2024.10.29 @ Looping in the Primordial Universe, CERN

Cancellation of Quantum Corrections on the Soft Curvature Perturbations

Yuichiro TADA (Nagoya U.)

w/ Motohashi 2303.16035
Terada & Tokuda 2308.04732

ζ-conservation @ tree

$\Updownarrow$

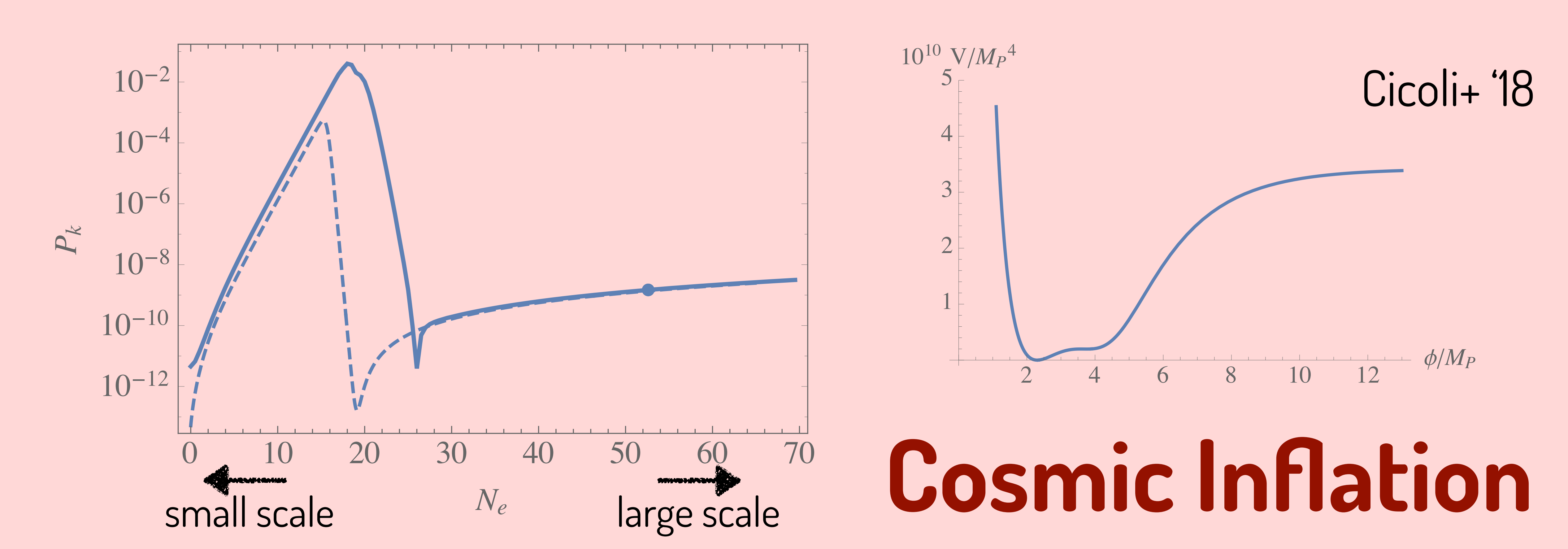
Maldacena's Consistency Relation

$\Updownarrow$

Cancellation of One-loop

Primordial BHs

Carr & Hawking '74



Primordial BHs

Dark Matter (Chapline '75)

LVK merger GW? (Sasaki+ '16)

SMBH seeds? (Düchting '04)

OGLE lensing obj.? (Niikura+ '19)

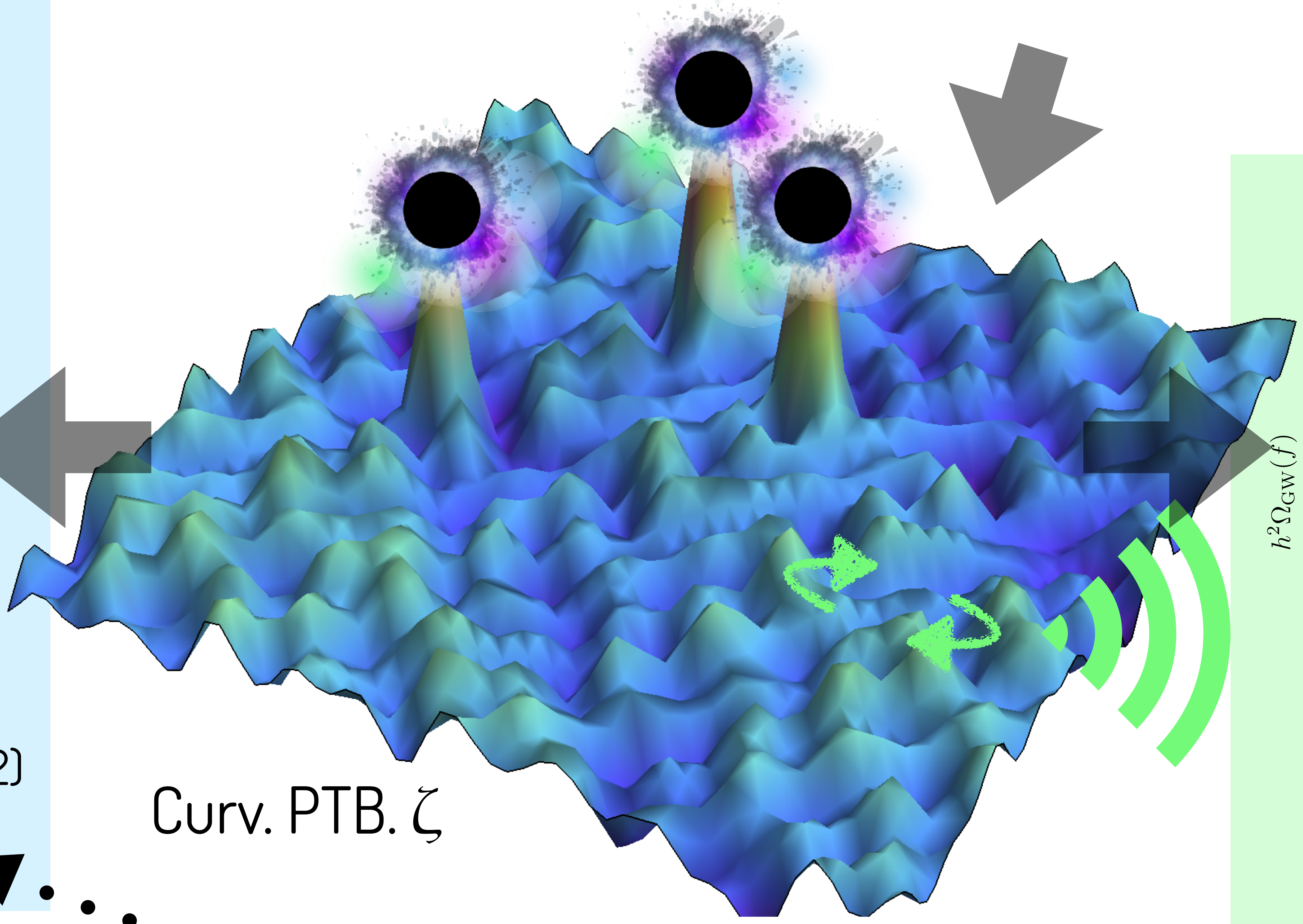
Planet 9? (Scholtz & Unwin '19)

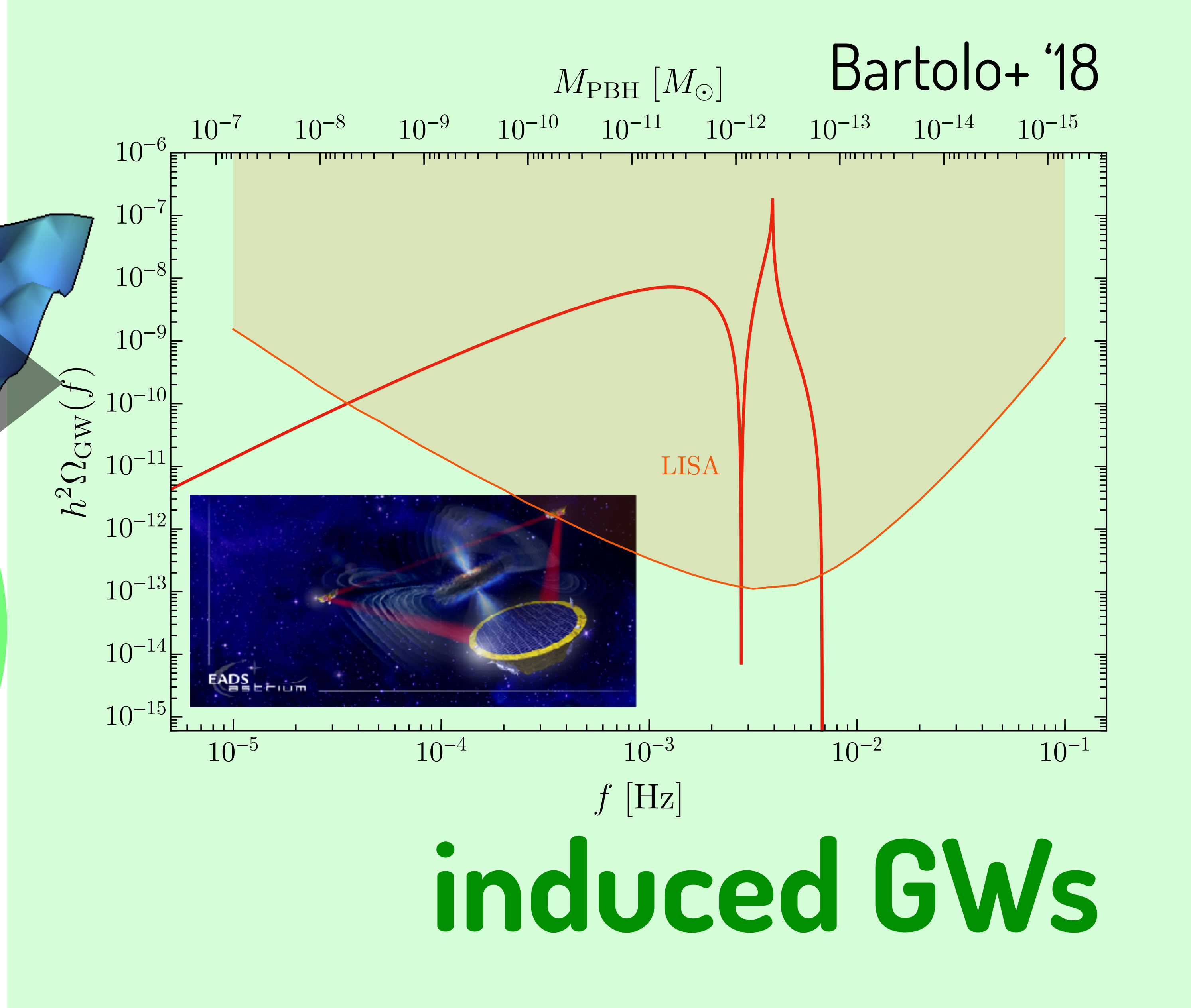
Trigger of r-process? (Fuller+ '17)

Baryogenesis? (Baumann+ '07)

JWST luminous gals? (Hutsi+ '22)

⋮





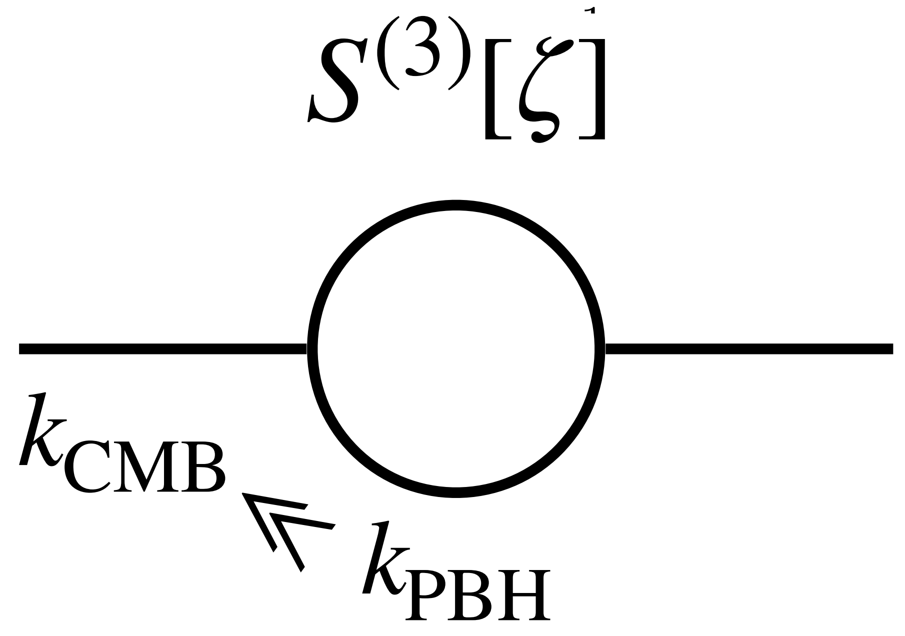
indirect evidence

PBH ruled out?

2211.03395

Ruling Out Primordial Black Hole Formation From Single-Field Inflation

Jason Kristiano[1, 2, *] and Jun'ichi Yokoyama[1, 2, 3, 4, †]



$$\mathscr{P}_\zeta^{\rm tree}(k_{\rm CMB}) \sim \Delta\mathscr{P}_\zeta^{1-\rm loop}(k_{\rm CMB})$$

PBH ruled out?

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Ruling Out Primordial Black Hole Formation From Single-Field Inflation

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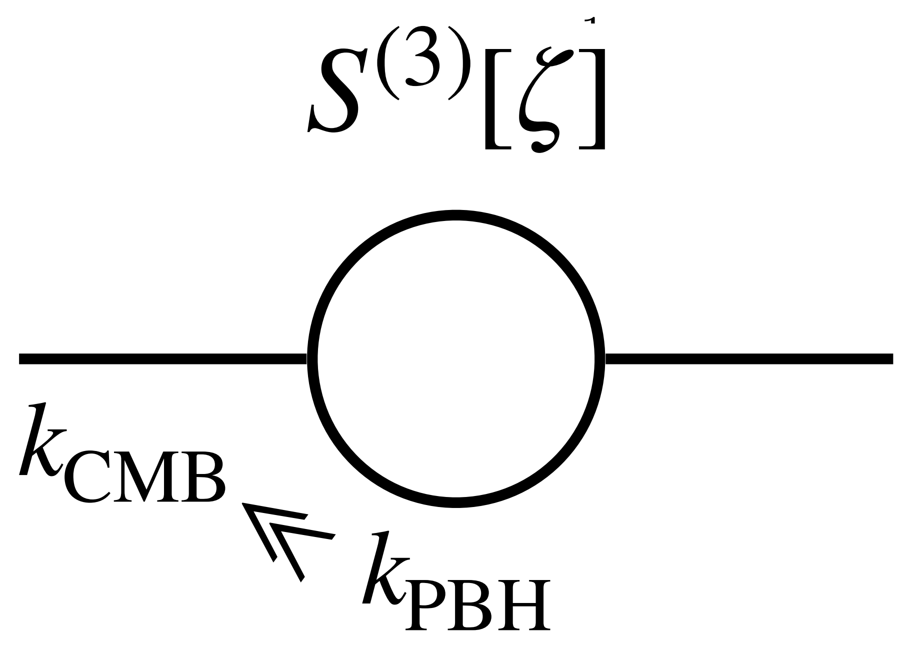
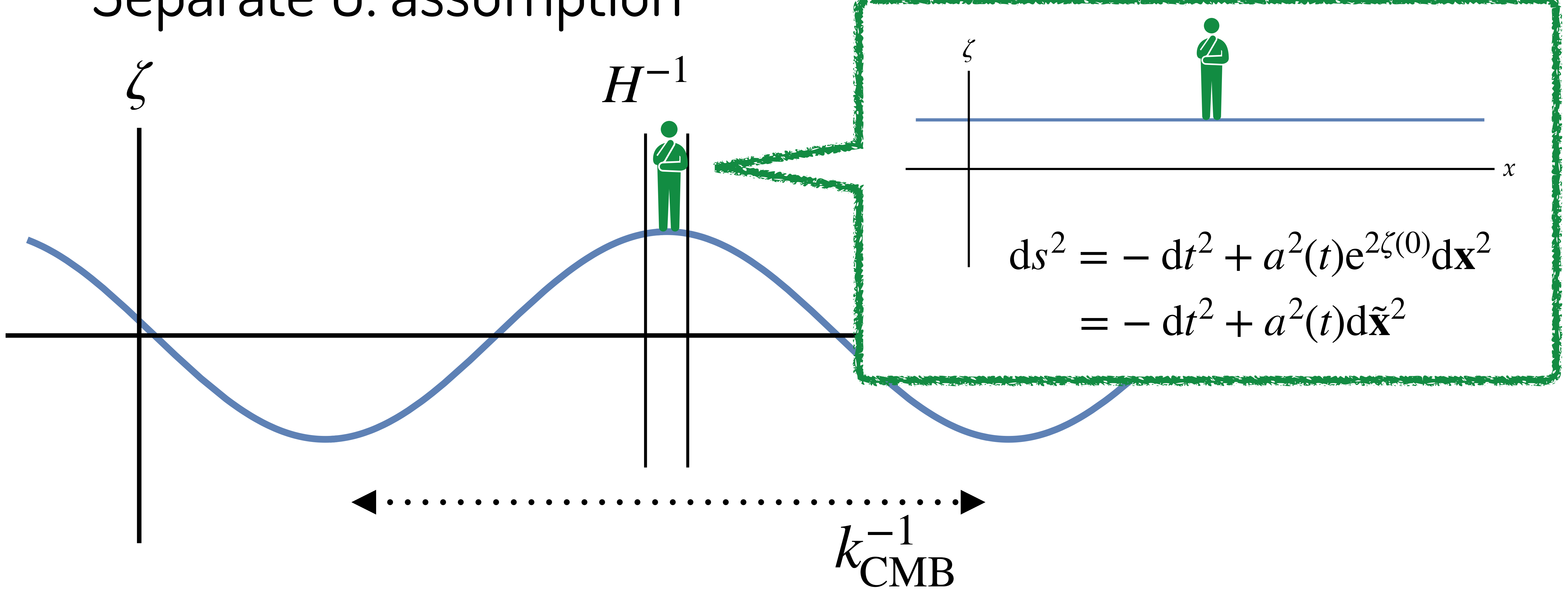


$$\mathscr{P}_\zeta^{\rm tree}(k_{\rm CMB}) \sim \Delta\mathscr{P}_\zeta^{1-{\rm loop}}(k_{\rm CMB})$$

