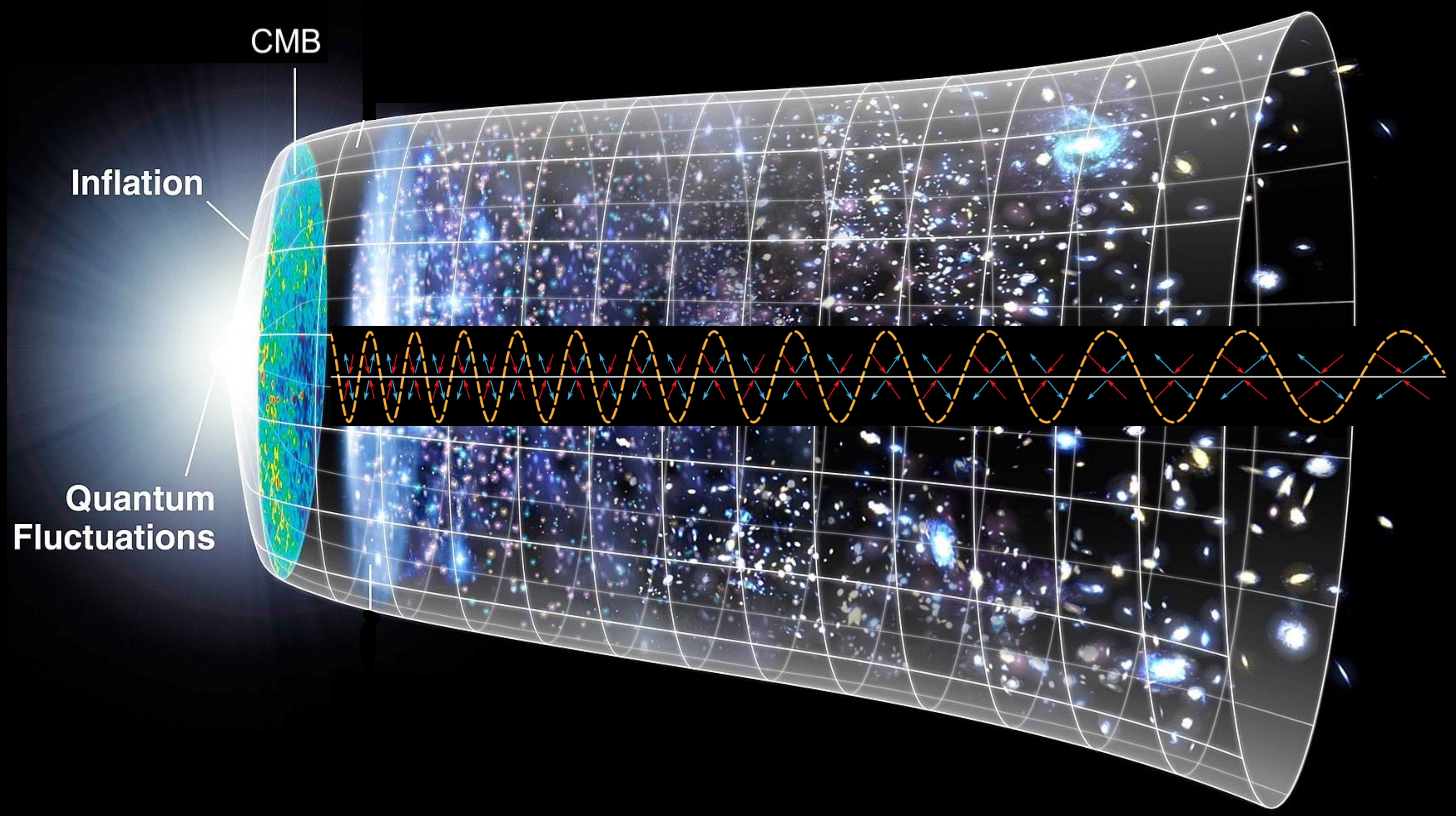
Renormalizing UV Divergences in the Early Universe

What can go wrong?

Anna Negro
In collaboration with
S. P. Patil



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(*ArXiv:2403.16806*)

Looping in the Primordial Universe

30/10/24

annahnegro@gmail.com

Looping in the Primordial Universe:

Looping in the Primordial Universe: Energy density

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Energy density $\rho := -\langle 0 | \hat{T}^{\phi}{}_0{}^0 | 0 \rangle = \langle 0 | \frac{1}{2}\partial_0 \hat{\phi} \partial_0 \hat{\phi} + \frac{1}{2}\partial_i \hat{\phi} \partial_i \hat{\phi} | 0 \rangle$

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Solving EOM $$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3}\left[\hat{a}_{\vec{k}} u_k + \hat{a}^\dagger_{\vec{k}} u^*_k\right], \quad u_k = \frac{e^{-ik_\mu x^\mu}}{\sqrt{2w_k}}$$

where $$k^\mu = (w_k, \vec{k}), \quad w_k^2 = |\vec{k}|^2, \quad \left[\hat{a}_{\vec{k}}, \hat{a}^\dagger_{\vec{k}'}\right] = (2\pi)^3\delta^3(\vec{k} - \vec{k}')$$

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New!

Curvature!

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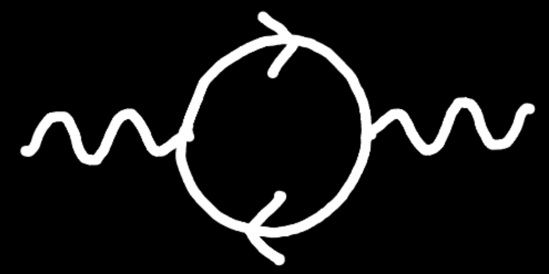
Divergences in terms of regulator: $\int^{k_{UV}} d^4k \longrightarrow$  $\sim \lim_{k_{UV}\to\infty} k_{UV}\,(\dots)$

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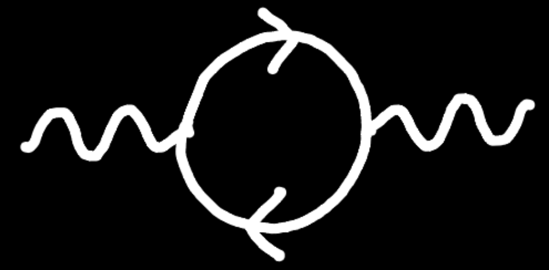

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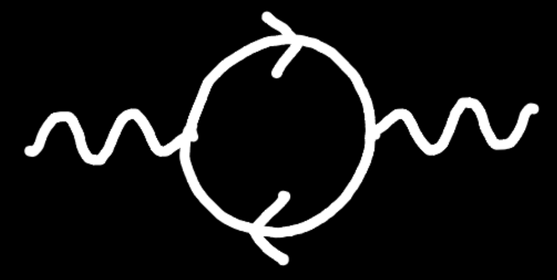
Divergences in terms of regulator: $\int^{k_{UV}} d^4k \longrightarrow$  $\sim \lim_{k_{UV}\to\infty} \cancel{k_{UV}(\ldots)}$

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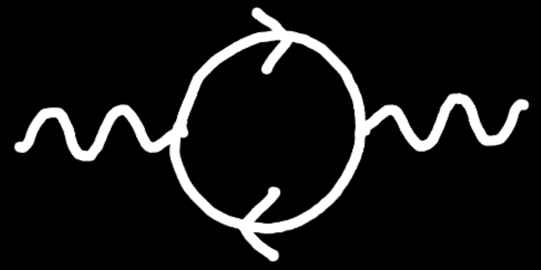
Divergences in terms of regulator: $\int^{k_{UV}} d^4k \longrightarrow$  $\sim \lim_{k_{UV}\to\infty} \cancel{k_{UV}} (\ldots)$

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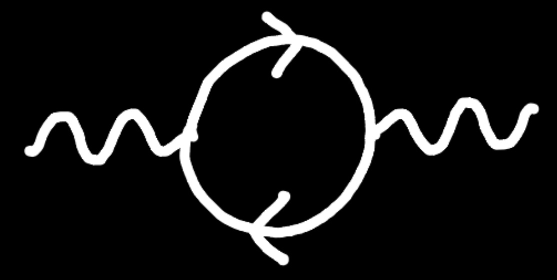
Divergences in terms of regulator: $\int^{k_{UV}} d^4k \longrightarrow$  $\sim \lim_{k_{UV}\to\infty} \cancel{k_{UV}} (\ldots)$

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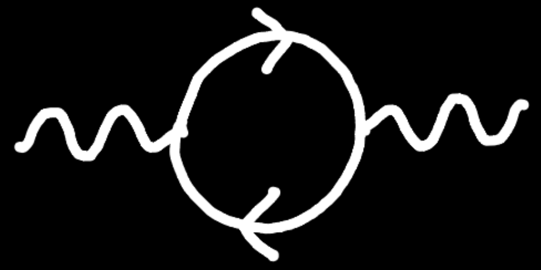
Divergences in terms of regulator: $\int^{k_{UV}} d^4k \longrightarrow$  $\sim \lim_{k_{UV}\to\infty} \cancel{k_{UV}} (\ldots)$

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...We have a predictive theory!

