Gravitational waves in ultra-slow-roll and their anisotropy at two loops

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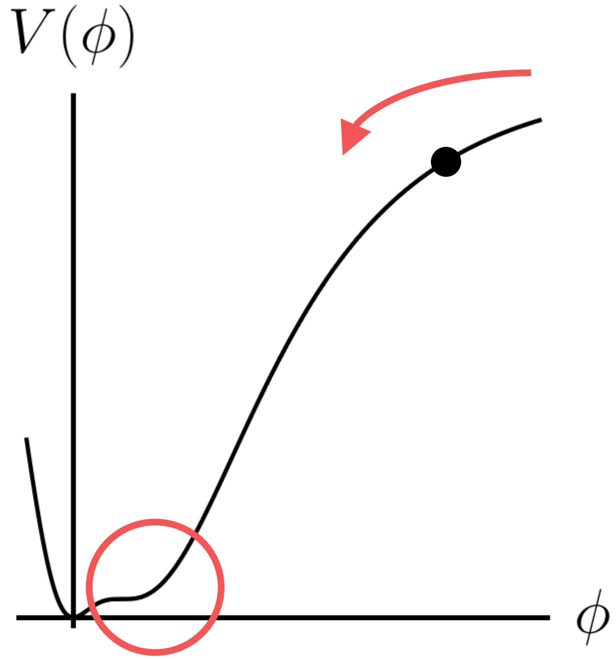
Sketch

Inflation has been studied for 40 years, mostly in slow-roll.
We can learn more by studying different regimes.

1 The field passes an inflection point.



2 Fluctuations grow.

$$\mathcal{P}_{\mathcal{R}} \propto H^4/\dot{\phi}^2$$

3 These fluctuations source tensor modes.★

$$\partial^2 h_{ij} \sim \left[\partial_i \mathcal{R} \partial_j \mathcal{R}\right]^{\mathrm{TT}}$$

4 The gravitational wave spectrum is...

$$\Omega_{\mathrm{GW}} \sim \langle h^2 \rangle \sim \langle \mathcal{R}^4 \rangle$$

We want to study the effect of non-Gaussianities.

★ Domenech 2109.01398

★ Ballesteros, Gambín 2404.07196
♠ Cai et al. 1810.11000, Li et al. 2309.07792

Interactions

Minimally-coupled inflaton.
We work in the $\delta\phi$ gauge

$$\Phi(t, \boldsymbol{x}) = \phi(t) + \delta\phi(t, \boldsymbol{x})$$

We need the fourth-order action.
There are many interactions because of curvature.
Most of them are suppressed by powers of $\dot{\phi}^2$.

Only the self-interactions survive.★

$$S = \int d^4x \left[\frac{a^3}{2} \delta\dot{\phi}^2 - \frac{a}{2} (\nabla \delta\phi)^2 - a^3 \sum_n \frac{V_n}{n!} \delta\phi^n \right]$$

The final step is performing a change of gauge

$$\mathcal{R} = -\frac{H}{\dot{\phi}} \delta\phi - \frac{1}{2}\eta \left(\frac{H}{\dot{\phi}} \delta\phi \right)^2 - \frac{1}{3} \left(\eta^2 + \frac{\dot{\eta}}{2H} \right) \left(\frac{H}{\dot{\phi}} \delta\phi \right)^3 + \mathcal{O}(\delta\phi^4)$$

$f_{\rm NL}$ $\qquad$ $g_{\rm NL}$

Two sources of NG

1. Intrinsic
2. Nonlinearities♠

The δN formalism

This gauge transformation is nothing but the δN formalism.