Separate U. assumption



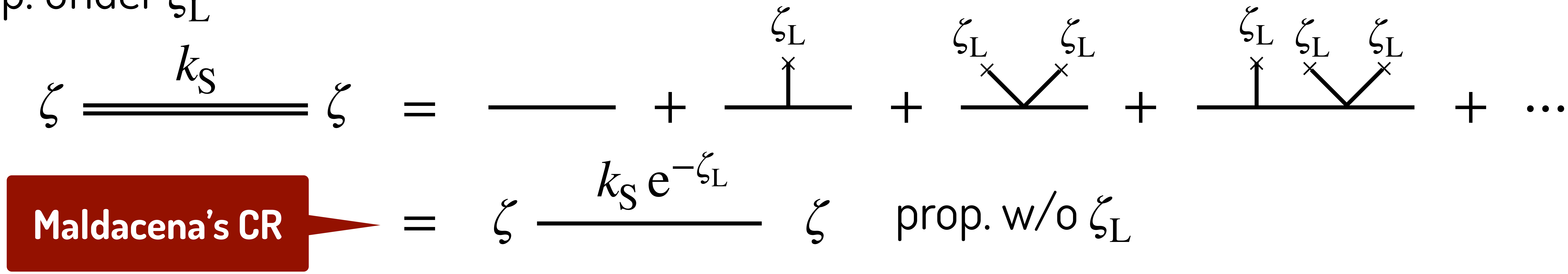
- ζ as NG boson of asymp. dilatation (e.g. Assassi, Baumann, Green '12)
- (classically) soft ζ is conserved (Lyth, Malik, Sasaki '05)
- Maldacena's consistency relation ('03)

$$S^{(3)}[\zeta] \;\Rightarrow\; \langle \zeta_{k_{\rm L}}\zeta_{k_{\rm S}}\zeta_{k_{\rm S}}\rangle \propto \mathscr{P}_\zeta(k_{\rm L})\frac{\mathrm{d}\mathscr{P}_\zeta(k_{\rm S})}{\mathrm{d}\ln k_{\rm S}}$$

merely scale-redefinition

Soft th. on loops

prop. under ζ_{L}



Soft th. on loops

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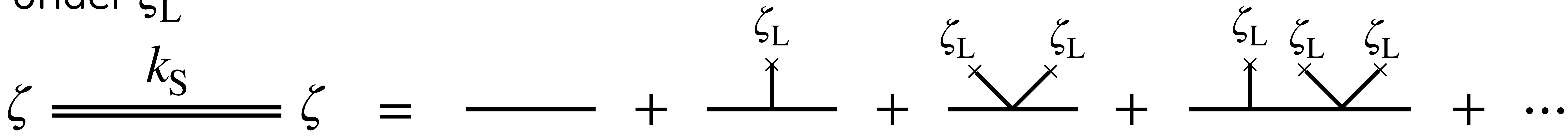
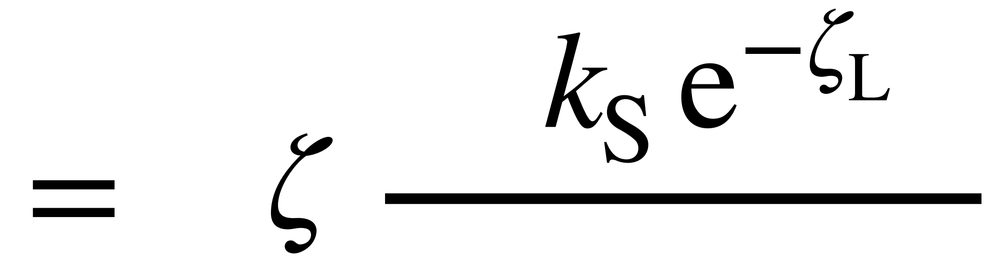


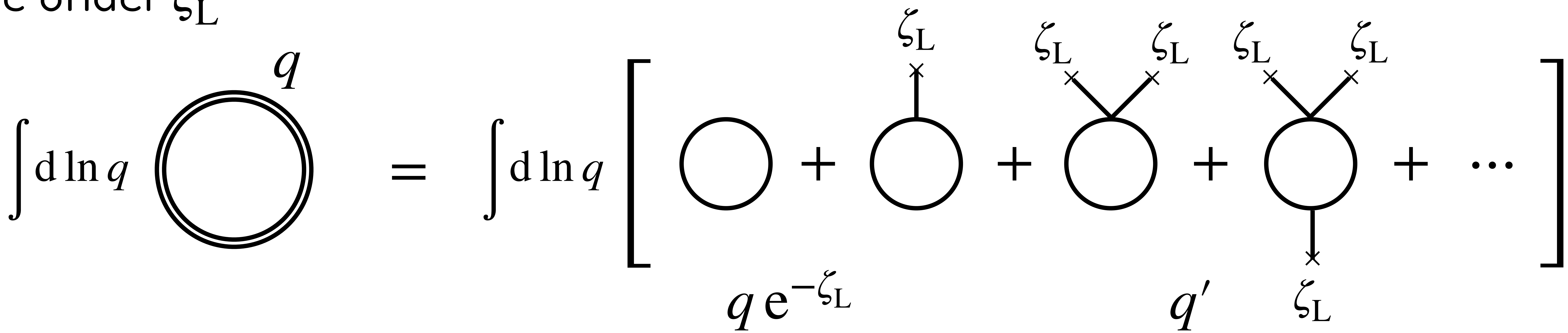
Maldacena's CR



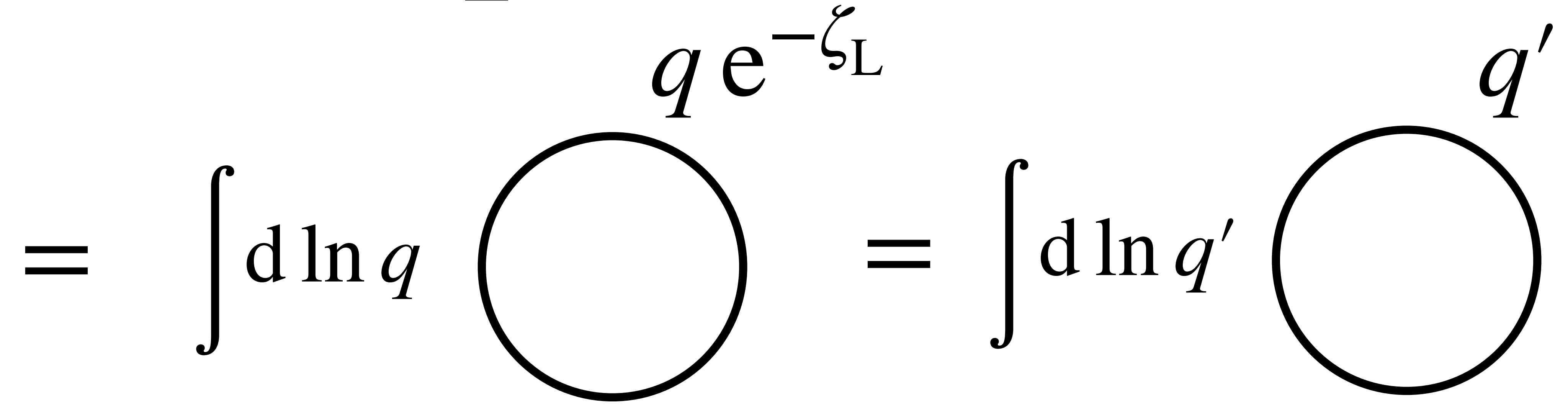
prop. w/o ζ_L

all 1-loop corrections on $n$pt. func.

bubble under ζ_L



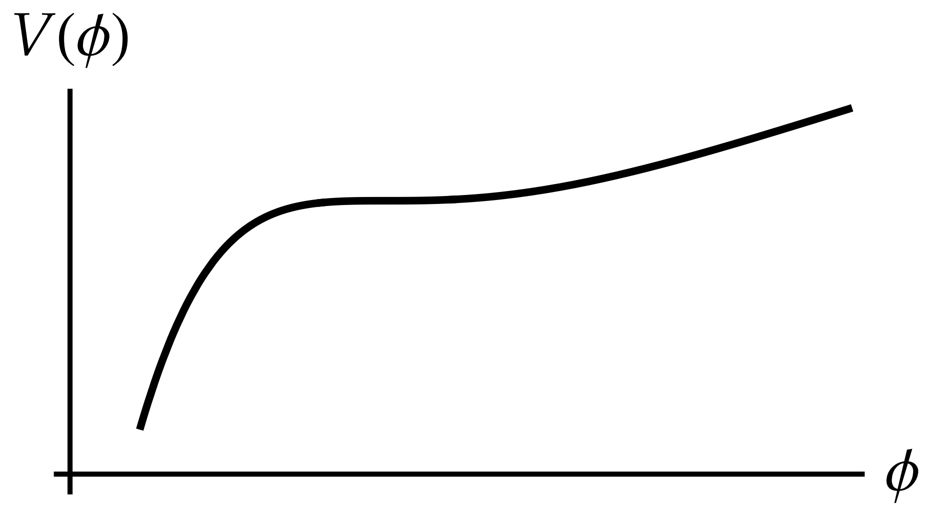
Maldacena's CR



In bubble-vanishing QFT, all 1-loop corrections vanish?

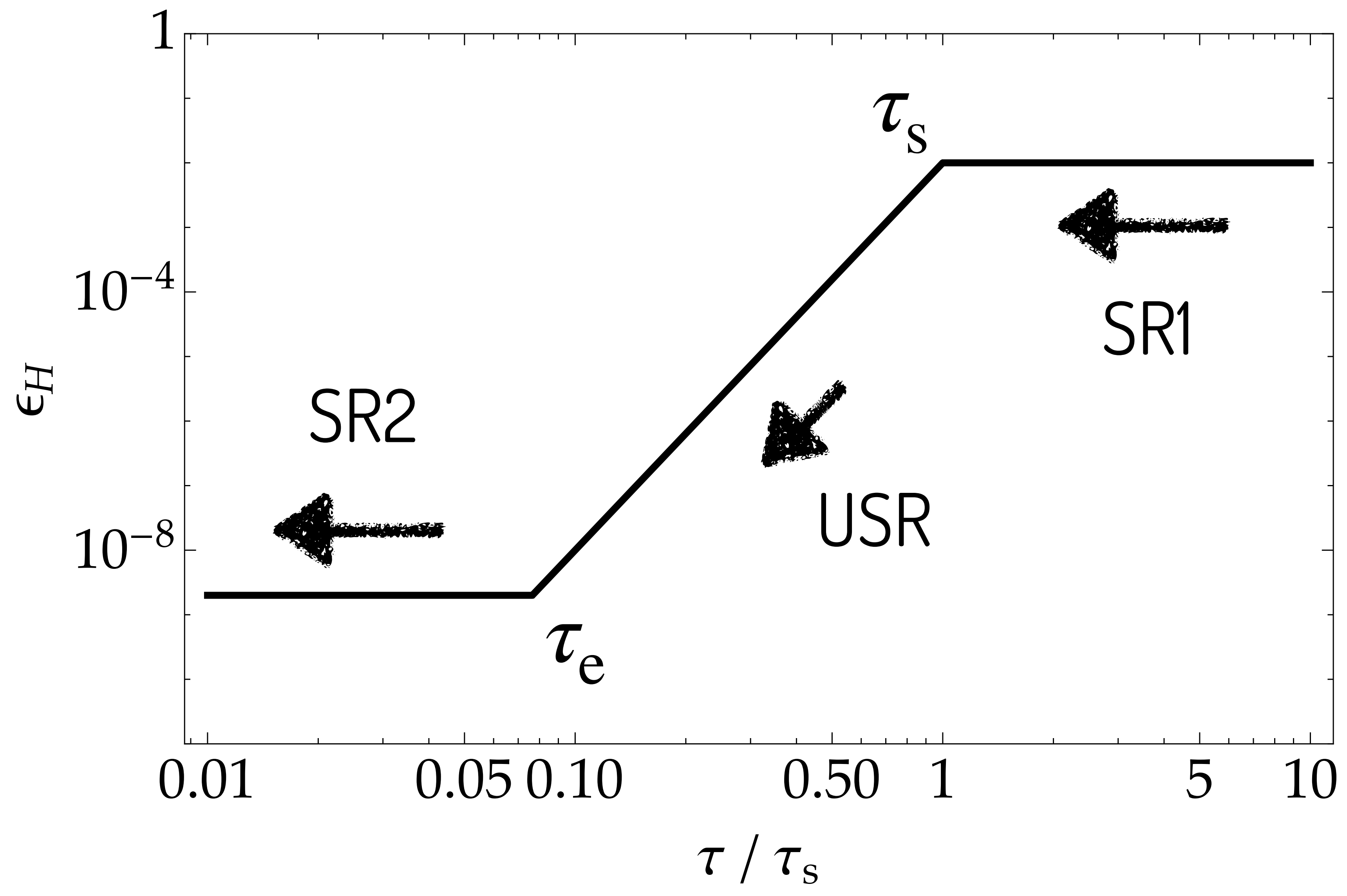
Transient USR

Kristiano & Yokoyama '22

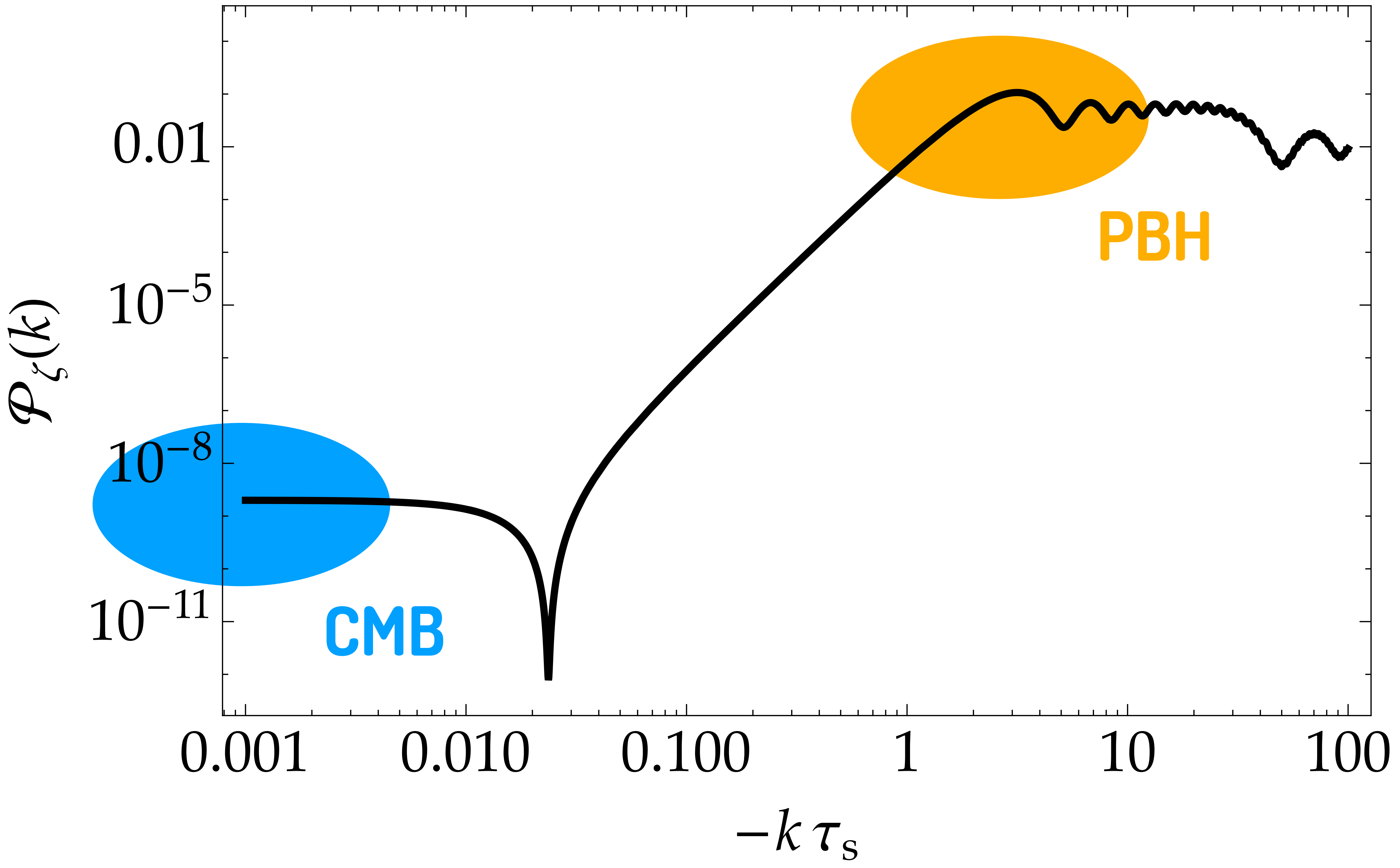


$$\eta = \frac{\dot{\epsilon}}{\epsilon H} = \begin{cases} 0 & \text{for } \tau \leq \tau_s, \\ -6 & \text{for } \tau_s \leq \tau < \tau_e, \\ 0 & \text{for } \tau_e \leq \tau\,. \end{cases}$$