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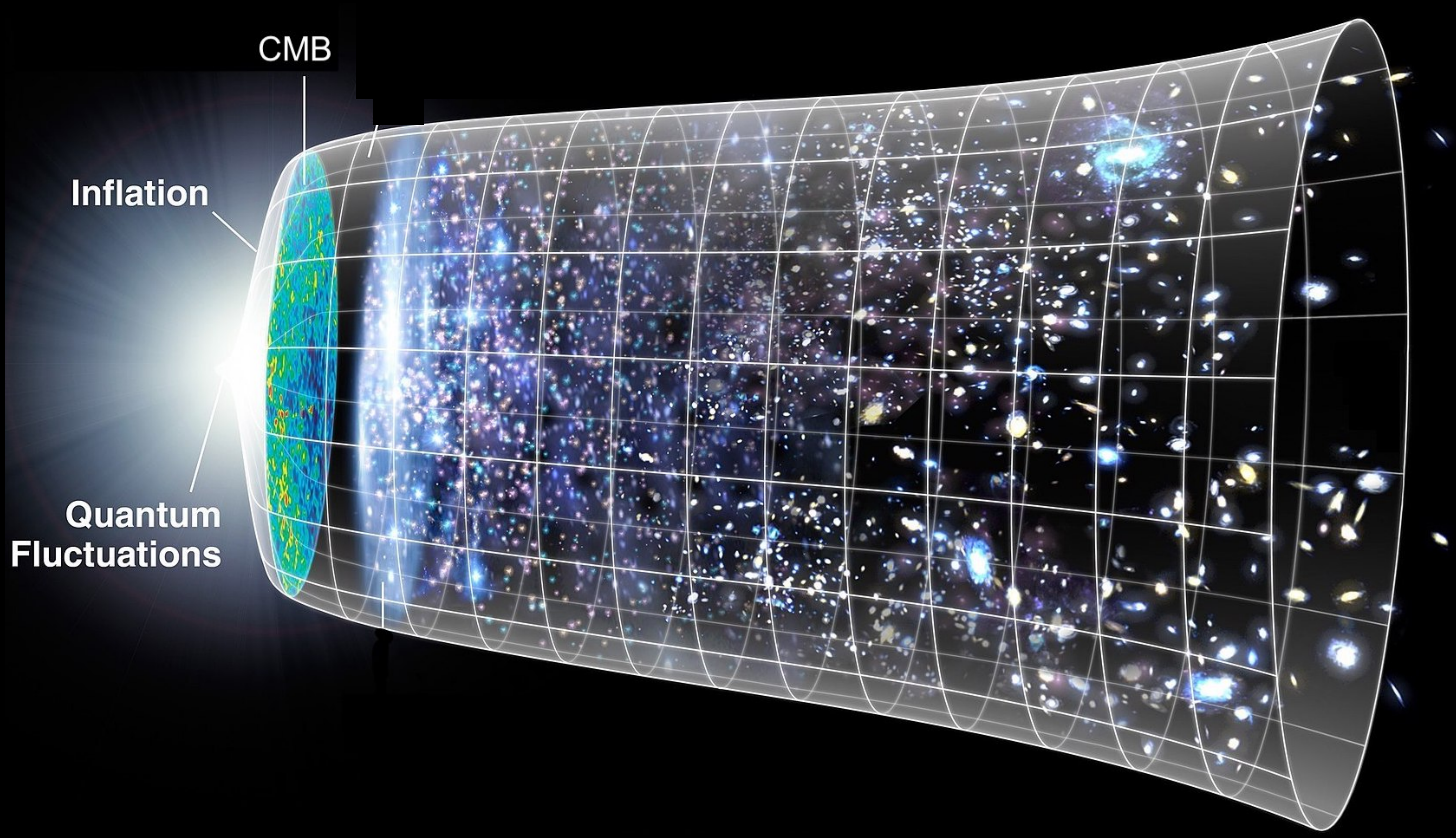

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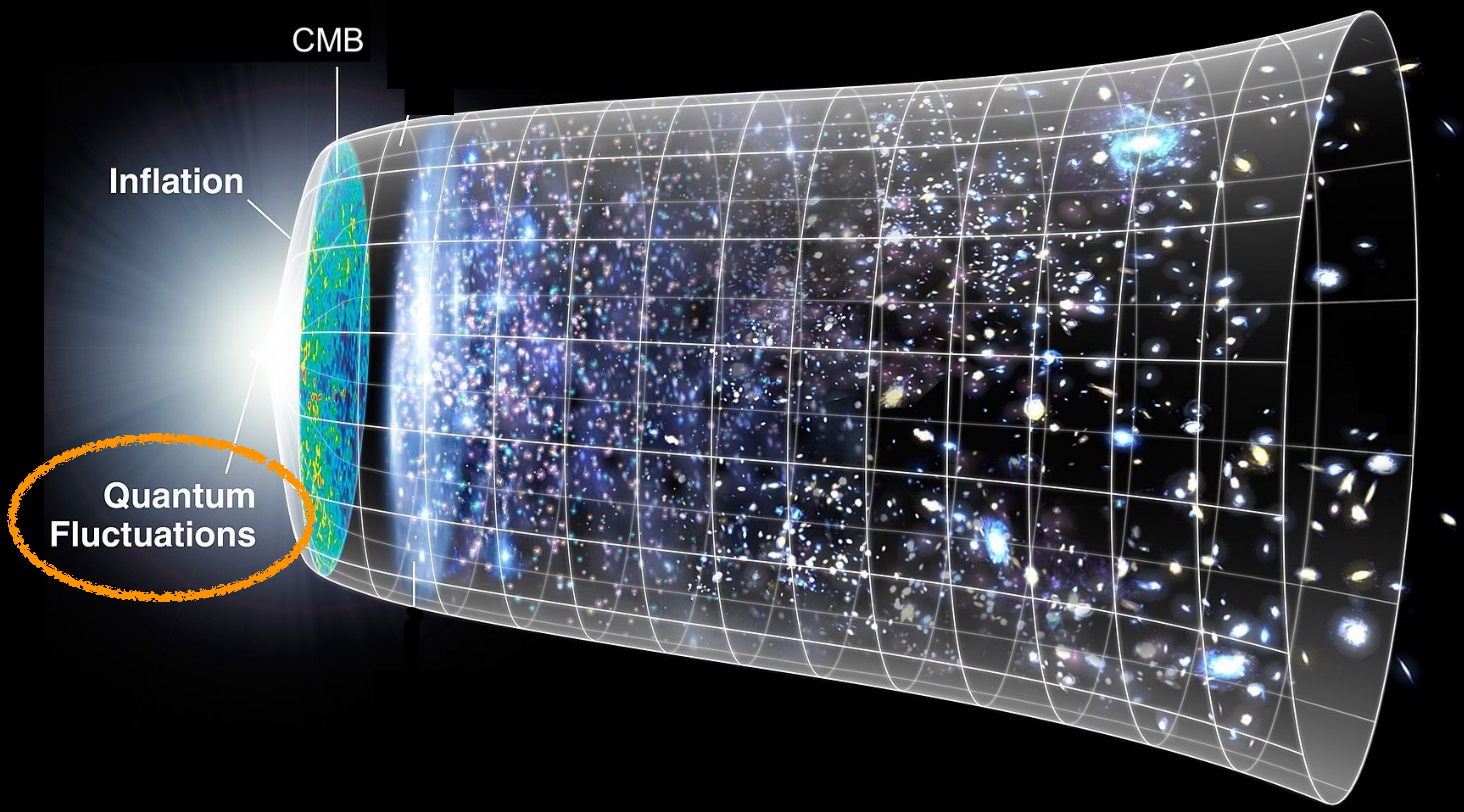

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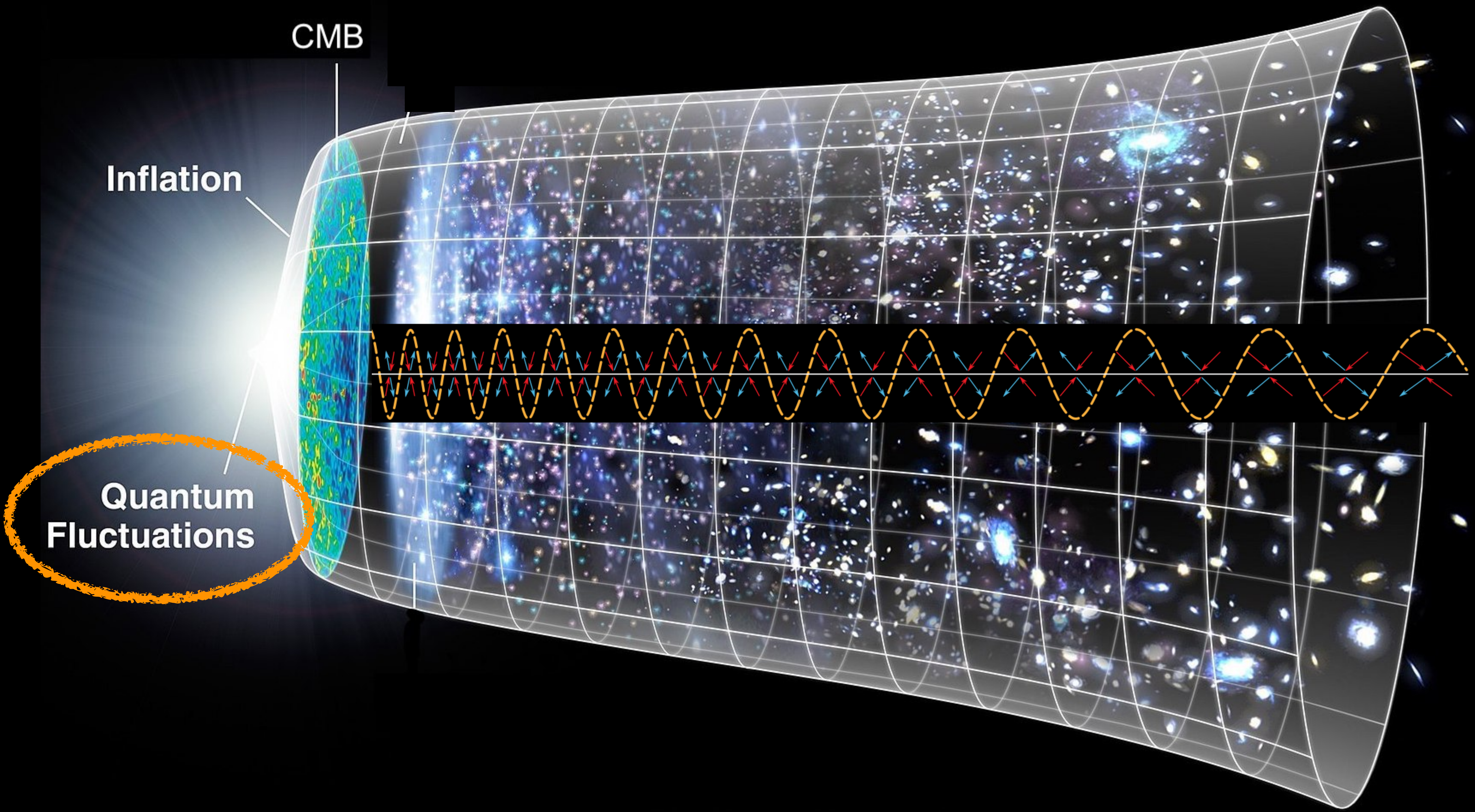