A change of coordinates gives

$$a(t)^2 e^{2\zeta} = a(t+\delta t)^2$$

$$\phi(t+\delta t) + \delta\phi(t+\delta t) = \phi(t)$$

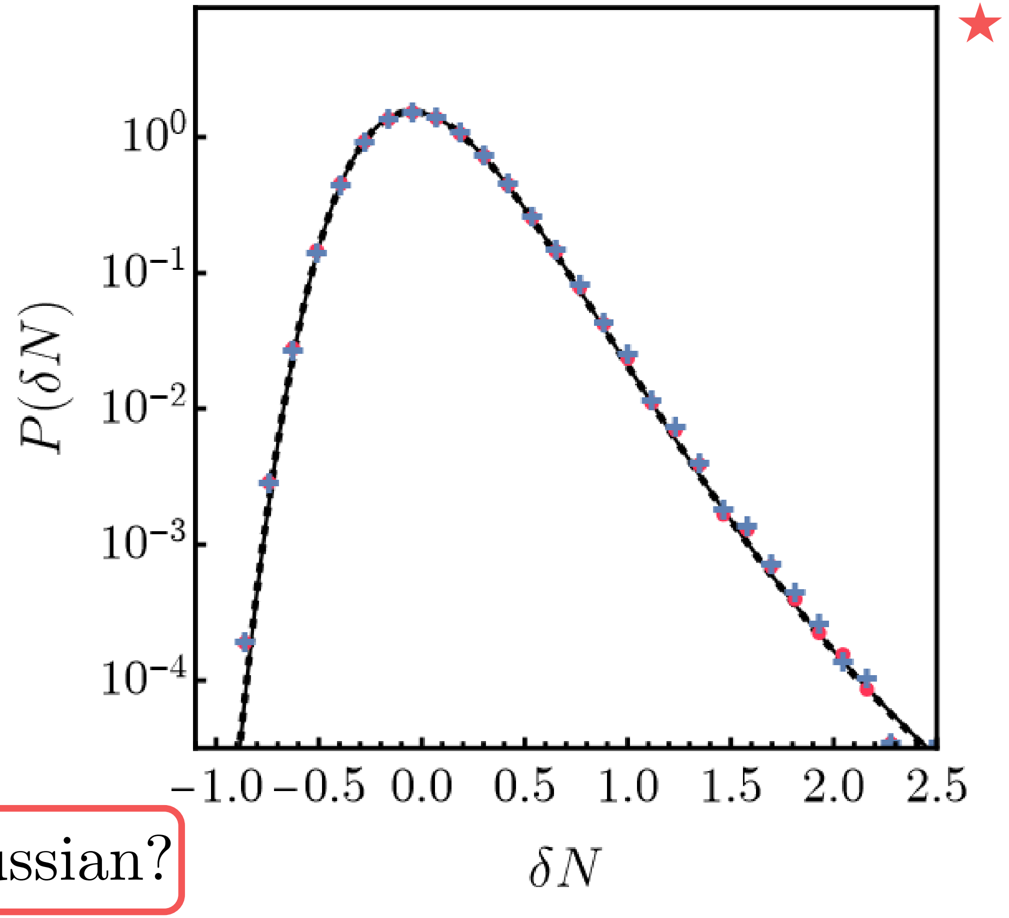
Assuming constant η

$$\mathcal{R} = \frac{1}{\eta}\left(1 - \eta\frac{H}{\dot\phi}\delta\phi\right)$$

This is nothing more than the resummation of

$$\mathcal{R} = \mathcal{R}_{\rm G} + f_{\rm NL}\mathcal{R}_{\rm G}^2 + \cdots$$

$\delta\phi$ Gaussian?

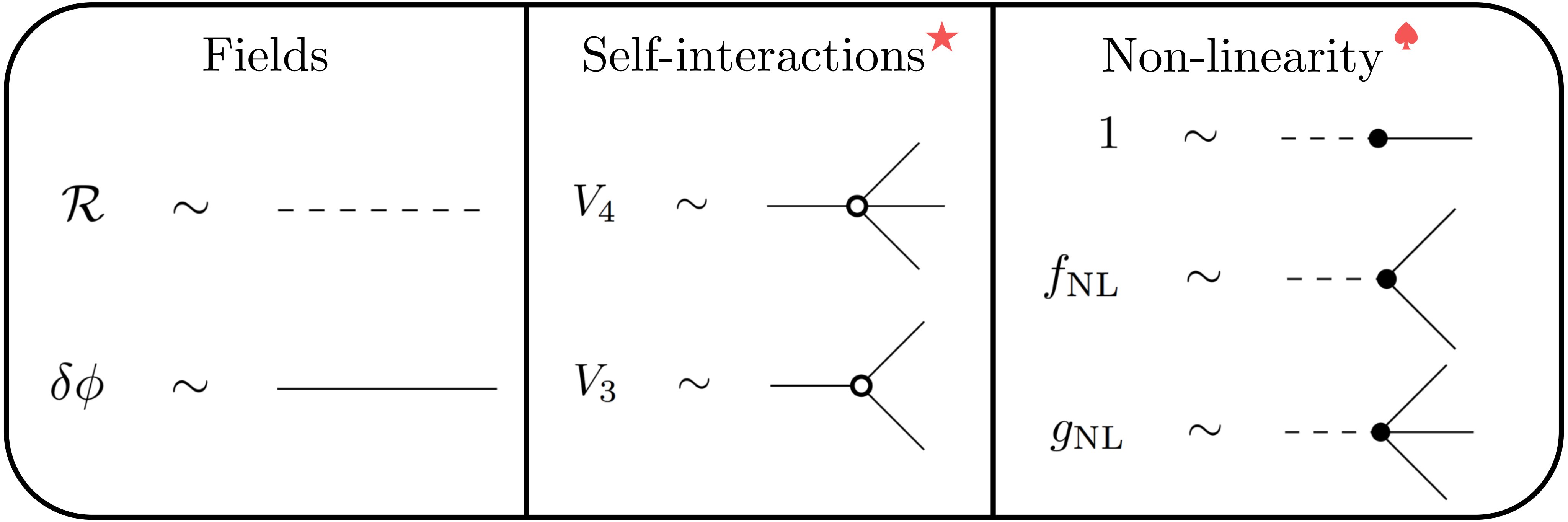


Takeaways: The δN formalism...

1. Is **not** a substitute for in-in
2. Does **not** predict the statistics of $\delta\phi$
3. Is **not** different from stochastic inflation

★ Ballesteros, **JR** et al. 2406.02417

Feynman rules



Correlators of $\mathcal{R}$ have external dashed lines.

