The Cosmological CPT Theorem

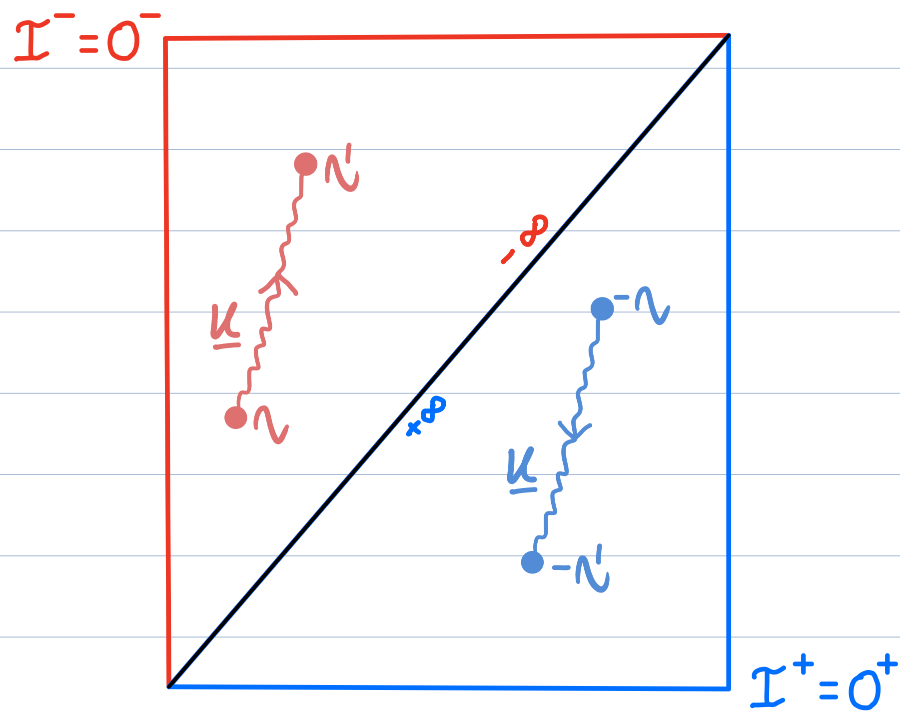
AYNGARAN THAVANESAN (HE/HIM)

H. Goodhew, **AT**, A. Wall arXiv:2408.17406,

AT arXiv:2410.xxxxx,

AT arXiv:2411.xxxxx .

$$\Psi\left[\bar{\phi}(\underline{k});\eta_0\right] = \Psi^*\left[\bar{\phi}^*(\underline{k});-\eta_0\right]$$



UNIVERSITY OF CAMBRIDGE

IOP Institute of Physics

CERN Oct 2024

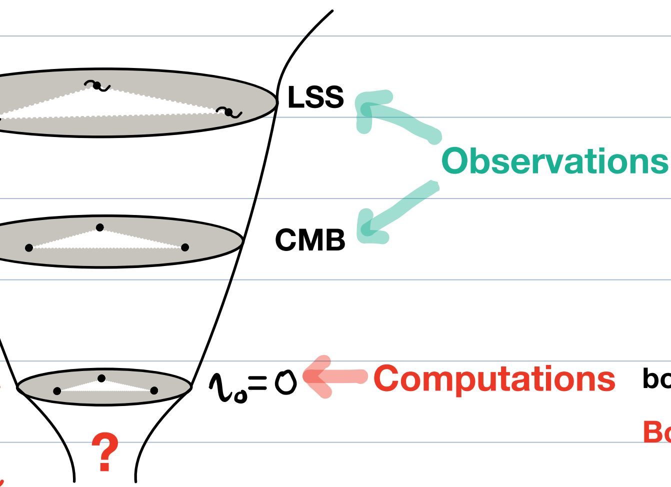
Where did we come from?

- A major goal for us as cosmologists is to understand where we came from.
- To answer this question we search for patterns (cosmological correlators) in our universe.
- My job as a theorist is to explain and guide the surveys carried out by my fellow observational cosmologists.
- So far we have made significant progress using continuous symmetries to constrain correlators.

$$\langle \bar{\phi}(\underline{x}_1) \dots \bar{\phi}(\underline{x}_n) \rangle$$

$$\longrightarrow \langle \bar{\phi}(\underline{k}_1) \dots \bar{\phi}(\underline{k}_n) \rangle$$

n-pt correlator of massless
spin-0 and spin-2 fields



boundary at end of inflation

Inflation
(quasi-de Sitter expansion)

Boundary $\Rightarrow$ holography?
dS/CFT?

- We have predicted and measured $\underbrace{\langle \zeta\zeta \rangle}^{\text{CMB}}$ and predict $\langle \gamma\gamma \rangle, \langle \zeta\zeta\gamma \rangle, \langle \zeta\gamma\gamma \rangle, \langle \gamma\gamma\gamma \rangle, \dots$

which we aim to measure soon. **BUT** why does our universe expand, why is there only matter?