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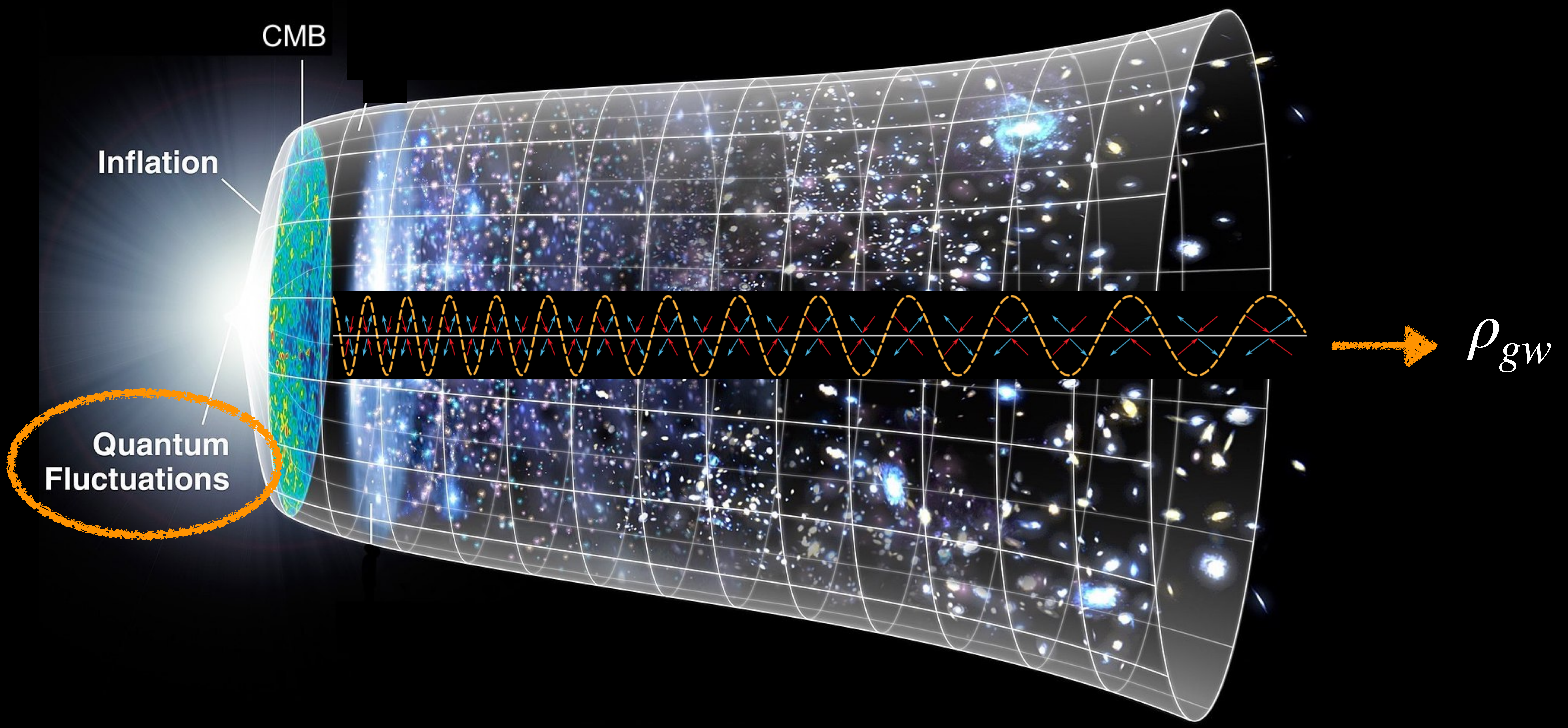

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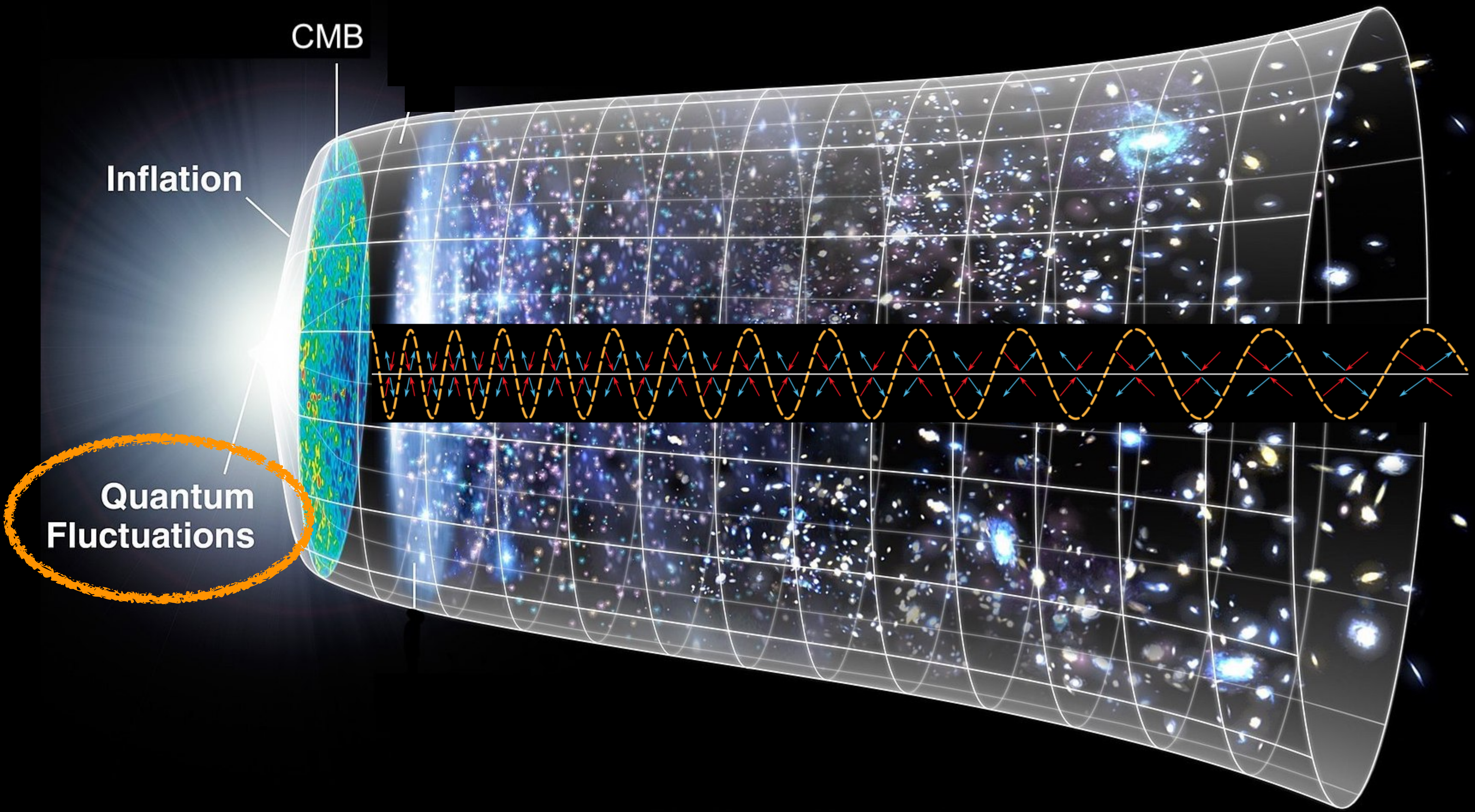

