ABSENCE OF ONE-LOOP EFFECTS ON LARGE SCALE FROM SMALL SCALES

JACOPO FUMAGALLI (ICCUB)

Looping in the primordial universe workshop
CERN,
29th October 2024

2305.19263, 2408.08296

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EXCELENCIA
MARÍA
DE MAEZTU
2020-2023

Could small scales perturbations lead to an effect on large scales?

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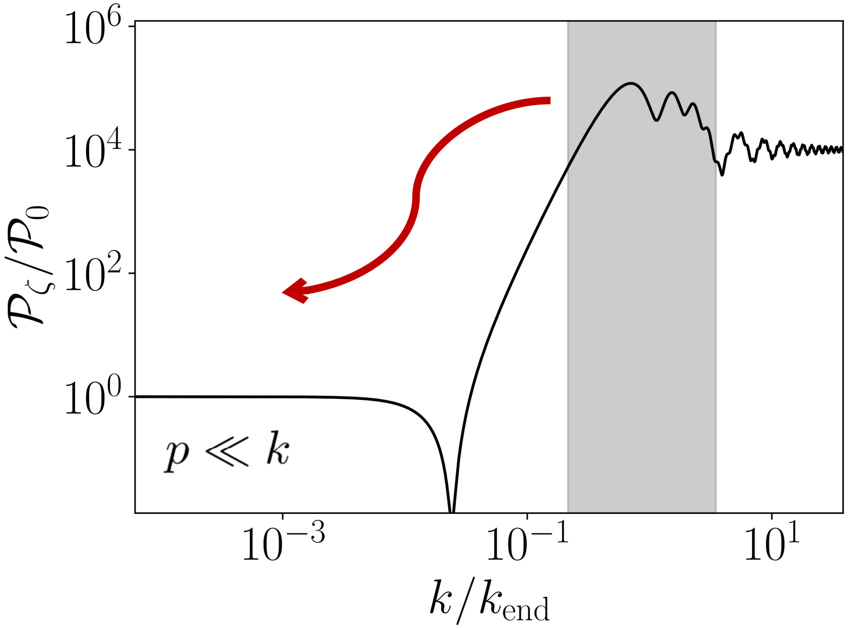
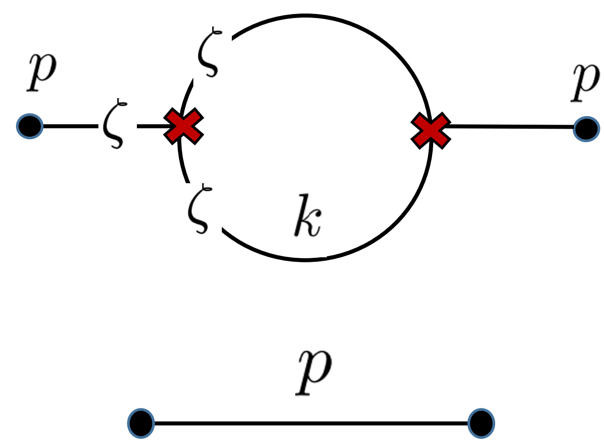
$$\mathcal{P}_\zeta = \mathcal{P}_\zeta^{\rm tree} + \mathcal{P}_\zeta^{1-\rm loop} + \dots\,,$$

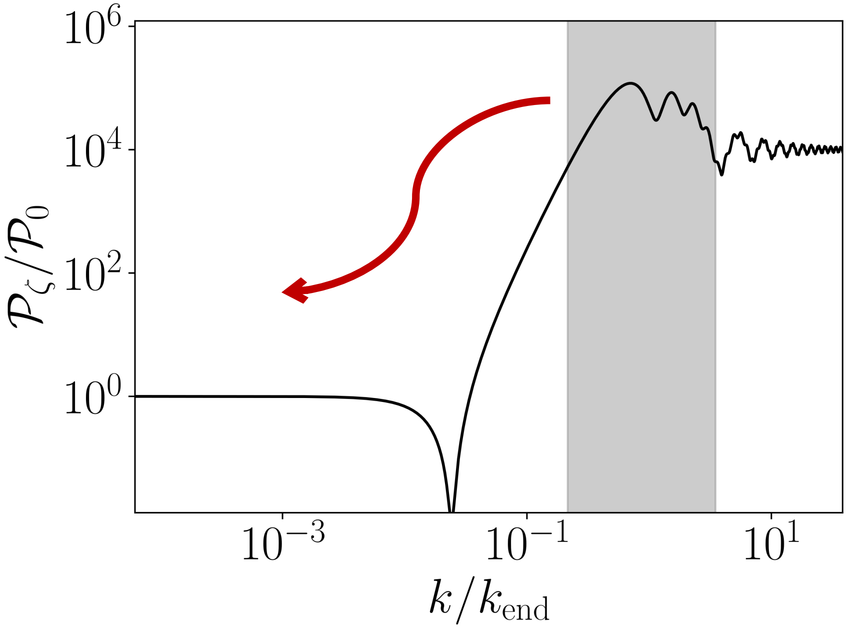


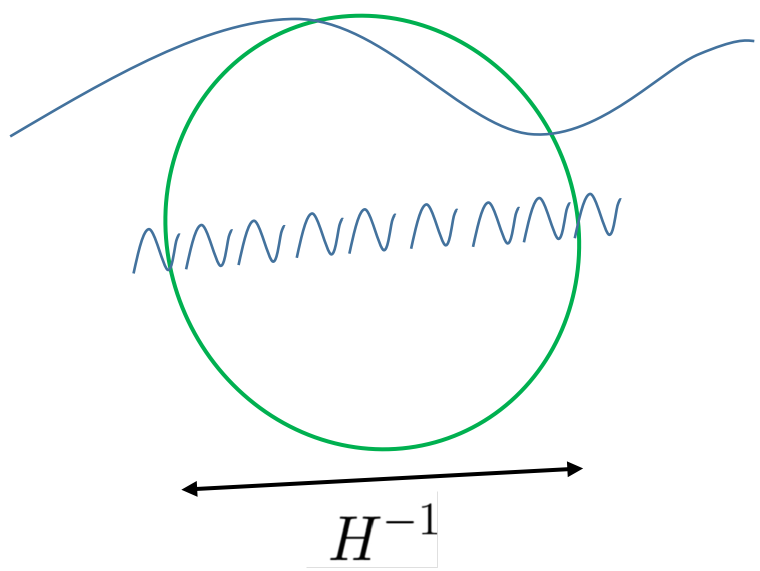


ONE-LOOP CORRECTIONS

$$\mathcal{P}_\zeta^{1-\text{loop}}(p) = \mathcal{P}^{\text{tree}}(p) \int d\ln k\, C(k) + O\left(\frac{p^3}{k^3}\right), \qquad p \ll k$$

J. Kristiano and J. Yokoama ‘22,
A. Riotto ‘23, H. Firouzjahi ‘23, A. Riotto and H. Firouzjahi ‘23, G. Franciolini et al. ‘23 ...





