Curvature Perturbations Protected Against One Loop

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Looping in the Primordial Universe@CERN, Oct. 29, 2024

One sentence summary

I am going to show that

superhorizon-limit curvature perturbations are constant **even at one loop level**

with the use of **spatially flat gauge.**

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I am going to show that

superhorizon-limit curvature perturbations are constant **even at one loop level**

with the use of **spatially flat gauge**.

(Yes, my conclusion is different from Kristiano's and Ballesteros's talks.)

Outline

- **Recent claim: Curvature is not conserved?**
- Conservation of curvature at one loop
- Summary

What is curvature perturbation?

curvature perturbation

$$\mathrm{d}^2 s = g_{00}\mathrm{d}\eta^2 + 2g_{0i}\mathrm{d}\eta\mathrm{d}x^i + a^2 \mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

(In the gauge where $g_{ij}|_{i\neq j} = 0$)

In the isotropic and homogeneous Universe (FLRW metric),

$$a^2\mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \to a^2\left(\frac{\mathrm{d}^2 r}{1-Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\Omega^2)\right)$$

$$R_3 = \frac{6K}{a^2} = 4\nabla^2\psi$$

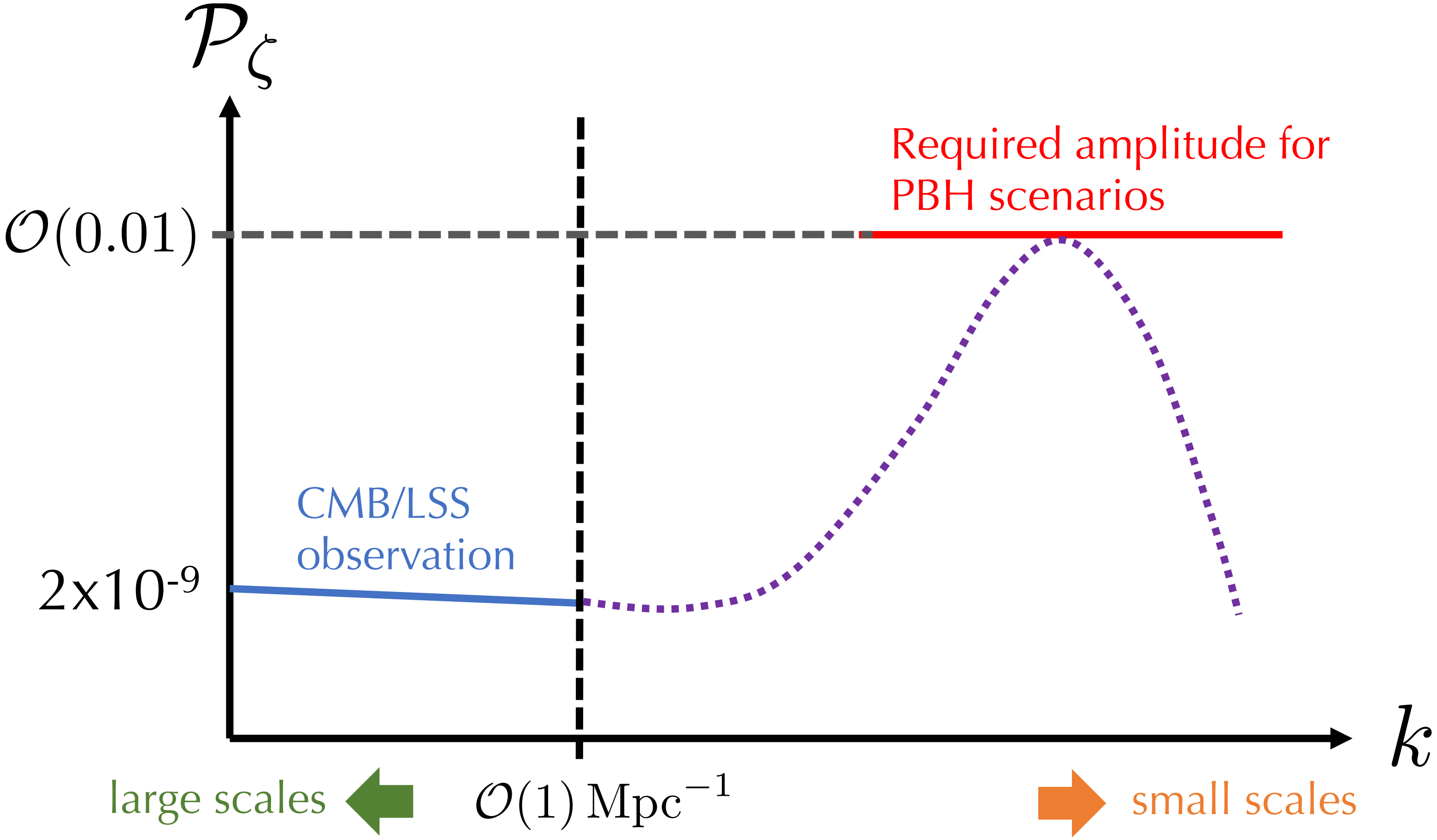
3-dim. Ricci scalar

Note that the spatially homogeneous part of ψ is a gauge artifact because it can be absorbed by coordinate rescaling.

$$\mathrm{d}x^i \to \mathrm{d}x^{i\prime} = \mathrm{e}^{-\psi}\mathrm{d}x^i$$

Large perturbations for PBH scenarios

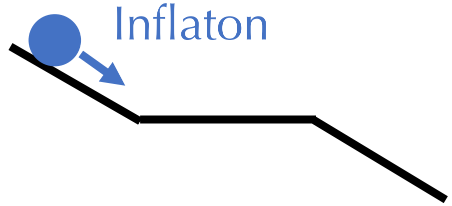
Primordial black holes are candidates of DM and BHs detected by LIGO-Virgo-KAGRA collaborations.



Inflaton potentials for large amplification