- In flat space there is a fundamental **discrete** symmetry known as **CPT** which the expansion of our universe and lack of anti-matter naïvely seems to break. **BUT** in this talk I will explain how **CPT** arises in cosmology.

Plan of talk

- Look at the original **CPT** theorem and understand what is required for **CPT**.
- Use the embedding space formalism to identify an analogous **CPT** theorem in cosmology.
- Derive a novel way to think about the **CPT** theorem to make converse statements.
- Explore constraints of **CPT** on cosmological correlators and the cosmological wavefunction.
- Demonstrate that **CPT invariance** strongly constrains parity violation in cosmology.
- Discuss implications of **COSMOLOGICAL CPT THEOREM** for early universe and quantum gravity.

KEY WORDS: Wavefunction of the Universe (WFU), cosmological correlators, CPT, parity violation.

Conventions

To be consistent with literature and (hopefully) avoid confusion:

- **CRT** involves flipping **ONE** spatial direction (+ rotations => **CPT** in even D).

- **CPT** involves flipping **ALL** spatial directions.

- Spacetime dimensions D=d+1 and d spatial dimensions.

- D=d+1 Lorentz invariant theory is invariant under $SO^{+}(1,d)$ (also known as the **proper orthochronous Lorentz group**, i.e. the transformations which preserve the orientation of time and space).

- D=d+1 de Sitter (dS) invariant theory is invariant under $SO^{+}(1,d+1)$.

- Working in **REAL** basis of fields, i.e. complex fields are broken up into real and imaginary parts;

to go from real to complex fields put an adjoint (dagger $\dagger$) on fields when there is a complex conjugation $*$.

Original CPT Theorem in Flat space

- **LORENTZ INVARIANCE + UNITARITY => CRT .**

- **NO CONVERSES** (so far…) **CRT + UNITARITY** $\not\Rightarrow$ **LORENTZ INVARIANCE .**

- Lorentz boosts $x^\mu = \begin{pmatrix} t' \\ x_1' \\ \vdots \\ x_d' \end{pmatrix} = \Lambda^\mu{}_\nu x^\nu = \begin{pmatrix} \cosh(\xi) & & & \sinh(\xi) \\ & 1 & & \\ & & \ddots & \\ & & & 1 & \\ \sinh(\xi) & & & \cosh(\xi) \end{pmatrix} \begin{pmatrix} t \\ x_1 \\ \vdots \\ x_d \end{pmatrix}$

$\xi \longrightarrow i\theta$
$t \longrightarrow -i\tau$
$t' \longrightarrow -i\tau'$

$$x^\mu = \begin{pmatrix} \tau' \\ x_1' \\ \vdots \\ x_d' \end{pmatrix} = \Lambda^\mu{}_\nu x^\nu = \begin{pmatrix} \cos(\theta) & & & -\sin(\theta) \\ & 1 & & \\ & & \ddots & \\ & & & 1 & \\ \sin(\theta) & & & \cos(\theta) \end{pmatrix} \begin{pmatrix} \tau \\ x_1 \\ \vdots \\ x_d \end{pmatrix} .$$

- Can Wick rotate quantum theory to Euclidean theory if:

1. The theory is Lorentz invariant.
2. The vacuum is Lorentz invariant.
3. The energy is bounded below.

Discrete Symmetries in Flat space and Cosmology

- **LORENTZ INVARIANCE** $SO^+(1,d)$ Wick rotates to $SO(d+1)$ => $SO(2)$ => $\theta = \pi$ rotation.

- $\theta = \pi$ rotation + **UNITARITY** => **CRT** in Lorentzian signature (+ rotations => **CPT** in even D).