1 Choose how to replace each factor of $\mathcal{R}$ by $\delta\phi$.

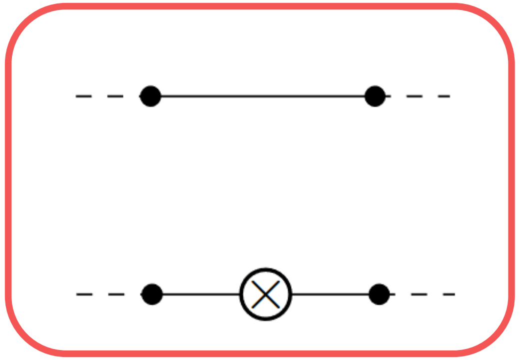
2 Contract the $\delta\phi$ using in-in to account for self-interactions.

★ Ballesteros, Gambín 2404.07196

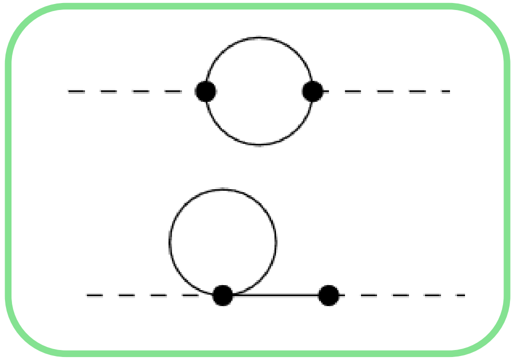
♠ Cai et al. 1810.11000, Li et al. 2309.07792

One-loop spectrum

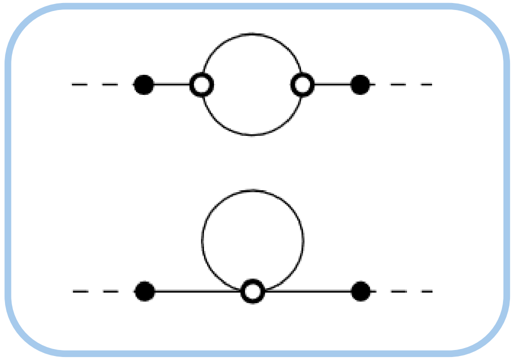
The one-loop scalar power spectrum receives contributions from seven different diagrams.



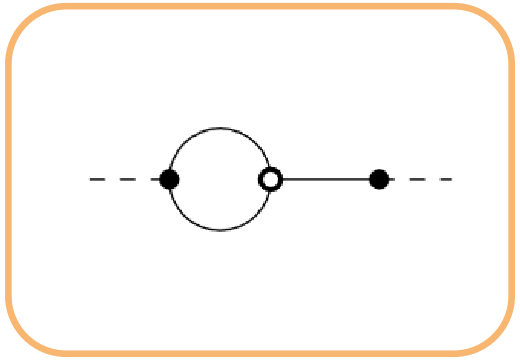
Tree-level & Counterterm



Non-linearities



Self-interactions



Mixed

We treat both kinds of interactions on the same footing so we can determine which one dominates.

1 Quartic loop is absorbed by the counterterm.★

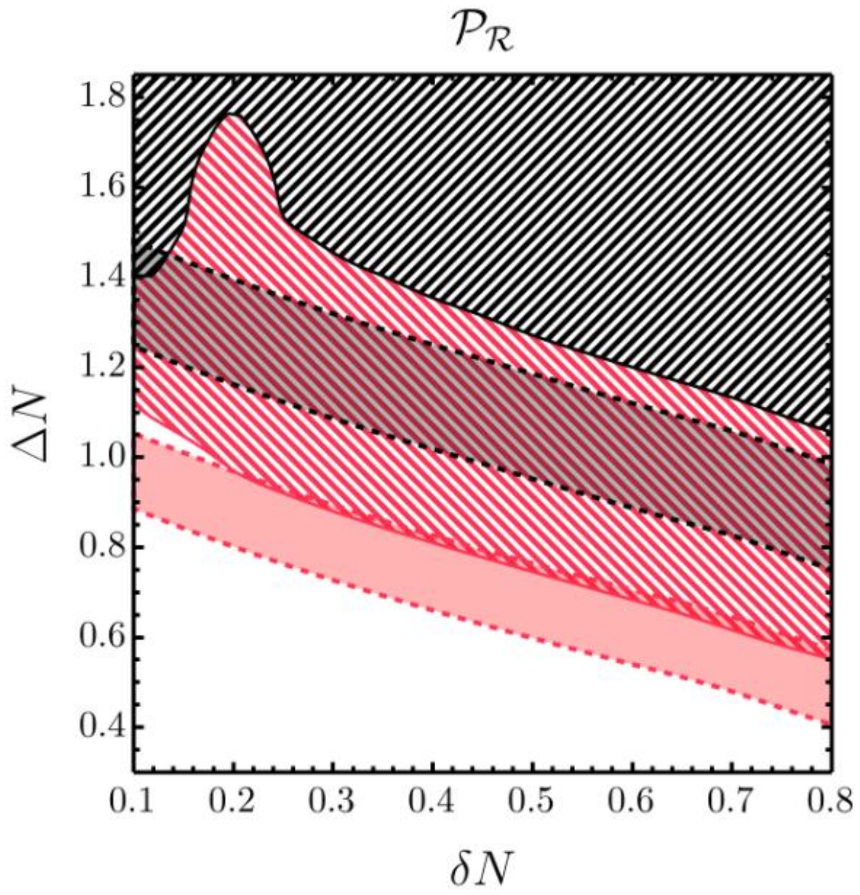
2 Cubic and mixed loops are finite.