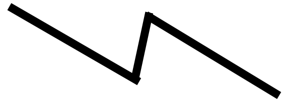
Single field models for large amplification of density perturbations:

flatter region



(Starobinsky 1992, Ivanov+ 1994,
Inoue and Yokoyama 2001, Kinney 2005)

step feature



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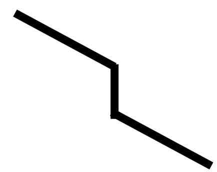
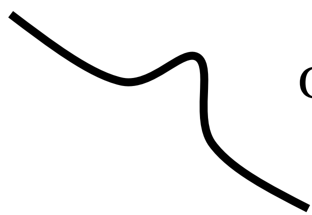


(Kefala+ 2020, Inomata+ 2021)

bump/dip feature



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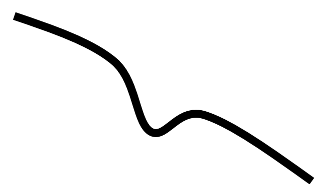
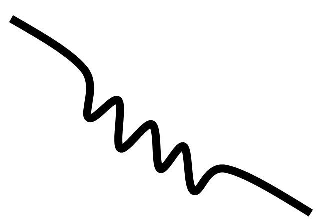


(Ozsoy+ 2018,
Mishra and Sahni 2019)

oscillatory feature



(R.G. Cai+ 2019, Zhou+ 2020,
Peng+ 2021)

One loop corrections

Lagrangian:
$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$$

E.o.m. for the inflaton fluctuations: (slow-roll-parameter suppressed terms neglected) $\quad (V_{(n)} \equiv \partial^n V/\partial\phi^n)$

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

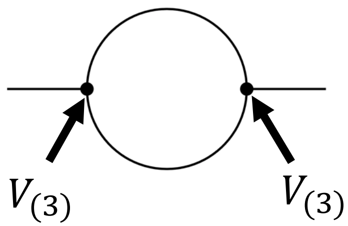
beyond liner order corrections

In-in formalism: (Jordan 1986, Calzetta and Hu 1987, Weinberg 2005)

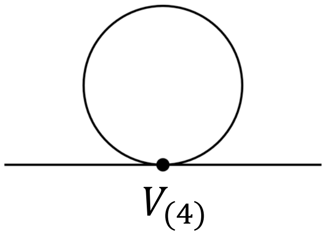
$$\langle\delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta)\rangle = \langle 0|\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta' H_{\mathrm{int}}(\eta')}\right)^\dagger \delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta)\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta'' H_{\mathrm{int}}(\eta'')}\right)|0\rangle$$

$$\left(H_{\mathrm{int},n} \equiv \int \mathrm{d}^3x\; a^4\mathcal{H}_n,\; \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

two vertices



one vertex



The lowest order corrections to linear power spectrum appear as one loops.

