Bispectrum at 1-loop in the EFT of Inflation

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IISER PUNE

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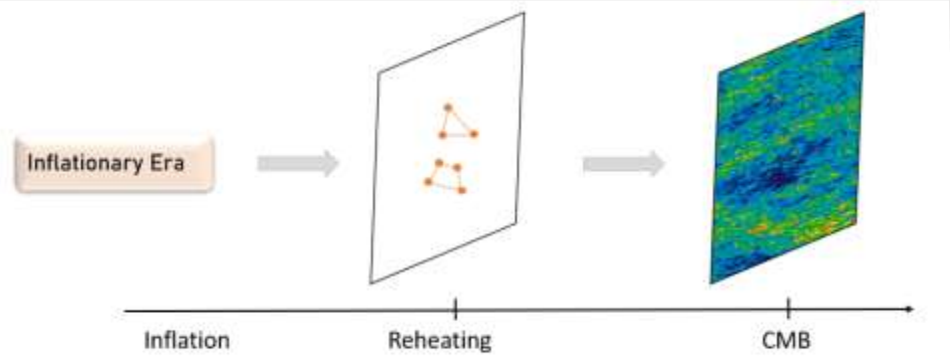
In collaboration with : Farman Ullah, Diptimoy Ghosh

Looping in the Primordial Universe

CERN

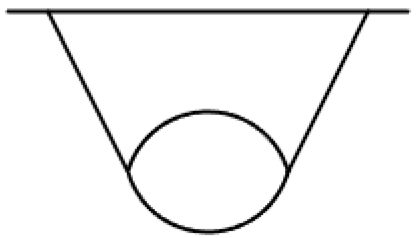
Motivation

➢ Inflationary correlations at late times are the observable output of inflation.



➢ Inflationary loop corrections expected to be small

The power spectrum at 1-loop is suppressed by *atleast* 10 orders of magnitude !



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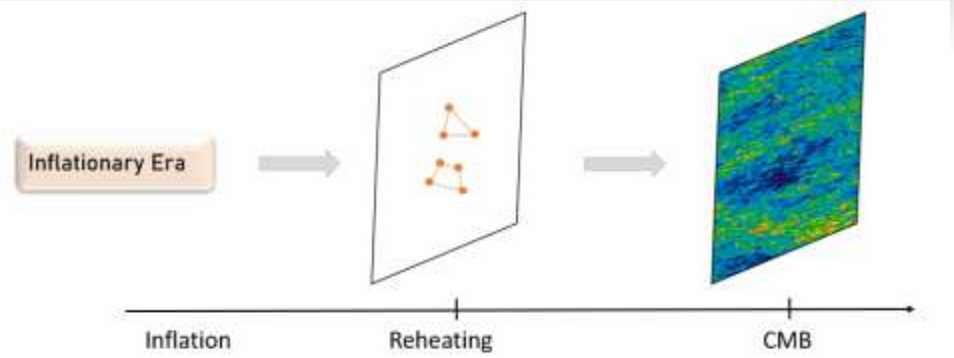

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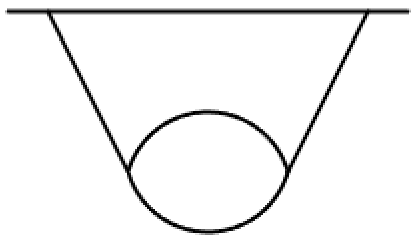

➢ Motivated to explore the theory even if predictions are unobservable, the first loop corrections were computed by Weinberg.

$$\langle \zeta_k^2 \rangle_{\text{loop}} \sim \frac{H^4}{M_{pl}^4} \frac{1}{k^3} \left(c + \log \frac{k}{\mu} \right)$$

Quantum Contributions to Cosmological Correlators, S. Weinberg, arXiv:0506236 [hep-th]

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- However, an error in the computation was pointed out, hinted by the log argument.

On Loops in Inflation, L. Senatore, M. Zaldarriaga, arXiv:0912.2734 [hep-th]

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- Log argument is comoving/physical scale : *a(t)* dependence in $\langle\, \zeta(0)\zeta(x) \,\rangle$

  but a(t) unobservable !

$$ds^2 = a^2[-d\eta^2 + d\vec{x}^2] = \left(\frac{a}{\lambda}\right)^2 [-d(\lambda\eta)^2 + d(\lambda\vec{x})^2]$$

- The catch : *d*-dimensional modes!

  Loop-divergences are regulated in dim. reg., with computations in $d = 3 + \delta$

- In de-Sitter, there is non-trivial dependence of modes on d .

Motivation

- Adding $\mathcal{O}(\delta)$ contribution cancels “unphysical” log, turns it into logarithm of ratios of physical scales.