$\rightarrow \rho_{gw}$

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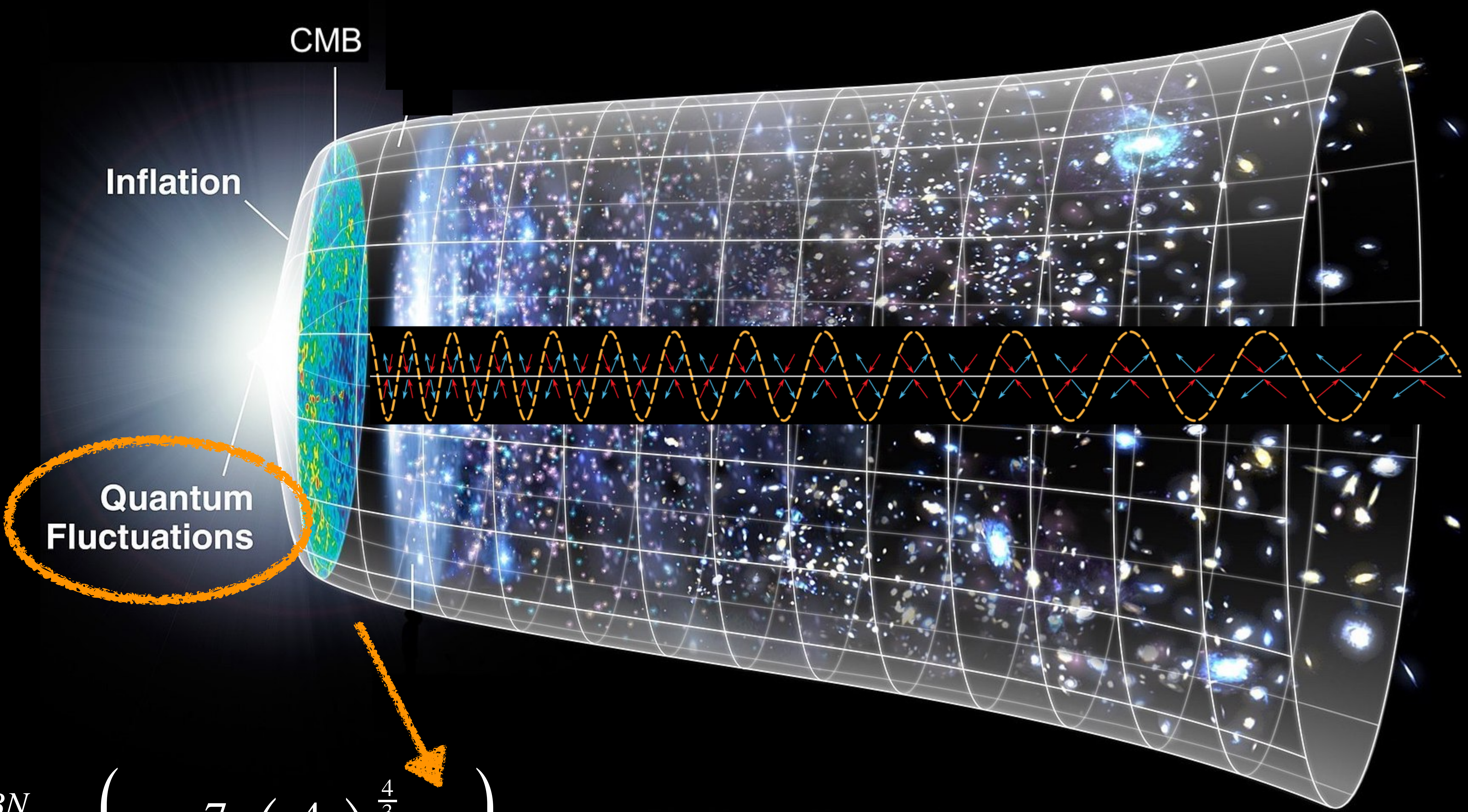

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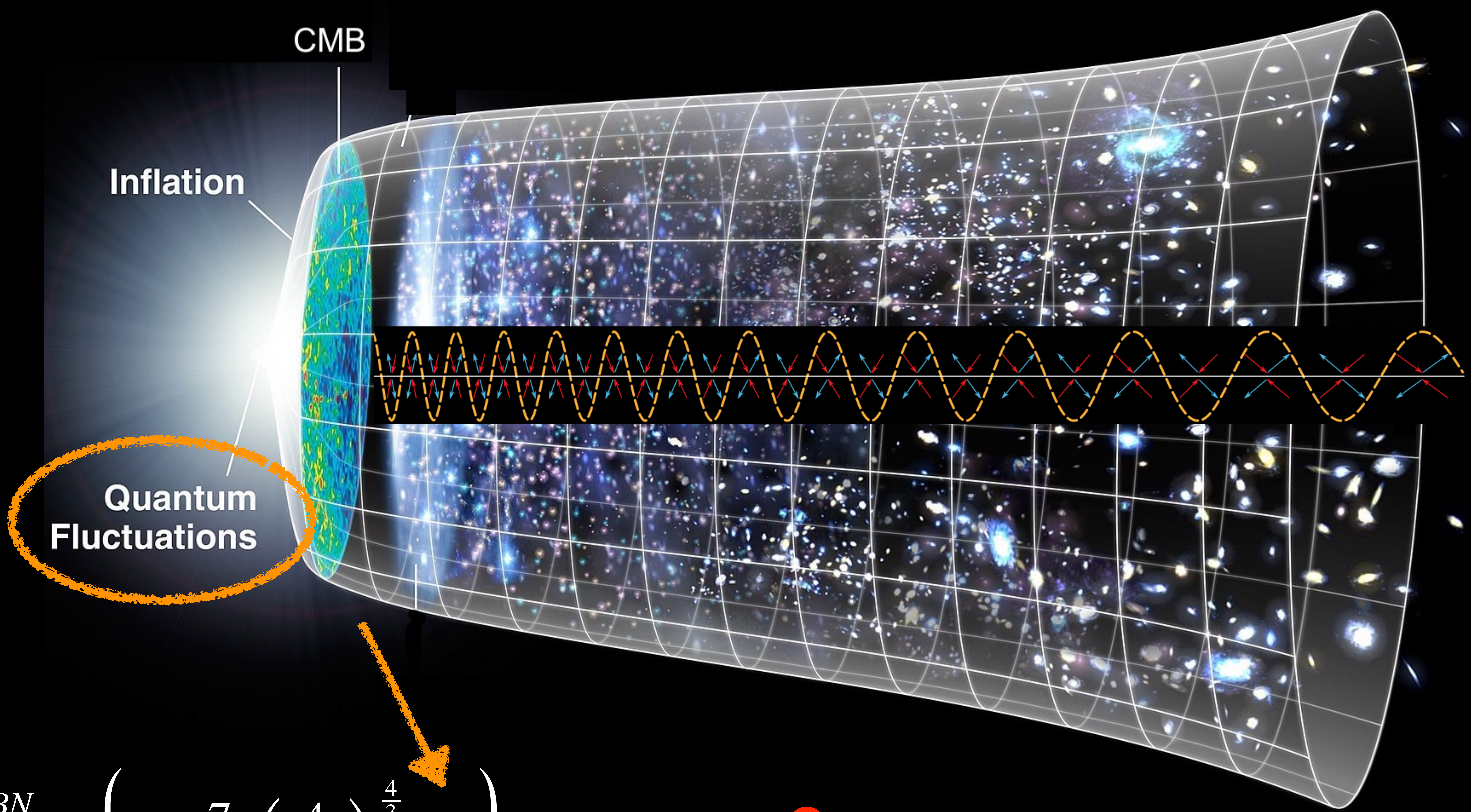

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$$^{(2)}\rho_{gw} = \frac{1}{32\pi G_N a^2}\left\langle h'_{ij}(\eta,k)h^{ij\prime}(\eta,k)\right\rangle \leftarrow \rho_{gw}$$

$^{(2)}$*M. Maggiore Gravitational waves vol 1*

Motivation

During radiation dominated era

$$^{(3)}\rho_{gw} = \frac{2H^2}{a^2}\int_0^\infty dk\, k \left(\frac{k}{k_*}\right)^{n_t} \left(\frac{\sin k\tau}{k^2\tau^2} - \frac{\cos k\tau}{k\tau}\right)^2 \longleftarrow \rho_{gw}$$

[3]Multi-wavelength constraints on the inflationary consistency relation *(2015)*

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k_{UV}**?**

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- **Renormalization conditions** VS predicted observables:

  Can $N_{\rm eff}$ bounds constrain vacuum GWs?

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Scalar case

(cf. **A.N S. Patil (2024)** GWs case)

IR divergences VS UV divergences

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Consider massless non-interacting test scalar field:

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UV and IR divergent!

IR divergences VS UV divergences

Consider massless non-interacting test scalar field:

$$\hat{\phi}(\tau, x) = \int \frac{d^3k}{(2\pi)^3} \hat{\phi}(\tau, k) e^{ik\cdot x} \,, \qquad \hat{\phi}(\tau, k) = \hat{a}_k \phi_k(\tau) + \hat{a}^\dagger_{-k} \phi^*_k(\tau)$$

Two point function rewritten in terms of power spectrum

$$\langle \hat{\phi}(\tau, x) \hat{\phi}(\tau, x) \rangle = \int_0^\infty \frac{dk}{k} P_\phi(\tau, k) \,, \qquad P_\phi(\tau, k) = \frac{k^3}{2\pi^2} |\phi_k(\tau)|^2$$

On a pure de Sitter background

$$\langle \hat{\phi}(\tau, x) \hat{\phi}(\tau, x) \rangle = \left(\frac{H}{2\pi} \right)^2 \int_0^\infty \frac{dk}{k} \left[1 + \left(\frac{k}{aH} \right)^2 \right]$$

- UV divergences: to be renormalized

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- IR divergences?

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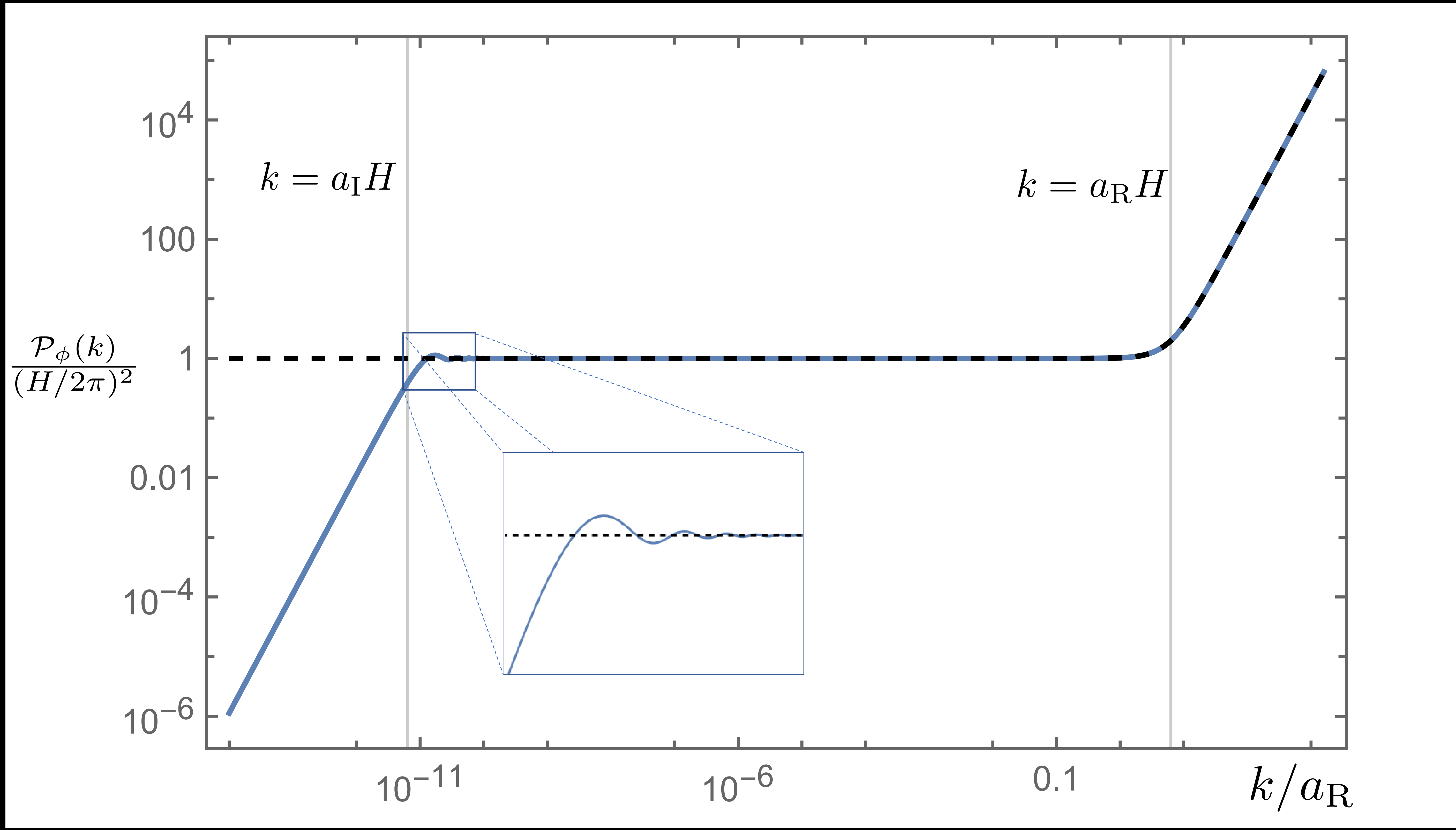
- UV divergences: to be renormalized
- IR divergences: *must disappear* in observables!