Superhorizon curvature evolves?

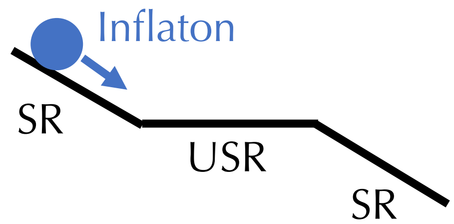
(Note: the conservation of linear ζ is well known.)

Recent claim:
Superhorizon-**limit** curvature perturbations **are not conserved at one-loop level** in the case of slow-roll (SR) → ultra-slow roll (USR) → SR.

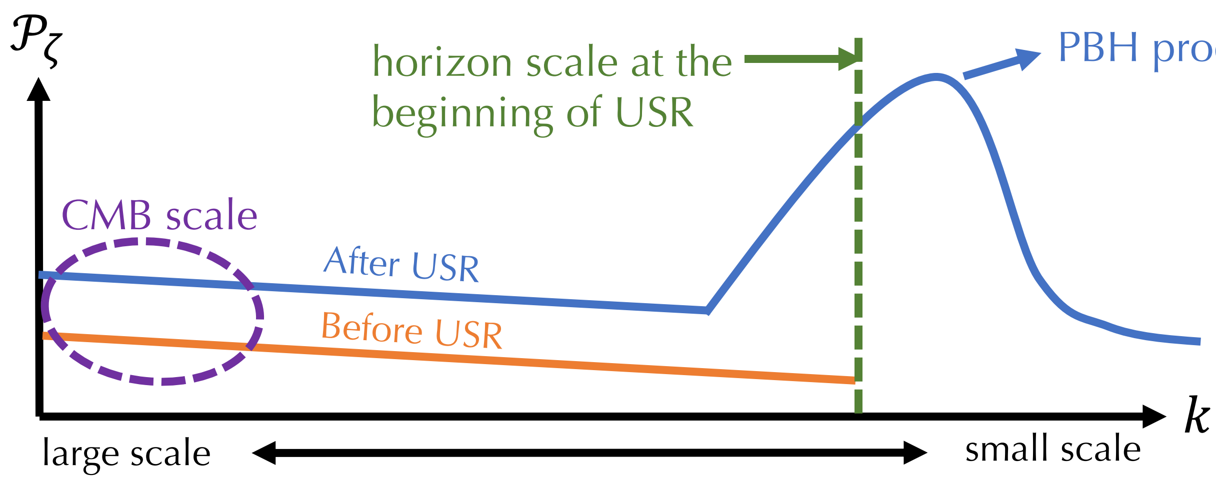
e.o.m.

$$3H\dot{\phi} + V'(\phi) = 0 \text{ (SR)}$$

$$\ddot{\phi} + 3H\dot{\phi} = 0 \text{ (USR)}$$



Kristiano & Yokoyama (2022), followed by
Riotto, Choudhury+, Firouzhahi, Motohashi & Tada, Franciolini+, Gianmassimo, Cheng+, Maity+, Davies+ (2023), Saburov & Ketov, Ballesteros & Gambín (2024)



If the one loop corrections can be comparable to the tree-level power spectrum in some PBH models, CMB spectrum changed. → models constrained?

However, this seems inconsistent with the separate Universe picture...

Why controversial?

Separate universe picture (= cosmological principle + causality)
If we consider a very large region (typically larger than Hubble distance), that region can be regarded as a homogeneous and isotropic Universe with the FLRW metric.
(Sasaki & Tanaka 1998, Wands+ 2000)

Separate universe in single-clock inflation
→ Superhorizon-limit curvature perturbations are conserved at non-perturbative level
(Lyth, Malik, and Sasaki, 2004)

Single-clock = The universe evolution is characterized only by ϕ (inflaton field value).

Single-clock during Ultra Slow Roll (USR)?
→ Depends on the scale of the region we consider.

For the superhorizon-limit region, which exits the horizon much before the USR, we can regard that region as a single-clock.

The recent claim violates the separate universe (cosmological principle or causality)?