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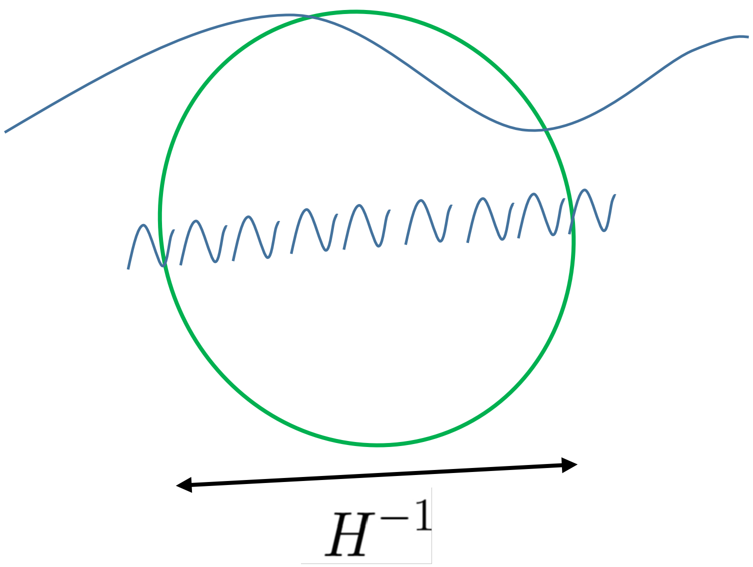
IMPLICATIONS

- Small scales / Large scales effect which is scales independent ?



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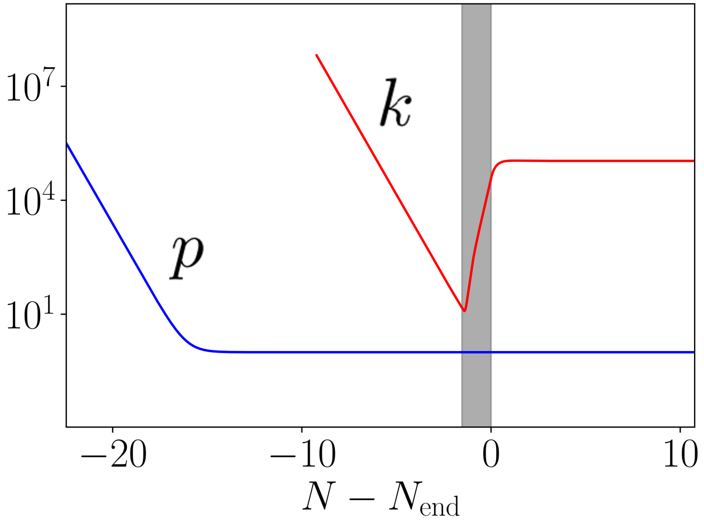

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J. Fumagalli 2305.19263, 2408.08296

See also Y. Tada, T. Terada and J. Tokuda '23, Keisuke I. '24 talks and works

and R. Kawaguchi, S. Tsujikawa and Y. Yamada 2407.19742

IMPLICATIONS

- Small scales / Large scales effect which is scales independent ?
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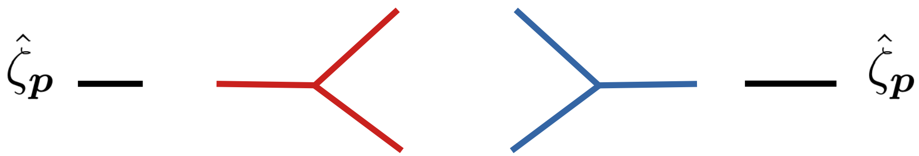
ORIGIN OF RELATIVE SCALE INVARIANT CORRECTIONS

$$\langle \hat{\zeta}_{\boldsymbol{p}}(\tau)\hat{\zeta}_{\boldsymbol{p}'}(\tau)\rangle_{H^{(3)}} = -\int_{\tau_0}^{\tau} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \langle [H^{(3)}(\tau_2), [H^{(3)}(\tau_1), \hat{\zeta}_{\boldsymbol{p}}(\tau)\hat{\zeta}_{\boldsymbol{p}'}(\tau)]]\rangle$$

$$H^{(3)} \supset \lambda(\tau)\cdot \zeta^A\zeta^B\zeta^C, \quad \zeta^X \equiv \{\zeta, \zeta', \partial_i\zeta\}$$



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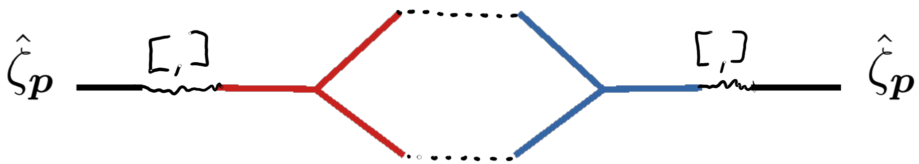

$$\mathcal{P}_\zeta^{1-\text{loop}}(p) \propto p^3\langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle \propto p^3\int \underbrace{d\boldsymbol{K}\lambda(\tau_1)\left[\hat{\zeta}_1^A, \hat{\zeta}_{\boldsymbol{p}}\right]\cdot\left[\hat{\zeta}_2^A, \hat{\zeta}_{\boldsymbol{p}}\right]\langle\hat{\zeta}_2^B\hat{\zeta}_2^C\hat{\zeta}_1^B\hat{\zeta}_1^C\rangle}_{\text{no dependence on the large scale}} \propto O\left(\frac{p^3}{k^3}\right)$$

In the limit $p\tau,\, p\tau_1 \ll 1,$ commutators are independent on the large scale

$$\left[\hat{\zeta}_{\boldsymbol{k}}(\tau_1),\, \hat{\zeta}_{\boldsymbol{p}}(\tau)\right]' = \mathcal{W}_{\tau_1}\, g_p\,(\tau,\,\tau_1) \qquad \mathcal{W}_{\tau_1} = \frac{i}{2\epsilon(\tau_1)\,a^2(\tau_1)}, \qquad g_p(\tau,\tau_1) \;\propto\; \tau_1$$

$$\left[\hat{\zeta}'_{\boldsymbol{k}}(\tau_1),\, \hat{\zeta}_{\boldsymbol{p}}(\tau)\right]' = \partial_{\tau_1}\left(\mathcal{W}_{\tau_1} g_p\,(\tau,\,\tau_1)\right) \simeq -\mathcal{W}_{\tau_1}.$$

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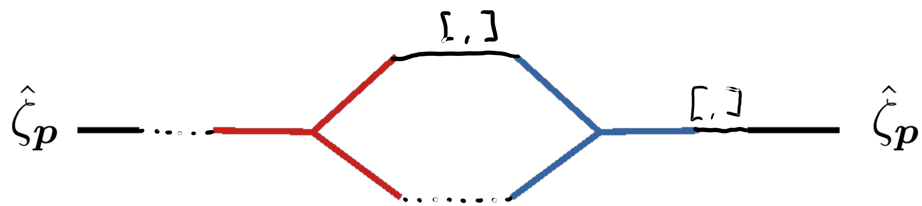

$$\mathcal{P}_\zeta^{1-\text{loop}}(p) \propto p^3\langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle \propto \underline{p^3}\int d\boldsymbol{K}\,\lambda(\tau_1)\left[\hat{\zeta}_1^A, \hat{\zeta}_{\boldsymbol{p}}\right]\cdot\left[\hat{\zeta}_2^A, \hat{\zeta}_{\boldsymbol{p}}\right]\langle\hat{\zeta}_2^B\hat{\zeta}_2^C\underline{\hat{\zeta}_1^B\hat{\zeta}_1^C}\rangle,$$

$$= \underline{\mathcal{P}^{\text{tree}}(p)}\int d\ln k\, C(k)$$

<u>Relative scale invariant correction</u>

In the limit $p\tau,\, p\tau_1 \ll 1,$ commutators are independent on the large scale

$$\left[\hat{\zeta}_{\boldsymbol{k}}(\tau_1),\, \hat{\zeta}_{\boldsymbol{p}}(\tau)\right]' = \mathcal{W}_{\tau_1}\, g_p\,(\tau,\, \tau_1) \qquad \mathcal{W}_{\tau_1} = \frac{i}{2\epsilon(\tau_1)\, a^2(\tau_1)}, \qquad g_p(\tau, \tau_1) \;\propto\; \tau_1$$

