the importance of decaying mode, even at late time for long-wavelength mode ζ_p.

Trispectrum



Total contributions to the trispectrum:

- Exchange diagram with two $H^{(3)}_{\delta\phi}$ vertices
  - s-channel: $s = |\mathbf{k}_1 + \mathbf{k}_2|$
  - t-channel: $t = |\mathbf{k}_1 + \mathbf{k}_3|$
  - u-channel: $u = |\mathbf{k}_1 + \mathbf{k}_4|$
- Contact diagram with $H^{(4)}_{\delta\phi}$ vertex

$$\langle \mathcal{O}(\tau)\rangle^{(3)} = \langle \mathcal{O}(\tau)\rangle^{\dagger}_{(0,2)} + \langle \mathcal{O}(\tau)\rangle_{(1,1)} + \langle \mathcal{O}(\tau)\rangle_{(0,2)}\,,$$

$$\langle \mathcal{O}(\tau)\rangle_{(1,1)} = \int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau} \mathrm{d}\tau_2 \left\langle H^{(3)}(\tau_1)\hat{\mathcal{O}}(\tau)H^{(3)}(\tau_2)\right\rangle,$$

$$\langle \mathcal{O}(\tau)\rangle_{(0,2)} = -\int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau_1} \mathrm{d}\tau_2 \left\langle \hat{\mathcal{O}}(\tau)H^{(3)}(\tau_1)H^{(3)}(\tau_2)\right\rangle.$$

$$\langle \mathcal{O}(\tau)\rangle^{(4)} = 2\int_{-\infty}^{\tau} \mathrm{d}\tau_1\ \mathrm{Im}\left\langle \hat{\mathcal{O}}(\tau)H^{(4)}(\tau_1)\right\rangle$$

Kristiano and Yokoyama (in preparation)

Quartic self-interaction

Cubic action: $S^{(3)}_{\delta\phi} = -\frac{1}{6}\int \mathrm{d}t\ \mathrm{d}^3x\ a^3 V_3 \delta\phi^3 = -\frac{1}{6} M_\mathrm{P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \left(a^2\epsilon_1\epsilon_2'\right)' \zeta_n^3$

Cubic Hamiltonian: $H^{(3)}_{\delta\phi} = \frac{1}{6} M_\mathrm{P}^2 \int \mathrm{d}^3x\ \left(a^2\epsilon_1\epsilon_2'\right)' \zeta_n^3$

Quartic action: $S^{(4)}_{\delta\phi} = -\frac{1}{24}\int \mathrm{d}t\ \mathrm{d}^3x\ a^3 V_4 \delta\phi^4 = -\frac{1}{24}\int \mathrm{d}t\ \mathrm{d}^3x\ (zM_\mathrm{P})^3 \frac{\dot{V}_3}{H} \zeta_n^4$

Quartic Hamiltonian: $H^{(4)}_{\delta\phi} = -\frac{1}{24} M_\mathrm{P}^2 \int \mathrm{d}^3x \left[\frac{1}{aH}\left(\epsilon_1 a^2 \epsilon_2'\right)'' - \left(4 + \frac{3}{2}\epsilon_2\right)\left(\epsilon_1 a^2\epsilon_2'\right)'\right] \zeta_n^4$

Seems easy and no problem!

Quartic-induced Hamiltonian

For the alternative cubic action: $S^{(3)}_{\delta\phi} = \frac{1}{2} M_{\mathrm{P}}^2 \int \mathrm{d}\tau \ \mathrm{d}^3 x \ a^2 \epsilon_1 \epsilon_2' \zeta_n' \zeta_n^2$

Quartic-induced Hamiltonian: $H^{(4\mathrm{I})}_{\delta\phi} = \frac{1}{16} M_{\mathrm{P}}^2 \int \mathrm{d}^3 x \ a^2 \epsilon_1 (\epsilon_2')^2 \zeta_n^4$

In case ϵ_2' is modelled as a Dirac-delta function of time, this $H^{(4\mathrm{I})}$ generates trispectrum (and one-loop power spectrum) proportional to Dirac-delta function of time (or inversely proportional to the duration of transition).

Very large and dangerous contribution!

However, this $H^{(4\mathrm{I})}$ must be added to the quartic Hamiltonian from quartic action $H^{(4)}_{\delta\phi}$.

Total quartic Hamiltonian

Quartic Hamiltonian: $H^{(4)}_{\delta\phi} = -\frac{1}{24}M_{\rm P}^2\int \mathrm{d}^3x\left[\frac{1}{aH}\left(\epsilon_1 a^2\epsilon_2'\right)'' - \left(4+\frac{3}{2}\epsilon_2\right)\left(\epsilon_1 a^2\epsilon_2'\right)'\right]\zeta_n^4$

Quartic-induced Hamiltonian: $H^{(4\mathrm{I})} = \frac{1}{16}M_{\rm P}^2\int \mathrm{d}^3x\ a^2\epsilon_1(\epsilon_2')^2\zeta_n^4$

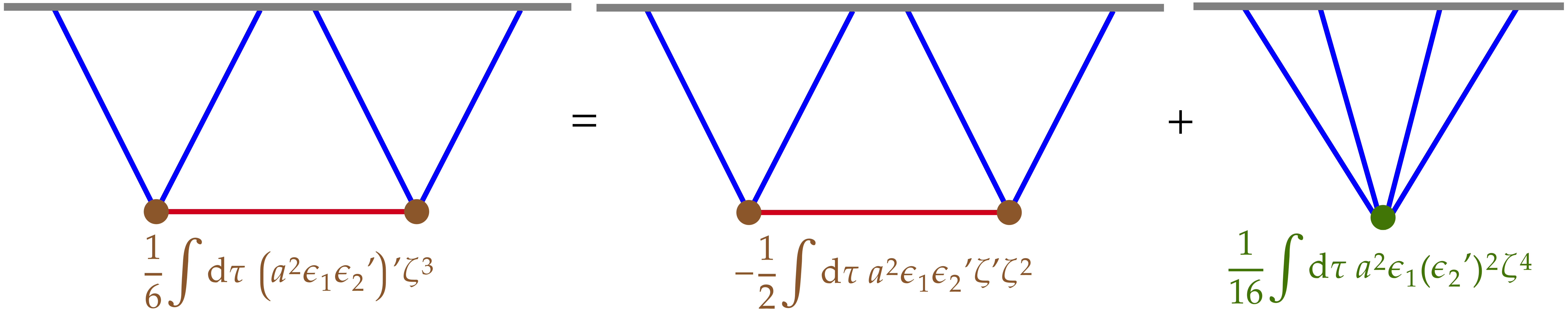
Total: $H^{(4)} = H^{(4)}_{\delta\phi} + H^{(4\mathrm{I})}_{\delta\phi} = -\frac{1}{24}M_{\rm P}^2\int \mathrm{d}^3x\left\{\frac{1}{aH}\left(\epsilon_1 a^2\epsilon_2'\right)'' - \left[\left(4+\frac{3}{2}\epsilon_2\right)\left(\epsilon_1 a^2\epsilon_2'\right)\right]'\right\}\zeta_n^4\ .$

The total quartic Hamiltonian is not proportional to $(\epsilon_2')^2$!

Total quartic Hamiltonian

If we start from $H^{(3)}_{\delta\phi} = \frac{1}{6} M_{\mathrm{P}}^2 \int \mathrm{d}^3x \left(a^2 \epsilon_1 \epsilon_2'\right)' \zeta_n^3$, substitute it to the second-order perturbation theory (and perform IBP) yields the quartic-induced Hamiltonian.

Take care of the total time derivative when performing IBP.