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- Motivated by this we explore **Structure of 1-loop correction to the Bispectrum**

- In particular, for Bispectrum one can have logarithm of ratios of comoving scales
Unlike power spectrum, which had only one scale due to translational invariance

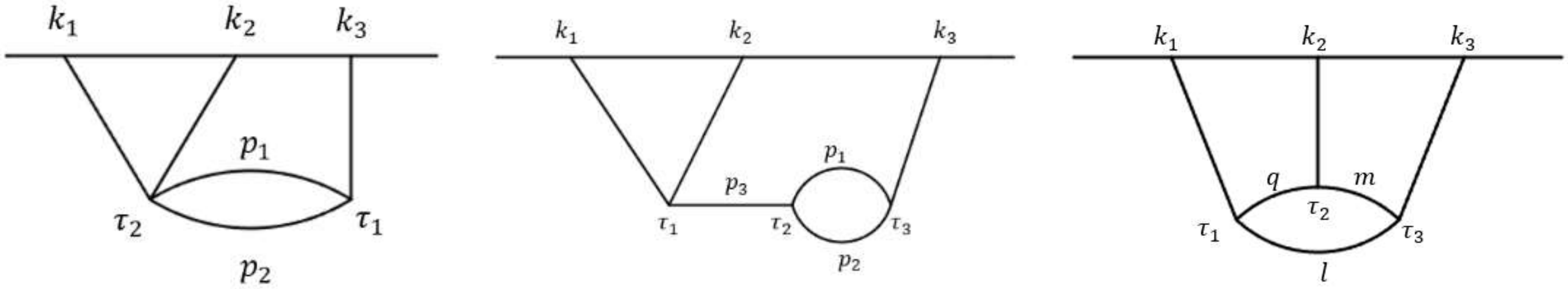
Bispectrum : Computational Scheme

➢ Interactions from Effective Field Theory of Inflation :

$$S = \int d^4x \sqrt{-g} \left[-M_{pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial\pi)^2}{a^2} \right) + g_3 \dot{\pi}^3 + g_{4,t} \dot{\pi}^4 + g_4 \dot{\pi}^2 \left(\frac{\partial\pi}{a} \right)^2 \right]$$

The Effective Field Theory of Inflation,
C. Cheung et al, arXiv:0709.0293 [hep-th]

➢ The following diagrams + other orderings.



$$\sim \frac{1}{\Lambda^6}$$

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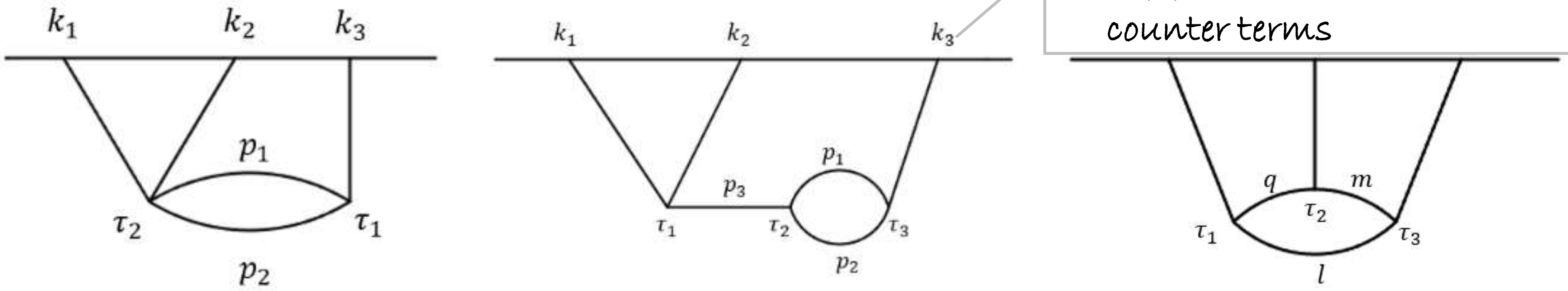
- The following diagrams + other orderings.