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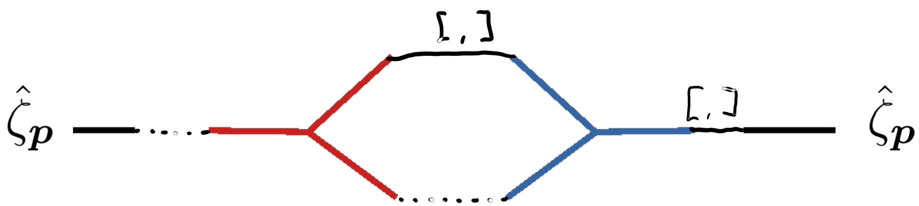
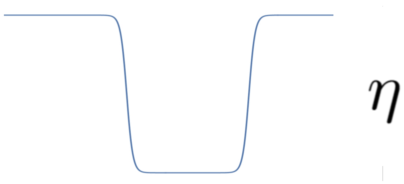
ORIGIN OF RELATIVE SCALE INVARIANT CORRECTIONS

$$\langle \hat{\zeta}_{\boldsymbol{p}}(\tau)\hat{\zeta}_{\boldsymbol{p}'}(\tau)\rangle_{H^{(3)}} = -\int_{\tau_0}^{\tau} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \langle [H^{(3)}(\tau_2), [H^{(3)}(\tau_1), \hat{\zeta}_{\boldsymbol{p}}(\tau)\hat{\zeta}_{\boldsymbol{p}'}(\tau)]]\rangle$$



<u>For instance:</u> $H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta'$

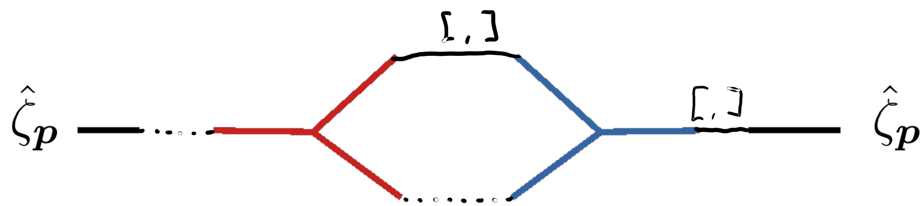


J. Kristiano and J. Yokoama '22,

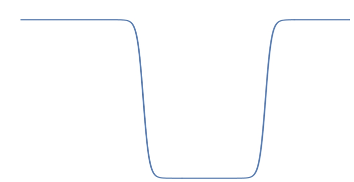
$$\mathcal{P}_{\zeta}^{1-\mathrm{loop}}(p) \cong \frac{|\Delta\eta|^2}{4}\mathcal{P}_{\zeta}^{\mathrm{tree}}(p)\int d\ln k\, \mathcal{P}_{\zeta}^{\mathrm{tree}}(k,\tau_{\mathrm{e}}).$$

ORIGIN OF RELATIVE SCALE INVARIANT CORRECTIONS

$$\langle \hat{\zeta}_{\boldsymbol{p}}(\tau)\hat{\zeta}_{\boldsymbol{p}'}(\tau)\rangle_{H^{(3)}} = -\int_{\tau_0}^{\tau} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \langle [\underline{H^{(3)}}(\tau_2), [\underline{H^{(3)}}(\tau_1), \hat{\zeta}_{\boldsymbol{p}}(\tau)\hat{\zeta}_{\boldsymbol{p}'}(\tau)]]\rangle$$



<u>For instance:</u> $H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right] + \ldots$



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J. Kristiano and J. Yokoama '22,
JF 2305.19263
H. Firouzjahi 2311.04080

$$\mathcal{P}_\zeta^{1-\text{loop}}(p) \cong \frac{|\Delta\eta|^2}{4}\mathcal{P}_\zeta^{\text{tree}}(p)\int d\ln k\, \mathcal{P}_\zeta^{\text{tree}}(k,\tau_{\text{e}}).$$

$$+ \quad \mathcal{P}_\zeta^{\text{tree}}(p)\int^{\tau} d\tau_1\, \eta^2 \cdot g_p(\tau,\tau') \int \frac{d\boldsymbol{k}}{(2\pi)^3} \left(|\zeta_k'(\tau_1)|^2 + k^2|\zeta_k(\tau_1)|^2\right)$$

ROADMAP

G. Pimentel, L. Senatore, M. Zaldarriaga'12

- One-loop diagrams as an intergal over three-point function (Quartic are crucial)

$$\mathcal{P}^{1-\text{loop}}(p) \propto \int_{\tau_0}^{\tau} d\tau_1 \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{p^3}{2\pi^2} \left\langle\!\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\Big|_{\boldsymbol{k},-\boldsymbol{k}} \hat{\zeta}_{\boldsymbol{p}} \right\rangle\!\!\right\rangle$$

- Maldacena consistency relations for arbitrary transient non-slow roll phase

$$\mathcal{P}^{1-\text{loop}}(p) \propto \mathcal{P}^{\text{tree}}(p) \int_{\tau_0}^{\tau} \int d\ln k \frac{d}{d\ln k} \left\langle\!\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\Big|_{\boldsymbol{k}} \right\rangle\!\!\right\rangle,$$

- Full one-loop computations

See also
Y. Tada, T. Terada and J. Tokuda '23
R. Kawaguchi, S. Tsujikawa and Y. Yamada '24

ONE-LOOP AS THREE-POINT FUNCTIONS

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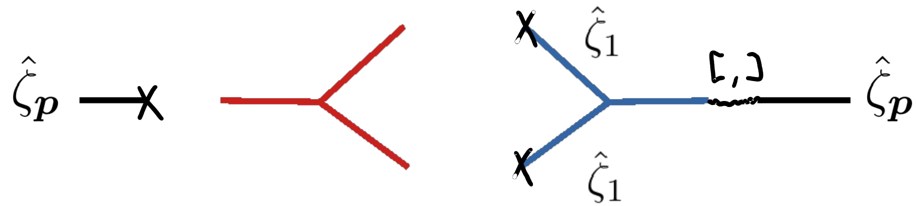

$$= \int d\tau_1 \int d\boldsymbol{K}_1 \cdot i[\hat{\zeta}_1^A, \hat{\zeta}_{\boldsymbol{p}'}] \cdot \lambda(\tau_1) \underbrace{\left(i \int^{\tau_1} d\tau_2 \langle [H^{(3)}(\tau_2), \hat{\zeta}_1^B \hat{\zeta}_1^C \hat{\zeta}_{\boldsymbol{p}}]\rangle \right)}_{\langle \zeta_1 \zeta_1 \zeta_{\boldsymbol{p}} \rangle},$$

<u>IF:</u> $\hat{\zeta}_1^B \hat{\zeta}_1^C = \hat{\zeta}_1 \hat{\zeta}_1$

ONE-LOOP AS THREE-POINT FUNCTIONS

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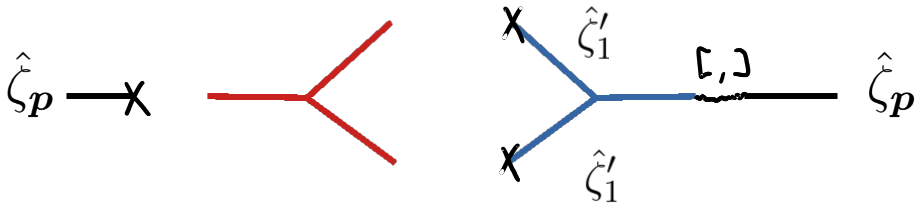