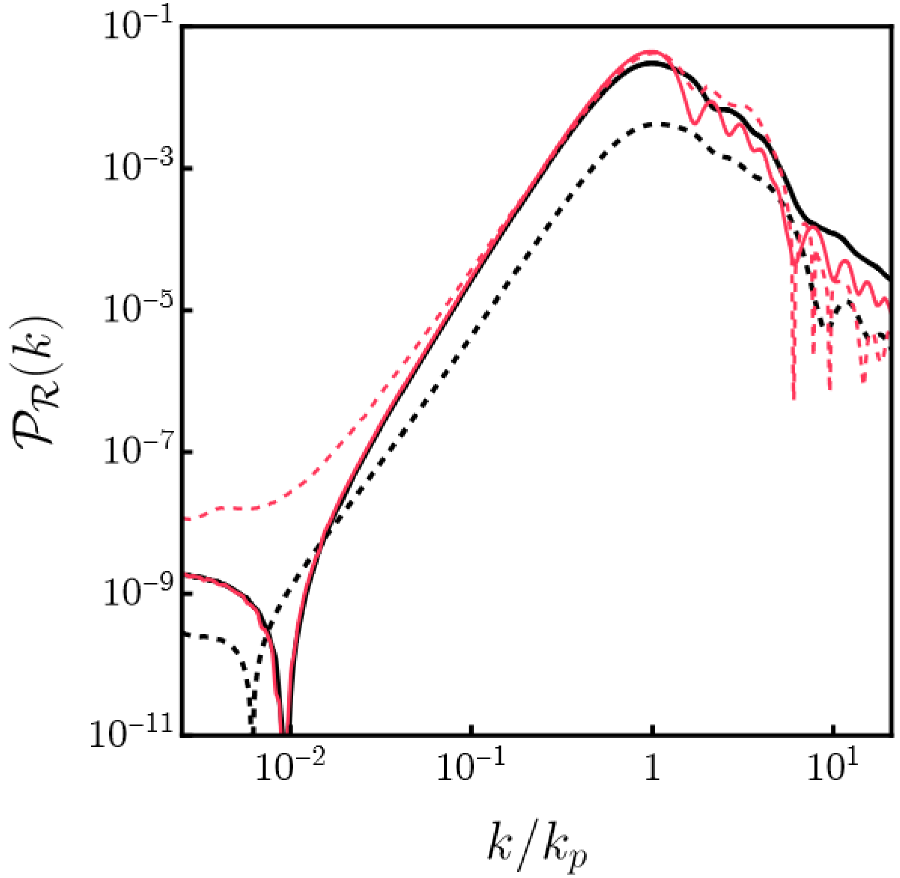
3 Non-linearity loops diverge. We use a cutoff.

★ Ballesteros, Gambín 2404.07196

Numerical results

Perturbation theory may be violated depending on the model parameters.
Self-interactions provide the dominant contribution!





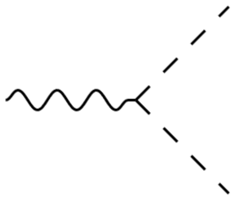
More Feynman rules

We add the following Feynman rules.

Gravitational waves

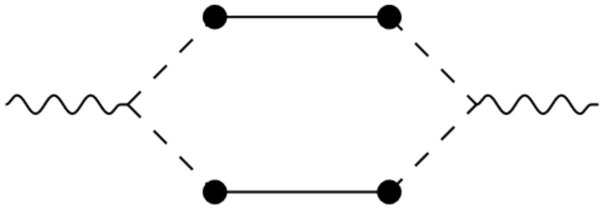
$h \quad \sim$ 

Interactions

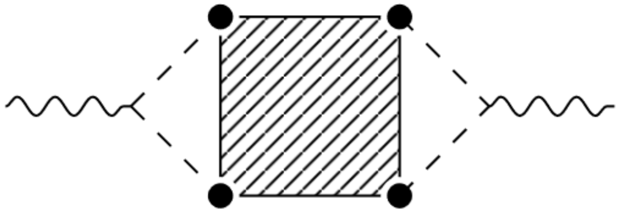
$h\mathcal{R}^2 \quad \sim$ 

We study only the effect on the initial conditions.
We neglect tensor-tensor interactions.

There is only one diagram at the Gaussian level.