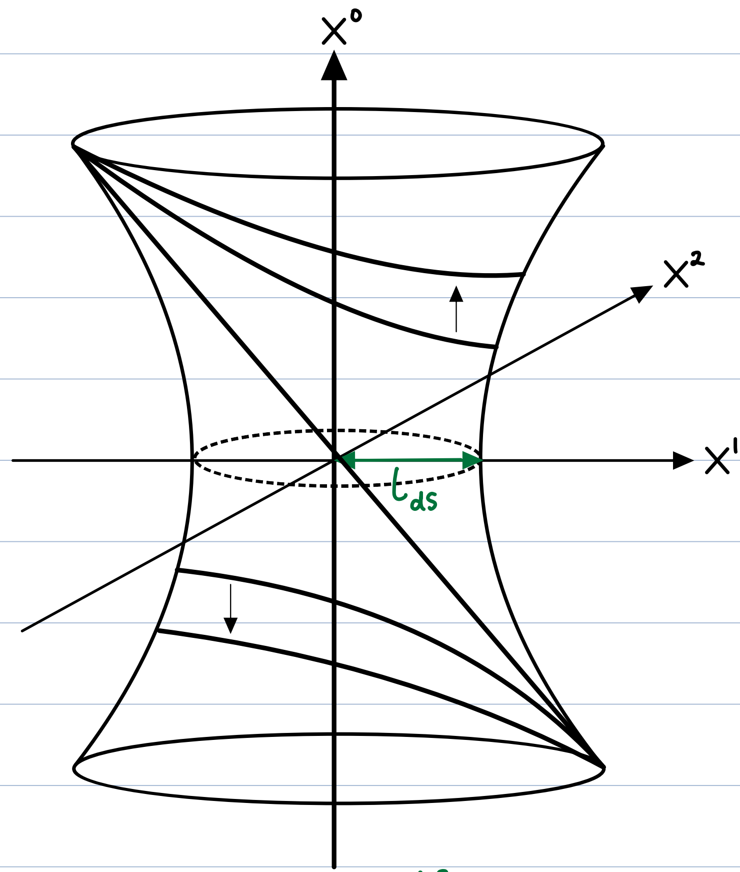
- **<u>ONLY</u>** need $SO(2)$ which comes from $SO^+(1,1)$ subgroup of $SO^+(1,d)$.

- **<u>BUT</u>** a D=d+1 dS invariant theory is invariant under $SO^+(1,d+1)$.

- $\therefore$ there must be an analogous **CPT** theorem in dS !

- **<u>BUT</u>** what is $\theta = \pi$ rotation and $SO^+(1,1)$ boost in cosmology?

Embedding (Ambient) space formalism



dS_2 in $\mathbb{R}^{1,2}$

- dS can be viewed as a timelike hyperbola embedded in D+1-dimensional Minkowski $\mathbb{R}^{1,D}$

$$X^A X_A = -(X^0)^2 + (X^i)^2 + (X^D)^2 = \ell_{dS}^2 = \frac{1}{H^2} .$$

- dS Poincaré patch co-ordinates and the embedding space co-ordinates are related as

$$X^0 = \frac{\eta^2 - x^2 - 1}{2H\eta} , \quad X^i = -\frac{x^i}{H\eta} , \quad X^D = \frac{x^2 - \eta^2 - 1}{2H\eta} ;$$

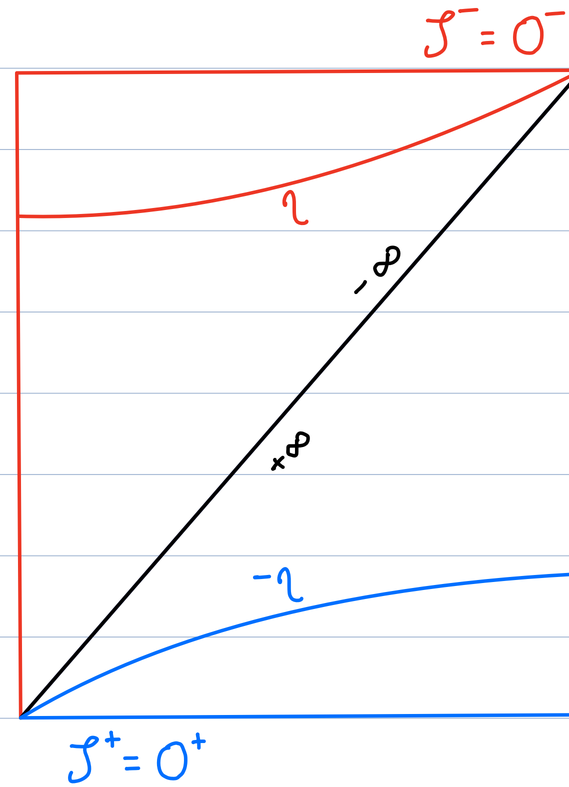
$$\eta = -\frac{1}{H(X^0 + X^D)} , \quad x^i = \frac{X^i}{X^0 + X^D} .$$

where the metric of the Poincaré patch is $ds^2 = \left(\frac{-1}{H\eta}\right)^2 (-d\eta^2 + dx^2) , \quad \eta \in [-\infty, 0]$.

- The generator of $\boldsymbol{SO^+(1,1)}$ is given by $L_{0D} = (X_0 \partial_D - X_D \partial_0) = (\eta \partial_\eta + x^i \partial_i) \equiv D$.

CPT in Cosmology

- ∴ in cosmology we analytically continue dilatations to get the 180° rotation, which takes us from one Poincaré patch to the other.



- **DILATATIONS** play the role of Lorentz boosts in the CPT theorem for cosmology, i.e. we can break dS boosts (SCT) and preserve CPT !

