Soft Metric Fluctuations During Inflation

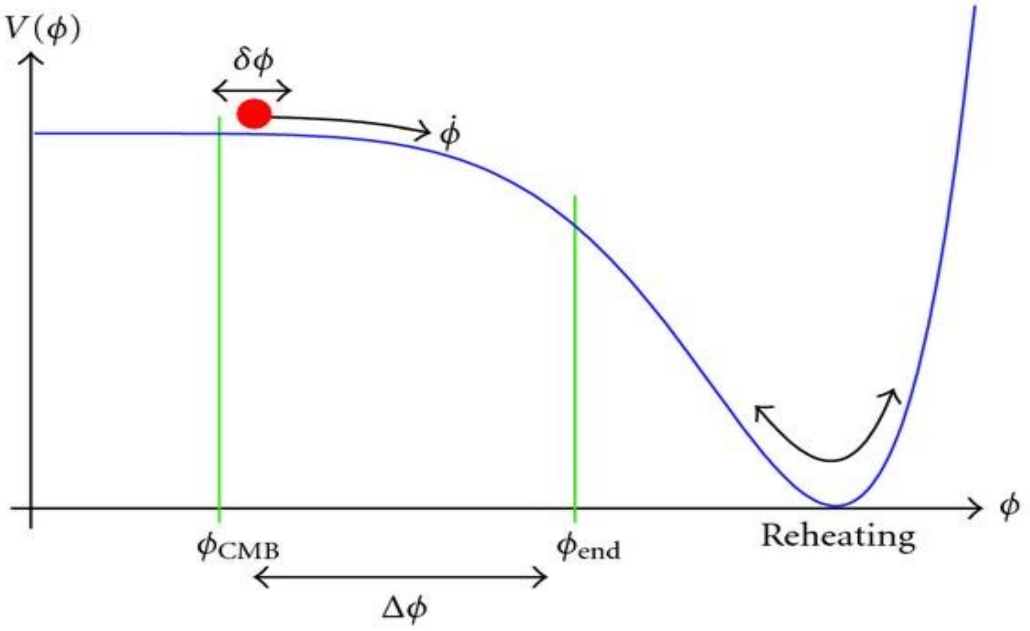
Kshitij Gupta, Daniel Green

(arxiv : 2410.11973)

INTRODUCTION

Review of inflationary perturbations

- During the era of inflation, we want to study metric perturbations that are produced due to quantum fluctuations.



Review of inflationary perturbations

- During the era of inflation, we want to study metric perturbations that are produced due to quantum fluctuations.
- A convenient choice to describe these fluctuations is the comoving gauge, where the inflaton is fixed $\phi = \phi(t)$ and acts as the clock, and all the perturbations are absorbed in the metric.
- We represent the perturbations as ζ, and the metric is given by

$$g_{ij} = a^2(t)e^{2\zeta(x,t)}\delta_{ij}$$

Conservation of ζ - Intuition

- We expect ζ to be conserved on super horizon scales.
- Conservation important for modern cosmological program – sets the initial conditions for the rest of cosmic evolution.
- Symmetries of ζ confirm this intuition – in the soft limit ζ should be equivalent to rescaling the coordinates

