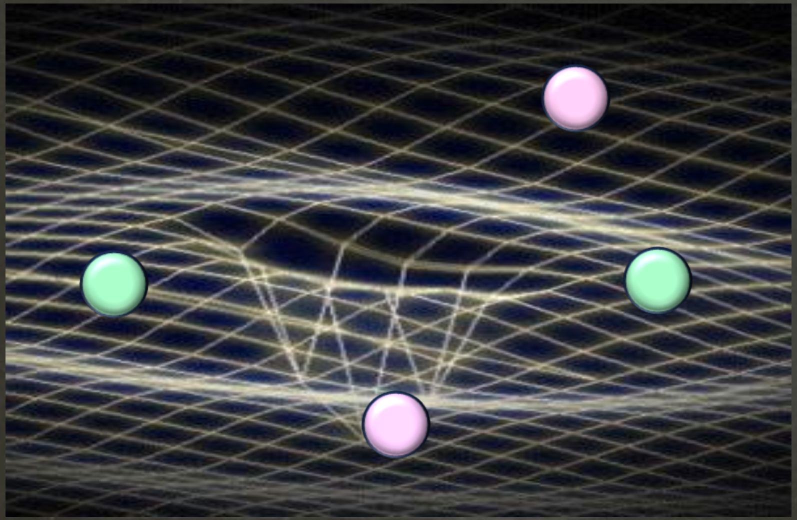
THE ROYAL SOCIETY

Weyl Fermion Creation by Cosmological Gravitational Wave Background at 1-loop!



Azadeh Malek-Nejad

King's College London

w/ Joachim Kopp

arXiv:2405.09723
arXiv:2406.01534

Looping in the Primordial Universe @ CERN   Oct-Nov 2024

Setup

1) Quantum Fluctuations in Cosmology

2) Gravitational Particle Production

3) GW-infuced Fermion Production

4) Outlook

w/ Joachim Kopp

arXiv:2405.09723
arXiv:2406.01534

Quantum Fluctuations in Cosmology

$$\hbar \neq 0$$

Quantum Vacuum $\hbar \neq 0$

Due to Uncertainty Principle

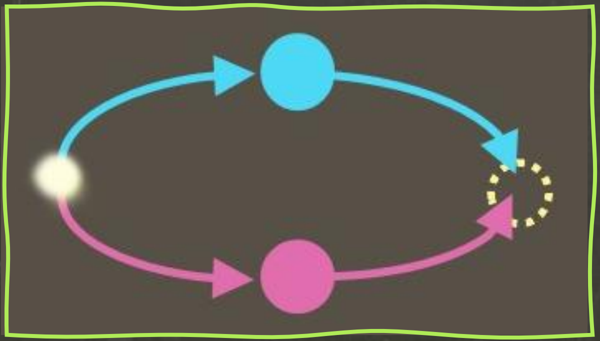
$$\Delta x \, \Delta p \geq \hbar/2$$

quantum vacuum is NOT nothing!

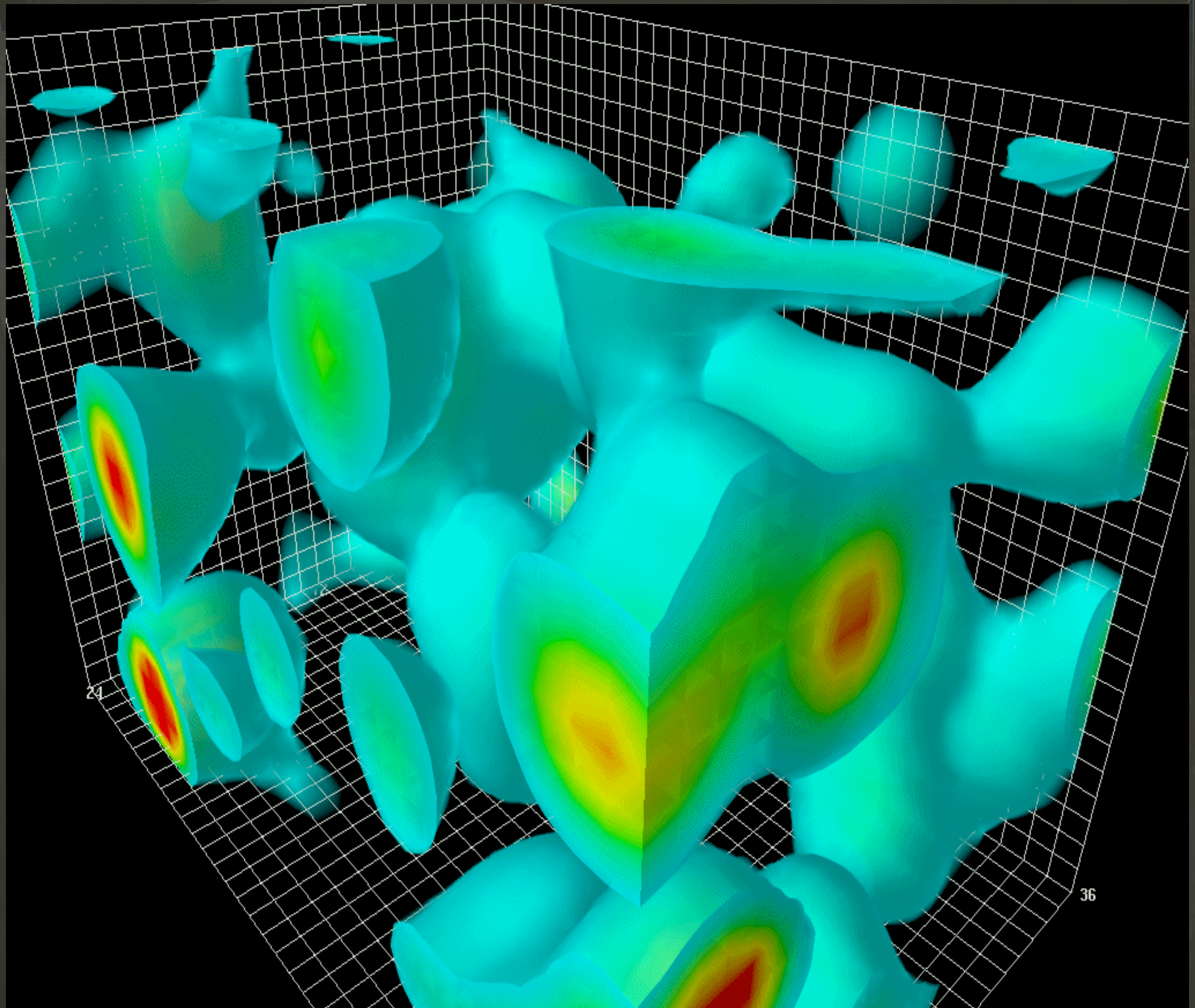
But, a vast ocean made of

Virtual particles

vacuum



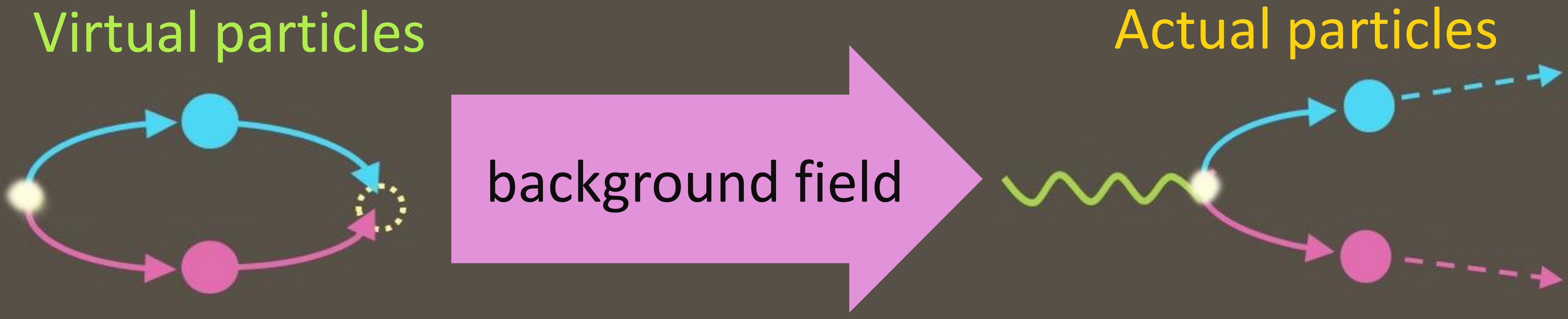
vacuum



Simulation credit: Derek Leinweber

Quantum Vacuum

Particle Production



Background field can upgrade them into actual particles!

$\langle J \rangle = 0$

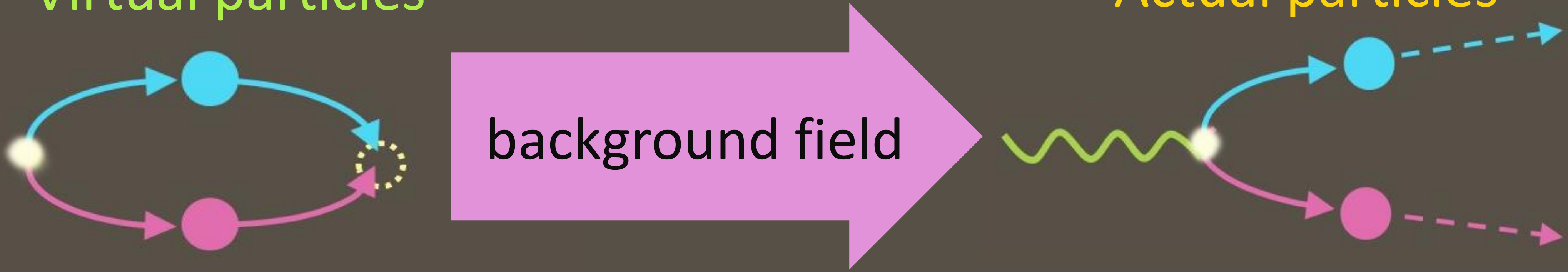
$\langle J \rangle \neq 0$

Quantum Vacuum

Particle Production

Virtual particles

Actual particles



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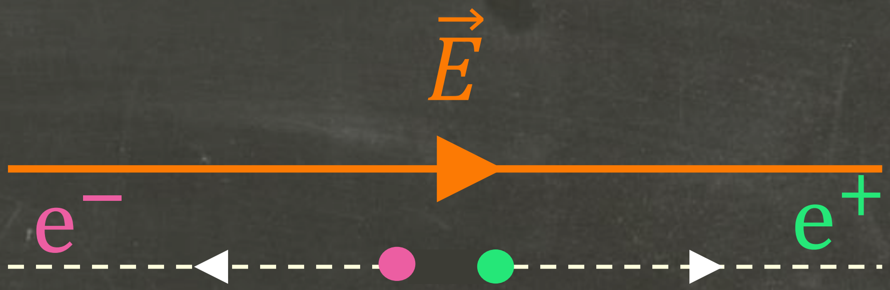
Examples of such BG fields:

1) Electric Field *Schwinger effect*



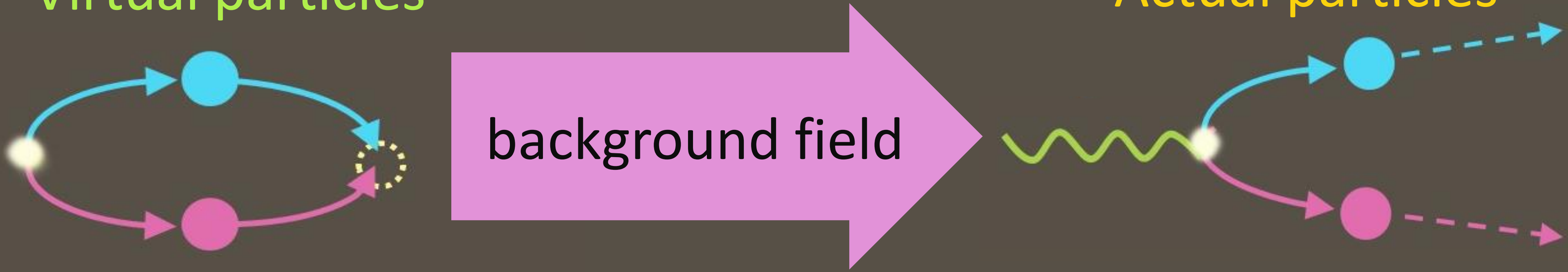
Work of the Lorentz force over Comptom wavelength $eE\,\lambda_{comp} = mc^2$ Rest energy of charged particle

J. Schwinger (1951)

Quantum Vacuum → Particle Production

Virtual particles

Actual particles



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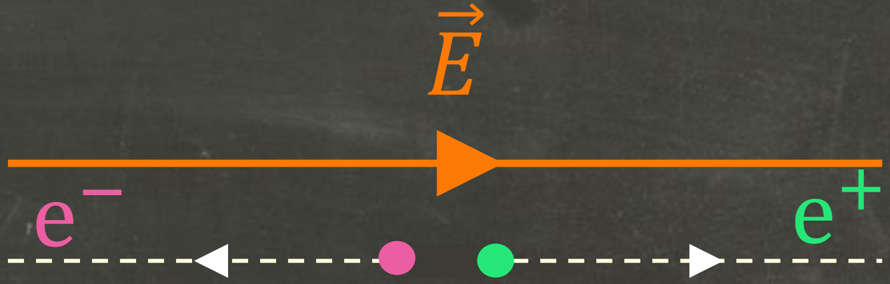

Work of the Lorentz force over Comptom wavelength $eE\,\lambda_{comp} = mc^2$ Rest energy of charged particle

The Electric field that can create electron pairs $E = \frac{m_e^2\, c^3}{e\hbar} = 10^{18}\ V/m$

J. Schwinger (1951)

What about
Schwinger Effect in Early Universe?