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<u>IF:</u> $\hat{\zeta}_1^B\hat{\zeta}_1^C = \hat{\zeta}'_1\hat{\zeta}'_1$

$$\neq \langle \hat{\zeta}'_1\hat{\zeta}'_1\hat{\zeta}_{\boldsymbol{p}}\rangle$$

E.g.

$$\langle \hat{\zeta}'_1\hat{\zeta}'_1\hat{\zeta}_{\boldsymbol{p}}\rangle = i\int^{\tau_1} d\tau_2 \langle [H^{(3)}(\tau_2), \hat{\zeta}'_1\hat{\zeta}'_1\hat{\zeta}_{\boldsymbol{p}}]\rangle$$

$$\underline{+\, i\langle [H^{(3)}(\tau_1), \hat{\zeta}_1]\hat{\zeta}'_1\hat{\zeta}_{\boldsymbol{p}}\rangle + i\langle \hat{\zeta}'_1[H^{(3)}(\tau_1), \hat{\zeta}_1]\hat{\zeta}_{\boldsymbol{p}}\rangle}$$

ONE-LOOP AS THREE-POINT FUNCTIONS

$$\langle \hat\zeta_{\boldsymbol p}(\tau)\hat\zeta_{\boldsymbol p'}(\tau)\rangle_{H^{(3)}} = -\int_{\tau_0}^{\tau} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \langle [H^{(3)}(\tau_2), [H^{(3)}(\tau_1), \hat\zeta_{\boldsymbol p}(\tau)\hat\zeta_{\boldsymbol p'}(\tau)]]\rangle$$

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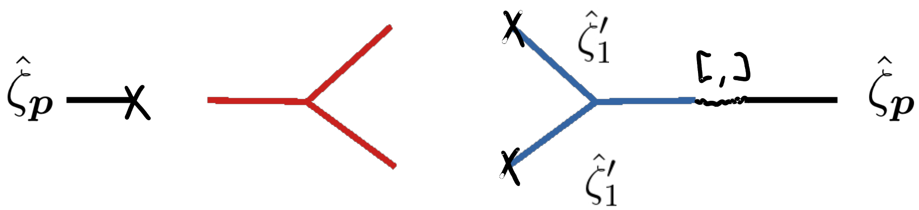

$$= \int d\tau_1 \int d\boldsymbol{K}_1 \cdot i[\hat\zeta_1^A, \hat\zeta_{\boldsymbol p'}] \cdot \lambda(\tau_1) \left(i \int^{\tau_1} d\tau_2 \langle [H^{(3)}(\tau_2), \hat\zeta_1'\hat\zeta_1'\hat\zeta_{\boldsymbol p}]\rangle \right),$$

<u>IF:</u> $\hat\zeta_1^B\hat\zeta_1^C = \hat\zeta_1'\hat\zeta_1'$

$$\neq \langle \hat\zeta_1'\hat\zeta_1'\hat\zeta_{\boldsymbol p}\rangle$$

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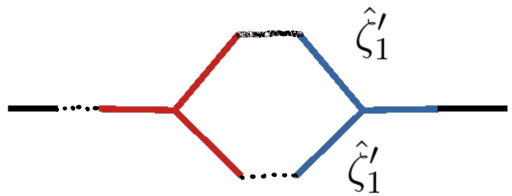
Miracle # 1 ➡

$$+ i\langle [H^{(3)}(\tau_1), \hat\zeta_1]\hat\zeta_1'\hat\zeta_{\boldsymbol p}\rangle + i\langle \hat\zeta_1'[H^{(3)}(\tau_1), \hat\zeta_1]\hat\zeta_{\boldsymbol p}\rangle$$

1-LOOP AS 3-POINT FUNCTIONS AND QUARTIC INTERACTIONS

- **MIRACLE #1:** Quartic induced Hamiltonian to build 3-point functions

$$\mathcal{H}_{\rm int} = -\mathcal{L}^{(3)} + \underline{\mathcal{H}_3^{(4)}} \qquad \underline{\mathcal{H}_3^{(4)}} = \frac{1}{2(2a^2\epsilon)M_{\rm Pl}^2}\left(\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'}\right)^2$$



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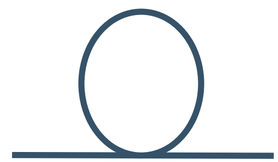

G.L. Pimentel, L. Senatore and M. Zaldarriaga, ‘12

$$\langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle = \langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle_{H^{(3)}} + \langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle_{H_3^{(4)}} \overset{\text{E.g.}}{=} -\int d\tau_1\,\langle\hat{\zeta}_1'\hat{\zeta}_1'\hat{\zeta}_{\boldsymbol{p}}\rangle$$

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G.L. Pimentel, L. Senatore and M. Zaldarriaga, 1203.6651

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- Caveat 1: Care to extract $\mathcal{H}_3^{(4)}$ from the cubic action written "a la Maldacena"

$$\mathcal{L}^{(3)} = \mathcal{L}^{(3)}_{\rm bulk} + \mathcal{L}^{(3)}_{\partial} + \mathcal{L}^{(3)}_{\rm eom}$$

J. Fumagalli, 2408.08296

1-LOOP AS 3-POINT FUNCTIONS AND QUARTIC INTERACTIONS

- MIRACLE #1: Quartic induced Hamiltonian to build 3-point functions

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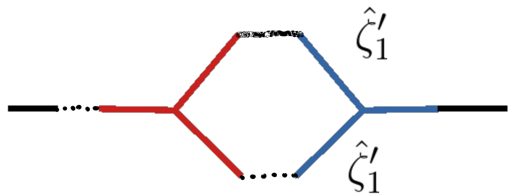

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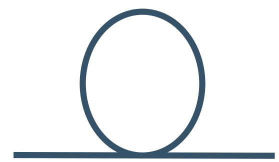


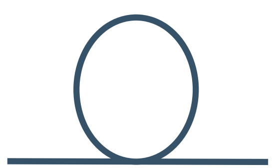
G.L. Pimentel, L. Senatore and M. Zaldarriaga, 1203.6651

$$\langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle = \langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle_{H^{(3)}} + \langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle_{H_3^{(4)}} = -\int d\tau_1 \quad \langle\hat{\zeta}_1'\hat{\zeta}_1\hat{\zeta}_{\boldsymbol{p}}\rangle$$

- Caveat 2: Spurious contribution from $\mathcal{H}_3^{(4)}$ when building

MIRACLE #2: They cancel exactly from the "tadpole induced Hamiltonian"

$$\mathcal{L}^{(1)}_{\rm tad} = c\,\zeta', \qquad c = -\left\langle\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'}\right\rangle, \qquad \mathcal{H}_{\rm int} = -\mathcal{L}^{(1)}_{\rm tad} + \mathcal{H}_1^{(2)}$$



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$= \;0$

J. Fumagalli, 2408.08296

ONE-LOOP AS THREE-POINT FUNCTIONS

$$\langle \hat{\zeta}_{\boldsymbol{p}} \hat{\zeta}_{\boldsymbol{p}'} \rangle = - \int d\tau_1 \int d\boldsymbol{K}_1 \cdot i[\hat{\zeta}_1^A, \hat{\zeta}_{\boldsymbol{p}'}] \cdot \left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A} \Big|_1 \hat{\zeta}_{\boldsymbol{p}} \right\rangle$$