Comparison PS massless scalar field

Dashed/bold line: Infinite/finite inflation



Power spectra evaluated at reheating $a = a_\mathrm{R}$,
where $a_\mathrm{I} = 10^{-12} a_\mathrm{R}$in units and H is set to 2π

Finite duration inflation

Finite duration inflation

$$a(\tau) = a_\mathrm{R}\left(2 - \frac{\tau}{\tau_\mathrm{I}}\right) e^{-\mathcal{N}_\mathrm{tot}} \qquad \tau < \tau_\mathrm{I}$$

Radiation pre-inflationary era

$$= a_\mathrm{R}\left(\frac{\tau_\mathrm{I}}{\tau}\right) e^{-\mathcal{N}_\mathrm{tot}} \qquad \tau_\mathrm{I} < \tau < \tau_\mathrm{R}$$

De Sitter inflation

$$= a_\mathrm{R}\left(2 - \frac{\tau}{\tau_\mathrm{R}}\right) \qquad \tau_\mathrm{R} < \tau$$

Radiation dominated era

$$H = -\frac{1}{a_\mathrm{R}\tau_\mathrm{R}}, \qquad \mathcal{N}_\mathrm{tot} = \log(a_\mathrm{R}/a_\mathrm{I}) = \log(\tau_\mathrm{I}/\tau_\mathrm{R})$$

Finite duration inflation

$$a(\tau) = a_{\rm R}\left(2 - \frac{\tau}{\tau_{\rm I}}\right) e^{-\mathcal{N}_{\rm tot}} \qquad \tau < \tau_{\rm I}$$ Radiation pre-inflationary era

$$= a_{\rm R}\left(\frac{\tau_{\rm I}}{\tau}\right) e^{-\mathcal{N}_{\rm tot}} \qquad \tau_{\rm I} < \tau < \tau_{\rm R}$$ De Sitter inflation

$$= a_{\rm R}\left(2 - \frac{\tau}{\tau_{\rm R}}\right) \qquad \tau_{\rm R} < \tau$$ Radiation dominated era

$$H = -\frac{1}{a_{\rm R}\tau_{\rm R}}, \qquad \mathcal{N}_{\rm tot} = \log(a_{\rm R}/a_{\rm I}) = \log(\tau_{\rm I}/\tau_{\rm R})$$

We have some preferred UV / IR scales … Do they appear as UV cutoffs?

Finite duration inflation

Energy density massless scalar field during post-inflationary radiation dominated era

$$\rho_\phi^{\mathrm{RD}} = \frac{1}{4\pi^2 a^2}\int_0^\infty k^2\,dk\left[k^2\,|\,\phi_k^{\mathrm{RD}}(\tau)\,|^2 + |\,\phi_k^{\mathrm{RD}\prime}(\tau)\,|^2\right]\ , \quad \phi_k^{\mathrm{RD}} = \frac{1}{a}\frac{1}{\sqrt{2k}}\left[\alpha_k^{\mathrm{R}} e^{-ik\tau_{\mathrm{R}}\left(2-\frac{a}{a_{\mathrm{R}}}\right)} + \beta_k^{\mathrm{R}} e^{ik\tau_{\mathrm{R}}\left(2-\frac{a}{a_{\mathrm{R}}}\right)}\right]$$

Finite duration inflation

Energy density massless scalar field during post-inflationary radiation dominated era

$$\rho_\phi^{\mathrm{RD}} = \frac{1}{4\pi^2 a^2}\int_0^\infty k^2\, dk \left[k^2 |\phi_k^{\mathrm{RD}}(\tau)|^2 + |\phi_k^{\mathrm{RD}\prime}(\tau)|^2\right], \quad \phi_k^{\mathrm{RD}} = \frac{1}{a}\frac{1}{\sqrt{2k}}\left[\alpha_k^{\mathrm{R}} e^{-ik\tau_{\mathrm{R}}\left(2-\frac{a}{a_{\mathrm{R}}}\right)} + \beta_k^{\mathrm{R}} e^{ik\tau_{\mathrm{R}}\left(2-\frac{a}{a_{\mathrm{R}}}\right)}\right]$$

Matching from initial Bunch Davies vacuum

$$\alpha_k^{\mathrm{R}} = \alpha_k^{\mathrm{I}}\left(-i + \frac{a_{\mathrm{R}}H}{k} + i\frac{a_{\mathrm{R}}^2H^2}{2k^2}\right) + i\beta_k^{\mathrm{I}}\frac{a_{\mathrm{R}}^2H^2}{2k^2}e^{-2i\frac{k}{a_{\mathrm{R}}H}} \qquad \alpha_k^{\mathrm{I}} = \left(i - i\frac{a_{\mathrm{I}}^2H^2}{2k^2} + \frac{a_{\mathrm{I}}H}{k}\right)$$

$$\beta_k^{\mathrm{R}} = -i\alpha_k^{\mathrm{I}}\frac{a_{\mathrm{R}}^2H^2}{2k^2}e^{2i\frac{k}{a_{\mathrm{R}}H}} + \beta_k^{\mathrm{I}}\left(i + \frac{a_{\mathrm{R}}H}{k} - i\frac{a_{\mathrm{R}}^2H^2}{2k^2}\right) \qquad \beta_k^{\mathrm{I}} = i\frac{a_{\mathrm{I}}^2H^2}{2k^2}e^{2i\frac{k}{a_{\mathrm{I}}H}}$$

Finite duration inflation

Energy density results

$$\rho_\phi^{\mathrm{RD}} = \frac{1}{8\pi^2 a^4}\int_0^\infty \frac{dk}{k}\left[k^4\left(2+\frac{a_{\mathrm{R}}^4 H^2}{a^2 k^2}\right)\left(1+\frac{a_{\mathrm{R}}^4 H^4 + a_{\mathrm{I}}^4 H^4}{2k^4}+\frac{a_{\mathrm{I}}^4 a_{\mathrm{R}}^4 H^8}{4k^8}\right)+\{\mathrm{osc}\}\right]$$

Finite duration inflation

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Does not contribute in UV limit

Finite duration inflation

Divergent contribution energy density results

$$\rho_\phi^{\mathrm{RD,div}} = \frac{1}{8\pi^4 a^4}\int_{-\infty}^{\infty} d^4k \left(1 + \frac{H^4 {a_\mathrm{R}}^4 + H^4 {a_\mathrm{I}}^4}{2k^4}\right) + \frac{1}{8\pi^4 a^6}\int_{-\infty}^{\infty} d^4k \frac{a_\mathrm{R}^4 H^2}{2k^2}$$

Finite duration inflation

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Scales corresponding to beginning / end of inflation *do not* regulate UV divergencies!

Finite duration inflation

Divergent contribution energy density results

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How do we regularize?

Regularization methods - Comparison

Divergent contribution energy density results

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Regularizing using hard cutoffs in physical momenta ($k_{\mathrm{UV}} = a\Lambda_{\mathrm{UV}}$)

$$\mathrm{c.t.} = \frac{\mu^4 - \Lambda_{\mathrm{UV}}^4}{16\pi^2} + \frac{H^4(1 + e^{-4\mathcal{N}_{\mathrm{tot}}})}{8\pi^2(a/a_{\mathrm{R}})^4}\log\frac{\mu}{\Lambda_{\mathrm{UV}}} + \frac{\mu^2 - \Lambda_{\mathrm{UV}}^2}{16\pi^2}\frac{H^2}{(a/a_{\mathrm{R}})^4}$$

Regularization methods - Comparison

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Regularizing with dimensional regularization

$$\mathrm{c.t.} = \frac{H^4(1 + e^{-4\mathcal{N}_{\mathrm{tot}}})}{8\pi^2 (a/a_{\mathrm{R}})^4}\left[-\frac{1}{\delta_{\mathrm{UV}}} + 1 - \gamma_E + \log\left(\frac{\mu}{H}\right)\right]$$

Regularization methods - Comparison

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Coefficient of the log matches!

Regularization methods - Comparison

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Coefficient of the log matches!