- *d-dim internal modes*
- *External modes taken in d=3, since $\mathcal{O}(\delta)$ contribution cancelled by counter terms*



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Bispectrum : Computational Scheme

Cosmological Cutting Rules, S. Melville and E. Pajer
arXiv:2103.09832]

- *d* - dimensional modes :

$d = 3 + \delta$, *massive* $\longrightarrow$ $\tau^{\frac{d-2}{2}} H_\nu^{(1)}(-k\tau)$ $\longrightarrow$ $m^2 = H^2 \frac{(d^2-9)}{4}$

with $\nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}$

- get back $\nu = \frac{3}{2}$
- massless mode functions

$$f_k(\tau) = (-H\tau)^{\delta/2} \frac{H}{\sqrt{-\dot{H} M_{pl}^2}} \frac{1}{\sqrt{2k^3}} (1 - ik\tau)\, e^{ik\tau}$$

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- The $\mathcal{O}(\delta)$ contribution arises from modes and measure :

$$f_k(\tau) = \left(1 + \frac{\delta}{2} \log(-H\tau)\right) f_k^{3d}(\tau)$$

$$a(\tau)^\delta = 1 - \delta \log(-H\tau)$$

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- The correlator splits into :

3-d + $\mathcal{O}(\delta)$ contribution : $D = D_1 + D_2$

Bispectrum : Computational Scheme

- Structure of D1 :

  scaling integrand by k_3

$$x_i = \frac{k_i}{k_3}$$

$$D_1(k_1, k_2, k_3) \equiv k_3^N \left\{ \frac{F_0(\{x_i\})}{\delta} + F_0(\{x_i\}) \left(\log k_3 - \frac{3}{2} \log \mu \right) + F_1(\{x_i\}) \right\} \text{ +permutations}$$

- Structure of D2 :

  same computation as that of D1 *alongwith* $\log(-H\tau)$

$$D_2(k_1, k_2, k_3) = \delta D_1 \sum_i n_i \log(H/k_T)$$

$\boldsymbol{n_i = -1}$ *from vertex (scale factor)*

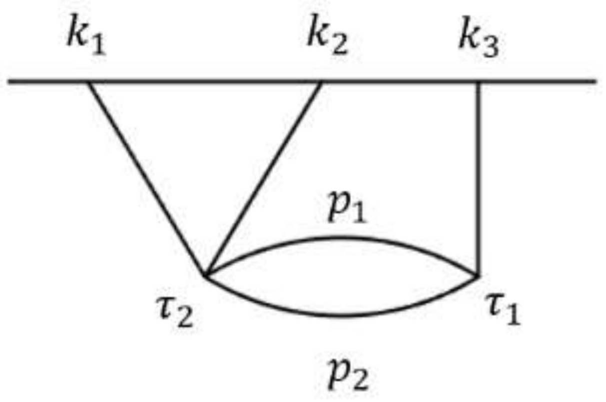
$\boldsymbol{n_i = \frac{1}{2}}$ *from internal modes*

Count no. of internal modes and vertices, add and multiply to D1 !

Results :

$$D = g_3 g_{4,t} \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{k_3(k_1+k_2)(k_3-2k_1-2k_2)}{k_1 k_2 k_T^7} \left\{ \frac{1}{\delta} + \frac{3}{2}\log \frac{H}{\mu} + \frac{1}{2}\log \frac{k_3}{k_T} - \frac{1}{2}\log k_T \right\} + \text{ NLf}$$

Non-log functions



Unphysical logs don't cancel from $\mathcal{O}(\delta)$ contributions

Results :

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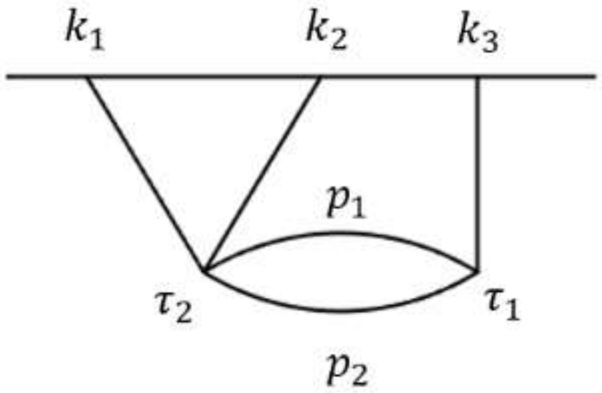
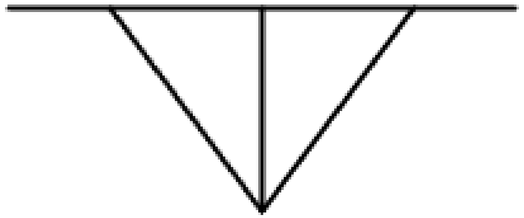


- Cancelling unphysical logs : Contribution from counter terms
- 3-pt contact, with Dim-10 Cubic counter terms



- 3 internal modes
- 1 vertex

$$\mathcal{O}^{\rm CT} \sim \frac{1}{\delta}\int \mu^{-\delta/2} a^{-3+\delta} \partial^7 \left(\pi^3\right) \longrightarrow \int d\tau \left(\frac{1}{\delta} - \frac{1}{2}\log\mu + \frac{1}{2}\log\left(-H\tau\right)\right)$$

$$\downarrow$$

$$\left(\frac{1}{\delta} + \frac{1}{2}\log\left(\frac{H}{\mu}\right) - \frac{1}{2}\log k_T\right)$$