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Consistency relations:

$$\mathcal{P}^{\text{tree}}(p) \frac{d}{d \ln k} \left\langle\!\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\bigg|_{\boldsymbol{k}} \right\rangle\!\!\right\rangle,$$

ONE-LOOP AS THREE-POINT FUNCTIONS

$$\langle \hat{\zeta}_{\boldsymbol{p}} \hat{\zeta}_{\boldsymbol{p}'} \rangle = -\int d\tau_1 \int d\boldsymbol{K}_1 \cdot i[\hat{\zeta}_1^A, \hat{\zeta}_{\boldsymbol{p}'}] \cdot \left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\Big|_1 \hat{\zeta}_{\boldsymbol{p}} \right\rangle$$

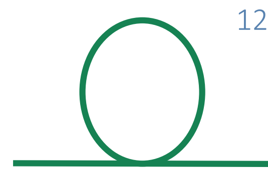
Unless $(\partial_i \zeta)^2$

MIRACLE #3: Include quartic interactions implied by residual diff. invariance

$$g_{ij} = a^2 e^{2\zeta} \qquad \zeta \to \zeta + b, \quad x^i \to x^i e^{-b} + C^i$$

Invariant building block: $e^{-\zeta}\partial_i\zeta$

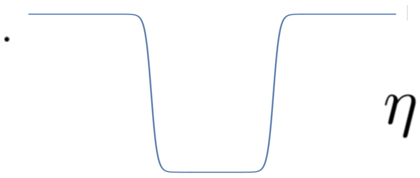
E.g. $\mathcal{L}^{(3)} \supset -c_1 \zeta'(\partial_i\zeta)^2 \Longrightarrow \mathcal{L}^{(4)}_{\text{diff}} \supset 2c_1\zeta'\zeta(\partial_i\zeta)^2$



Y. Urakawa and T. Tanaka 0902.3209, 1007.0468
G.L. Pimentel, L. Senatore and M. Zaldarriaga, 1203.6651

CONSISTENCY RELATIONS IN TRANSIENT NON-SLOW-ROLL

$$\mathcal{L}^{(3)} = \frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta' + \frac{\epsilon a^2}{aH}\zeta'^2\zeta\right] + \mathcal{E}_\zeta\left(\frac{\eta}{2}\zeta^2 + \frac{2}{aH}\zeta'\zeta\right).$$



$p \ll k \simeq k'$,

$$\langle\langle \hat{\zeta}_{\boldsymbol{k}}(\tau)\hat{\zeta}_{\boldsymbol{k}'}(\tau)\hat{\zeta}_{\boldsymbol{p}}(\tau)\rangle\rangle = -\frac{d\ln\mathcal{P}_\zeta(k,\tau)}{d\ln k}\frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k,\tau)\frac{2\pi^2}{p^3}\mathcal{P}_\zeta(p,\tau)$$

$$\langle\langle \hat{\zeta}'_{\boldsymbol{k}}(\tau)\hat{\zeta}'_{\boldsymbol{k}'}(\tau)\hat{\zeta}_{\boldsymbol{p}}(\tau)\rangle\rangle = -\frac{d\ln\mathcal{P}_{\zeta'}(k,\tau)}{d\ln k}\frac{2\pi^2}{k^3}\mathcal{P}_{\zeta'}(k,\tau)\frac{2\pi^2}{p^3}\mathcal{P}_\zeta(p,\tau)$$

. ...

J. Fumagalli, 2408.08296

Hint / Hankel functions :

$$\zeta_k(\tau) = C_1\bar{x}^{1+\frac{3}{2}-\nu}x^\nu\frac{i\pi}{4}\left(\mathcal{A}_1(\bar{x})\cdot H^1_\nu(x) - \mathcal{A}_2(\bar{x})\cdot H^2_\nu(x)\right),$$

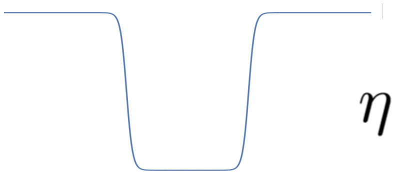
$$\frac{d}{dx}\{H^i_\alpha(x), H^j_\beta(x)\} = -\frac{\{H^i_\alpha(x), H^j_\beta(x)\}}{x} + \frac{\beta-\alpha}{x}(H^i_\alpha(x), H^j_\beta(x)),$$

Where $(H^i_\alpha, H^j_\beta) \equiv H^i_\alpha \cdot H^j_{\beta-1} + H^i_{\alpha-1}\cdot H^j_\beta.$

$$\{H^i_\alpha, H^j_\beta\} \equiv H^i_\alpha \cdot H^j_{\beta-1} - H^i_{\alpha-1}\cdot H^j_\beta,$$

CONSISTENCY RELATIONS IN TRANSIENT NON-SLOW-ROLL

$$\mathcal{L}^{(3)} = \frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta' + \frac{\epsilon a^2}{aH}\zeta'^2\zeta\right] + \mathcal{E}_\zeta\left(\frac{\eta}{2}\zeta^2 + \frac{2}{aH}\zeta'\zeta\right).$$



$$\mathcal{A}_0 = |\zeta_p|^{-2}\langle\langle\hat{\zeta}_{\boldsymbol{k}}(\tau_1)\hat{\zeta}_{\boldsymbol{k}'}(\tau_1)\hat{\zeta}_{\boldsymbol{p}}(\tau_1)\rangle\rangle = -\frac{2\pi^2}{k^3}\cdot\frac{d\mathcal{P}_\zeta(k,\tau_1)}{d\ln k}$$

$$= \eta|\zeta_k(\tau_1)|^2 + \frac{2\mathrm{Re}(\zeta_k(\tau_1)\zeta_k'^*(\tau_1))}{aH} + \mathcal{F}(\tau_s,\tau_1) - \mathcal{F}(\tau_e,\tau_1)\theta(\tau_1-\tau_e)$$

$$\mathcal{A} = |\zeta_p|^{-2}\partial_{\tau_1}\langle\langle\hat{\zeta}_{\boldsymbol{k}}(\tau_1)\hat{\zeta}_{\boldsymbol{k}'}(\tau_1)\hat{\zeta}_{\boldsymbol{p}}(\tau_1)\rangle\rangle$$

$$= \frac{2}{aH}|\zeta_k'(\tau_1)|^2 - 6\,\mathrm{Re}(\zeta_k(\tau_1)\zeta_k'^*(\tau_1)) - \frac{2k^2}{aH}|\zeta_k(\tau_1)|^2 + \partial_{\tau_1}\mathcal{F}(\tau_1,\tau_s) - \partial_{\tau_1}\mathcal{F}(\tau_1,\tau_e)\theta(\tau_1-\tau_e)$$

$$\mathcal{B} = |\zeta_p|^{-2}\langle\langle\hat{\zeta}'_{\boldsymbol{k}}(\tau_1)\hat{\zeta}'_{\boldsymbol{k}'}(\tau_1)\hat{\zeta}_{\boldsymbol{p}}(\tau_1)\rangle\rangle = -\frac{2\pi^2}{k^3}\frac{d\mathcal{P}_{\zeta'}(k,\tau_1)}{d\ln k}$$

$$= -(\eta+6)|\zeta_k'(\tau_1)|^2 - \frac{2k^2}{aH}\mathrm{Re}(\zeta_k(\tau_1)\zeta_k'^*(\tau_1)) + \widetilde{\mathcal{F}}(\tau_1,\tau_s) - \widetilde{\mathcal{F}}(\tau_1,\tau_e)\theta(\tau_1-\tau_e)$$

$$\mathcal{C} = -\frac{2\pi^2}{k^3}\frac{d\left(k^2\mathcal{P}_\zeta(k,\tau_1)\right)}{d\ln k} = k^2\mathcal{A}_0 - 2k^2|\zeta_k(\tau_1)|^2$$