- Phenomenologically relevant models of inflation break dS boosts !

- ∴ in FLAT SPACE: LORENTZ INVARIANCE + UNITARITY => CRT

  and in COSMOLOGY: SCALE INVARIANCE + UNITARITY => CPT .

- In cosmology we flip all co-ordinates regardless of dimension D; i.e. **CRT** is actually **CPT** !

Rethinking the CPT Theorem for Flat Space

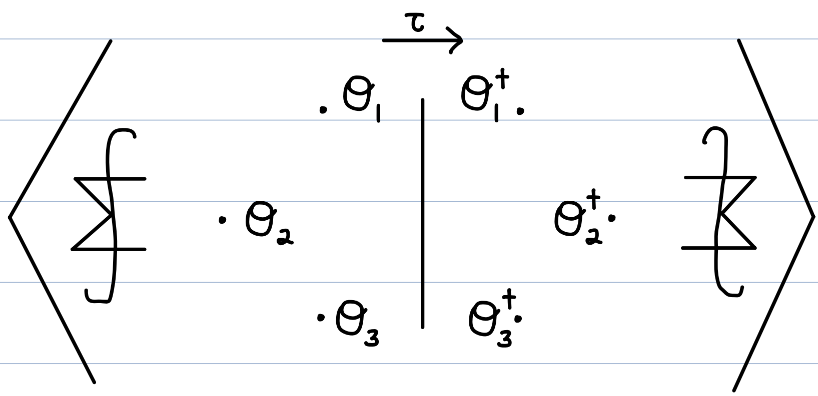
- In Euclidean signature **UNITARITY => REFLECTION POSITIVITY** written as

$$\xrightarrow{\tau}$$

$$\left\langle \int\!\!\!\!\!\!\sum \;\; \cdot\,\mathcal{O}_1 \;\; \cdot\,\mathcal{O}_2 \;\; \cdot\,\mathcal{O}_3 \;\;\Bigg|\;\; \mathcal{O}_1^\dagger\cdot \;\; \mathcal{O}_2^\dagger\cdot \;\; \mathcal{O}_3^\dagger\cdot \;\; \int\!\!\!\!\!\!\sum \right\rangle \in \mathbb{R} > 0 .$$

Rethinking the CPT Theorem for Flat Space

- In Euclidean signature **UNITARITY => REFLECTION POSITIVITY** written as



$\in \mathbb{R} > 0.$

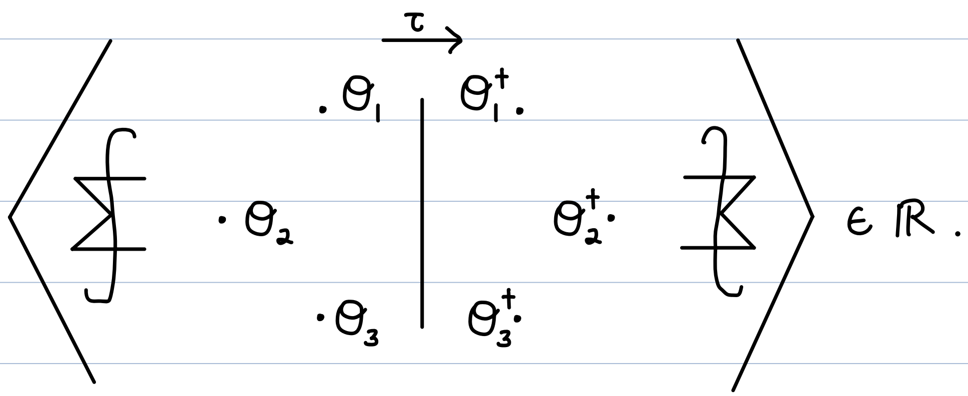
$$\langle \Theta_1(-\tau_1)\, \Theta_2(-\tau_2)\, \Theta_2(\tau_2)\, \Theta_1(\tau_1) \rangle \in \mathbb{R} > 0.$$

$$\left\langle \cdot\, \Theta_1(-\tau_1) \; + \begin{array}{c} \cdot\, \Theta_2(-\tau_2) \\ \\ \cdot\, \Theta_3(-\tau_3) \end{array} \;\Bigg|\; \begin{array}{c} \cdot\, \Theta_2(\tau_2) \\ \\ \cdot\, \Theta_3(\tau_3) \end{array} + \cdot\, \Theta_1(\tau_1) \right\rangle \in \mathbb{R} > 0$$

$$= \Big(\langle \Theta_1(-\tau_1)\, \Theta_2(\tau_2)\, \Theta_3(\tau_3) \rangle + \langle \Theta_1(-\tau_1)\, \Theta_1(\tau_1) \rangle + \langle \Theta_2(-\tau_2)\, \Theta_3(-\tau_3)\, \Theta_2(\tau_2)\, \Theta_3(\tau_3) \rangle$$
$$+ \langle \Theta_2(-\tau_2)\, \Theta_3(-\tau_3)\, \Theta_1(\tau_1) \rangle \Big) \in \mathbb{R} > 0.$$

Rethinking the CPT theorem for Flat space (cont.)