Non-Gaussian corrections are obtained by joining the corners of the following box ♠

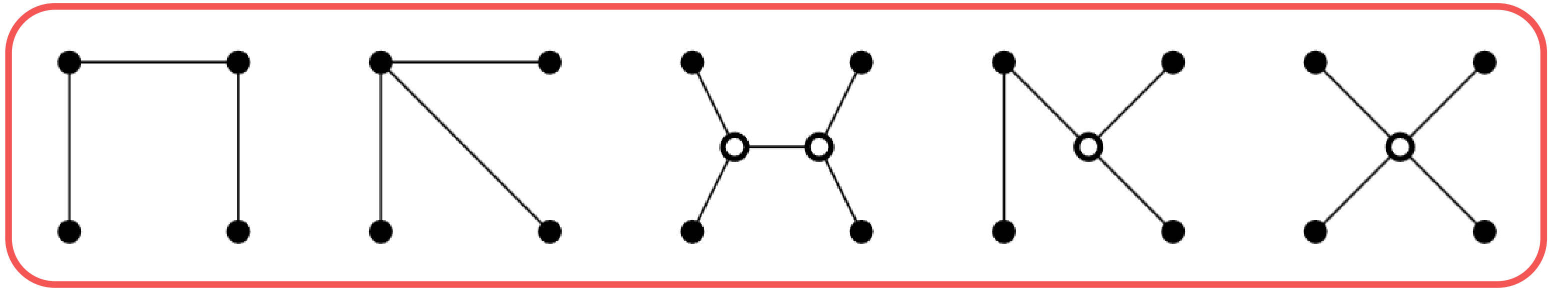


♠ Adshead et al. 2105.01659, Li et al. 2309.07792

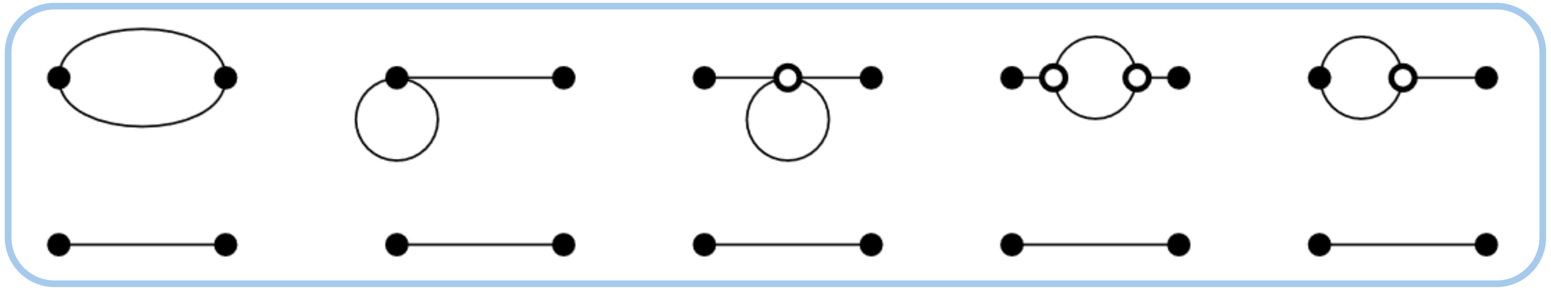
Induced gravitational wave spectrum

The leading contribution is at one loop, so we must go to two loops.

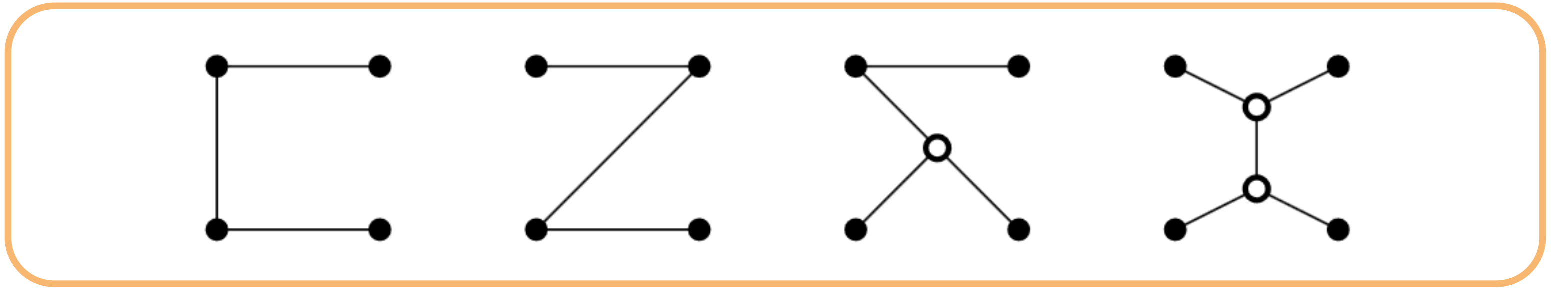
These diagrams violate helicity conservation.



These are obtained by replacing the one-loop scalar spectrum.



These involve computing 6D integrals.