However…

Regularization methods - Comparison

Consider energy density and pressure of massless scalar field on de Sitter

Regularization methods - Comparison

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$$\rho = \frac{H^4}{8\pi^2}\int_0^\infty \frac{dk}{k}\left[\left(\frac{k}{aH}\right)^2 + 2\left(\frac{k}{aH}\right)^4\right] \quad , \quad p = \frac{H^4}{8\pi^2}\int_0^\infty \frac{dk}{k}\left[-\frac{1}{3}\left(\frac{k}{aH}\right)^2 + \frac{2}{3}\left(\frac{k}{aH}\right)^4\right]$$

Regularization methods - Comparison

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Regularizing using hard cutoffs:

required counterterms cannot be constructed from geometric invariants ($\not\propto g_{\mu\nu}, R_{\mu\nu} \ldots$)

Regularization methods - Comparison

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From now on we *only* use dimensional regularization

Outline

Renormalizing in the Early Universe: *what can go wrong?*

- IR divergences VS UV divergences
- **Regularization:** UV cutoffs VS physical UV scales
- **Renormalization:** hard cutoffs VS dimensional regularization counterterterms

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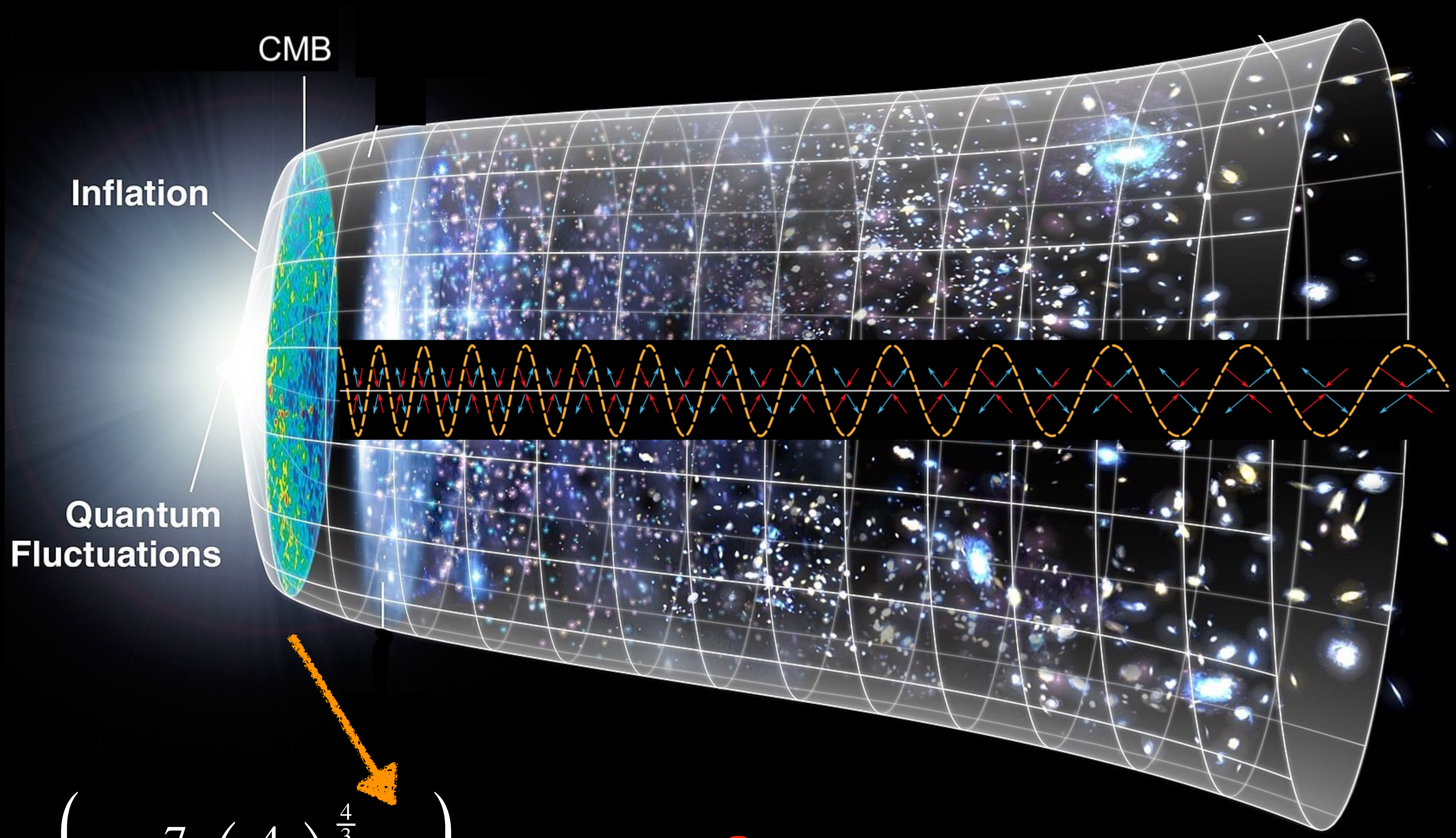
Outline

Renormalizing in the Early Universe: *what can go wrong?*

- **Renormalization conditions** VS predicted observables

  Can $N_{\rm eff}$ bounds constrain vacuum GWs?

REMINDER !



ρ_{gw}

$$\frac{\rho_{tot}^{RD,BBN}}{\rho_\gamma} = \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} N_{eff}\right) \quad \overset{?}{\longleftrightarrow} \quad \rho_{tot}^{RD,BBN} = \rho_\gamma + \rho_\nu + \rho_{gw} \sim \infty$$

ρ_{gw} definition

ρ_{gw} definition

$$^{(4)}\rho_{gw} = \frac{1}{32\pi G_N a^2}\left\langle h'_{ij}(\eta, k) h^{ij'}(\eta, k)\right\rangle$$

[4] *M. Maggiore Gravitational waves vol 1*

ρ_{gw} definition

[4]
$$\rho_{gw} = \frac{1}{32\pi G_N a^2}\left\langle h'_{ij}(\eta,k)h^{ij'}(\eta,k)\right\rangle$$

[5] **Isaacson definition**

assumptions:

- High frequency GWs (wrt to curvature radius)
- $\sim$ Flat spacetime

[4] *M. Maggiore Gravitational waves vol 1*

[5] *Gravitational Radiation in the Limit of High Frequency. II. Nonlinear Terms and the Effective Stress Tensor (1968)*

ρ_{gw} RE-definition

$$\rho_{gw} = \frac{1}{64\pi a^2 G_N}\left\langle \hat{h}'_{ij}\hat{h}'^{ij} - 3\partial_k \hat{h}_{ij}\partial^k \hat{h}^{ij} - 4\hat{h}_{ij}\partial_k\partial^k \hat{h}^{ij} + 2\partial_k \hat{h}_{ij}\partial^j \hat{h}^{ik} + 8\mathcal{H}\hat{h}_j{}^i \hat{h}'_i{}^j \right\rangle$$

Isaacson definition

assumptions:

- ~~High frequency GWs~~
- ~~Flat spacetime~~

ρ_{gw}- Finite duration inflation

Consider tensor vacuum perturbations ($\nabla_i h^{ij} = 0,\, h^i_i = 0$)

$$\hat{h}_{ij}(\tau, x) = \sum_{r=+,x} \int \frac{d^3k}{M_{\rm pl}\,(2\pi)^3} e^{ix\cdot k} \left[\epsilon^r_{ij}(k)\hat{a}_k\gamma_k(\tau) + \epsilon^{r*}_{ij}(-k)\hat{a}^\dagger_{-k}\gamma^*_k(\tau)\right]$$

ρ_{gw}- Finite duration inflation

Consider tensor vacuum perturbations during post-inflationary radiation dominated era

$$\hat{h}_{ij}^{\mathrm{RD}}(\tau, x) = \sum_{r=+,x} \int \frac{d^3k}{M_{\mathrm{pl}}\,(2\pi)^3} e^{ix\cdot k} \left[\epsilon_{ij}^{r}(k)\hat{a}_k \gamma_k^{\mathrm{RD}}(\tau) + \epsilon_{ij}^{r*}(-k)\hat{a}_{-k}^{\dagger}\gamma_k^{*\mathrm{RD}}(\tau)\right]$$

where

$$\gamma_k^{\mathrm{RD}}(\tau) = \frac{1}{a}\frac{1}{\sqrt{2k}}\left[\alpha_k^{\mathrm{R}} e^{-ik\tau_{\mathrm{R}}\left(2-\frac{a}{a_{\mathrm{R}}}\right)} + \beta_k^{\mathrm{R}} e^{ik\tau_{\mathrm{R}}\left(2-\frac{a}{a_{\mathrm{R}}}\right)}\right]$$

$$\gamma_k^{\prime\,\mathrm{RD}}(\tau) = \frac{a_{\mathrm{R}}}{a^2\tau_{\mathrm{R}}\sqrt{2k}}\left[\alpha_k^{\mathrm{R}} e^{-ik\tau_{\mathrm{R}}\left(2-\frac{a}{a_{\mathrm{R}}}\right)}\left(1-\frac{iak\tau_R}{a_{\mathrm{R}}}\right) + \beta_k^{\mathrm{R}} e^{ik\tau_{\mathrm{R}}\left(2-\frac{a}{a_{\mathrm{R}}}\right)}\left(1+\frac{ik\tau_{\mathrm{R}}a}{a_{\mathrm{R}}}\right)\right]$$

ρ_{gw}- Finite duration inflation

ρ_{gw} results

$$\rho_{gw} = \frac{1}{2\pi^2 a^4} \int_0^\infty \frac{dk}{k} k^4 \left[\left(1 - \frac{7 a_{\mathrm{R}}^4 H^2}{2 a^2 k^2} \right) \left(1 + \frac{a_{\mathrm{R}}^4 H^4 + a_{\mathrm{I}}^4 H^4}{2k^4} + \frac{a_{\mathrm{I}}^4 a_{\mathrm{R}}^4 H^8}{4k^8} \right) + \{\mathrm{osc}\} \right]$$

ρ_{gw}- Finite duration inflation

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Does not contribute in UV limit