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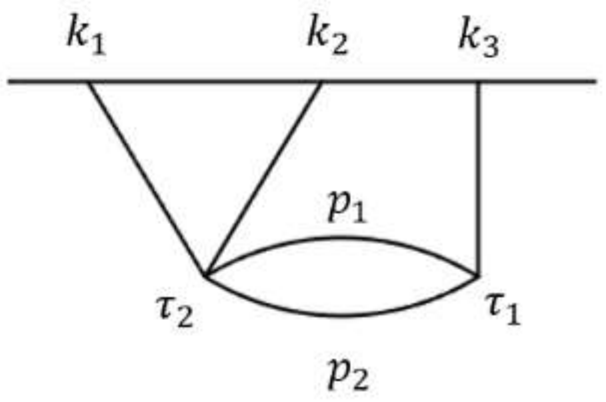

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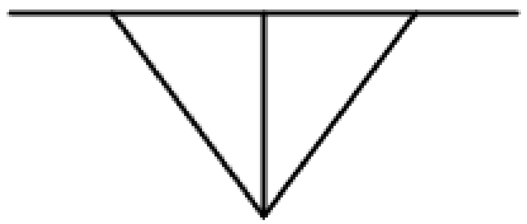

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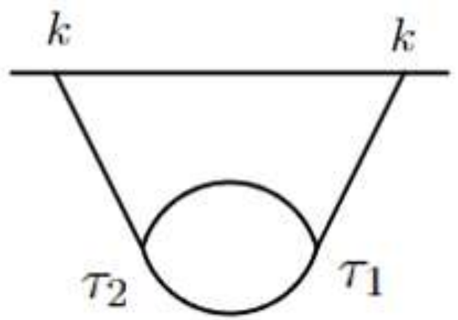
Results :

- **Since counter terms absorb divergences, unphysical logs cancel**

- Cancellation in 2pt case did not require renormalization : a happy accident…



$$\sim \frac{1}{\Lambda^4}$$

$$D_1(k) \sim F_0 \left(\frac{1}{\delta} + \log \frac{k}{\mu} \right) + \text{ finite}$$

$$D_2(k) \sim \delta \left\{ \left(\frac{3}{2} - 1 \right) + \left(\frac{3}{2} - 1 \right) \right\} D_1 \log \frac{H}{k} + \text{ finite}$$

3 int modes, 1 a(t) at each vertex

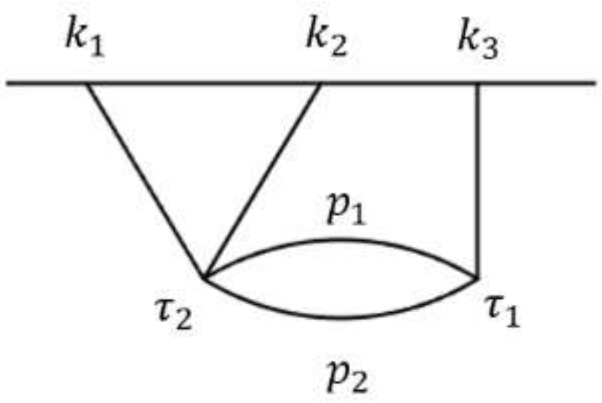
$$D(k) \sim F_0 \left(\frac{1}{\delta} + \log \frac{H}{\mu} \right) + \text{ finite}$$

- *Counter terms in 2pt case do not produce unphysical log*

  Contact Diagram from Dim 8 quadratic operator :

  2 internal modes + 1 vertex : $\sum n_i = 0$

Results :