Schwinger effect in scalar QED in 4d de Sitter

- T. Kobayashi, N. Afshordi 2014

How about Axion-inflation?!

i) a natural candidate for the inflaton field

ii) Naturally coupled to gauge fields

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Schwinger effect in axion-inflation

K. Lozanov

E. Komatsu

- K. Lozanov, **A. M.**, E. Komatsu **2018**
- **A. M.**, E. Komatsu **2019**
- V. Domcke, Y. Ema, K. Mukaida, R. Sato 2019
- L. Mirzagholi, **A. M.**, K. Lozanov **2019**
- ….
- *Many many more*

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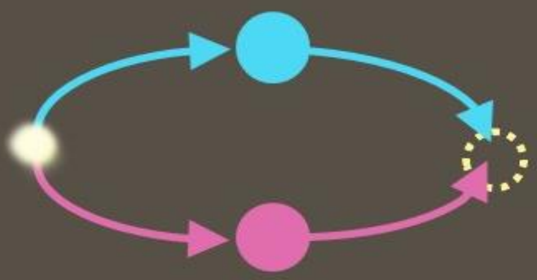
- E. Komatsu **2022** nature reviews physics

New physics from the polarized light of the cosmic microwave background

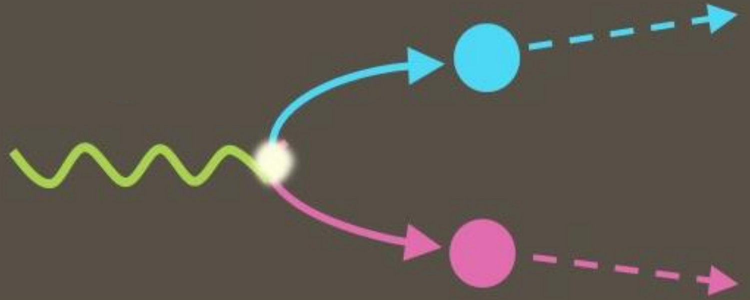
Quantum Vacuum → Particle Production

Virtual particles



background field

Actual particles



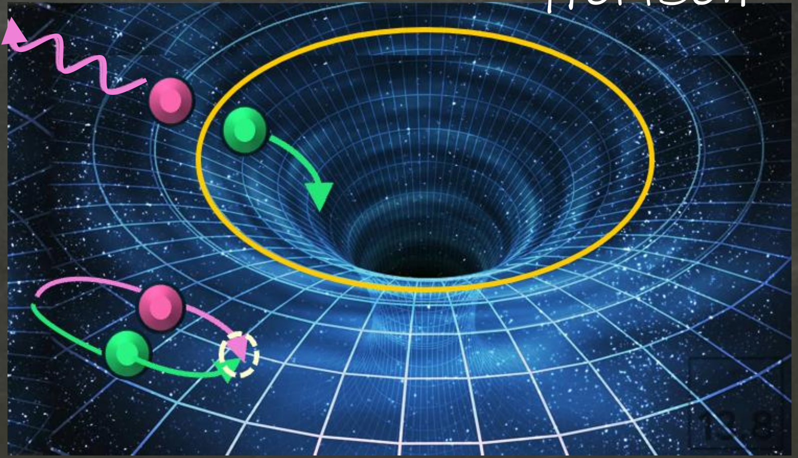
Background field can upgrade them into actual particles!

Examples of such BG fields:

2) Gravitational

Hawking radiation

Horizon



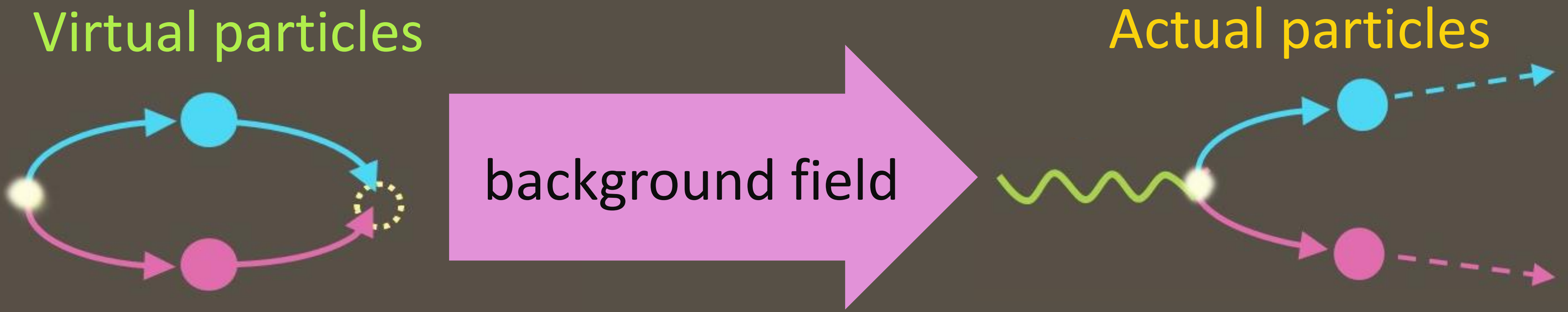
one particle fall into the BH, while the other escapes…

S. Hawking (1974)

Power BH emitted is

$$P = \frac{\pi c^3 M_{pl}^4}{240} \frac{1}{M^2}$$

Quantum Vacuum — Particle Production



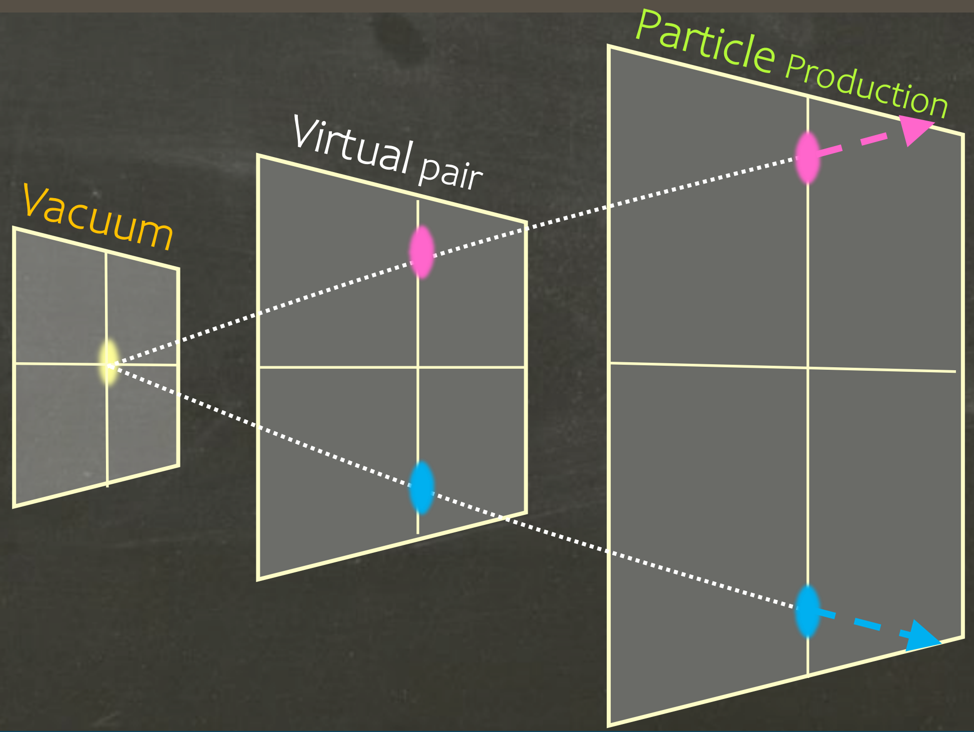
Background field can upgrade them into actual particles!

Examples of such BG fields:

1) Electric Field *Schwinger effect*

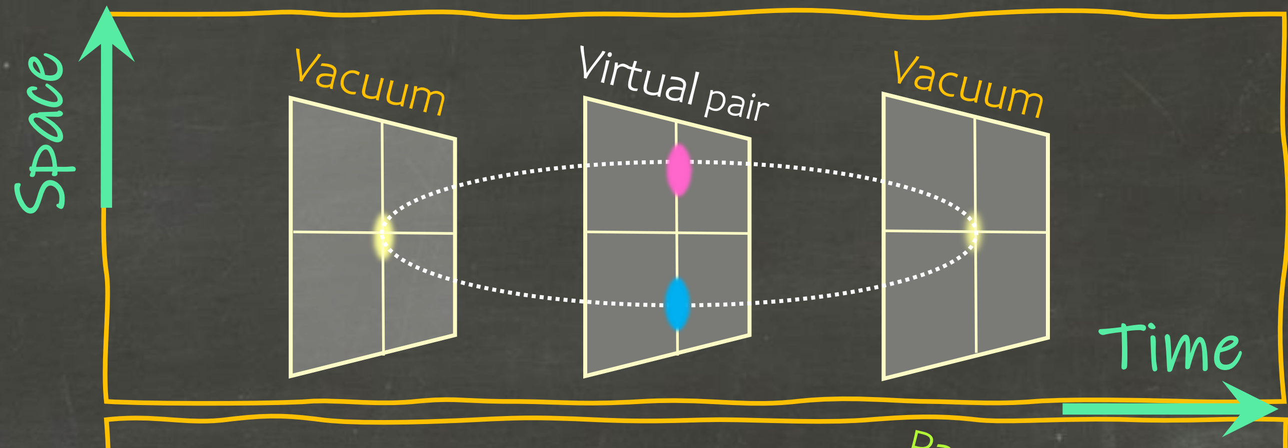
2) Gravitational *Hawking radiation*

or expansion of the Universe!



Expanding Universe Produces Particles!

Flat Space:

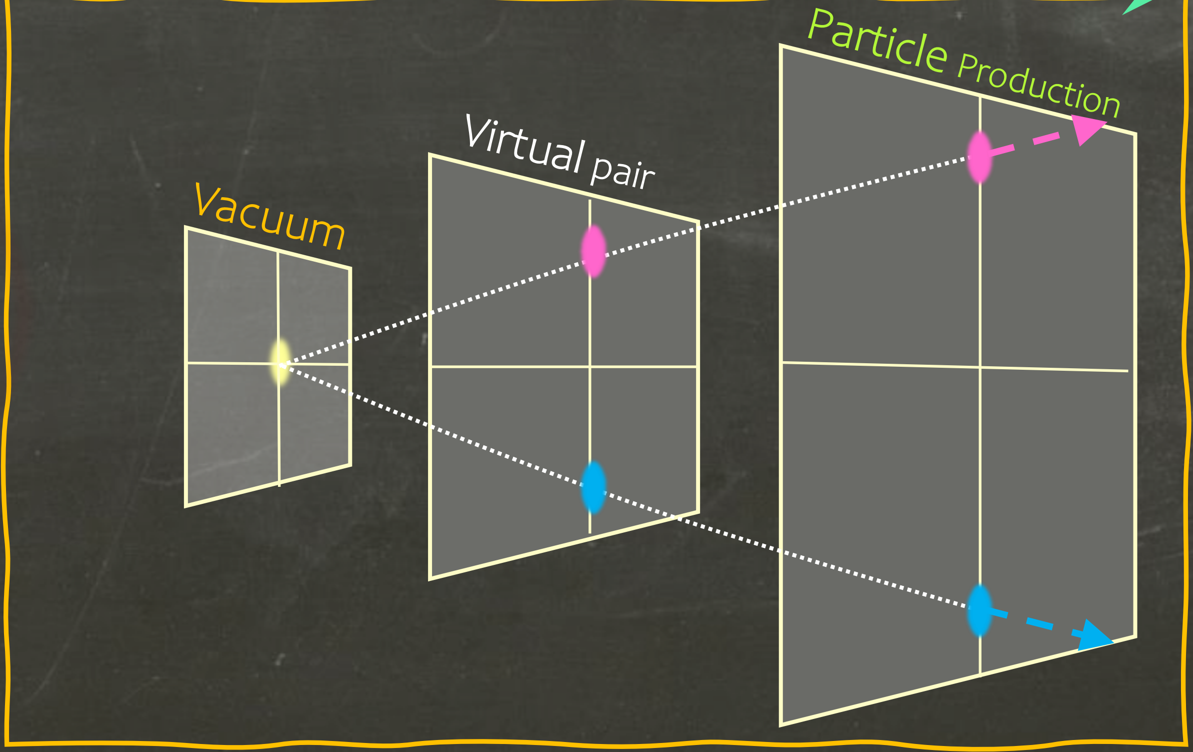


Vacuum

Expanding space:

E. Schrödinger (1939)

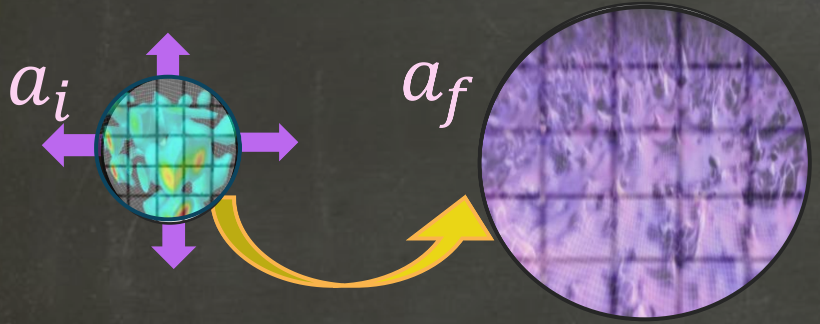
Shocked by his discovery,
Schrödinger found it
an alarming phenomenon!