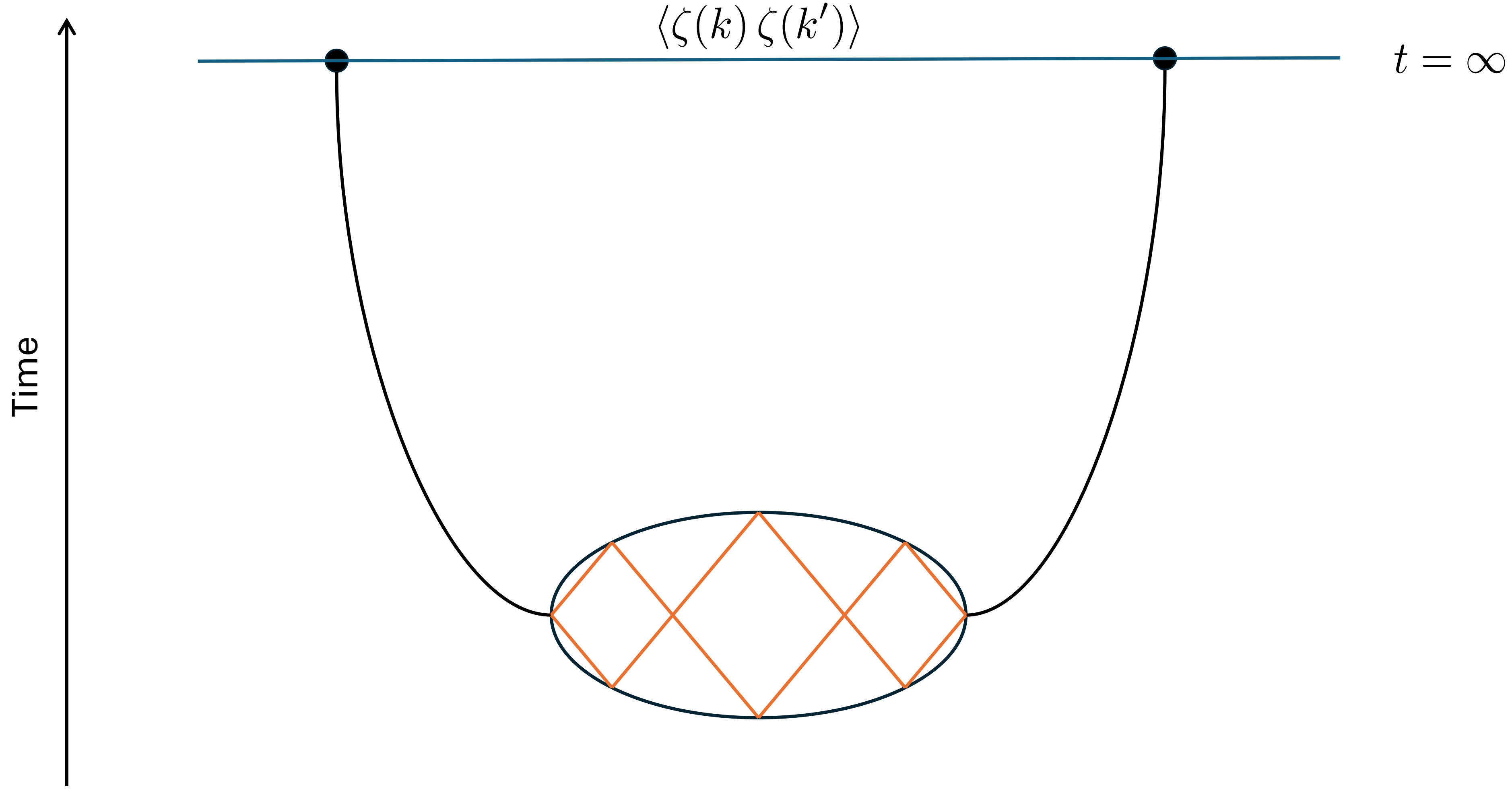
Conservation of ζ - Technical Details

- But, it is technically hard to see – because of loops.
- It has been proven to all orders.