Quartic $g_4\dot{\pi}^2\left(\frac{\partial\pi}{a}\right)^2$ interaction

$$D = g_3 g_4 \frac{H^9}{\left(-\dot{H}M_{pl}^2\right)^5}\frac{k_3\{\vec{k}_1\cdot\vec{k}_2\}}{8k_1^3k_2^3 15k_T^7}(40k_1^4 + 280k_2k_1^3 + 55k_3k_1^3 + 480k_2^2k_1^2 + 21k_3^2k_1^2$$
$$+ 105k_2k_3k_1^2 + 280k_2^3k_1 + 7k_3^3k_1 + 42k_2k_3^2k_1 + 105k_2^2k_3k_1 + 40k_2^4 + k_3^4 + 7k_2k_3^3$$
$$+ 21k_2^2k_3^2 + 55k_2^3k_3)\left(\frac{2}{\delta} + \log\left(\frac{k_3}{k_T}\right) + 2\log\left(\frac{k_{12}}{k_T}\right) - \log k_T + 3\log\left(\frac{H}{\mu}\right)\right)$$
$$+ g_3 g_4 \frac{H^9}{\left(-\dot{H}M_{pl}^2\right)^5}\frac{1}{8k_1^3k_2^3k_3}\frac{1}{120k_T^7}\left\{\frac{1}{\delta} + \frac{1}{2}\log\left(\frac{k_3}{k_T}\right) - \frac{1}{2}\log k_T + \frac{3}{2}\log\left(\frac{H}{\mu}\right)\right\}$$
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$$- 1904k_2k_3^2k_1^5 + 560k_2^2k_3k_1^5 - 440k_2^4k_1^4 + 260k_3^4k_1^4 + 56k_2k_3^3k_1^4 - 248k_2^2k_3^2k_1^4$$
$$- 1540k_2^3k_3k_1^4 - 140k_2^5k_1^3 + 124k_3^5k_1^3 + 1764k_2k_3^4k_1^3 - 2592k_2^2k_3^3k_1^3 + 3008k_2^3k_3^2k_1^3$$
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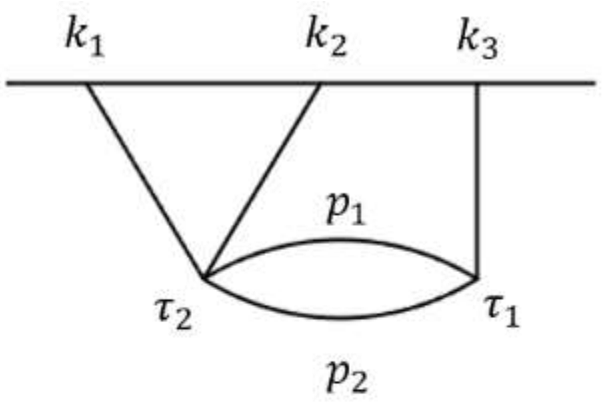

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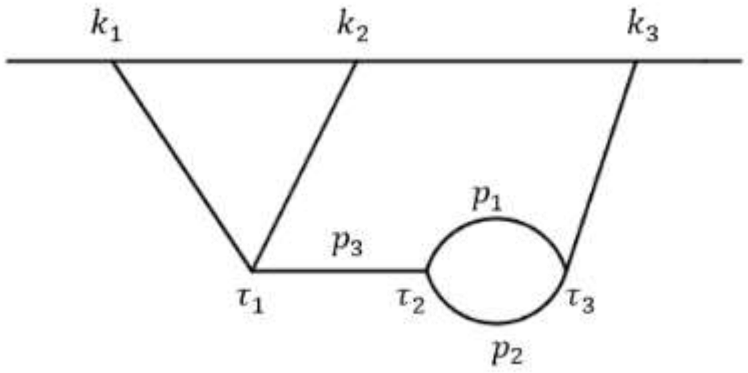


$$D = -g_3^3 \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^6} \frac{1}{8k_1k_2k_3k_T^8} \frac{1}{15} \left\{ \frac{1}{\delta} + \log\left(\frac{k_3}{k_T}\right) - \frac{1}{2}\log k_T + \frac{3}{2}\log\left(\frac{H}{\mu}\right) \right\}$$

$$(k_1^5 + 5k_2k_1^4 + 11k_3k_1^4 + 10k_2^2k_1^3 + 58k_3^2k_1^3 + 44k_2k_3k_1^3 + 10k_2^3k_1^2 + 198k_3^3k_1^2$$

$$+ 174k_2k_3^2k_1^2 + 66k_2^2k_3k_1^2 + 5k_2^4k_1 - 99k_3^4k_1 + 396k_2k_3^3k_1 + 174k_2^2k_3^2k_1$$

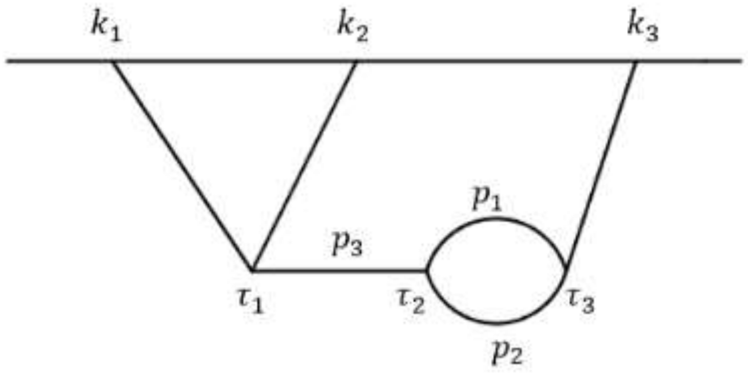
$$+ 44k_2^3k_3k_1 + k_2^5 + 3k_3^5 - 99k_2k_3^4 + 198k_2^2k_3^3 + 58k_2^3k_3^2 + 11k_2^4k_3)$$