- Can weaken reflection positivity to **REFLECTION REALITY (RR)**



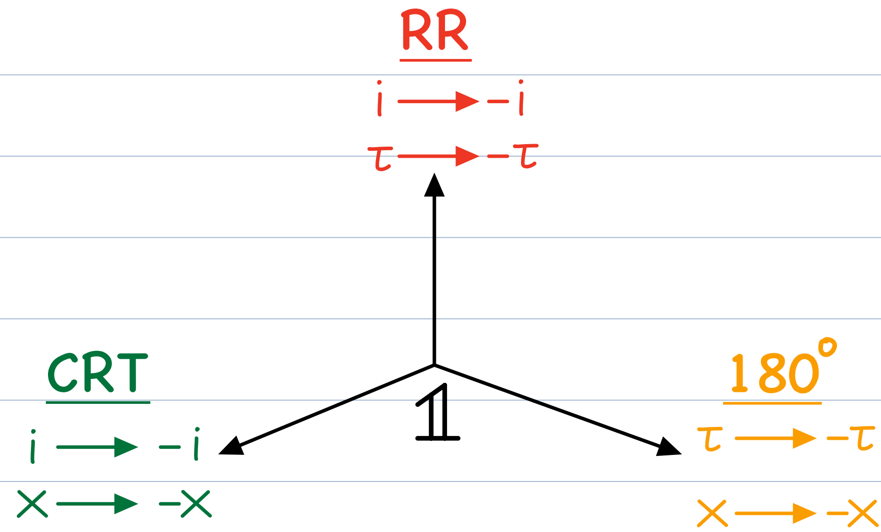
- All n-pt correlators in a reflection real theory are invariant under a Z2 symmetry which we will call

  REFLECTION REALITY (RR) $\quad i \longrightarrow -i$

  $\tau \longrightarrow -\tau$.

- Can use **RR** to think of the **CPT** theorem in terms of a Z2 x Z2 group structure.

Rethinking the CPT theorem for Flat space (cont.)



∴ in **FLAT SPACE**, we have original CPT Theorem **LORENTZ INVARIANCE + UNITARITY => CRT**

AS WELL AS ***$SO^+(1,1)$*** **+ REFLECTION REALITY => CRT**

and **CRT INVARIANCE +** ***$SO^+(1,1)$*** **=> REFLECTION REALITY**

and **CRT INVARIANCE + REFLECTION REALITY => DISCRETE 180° ROTATION .**

Flat space correlators

Symmetry	Euclidean	Implied by	Lorentzian signature
RR	$i \rightarrow -i$ $\tau \rightarrow -\tau$	UNITARITY	$\langle \bar{T}\, \phi_1^\dagger(t_1,\underline{x}_1)\ldots\phi_n^\dagger(t_n,\underline{x}_n)\rangle^*$
180°	$\tau \rightarrow -\tau$ $x \rightarrow -x$	LORENTZ INVARIANCE	$\langle T\, \phi_1(t_1,x_1)\ldots\phi_n(t_n,x_n)\rangle$ $x\rightarrow -x$, $t \rightarrow -t$ (no $*$)
CRT	$i \rightarrow -i$ $x \rightarrow -x$	—	$\langle \bar{T}\, \phi_1^\dagger(t_1,x_1)\ldots\phi_n^\dagger(t_n,x_n)\rangle^*$ $x\rightarrow -x$, $t \rightarrow -t$

- We can construct a table for cosmological correlators where **DILATATIONS** provide the 180° rotation instead of **LORENTZ INVARIANCE**.
- **ALL** co-ordinates are flipped by **DILATATIONS** **NOT** just one co-ordinate so there is only **CPT**.

Cosmological Wavefunction

QFT in curved spacetime path integral

$$\Psi\left[\bar{\phi}(\underline{k}_a); \eta_0\right] = \int_{\phi(-\infty)=\Omega_{BD}}^{\phi(\eta_0)=\bar{\phi}} D\phi \; e^{iS[\phi]} = \exp\left\{-\sum_{n=2}^{\infty} \frac{1}{n!} \int_{\underline{k}_1,\dots,\underline{k}_n} \psi_n(\omega_a, \underline{k}_a; \eta_0)\, \bar{\phi}_{\underline{k}_1} \cdots \bar{\phi}_{\underline{k}_n}\right\}$$

any spinning field

quadratic/free part

$$\frac{\partial}{\partial \eta}\left(\frac{\delta L_2}{\delta \phi'} + \frac{\delta L_{int}}{\delta \phi'}\right) - \left(\frac{\delta L_2}{\delta \phi} + \frac{\delta L_{int}}{\delta \phi}\right) = 0$$

interacting part

"energies"

where $\omega_a = |\underline{k}_a|$ $\qquad \psi_n = \psi_n'(\underline{k}_1,\dots,\underline{k}_n)(2\pi)^d \delta^{(d)}(\sum \underline{k}_a)$

for massless fields
in d=3 (our universe) $\qquad \psi_n' \propto |\underline{k}|^3 = \omega^3$ $\qquad$ by dimensional analysis
and/or scale invariance