Paradox in recent claim

The recent claim says the total ζ evolves as: ($\zeta^{(1)}$ is the superhorizon limit mode)

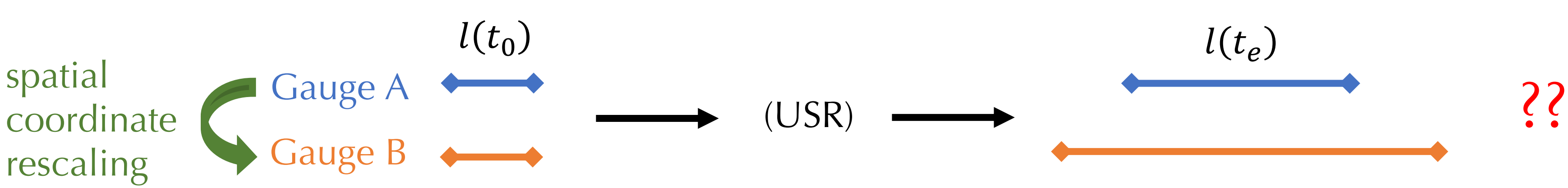
$$\zeta^{(1)} \text{ (first SR)} \to \text{(ultra-slow-roll)} \to \zeta^{(1)}(1+C) \text{ (second SR)}$$

C: some constant proportional to $\mathcal{O}\left(\mathcal{P}_\zeta(k_{peak})\right)$.

Let's consider the distance between two points given by $l^2 \equiv g_{ij}\Delta x^i \Delta x^j$ inside the horizon. l^2 should be invariant under the spatial coordinate rescaling:

$$\Delta \boldsymbol{x} \to \Delta \bar{\boldsymbol{x}} = \mathrm{e}^{\zeta}\Delta \boldsymbol{x}, \;\; g_{ij} \to \bar{g}_{ij} = \mathrm{e}^{-2\zeta} g_{ij}$$

However, if the recent claim is correct, the evolution of l^2 is gauge dependent because the evolution of ζ depends on $\zeta^{(1)}$, which is a gauge artifact for a local observer.



(The time slice is the same in gauge A and B)

Does the universe expansion depend on the spatial coordinate rescaling?

Outline

- Recent claim: Curvature is not conserved?
- **Conservation of curvature at one loop**
- Summary

Two gauges

$$\mathrm{d}^2 s = g_{00}\mathrm{d}\eta^2 + 2g_{0i}\mathrm{d}\eta\mathrm{d}x^i + a^2\mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Inflaton fluctuation

Comoving gauge:

$$\psi \neq 0, \delta\phi = 0$$

Kristiano's, Tada's, and Fumagalli's talk

Spatially-flat gauge:

$$\psi = 0, \delta\phi \neq 0$$

Ballesteros's talk and **my talk**

Strategy of this work

In this work, **spatially-flat gauge ($\psi = 0$)** is taken, where $\delta\phi$ is the basic quantity.

The metric perturbations are suppressed by slow-roll parameter ϵ, compared to $V_{(n)}$ terms.

($\epsilon \to 0$ is known as the decoupling limit in effective field theory of inflation.)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right]$$

$$\downarrow \qquad \left(V_{(n)} \equiv \frac{\partial^n V(\phi)}{\partial\phi^n}\right)$$

$$S_n = -\int \mathrm{d}^4x\, a^4 \frac{V_{(n)}(\bar{\phi})}{n!}\delta\phi^n$$

Simple!

Advantage: The higher order action can be easily obtained.
$\rightarrow$ no need to worry about boundary terms!

Strategy: We first calculate the one-loop power spectrum of $\delta\phi$.
Then, we connect it to the one loop-power spectrum of ζ.

One loop calculation

$(\phi(\boldsymbol{x}) = \bar{\phi} + \delta\phi^{(1)}(\boldsymbol{x}) + \delta\phi^{(2)}(\boldsymbol{x}) + \delta\phi^{(3)}(\boldsymbol{x}) + \cdots \text{ with } \bar{\phi} = \langle \phi(\boldsymbol{x}) \rangle)$

Equation of motion:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

$$\hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2 V_{(2)}(\bar{\phi})$$

$$\delta\phi^{(1)}(\boldsymbol{x},\eta) = \int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}\delta\phi^{(1)}_{\boldsymbol{k}}(\eta) = \int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}\left[U_k(\eta)\hat{a}(\boldsymbol{k}) + U^*_k(\eta)\hat{a}^\dagger(-\boldsymbol{k})\right]$$

$$\hat{\mathcal{N}}_k\delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

$$\hat{\mathcal{N}}_k\delta\phi^{(2)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}},$$

$$\hat{\mathcal{N}}_k\delta\phi^{(3)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}(\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(2)}_{\boldsymbol{p}} + \delta\phi^{(2)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}}) - \frac{a^2}{6}V_{(4)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int\frac{\mathrm{d}^3p'}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}'}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}'}$$