Particle
Production

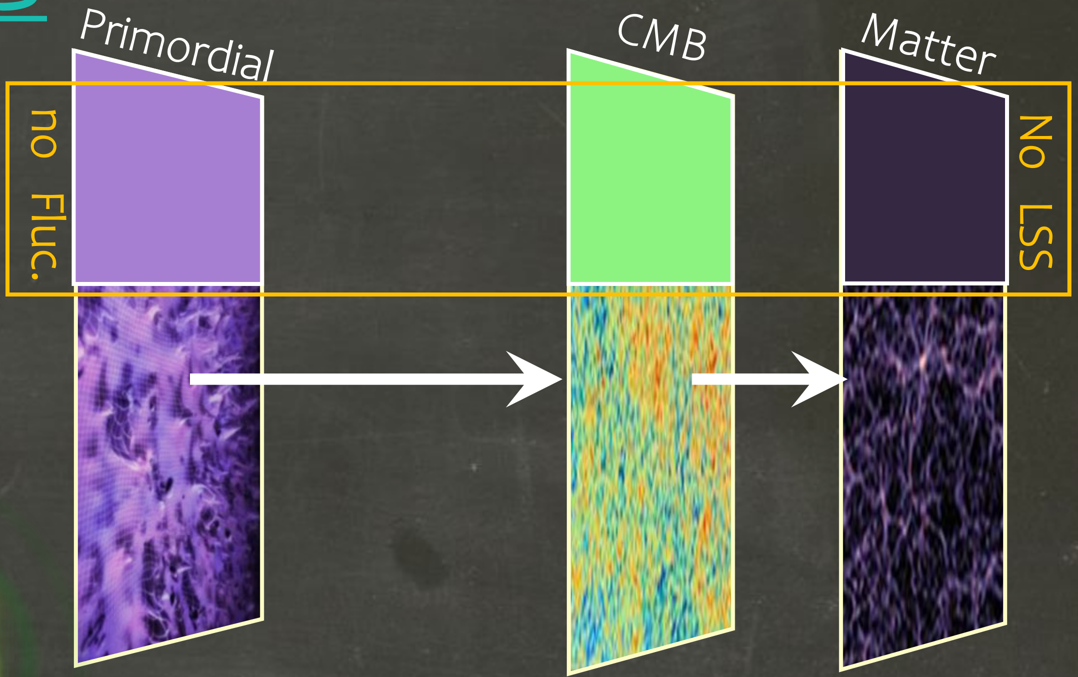
Cosmic Perturbations

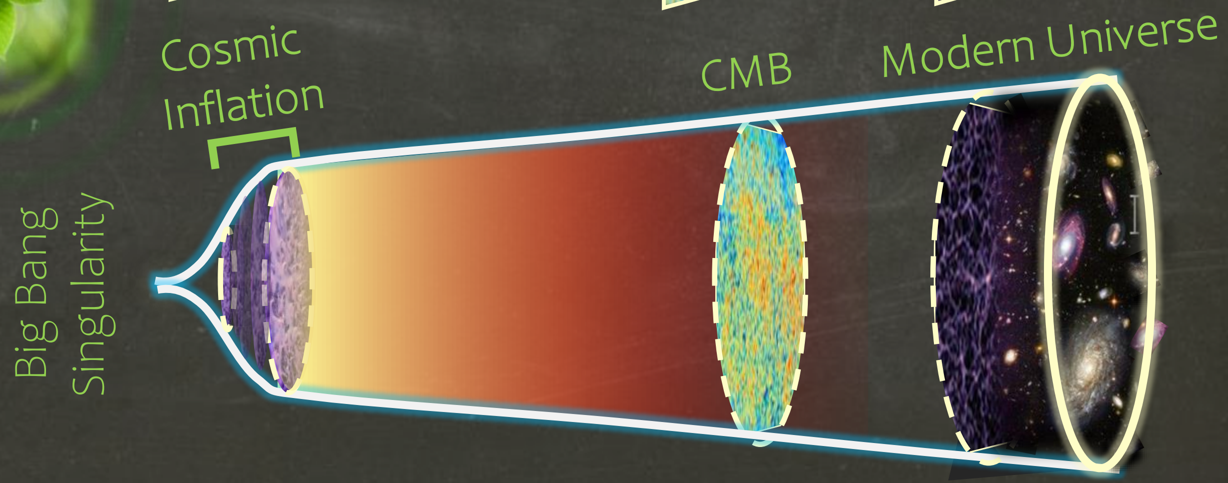
Exponential expansion turns initial quantum vacuum fluctuations into



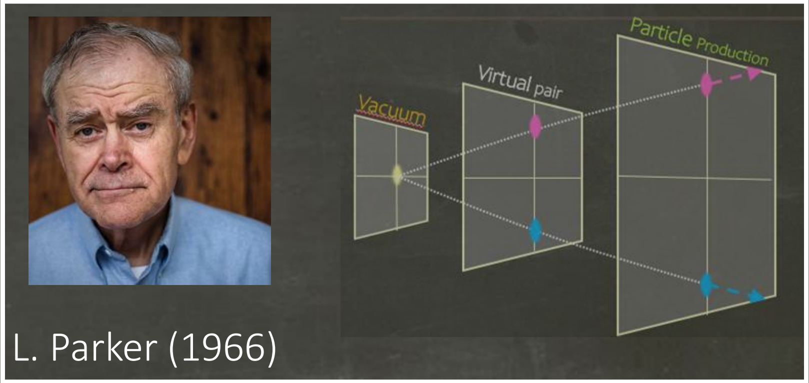
actual cosmic perturbations!

We are the product of quantum fluctuations in the very early universe!





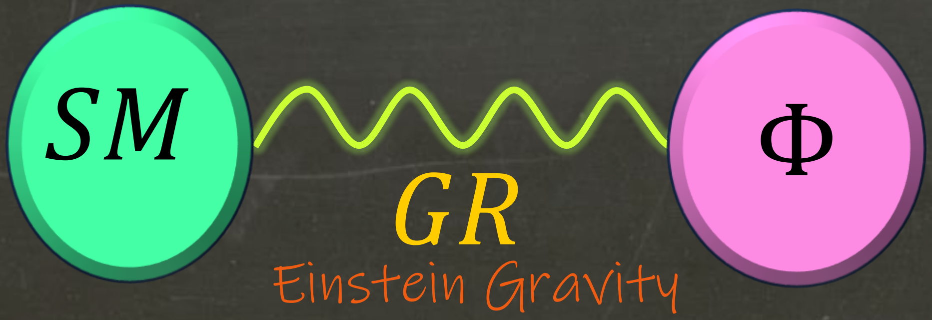
Cosmological Gravitational Particle Production (CGPP)



L. Parker (1966)

The expansion of the Universe creates pair production in FRW geometry.

But conformal fields in 4d will not be produced since FRW is conformally flat!



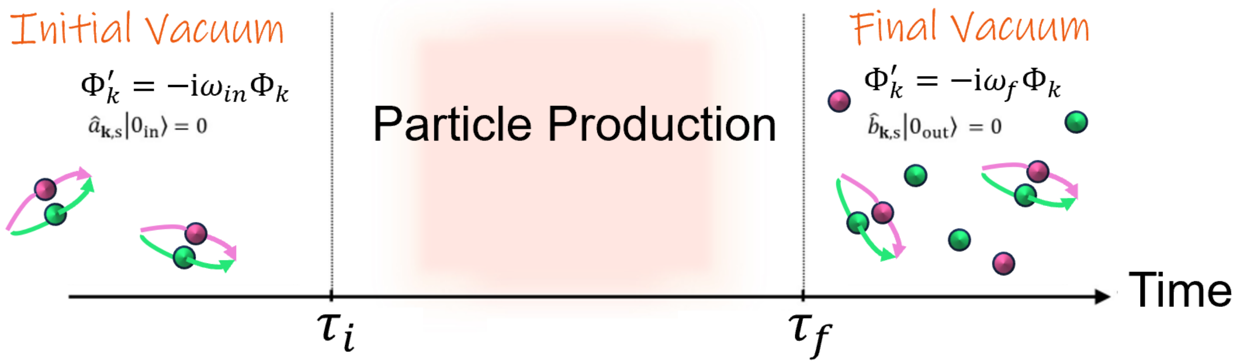
Scalar Field in Expanding Universe

Consider a scalar field $\mathcal{L} = \frac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} m^2 \Phi^2$

In cosmological background $g_{\mu\nu} = a^2(\tau)\mathrm{diag}(-1, 1, 1, 1)$

The field equation is $\Phi_k'' + {\omega_k}^2(\tau)\Phi_k = 0$

effective frequency $${\omega_k}^2(\tau) = k^2 + a^2(\tau)\left(m^2 + \frac{R(\tau)}{6}\right)$$



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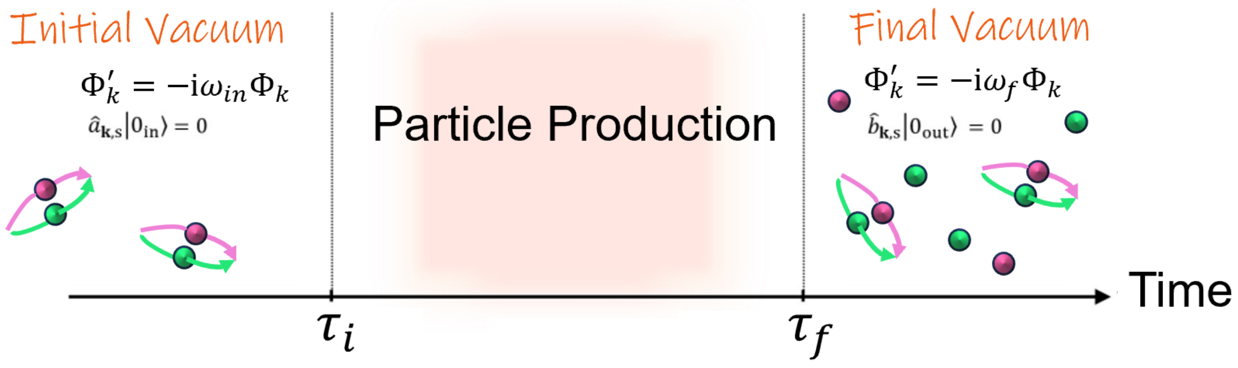