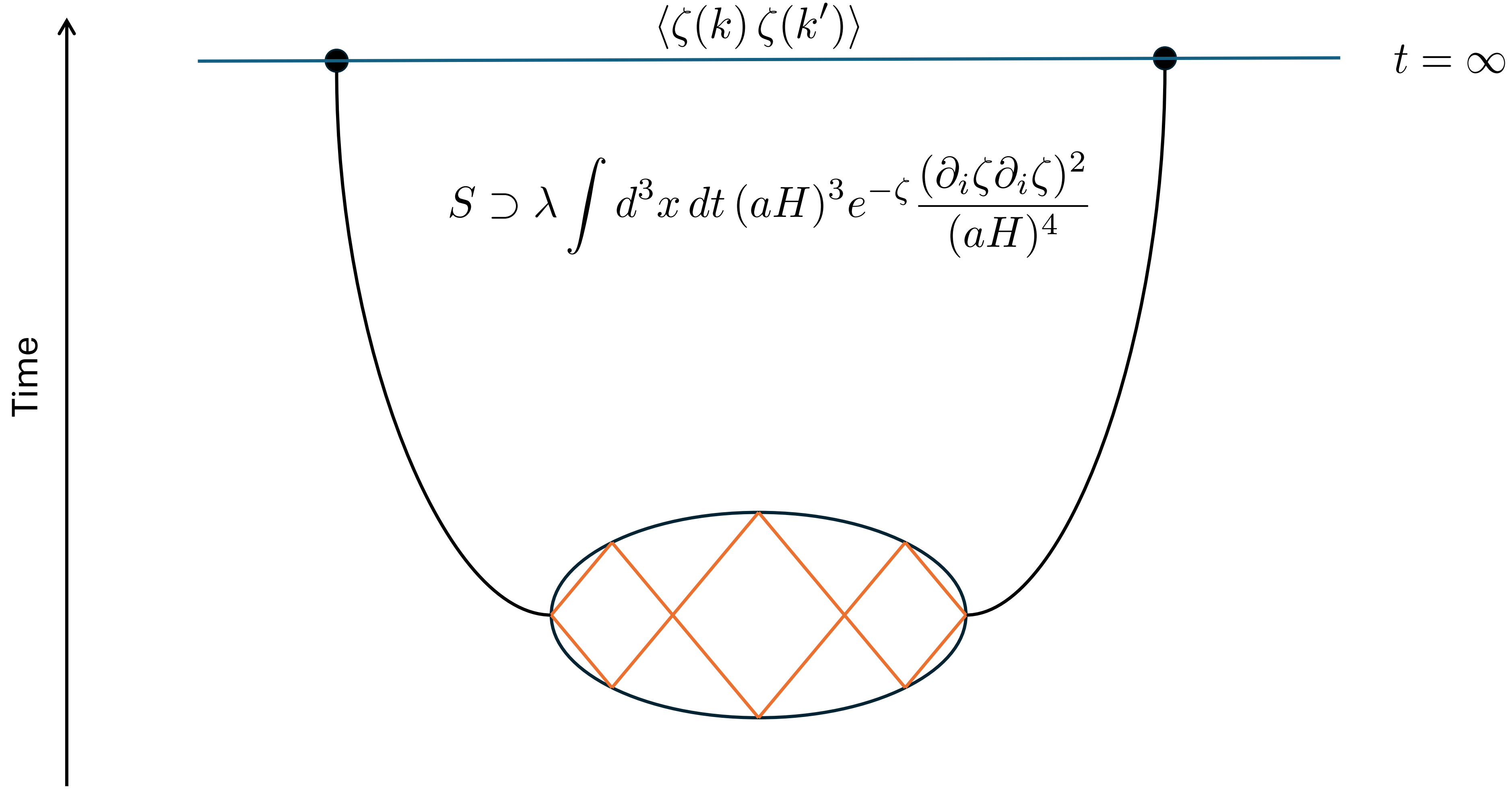
Senatore and Zaldarriaga; Assassi, Baumann, Green

- The arguments are diagrammatic in nature.