- $\zeta_k(\tau) = C_k\sqrt{-k\tau}H^{(1)}_{3/2}(-k\tau) + D_k\sqrt{-k\tau}H^{(2)}_{3/2}(-k\tau)$ @ each stage
- $\zeta(\tau-0) = \zeta(\tau+0)$, $\zeta'(\tau-0) = \zeta'(\tau+0)$ @ τ_s, τ_e



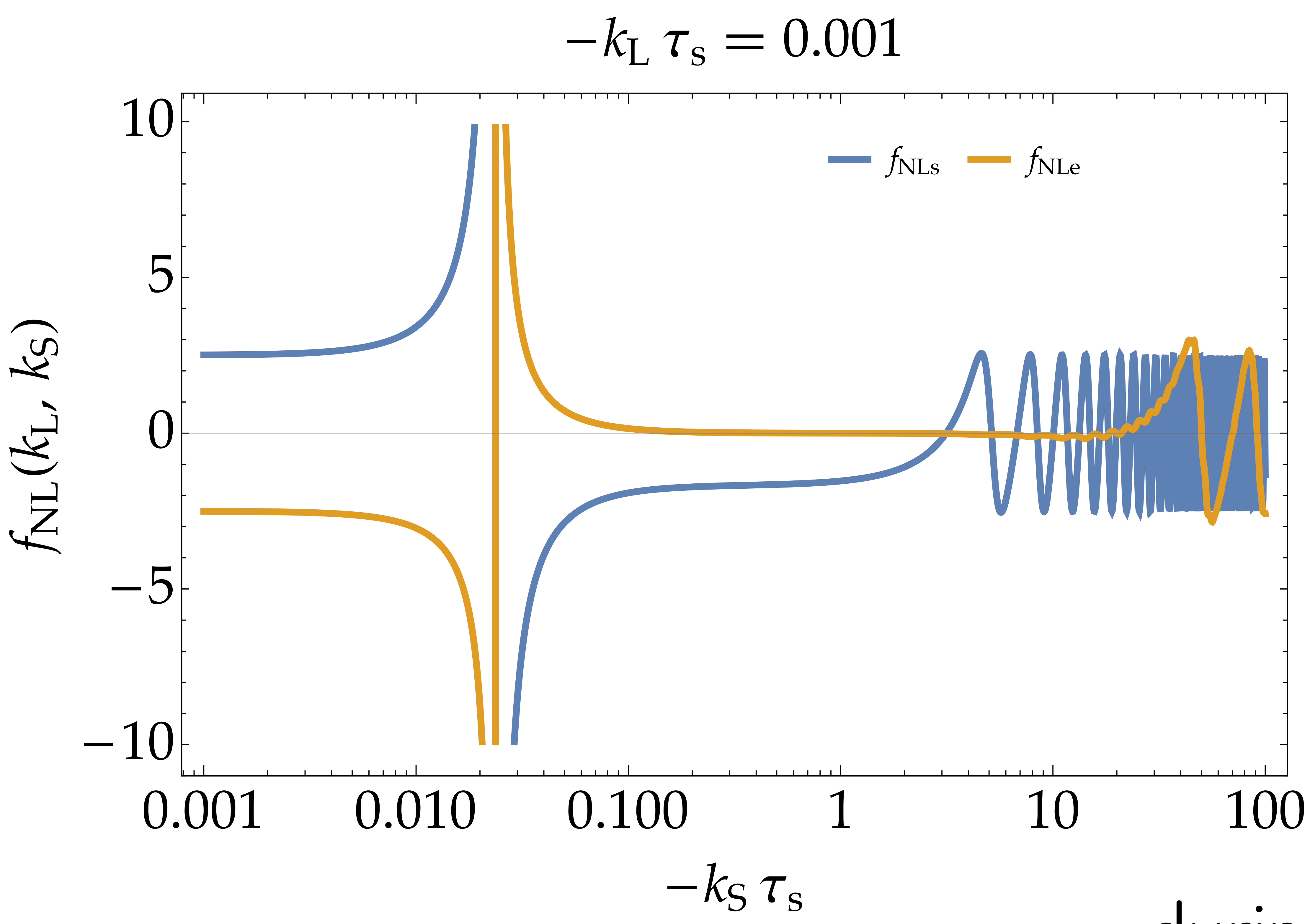
(see Appendix of Motohashi & YT '23 for V reconstruction)

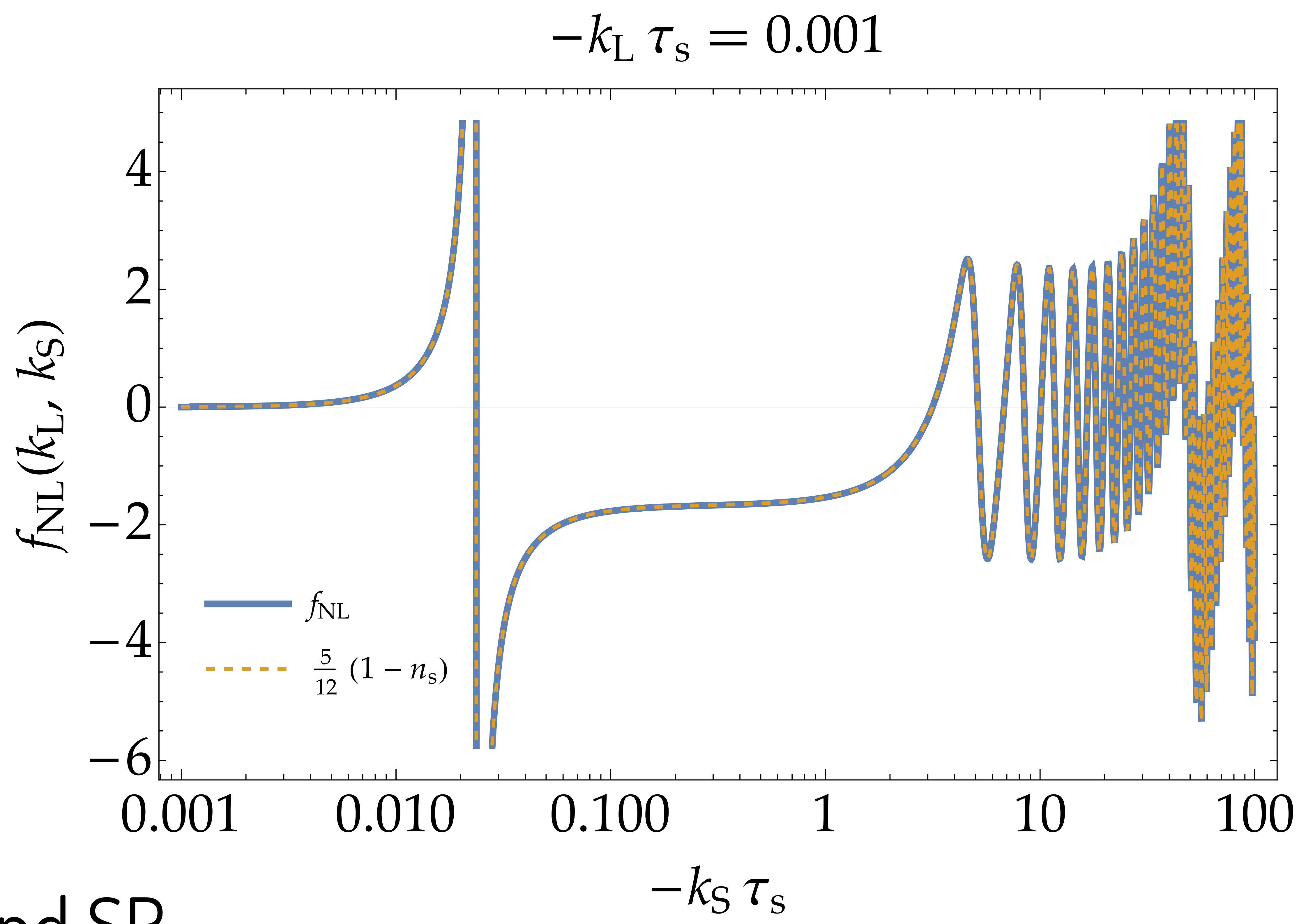


Squeezed Bispectrum

Motohashi & YT '23

$$S^{(3)} \supset \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' \right] \quad \Rightarrow \quad \langle \zeta_{\mathbf{k}_\mathrm{L}}\zeta_{\mathbf{k}_\mathrm{S}}\zeta_{\mathbf{k}_\mathrm{S}} \rangle = (2\pi)^3\delta^{(3)}(\mathbf{K})\frac{12}{5}f_\mathrm{NL}(k_\mathrm{L},k_\mathrm{S})P_\zeta(k_\mathrm{L})P_\zeta(k_\mathrm{S})$$





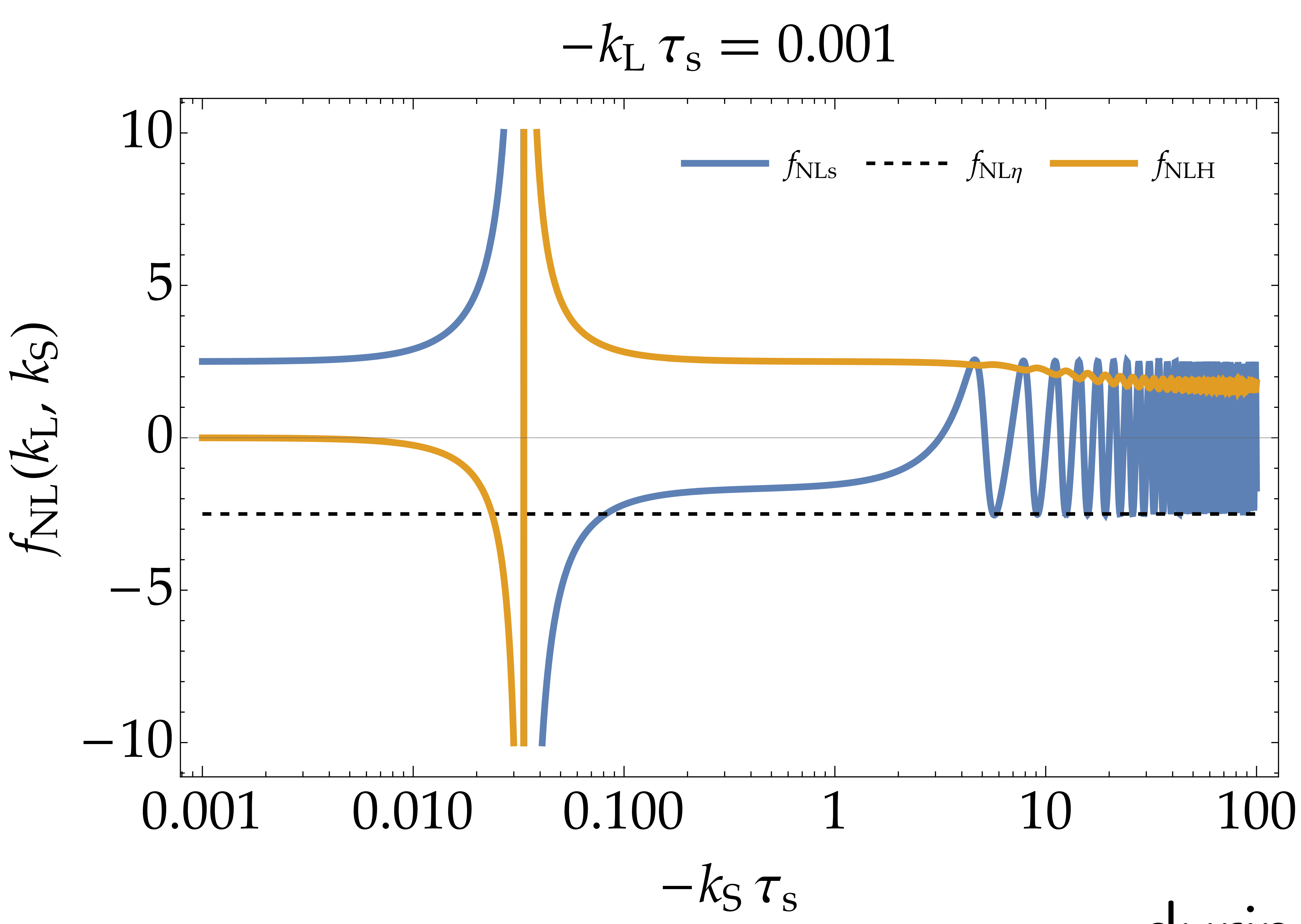
during 2nd SR

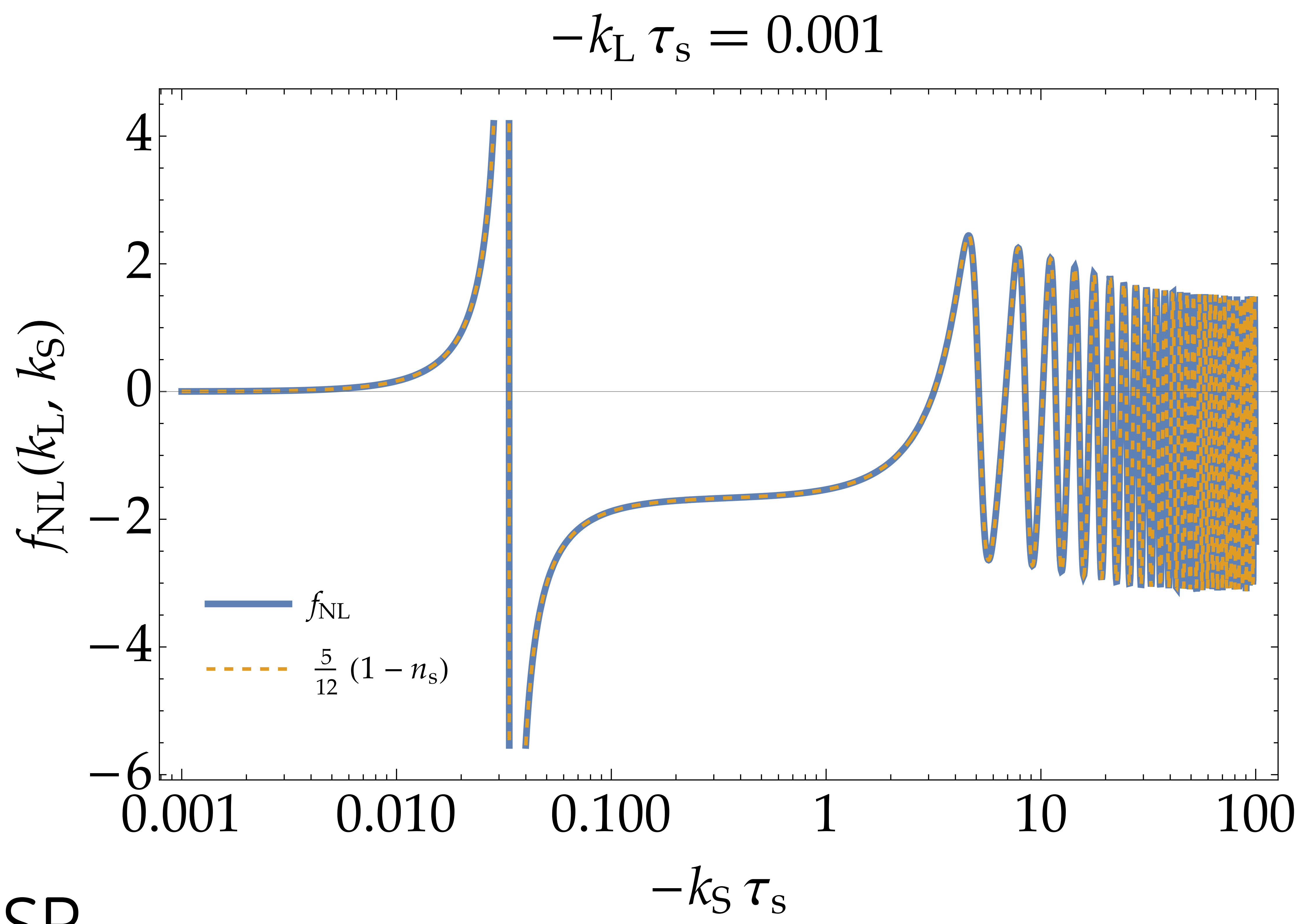
Squeezed Bispectrum

Motohashi & YT '23

cf. Arroja & Tanaka '11

$$S^{(3)} \supset \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{a^2\epsilon}{2}\eta\zeta^2\zeta' + \frac{a\epsilon}{H}\zeta\zeta'^2 \right) \right] \Rightarrow \langle \zeta_{\mathbf{k}_\mathrm{L}}\zeta_{\mathbf{k}_\mathrm{S}}\zeta_{\mathbf{k}_\mathrm{S}} \rangle = (2\pi)^3\delta^{(3)}(\mathbf{K})\frac{12}{5}f_\mathrm{NL}(k_\mathrm{L},k_\mathrm{S})P_\zeta(k_\mathrm{L})P_\zeta(k_\mathrm{S})$$