Scalar field can feel
the expansion of Universe

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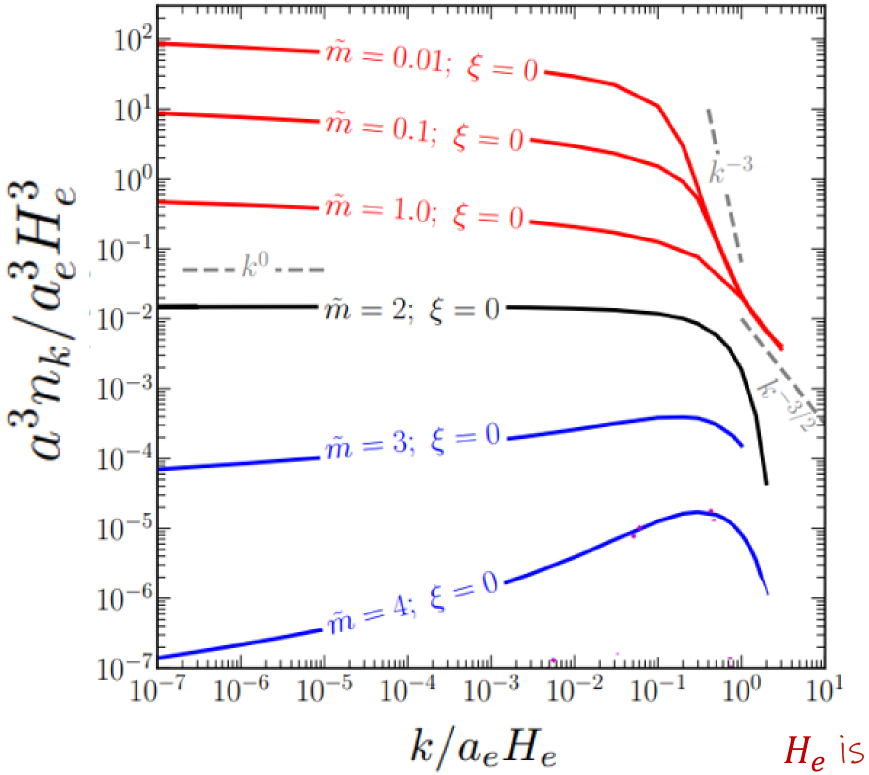
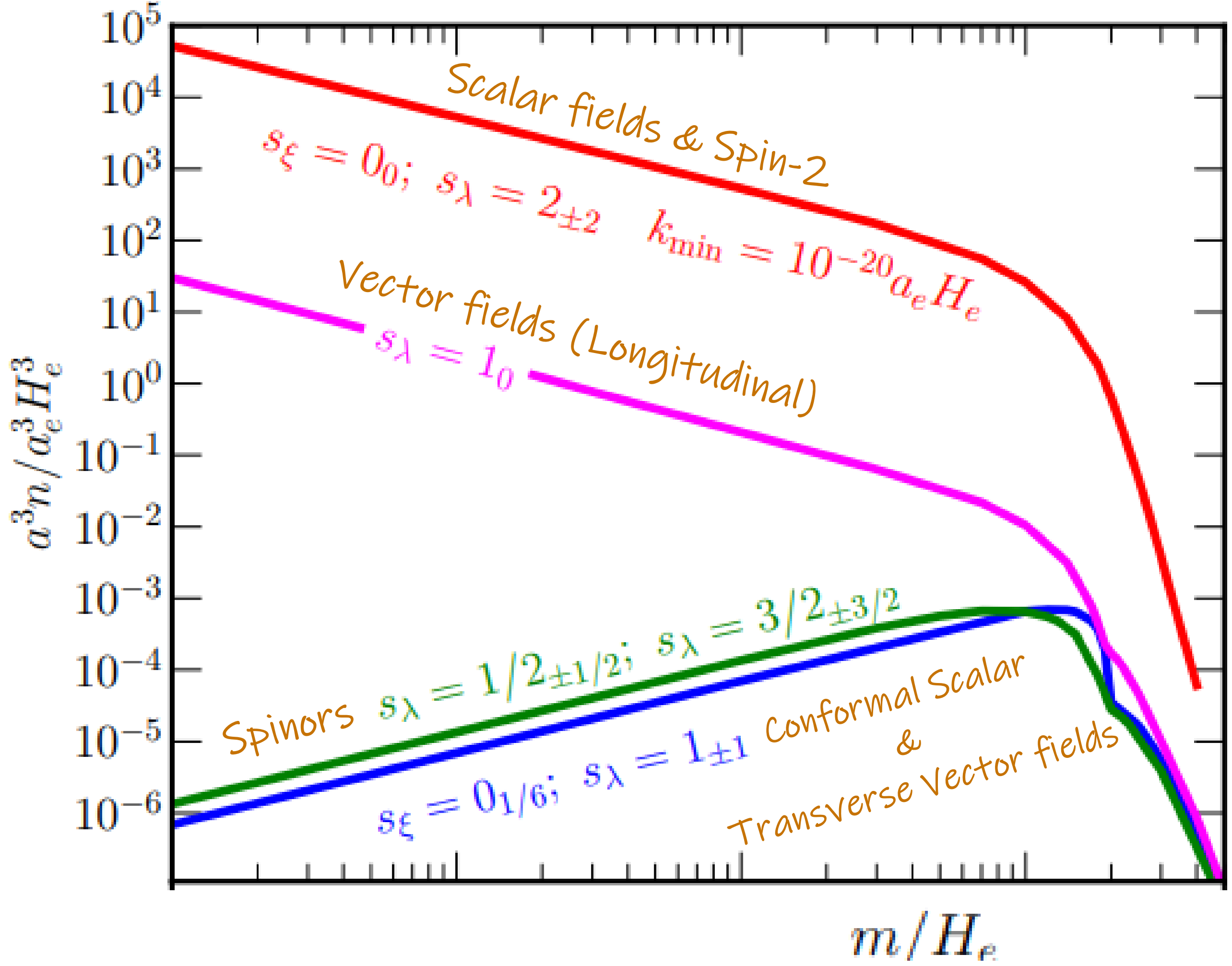


An example of **Cosmological Gravitational Particle Production (CGPP)**

Plot credit: Kolb & Long Reviews of Modern Physics 2023

H_e is Hubble scale at the end of inflation

Spinning Fields in Expanding Universe



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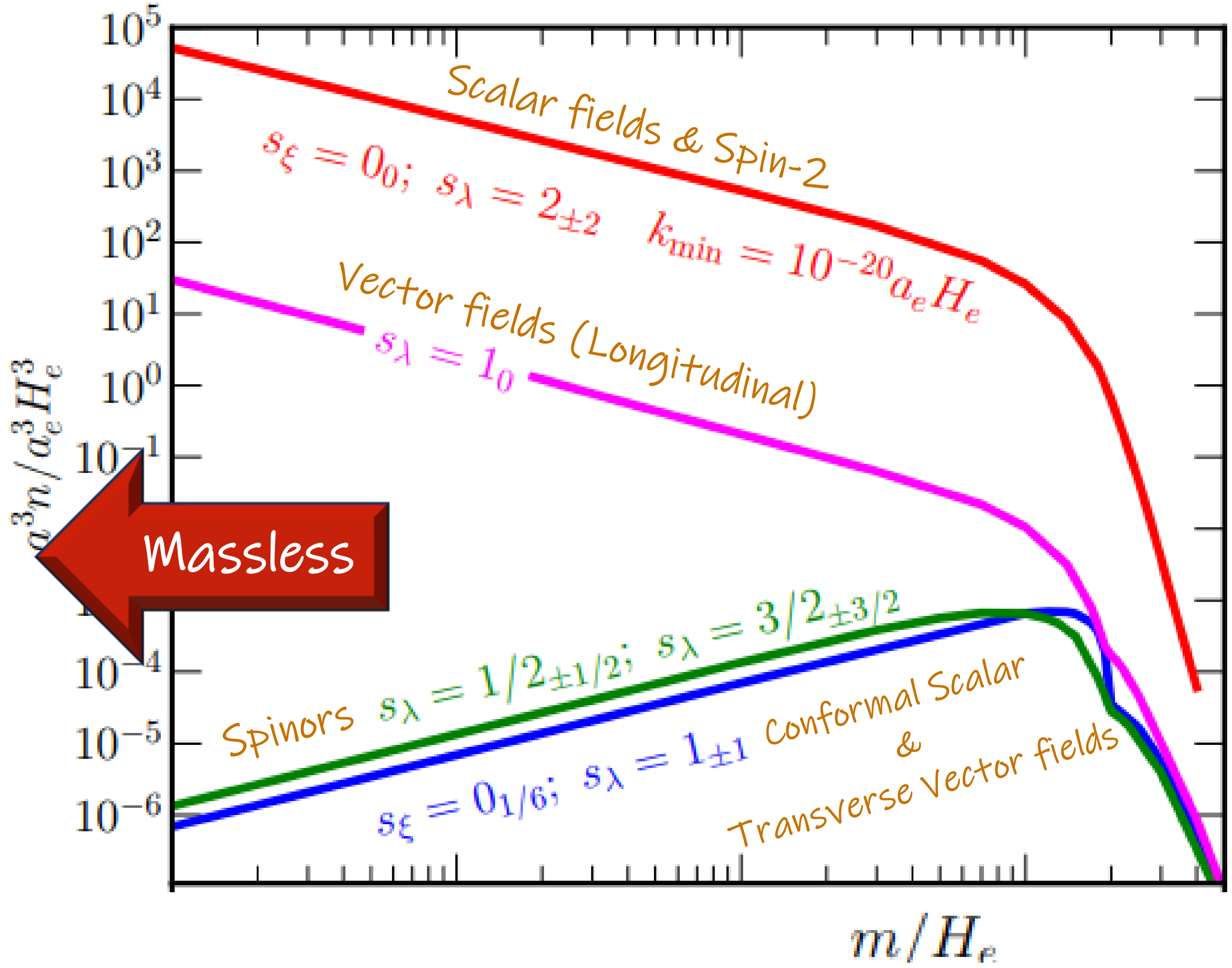

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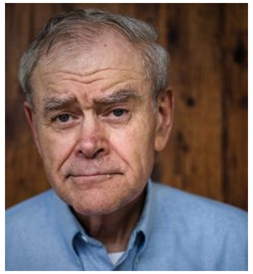

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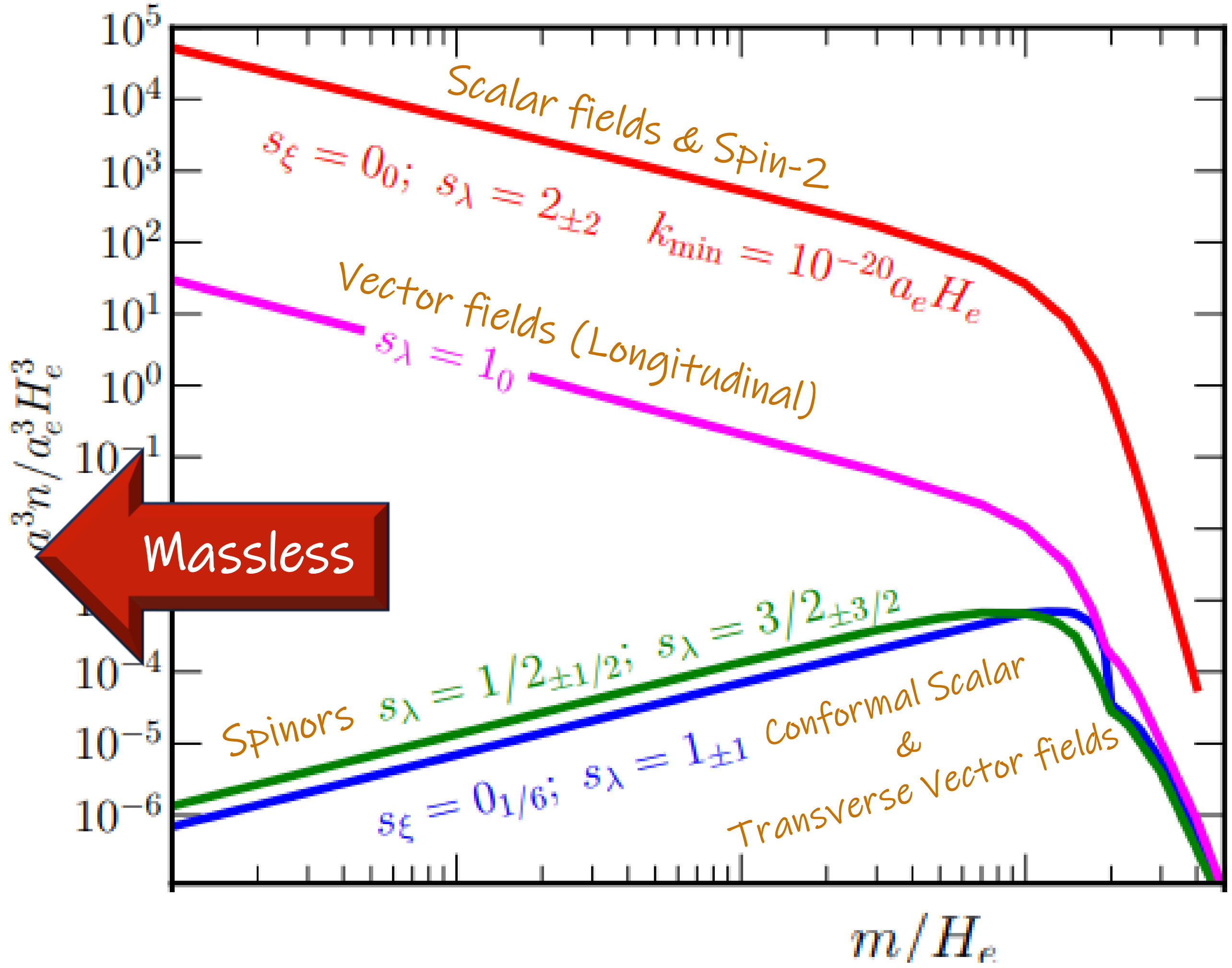

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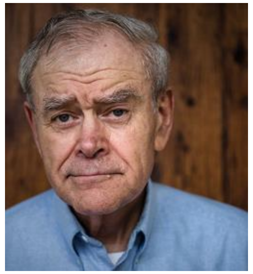

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No CGPP for massless fermions!