The worry

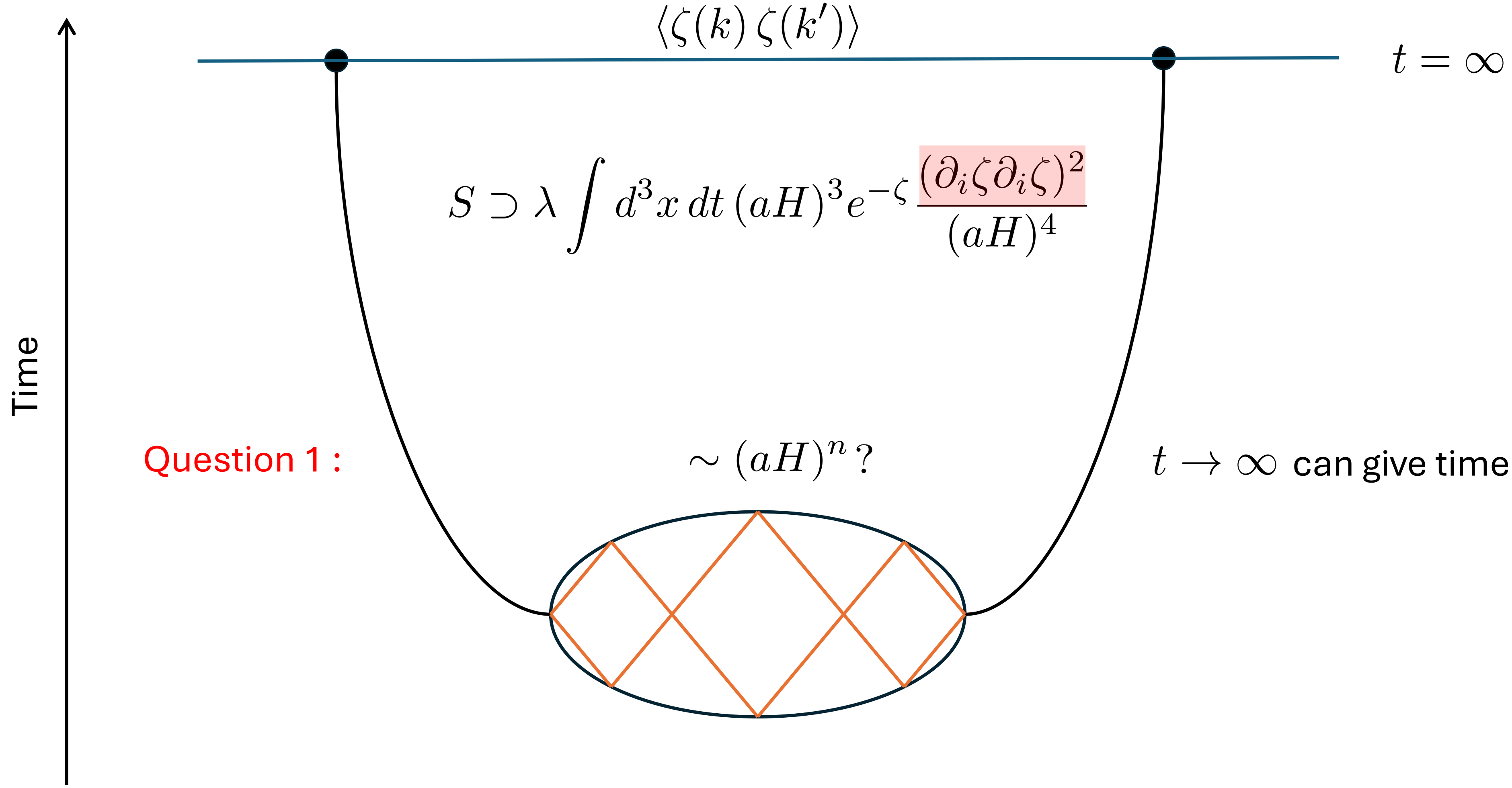


The worry



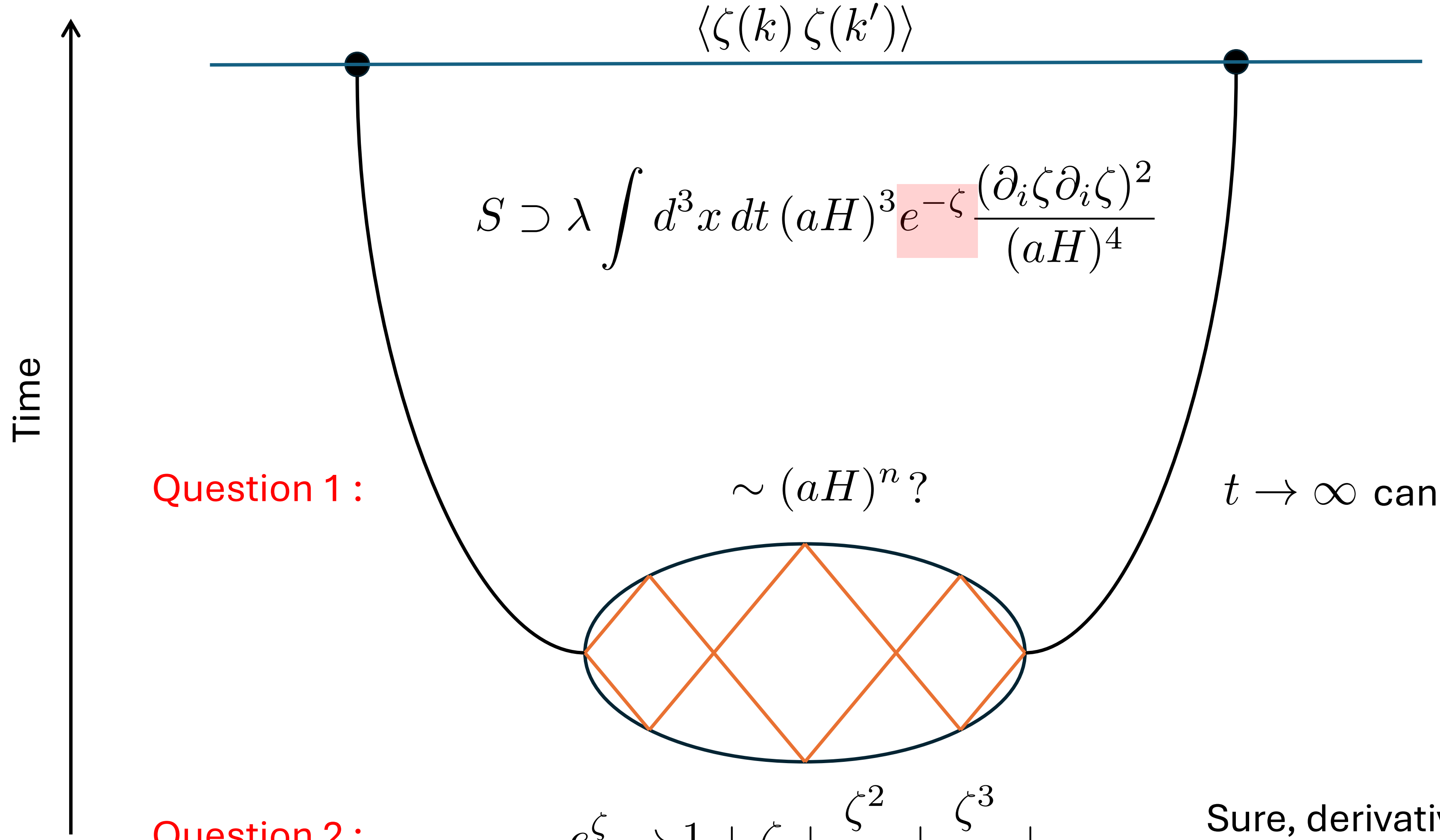
$\langle\zeta(k)\,\zeta(k')\rangle$
$t=\infty$
$S \supset \lambda \int d^3x\,dt\,(aH)^3 e^{-\zeta}\frac{(\partial_i\zeta\partial_i\zeta)^2}{(aH)^4}$
Time

The worry



$t \to \infty$ can give time dependence

The worry



$$\langle \zeta(k)\, \zeta(k') \rangle$$

$t = \infty$