- The wavefunction gives us a probability distribution for the state of the universe as we observe in the sky.
- We then use this probability distribution to compute cosmological correlators.
- I am now going to explain some powerful constraints on the wavefunction which come from **CPT**.
- From these constraints on the wavefunction I will be able to make robust predictions about correlators.

Cosmological Wavefunction (cont.)

for wavefunction need **LOCAL** and global discrete symmetries!

$$\Psi\left[\bar\phi(\underline{k}_a);\eta_0\right] = \langle \bar\phi;\eta_0|\Omega\rangle = \int_{\phi(-\infty(1-i\epsilon))=\Omega_{BD}}^{\phi(\eta_0)=\bar\phi} D\phi\; e^{iS[\phi]}$$

$$= \exp\left\{-\sum_{n=2}^{\infty}\frac{1}{n!}\int_{\underline{k}_1,\dots,\underline{k}_n}\psi_n(\omega_a,\underline{k}_a;\eta_0)\,\bar\phi_{\underline{k}_1}\cdots\bar\phi_{\underline{k}_n}\right\}$$

working in Fourier space

- **DILATATIONS**: $\omega_a \longrightarrow \lambda\,\omega_a,\ \underline{k}_a \longrightarrow \lambda \underline{k}_a,\ \eta \longrightarrow \lambda^{-1}\eta$ for $\lambda \in \mathbb{R}^+$

analytically continues to **180°** $\omega_a \longrightarrow e^{-i\pi}\omega_a,\ \underline{k}_a \longrightarrow e^{-i\pi}\underline{k}_a,\ \eta \longrightarrow e^{i\pi}\eta$ for $\lambda = e^{-i\pi}$.

Symmetry	Implied by	Lorentzian signature
REFLECTION REALITY	UNITARITY	$\psi_n^{(L)*} = e^{i\pi((d+1)L-1)}\,\psi_n^{(L)}(e^{-i\pi}\omega_a, e^{-i\pi}\underline{k}_a;\eta_0)$
180°	DILATATIONS	$\psi_n^{(L)} = e^{\pm i\pi d}\,\psi_n(e^{\mp i\pi}\omega_a, e^{\mp i\pi}\underline{k}_a; e^{\pm i\pi}\eta_0)$ (no $*$)
CPT	—	$\psi_n^{(L)*} = e^{i\pi(d+1)(L-1)}\,\psi_n(\omega_a,\underline{k}_a;e^{-i\pi}\eta_0)$

Cosmological CPT Theorem and No-go theorem

- We have derived a CPT theorem in cosmology **SCALE INVARIANCE + UNITARITY => CPT**

as well as **SCALE INVARIANCE + REFLECTION REALITY => CRT**

and **CPT INVARIANCE + SCALE INVARIANCE => REFLECTION REALITY**

and **CPT INVARIANCE + REFLECTION REALITY => DISCRETE SCALE INVARIANCE .**

- Using the constraints the **Cosmological CPT Theorem** we can completely determine the phase of the wavefunction (WFU) coefficients $e^{i\arg(\Psi_n^{(L)})}$ at the end of inflation $\mathcal{I}^-$, where $\eta = 0^-$.

spatial dimensions
(holds for non-integer d!)

number of loops

mass of field

$$e^{i\arg(\Psi_n^{(L)})} = \pm(-i)^{\left[(d+1)(L-1) + d\,n - \sum_\alpha \left(\frac{d}{2} + \sqrt{\frac{d^2}{4} - \frac{m_\alpha^2}{H^2}}\right)\right]} \qquad \equiv \Delta_\alpha = \frac{d}{2} + \nu$$

n-pt correlator

Hubble parameter

"conformal dimension"

- For **massless** scalar fields and **gravitons** $$e^{i\arg(\Psi_n^{(L)})} = \pm(-i)^{[(d+1)(L-1)]}$$

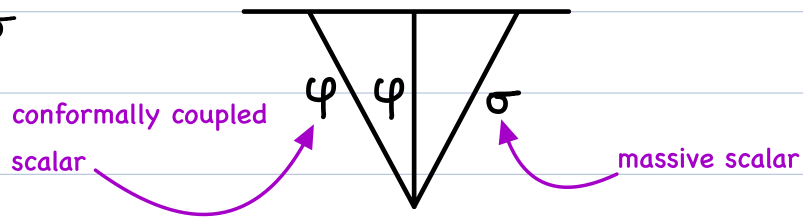
i.e. WFU coefficient is real in d=3 spatial dimensions => **NO BOUNDARY PARITY ODD CORRELATORS !**

(for non-UV finite loop diagrams, parity-odd correlator arises from expansion in "fractional dimensions".)