during USR

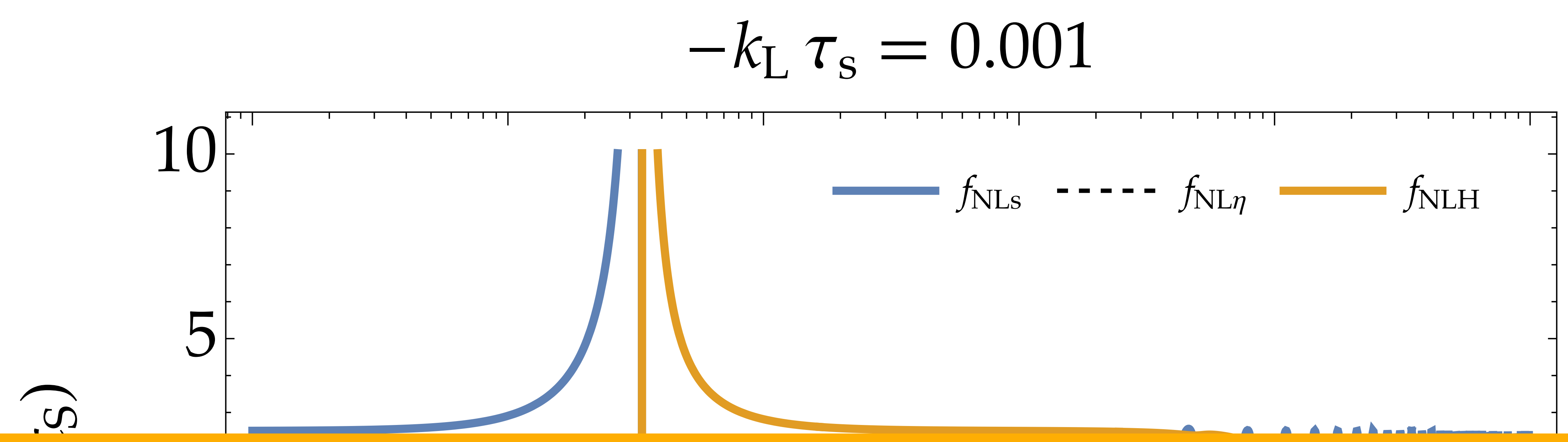
Squeezed Bispectrum

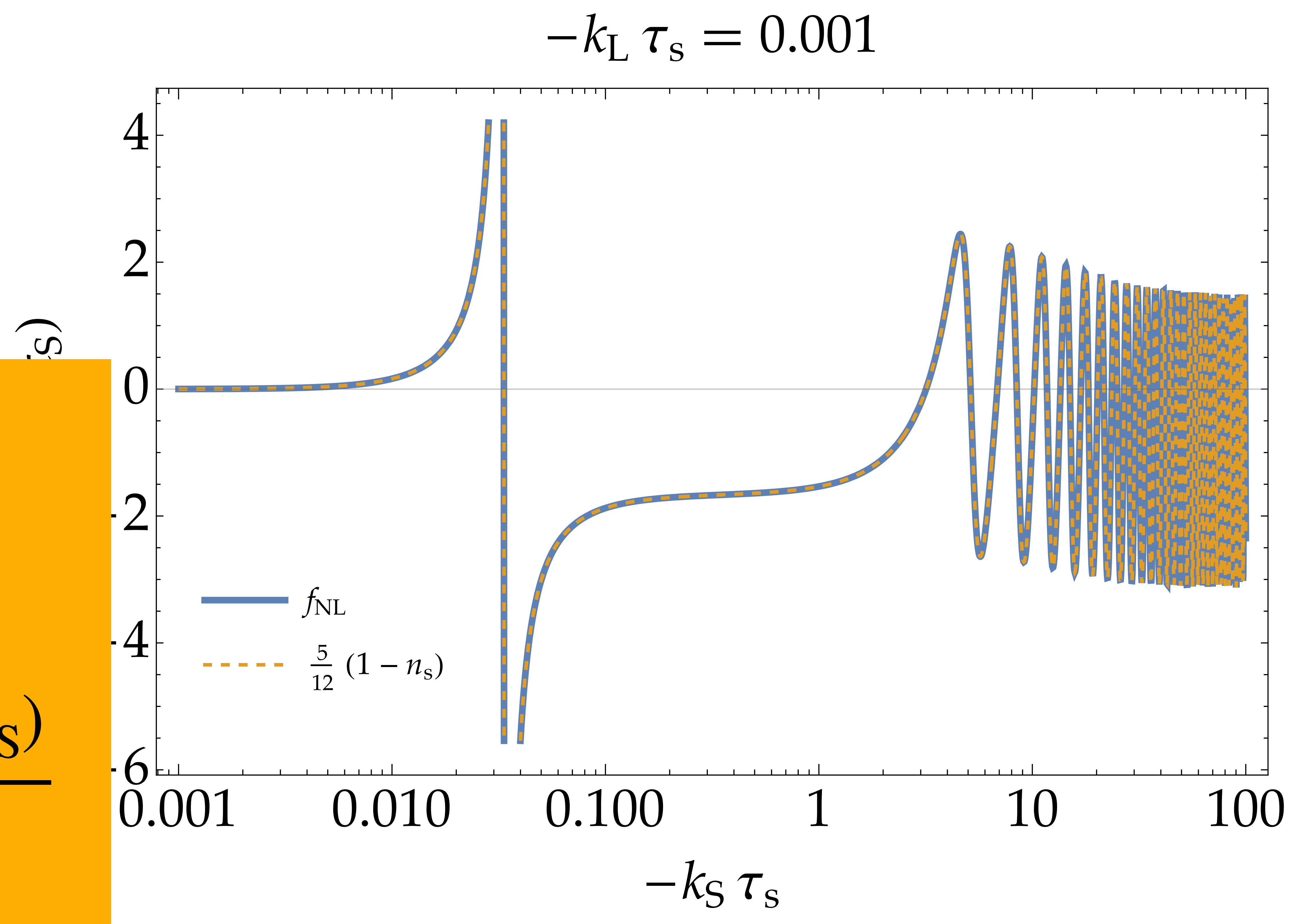
Motohashi & YT '23

cf. Arroja & Tanaka '11

$$S^{(3)} \supset \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{a^2\epsilon}{2}\eta\zeta^2\zeta' + \frac{a\epsilon}{H}\zeta\zeta'^2 \right) \right]$$

$$\Rightarrow \langle \zeta_{\mathbf{k}_\mathrm{L}}\zeta_{\mathbf{k}_\mathrm{S}}\zeta_{\mathbf{k}_\mathrm{S}} \rangle = (2\pi)^3\delta^{(3)}(\mathbf{K})\frac{12}{5}f_\mathrm{NL}(k_\mathrm{L},k_\mathrm{S})P_\zeta(k_\mathrm{L})P_\zeta(k_\mathrm{S})$$



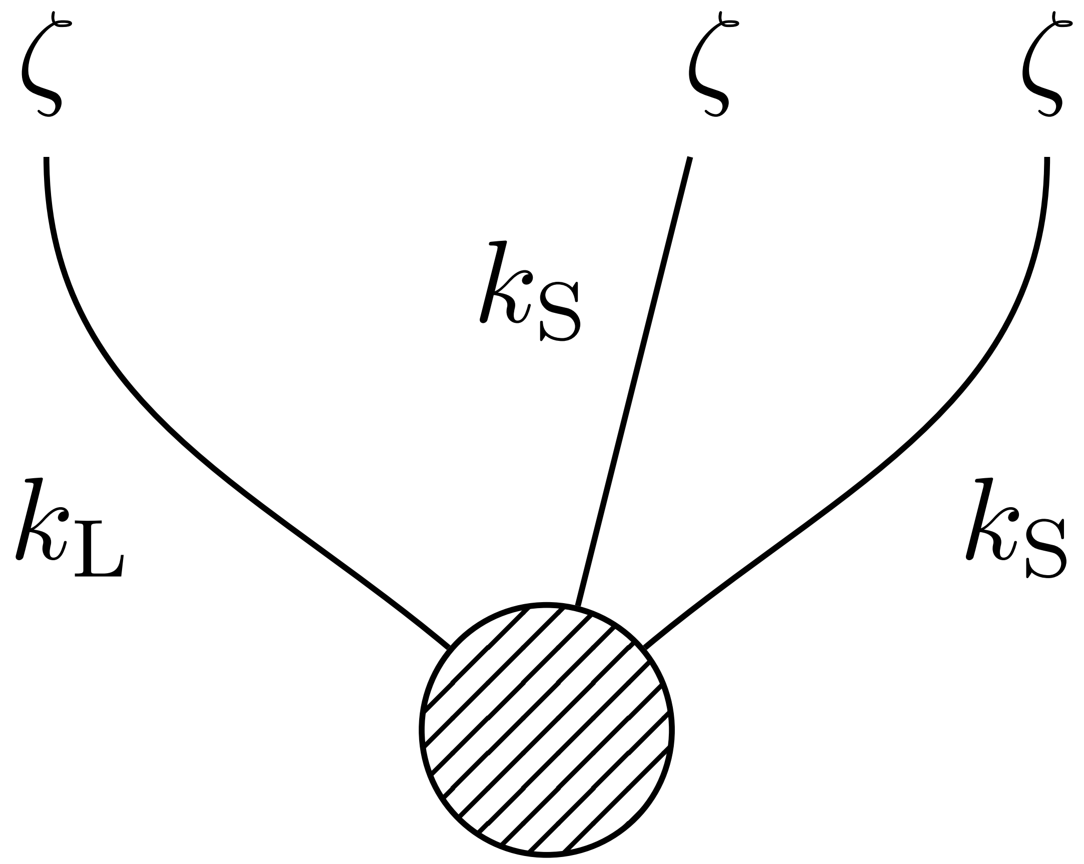


- Boundary terms are relevant!
- Maldacena's CR holds at any time!

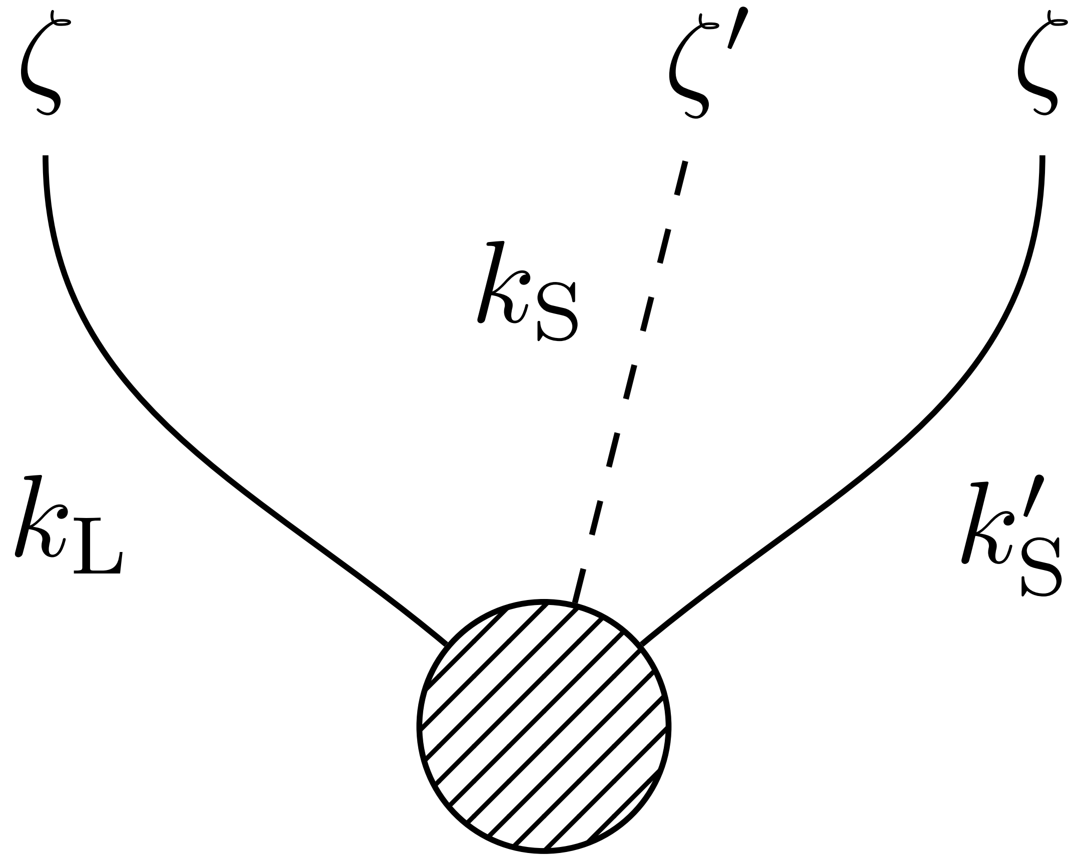
$$f_\mathrm{NL}(k_\mathrm{L},k_\mathrm{S}) = \frac{5}{12}\left(1-n_\mathrm{s}(k_\mathrm{S})\right) = -\frac{5}{12}\frac{\mathrm{d}\ln\mathcal{P}_\zeta(k_\mathrm{S})}{\mathrm{d}\ln k_\mathrm{S}}$$

CR in diagrams

YT, Terada, Tokuda ‘23



$$= -\frac{2\pi^2}{k_{\rm S}^3} P_\zeta(k_{\rm L}) \frac{\mathrm{d}\mathcal{P}_\zeta(k_{\rm S})}{\mathrm{d}\ln k_{\rm S}}$$

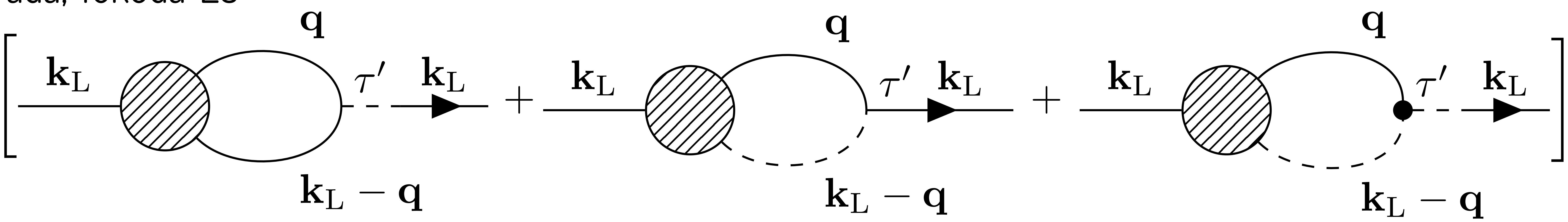


$$+ \ (k_{\rm S} \leftrightarrow k'_{\rm S}) = -\frac{2\pi^2}{k_{\rm S}^3} P_\zeta(k_{\rm L}) \frac{\mathrm{d}(\partial_\tau \mathcal{P}_\zeta(k_{\rm L}))}{\mathrm{d}\ln k_{\rm L}}$$