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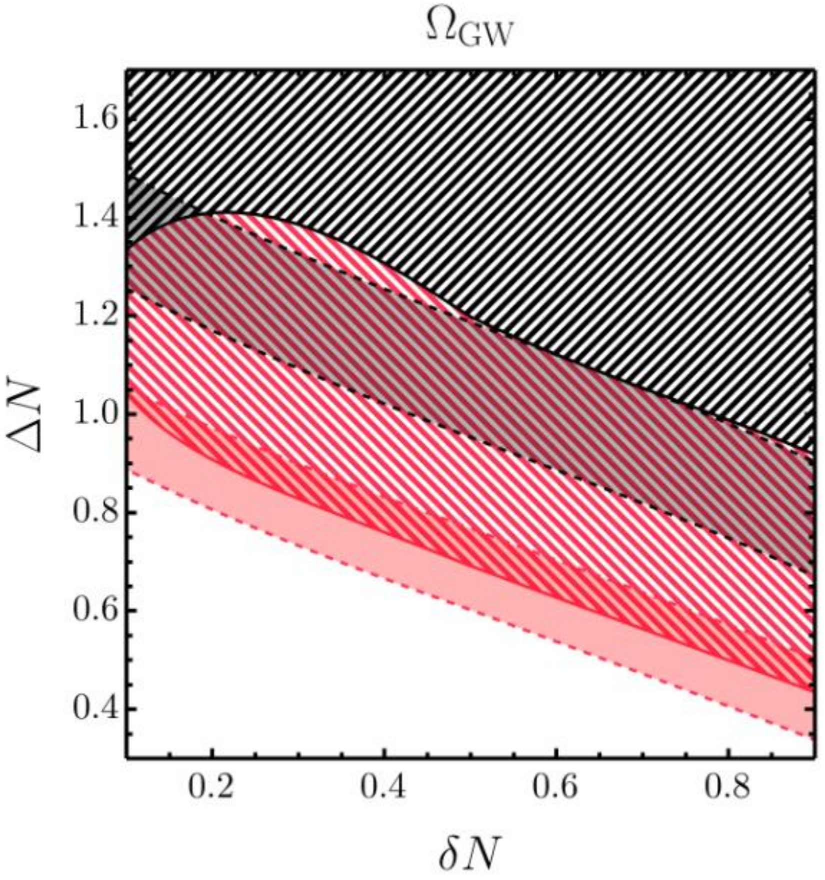
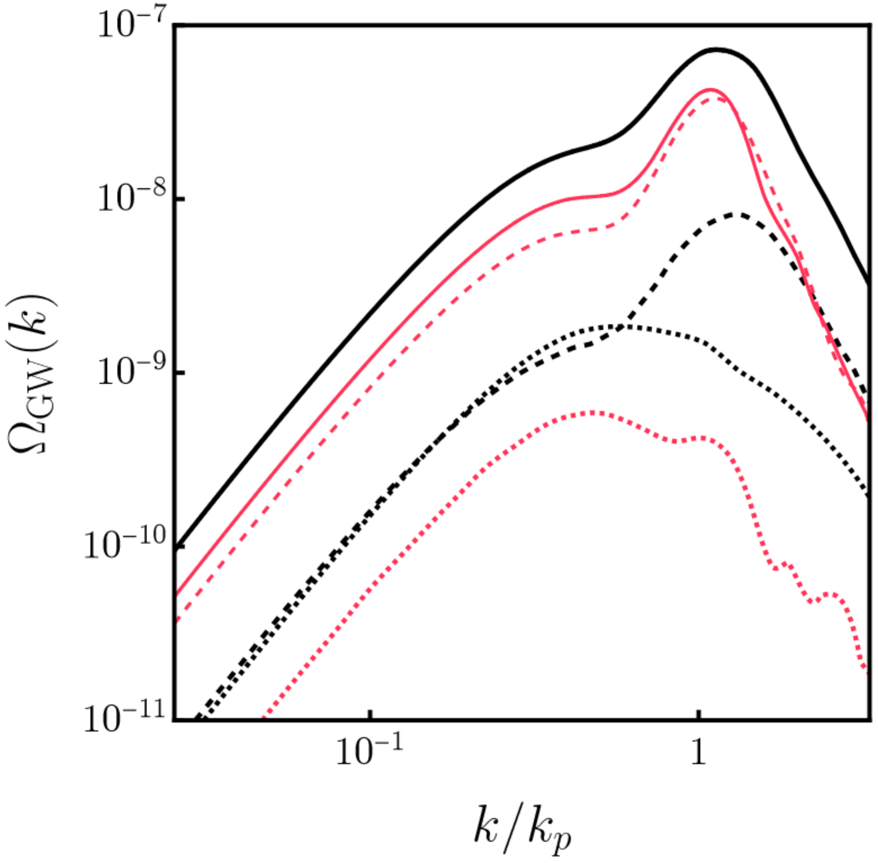




Anisotropies

The GW density contrast is $\delta_{\rm GW} = \delta\rho_{\rm GW}/\langle\rho_{\rm GW}\rangle \sim h^2$

Anisotropies are encoded in $\langle\delta^2_{\rm GW}\rangle \sim \langle\mathcal{R}^8\rangle$