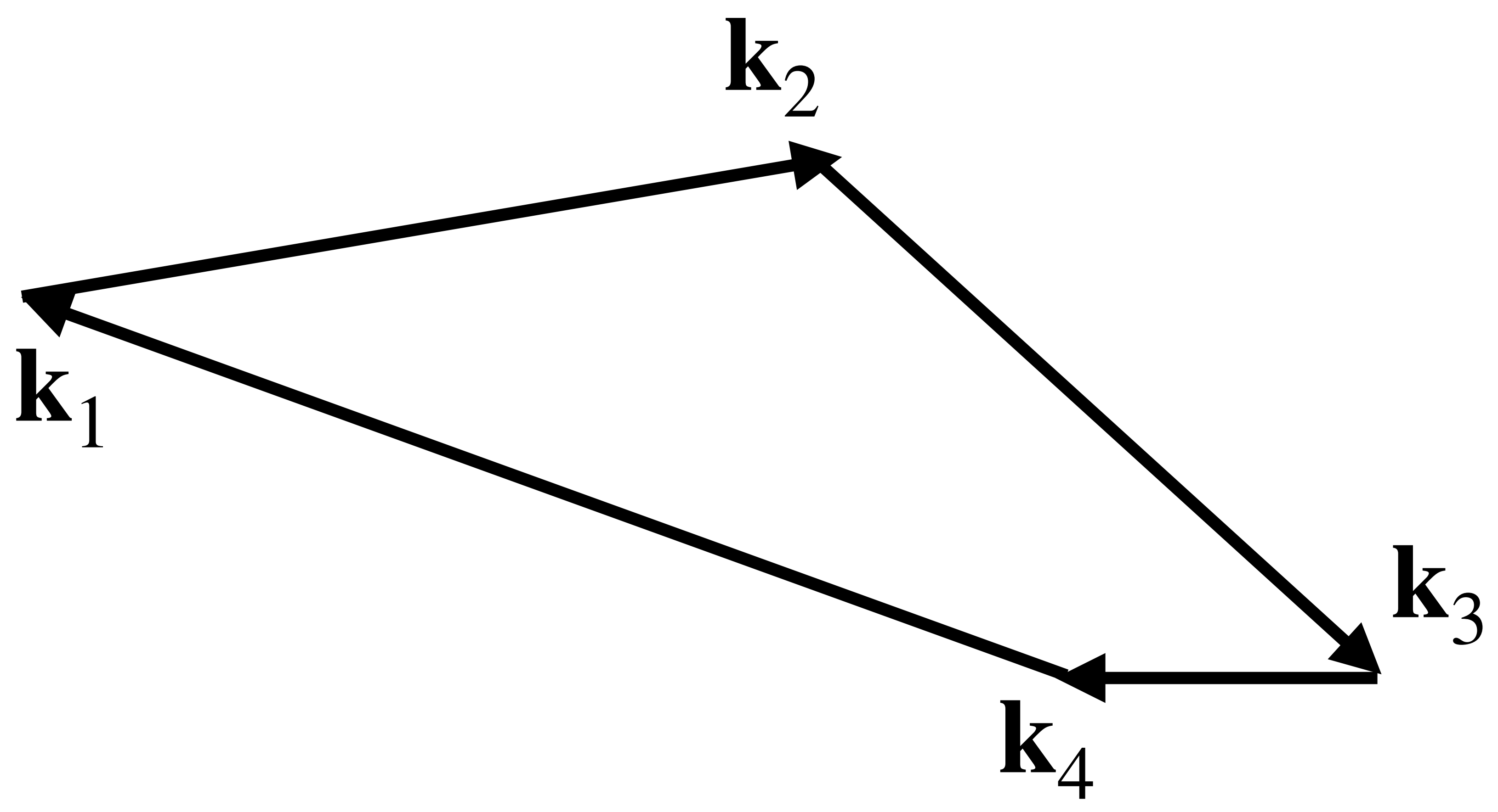


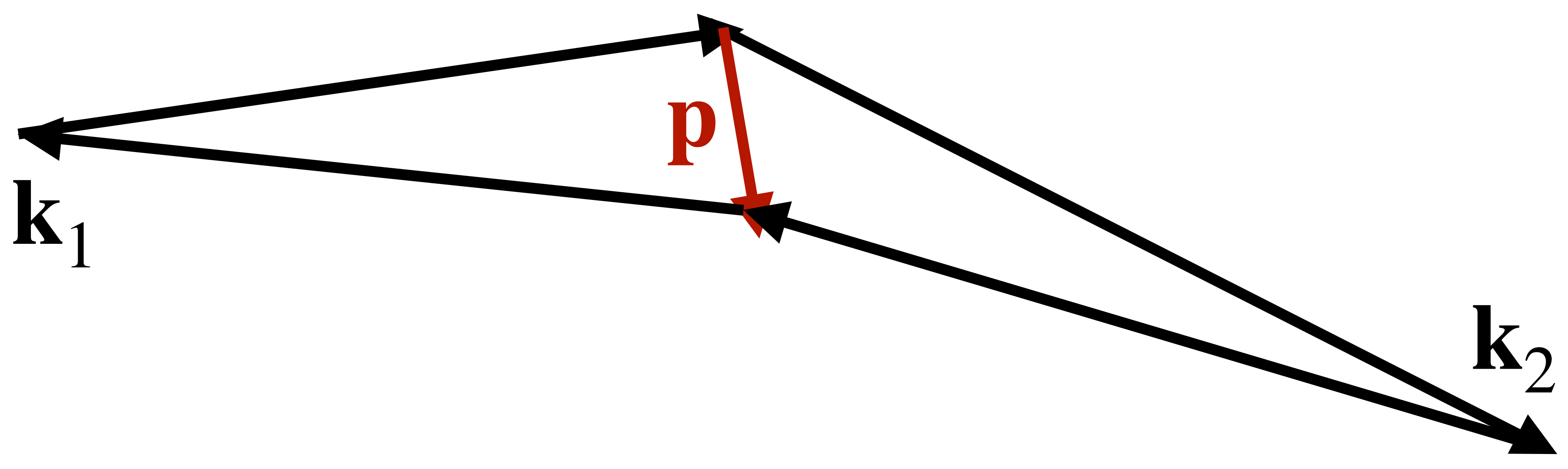
Trispectrum

Explicit form of trispectrum is too long, but it satisfies Maldacena's theorem.

Squeezed: $$\lim_{k_4 \to 0} \frac{-1}{|\zeta_{k_4}(\tau)|^2} \langle\langle \zeta_{\mathbf{k}_1}(\tau)\zeta_{\mathbf{k}_2}(\tau)\zeta_{\mathbf{k}_3}(\tau)\zeta_{\mathbf{k}_4}(\tau) \rangle\rangle = \left(6 + \sum_{n=1}^{3} k_n \frac{\partial}{\partial k_n}\right) \langle\langle \zeta_{\mathbf{k}_1}(\tau)\zeta_{\mathbf{k}_2}(\tau)\zeta_{\mathbf{k}_3}(\tau) \rangle\rangle.$$

Collapsed: $$\lim_{p \to 0} \frac{-1}{|\zeta_{k_4}(\tau)|^2} \langle\langle \zeta_{\mathbf{k}_1}(\tau)\zeta_{\mathbf{p}-\mathbf{k}_1}(\tau)\zeta_{-\mathbf{k}_2}(\tau)\zeta_{\mathbf{k}_2-\mathbf{p}}(\tau) \rangle\rangle = (n_s(k_1,\tau) - 1)(n_s(k_2,\tau) - 1)|\zeta_{k_1}(\tau)|^2 |\zeta_{k_2}(\tau)|^2.$$