$$S \supset \lambda \int d^3x\, dt\, (aH)^3 e^{-\zeta} \frac{(\partial_i\zeta\partial_i\zeta)^2}{(aH)^4}$$

Question 1 : $\sim (aH)^n\,?$

$t \to \infty$ can give time dependence

Question 2 : $e^{\zeta} \to 1 + \zeta + \frac{\zeta^2}{2} + \frac{\zeta^3}{6} + \ldots$

Sure, derivatives tend to 0, but what about this object?

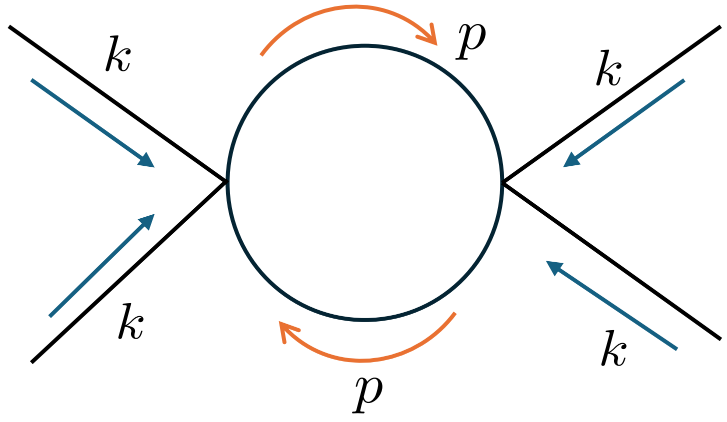
Regulators

- What makes these calculations more confusing is the lack of obviously nice regulators.