Density contrast

$\delta_{\rm GW} \sim$ 

1 We can only measure anisotropies on large scales.

2 The generation of GWs is a local phenomenon.

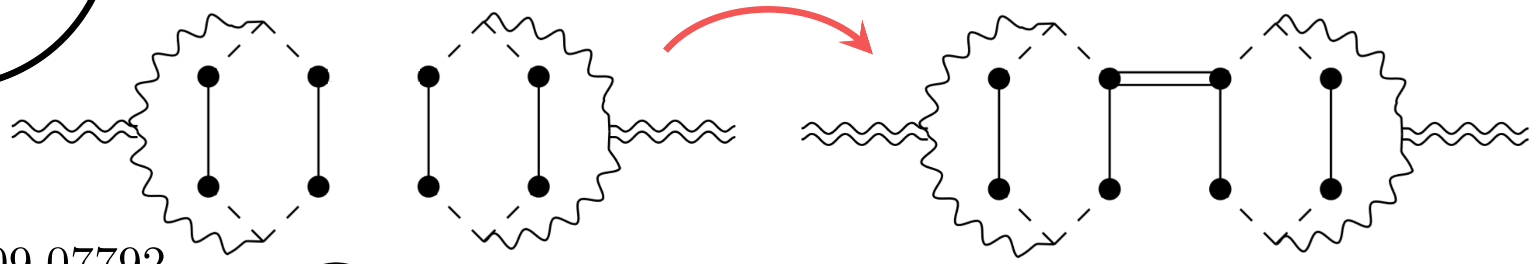
3 Distant patches are uncorrelated.◆

To gain intuition, let us neglect self-interactions.

$$\mathcal{R} \sim f_{\rm NL}\delta\phi^2$$

Two short modes can make a long one!

If the non-Gaussianity is strong enough, we can correlate distant patches.◆

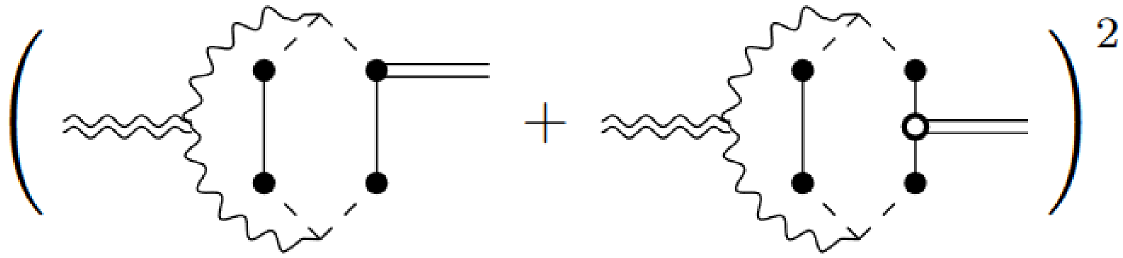


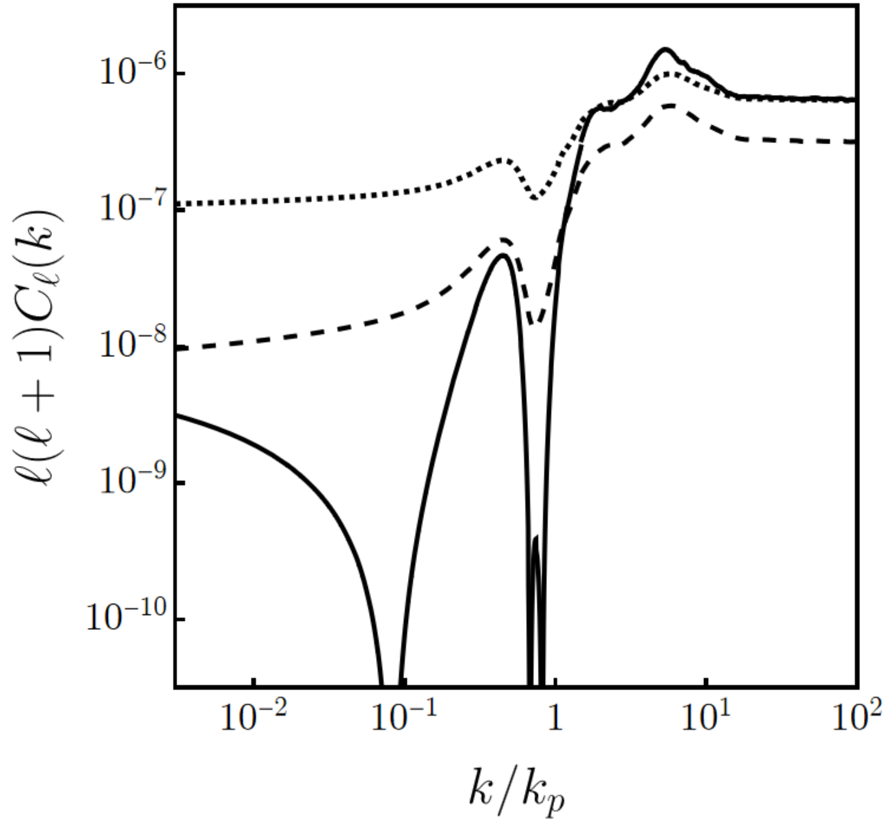
◆ Bartolo et al. 1909.12619, Li et al. 2309.07792

Angular coefficients

Self-interactions allow for a new Non-Gaussian bridge.