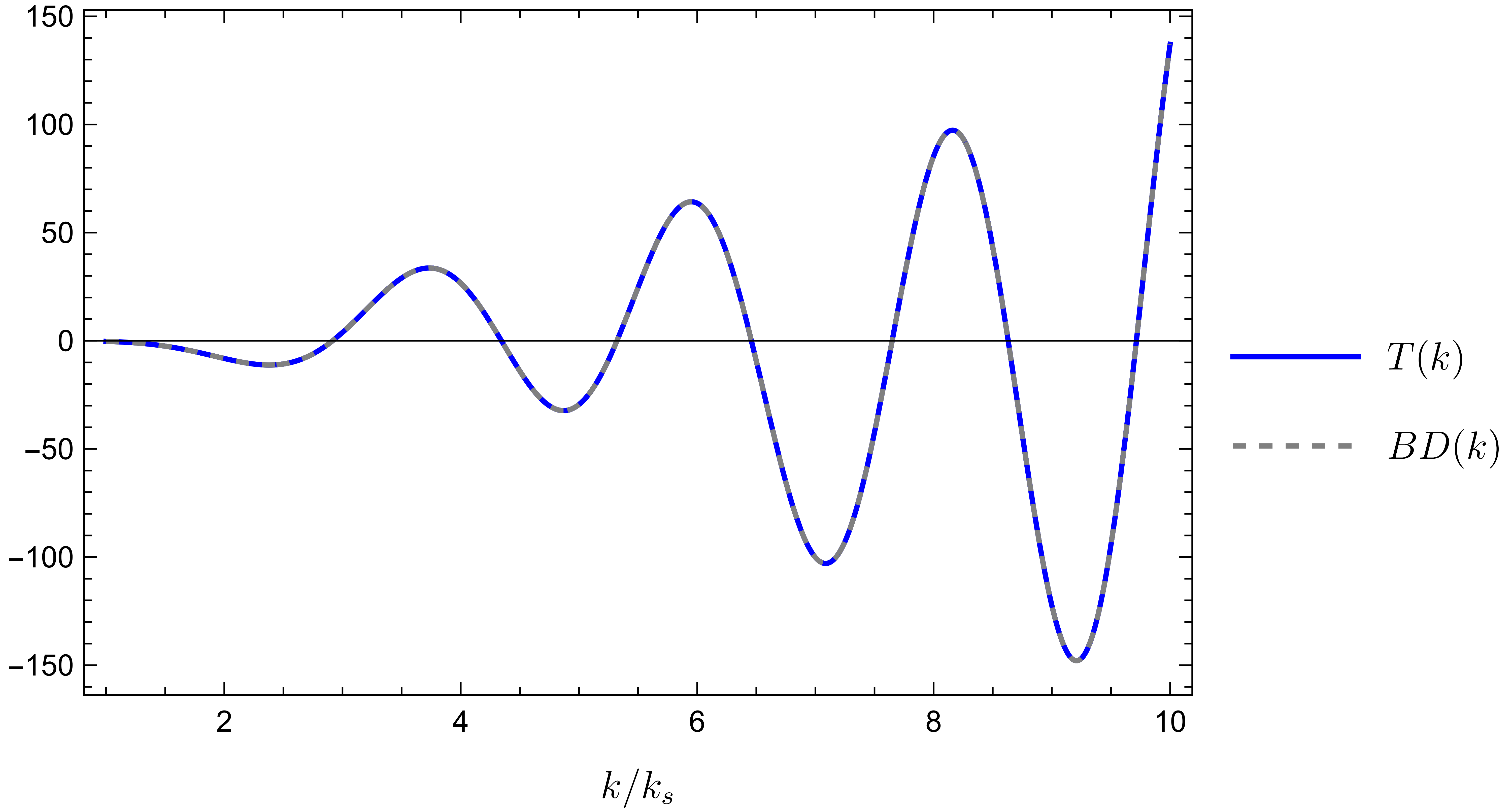


Kristiano and Yokoyama (in preparation)

Trispectrum

$T(k)$ = soft limit of trispectrum evaluated at equilateral shape

$BD(k)$ = running of bispectrum evaluated at equilateral shape



Trispectrum

Second-order perturbation:

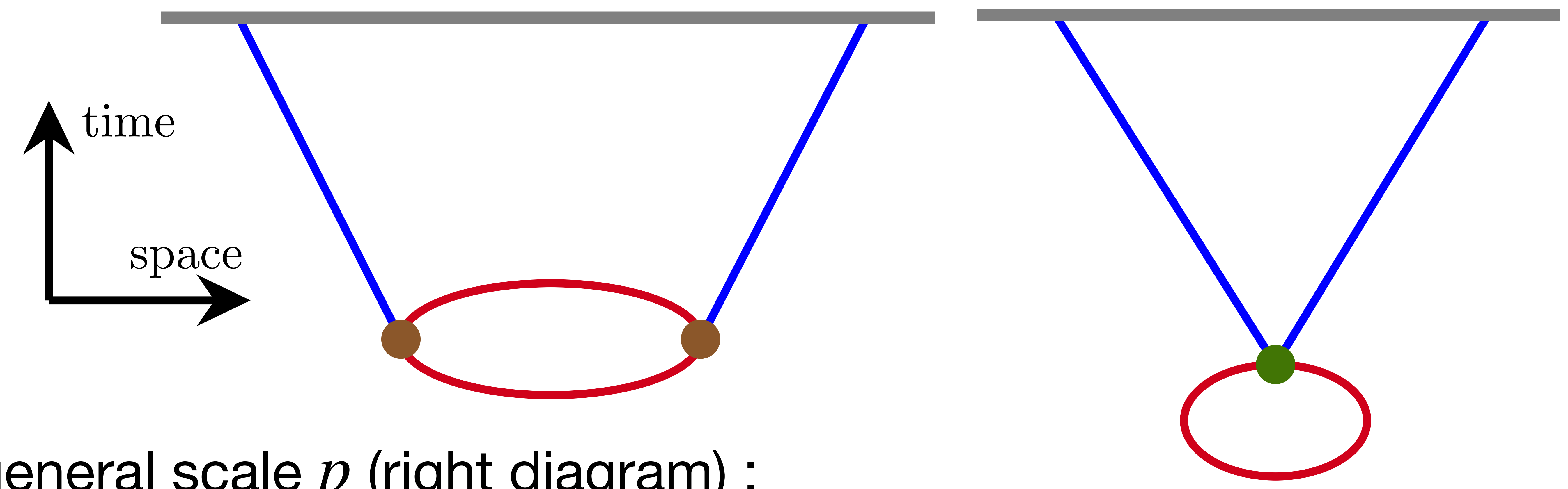
$$\langle \mathcal{O}(\tau) \rangle = \int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau} \mathrm{d}\tau_2 \left\langle H^{(3)}(\tau_1)\hat{\mathcal{O}}(\tau)H^{(3)}(\tau_2) \right\rangle - 2\mathrm{Re} \int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau_1} \mathrm{d}\tau_2 \left\langle \hat{\mathcal{O}}(\tau)H^{(3)}(\tau_1)H^{(3)}(\tau_2) \right\rangle$$

$$+2\int_{-\infty}^{\tau} \mathrm{d}\tau_1 \ \mathrm{Im} \left\langle \hat{\mathcal{O}}(\tau)H^{(4)}(\tau_1) \right\rangle$$

- Trispectrum: $\hat{\mathcal{O}} = \zeta^4$
- Bare one-loop power spectrum: $\hat{\mathcal{O}} = \zeta^2$

If one claims set of $H^{(3)}$ and $H^{(4)}$ that are consistent for one-loop power spectrum, one should be able to show that the set generate consistent trispectrum.