ρ_{gw}- Finite duration inflation

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Divergent contribution that necessitates subtraction

$$\rho_{\mathrm{gw,div}} = \frac{1}{4\pi^4 a^4}\int_{-\infty}^{\infty} d^4k \left(1 + \frac{a_{\mathrm{R}}^4 H^4 + a_{\mathrm{I}}^4 H^4}{2k^4} - \frac{7\, a_{\mathrm{R}}^4 H^2}{2a^2k^2}\right)$$

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Regularising using dimensional regularization

$$\rho_{\mathrm{gw,div}} = \lim_{\delta_{\mathrm{UV}}\to 0}\left\{\frac{H^4\left(1+e^{-4\mathcal{N}_{\mathrm{tot}}}\right)}{4\pi^2(a/a_R)^4}\left[\frac{1}{\delta_{\mathrm{UV}}}+1-\gamma_E+\log\left(\frac{\mu}{H}\right)\right]\right\}$$

ρ_{gw}- Finite duration inflation

Absorbing divergences, Einstein equations during radiation domination

$$\frac{1}{8\pi G_B}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = T^{\rm bg}_{\mu\nu} + \langle \hat{T}^{gw}_{\mu\nu}\rangle + \langle \hat{T}^{\rm ct}_{\mu\nu}\rangle$$

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00 component results

$$-\frac{R_0{}^0}{8\pi G_B} = \rho^{\mathrm{cl}}_{\mathrm{bg}} + \rho_{gw} + \rho_{\mathrm{ct}}$$

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$$\frac{1}{8\pi G_B}\frac{3H^2}{(a/a_R)^4}=\rho^{\rm cl}_{\rm bg}+\frac{H^4\left(1+e^{-4\mathcal{N}_{\rm tot}}\right)}{4\pi^2(a/a_R)^4}\left[\frac{1}{\delta_{\rm UV}}+1-\gamma_E+\log\left(\frac{\mu}{H}\right)\right]+\rho_{\rm ct}+\rho_{gw,\rm finite}$$

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ρ_{gw}- Finite duration inflation

Finite 00 component results

$$\frac{1}{8\pi G_B}\left(\frac{3H^2}{(a/a_R)^4}\right) = \rho_{\rm bg}^{\rm cl} + \rho_{gw,\rm finite} + \left(\frac{3H^2}{(a/a_R)^4}\right)\left[\frac{H^2\left(1+e^{-4\mathcal{N}_{\rm tot}}\right)}{12\pi^2}\left(1-\gamma_E+\log\left(\frac{\mu}{H}\right)\right)+B_0\right]$$

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We define finite renormalized Newtonian constant as

$$\frac{1}{8\pi G_N(\mu)} = \frac{1}{8\pi G_B} - B_0 - \frac{H^2}{12\pi^2}(1+e^{-4\mathcal{N}_{\rm tot}})\left[1-\gamma_E+\log\left(\frac{\mu}{H}\right)\right]$$

ρ_{gw}- Finite duration inflation

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$$\frac{1}{8\pi G_N(\mu)}\frac{3H^2}{(a/a_R)^4} = \rho_{\rm bg}^{\rm cl} + \rho_{gw,\rm finite}$$

Can we use this result to constrain $\rho_{gw,\rm finite}$?

ρ_{gw}- Finite duration inflation

Finite 00 component results

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So that 00 component results

$$\frac{1}{8\pi G_N(\mu)}\frac{3H^2}{(a/a_R)^4} = \rho_{\rm bg}^{\rm cl} + \rho_{gw,\rm finite}$$

$\rho_{gw,\rm finite}$ **and** $G_N(\mu)$

to be fixed by Renormalization Conditions

Can we constrain primordial GWs with $N_{\rm eff}$ bounds?

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No!
Primordial GWs are *indistinguishable* from the “classical background”

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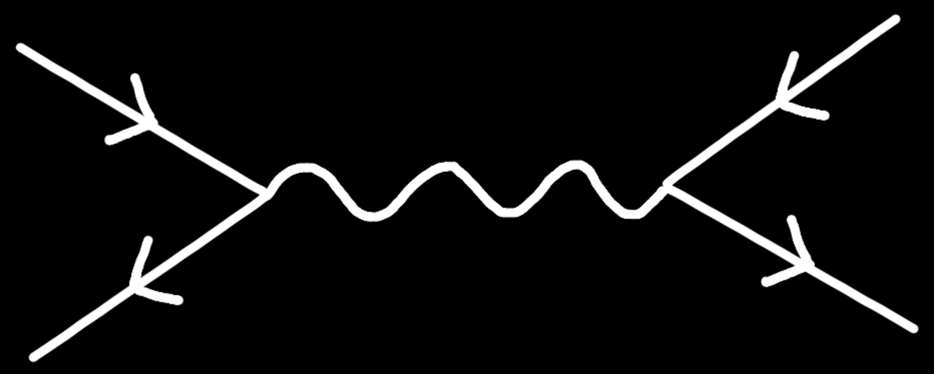
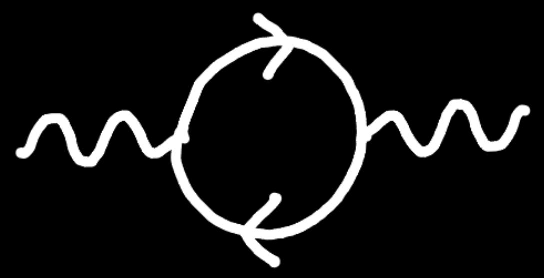
$\sigma_{\mathrm{meas}} =$  $+$  $+ \ldots$

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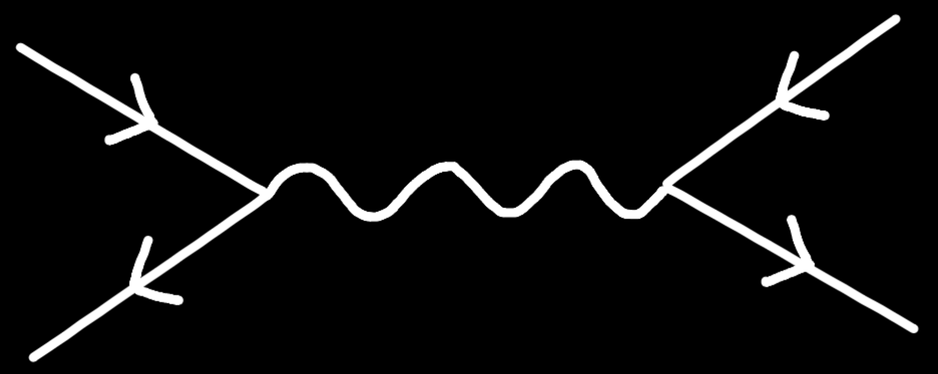
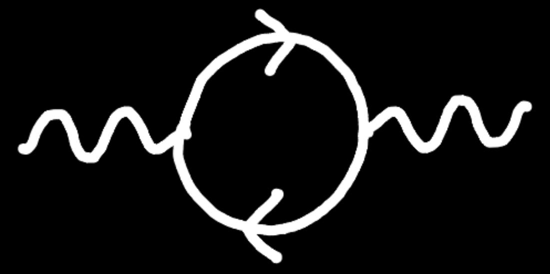
$\sigma_{\text{meas}} =$  $+$  $+ \ldots$

We *do not* separately measure “classical quantities” and “quantum corrections”

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Pro:

Cons:

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Pro: Less constraints, more freedom!

Cons:

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Pro: Less constraints, more freedom!

Cons: More freedom, less constraints…

Can we constrain primordial GWs with $N_{\rm eff}$ bounds?

No!
Primordial GWs are *indistinguishable* from the “classical background”

Pro: Less constraints, more freedom!

Cons: More freedom, less constraints… *Working progress*

Take home message

Take home message

Infinities appear in cosmology

Take home message

Infinities appear in cosmology: UV or IR?

Take home message

Infinities appear in cosmology: IR divergence

→ It should not be there

Take home message

Infinities appear in cosmology: UV divergence

→ Keep calm and use the Renormalization Procedure

Take home message

Infinities appear in cosmology: UV divergence

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- **Regularize:**

Take home message

Infinities appear in cosmology: UV divergence

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- **Regularize:**

- **Renormalize:**

Take home message

Infinities appear in cosmology: UV divergence

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- **Regularize:**
- **Renormalize:**
- **Fix Renormalization Conditions:**

Take home message

Infinities appear in cosmology: UV divergence

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- **Regularize:**
- **Renormalize:**
- **Fix Renormalization Conditions:**
- **Predictive theory!**

Take home message

Infinities appear in cosmology: UV divergence

→ Keep calm and use the Renormalization Procedure

- **Regularize:**

→ Physical scales are not UV cutoffs (and the other way around!)

- **Renormalize:**

- **Fix Renormalization Conditions:**

- **Predictive theory!**

Take home message

Infinities appear in cosmology: UV divergence

→ Keep calm and use the Renormalization Procedure

- **Regularize:**

  → Physical scales are not UV cutoffs (and the other way around!)

- **Renormalize:**

  → Using counterterms that respect symmetries of background

- **Fix Renormalization Conditions:**

- **Predictive theory!**

Take home message

Infinities appear in cosmology: UV divergence

→ Keep calm and use the Renormalization Procedure

- **Regularize:**

→ Physical scales are not UV cutoffs (and the other way around!)

- **Renormalize:**

→ Using counterterms that respect symmetries of background

- **Fix Renormalization Conditions:**

→ We measure fully quantum corrected observables

- **Predictive theory!**

Take home message

Infinities appear in cosmology: UV divergence

→ Keep calm and use the Renormalization Procedure

- **Regularize:**

→ Physical scales are not UV cutoffs (and the other way around!)

- **Renormalize:**

→ Using counterterms that respect symmetries of background

- **Fix Renormalization Conditions:**

→ Vacuum GWs cannot be constrained by $N_{\rm eff}$ bounds

- **Predictive theory!**

Take home message

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→ Keep calm and use the Renormalization Procedure

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→ Physical scales are not UV cutoffs (and the other way around!)