(conformal symmetry)

Fermions in Expanding Universe

Consider spin $\frac{1}{2}$ massless fermions $\mathcal{L}_\psi = i\psi^\dagger\, \gamma^\mu \mathcal{D}_\mu \psi$

Spinor covariant derivative $\mathcal{D}_\mu = \nabla_\mu - \omega_\mu$ Spin connection

In cosmological background $g_{\mu\nu} = a^2(\tau)\mathrm{diag}(-1,1,1,1)$

The field equation of fermion is $\left(\gamma^0(\partial_0 + \frac{3}{2}H) + \frac{1}{a}\gamma^i\partial_i\right)\psi = 0.$

Effect of gravity

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Effect of gravity

The effect of FRW gravity (conformally flat geometry) can be absorbed as

$$\Psi \equiv a^{3/2}\psi,$$

canonically renormalized field lives in flat space!

How to Create Fermions in Expanding Universe?

Breaking the conformal symmetry of Weyl fermions by interactions, e.g.

(dilatation transformation)

- Couple your Weyl fermion with { Inflaton field, Standard Model, Dark sector coupled to thermal bath
- make the fermion massive to produce them gravitationally! (CGPP)

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Gravitational Particle Production Mechanisms

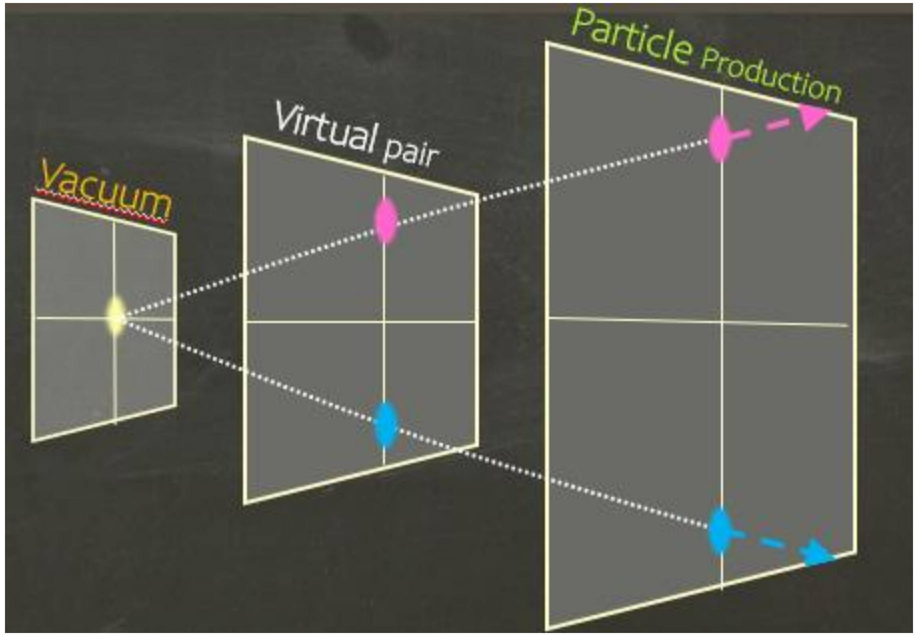
Gravitational Particle Production Mechanisms

	Production Mechanism	Underlying Physics	Conditions
I)	Cosmological Gravitational Particle Production (CGPP)	Cosmic expansion	super-massive fields $M > 10^{13}\ GeV$

Kolb & Long 2017

Relic density by CGPP

$$\frac{\rho_{\Psi,0}^{\rm CGPP}}{\rho_{\rm DM,0}} \sim 7 \times \left(\frac{M}{10^{11}{\rm GeV}}\right)^2 \left(\frac{T_{\rm reh}}{10^9{\rm GeV}}\right),$$



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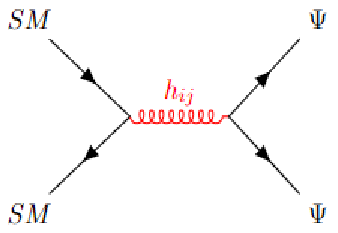
II)	Graviton-Mediated Annihilation (GMA)		Super-massive field High temperature plasma $T_{reh} > 10^{13}\ GeV$	M. Garny, et al 2016 Bernal, et. al. 2018 Clery et. al. 2022

Relic density by GMA

$$\frac{\rho_{\Psi,0}^{\mathrm{GMA}}}{\rho_{\mathrm{DM},0}} \sim 5\left(\frac{M}{10^{12}\mathrm{GeV}}\right)\left(\frac{T_{\mathrm{reh}}}{10^{13}\mathrm{GeV}}\right)^3,$$

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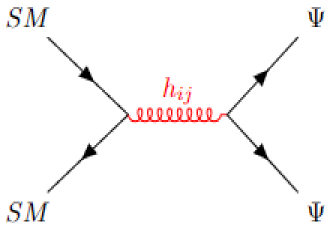
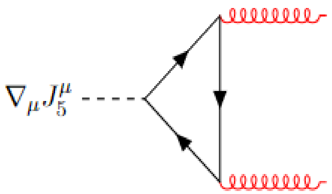
II)	Graviton-Mediated Annihilation (GMA)		Super-massive field High temperature plasma $T_{reh} > 10^{13}$ GeV	M. Garny, et al 2016 Bernal, et. al. 2018 Clery et. al. 2022
III)	Gravitational Leptogenesis		Parity violation $h_L \neq h_R$ Chiral GWs Chiral fermions	Alexander et. al. 2006 **A.M.** 2014 & 2016

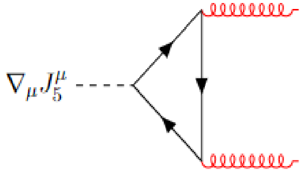
Chiral fermions

(global) Gravitational anomaly

$$\nabla_\mu J_5^\mu = \frac{N_L - N_R}{16\pi^2} R\tilde{R}$$

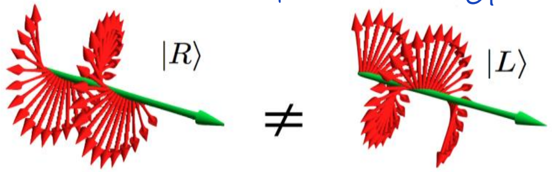
Parity violation in inflation

Matter Asymmetry by Gravitational Anomaly: $\langle R\tilde{R}\rangle \neq 0!$



What makes Chiral Gravitational Waves?

To generate circularly polarized GWs, we need Parity violation in inflation.
Two possible models are



1) Chern-Simons Gravity $\mathcal{L}_{eff} = \frac{1}{\Lambda}\varphi R\tilde{R}$

Alexander, Peskin, Sheikh-Jabbari 2006

2) Non-Abelian Gauge fields in axion-inflation $\mathcal{L}_{eff} = \frac{1}{\Lambda}\varphi F\tilde{F}$ (Chiral Gauge Field Chiral GWs)

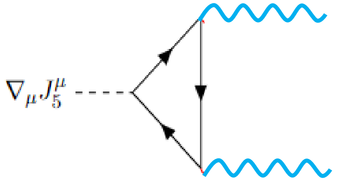
A.M., Noorbala, Sheikh-Jabbari 2012
A.M. 2014 & 2016
Caldwell, Devulder 2017
Adshead, Long, Sfakianakis 2017
Alexander, McDonough, Spergel 2018
Kamada, Kume, Yamada, Yokoyama 2019

Axion-inflation is a generic setting for leptogenesis
(All the Sakharov conditions are satisfied)
A.M. 2014

~~3) U(1) Gauge fields in axion-inflation~~

Papageorgiou, Peloso 2017

Matter Asymmetrey by Chiral Anomaly: $\langle F\tilde{F}\rangle \neq 0!$



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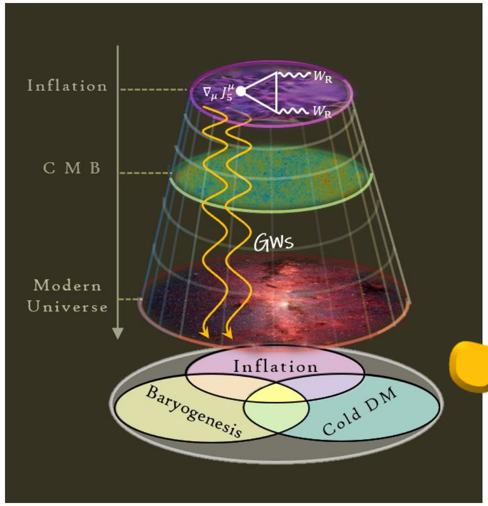
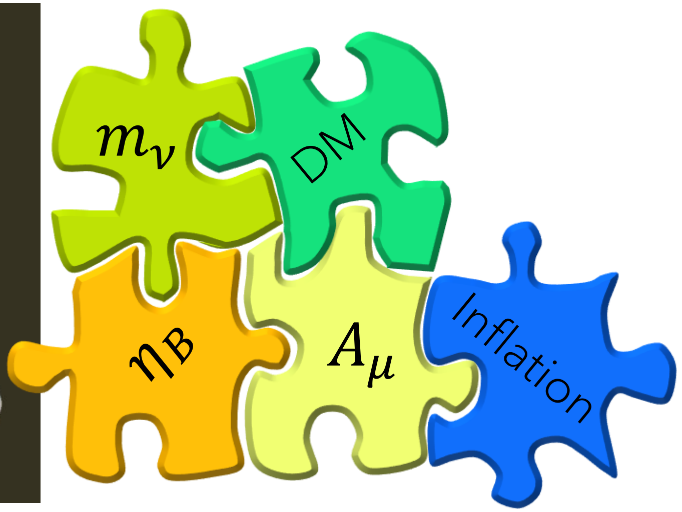
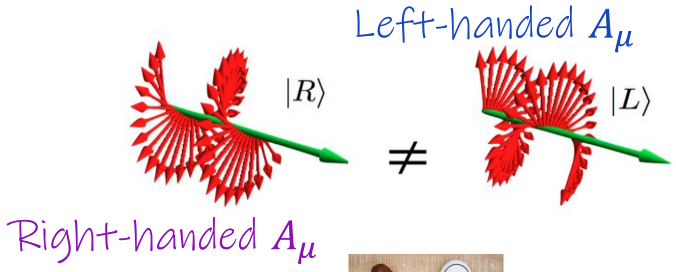
- Non-Abelian Gauge fields in axion-inflation

A.M. 2019

A.M. 2020, 2021







- U(1) Gauge fields in axion-inflation

Domcke, Harling, Morgante, Mukaida 2019

Domcke, Kamada, Mukaida, Schmitz, Yamada 2020

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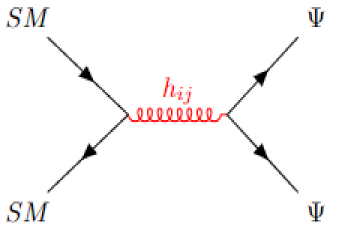
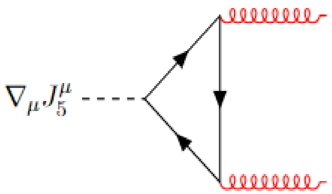
II)	Graviton-Mediated Annihilation (GMA)		Super-massive field High temperature plasma $T_{reh} > 10^{13}\ GeV$	M. Garny, et al 2016 Bernal, et. al. 2018 Clery et. al. 2022
III)	Gravitational Leptogenesis		Parity violation Chiral GWs Chiral fermions	Alexander et. al. 2006 **A.M.** 2014 & 2016

$h_L \neq h_R$

What does Unpolarized Gravitational Waves do!?