Examples

d=3, D=d+1

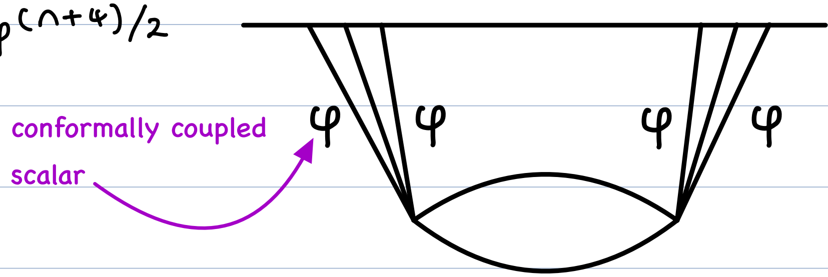
$\Psi^{(L=0,\text{tree})}_{3,\varphi\varphi\sigma}$



$$\arg\left(\Psi^{(L=0,\text{tree})}_{3,\varphi\varphi\sigma}\right) = \frac{\pi}{2}\left(\nu + \frac{1}{2}\right)$$

d=3+δ, D=d+1

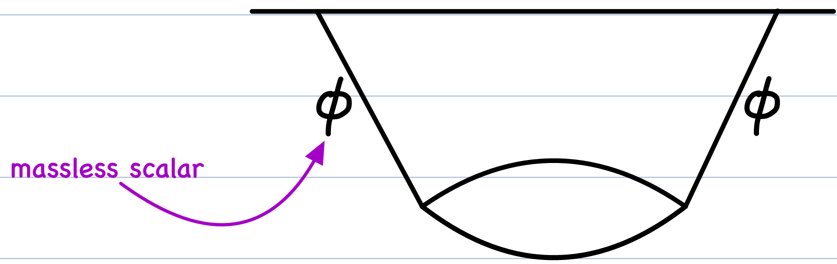
$\Psi^{(L=1)}_{n,\varphi^{(n+4)/2}}$



$$\arg\left(\Psi^{(L=1)}_{n,\varphi^{(n+4)/2}}\right) = -\frac{n\pi}{4}\left(2+\delta\right)$$

d=3+δ, D=d+1

$\Psi^{(L=1)}_{2,\dot{\phi}^3}$



$$\arg\left(\Psi^{(L=1)}_{2,\dot{\phi}^3}\right) = -\frac{\pi\delta}{2}$$

Conclusions

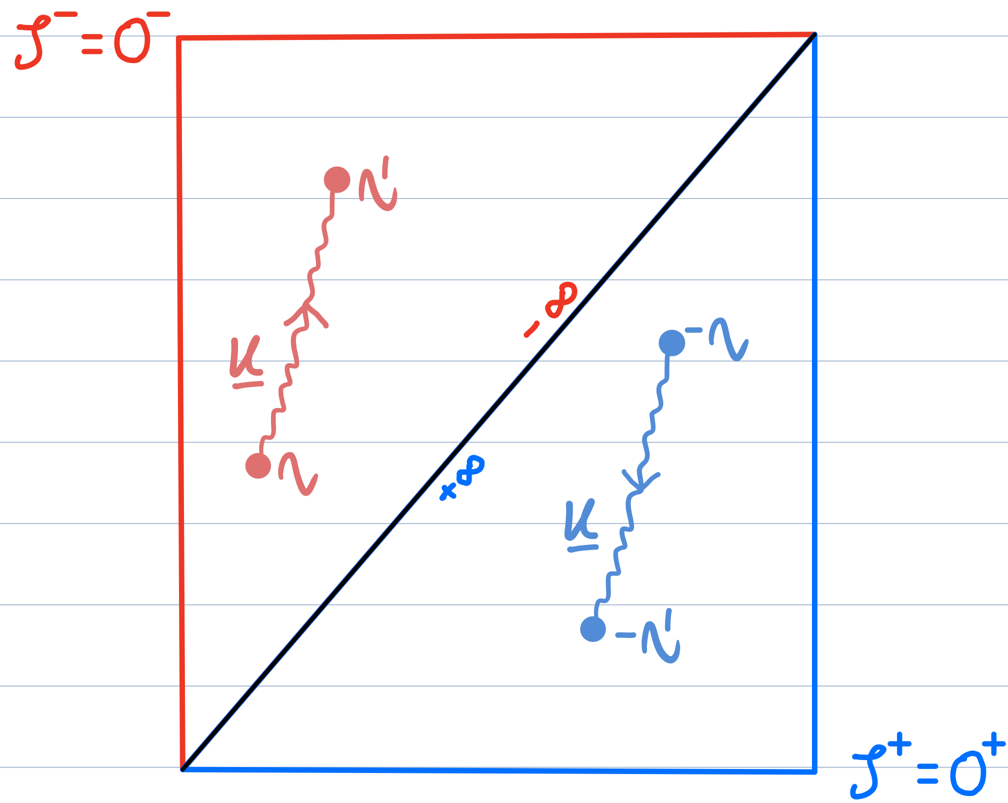
- Showed that an analogous CPT theorem exists in **cosmology**.
- Derived novel way to think about the CPT theorem to make converse statements.
- Showed non-perturbative constraints of **CPT** on the full cosmological wavefunction and the wavefunction coefficients, and expressed them in perturbation theory.
- Used constraints to completely determine the phase of the wavefunction coefficients for any theory and in turn derive a no-go theorem for parity violation.