$$+ g_3^3 \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^6} \frac{1}{8k_1k_2k_3k_T^8} \frac{1}{15} \frac{2k_3^4}{(k_3 - k_1 - k_2)^3} \log\left(\frac{k_T}{k_3}\right) (150k_1^4 + 600k_2k_1^3$$

$$- 195k_3k_1^3 + 900k_2^2k_1^2 + 133k_3^2k_1^2 - 585k_2k_3k_1^2 + 600k_2^3k_1 - 25k_3^3k_1 + 266k_2k_3^2k_1$$

$$- 585k_2^2k_3k_1 + 150k_2^4 + k_3^4 - 25k_2k_3^3 + 133k_2^2k_3^2 - 195k_2^3k_3) + \text{NLf}$$

Results :



$$
\begin{aligned}
D = &-g_3^3 \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^6} \frac{1}{8 k_1 k_2 k_3 k_T^8} \frac{1}{15} \left\{ \frac{1}{\delta} + \log\left(\frac{k_3}{k_T}\right) - \frac{1}{2} \log k_T + \frac{3}{2} \log\left(\frac{H}{\mu}\right) \right\} \\
&(k_1^5 + 5k_2k_1^4 + 11k_3k_1^4 + 10k_2^2k_1^3 + 58k_3^2k_1^3 + 44k_2k_3k_1^3 + 10k_2^3k_1^2 + 198k_3^3k_1^2 \\
&+ 174k_2k_3^2k_1^2 + 66k_2^2k_3k_1^2 + 5k_2^4k_1 - 99k_3^4k_1 + 396k_2k_3^3k_1 + 174k_2^2k_3^2k_1 \\
&+ 44k_2^3k_3k_1 + k_2^5 + 3k_3^5 - 99k_2k_3^4 + 198k_2^2k_3^3 + 58k_2^3k_3^2 + 11k_2^4k_3) \\
&+ g_3^3 \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^6} \frac{1}{8 k_1 k_2 k_3 k_T^8} \frac{1}{15} \frac{2k_3^4}{(k_3 - k_1 - k_2)^3} \log\left(\frac{k_T}{k_3}\right) (150k_1^4 + 600k_2k_1^3 \\
&- 195k_3k_1^3 + 900k_2^2k_1^2 + 133k_3^2k_1^2 - 585k_2k_3k_1^2 + 600k_2^3k_1 - 25k_3^3k_1 + 266k_2k_3^2k_1 \\
&- 585k_2^2k_3k_1 + 150k_2^4 + k_3^4 - 25k_2k_3^3 + 133k_2^2k_3^2 - 195k_2^3k_3) + \text{NLf}
\end{aligned}
$$

Results :

Structure of Bispectrum at 1-loop

$$D(k_1,k_2,k_3) = g_3 g_4 \frac{H^9}{\left(\dot{H} M_{pl}^2\right)^5} f(k_i) \left\{ \log\left(\frac{k_i}{k_T}\right) + \log\left(\frac{H}{\mu}\right) \right\}$$

$$f(k) \sim \frac{1}{k^6}$$

Conclusion

- Computation of loop corrections to 3 pt.
- Unphysical Logs cancel only after renormalization
- Loop correction : features scale invariance
- Rich NG shapes $f(k_i)$, log structures : $\log(k_i/k_T)$, $\log(H/\mu)$

Results :

Structure of Bispectrum at 1-loop

$$D(k_1,k_2,k_3) = g_3 g_4 \frac{H^9}{\left(\dot{H} M_{pl}^2\right)^5} f(k_i) \left\{ \log\left(\frac{k_i}{k_T}\right) + \log\left(\frac{H}{\mu}\right) \right\}$$

Some subtleties involved in implementing dim. reg., hence a cross check

Cross-check using cutoff regularisation

- Tree level contributions from counter terms with 3-d modes will not produce additional logs

$$D(k_1,k_2,k_3) = g_3 g_4 \frac{H^9}{\left(\dot{H} M_{pl}^2\right)^5} f(k_i) \left\{ \log\left(\frac{k_i}{k_T}\right) + \log \frac{H}{\Lambda} \right)\right\}$$

+power law divergences in Λ + finite

Comments and Outlook :

Singularity structure of correlations at 1-loop (2-site)

- “To KLN, or not to KLN” : The Cosmological KLN theorem – “For Massless fields, equal time correlations may only have poles in total energy.”