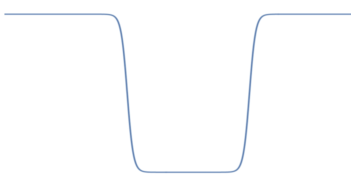
$$\mathcal{D} = -\frac{2\pi^2}{k^3}\cdot\partial_{\tau_1}\left(\frac{k^2 d\mathcal{P}_\zeta(k,\tau_1)}{d\ln k}\right) = k^2\mathcal{A} - 4k^2\mathrm{Re}(\zeta_k(\tau_1)\zeta_k'^*(\tau_1)),$$

J. Fumagalli, 2408.08296

SUMMARY

$$\mathcal{L}^{(3)} = \mathcal{L}^{(3)}_{\rm bulk} + \mathcal{L}^{(3)}_{\partial} + \mathcal{L}^{(3)}_{\rm eom}$$

J. Fumagalli, 2408.08296

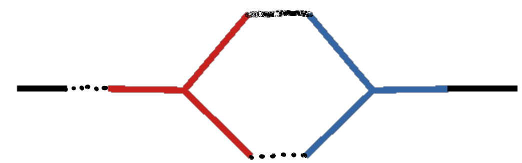


η

$$\mathcal{L}^{(3)} = \frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta' + \frac{\epsilon a^2}{aH}\zeta'^2\zeta\right] + \mathcal{E}_\zeta\left(\frac{\eta}{2}\zeta^2 + \frac{2}{aH}\zeta'\zeta\right).$$

- Cubic Hamiltonian

$$\mathcal{H}^{(3)}_a = -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta', \quad \mathcal{H}^{(3)}_b = \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right], \quad \mathcal{H}^{(3)}_e = \frac{d}{d\tau}\left[\frac{a\epsilon}{H}\zeta\zeta'^2\right]$$



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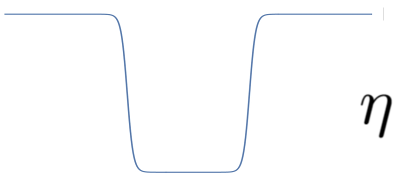
J. Fumagalli, 2408.08296



η

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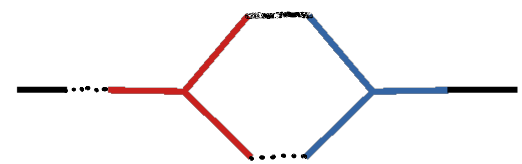
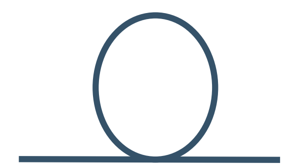


- Quartic induced Hamiltonian

$$\mathcal{H}^{(4)}_A = 9\epsilon a^2\zeta'^2\zeta^2, \qquad \mathcal{H}^{(4)}_B = \frac{\epsilon a^2}{(aH)^2}\zeta^2(\partial^2\zeta)^2, \qquad \mathcal{H}^{(4)}_C = -\frac{9\epsilon a^2}{aH}\zeta'^3\zeta,$$

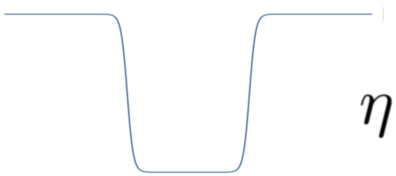
$$\mathcal{H}^{(4)}_D = -\frac{6\epsilon a^2}{aH}\zeta^2\zeta'\partial^2\zeta, \qquad \mathcal{H}^{(4)}_E = \frac{3\epsilon a^2}{(aH)^2}\zeta\zeta'^2\partial^2\zeta, \qquad \mathcal{H}^{(4)}_F = \frac{9\epsilon a^2}{4(aH)^2}\zeta'^4$$



SUMMARY

$$\mathcal{L}^{(3)} = \mathcal{L}^{(3)}_{\text{bulk}} + \mathcal{L}^{(3)}_{\partial} + \mathcal{L}^{(3)}_{\text{eom}}$$

J. Fumagalli, 2408.08296



η

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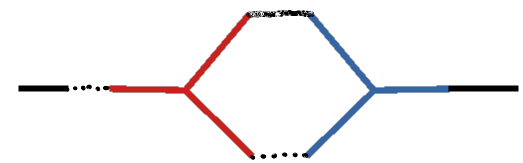
- Cubic Hamiltonian

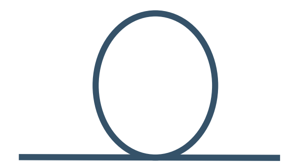
$$\mathcal{H}^{(3)}_a = -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta', \quad \mathcal{H}^{(3)}_b = \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right], \quad \mathcal{H}^{(3)}_e = \frac{d}{d\tau}\left[\frac{a\epsilon}{H}\zeta\zeta'^2\right]$$



- Quartic induced Hamiltonian

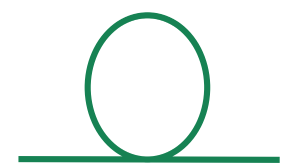
$$\mathcal{H}^{(4)}_A = 9\epsilon a^2\zeta'^2\zeta^2, \qquad \mathcal{H}^{(4)}_B = \frac{\epsilon a^2}{(aH)^2}\zeta^2(\partial^2\zeta)^2, \qquad \mathcal{H}^{(4)}_C = -\frac{9\epsilon a^2}{aH}\zeta'^3\zeta,$$

$$\mathcal{H}^{(4)}_D = -\frac{6\epsilon a^2}{aH}\zeta^2\zeta'\partial^2\zeta, \qquad \mathcal{H}^{(4)}_E = \frac{3\epsilon a^2}{(aH)^2}\zeta\zeta'^2\partial^2\zeta, \qquad \mathcal{H}^{(4)}_F = \frac{9\epsilon a^2}{4(aH)^2}\zeta'^4$$



- Diff. deduced Hamiltonian

$$\mathcal{H}^{(4)}_{\text{diff},\,A} = a^2\epsilon\eta\,\zeta^2(\partial\zeta)^2, \qquad \mathcal{H}^{(4)}_{\text{diff},\,B} = \frac{4a^2\epsilon}{aH}\zeta\zeta'(\partial\zeta)^2, \qquad \mathcal{H}^{(4)}_{\text{diff},\,C} = \frac{2a^2\epsilon}{aH}\zeta^2\partial\zeta\partial\zeta'.$$



- Tadpole induced Hamiltonian

$$\mathcal{H}^{(2)}_1 = -\frac{1}{2a^2\epsilon}\left\langle\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'}\right\rangle\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'} = -18\epsilon a^2\langle\zeta'\zeta\rangle\zeta'\zeta + \frac{9\epsilon a^2}{aH}\langle\zeta'^2\rangle\zeta'\zeta + \frac{6\epsilon a^2}{aH}\langle\zeta\,\partial^2\zeta\rangle\zeta'\zeta.$$