One-loop correction



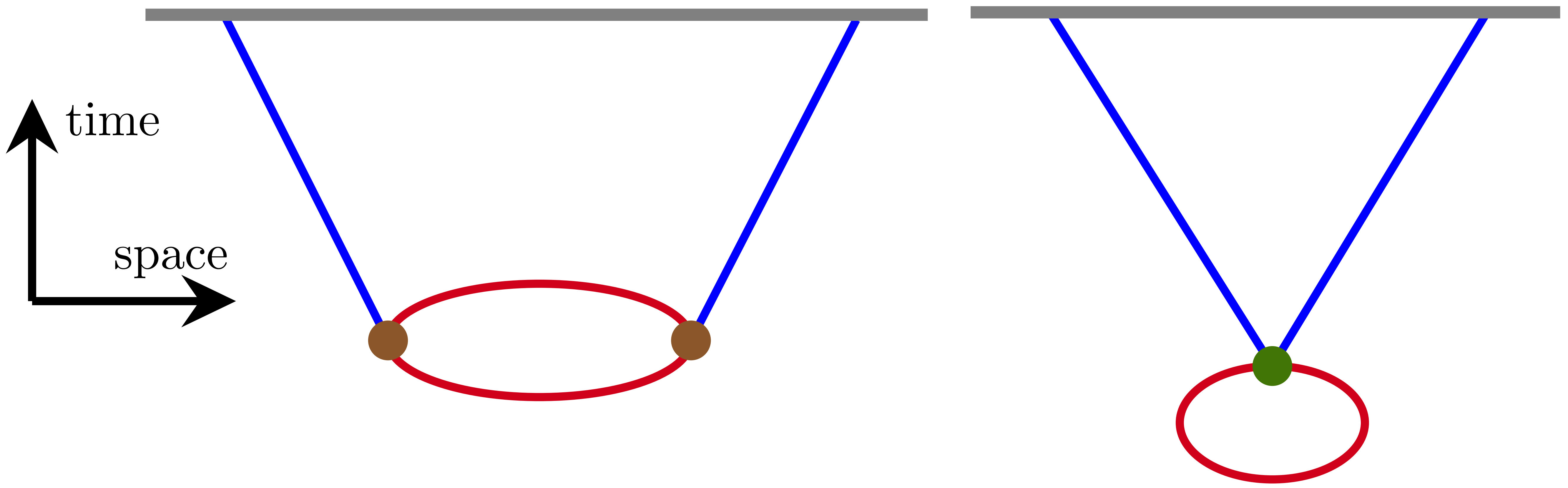
One-loop correction to general scale p (right diagram) :

$$\langle \zeta_{\mathbf{p}}(\tau_0)\zeta_{-\mathbf{p}}(\tau_0)\rangle_{(1)} = -M_{\mathrm{P}}^2\epsilon_1(\tau_e)a^2(\tau_e)(\Delta\epsilon_2)\int \frac{\mathrm{d}^3k}{(2\pi)^3} f(p,k;\tau_e)\ ,$$

$$f(p,k;\tau_e) = \mathrm{Im}[\zeta_p^2(\tau_0)\zeta_p^*(\tau_e)\zeta_p'^*(\tau_e)]\left(-\frac{1}{2}\Delta\epsilon_2|\zeta_k(\tau_e)|^2 + \frac{8}{k_e}\mathrm{Re}[\zeta_k(\tau_e)\zeta_k'^*(\tau_e)]\right) + \frac{2}{k_e}\mathrm{Im}[\zeta_p^2(\tau_0)\zeta_p'^{*2}(\tau_e)]|\zeta_k(\tau_e)|^2$$

$$+\mathrm{Im}[\zeta_p^2(\tau_0)\zeta_p^{*2}(\tau_e)]\left(\frac{2}{k_e}|\zeta_k'(\tau_e)|^2 - \frac{1}{2}\Delta\epsilon_2\mathrm{Re}[\zeta_k(\tau_e)\zeta_k'^*(\tau_e)] - 2\frac{p^2+k^2}{k_e}|\zeta_k(\tau_e)|^2\right).$$

One-loop correction



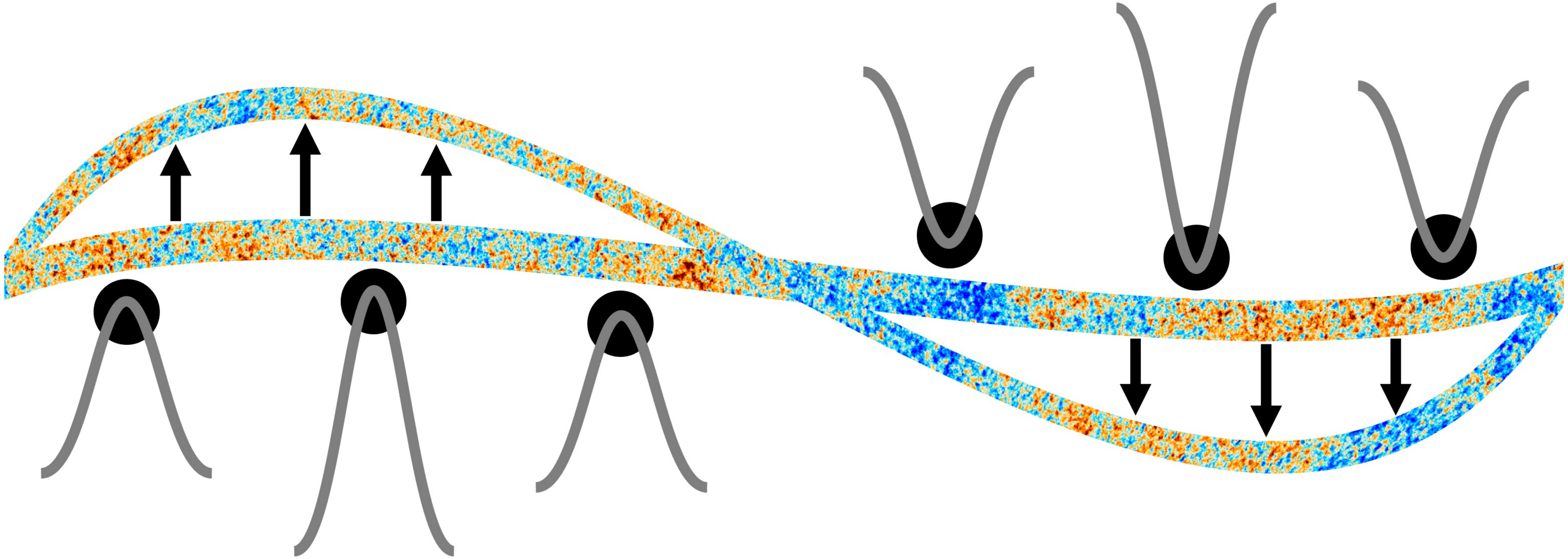
For large loop momentum $k \gg p$ and summing with contribution from cubic Hamiltonian:

$$\Delta^2_{s(1)}(p,\tau_0) = \Delta^2_{s(0)}(p,\tau_0)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left\{\Delta\epsilon_2\left(\frac{3}{8}\Delta\epsilon_2-\frac{k^2}{3k_e^2}\right)|\zeta_k(\tau_e)|^2-\Delta\epsilon_2\left(2+\frac{\Delta\epsilon_2}{12}\right)\mathrm{Re}\left[\zeta_k(\tau_e)\zeta_k'^*(\tau_e)\right]-\frac{\Delta\epsilon_2}{3k_e^2}|\zeta_k'(\tau_e)|^2\right\}$$

$$\Delta^2_{s(1)}(p,\tau_0) = -\left(13.8+13.5\log\frac{k_e}{k_s}\right)\Delta^2_{s(\mathrm{PBH})}\Delta^2_{s(0)}(p,\tau_0)$$