One-loop Cancellation

YT, Terada, Tokuda '23

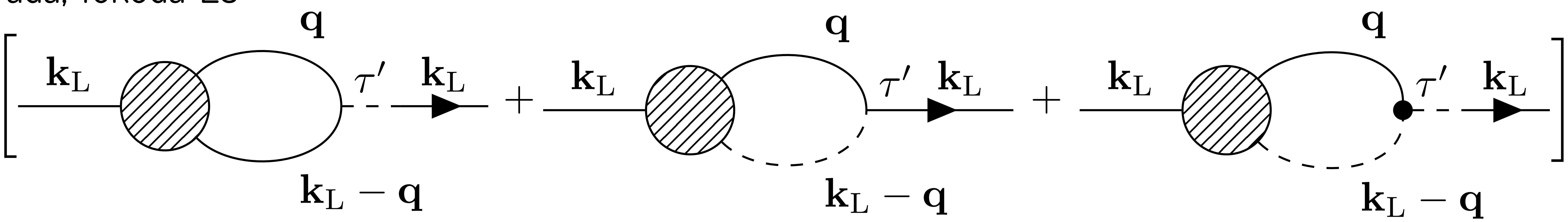
$$P_\zeta^{(\mathrm{soft})}(k_\mathrm{L}) = \int^\tau \mathrm{d}\tau' \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \left[\cdots \right]$$



One-loop Cancellation

YT, Terada, Tokuda '23

$$P_\zeta^{(\mathrm{soft})}(k_\mathrm{L}) = \int^\tau \mathrm{d}\tau' \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\Bigg[\quad\Bigg]$$



$$\propto P_\zeta(k_\mathrm{L})\int \mathrm{d}\ln q \frac{\mathrm{d}\mathscr{P}_\zeta(q)}{\mathrm{d}\ln q} \qquad \propto P_\zeta(k_\mathrm{L})\int \mathrm{d}\ln q \frac{\mathrm{d}(\partial_\tau\mathscr{P}_\zeta(q))}{\mathrm{d}\ln q} \qquad \propto P_\zeta(k_\mathrm{L})\int \mathrm{d}\ln q \frac{\mathrm{d}(\partial_\tau\mathscr{P}_\zeta(q))}{\mathrm{d}\ln q}$$

One-loop Cancellation

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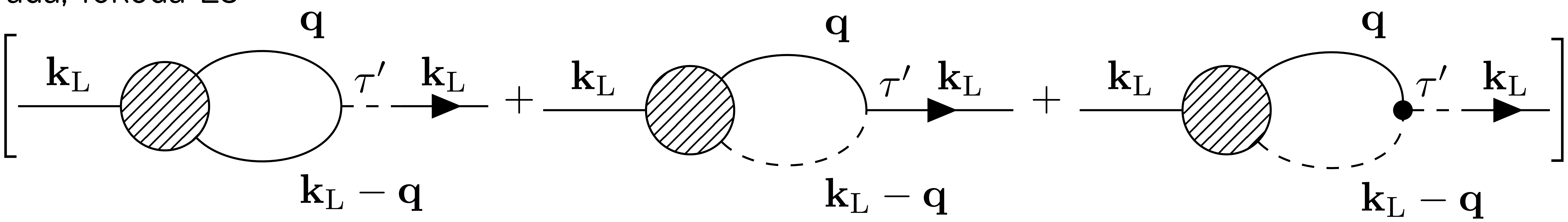


$$\Big]$$

$$\propto P_\zeta(k_\mathrm{L}) \int \mathrm{d}\ln q \frac{\mathrm{d}\mathscr{P}_\zeta(q)}{\mathrm{d}\ln q} \qquad \propto P_\zeta(k_\mathrm{L}) \int \mathrm{d}\ln q \frac{\mathrm{d}(\partial_\tau \mathscr{P}_\zeta(q))}{\mathrm{d}\ln q} \qquad \propto P_\zeta(k_\mathrm{L}) \int \mathrm{d}\ln q \frac{\mathrm{d}(\partial_\tau \mathscr{P}_\zeta(q))}{\mathrm{d}\ln q}$$

→ 0 by $i\varepsilon$ (i.e. $\tau \to (1+i\varepsilon)\tau$)

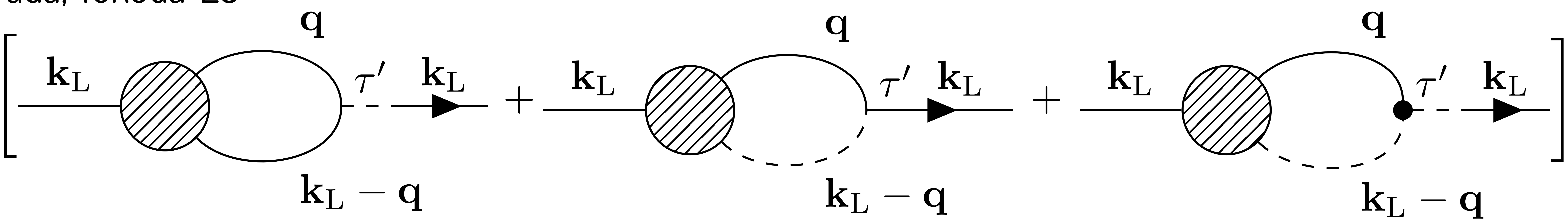
$$= P_\zeta(k_\mathrm{L}) \mathscr{P}_\zeta(q) \Big|_{q\to k_\mathrm{L}}^{q\to\infty} \qquad = P_\zeta(k_\mathrm{L}) (\partial_\tau \mathscr{P}_\zeta(q)) \Big|_{q\to k_\mathrm{L}}^{q\to\infty} \qquad = P_\zeta(k_\mathrm{L}) (\partial_\tau \mathscr{P}_\zeta(q)) \Big|_{q\to k_\mathrm{L}}^{q\to\infty}$$

$$\sim \mathscr{O}(10^{-9}) \qquad \to 0 \qquad \to 0$$

One-loop Cancellation

YT, Terada, Tokuda '23

$$P_\zeta^{(\mathrm{soft})}(k_\mathrm{L}) = \int^\tau \mathrm{d}\tau' \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\Bigg[$$



$$\Bigg]$$

$$\propto P_\zeta(k_\mathrm{L})\int \mathrm{d}\ln q \frac{\mathrm{d}\mathscr{P}_\zeta(q)}{\mathrm{d}\ln q} \qquad \propto P_\zeta(k_\mathrm{L})\int \mathrm{d}\ln q \frac{\mathrm{d}(\partial_\tau\mathscr{P}_\zeta(q))}{\mathrm{d}\ln q} \qquad \propto P_\zeta(k_\mathrm{L})\int \mathrm{d}\ln q \frac{\mathrm{d}(\partial_\tau\mathscr{P}_\zeta(q))}{\mathrm{d}\ln q}$$

→ 0 by $i\varepsilon$ (i.e. $\tau \to (1+i\varepsilon)\tau$)

$$= P_\zeta(k_\mathrm{L})\mathscr{P}_\zeta(q)\Big|_{q\to k_\mathrm{L}}^{q\to\infty} \qquad = P_\zeta(k_\mathrm{L})(\partial_\tau\mathscr{P}_\zeta(q))\Big|_{q\to k_\mathrm{L}}^{q\to\infty} \qquad = P_\zeta(k_\mathrm{L})(\partial_\tau\mathscr{P}_\zeta(q))\Big|_{q\to k_\mathrm{L}}^{q\to\infty}$$

$$\sim \mathcal{O}(10^{-9}) \qquad \to 0 \qquad \to 0$$

$$\sim \mathcal{O}(10^{-9}) \times P_\zeta(k_\mathrm{L}) \ll P_\zeta(k_\mathrm{L})$$ ✔

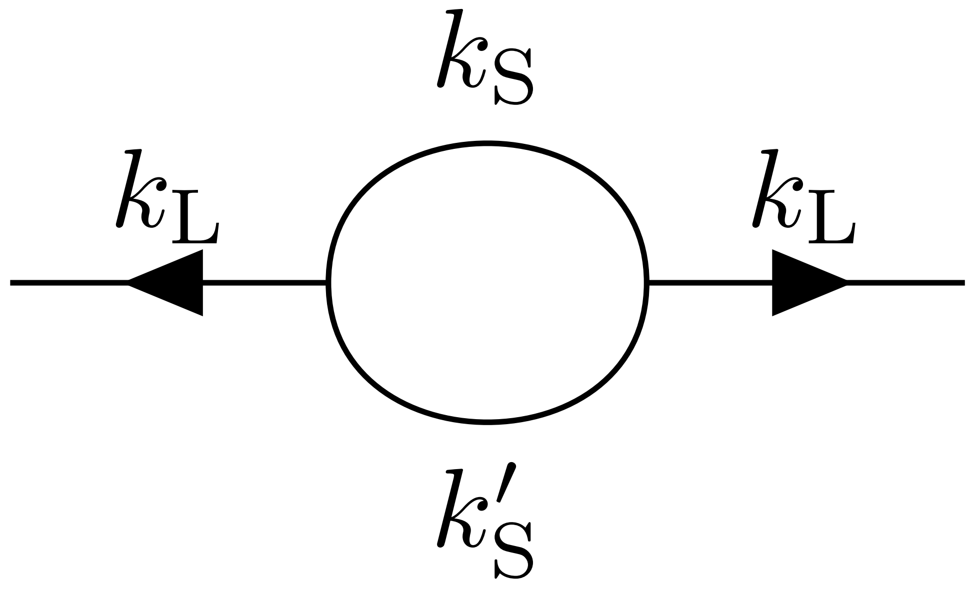
Conserved ζ @ tree ⇔ Maldacena's CR ⇔ Cancellation of One-loop

cf. Ward—Takahashi by asymptotic dilatation

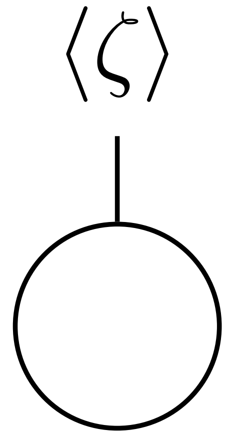
Comments

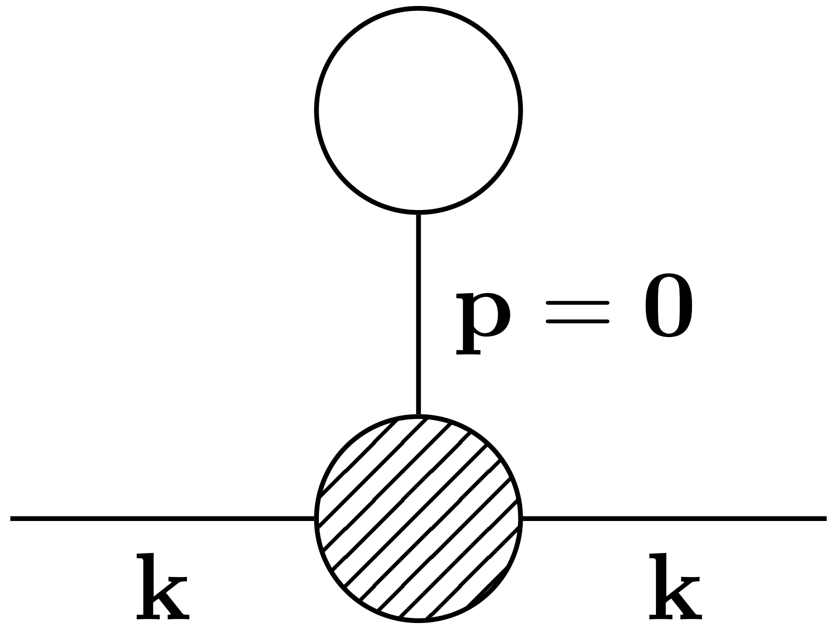
YT, Terada, Tokuda '23

- induced scalar (cf. induced GW)

$P_\zeta^{(\mathrm{induced})}(k_\mathrm{L}) \propto$  $\propto \left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^3$

- tadpole





$= -\langle\zeta\rangle \frac{\mathrm{d}\mathcal{P}_\zeta(k)}{\mathrm{d}\ln k}$

$P_\zeta^{\mathrm{ren}}(k) := P_\zeta(k e^{-\langle\zeta\rangle})$

cf. stochastic-δN

$\langle\zeta\rangle = \langle\mathcal{N}\rangle - N_\mathrm{cl}$

- tensor (Ota, Sasaki, Wang '22x2)

not total derivative...

(2311.11053; no mass correction in dS?)

$$B_{h_\lambda\zeta\zeta}(k_\mathrm{L}, k_\mathrm{S}, k_\mathrm{S}) = -\frac{1}{2} P_{h_\lambda}(k_\mathrm{L}) P_\zeta(k_\mathrm{S}) e_{ij}^\lambda(\hat{\mathbf{k}}_\mathrm{L}) \hat{k}_\mathrm{S}^i \hat{k}_\mathrm{S}^j \frac{\mathrm{d}\ln P_\zeta(k_\mathrm{S})}{\mathrm{d}\ln k_\mathrm{S}}$$

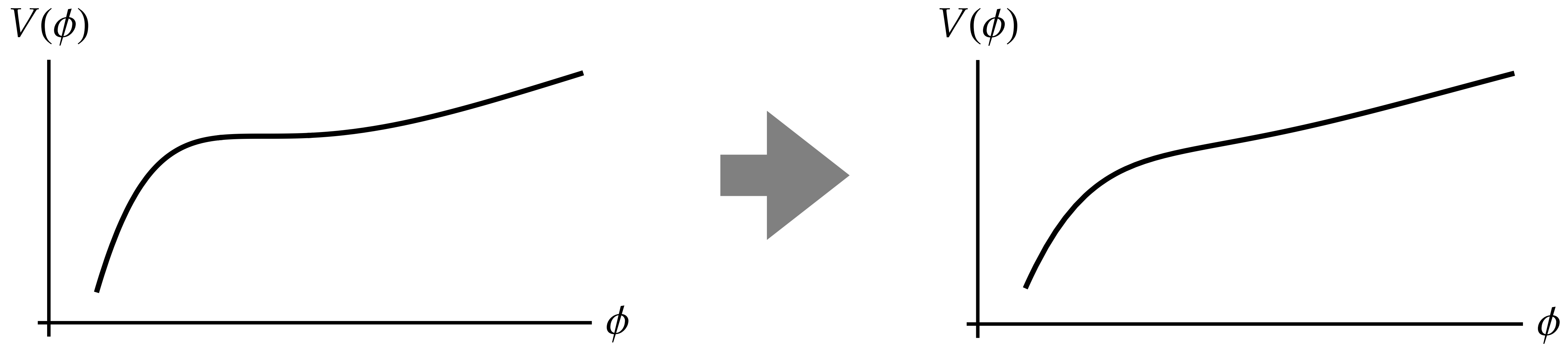
$$P_h^{(\mathrm{soft})}(k_\mathrm{L}) \propto P_h(k_\mathrm{L}) \int \mathrm{d}\ln q\, q^5 \frac{\mathrm{d}P_\zeta(q)}{\mathrm{d}\ln q}$$

Comments

- other works

e.g. Firouzjahi 2311.0408

$S_{\mathrm{EFT}}[\delta\phi]$: large corrections on coupling consts. are possible
but it simultaneously changes B.G. and relation $\delta\phi \leftrightarrow \zeta$



$V(\phi)$ may change but ζ-conservation should NOT depend on the details of $V(\phi)$

cf. Inomata '24

Riotto, Choundhury+, Fumagalli, Tasinato, Maity+, Mulryne+, etc., etc., ...

See also Kawaguchi, Tsujikawa, Yamada '24 for the path integral formalisation.

There, the EoM terms $2f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1$ are relevant rather than the boundary terms.

Comments

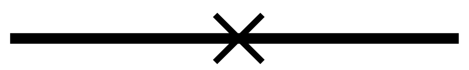
- 4pt couplings

tadpole cancellation: $\tilde{\zeta} := \zeta - \langle \zeta \rangle$ so that $\langle \tilde{\zeta} \rangle = 0$

dilatation sym. counter term

tadpole counter term

mass counter term



cancel

Pimentel, Senatore, Zaldarriaga '12
Kawaguchi, Tsujikawa, Yamada '24

- symmetry?

e.g. Soft dS Effective Theory by Cohen & Green '20

"conservation of 'const.'-modes of ζ and h is preserved @ all orders"

Summary

- Asymp. Dilatation ensures One-loop Cancellation
- Loops in Lattice (e.g., Butterfly-effect, STOLAS, ...)?
- Tensor may be serious...

Appendices

Field Redefinition?

Kristiano & Yokoyama '22

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$ & comoving gauge ($\delta\phi = 0$)

$$\Rightarrow S^{(3)}[\zeta] = \int \mathrm{d}^4x \text{ (tedious terms.)}$$

Arroja & Tanaka '11

$$= \int \mathrm{d}^4x \left[\mathcal{O}(\epsilon^2) + \frac{a^3\epsilon}{2}\dot{\eta}\zeta^2\dot{\zeta} + 2f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left((\zeta^3\text{-terms}) - \frac{a^3\epsilon}{2}\eta\zeta^2\dot{\zeta} - \frac{a^3\epsilon}{H}\zeta\dot{\zeta}^2 + \mathcal{O}(\epsilon^2)\right)\right]$$

$$\left.\frac{\delta L}{\delta\zeta}\right|_1 = 0 : \text{tree EoM}, \quad f(\zeta) = \frac{\eta}{4}\dot{\zeta}^2 + \frac{1}{H}\zeta\dot{\zeta} + \cdots$$

field redef. : $\zeta = \tilde{\zeta} + f(\tilde{\zeta})$ Maldacena '02

$$\Rightarrow S^{(3)}[\tilde{\zeta}] = \int \mathrm{d}^4x \left[\mathcal{O}(\epsilon^2) + \frac{a^3\epsilon}{2}\dot{\eta}\tilde{\zeta}^2\dot{\tilde{\zeta}}\right]$$

unique relevant vertex $\to H_{\mathrm{int}}$

Field Redefinition?