RESULT

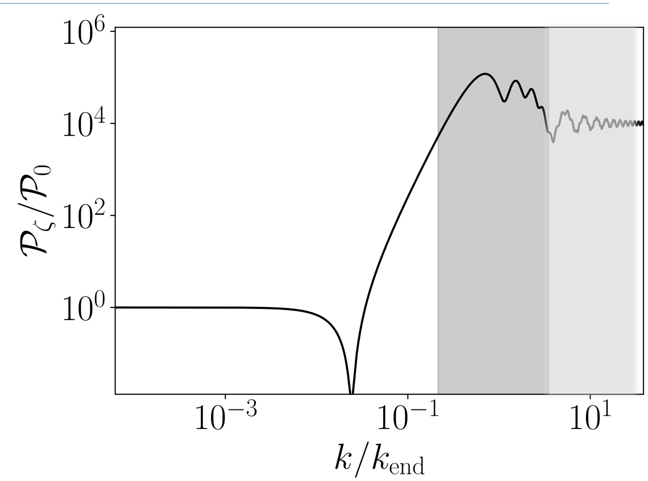
J. Fumagalli, 2408.08296

Full one-loop result from small scale to large scale in non-slow-roll dynamics

$$\mathcal{P}_\zeta^{1-\mathrm{loop}}(p,\tau) = \mathcal{P}_\zeta^{\mathrm{tree}}(p,\tau)\int_{\tau_0}^{\tau} d\tau_1 \int dk\, C(k,\tau_1),$$

$$C(k,\tau_1) = \frac{d}{dk}\left(3\partial_{\tau_1}\mathcal{P}_\zeta(k,\tau_1) - \frac{3}{aH}\mathcal{P}_{\zeta'}(k,\tau_1) + \frac{2}{aH}k^2\mathcal{P}_\zeta(k,\tau_1)\right) + \frac{d}{dk}\left(g_p(\tau,\tau_1)\left(-3\mathcal{P}_{\zeta'}(k,\tau_1) - \eta k^2\mathcal{P}_\zeta(k,\tau_1) - \frac{1}{aH}\partial_{\tau_1}(k^2\mathcal{P}_\zeta(k,\tau_1))\right)\right).$$

NO DEPENDENCE ON THE ENHANCED SHORT MODES!



RESULT

J. Fumagalli, 2408.08296

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$$C(k,\tau_1) = \frac{d}{dk}\left(3\partial_{\tau_1}\mathcal{P}_\zeta(k,\tau_1) - \frac{3}{aH}\mathcal{P}_{\zeta'}(k,\tau_1) + \frac{2}{aH}k^2\mathcal{P}_\zeta(k,\tau_1)\right) + \frac{d}{dk}\left(g_p(\tau,\tau_1)\left(-3\mathcal{P}_{\zeta'}(k,\tau_1) - \eta k^2\mathcal{P}_\zeta(k,\tau_1) - \frac{1}{aH}\partial_{\tau_1}(k^2\mathcal{P}_\zeta(k,\tau_1))\right)\right).$$

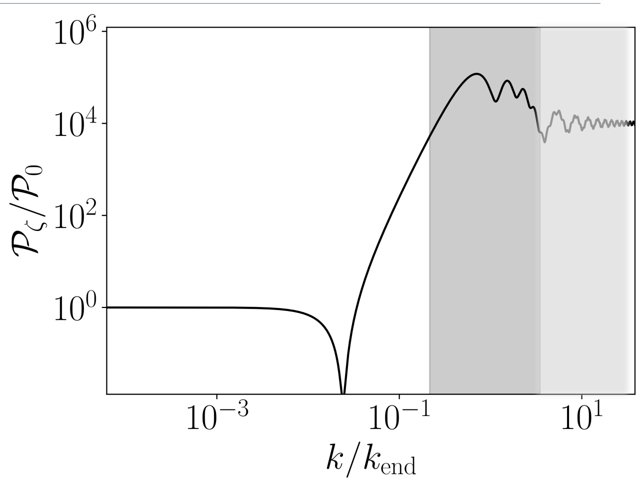
NO DEPENDENCE ON THE ENHANCED SHORT MODES!

Further

IR $\propto (\ldots)\,|_{k_{\rm IR}} \propto k_{\rm IR}\tau_{\rm int}k^\delta \ll 1$

UV – dim reg. $\propto \int d^{3+\delta}k \frac{1}{k^{3+\delta}}\frac{d}{d\ln k}\left(k^{3+\delta}\left\langle\!\left\langle \frac{\delta\mathcal{L}^{(3)}}{\delta\zeta^A}\right\rangle\!\right\rangle\right) \longrightarrow 0$

G.L. Pimentel, L. Senatore and M. Zaldarriaga, 1203.6651
R. Kawaguchi, S. Tsujikawa and Y. Yamada 2407.19742



(NON-Exhaustive) COMPARISON WITH LITERATURE

$$H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta'$$

J. Kristiano and J. Yokoama ‘22,

A. Riotto ‘23, G. Franciolini et al. ‘23……… : sizeable or not

(NON-Exhaustive) COMPARISON WITH LITERATURE

$$H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right],$$

J. Kristiano and J. Yokoama '22,

A. Riotto '23, G. Franciolini et al. '23........ : sizeable or not

JF 2305.19263 : Boundary terms

(NON-Exhaustive) COMPARISON WITH LITERATURE

$$H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right],$$

$$H^{(3)} \supset \quad a^2\epsilon\eta\zeta'^2\zeta \quad + \quad \frac{1}{2}a^2\epsilon\eta\zeta^2\partial^2\zeta,$$

J. Kristiano and J. Yokoama ‘22,

A. Riotto ‘23, G. Franciolini et al. ‘23……… : sizeable or not

H. Firouzjahi ‘23: EFT of inflation and quartic interactions

JF 2305.19263 : Boundary terms

(NON-Exhaustive) COMPARISON WITH LITERATURE

$$H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right] + \frac{d}{d\tau}\left[\frac{a\epsilon}{H}\zeta\zeta'^2\right]$$

$$H^{(3)} \supset a^2\epsilon\eta\zeta'^2\zeta + \frac{1}{2}a^2\epsilon\eta\zeta^2\partial^2\zeta,$$

J. Kristiano and J. Yokoama ‘22,

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Y. Tada, T. Terada and J. Tokuda 2308.04732: boundary terms and consistency relations but no quartic

(NON-Exhaustive) COMPARISON WITH LITERATURE

$$H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right] + \frac{d}{d\tau}\left[\frac{a\epsilon}{H}\zeta\zeta'^2\right] + \dots$$

$$H^{(3)} \supset a^2\epsilon\eta\zeta'^2\zeta + \frac{1}{2}a^2\epsilon\eta\zeta^2\partial^2\zeta,$$

J. Kristiano and J. Yokoama '22,

A. Riotto '23, G. Franciolini et al. '23……… : sizeable or not

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H. Firouzjahi 2311.04080: ..

R. Kawaguchi, S. Tsujikawa and Y. Yamada 2407.19742: Path integral formalism and absence of corrections

JF 2408.08296: THIS TALK

(NON-Exhaustive) COMPARISON WITH LITERATURE

$$H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right] + \frac{d}{d\tau}\left[\frac{a\epsilon}{H}\zeta\zeta'^2\right]$$

$$H^{(3)} \supset a^2\epsilon\eta\zeta'^2\zeta + \frac{1}{2}a^2\epsilon\eta\zeta^2\partial^2\zeta,$$

J. Kristiano and J. Yokoama '22,

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JF 2408.08296: THIS TALK

Different approaches

Delta-Phi gauge: K. Inomata, '24, G. Ballesteros and J.G. Egea '24,
Separate Universe approch: L. Iacconi, D. Mulryne and D. Seery '23
Large-eta approach: G. Tasinato '23

CONCLUSIONS

In non-slow-roll / any dynamics, inflation is safe:

$$\mathcal{P}_\zeta^{1-\text{loop}}(p) = \mathcal{P}^{\text{tree}}(p) \int d\ln k\, C(k) + O\left(\frac{p^3}{k^3}\right), \qquad p \ll k$$

Future directions:

- Agree / Pandora box / Lesson from other gauges

- One-loop corrections at all scales
  → Observable effects?
  → Multi-field / Higher loops ?