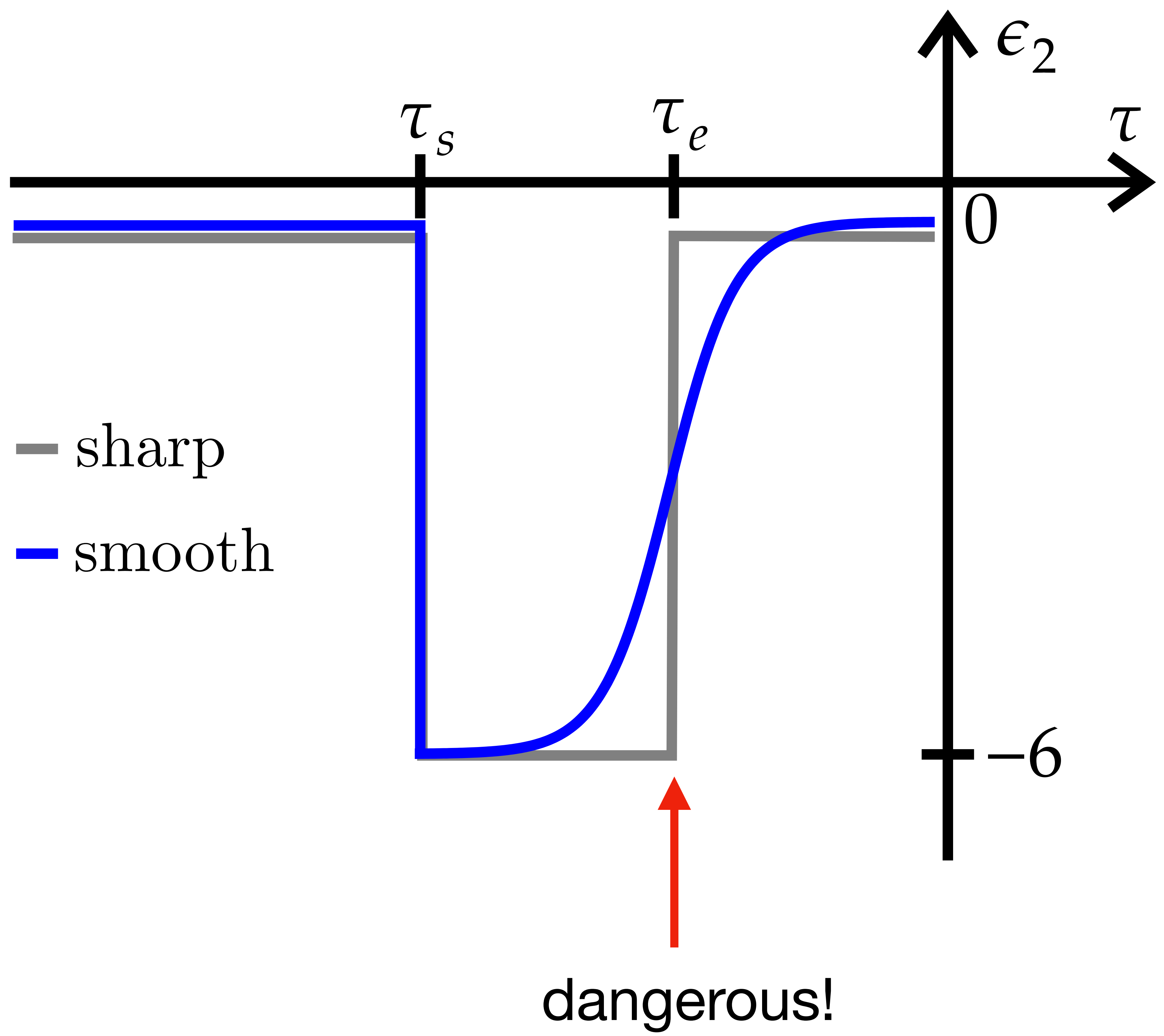
Kristiano and Yokoyama (in preparation)

Physical picture



At the transition, small-scale perturbations coherently kick the large-scale perturbation

Smooth transition



Wands duality

$$\frac{z''}{z} = (aH)^2\left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{1}{2}\epsilon_2\epsilon_3\right)$$

Almost constant $\epsilon_2 \longrightarrow |\epsilon_3| \ll 1$

$$\frac{z''}{z} \simeq \frac{2}{\tau^2}$$

SR: $\epsilon_1, |\epsilon_2| \ll 1$

USR: $\epsilon_1 \ll 1, \epsilon_2 \simeq -6$

Dynamical $\epsilon_2 \longrightarrow |\epsilon_3| \sim \mathcal{O}(1)$

$$\frac{z''}{z} \simeq \frac{2}{\tau^2}$$

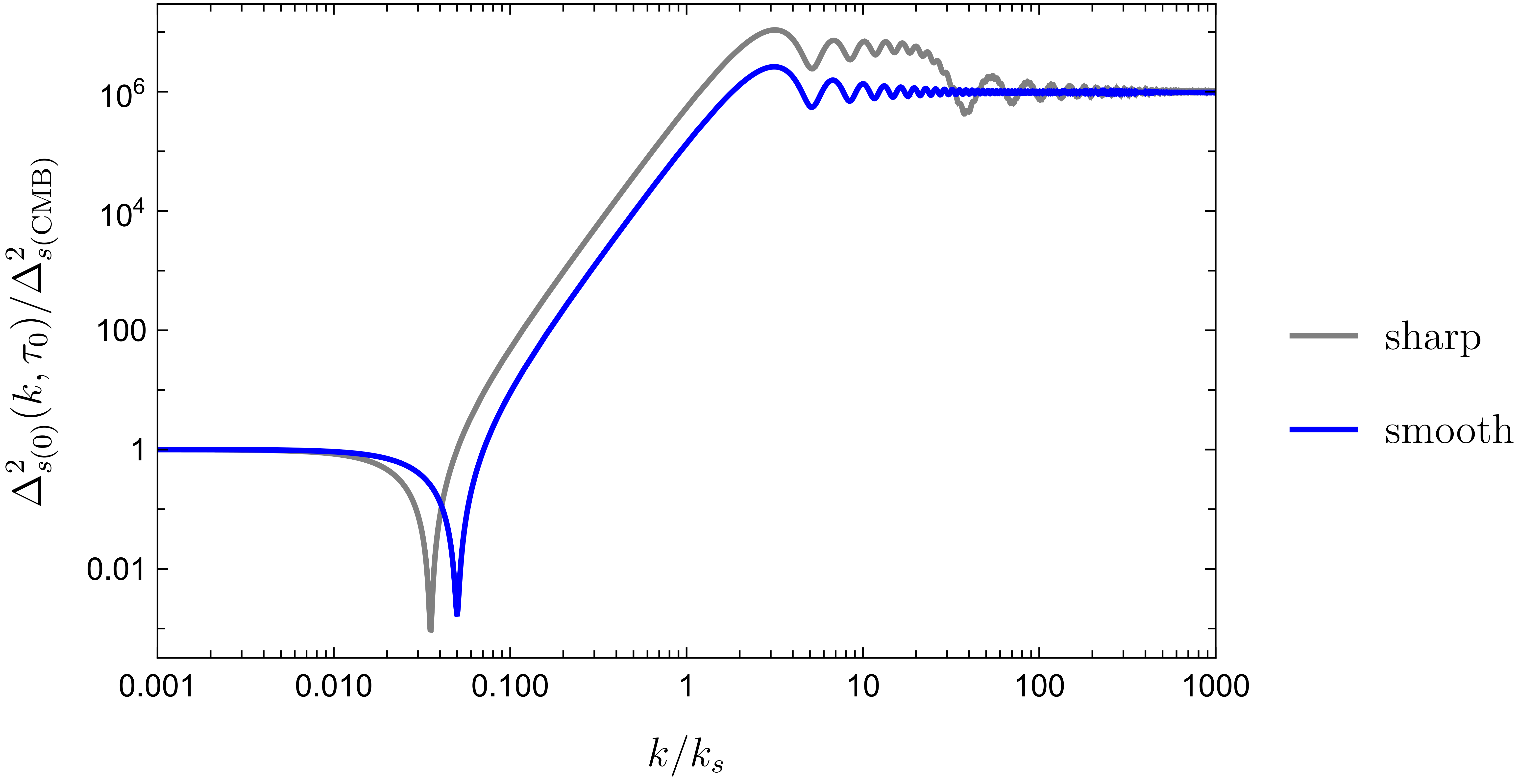
SR: $\epsilon_1, |\epsilon_2| \ll 1$

USR: $\epsilon_1 \ll 1, \epsilon_2 \simeq -6$

Transition: $\frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{\dot{\epsilon}_2}{2H} \simeq 0$

Two-point functions

Comparing power spectrum:



More on Wands duality

Differential equation: $\frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{\dot{\epsilon}_2}{2H} = \text{constant.}$

Taking time derivative: $0 = 2\epsilon_2' + \epsilon_2'\epsilon_2 - \epsilon_2''\tau$.