Kristiano & Yokoyama '22

Weinberg '05

$$\langle \hat{\tilde{\zeta}}_{\mathbf{k}_L}(\tau_e)\hat{\tilde{\zeta}}_{\mathbf{k}_L'}(\tau_e)\rangle_{(1)} = \left\langle \left[\bar{T}\exp\left(i\int_{-\infty}^{\tau_e} \hat{H}_{\rm int}(\tau)\mathrm{d}\tau \right)\right] \hat{\tilde{\zeta}}_{\mathrm{I},\mathbf{k}_L}(\tau_e)\hat{\tilde{\zeta}}_{\mathrm{I},\mathbf{k}_L'}(\tau_e)\left[T\exp\left(-i\int_{-\infty}^{\tau_e} \hat{H}_{\rm int}(\tau)\mathrm{d}\tau \right)\right] \right\rangle_{(1)}$$

$$= i^2\int^{\tau_e}\mathrm{d}\tau_1\int^{\tau_1}\mathrm{d}\tau_2\langle[\hat{H}_{\rm int}(\tau_1),[\hat{H}_{\rm int}(\tau_2),\hat{\tilde{\zeta}}_{\mathrm{I},\mathbf{k}_L}(\tau_e)\hat{\tilde{\zeta}}_{\mathrm{I},\mathbf{k}_L'}(\tau_e)]]\rangle$$

$$\sim \eta^2 P_\zeta(k_L)\int_{k_s}^{k_e}\mathrm{d}\ln k\,\mathscr{P}_\zeta(k) \sim P_\zeta(k_L)$$

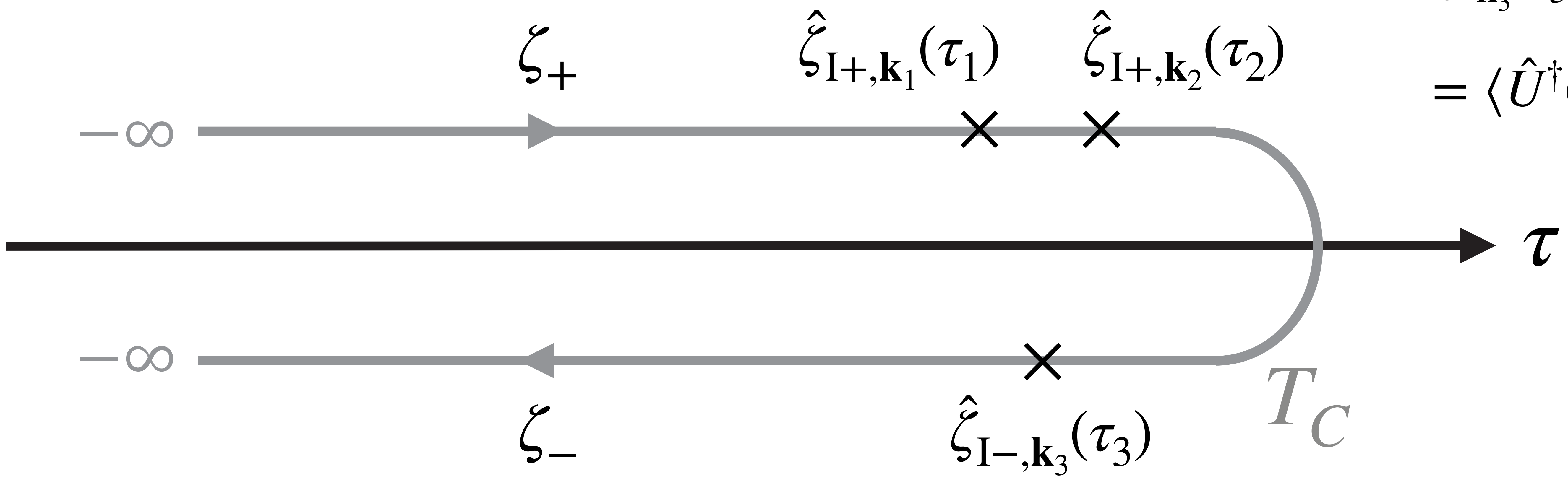
PTB approach breaks down even on CMB scale?

Rmk:

- $\tilde{\zeta}(\tau) \to \zeta(\tau)$ @ SR2
- $S^{(4)}[\tilde{\zeta}]$?
- non-lin. field-redef. works @ one-loop?
- $\cdots$

Closed Time Path

Schwinger '65, Keldysh '65



Example

$$\langle \hat{\zeta}_{\mathbf{k}_3}(\tau_3)\hat{\zeta}_{\mathbf{k}_2}(\tau_2)\hat{\zeta}_{\mathbf{k}_1}(\tau_1)\rangle = \left\langle T_C\,\hat{\zeta}_{\mathrm{I}-,\mathbf{k}_3}(\tau_3)\hat{\zeta}_{\mathrm{I}+,\mathbf{k}_2}(\tau_2)\hat{\zeta}_{\mathrm{I}+,\mathbf{k}_1}(\tau_1)\mathrm{e}^{-i\int \hat{H}_{\mathrm{int}}\mathrm{d}\tau}\right\rangle$$

$$= \langle \hat{U}^\dagger(\tau_3,-\infty)\hat{\zeta}_{\mathrm{I},\mathbf{k}_3}(\tau_3)\hat{U}^\dagger(\tau_2,\tau_3)\hat{\zeta}_{\mathrm{I},\mathbf{k}_2}(\tau_2)\hat{U}(\tau_2,\tau_1)\hat{\zeta}_{\mathrm{I},\mathbf{k}_1}(\tau_1)\hat{U}(\tau_1,-\infty)\rangle$$

Propagator

$$G_{ab}(x,x') = \langle T_C\,\hat{\zeta}_{\mathrm{I}a}(x)\hat{\zeta}_{\mathrm{I}b}(x')\rangle$$

$$= \begin{cases} \Theta(\tau-\tau')\langle\hat{\zeta}_{\mathrm{I}}(x)\hat{\zeta}_{\mathrm{I}}(x')\rangle + \Theta(\tau'-\tau)\langle\hat{\zeta}_{\mathrm{I}}(x')\hat{\zeta}_{\mathrm{I}}(x)\rangle, & (a,b)=(+,+) \\ \langle\hat{\zeta}_{\mathrm{I}}(x')\hat{\zeta}_{\mathrm{I}}(x)\rangle, & (a,b)=(+,-) \\ \langle\hat{\zeta}_{\mathrm{I}}(x)\hat{\zeta}_{\mathrm{I}}(x')\rangle, & (a,b)=(-,+) \\ \Theta(\tau'-\tau)\langle\hat{\zeta}_{\mathrm{I}}(x)\hat{\zeta}_{\mathrm{I}}(x')\rangle + \Theta(\tau-\tau')\langle\hat{\zeta}_{\mathrm{I}}(x')\hat{\zeta}_{\mathrm{I}}(x)\rangle, & (a,b)=(-,-) \end{cases}$$

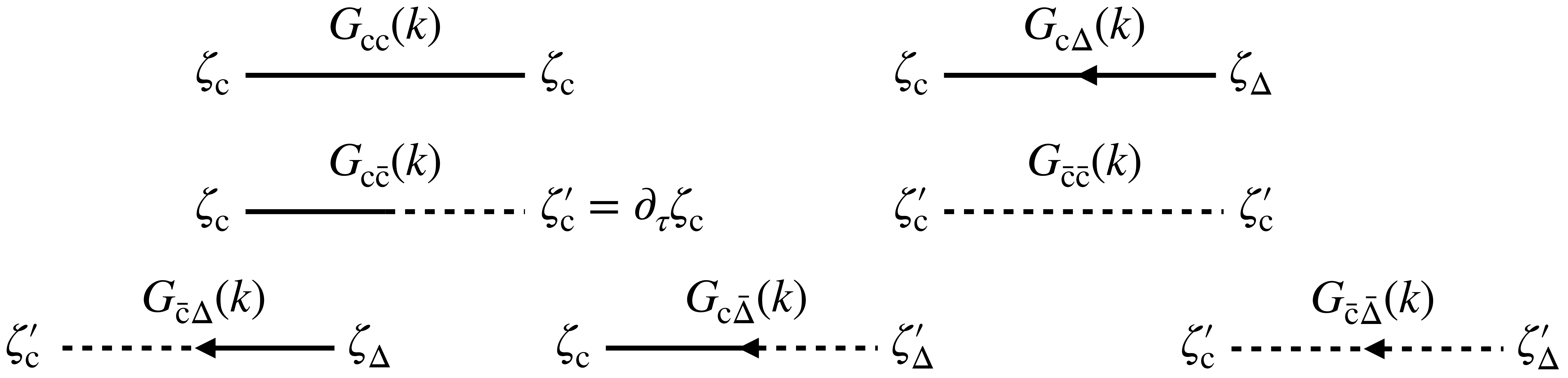
Closed Time Path

Schwinger '65, Keldysh '65

Shcwinger—Keldysh basis $\qquad \zeta_{\rm c} = \dfrac{\zeta_+ + \zeta_-}{2}, \quad \zeta_\Delta = \zeta_+ - \zeta_-$

$$G_{\alpha\beta}(x,x') = \begin{cases} \frac{1}{2}\langle\{\hat{\zeta}_{\rm I}(x), \hat{\zeta}_{\rm I}(x')\}\rangle, & (\alpha,\beta)=({\rm c},{\rm c}) \\ \Theta(\tau-\tau')\langle[\hat{\zeta}_{\rm I}(x), \hat{\zeta}_{\rm I}(x')]\rangle, & (\alpha,\beta)=({\rm c},\Delta) \\ \Theta(\tau'-\tau)\langle[\hat{\zeta}_{\rm I}(x'), \hat{\zeta}_{\rm I}(x)]\rangle, & (\alpha,\beta)=(\Delta,{\rm c}) \\ 0, & (\alpha,\beta)=(\Delta,\Delta) \end{cases}$$

$$G_{\alpha\beta}(\tau,\tau';k) = \begin{cases} \Re\zeta_k(\tau)\zeta_k^*(\tau'), & (\alpha,\beta)=({\rm c},{\rm c}) \\ 2i\Theta(\tau-\tau')\Im\zeta_k(\tau)\zeta_k^*(\tau'), & (\alpha,\beta)=({\rm c},\Delta) \\ 2i\Theta(\tau'-\tau)\Im\zeta_k(\tau')\zeta_k^*(\tau), & (\alpha,\beta)=(\Delta,{\rm c}) \\ 0, & (\alpha,\beta)=(\Delta,\Delta) \end{cases}$$



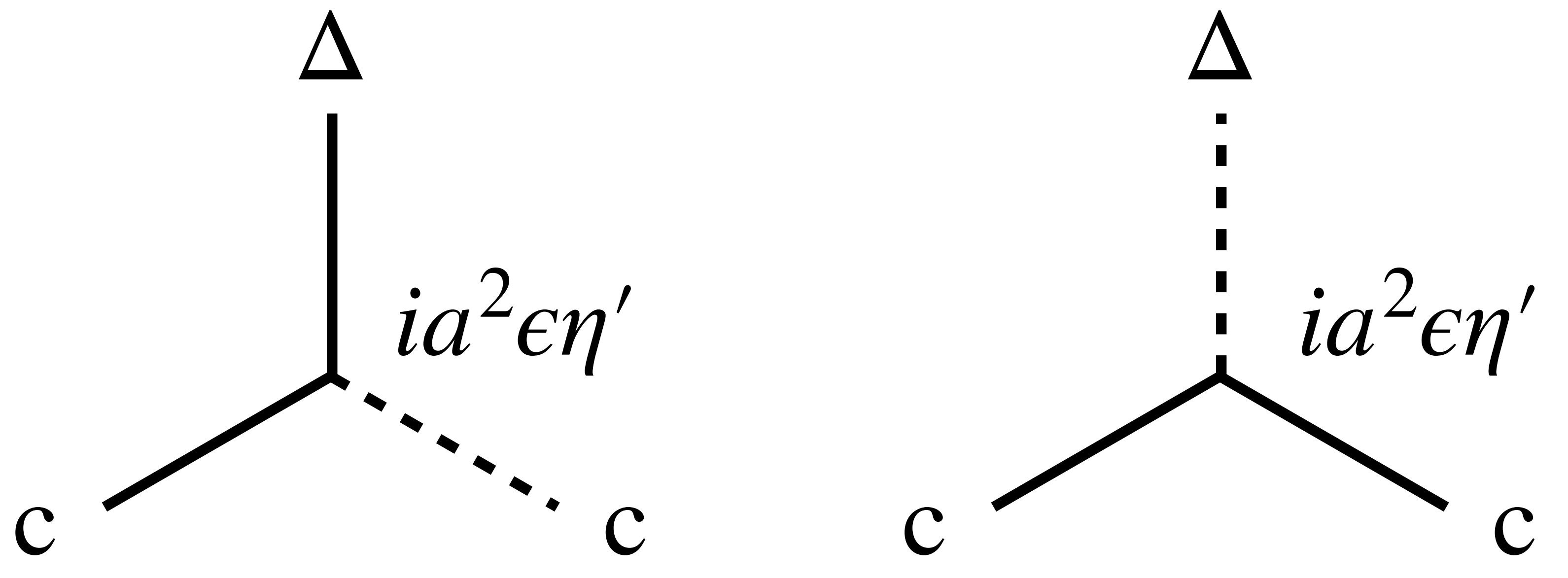
Boundary Vertices

Motohashi & YT '23

$$S^{(3)} \supset \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{a^2\epsilon}{2}\eta\zeta^2\zeta' + \frac{a\epsilon}{H}\zeta\zeta'^2 \right) \right]$$

$$\mathscr{L}^{(3)}_{\text{bulk}} = \frac{a^2\epsilon}{2}\eta'(\zeta_+^2\zeta_+' - \zeta_-^2\zeta_-')$$

$$= a^2\epsilon\eta' \left(\zeta_c\zeta_\Delta\zeta_c' + \frac{1}{2}\zeta_c^2\zeta_\Delta' \right) + \mathscr{O}(\zeta_\Delta^3)$$



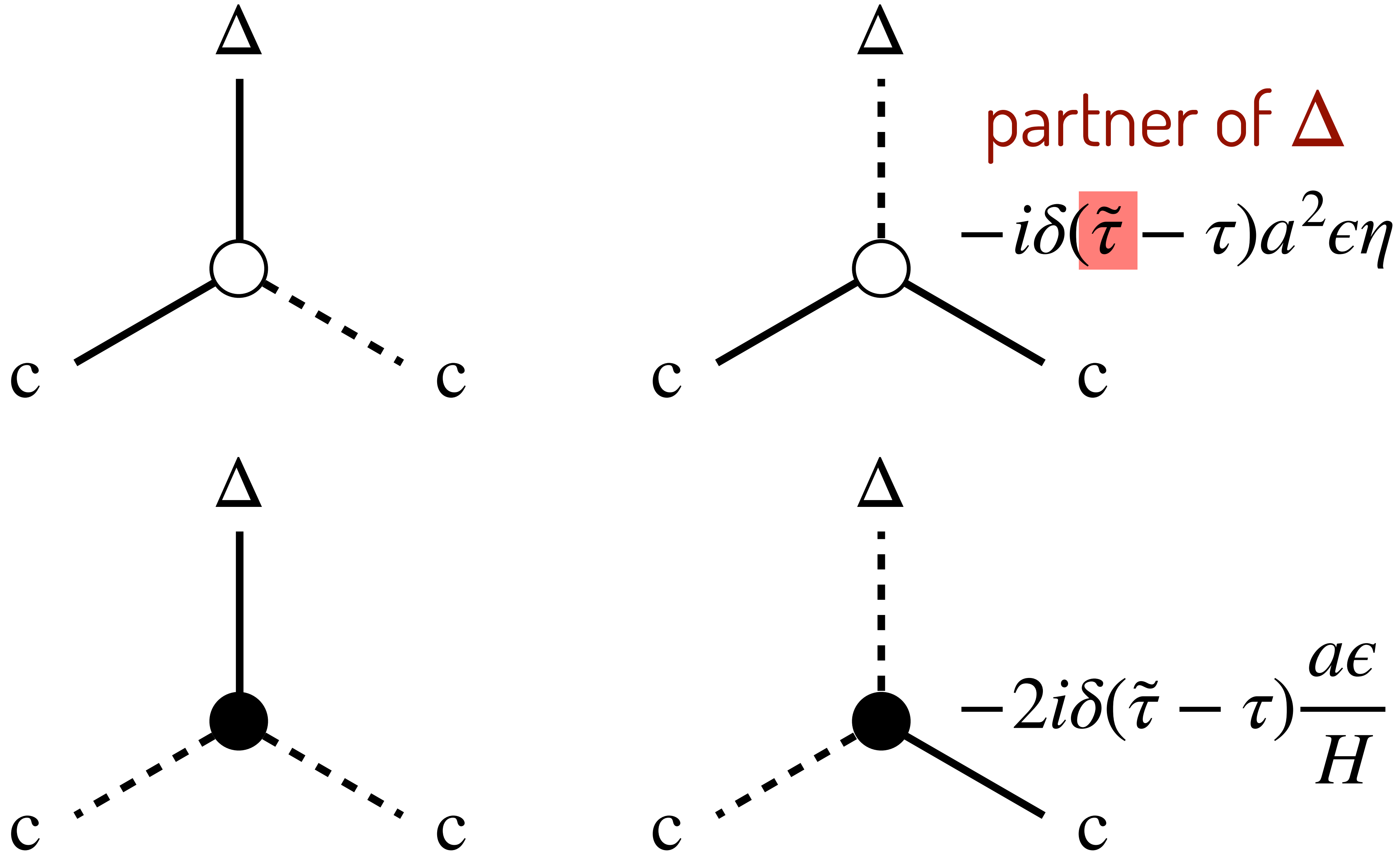
Boundary Vertices $\quad \mathscr{L}^{(3)}_{\text{B}} = \frac{\mathrm{d}}{\mathrm{d}\tau}\mathscr{B}$

$$\left\langle T_C\,\hat{\mathscr{O}}(\tau_1, \tau_2, \cdots)\frac{\mathrm{d}}{\mathrm{d}\tau}\hat{\mathscr{B}}(\tau) \right\rangle$$

$$= \frac{\mathrm{d}}{\mathrm{d}\tau}\langle T_C\,\hat{\mathscr{O}}(\tau_1, \tau_2, \cdots)\hat{\mathscr{B}}(\tau)\rangle$$

$$+\delta(\tau_1 - \tau)G_{c\Delta}(\tau_1, \tau)G_{cc}(\tau_2, \tau)\cdots + \cdots$$