CONSISTENCY RELATIONS AND SPATIAL DERIVATIVES

$$\mathcal{L}^{(3)} \supset -c_1\zeta(\partial_i\zeta)^2 \Longrightarrow \mathcal{L}^{(4)}_{\text{diff}} \supset c_1\zeta^2(\partial_i\zeta)^2$$

$$\begin{aligned}\langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle_{H^{(3)}} &= -\int^{\tau} d\tau_1 \int^{\tau_1} d\tau_2 \int d\boldsymbol{K}_1 \langle[H^{(3)}(\tau_2), [c_1\hat{\zeta}_{\boldsymbol{k}_{1,1}}(\partial_i\hat{\zeta})_{\boldsymbol{k}_{1,2}}(\partial_i\hat{\zeta})_{\boldsymbol{k}_{1,3}}, \hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}]]\rangle \\ &= 2i\int^{\tau} d\tau_1 \int d\boldsymbol{K}_1 c_1[\hat{\zeta}_{\boldsymbol{k}_{1,1}}, \hat{\zeta}_{\boldsymbol{p}'}] \cdot \left(\underline{\langle(\partial_i\hat{\zeta})_{\boldsymbol{k}_{1,2}}(\partial_i\hat{\zeta})_{\boldsymbol{k}_{1,3}}\hat{\zeta}_{\boldsymbol{p}}\rangle}\right),\end{aligned}$$

(

$$\begin{aligned}\langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle_{H^{(4)}_{\text{diff}}} &= i\int^{\tau} d\tau_1 \langle[H^{(4)}_{\text{diff}}(\tau_1), \hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}]\rangle \\ &= 2i\int^{\tau} d\tau_1 \int d\boldsymbol{K}_1 c_1[\hat{\zeta}_{\boldsymbol{k}_{1,1}}, \hat{\zeta}_{\boldsymbol{p}'}] \cdot \left(-2\underline{\langle\hat{\zeta}_{\boldsymbol{k}_{1,1}}(\partial_i\hat{\zeta})_{\boldsymbol{k}_{1,2}}(\partial_i\hat{\zeta})_{\boldsymbol{k}_{1,3}}\hat{\zeta}_{\boldsymbol{p}}\rangle}\right),\end{aligned}$$

$$\langle\!\langle(\partial\zeta)_{\boldsymbol{k}}(\partial\zeta)_{\boldsymbol{k}'}\zeta_{\boldsymbol{p}}\rangle\!\rangle - 2\langle\!\langle(\partial_i\hat{\zeta})_{\boldsymbol{k}'}(\partial_i\hat{\zeta})_{\boldsymbol{k}}\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle\!\rangle = \left(-|\zeta_p(\tau)|^2\frac{2\pi^2}{k^3}\right) \cdot \frac{d\left(k^2\mathcal{P}_\zeta(k,\tau)\right)}{d\ln k}.$$

EQUIVALENT CUBIC ACTIONS

J. Fumagalli, 2408.08296

$$\mathcal{L}^{(3)} = -\eta\epsilon a^2\zeta\zeta'^2 + \eta\epsilon a^2\zeta(\partial\zeta)^2 + \frac{d}{d\tau}\left[-\frac{\epsilon a^2}{aH}\zeta'^2\zeta\right] + \mathcal{E}_\zeta\frac{2}{aH}\zeta'\zeta.$$

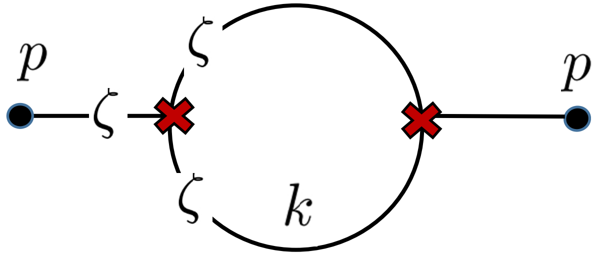
$$\mathcal{L}^{(3)} = \frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta' + \frac{\epsilon a^2}{aH}\zeta'^2\zeta\right] + \mathcal{E}_\zeta\left(\frac{\eta}{2}\zeta^2 + \frac{2}{aH}\zeta'\zeta\right).$$

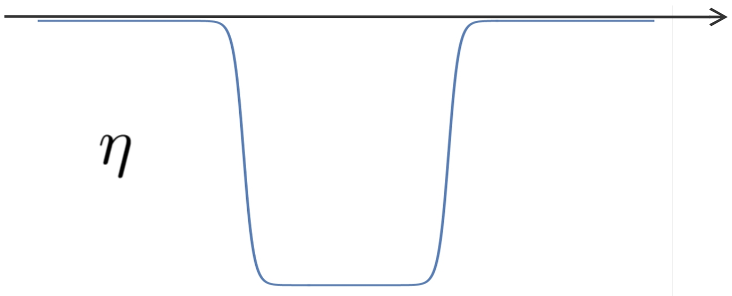
$$\mathcal{L}^{(3)} = \eta\epsilon a^2\zeta(\partial\zeta)^2 - \left(\frac{\epsilon a^2}{aH}\zeta'^3 - 3\epsilon a^2\zeta'^2\zeta + \frac{2a^2\epsilon}{aH}\zeta'\zeta\partial^2\zeta\right).$$

where we used

$$\frac{d}{d\tau}\left[-\frac{\epsilon a^2}{aH}\zeta'^2\zeta\right] = -\frac{2}{aH}\zeta\zeta'(\epsilon a^2\zeta')' - \frac{\epsilon a^2}{aH}\zeta'^3 + (\eta + 3 - \epsilon)\epsilon a^2\zeta'^2\zeta,$$

CUBIC ACTION & BOUNDARY TERMS





Maldacena´02 (selected terms from jump in eta) + boundary terms

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2 \frac{a^3\epsilon}{2}\dot{\eta}\zeta^2\dot{\zeta} + \underline{M_{\rm Pl}^2 \frac{d}{dt}\left[-\frac{a^3\epsilon\eta}{2}\zeta^2\dot{\zeta} + \ldots\right]} + f(\zeta)\frac{\delta\mathcal{L}^{(2)}}{\delta\zeta} + \ldots$$

- Field redefinition $\zeta \to \zeta_n + f(\zeta_n)$, nd then link correlators of the two variables

Maldacena '02,...

- Or work in terms of the original variable but crucially including boundary terms

F. Arroja and T. Tanaka '11,
C. Burrage, R.H. Ribeiro and D. Seery '11,...
S. Garcia-Saenz, L. Pinol and S. Renaux-Petel '20

even on scales much longer than the horizon. This would be very surprising and would create serious problems for the theory of inflation. In fact the predictivity of inflation relies on the fact that the perturbation ζ is expected to be constant on scales much longer than the horizon independently of what happens on scales of order of the horizon. This is very important because for some epochs such as reheating or a GUT phase transition (if this exist), we have almost no idea of what happens on scales of the order or shorter than the horizon. In principle large fluctuations on horizon scales can exist during these epochs. They are correlated only over horizon scale distances and so one expects they cannot affect a ζ mode of much longer wavelength. Instead a time dependence from the one loop corrections would imply the contrary. Notice that the situation can become really worrisome. The effect of the σ fields, even

"On loops in inflation"
L. Senatore, M. Zaldarriaga '09