Prove that $\epsilon_1(\tau)a^2(\tau)\epsilon_2'(\tau) = \text{constant}$:

$$(\epsilon_1 a^2 \epsilon_2')' = \epsilon_1 a^3 H \left(2\epsilon_2' + \epsilon_2\epsilon_2' - \epsilon_2''\tau\right) = 0.$$

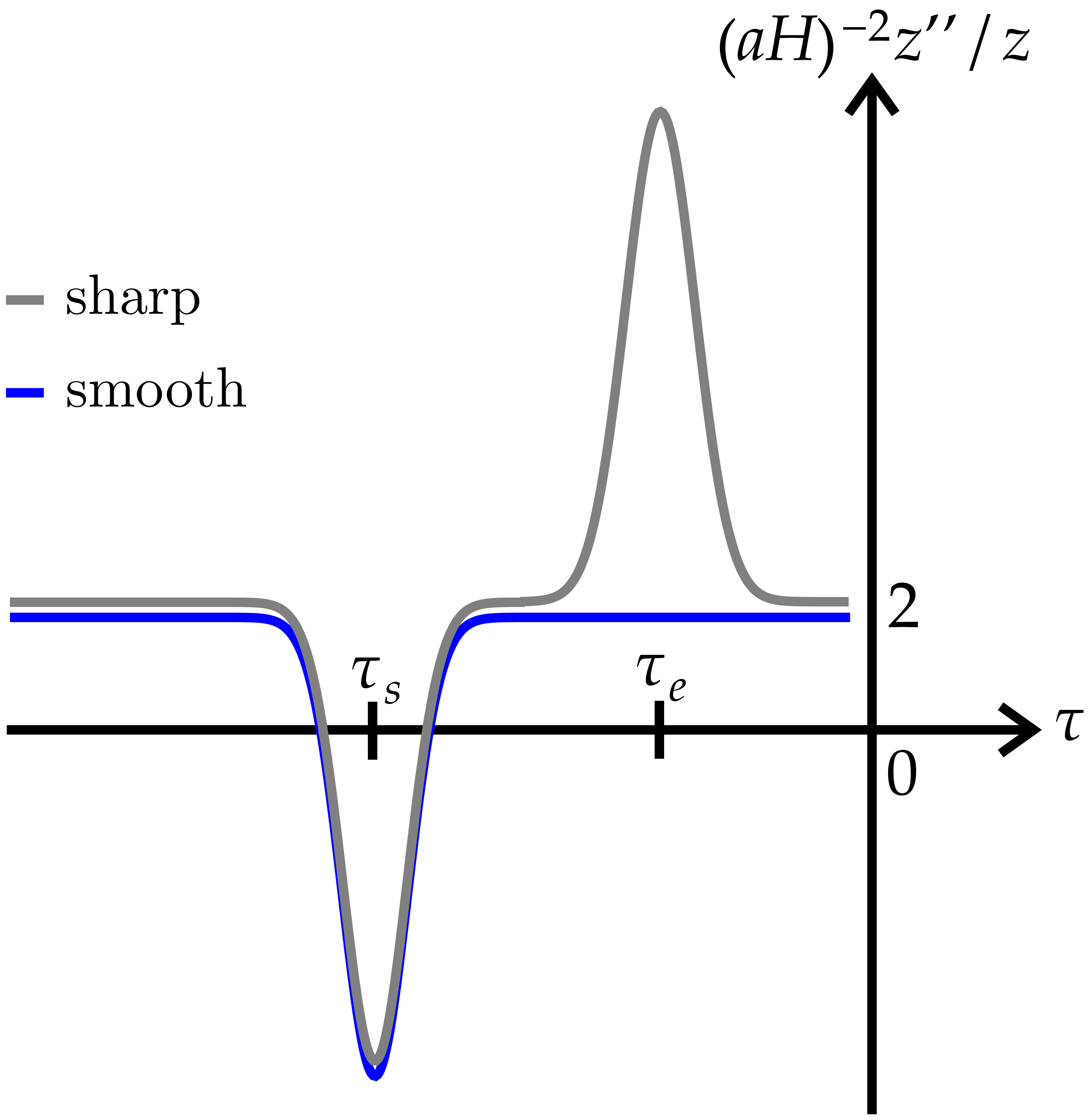
Therefore in this setup: $H^{(3)}_{\delta\phi} = \frac{1}{6}M_\mathrm{P}^2 \int \mathrm{d}^3x \left(a^2\epsilon_1\epsilon_2'\right)' \zeta_n^3 = 0$

$$H^{(4)}_{\delta\phi} = -\frac{1}{24}M_\mathrm{P}^2 \int \mathrm{d}^3x \left[\frac{1}{aH}\left(\epsilon_1 a^2 \epsilon_2'\right)'' - \left(4 + \frac{3}{2}\epsilon_2\right)\left(\epsilon_1 a^2 \epsilon_2'\right)'\right]\zeta_n^4 = 0$$

Bigger picture

Deviation from Wands duality condition generates higher-order correction to the correlation functions.

Possible guidance for bootstrap?



Direct approach

As an introduction, consider bispectrum in pure USR.

Field redefinition: $\zeta = \zeta_n + \frac{1}{4}\epsilon_2 \zeta_n^2 + \frac{1}{H}\dot{\zeta}_n \zeta_n$

Bispectrum of ζ_n is 0, so the whole contributions come from the field redefinition terms.

At the end of inflation $\dot{\zeta}_k(\tau_0) = 3H\zeta_k(\tau_0)$, so we have

$$\langle\langle \zeta_{\mathbf{k}_1}(\tau)\zeta_{\mathbf{k}_2}(\tau)\zeta_{\mathbf{k}_3}(\tau)\rangle\rangle = 2\left(\frac{1}{4}\epsilon_2 + 3\right)\left[\,|\zeta_{k_1}(\tau_0)|^2|\zeta_{k_2}(\tau_0)|^2 + |\zeta_{k_1}(\tau_0)|^2|\zeta_{k_3}(\tau_0)|^2 + |\zeta_{k_2}(\tau_0)|^2|\zeta_{k_3}(\tau_0)|^2\right]$$

Direct approach

As an alternative, we can start from cubic action of ζ (rather than ζ_n):

$$S^{(3)}[\zeta] = M_{\rm P}^2 \int {\rm d}\tau\ {\rm d}^3 x \left[\frac{1}{2}a^2\epsilon_1\epsilon_2'\zeta'\zeta^2 - \left(\frac{1}{2}a^2\epsilon_1\epsilon_2\zeta'\zeta^2 + \frac{a}{H}(\zeta')^2\zeta\right)'\right]$$