- Nice regulators make our lives a lot easier by only keeping track of as much physics as possible, while throwing away scheme dependent things as much as possible.

- Moreover, we want to choose regulators and schemes that make our life as simple as possible. However, in de Sitter, this choice is not obvious.

Power of Regulators



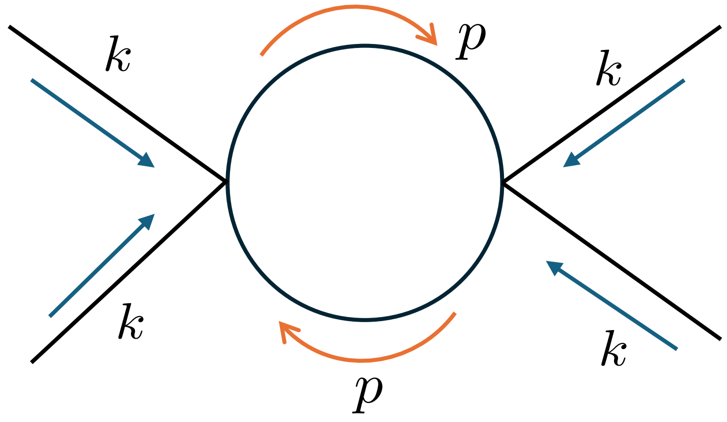
- Consider a simple one-loop diagram for a scalar in flat spacetime.

- Action ➡ $\mathcal{L}_{int} = \frac{g}{\Lambda^4}(\partial_\mu\phi\,\partial^\mu\phi)^2$, Λ some EFT scale

- If we use UV cutoff, loop result gives $$\frac{g^2k^4}{\Lambda^8}\int^{\Lambda_{\rm UV}}\frac{d^4p}{(2\pi)^4} \to g^2\frac{k^4}{\Lambda^4}\frac{\Lambda_{\rm UV}^4}{\Lambda^4}$$

- Makes decoupling of scales hard to see. Really, this term should be absorbed by a local counterterm, and gives no RG/logs.

Power of Regulators



- Consider a simple one-loop diagram for a scalar in flat spacetime.
- Action ⇒ $\mathcal{L}_{int} = \frac{g}{\Lambda^4}(\partial_\mu\phi\,\partial^\mu\phi)^2$, Λ some EFT scale
- Dim reg sets it to 0, making no RG obvious $\frac{g^2 k^4}{\Lambda^8}\int\frac{d^4p}{(2\pi)^4} \to 0$

Dim reg issues for in-in correlators

- In our in-in picture, dim reg isn't as great. In flat space, consider

$$\int \frac{d^3p}{(2\pi)^3} p^n e^{impt}$$

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- Now for $t = 0$, we get a Power Law divergence, which dim reg automatically sets to 0, so no logs

$$\int \frac{d^3p}{(2\pi)^3} p^n = 0$$

Dim reg issues for in-in correlators

- In our in-in picture, dim reg isn't as great. In flat space, consider

$$\int \frac{d^3p}{(2\pi)^3} p^n e^{impt}$$

- But, if we first integrate p, we get

$$\int \frac{d^3p}{(2\pi)^3} p^n e^{impt-\epsilon mp|t|} \to \frac{1}{2\pi^2} \frac{\Gamma[3+n]}{(-imt)^{3+n}}$$

- Now taking $t \to 0$ limit gives a divergence. The original divergence therefore was never really regulated.

Try to understand loops and IR divergences

- So, we want to develop techniques to make the physics in the IR more manifest.

- This goes hand-in-hand with understanding how to regulate loops in a more convenient way.

Game plan

- Two ways to accomplish these :
- Writing an EFT description for the long wavelength modes – that will make the IR behavior more manifest, as well as offer a convenient way of regulating loops in the IR.

- For the full theory, introduce the Mellin representation as a technique for calculating loops in a dim reg fashion. An added advantage that it is a natural scheme to match onto the EFT.