The Cosmological Tree Theorem,
S. Salcedo, S. Melville, arXiv:2308.00680 [hep-th]

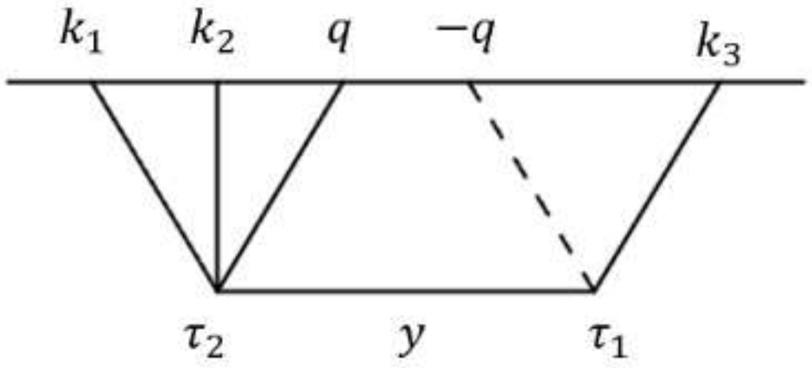
The correlator will have wavefunction contributions including loops and trees. Tree theorem cuts open loops to give (integral over) sum of trees, with poles of partial and total energies. The singularities in the integrand don’t mix total energy and loop momenta – so no logs of total energy?

But $\log k_T$ can arise from mixing due to integration limits

Comments and Outlook :

Singularity structure of correlations at 1-loop (2-site)

But $\log k_T$ can arise from mixing due to integration limits



$$\sim \int \frac{qy}{(q+y+k_{12})^5(-q+y+k_3)^4} d^3q$$
$$= (2\pi) \int_{q=0}^{\infty} \int_{y=|k_3-q|}^{k_3+q} \frac{q^2y^2}{(q+y+k_{12})^5(-q+y+k_3)^4} dqdy$$

Wavefunction with a quartic $\dot{\pi}^4$ and a cubic interaction $\dot{\pi}^3$

- A systematic understanding of the singularity structure – poles, branch cuts, thresholds

(Work in progress)

Appendix : Bispectrum at 1-loop : Computational Scheme

- Computing D_1
- Scale by k_3 : $D_1(k_1,k_2,k_3) \equiv \mu^{-\frac{3\delta}{2}} k_3^{\delta+N} R(d,x_i)$ $\qquad x_i = \frac{k_i}{k_3}$
- Expand around $\delta = 0$

Finite + Log

$$R(3+\delta,\{x_i\}) = \frac{F_0(\{x_i\})}{\delta} + F_1(\{x_i\}) + \mathcal{O}(\delta)$$

$$D_1(k_1,k_2,k_3) \equiv k_3^N \left\{ \frac{F_0(\{x_i\})}{\delta} + F_0(\{x_i\}) \left(\log k_3 - \frac{3}{2}\log\mu \right) + F_1(\{x_i\}) \right\}$$

Appendix : Bispectrum at 1-loop : Computational Scheme

- Computing D_2

$$D_2(k_1,k_2,k_3) = \delta\left[\int d^d\vec{p}_1 d^d\vec{p}_2 P(p_1,p_2,\{k_i\},\{\tau_i\})\delta^{(d)}(\vec{p}_1+\vec{p}_2-\vec{k}_3)\prod_i d\tau_i\right]\sum_i n_i \log(-H/k_T)$$

(m,n) $(2,2)$ $(2,3)$ $(2,4)$

2 Vertex

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2\ \tau_1^n \tau_2^m e^{ia\tau_1} e^{ib\tau_2}$$

$m,n,l = 2$

3 Vertex

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3\ \tau_1^m \tau_2^n \tau_3^l e^{ia\,\tau_1} e^{ib\,\tau_2} e^{ic\,\tau_3}$$

$$\times\ \delta\ \log(-H\tau_1)$$

$$\int_{-\infty}^{0} d\tau_1\ \tau_1^m e^{ik_T\tau_1} g(k_i,p_j,\tau_1)\log\tau_1 \;=\; f_1(k_i,p_j) + \log\left(\frac{1}{k_T}\right)\int_{-\infty}^{0} d\tau_1\ \tau_1^m e^{ik_T\tau_1} g(k_i,p_j,\tau_1)$$

Appendix : Bispectrum at 1-loop : Computational Scheme

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(m,n) $(2,2)$ $(2,3)$ $(2,4)$

2 Vertex

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2\ \tau_1^n \tau_2^m e^{ia\tau_1} e^{ib\tau_2}$$