Future/Outloook

- In progress work:
  - CPT and Reflection Reality give us the constraints required for a bootstrap approach to find dS/CFT.
  - We understand the analytic properties of loops => cosmological generalised unitarity.
  - Phase formula for flat-space amplitudes from flat space limit of Feynman-Witten diagrams.
- Fermions in cosmology. 180° rotation enhanced to Z4 symmetry for spinors.
- We hope the evolution of our universe is unitary **BUT** it slightly breaks scale invariance and thus slightly breaks **CPT** invariance. What does it mean for the universe to slightly break **CPT**?
- Implications for matter-antimatter asymmetry, the arrow of time, …? Is **CPT** a symmetry of quantum gravity?

Thank you for listening!

$$\Psi\left[\bar{\phi}(\underline{k}); \eta_0\right] = \Psi^*\left[\bar{\phi}^*(\underline{k}); -\eta_0\right]$$



References

For interested watchers/readers here is a (**non-comprehensive**) list of references to read for more details regarding the topics discussed in my talk:

1. H. Goodhew, **A. Thavanesan,** A. Wall ; The Cosmological CPT Theorem ; arXiv:2408.17406.
2. **A. Thavanesan** ; No-go theorem for cosmological parity violation ; arXiv:2410.xxxxx.
3. **A. Thavanesan** ; Going through phases of UV and IR divergences ; arXiv:2411.xxxxx.
4. H. Goodhew, S. Jazayeri, E. Pajer ; The Cosmological Optical Theorem ; arXiv:2009.02898.
5. S. Céspedes, A.-C. Davis, S. Melville ; On the time evolution of cosmological correlators ; arXiv:2009.07874.
6. R.N. Cahn, Z. Slepian, J. Hou ; Test for Cosmological Parity Violation Using the 3D Distribution of Galaxies ; arXiv:2110.12004.
7. O.H.E. Philcox ; Probing parity violation with the four-point correlation function of BOSS galaxies ; arXiv:2206.04227.
8. J. Hou, Z. Slepian, R.N. Cahn ; Measurement of parity-odd modes in the large-scale 4-point correlation function of Sloan Digital Sky Survey Baryon Oscillation Spectroscopic Survey twelfth data release CMASS and LOWZ galaxies ; arXiv:2206.03625.

9. O.H.E. Philcox ; Do the CMB Temperature Fluctuations Conserve Parity? ; arXiv:2303.12106.
10. D. Stefanyszyn, X. Tong, Y. Zhu ; Cosmological Correlators Through the Looking Glass: Reality, Parity, and Factorisation ; arXiv:2309.07769.
11. D. Stefanyszyn, X. Tong, Y. Zhu ; There and Back Again: Mapping and Factorising Cosmological Observables ; arXiv:2406.00099.
12. G. Cabass, S. Jazayeri, E. Pajer, D. Stefanyszyn ; Parity violation in the scalar trispectrum: no-go theorems and yes-go examples ; arXiv:2210.02907.
13. M.H.G. Lee, C. McCulloch, E. Pajer ; Leading loops in cosmological correlators ; arXiv:2305.11228.
14. J.-O. Gong, M. Mylova, M. Sasaki ; New shape of parity-violating graviton non-Gaussianity ; arXiv:2303.05178.
15. G. Cabass, E. Pajer, D. Stefanyszyn, J. Supel ; Bootstrapping large graviton non-Gaussianities ; arXiv:2109.10189.
16. G. Cabass, D. Stefanyszyn, J. Supel, **A. Thavanesan** ; On graviton non-Gaussianities in the Effective Field Theory of Inflation ; arXiv:2209.00677