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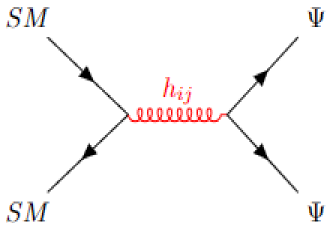
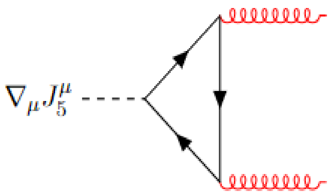
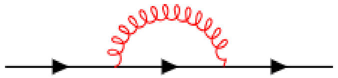
	Production Mechanism	Underlying Physics	Conditions	
I)	Cosmological Gravitational Particle Production (CGPP)	Cosmic expansion	super-massive fields $M > 10^{13}\ GeV$	Kolb & Long 2017
II)	Graviton-Mediated Annihilation (GMA)		Super-massive field High temperature plasma $T_{reh} > 10^{13}\ GeV$	M. Garny, et al 2016 Bernal, et. al. 2018 Clery et. al. 2022
III)	Gravitational Leptogenesis		Parity violation $h_L \neq h_R$ Chiral GWs Chiral fermions	Alexander et. al. 2006 **A.M.** 2014 & 2016
IV)	GW-Induced Freeze-In		GWs Background	**A.M.** & Kopp 2024

Gravitational Wave-induced Freeze-in

In a nutshell it is the production of Weyl fermions by a stochastic cosmic perturbations

Based on
A.M. & Kopp 2024

arXiv:2405.09723
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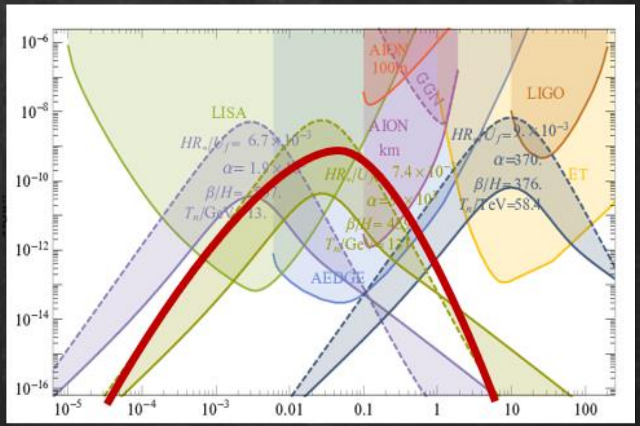
As the lowest hanging fruit of this mechanism, consider:
unpolarized stochastic background of GWs
with broken power-law spectrum Ω_{GW} in radiation ear

Ω_{GW}



$f\ [Hz]$

Plot credit: Ellis et. Al. 2020

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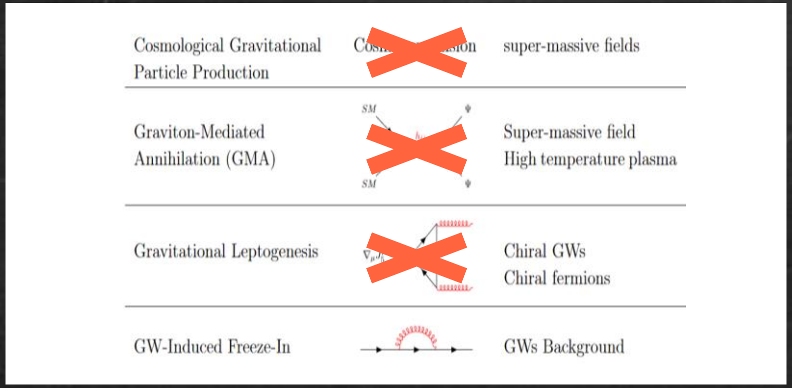

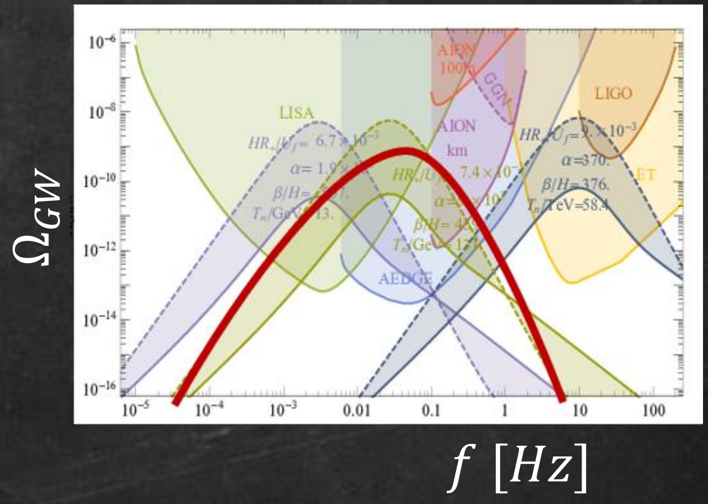

Plot credit: Ellis et. Al. 2020

The graviton–fermion Interaction

Cosmological background with Gravitational Waves

transverse-traceless

$$ds^2 = -dt^2 + a^2(t)\,\hat{g}_{ij}\,dx_i dx_j,$$

$$\hat{g}_{ij} = \left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{jk} + \dots\right)$$

Consider spin $\frac{1}{2}$ Weyl fermions

$$\mathcal{L}_\psi = i\boldsymbol{\psi}_D^\dagger \gamma^\mu \mathcal{D}_\mu \boldsymbol{\psi}_D,$$

Free massless fermions can be written as $\boldsymbol{\Psi} \equiv a^{3/2}\boldsymbol{\psi},$

Does not feel the expansion of Universe

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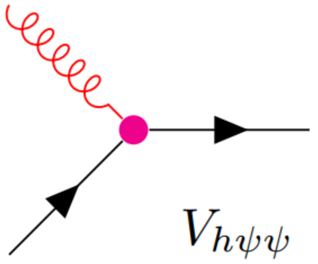
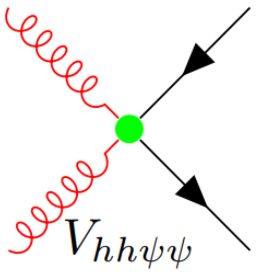
$$\mathcal{L}_{\rm int}^{(1)} = -\frac{i}{2a^4} h_{ij} \bar{\boldsymbol{\Psi}}_D \gamma^i \overleftrightarrow{\partial}_j \boldsymbol{\Psi}_D.$$

Cubic vertex



$$\mathcal{L}_{\rm int}^{(2)} = -\frac{i}{16a^3} \mathbf{e}^\mu{}_\alpha h_{ij} \partial_\mu h_{ik} \bar{\boldsymbol{\Psi}}_D \Gamma^{\alpha jk} \boldsymbol{\Psi}_D,$$

Quartic vertex



The graviton–fermion Interaction

We use In-In formalism to compute the energy density of Weyl fermions

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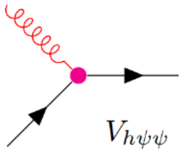
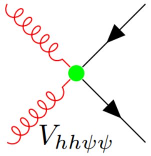


$$\mathcal{L}^{(2)}_{\rm int} = -\frac{i}{16a^3} \mathbf{e}^{\mu}{}_{\alpha} h_{ij} \partial_\mu h_{ik} \bar{\mathbf{\Psi}}_D \Gamma^{\alpha jk} \mathbf{\Psi}_D ,$$



Expectation value of an arbitrary operator in In-In formalism

$$\langle Q(t) \rangle = \left\langle \bar{\mathrm{T}} \exp\left[i \int_{t_i^-}^{t} dt''\, H_{\rm int}(t'') \right] Q_I(t)\, \mathrm{T} \exp\left[-i \int_{t_i^+}^{t} dt'\, H_{\rm int}(t') \right] \right\rangle ,$$

Interaction Hamiltonian

$$H_{\rm int}(t) = -\int d^3x\, a^3(t)\, \mathcal{L}_{\rm int}(t, \mathbf{x}),$$

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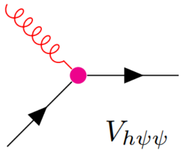

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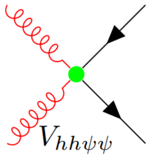

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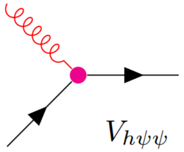
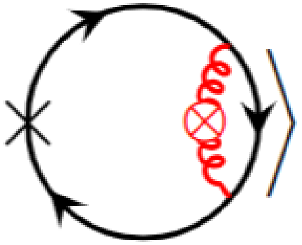
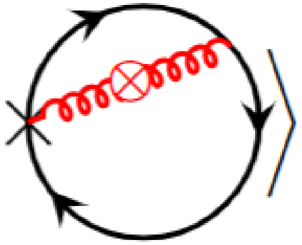


$\varrho_1 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle ,$

$\varrho_2 = \Big\langle \rho_\psi^{(1)}$  $\Big\rangle ,$

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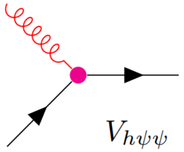
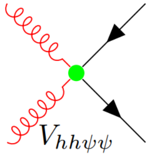
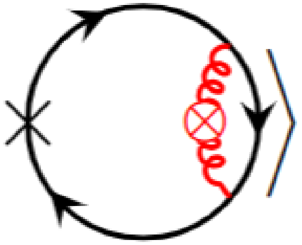
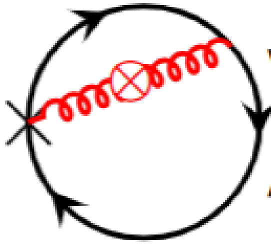
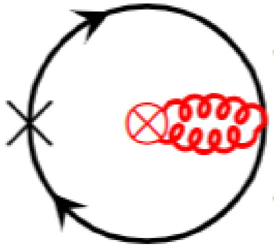
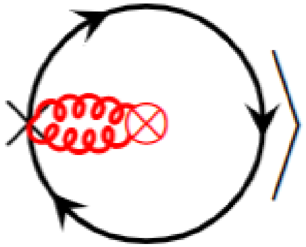




$\varrho_1 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$,

$\varrho_2 = \Big\langle \rho_\psi^{(1)}$  $\Big\rangle$,

$\varrho_3 = \Big\langle \rho_\psi^{(0)}$  $\Big\rangle$,

$\varrho_4 = \Big\langle \rho_\psi^{(2)}$  $\Big\rangle$.

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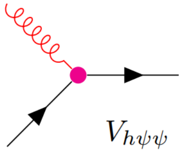
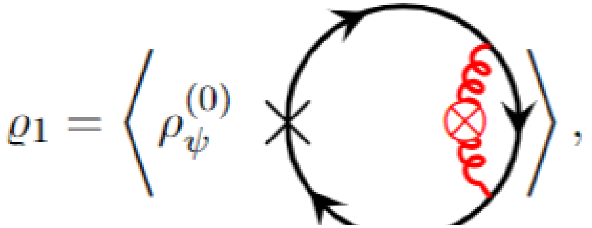
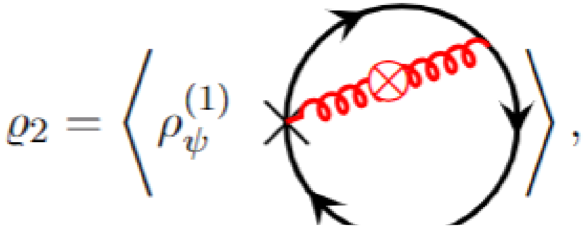
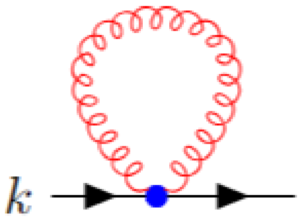
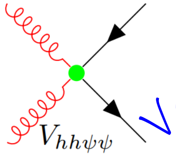




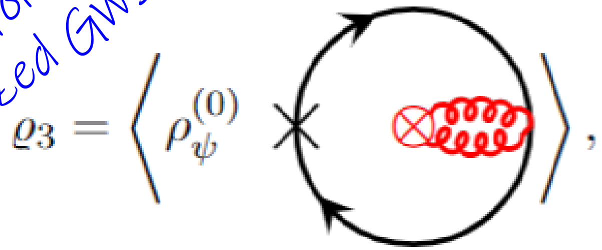


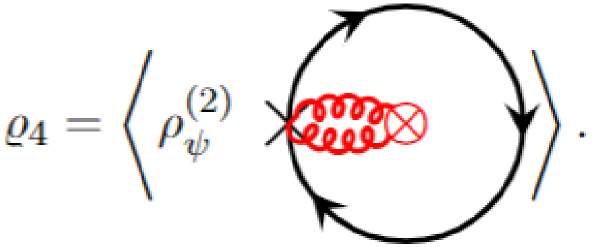
$\sum_s s \langle \hat{h}^\dagger_{s,\mathbf{q}}(\tau) \hat{h}_{s,\mathbf{q}}(\tau) \rangle$





Vanishes for unpolarized GWs





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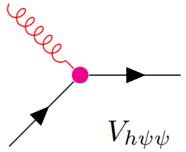

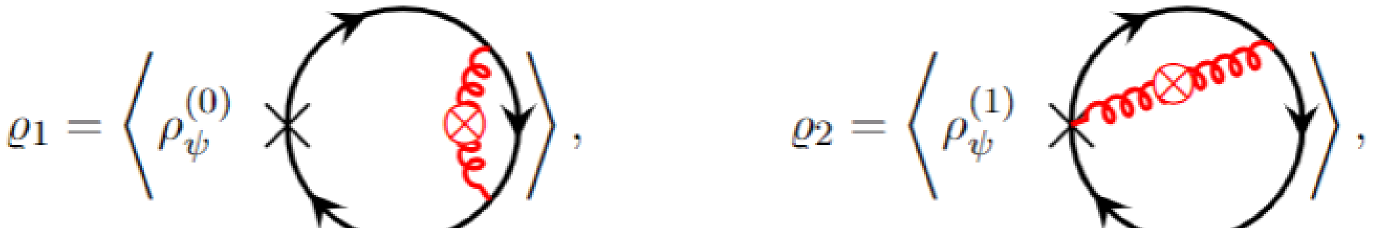

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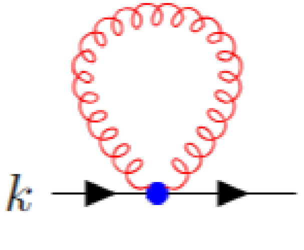

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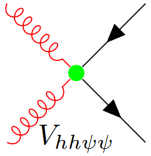