$m,n,l=2$

3 Vertex

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3\ \tau_1^m \tau_2^n \tau_3^l e^{ia\ \tau_1} e^{ib\ \tau_2} e^{ic\ \tau_3}$$

$$\times\ \delta\ \log(-H\tau_2)$$

$$\int_{-\infty}^{0} d\tau_1\ \tau_1^m e^{ia\ \tau_1}\{\mathrm{Ei}\,(i(b+c)\tau_1)\, g_1(k_i,p_j) + e^{i(b+c)\ \tau_1} g_2(k_i,p_j,\tau_1)\ + e^{i(b+c)\ \tau_1}\log\tau_1\ g_3(k_i,p_j,\tau_1)\}$$

$\tau_1 \downarrow$ Finite + Logs

$\tau_1 \downarrow$ Finite

Result of integrals without log

Appendix : Bispectrum at 1-loop : Computational Scheme

- Computing D_2

$$D_2(k_1,k_2,k_3) = \delta\left[\int d^d\vec{p}_1 d^d\vec{p}_2 P(p_1,p_2,\{k_i\},\{\tau_i\})\delta^{(d)}(\vec{p}_1+\vec{p}_2-\vec{k}_3)\prod_i d\tau_i\right]\sum_i n_i \log(-H/k_T)$$

(m,n) $(2,2)$ $(2,3)$ $(2,4)$

2 Vertex

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2\ \tau_1^n \tau_2^m e^{ia\tau_1} e^{ib\tau_2}$$

$m,n,l = 2$

3 Vertex

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3\ \tau_1^m \tau_2^n \tau_3^l e^{ia\ \tau_1} e^{ib\ \tau_2} e^{ic\ \tau_3}$$

$$\times\ \delta\ \log(-H\tau_3)$$

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \tau_1^m \tau_2^n \{\mathrm{Ei}\,(ic\tau_2)\, h_1(k_i,p_j) + e^{ic\tau_1} h_2(k_i,p_j,\tau_2)\ + e^{ic\tau_2}\log\tau_2\ h_3(k_i,p_j,\tau_2)\}$$

$\downarrow \tau_2$ Finite + Ei $\xrightarrow{\tau_1}$ Finite + Logs

$\tau_2 \downarrow$ Finite

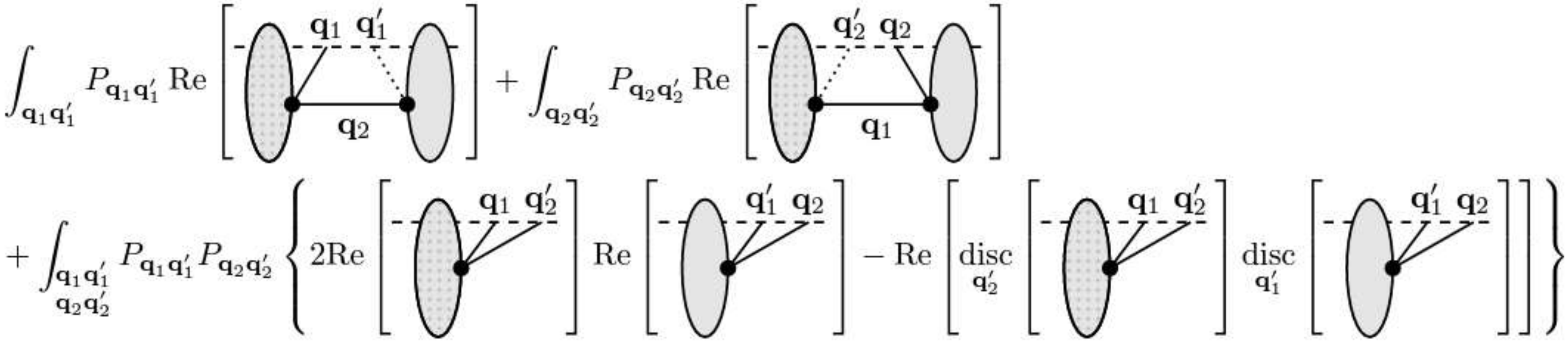
Result of integrals without log

Appendix:Singularity structure of 1-loop (2-site) corrections

The Cosmological Tree Theorem,
S. Salcedo, S. Melville, arXiv:2308.00680 [hep-th]

- The Cosmological KLN theorem

Wavefunction contributions to a correlator at 1-loop using cosmological tree theorem



- The integrands have poles in partial energies or total energy (independent of loop energy)
- No mixing of loop momentum and total energy : no branch cuts of total energy expected

…or is it?

Appendix:Singularity structure of 1-loop (2-site) corrections

The Cosmological Tree Theorem,
S. Salcedo, S. Melville, arXiv:2308.00680 [hep-th]

- The Cosmological KLN theorem

Wavefunction contributions to a correlator at 1-loop