EFFECTIVE FIELD THEORY

Overview

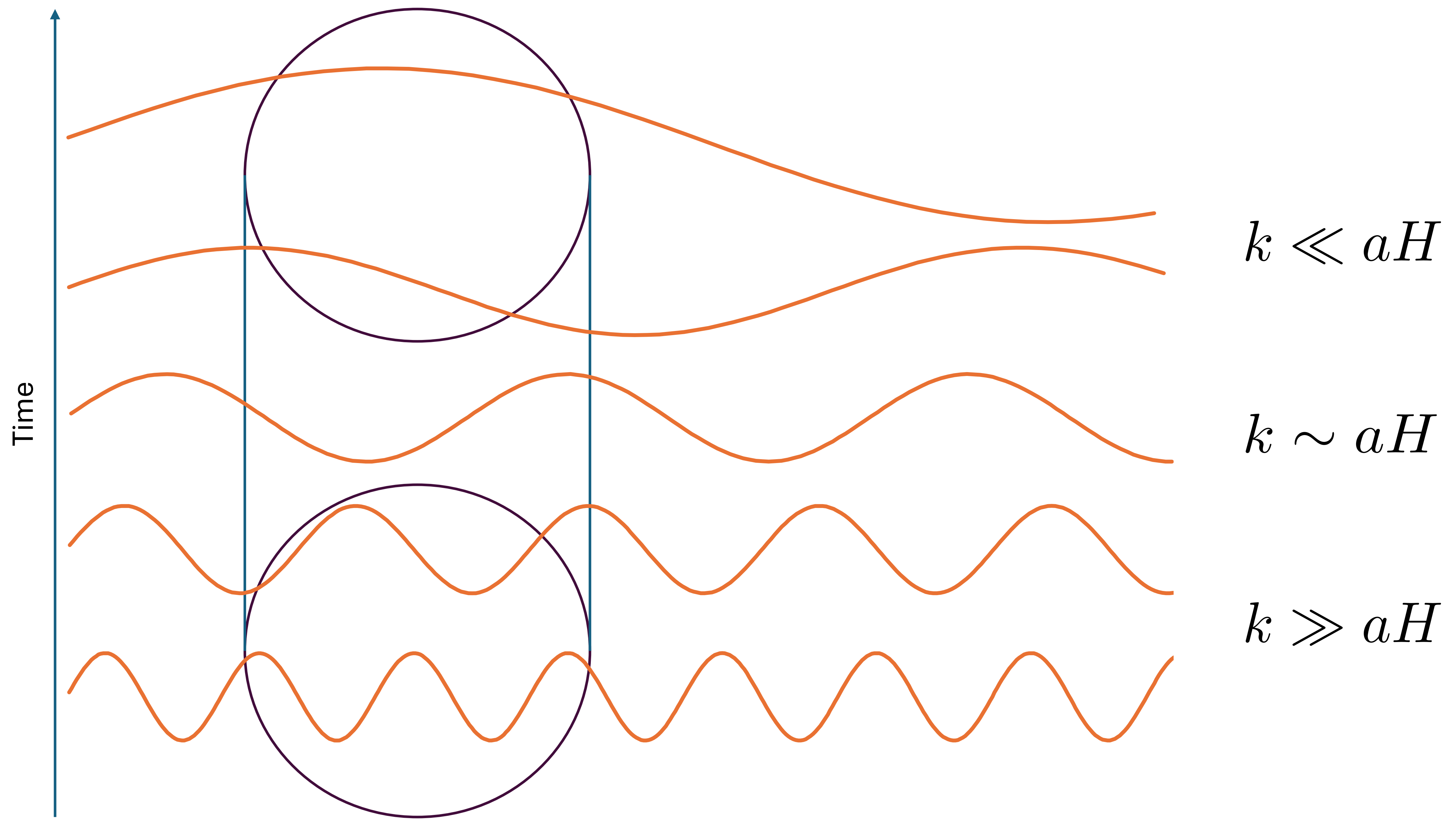
- We want to construct an effective theory for the metric modes ζ that reproduce the IR behavior (in particular divergences)
- In generic models we have that ζ propagates with a speed of sound $c_s \leq 1$.
- For the purposes of our discussion, we will set $c_s = 1$. It doesn't change any of our results, and makes it conceptually simpler to follow.