In-in formalism = equation-of-motion approach: (Musso (2013), Inomata+ (2022))

$$\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle = \langle 0|\left(T\mathrm{e}^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta' H_{\rm int}(\eta')}\right)^\dagger \delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}\left(T\mathrm{e}^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta'' H_{\rm int}(\eta'')}\right)|0\rangle$$

$$= \ldots \qquad \left(H_{\mathrm{int},n} \equiv \int\mathrm{d}^3x\, a^4\mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

$$= \langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(2)}_{\mathbf{k}}\delta\phi^{(2)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(3)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(3)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle$$

One loop calculation

$\left(\phi(\boldsymbol{x}) = \bar{\phi} + \delta\phi^{(1)}(\boldsymbol{x}) + \delta\phi^{(2)}(\boldsymbol{x}) + \delta\phi^{(3)}(\boldsymbol{x}) + \cdots \text{ with } \bar{\phi} = \langle \phi(\boldsymbol{x}) \rangle\right)$

Equation of motion:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

$$\hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2 V_{(2)}(\bar{\phi})$$

$$\delta\phi^{(1)}(\boldsymbol{x},\eta) = \int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}\delta\phi^{(1)}_{\boldsymbol{k}}(\eta) = \int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}\left[U_k(\eta)\hat{a}(\boldsymbol{k}) + U_k^*(\eta)\hat{a}^\dagger(-\boldsymbol{k})\right]$$

$$\hat{\mathcal{N}}_k\delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

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$$\hat{\mathcal{N}}_k\delta\phi^{(3)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left(\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(2)}_{\boldsymbol{p}} + \delta\phi^{(2)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}}\right) - \frac{a^2}{6}V_{(4)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int\frac{\mathrm{d}^3p'}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}'}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}'}$$

In-in formalism = equation-of-motion approach: (Musso (2013), Inomata+ (2022))

$$\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle = \langle 0|\left(T\mathrm{e}^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta' H_{\rm int}(\eta')}\right)^\dagger \delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}\left(T\mathrm{e}^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta'' H_{\rm int}(\eta'')}\right)|0\rangle$$

$$= \dots \qquad \left(H_{{\rm int},n} \equiv \int \mathrm{d}^3x\, a^4\mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

$$= \underbrace{\langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle}_{\text{tree}} + \underbrace{\langle 0|\delta\phi^{(2)}_{\mathbf{k}}\delta\phi^{(2)}_{\mathbf{k}'}|0\rangle}_{\text{Poisson fluctuations}} + \underbrace{\langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(3)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(3)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle}_{\text{nonzero even in superhorizon limit}}$$

tree

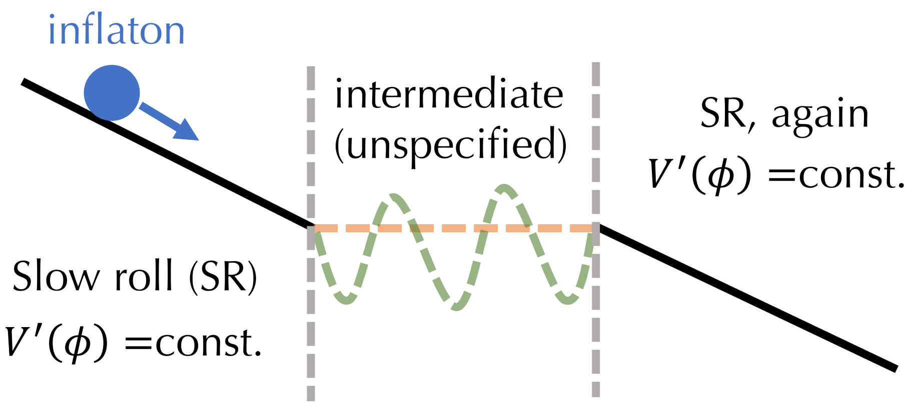
Poisson fluctuations
→ 0 in superhorizon limit

nonzero even in superhorizon limit

Superhorizon curvature perturbations evolve at one loop? → **There is a trick!**

Conservation of curvature

The trick lies in the relation between $\delta\phi$ and ζ.
Consider the simplest case:



During the intermediate period,
ζ is enhanced.
→ Peak power spectrum

We assume the separate Universe satisfied at least during the SR periods **(We do not assume separate Universe during the intermediate period)**.