$\langle \delta^2_{\mathrm{GW}} \rangle \sim$





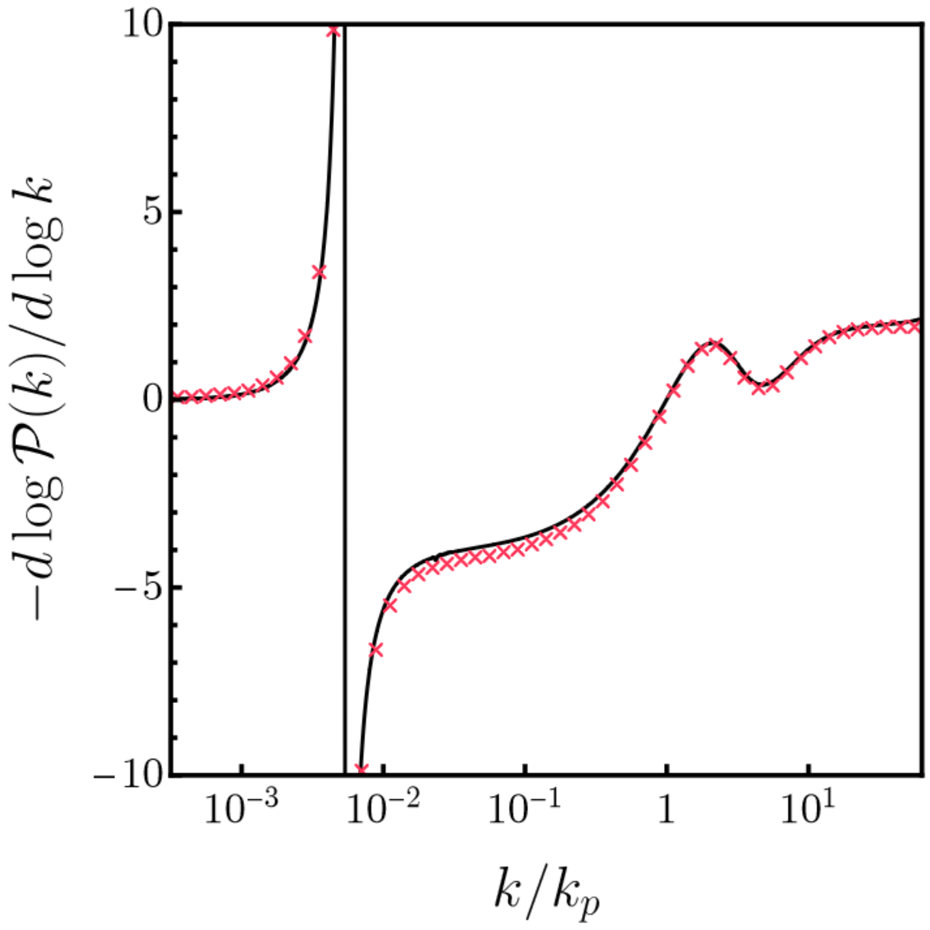
The **dotted line** is the δN prediction.

Self-interactions change the angular power spectrum.

Taking large non-linearity parameters is not justified!

With anisotropies we can know if the GWs are scalar-induced.

Consistency relation



× Squeezed bispectrum

The consistency relation is satisfied even when there is a USR phase.

It works as long as the bispectrum is evaluated after USR (not during!).

This shows we have included all the relevant interactions.

Consistency relation for anisotropies

Anisotropies also obey a consistency relation.

$$C_\ell(\tau_\star, x_\star, q) = \frac{2\pi \mathcal{P}_\mathcal{R}^\mathrm{L}}{\ell(\ell+1)} \left\{ \frac{\Omega_\mathrm{NG}(\tau_\star, \boldsymbol{x}_\star, q)}{\Omega_\mathrm{GW}(\tau_\star, q)} + \frac{3}{5}\left[4 - \frac{\partial \log \Omega_\mathrm{GW}(\tau_\star, q)}{\partial \log q}\right]\right\}^2$$

The angular power spectrum from induced GWs in single-field inflation is completely determined by the **scalar and tensor tilt**

$$\Omega_\mathrm{NG}(t, q) = -\frac{2}{24}\frac{q^2}{\mathcal{H}^2}\frac{q^3}{2\pi^2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{q^4}\left[\boldsymbol{p}\cdot\boldsymbol{e}^s(\boldsymbol{q})\cdot\boldsymbol{p}\right]^2 \overline{I_q(p, |\boldsymbol{q}-\boldsymbol{p}|)^2}|\varphi_{|\boldsymbol{q}-\boldsymbol{p}|}(t_e)|^2|\varphi_p(t_e)|^2 \frac{d\log \mathcal{P}_\mathcal{R}(p)}{d\log p}$$

We have shown it only for the self-interaction term.
But it **also holds when all interactions are kept!** Proof in [2411.XXXXX].

Summary

1. Non-Gaussianities in USR are not local. The exponential tail is not enough to make accurate predictions. δN does not capture all the physics.

2. Nonlinearity parameters should be taken of order 1. In this case, self-interactions dominate the loop corrections.

3. Perturbation theory may be violated depending on the parameter choices. The gravitational wave spectrum inherits this property.

4. Non-Gaussianities change the angular power spectrum. Scalar-induced anisotropies obey a consistency relation.

Thanks!