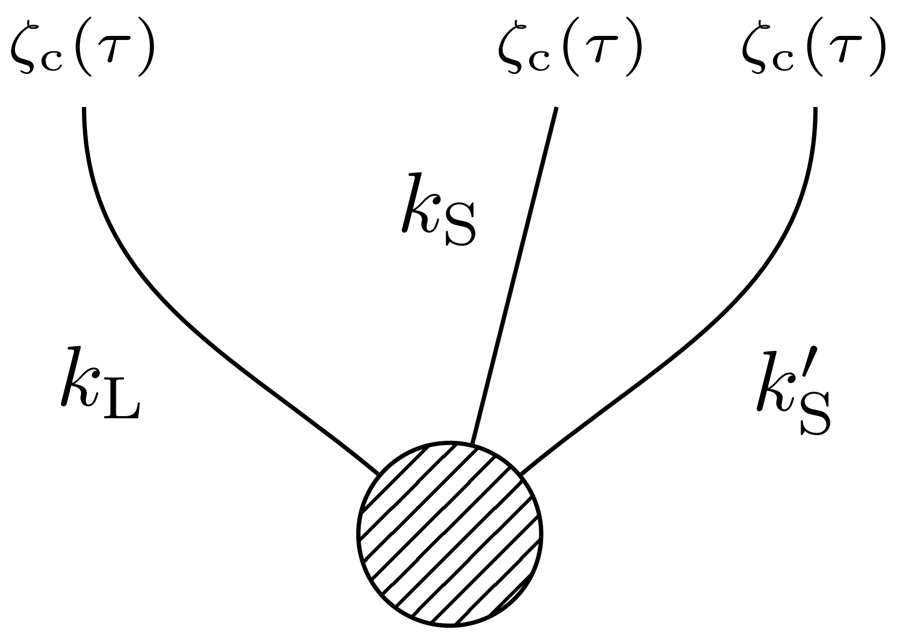
$\frac{\mathrm{d}}{\mathrm{d}\tau}\Theta(\tau_1 - \tau)$ in $G_{c\Delta}(\tau_1, \tau)$



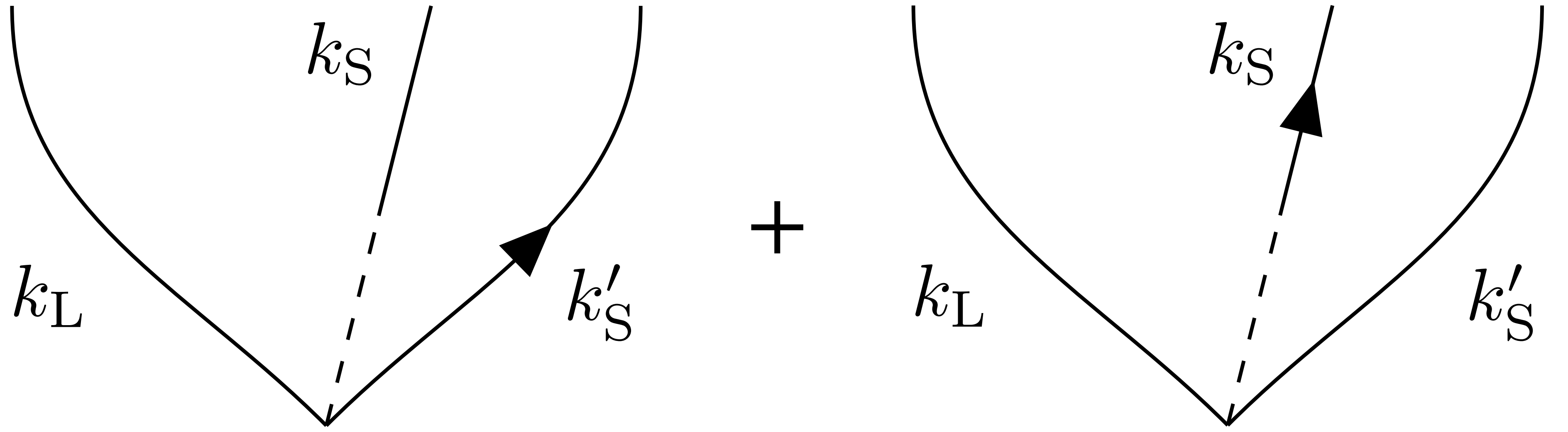
Bispectrum

Motohashi & YT '23

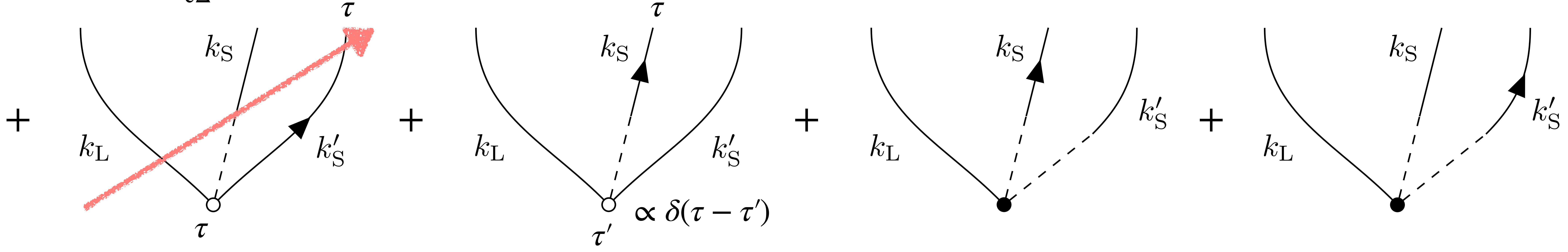
$B_{\zeta\zeta\zeta}(k_{\rm L}, k_{\rm S}, k_{\rm S}'; \tau) =$



$= \quad \sim P_\zeta(k_{\rm L}) \propto \dfrac{1}{k_{\rm L}^3}$



$\propto G_{c\Delta}(\tau, \tau) = 0$

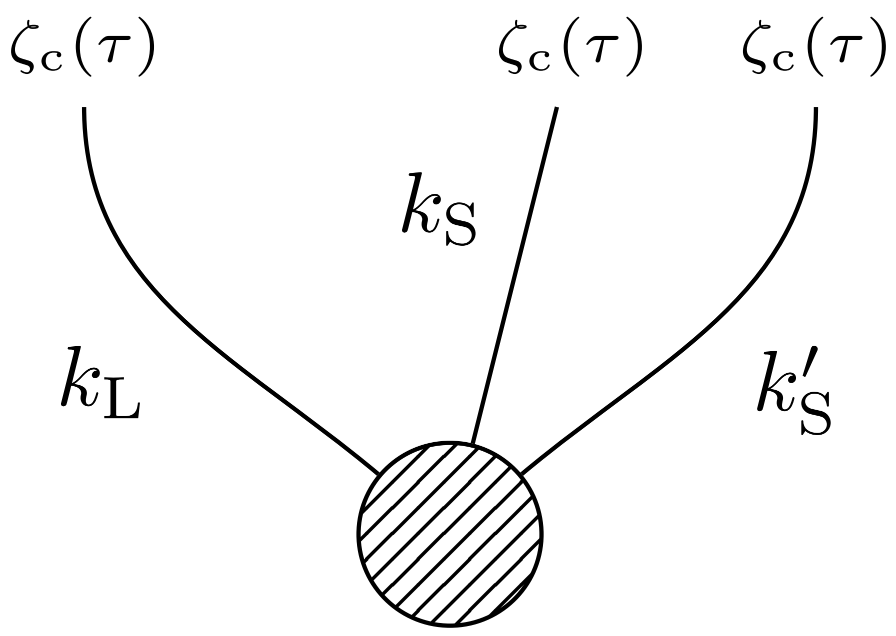


Bispectrum

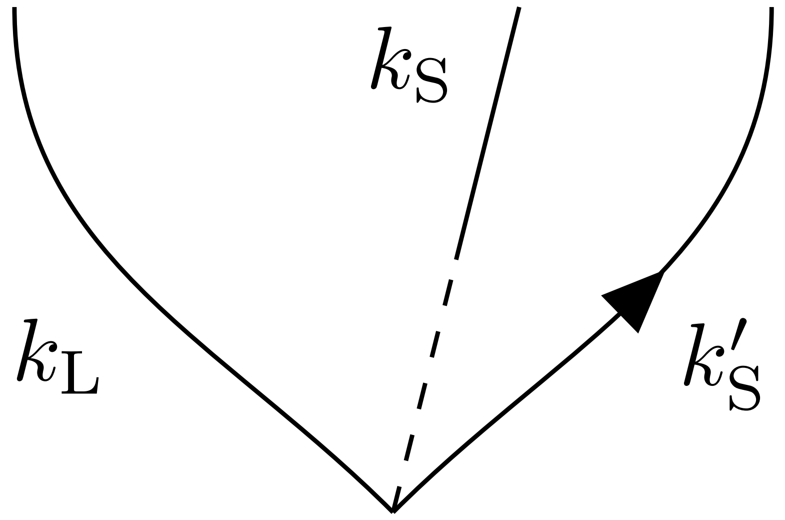
Motohashi & YT '23

during SR2 $\tau \gg \tau_e$

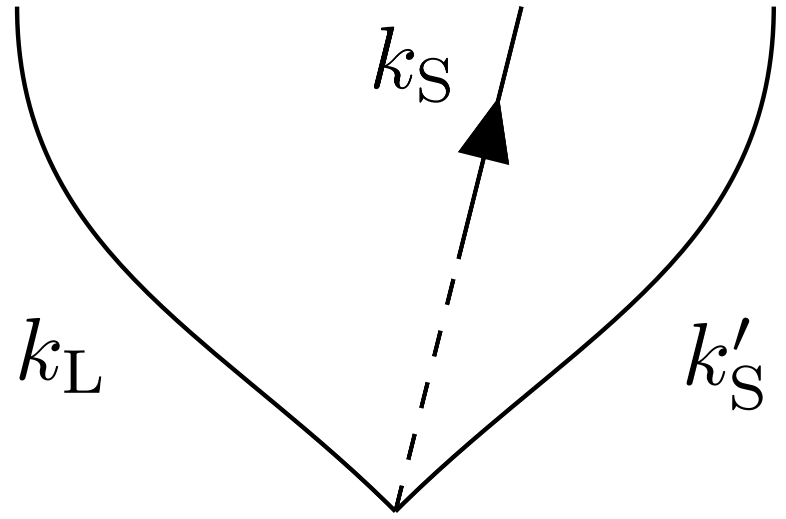
$B_{\zeta\zeta\zeta}(k_L, k_S, k'_S; \tau) =$



$= \quad \sim P_\zeta(k_L) \propto \frac{1}{k_L^3}$

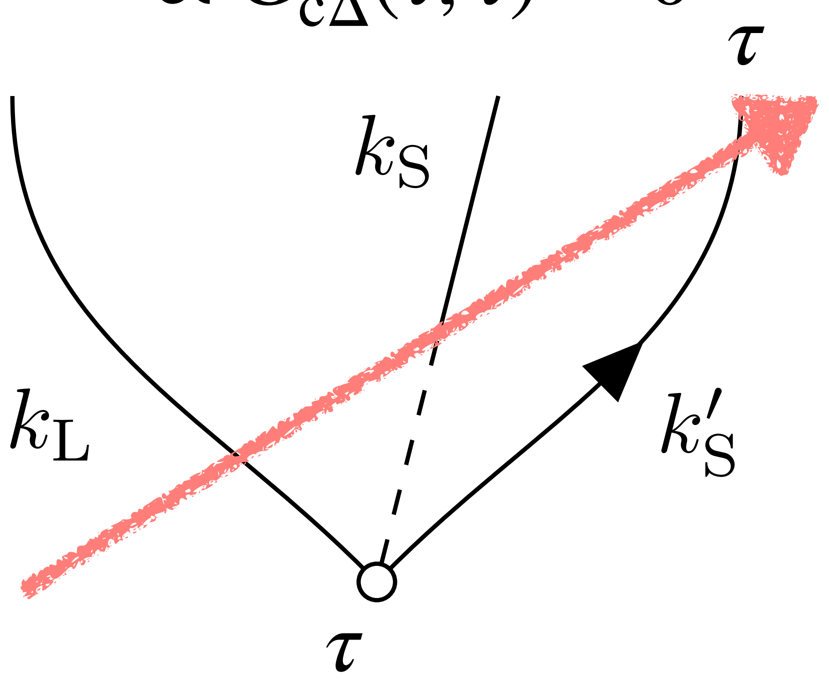


$+$



$\propto G_{c\Delta}(\tau, \tau) = 0$

$+$



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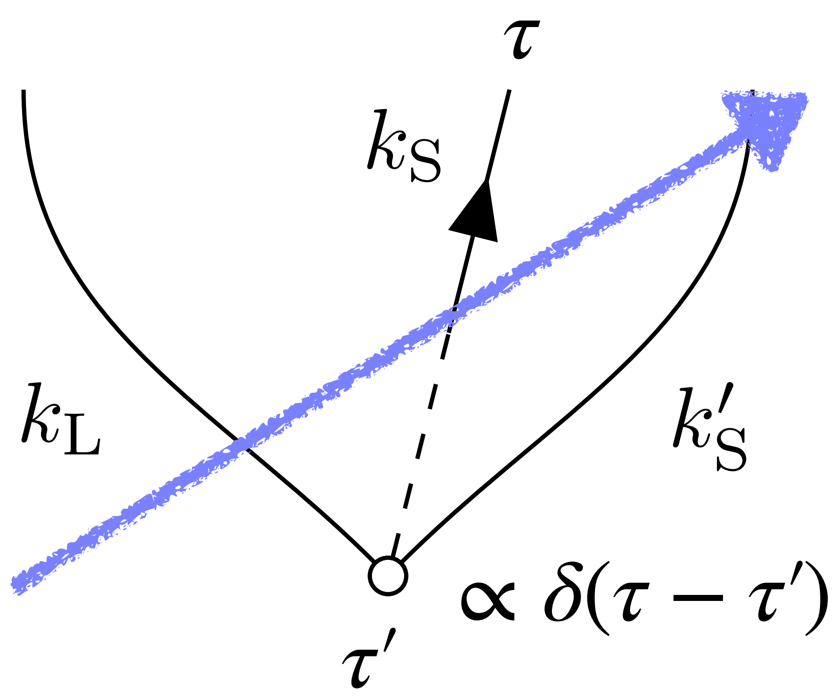
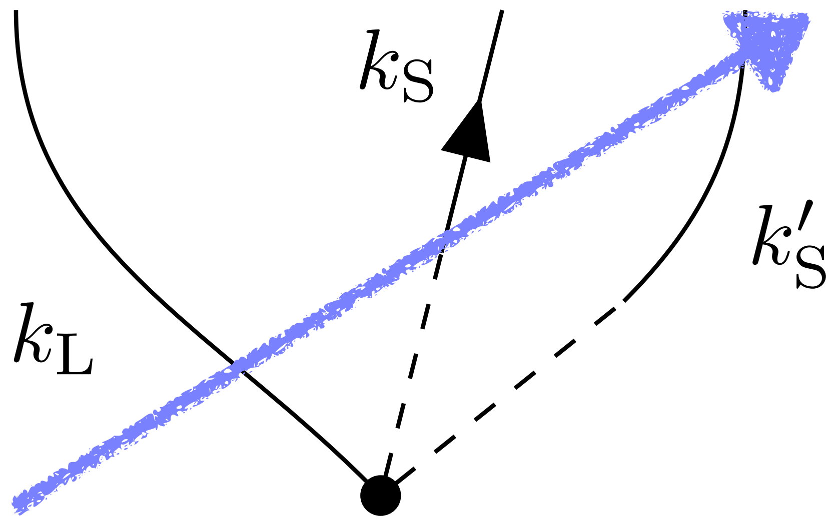