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→ Using counterterms that respect symmetries of background

- **Fix Renormalization Conditions:**

→ Vacuum GWs cannot be constrained by $N_{\rm eff}$ bounds

- **Predictive theory!**

Thank you!

Backup slides

Scaleless integrals and dim-reg

Two point function massless non interacting scalar field on pure de Sitter (late time)

$$\lim_{\tau\to 0}\langle\hat{\phi}(\tau,x)\hat{\phi}(\tau,x)\rangle = \frac{H^2}{8\pi^4}\int_{-\infty}^{\infty}\frac{d^4k}{k^4} = \frac{H^2}{8\pi^4}\int_{-\infty}^{\infty} d^4k\left[\frac{1}{k^2(k^2+m^2)} + \frac{m^2}{k^4(k^2+m^2)}\right]$$

Using dimensional regularization we obtain equal but opposite contributions

$$+/-\ \frac{H^2}{4\pi^2}\left[\frac{1}{\delta} - \frac{1}{2}\left(\log\frac{m^2}{4\pi\mu^2} + \gamma_E - 1\right)\right]$$

Keeping separate UV/IR contributions (assuming IR div disappear in well defined obs)

$$\lim_{\tau\to 0}\langle\hat{\phi}(\tau,x)\hat{\phi}(\tau,x)\rangle = \left(\frac{H}{2\pi}\right)^2\left[\frac{1}{\delta_{\text{UV}}} - \frac{1}{\delta_{\text{IR}}} + \log\frac{\mu_{\text{UV}}}{\mu_{\text{IR}}}\right] \text{“=“} \left(\frac{H}{2\pi}\right)^2\left[\frac{1}{\delta_{\text{UV}}} + \log\frac{4\pi\mu_{\text{UV}}}{m}\right]$$

Massless field on de Sitter

Two point function

$$\langle \hat{\phi}(\tau, x)\hat{\phi}(\tau, x)\rangle = \left(\frac{H}{2\pi}\right)^2 \int_0^\infty \frac{dk}{k}\left[1 + \left(\frac{k}{aH}\right)^2\right]$$

- UV divergent
- IR divergent

Regularizing using dimensional regularization

$$\langle \hat{\phi}(\tau, x)\hat{\phi}(\tau, x)\rangle = \frac{H^2}{8\pi^4}\int_{-\infty}^{\infty} \frac{d^4q}{q^4}(1+q^2) = \frac{H^2}{8\pi^4}\int_{-\infty}^{\infty} d^4q \left[\frac{1}{(q^2+\widetilde{m}^2)} + \frac{1+\widetilde{m}^2}{q^2(q^2+\widetilde{m}^2)} + \frac{\widetilde{m}^2}{q^4(q^2+\widetilde{m}^2)}\right]$$

Counterterm that subtracts UV divergence results (ignoring IR divergences)

$$\mathrm{c.t.} = \left(\frac{H}{2\pi}\right)^2 \left[\log\left(\frac{\mu}{H}\right) - \frac{1}{\delta_{\mathrm{UV}}} + \frac{1}{2}\left(\gamma_E - 1 - \log 4\pi\right)\right]$$

Massless field on de Sitter

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- UV divergent
- IR divergent

Regularizing using dimensional regularization ($q := k/(aH)$ and $\widetilde{m} := \mu/H$):

$$\langle \hat{\phi}(\tau,x)\hat{\phi}(\tau,x)\rangle = \frac{H^2}{8\pi^4}\int_{-\infty}^{\infty}\frac{d^4q}{q^4}(1+q^2) = \frac{H^2}{8\pi^4}\int_{-\infty}^{\infty} d^4q\left[\frac{1}{(q^2+\widetilde{m}^2)} + \frac{1+\widetilde{m}^2}{q^2(q^2+\widetilde{m}^2)} + \frac{\widetilde{m}^2}{q^4(q^2+\widetilde{m}^2)}\right]$$

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Finite duration inflation

Two point function massless non interacting scalar field

$$\langle \hat{\phi}^{\mathrm{RD}}(\tau,x)\hat{\phi}^{\mathrm{RD}}(\tau,x)\rangle = \frac{1}{2\pi^2 a^2}\int_0^\infty \frac{dk}{k}\frac{k^2}{2}\left[1+2\,|\beta_k^{\mathrm{R}}|^2+\alpha_k^{\mathrm{R}}\beta_k^{\mathrm{R}*}e^{\frac{2ik}{a_{\mathrm{R}}H}\left(2-\frac{a}{a_{\mathrm{R}}}\right)}+\alpha_k^{\mathrm{R}*}\beta_k^{\mathrm{R}}e^{-\frac{2ik}{a_{\mathrm{R}}H}\left(2-\frac{a}{a_{\mathrm{R}}}\right)}\right]$$

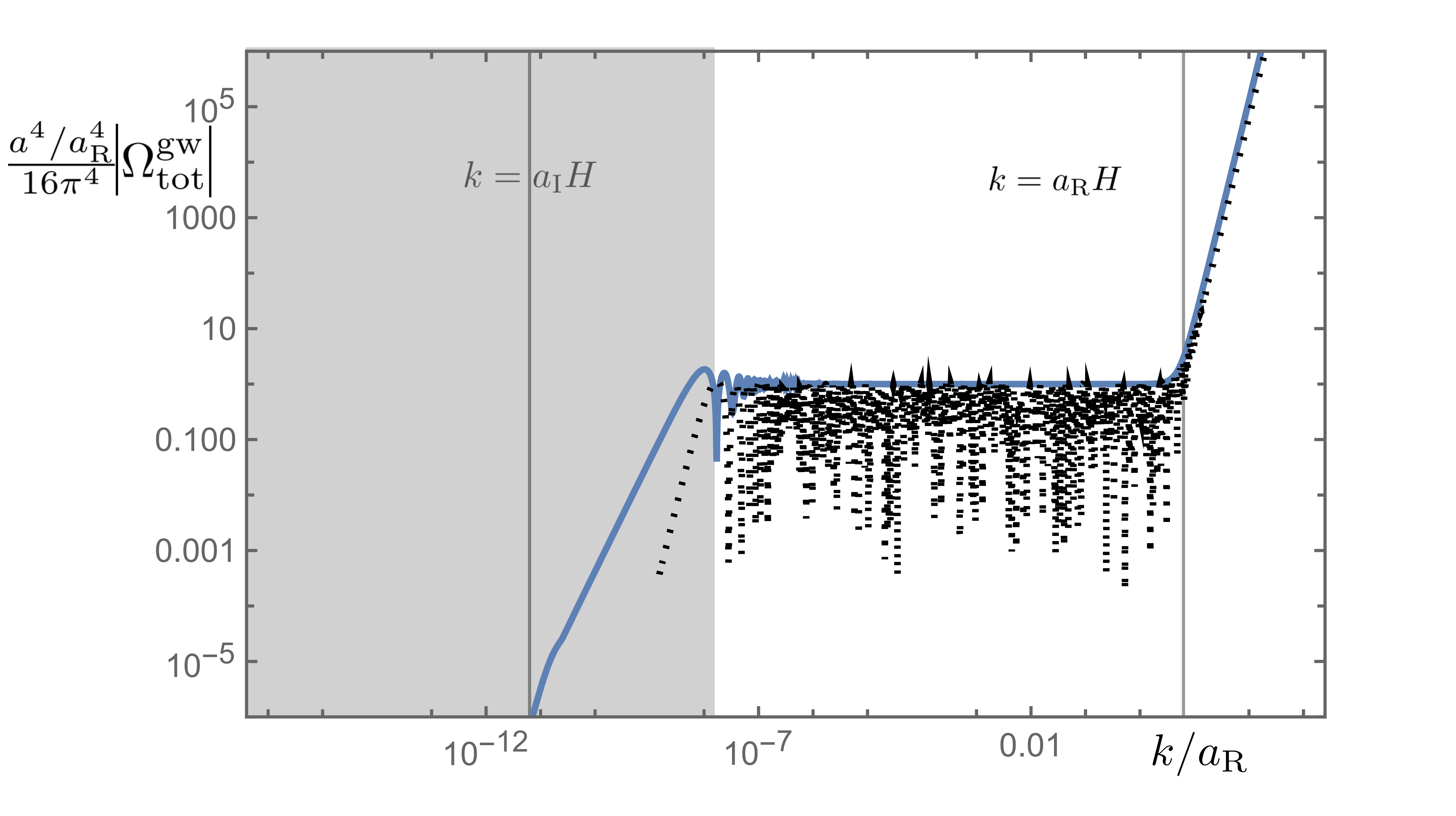
$$= \frac{1}{2\pi^2 a^2}\int_0^\infty \frac{dk}{k}\frac{k^2}{2}\left[1+\cancel{\frac{a_{\mathrm{R}}^4H^4+a_{\mathrm{I}}^4H^4}{2k^4}}+\frac{a_{\mathrm{I}}^4a_{\mathrm{R}}^4H^8}{4k^8}+\{\mathrm{osc}\}\right]$$

$$\lim_{k\to 0}\{\mathrm{osc}\} = -1-\cancel{\frac{a_{\mathrm{R}}^4H^4+a_{\mathrm{I}}^4H^4}{2k^4}}-\frac{a_{\mathrm{I}}^4a_{\mathrm{R}}^4H^8}{4k^8}+\frac{(3a_{\mathrm{I}}^3a_{\mathrm{R}}+2a(a_{\mathrm{R}}^3-a_{\mathrm{I}}^3))^2}{9a_{\mathrm{I}}^2a_{\mathrm{R}}^6}$$

Having a pre-inflationary era *cures* IR divergences!

Comparison Isaacson ρ_{gw} and Improved ρ_{gw}

Result energy density from Isaacson stress tensor (dashed lines) compared with improved result (blue line).



Grey shaded regions correspond to super-horizon scales (outside domain of validity of Isaacson stress tensor, and also where improved becomes negative)