Bispectrum generated by the first total time derivative term:

$$\langle\!\langle \zeta_{\mathbf{k}_1}(\tau_0)\zeta_{\mathbf{k}_2}(\tau_0)\zeta_{\mathbf{k}_3}(\tau_0)\rangle\!\rangle = 2M_{\rm P}^2 {\rm Im}\int_{-\infty}^{\tau_0} {\rm d}\tau \left\{\epsilon_1(\tau)\epsilon_2(\tau)a^2(\tau)\left[\zeta_{k_1}(\tau_0)\zeta_{k_2}(\tau_0)\zeta_{k_3}(\tau_0)\zeta^*_{k_1}(\tau)\zeta^*_{k_2}(\tau)\zeta'^*_{k_3}(\tau) + {\rm perm}\right]\right\}'$$

$$= 2M_{\rm P}^2\epsilon_1(\tau_0)\epsilon_2(\tau_0)a^2(\tau_0)\left\{|\zeta_{k_1}(\tau_0)|^2|\zeta_{k_2}(\tau_0)|^2 {\rm Im}\left[\zeta_{k_3}(\tau_0)\zeta'^*_{k_3}(\tau_0)\right] + {\rm perm}\right\}$$

$$= 2\frac{\epsilon_2}{4}\left\{|\zeta_{k_1}(\tau_0)|^2|\zeta_{k_2}(\tau_0)|^2 + {\rm perm}\right\}$$ (same as field redefinition approach)

Direct approach

However, there is a subtlety at second-order perturbation theory.

$$\langle \mathcal{O}(\tau) \rangle^{(3)} = \langle \mathcal{O}(\tau) \rangle^{\dagger}_{(0,2)} + \langle \mathcal{O}(\tau) \rangle_{(1,1)} + \langle \mathcal{O}(\tau) \rangle_{(0,2)}$$

$$\langle \mathcal{O}(\tau) \rangle_{(1,1)} = \int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau} \mathrm{d}\tau_2 \left\langle H^{(3)}(\tau_1) \hat{\mathcal{O}}(\tau) H^{(3)}(\tau_2) \right\rangle$$

$$\langle \mathcal{O}(\tau) \rangle_{(0,2)} = - \int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau_1} \mathrm{d}\tau_2 \left\langle \hat{\mathcal{O}}(\tau) H^{(3)}(\tau_1) H^{(3)}(\tau_2) \right\rangle$$

$$H^{(3)} = \int \mathrm{d}^3x \left[\mathscr{B}^{(3)}(\zeta, \zeta') \right]'$$

Due to time-ordered integral, even total time derivative Hamiltonian induces correlation evaluated at the bulk.

Warm up

Consider $\mathscr{B}(\mathbf{x},\tau) = c(\tau)\zeta^3(\mathbf{x},\tau)$

$$\langle \mathcal{O}(\tau_0)\rangle^{(3)} = -\int \mathrm{d}^3x\mathrm{d}^3y \int_{\tau_s}^{\tau_0} \mathrm{d}\tau_1 \mathrm{Re}\left\langle \hat{\mathcal{O}}(\tau_0)\left[\mathscr{B}'(\zeta(\mathbf{x},\tau_1)), \mathscr{B}(\zeta(\mathbf{y},\tau_1))\right]\right\rangle$$

$$\langle \mathcal{O}(\tau_0)\rangle^{(3)} = \int \mathrm{d}^3x \int_{\tau_s}^{\tau_0} \mathrm{d}\tau_1 \mathrm{Re}\left\langle \hat{\mathcal{O}}(\tau_0)\frac{9ic^2(\tau)}{2M_\mathrm{P}^2a^2\epsilon_1(\tau)}\zeta^4(\mathbf{x},\tau)\right\rangle$$

Quartic-induced: $\langle \mathcal{O}(\tau_0)\rangle^{(4\mathrm{I})} = 2\int_{\tau_s}^{\tau_0} \mathrm{d}\tau_1 \mathrm{Im}\left\langle \hat{\mathcal{O}}(\tau)H^{(4\mathrm{I})}(\tau_1)\right\rangle = \int \mathrm{d}^3x \int_{\tau_s}^{\tau_0} \mathrm{d}\tau_1 \mathrm{Im}\left\langle \hat{\mathcal{O}}(\tau_0)\frac{9ic^2(\tau)}{2M_\mathrm{P}^2a^2\epsilon_1(\tau)}\zeta^4(\mathbf{x},\tau)\right\rangle$

Total: $\langle \mathcal{O}(\tau_0)\rangle^{(3)} + \langle \mathcal{O}(\tau_0)\rangle^{(4\mathrm{I})} = 0$

Warm up

Contribution of cubic total time derivative interaction at second-order perturbation mimics contribution of bulk quartic interaction at first-order perturbation.

For $\mathscr{B} \propto \zeta^3$, such contribution is cancelled by quartic-induced Hamiltonian.

How about $\mathscr{B} \propto \zeta'\zeta^2$ and $\mathscr{B} \propto (\zeta')^2\zeta$? Will it generate contributions that cannot be captured by field redefinition (at second-order)? To know the answer, we must know the whole quartic interaction of ζ (not ζ_n).

This subtlety comes from time-ordered integral at second-order perturbation. It means that it is not only about one-loop power spectrum $\mathscr{O} = \zeta^2$, but also tree-level trispectrum $\mathscr{O} = \zeta^4$.

Quartic total time derivative interactions are evaluated at first-order perturbation, so their contribution will be evaluated at the end of inflation (and we should not worry about them).

Warm up

Cubic action: $S^{(3)}[\zeta] = M_{\rm P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \left[\frac{1}{2} a^2 \epsilon_1 \epsilon_2' \zeta' \zeta^2 - \left(\frac{1}{2} a^2 \epsilon_1 \epsilon_2 \zeta' \zeta^2 + \frac{a}{H} (\zeta')^2 \zeta \right)' \right]$

To compute bispectrum at intermediate time (USR), field redefinition (or total time derivative terms) become important.

$$\lim_{k_L \to 0} \langle\langle \zeta_{\mathbf{k}_L}(\tau) \zeta_{\mathbf{k}_S}(\tau) \zeta_{-\mathbf{k}_S}(\tau) \rangle\rangle = -(n_s(k_S, \tau) - 1) \, |\, \zeta_{k_S}(\tau) \,|^2 \, |\, \zeta_{k_L}(\tau) \,|^2$$

Claimed in J. Fumagalli (2305.19263, 2408.08296), Y. Tada, T. Terada, and J. Tokuda (2308.04732), R. Kawaguchi, S. Tsujikawa, and Y. Tamada (2407.19742)

$$\langle \zeta_{\mathbf{p}}(\tau_0) \zeta_{-\mathbf{p}}(\tau_0) \rangle^{(3)} \propto \int_{k_{\rm IR}}^{k_{\rm UV}} \mathrm{d}k \frac{\mathrm{d}\Delta^2_{s(0)}(k, \tau_e)}{\mathrm{d}k} \longrightarrow 0$$

Direct approach

Cubic action: $S^{(3)}[\zeta] = S^{(3)}_{\delta\phi}[\zeta] + S^{(3)}_{\mathscr{B}}[\zeta] + S^{(3)}_{\mathrm{EoM}}[\zeta]$

$$S^{(3)}_{\delta\phi}[\zeta] = M_\mathrm{P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \frac{1}{2} a^2 \epsilon_1 \epsilon_2' \zeta' \zeta^2$$

$$S^{(3)}_{\mathscr{B}}[\zeta] = M_\mathrm{P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \left(\frac{1}{2} a^2 \epsilon_1 \epsilon_2 \zeta' \zeta^2 + \frac{a}{H} (\zeta')^2 \zeta \right)'$$

$$S^{(3)}_{\mathrm{EoM}}[\zeta] = M_\mathrm{P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x 2 \left(\frac{1}{4} \epsilon_2 \zeta^2 + \frac{1}{aH} \zeta' \zeta \right) \left[(\epsilon_1 a^2 \zeta')' - \epsilon_1 a^2 \partial^2 \zeta \right]$$

Field redefinition/gauge transformation: $S^{(2)}[\zeta] + S^{(3)}[\zeta] = S^{(2)}[\zeta_n] + S^{(3)}_{\delta\phi}[\zeta_n]$

The differences between cubic action of ζ and ζ_n are total time derivative terms.