From δN formalism, curvature perturbations during the SR periods are:

$$-\zeta|_{\leq 1\text{-loop}} = \left.\frac{H\delta\phi}{\dot{\phi}}\right|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\phi}^{(0)} + \dot{\phi}^{(2)}} \quad \text{(SR)}$$

On the other hand, $\frac{H\delta\phi}{\dot{\phi}}$ is **always** constant:

$$\left.\frac{H\delta\phi}{\dot{\phi}}\right|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\phi}^{(0)} + \dot{\phi}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\phi}^{(0)}} = \text{const.}$$

Point: $\dot{\phi}$ gets the one-loop backreaction, $\dot{\phi}^{(2)}$, which cancels $\delta\phi^{(3)}$. (Backreaction is ignored in Ballesteros's talk)

ζ during the first and the second SR periods coincide. → ζ is conserved!

One loop backreaction

$$\ddot{\bar\phi} + 3H\dot{\bar\phi} + V_{(1)}(\bar\phi) = -\frac{1}{2}V_{(3)}(\bar\phi)\left\langle(\delta\phi^{(1)})^2\right\rangle$$

Take time derivative

$$\left(\bar\Pi \equiv \dot{\bar\phi},\ \hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2 V_{(2)}(\bar\phi)\right)$$

$$\hat{\mathcal{N}}_0 \bar\Pi^{(0)} = 0,$$

$$\hat{\mathcal{N}}_0 \bar\Pi^{(2)} = -\frac{a^2}{2}\left(V_{(3)}\left\langle\delta\phi^2\right\rangle^{\cdot} + V_{(4)}\left\langle\delta\phi^2\right\rangle\bar\Pi^{(0)}\right)$$

Recap:

$$\hat{\mathcal{N}}_k \delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

$$\hat{\mathcal{N}}_k \delta\phi^{(3)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left(\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(2)}_{\boldsymbol{p}} + \delta\phi^{(2)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}}\right) - \frac{a^2}{6}V_{(4)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int\frac{\mathrm{d}^3p'}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}'}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}'}$$

After some (easy) calculation, we find

$$\hat{\mathcal{N}}_q \delta\phi^{(3)}_{\boldsymbol{q}}\Big|_{q\to0} = -\frac{a^2}{2}\left(V_{(3)}\left\langle\delta\phi^2\right\rangle^{\cdot} + V_{(4)}\left\langle\delta\phi^2\right\rangle\bar\Pi^{(0)}\right)\frac{\delta\phi^{(1)}_{\boldsymbol{q}}}{\bar\Pi^{(0)}}$$

We finally obtain

$$\delta\phi^{(3)}_{\boldsymbol{q}} = \frac{\dot{\bar\phi}^{(2)}}{\dot{\bar\phi}^{(0)}}\delta\phi^{(1)}_{\boldsymbol{q}} \quad\Rightarrow\quad \frac{H\delta\phi}{\dot{\bar\phi}}\Bigg|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar\phi}^{(0)} + \dot{\bar\phi}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar\phi}^{(0)}} = \text{const.}$$

This relation is always satisfied.

Renormalization

In general, the loop contributions have divergence, which must be cancelled out by counter terms.

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(2)} = -\frac{a^2}{2}\left(V_{(3)} \left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)} \left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)}\right)$$

$$\hat{\mathcal{N}}_q \delta\phi_{\boldsymbol{q}}^{(3)}\Big|_{q\to 0} = -\frac{a^2}{2}\left(V_{(3)} \left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)} \left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)}\right) \frac{\delta\phi_{\boldsymbol{q}}^{(1)}}{\bar{\Pi}^{(0)}}$$

Counter terms are introduced in the same way for $\bar{\Pi}^{(2)} (= \dot{\bar{\phi}}^{(2)})$ and $\delta\phi_q^{(3)}$ through $\hat{\mathcal{N}}_0$.

$$\hat{\mathcal{N}}_0 \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + a^2 V_{(2)}(\bar{\phi}) + \underline{a^2 m_{\text{ct.}}^2}$$

counter term

The introduction of the counter terms does not break the following relations:

$$\delta\phi_{\boldsymbol{q}}^{(3)} = \frac{\dot{\bar{\phi}}^{(2)}}{\dot{\bar{\phi}}^{(0)}} \delta\phi_{\boldsymbol{q}}^{(1)} \qquad -\zeta = \frac{H\delta\phi}{\dot{\bar{\phi}}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$

Point: Counter terms for $\delta\phi^{(3)}$ are not independent of those for the inflaton potential.