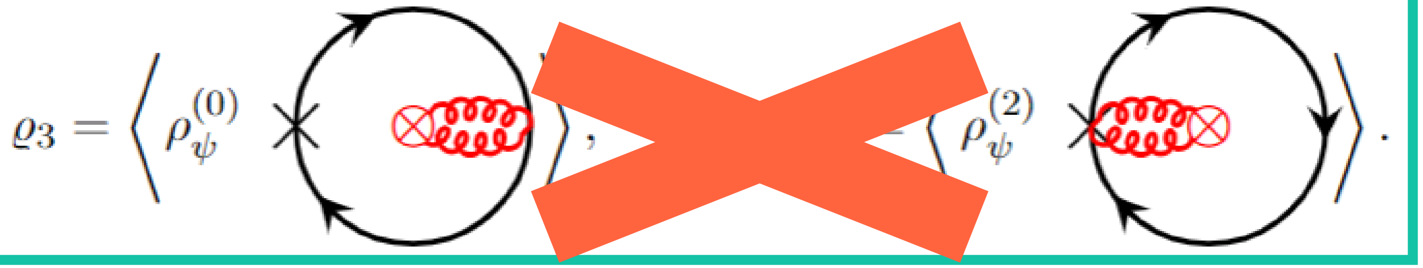

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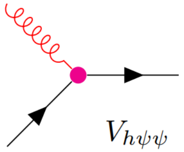

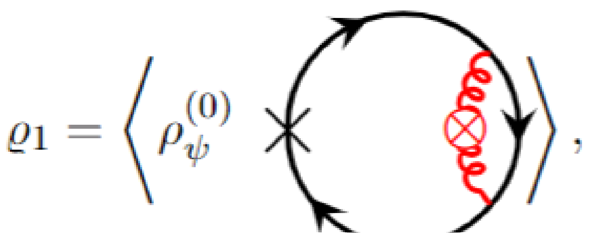

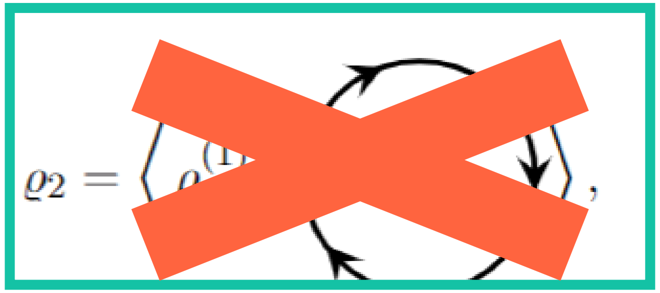

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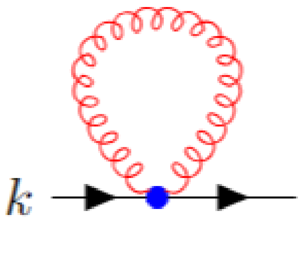

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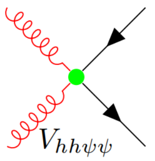
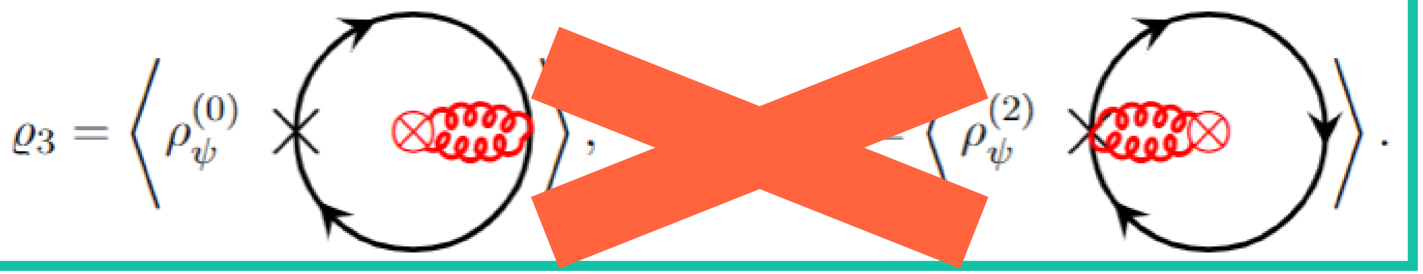
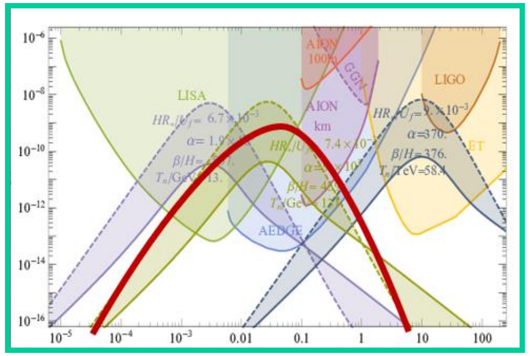




GW-induced Freeze-in



The energy density of Weyl fermions

$$\langle \rho_\psi(\tau)\rangle = \frac{1}{4\pi}\frac{1}{a^4(\tau)}\int_{\tau_i^+}^{\tau} d\tau' \int_{\tau_i^-}^{\tau} d\tau'' \int q^2 dq \, \langle \mathrm{h}_q(\tau'')\mathrm{h}_q^*(\tau')\rangle \int k^4 dk \int d\theta \, \sin^3\theta \, e^{i(k+\omega)(\tau'-\tau'')}$$

$$\times (k+\omega)\left[2 - \frac{\sin^2\theta((\omega+k)^2+q^2)}{2\omega(\omega+k-q\cos\theta)}\right] + c.c. \, .$$

Unequal time power spectrum of GWs

It acts like radiation

$$\langle P_\psi(\tau)\rangle = \frac{1}{3}\langle \rho_\psi(\tau)\rangle \propto \frac{1}{a^4(\tau)} .$$

It depends on the degree of temporal coherency of GWs background

$$\langle \mathrm{h}_q^*(\tau'')\mathrm{h}_q(\tau')\rangle = \gamma_q(|\tau'-\tau''|)\sqrt{\langle|\mathrm{h}_q(\tau')|^2\rangle \, \langle|\mathrm{h}_q(\tau'')|^2\rangle},$$

Fully incoherent

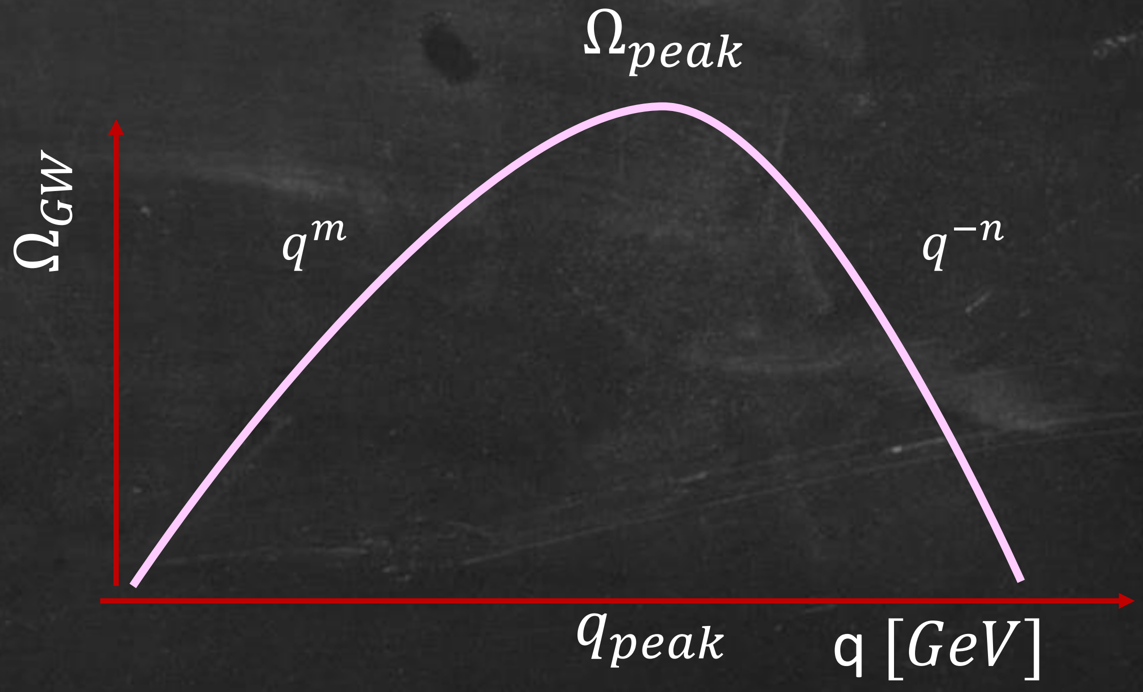
$$\gamma_q(|\tau'-\tau''|) = \Delta\eta \, \delta(\tau'-\tau''),$$

$$\gamma_q(|\tau'-\tau''|) = 1.$$

Fully coherent

Phenomenological Model for Ω_{GW}

Broken Power-law Spectrum



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The energy density of Weyl fermions

Unequal time power spectrum of GWs

$$\langle \rho_\psi(\tau) \rangle = \frac{1}{4\pi} \frac{1}{a^4(\tau)} \int_{\tau_i^+}^{\tau} d\tau' \int_{\tau_i^-}^{\tau} d\tau'' \int q^2 dq \, \langle \mathrm{h}_q(\tau'') \mathrm{h}_q^*(\tau') \rangle \int k^4 dk \int d\theta \, \sin^3\theta \, e^{i(k+\omega)(\tau'-\tau'')}$$
$$\times (k+\omega) \left[2 - \frac{\sin^2\theta((\omega+k)^2+q^2)}{2\omega(\omega+k-q\cos\theta)} \right] + c.c. \, .$$

It acts like radiation $\langle P_\psi(\tau) \rangle = \frac{1}{3} \langle \rho_\psi(\tau) \rangle \propto \frac{1}{a^4(\tau)}.$

It depends on the degree of temporal coherency of GWs background

$$\langle \mathrm{h}_q^*(\tau'') \mathrm{h}_q(\tau') \rangle = \gamma_q(|\tau' - \tau''|) \sqrt{\langle |\mathrm{h}_q(\tau')|^2 \rangle \langle |\mathrm{h}_q(\tau'')|^2 \rangle},$$

Fully incoherent

$$\gamma_q(|\tau' - \tau''|) = \Delta\eta \, \delta(\tau' - \tau''),$$

$$\gamma_q(|\tau' - \tau''|) = 1.$$ Fully coherent

GW-induced Freeze-in

The energy density of Weyl fermions

Unequal time power spectrum of GWs

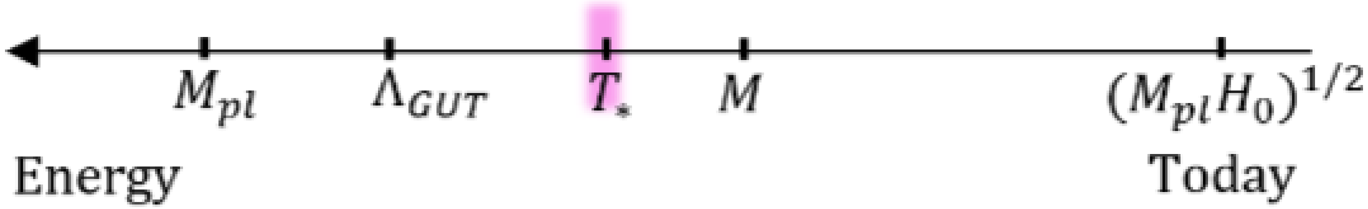
$$\langle\rho_\psi(\tau)\rangle = \frac{1}{4\pi}\frac{1}{a^4(\tau)}\int_{\tau_i^+}^{\tau} d\tau' \int_{\tau_i^-}^{\tau} d\tau'' \int q^2 dq \, \langle \mathrm{h}_q(\tau'')\mathrm{h}_q^*(\tau')\rangle \int k^4 dk \int d\theta \, \sin^3\theta \, e^{i(k+\omega)(\tau'-\tau'')}$$
$$\times (k+\omega)\left[2 - \frac{\sin^2\theta((\omega+k)^2+q^2)}{2\omega(\omega+k-q\cos\theta)}\right] + c.c. \, .$$

It acts like radiation $\langle P_\psi(\tau)\rangle = \frac{1}{3}\langle\rho_\psi(\tau)\rangle \propto \frac{1}{a^4(\tau)}.$

Final result $\langle\rho_\psi(\tau)\rangle = \left(\frac{q_{\text{peak}}}{a(\tau)}\right)^4 \left(\frac{H_0}{\mathcal{H}_*}\right)^2 z_*^2 \, C \quad \Omega_{\text{peak}},$