Direct approach

Quartic action: $S^{(4)}[\zeta] = S^{(4)}_{\delta\phi}[\zeta] + S^{(4)}_{\rm g}[\zeta]$

$$S^{(4)}_{\delta\phi}[\zeta] = \frac{1}{24}M_{\rm P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \left[\frac{1}{aH}\left(\epsilon_1\epsilon_2' a^2\right)'' - \left(4 + \frac{3}{2}\epsilon_2\right)\left(\epsilon_1\epsilon_2' a^2\right)'\right]\zeta^4$$

$$S^{(4)}_{\rm g}[\zeta] = M_{\rm P}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \left[-\frac{1}{2}\epsilon_1\epsilon_2' a^2(f'\zeta^2 + 2f\zeta'\zeta) + \epsilon_1 a^2 (f')^2 - \epsilon_1 a^2(\partial f)^2\right],\ f = \frac{1}{4}\epsilon_2\zeta^2 + \frac{1}{aH}\zeta'\zeta$$

Field redefinition/gauge transformation: $S^{(2)}[\zeta] + S^{(3)}[\zeta] + +S^{(4)}[\zeta] = S^{(2)}[\zeta_n] + S^{(3)}_{\delta\phi}[\zeta_n] + +S^{(4)}_{\delta\phi}[\zeta_n]$

The differences between cubic action of ζ and ζ_n are (not only) total time derivative terms.

Kristiano and Yokoyama (in preparation)

Direct approach

Cubic Hamiltonian: $H^{(3)} = H^{(3)}_{\delta\phi} + H^{(3)}_{\mathscr{B}} + H^{(3)}_{\rm EoM}\,,\, H^{(3)}_{\mathscr{B}} = B' = \int {\rm d}^3x \mathscr{B}'$

Substitute to second-order perturbation: $\langle \mathscr{O}(\tau)\rangle^{(3)} = \langle \mathscr{O}(\tau)\rangle^{(3)}_{\delta\phi} + \langle \mathscr{O}(\tau)\rangle^{(3)}_{\mathscr{B}}$

Claim: $\langle \mathscr{O}(\tau)\rangle^{(3)}_{\mathscr{B}}$ effectively behaves as quartic contribution

$$\langle \mathscr{O}(\tau_0)\rangle^{(3)}_{\mathscr{B}} = 2{\rm Im}\int_{-\infty}^{\tau_0} {\rm d}\tau \langle \mathscr{O}(\tau_0) H^{(4)}_{\rm ef}(\tau)\rangle,$$

$$H^{(4)}_{\rm ef}(\tau) = -\,i\left[H^{(3)}_{\delta\phi}(\tau) + \frac{1}{2}B'(\tau) + \frac{1}{2}H^{(3)}_{\rm EoM}(\tau), B(\tau)\right]$$

Kristiano and Yokoyama (in preparation)

Direct approach

Expanding $S^{(3)}[\zeta]$ (without doing IBP) yields

$$S^{(3)}[\zeta] = M_{\mathrm{P}}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x \left[-\frac{1}{H}\epsilon_1 a(\zeta')^3 + 3\epsilon_1 a^2(\zeta')^2\zeta - \frac{2}{H}\epsilon_1 a\zeta'\zeta\partial^2\zeta - \frac{1}{2}\epsilon_1\epsilon_2 a^2\zeta^2\partial^2\zeta \right]$$

Quartic-induced Hamiltonian

$$H^{(4\mathrm{I})} = M_{\mathrm{P}}^2 \int \mathrm{d}^3x \left[\frac{9}{4H^2}\epsilon_1(\zeta')^4 - \frac{9}{H}\epsilon_1 a(\zeta')^3\zeta + \frac{3}{H^2}\epsilon_1(\zeta')^2\zeta\partial^2\zeta + 9\epsilon_1 a^2(\zeta')^2\zeta^2 - \frac{6}{H}\epsilon_1 a\zeta'\zeta^2\partial^2\zeta + \frac{1}{H^2}\epsilon_1\zeta^2(\partial^2\zeta)^2 \right]$$

Kristiano and Yokoyama (in preparation)

Direct approach

Total: $\langle \mathcal{O}(\tau_0) \rangle = \langle \mathcal{O}(\tau_0) \rangle_{\delta\phi}^{(3)} + \langle \mathcal{O}(\tau_0) \rangle_{\mathcal{B}}^{(3)} + \langle \mathcal{O}(\tau_0) \rangle^{(4\mathrm{I})} + \langle \mathcal{O}(\tau_0) \rangle_{\delta\phi}^{(4)} + \langle \mathcal{O}(\tau_0) \rangle_{\mathrm{g}}^{(4)}$

$$\langle \mathcal{O}(\tau_0) \rangle = \langle \mathcal{O}(\tau_0) \rangle_{\delta\phi}^{(3)} + 2 \int_{-\infty}^{\tau_0} \mathrm{d}\tau \ \mathrm{Im} \langle \mathcal{O}(\tau_0) \Big[H_{\mathrm{ef}}^{(4)}(\tau) + H^{(4\mathrm{I})}(\tau) + H_{\mathrm{g}}^{(4)}(\tau) + H_{\delta\phi}^{(4)}(\tau) \Big] \rangle$$

Claim (very technical details): $H_{\mathrm{ef}}^{(4)} + H^{(4\mathrm{I})} + H_{\mathrm{g}}^{(4)} = H_{\delta\phi}^{(4\mathrm{I})}$

Therefore $\langle \mathcal{O}(\tau_0) \rangle = \langle \mathcal{O}(\tau_0) \rangle_{\delta\phi}^{(3)} + \langle \mathcal{O}(\tau_0) \rangle_{\delta\phi}^{(4\mathrm{I})} + \langle \mathcal{O}(\tau_0) \rangle_{\delta\phi}^{(4)}$, consistent with field redefinition approach.

Cubic total time derivative interactions do not give contribution evaluated in the bulk, even at second-order perturbation.

No advantage of using direct approach, it just give us extra complication that can be extremely simplified by field redefinition/gauge transformation.

Kristiano and Yokoyama (in preparation)

EFT of inflation

Substituting $\zeta = -H\pi + H\dot{\pi}\pi + \mathcal{O}(\pi^3)$ to $S^{(2)}[\zeta] + S^{(3)}[\zeta] + +S^{(4)}[\zeta]$, we obtain

$$S^{(3)}[\pi] = M_{\mathrm{P}}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x H^3 \epsilon_1 \epsilon_2 a^2 \left[\pi(\pi')^2 - \pi(\partial\pi)^2\right]$$

$$S^{(4)}[\pi] = \frac{1}{2} M_{\mathrm{P}}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x H^4 \epsilon_1 a^2 \left(\epsilon_2^2 + \frac{\epsilon_2'}{aH}\right) \left[(\pi')^2\pi^2 - \pi^2(\partial\pi)^2\right]$$

Reproduces EFT of inflation by Firouzjahi (2303.12025, 2311.04080, 2403.03841)

Kristiano and Yokoyama (in preparation)

Conclusion

- Precision cosmology for inflation model with large fluctuations has just begun!

- We have presented cubic and quartic self-interactions that yields consistent correlation function at second-order perturbation theory: bispectrum, trispectrum, and one-loop power spectrum.

- We have clarified two different approaches on computing correlation function at second-order perturbation: field redefinition/gauge transformation and direct computation from action of ζ. Cubic total time derivative interactions do not give contribution evaluated in the bulk.

- This is important to go further. After we know consistent interactions, we can proceed to compute one-loop power spectrum at small scale to make prediction for the whole spectrum.