QFT: static background ⟷ Cosmology: nonstatic background

Outline

- Recent claim: Curvature is not conserved?
- Conservation of curvature at one loop
- **<u>Summary</u>**

Summary

Non-conservation of superhorizon curvature perturbations at one-loop level has been claimed recently.

However, the claim seems inconsistent with the separate Universe picture.

I have taken the spatially-flat gauge and focused on $\delta\phi$ evolution at one loop level.

I have finally found that the superhorizon curvature is conserved if we carefully consider the one-loop backreaction.

(Note added: Apart from the talks in this workshop, conservation of curvature has also been shown in Kawaguchi+ 2024 with the path integral method.)

One sentence summary

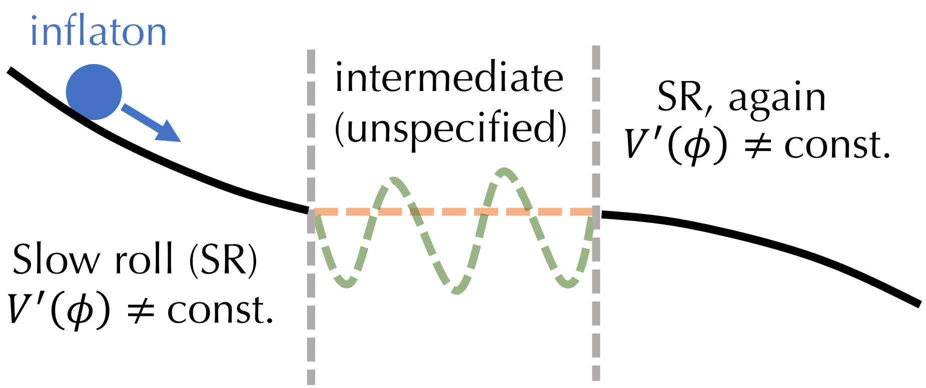
I have shown that

superhorizon-limit curvature perturbations are constant **even at one loop level**

with the use of **spatially flat gauge.**

Backup

In general SR potentials



Again, we assume the separate Universe satisfied at least during the SR periods **(do not assume that during the intermediate period)**.

When the separate Universe is satisfied, the curvature perturbations are conserved even at non-perturbative (including one-loop) level. (Lyth, Malik, and Sasaki, 2004)

This means that, if we find one concrete SR potential for the conservation of ζ, the conservation is secured for any types of SR potential.

One concrete example: the SR potentials that have region of $V'(\phi) =$ const.

Background equation of motion

We define the background as the homogeneous mode of the inflaton:

$$\phi(\boldsymbol{x}) = \bar{\phi} + \delta\phi^{(1)}(\boldsymbol{x}) + \delta\phi^{(2)}(\boldsymbol{x}) + \delta\phi^{(3)}(\boldsymbol{x}) + \cdots$$

The background can be expressed with the spatial average of the inflaton:

$$\bar{\phi} = \langle \phi(\boldsymbol{x}) \rangle$$

By definition, $< \delta\boldsymbol{\phi}(\boldsymbol{x}) > = 0$ is satisfied.

The equation of motion is given by

$$\phi'' + 2\mathcal{H}\phi' - \nabla^2\phi + a^2 V_{(1)}(\phi) = 0$$

Extracting the homogeneous mode, we obtain the background EOM:

$$\begin{aligned}
&\left\langle \phi'' + 2\mathcal{H}\phi' - \nabla^2\phi + a^2 V_{(1)}(\phi) \right\rangle = 0 \\
\Rightarrow\ &\bar{\phi}'' + 2\mathcal{H}\bar{\phi}' + a^2 \left\langle V_{(1)}(\bar{\phi} + \delta\phi^{(1)} + \cdots) \right\rangle = 0 \\
\Rightarrow\ &\bar{\phi}'' + 2\mathcal{H}\bar{\phi}' + a^2 \left\langle V_{(1)}(\bar{\phi}) \right\rangle = -\frac{a^2}{2} V_{(3)}(\bar{\phi}) \left\langle (\delta\phi^{(1)})^2 \right\rangle
\end{aligned}$$

What people call curvature perturbation

$$\mathrm{d}^2 s = g_{00}\mathrm{d}\eta^2 + 2g_{0i}\mathrm{d}\eta\mathrm{d}x^i + a^2 \mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

ψ itself is gauge dependent (depends on the coordinate choice).