$\propto \delta(\tau - \tau')$

$+$



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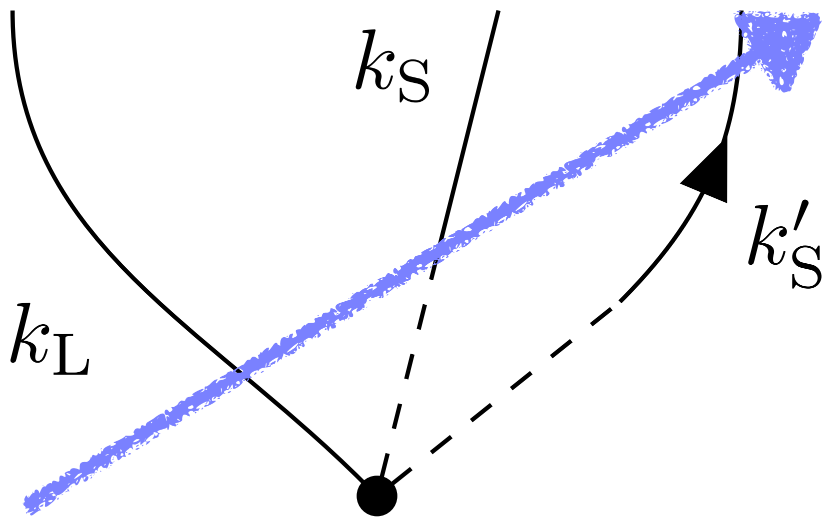
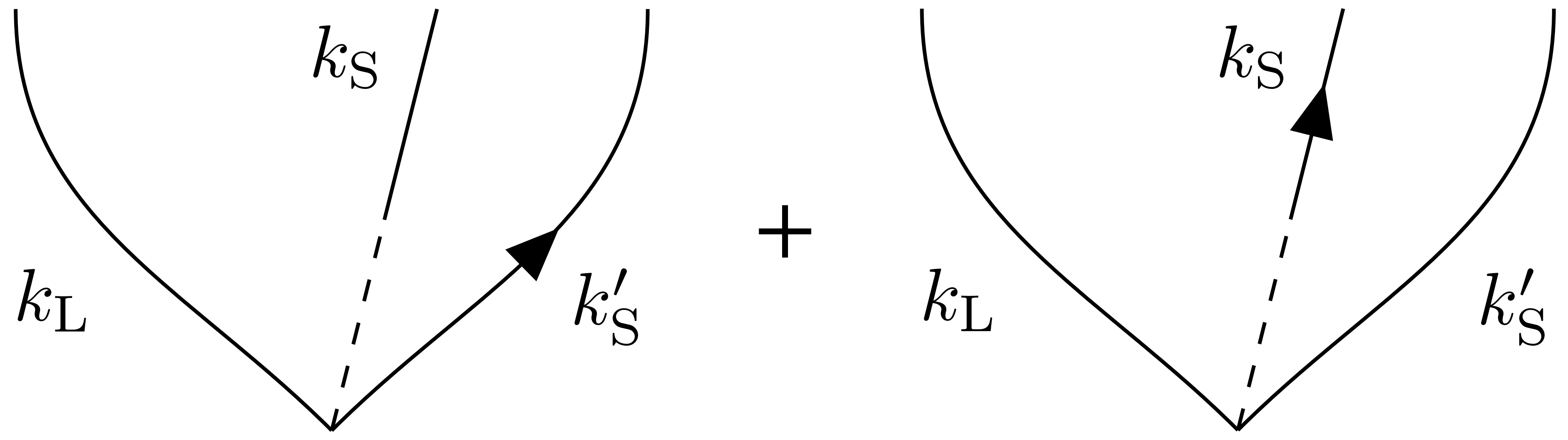
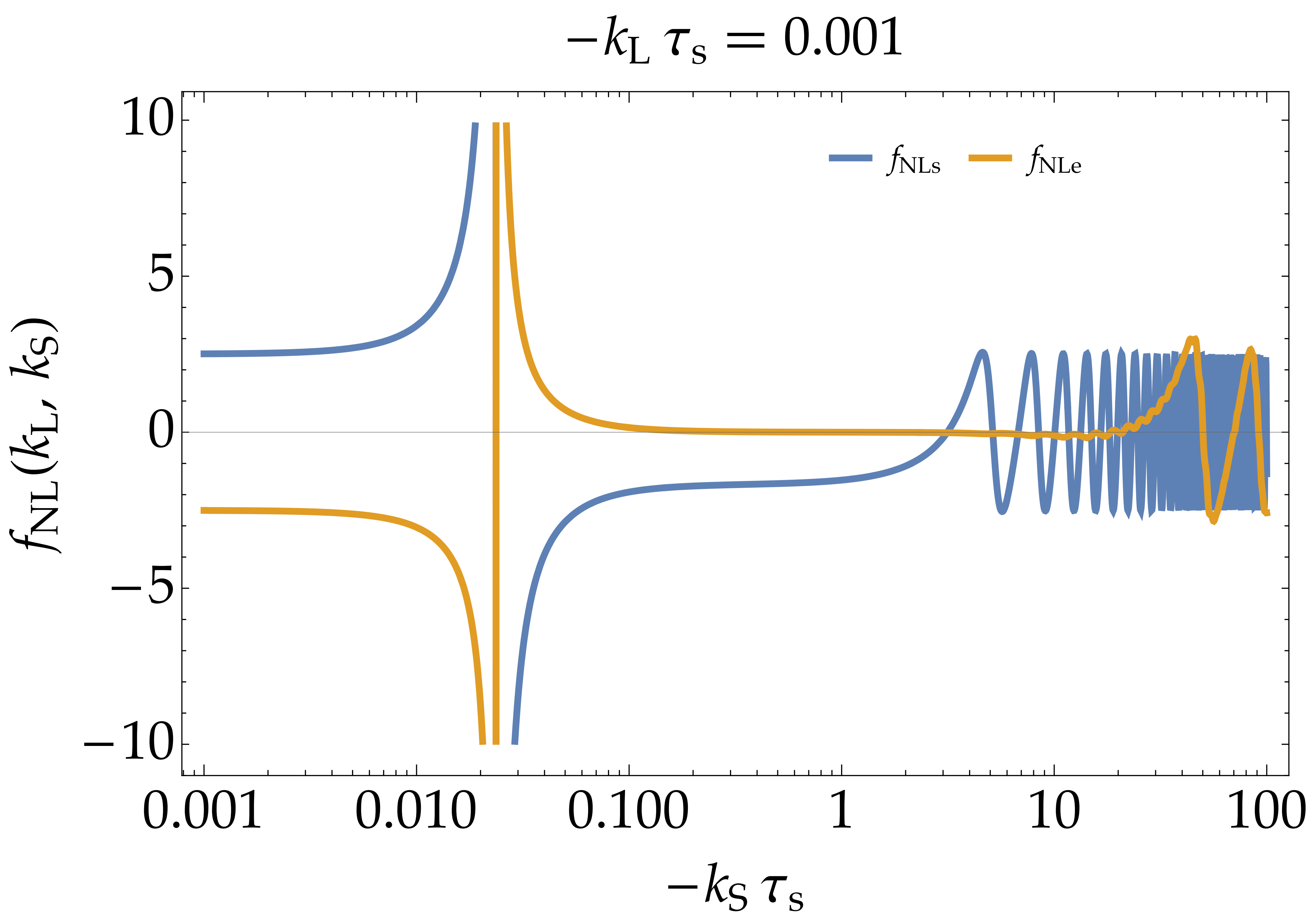


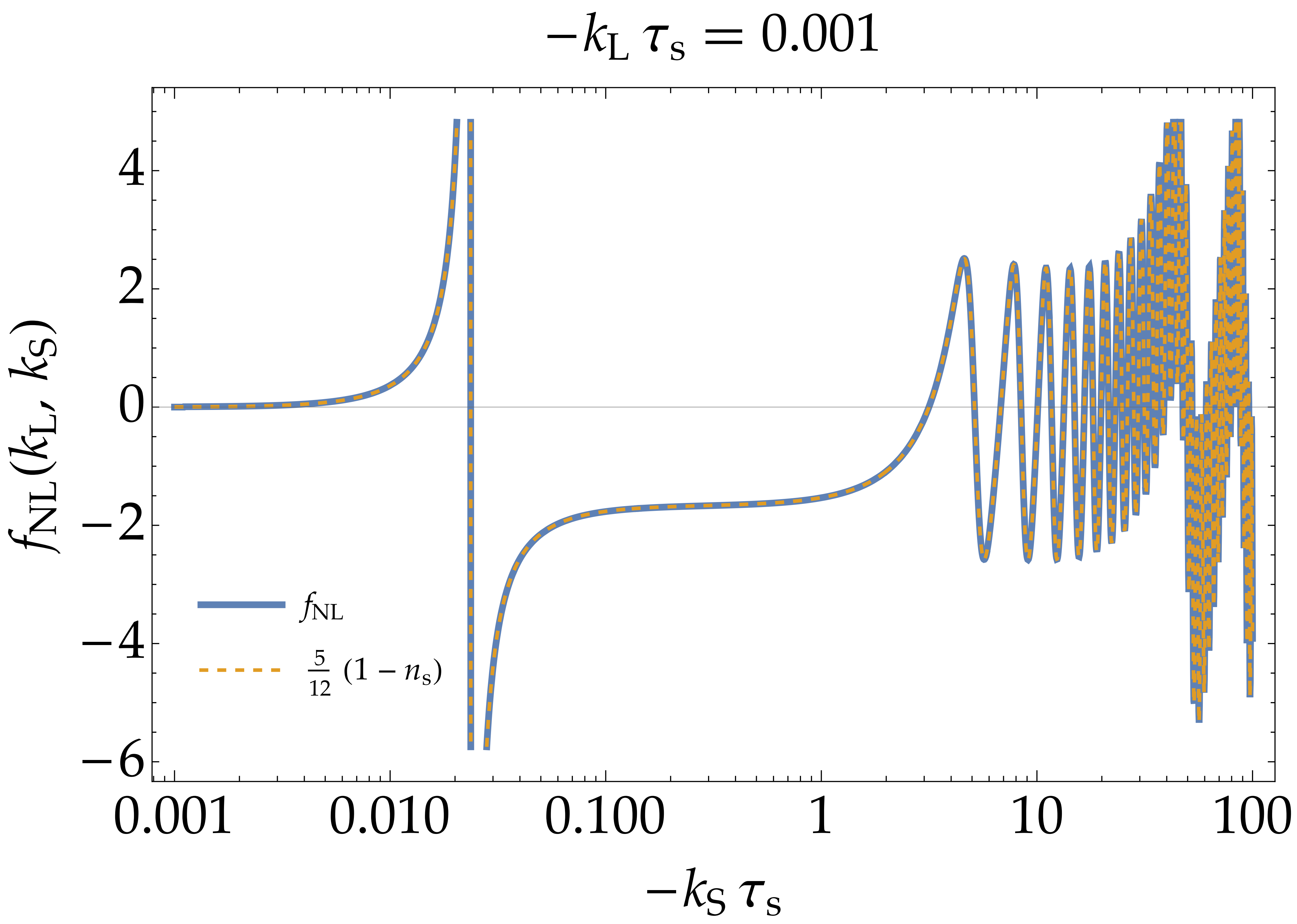
Bispectrum

Motohashi & YT '23

during SR2 $\tau \gg \tau_e$



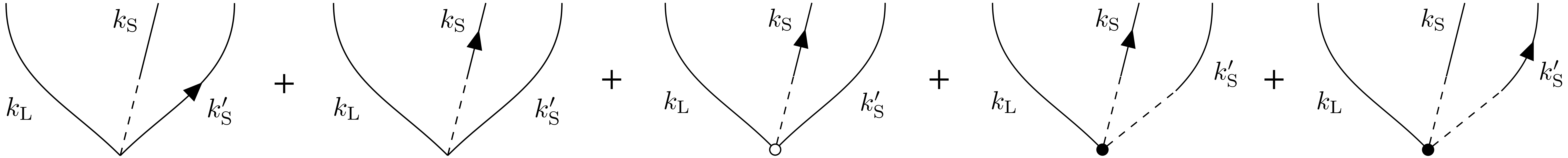




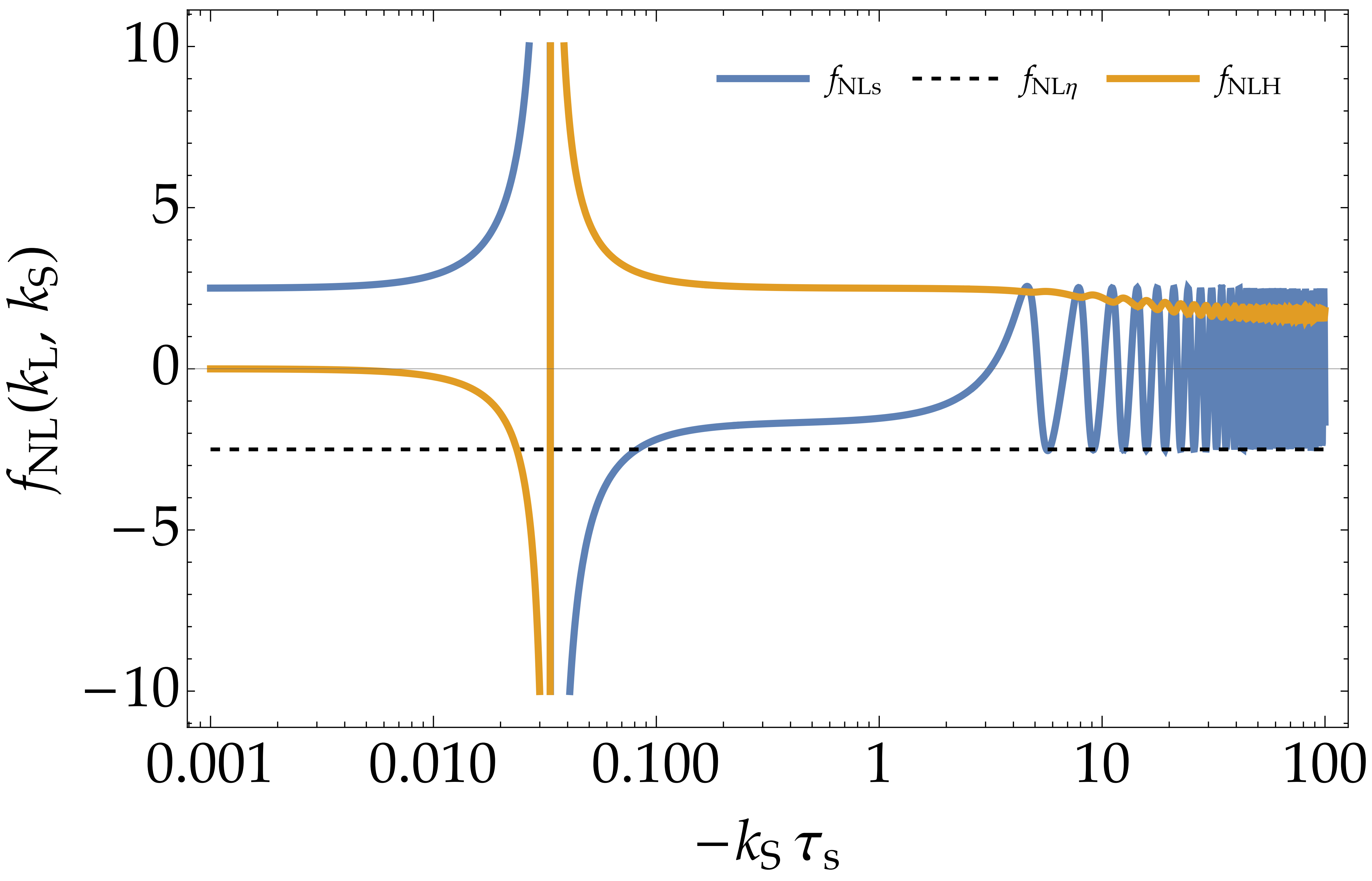
Bispectrum

Motohashi & YT '23

during USR $\tau_s < \tau < \tau_e$



$-k_L\,\tau_s = 0.001$



$-k_L\,\tau_s = 0.001$