GW-induced Freeze-in & Dark Matter

(effectively) massless during production

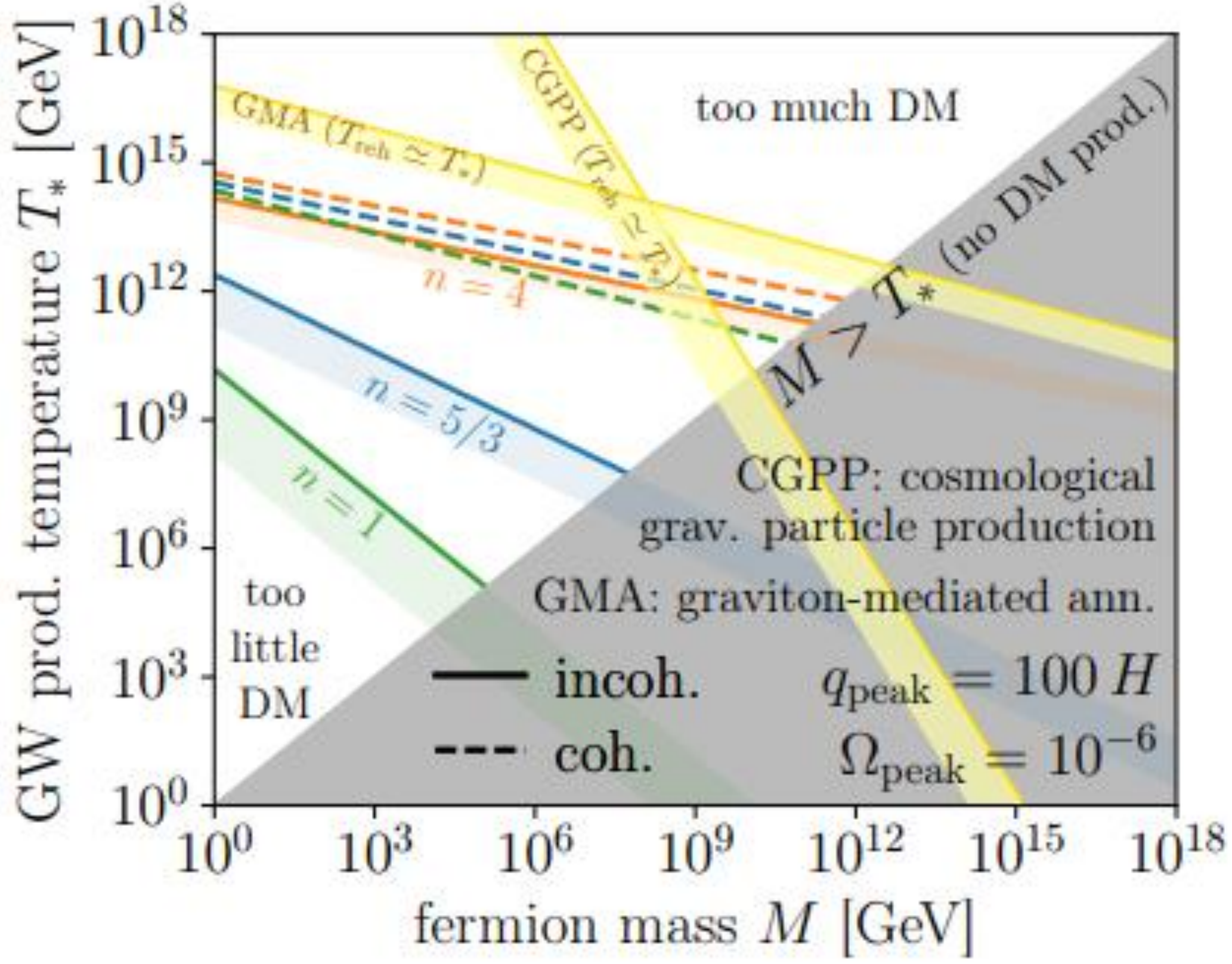


Fermion eventually becomes massive with mass M

$$\Omega_{\psi,0} \simeq 0.36 \times \mathcal{C} \left(\frac{M}{T_*}\right) \left(\frac{q_{\rm peak}/\mathcal{H}_*}{100}\right)^4 \left(\frac{g_*(\tau_*)}{106.75}\right)^{4/3} \left(\frac{T_*}{3 \times 10^{11}\,{\rm GeV}}\right)^5 \left(\frac{\Omega_{\rm peak}}{10^{-6}}\right).$$

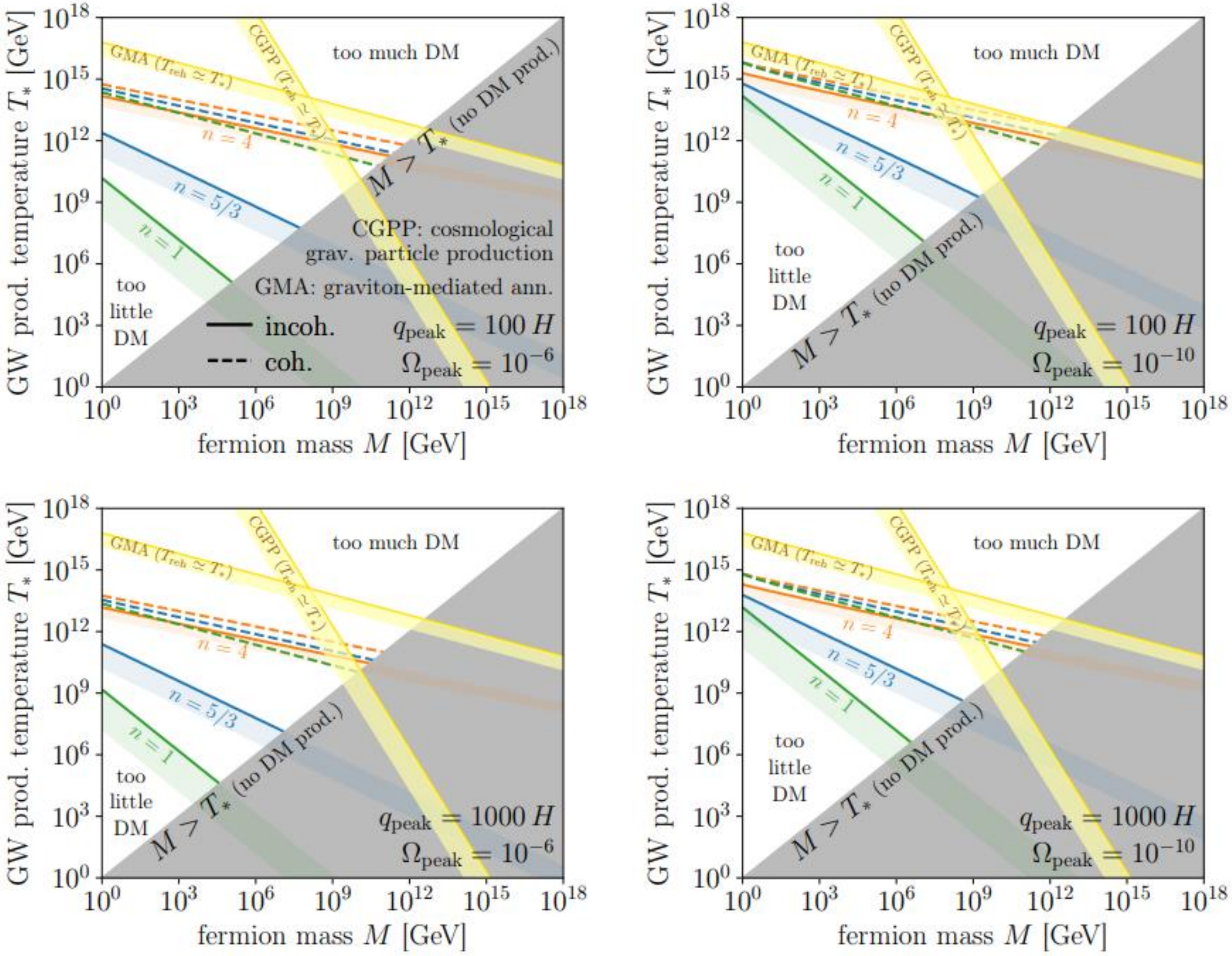
Parameter space of GW-induced freeze-in of fermion

A.M. & Kopp 2024

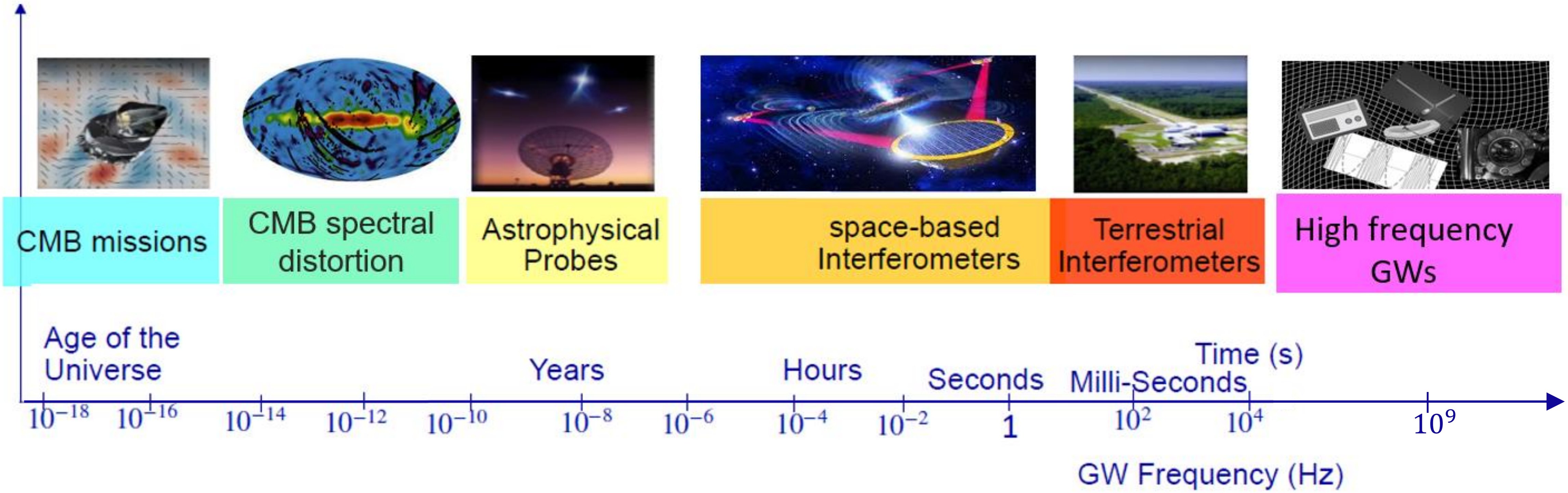


Parameter space of GW-induced freeze-in of fermion

A.M. & Kopp 2024



Gravitational Waves Spectrum

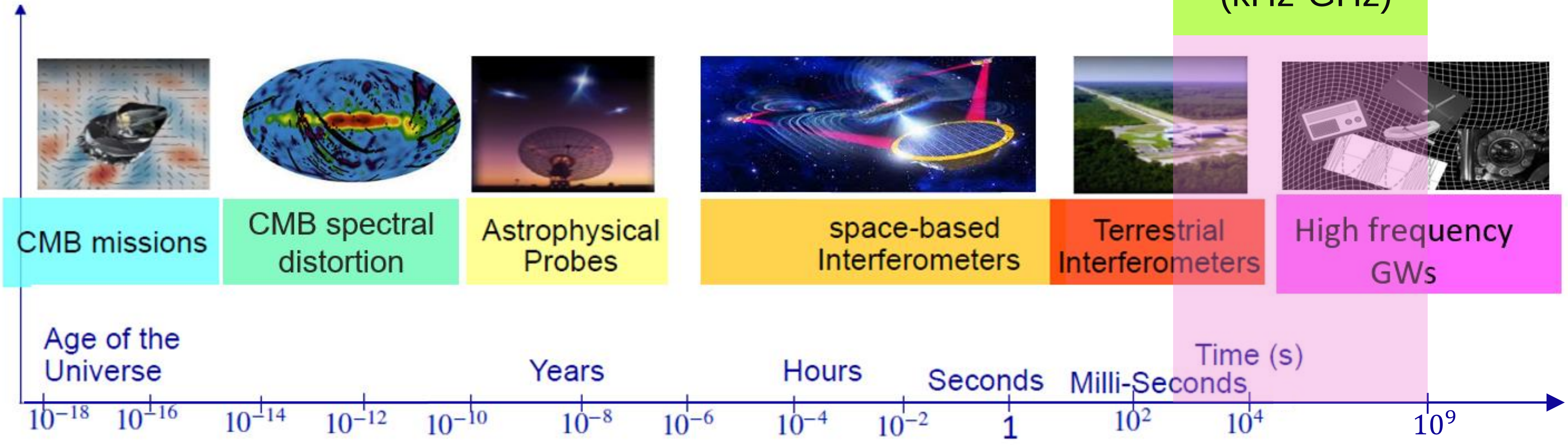


Age of the Universe = Billions of Years

Gravitational Waves Spectrum

GW-induced freeze-in mechanism requires a GWs spectrum with peak frequency

$f_{peak} \in$ (kHz-GHz)



Age of the Universe = Billions of Years

Summary

Gravity and Quantum Effects in Cosmology can still surprise us:
We discussed an effect that is zero at tree level and non-zero at 1-loop in cosmic perturbations!

Cosmic Perturbations (like GWs) naturally break the conformal symmetry of Weyl Fermions in Cosmology

It leads to a new mechanism for dark matter production in early universe, i.e. GW-induced freeze-in of fermionic dark matter.

Questions?!