In Cosmology, the following gauge-invariant quantities are often used.

$$\zeta = -\psi + \frac{\delta\rho}{3(\rho + P)}$$

ζ coincides the curvature
in uniform density gauge, $\delta\rho = 0$.

$$\mathcal{R} = -\psi - \frac{H\delta\phi}{\dot{\phi}}$$

$\mathcal{R}$ coincides the curvature
in comoving gauge, $\delta\phi = 0$.

In the superhorizon limit,

$$\zeta = \mathcal{R}$$

People often call ζ and $\mathcal{R}$
curvature perturbations.

Ongoing debates

The higher order action is needed for one loop calculation.

Comoving gauge ($\boldsymbol{\delta\phi}$=0) is often taken, where ζ appears as a metric perturbation.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right]$$

astro-ph/0210603, Maldacena

$$\begin{aligned} S_3 = \int & \frac{1}{4}\frac{\dot\phi^4}{\dot\rho^4}[e^{3\rho}\dot\zeta^2\zeta + e^\rho(\partial\zeta)^2\zeta] - \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\partial_i\chi\partial_i\zeta + \\ & -\frac{1}{16}\frac{\dot\phi^6}{\dot\rho^6}e^{3\rho}\dot\zeta^2\zeta + \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\zeta^2\frac{d}{dt}\left[\frac{1}{2}\frac{\ddot\phi}{\dot\phi\dot\rho} + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}\right] + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\partial_i\partial_j\chi\partial_i\partial_j\chi\zeta \\ & + f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1 \end{aligned}$$

arXiv:0709.2708, Jarnhus & Sloth

$$\begin{aligned} S^{(4)} = \frac{1}{2}\int \mathrm{d}t\mathrm{d}^3x a^3 \Bigg\{ & -\frac{1}{3}\zeta^3\partial^2\zeta - 2\alpha^{(1)}(\zeta\partial_i\zeta\partial^i\zeta + \zeta^2\partial^2\zeta) + \dot\phi_c^2\alpha^{(1)2}\left[\frac{9}{2}\zeta^2 - 3\zeta\alpha^{(1)} + \alpha^{(1)2}\right] \\ & \left[\frac{1}{2}\zeta^2 + \zeta\alpha^{(1)} + \alpha^{(1)2}\right]\left[\partial_i\partial_j\chi^{(1)}\partial^i\partial^j\chi^{(1)} - \partial^2\chi^{(1)}\partial^2\chi^{(1)}\right] + (6H^2 - \dot\phi^2)\alpha^{(2)2} \\ & - 2[\zeta + \alpha^{(1)}]\left[\partial_i\partial_j\chi^{(1)}\partial^i\partial^j\chi^{(2)} - \partial^2\chi^{(1)}\partial^2\chi^{(2)} - 2\partial_i\partial_j\chi^{(1)}\partial^i\chi^{(1)}\partial^j\zeta\right] \\ & - 2\left[2\partial_i\partial_j\chi^{(2)}\partial^i\chi^{(1)}\partial^j\zeta + 2\partial_i\partial_j\chi^{(1)}\partial^i\chi^{(2)}\partial^j\zeta - \partial_j\chi^{(1)}\partial_i\zeta\partial^i\chi^{(1)}\partial^j\zeta\right] \\ & + \frac{1}{2}\partial_i\beta_j{}^{(2)}\partial^i\beta^{j\,(2)} - 2\alpha^{(1)}\partial_i\partial_j\chi^{(1)}\partial^i\beta^{j\,(2)} \Bigg\} \end{aligned}$$

However, these expressions neglect the boundary terms.

e.g. $$\int \mathrm{d}t A\dot B = -\int \mathrm{d}t\dot A B + \underline{\int \mathrm{d}t\frac{\mathrm{d}}{\mathrm{d}t}(AB)}$$

boundary term

Ongoing debate: the missing boundary terms lead to the curvature conservation?

Fumagalli (2023), Tada+ (2023), Firouzhahi (2023), Braglia & Pinol (2024)