Technical motivation for constructing EFT

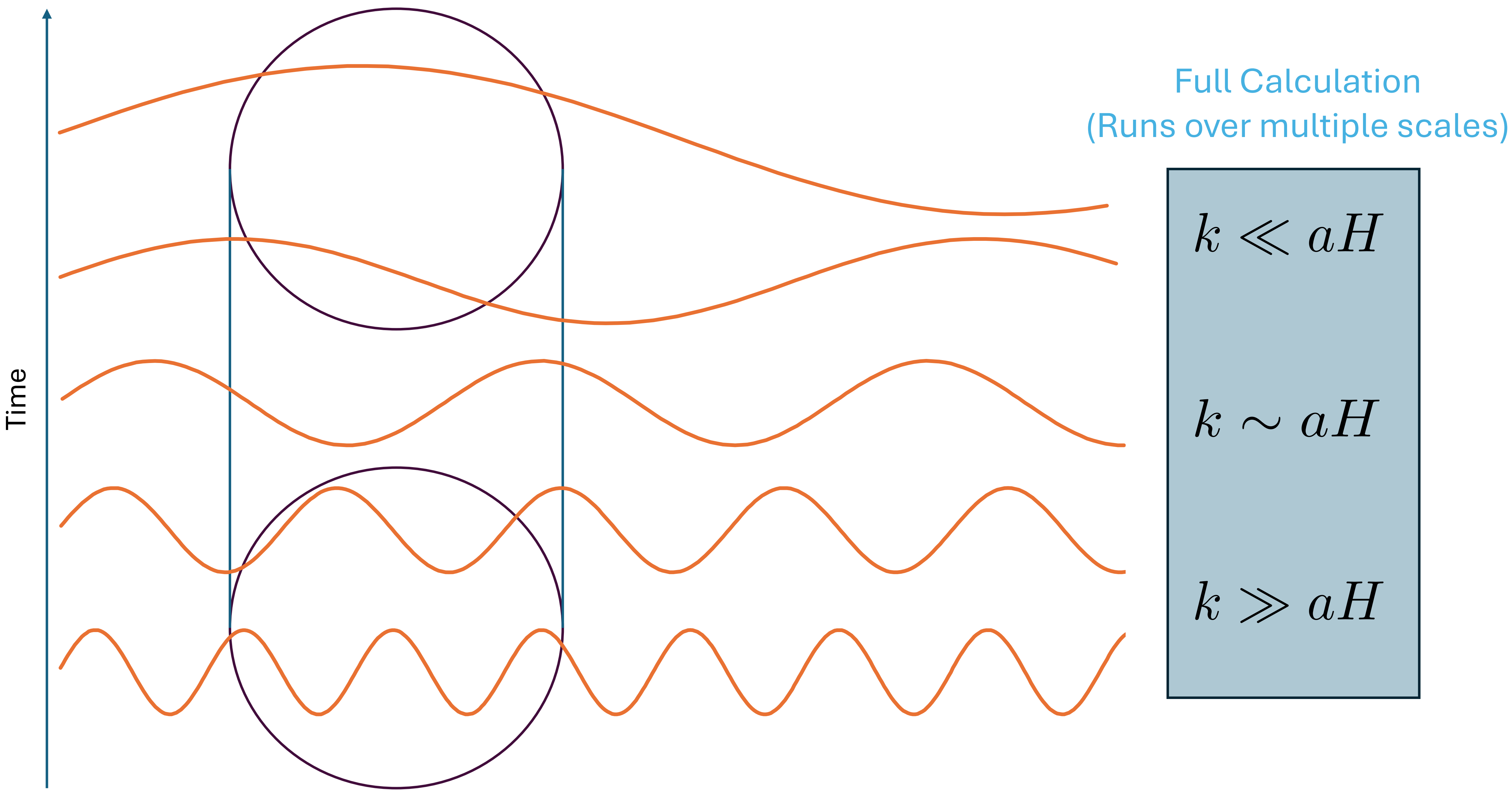
- The technical motivation starts with the idea that the IR logs we see are of the form $k/(aH)$.

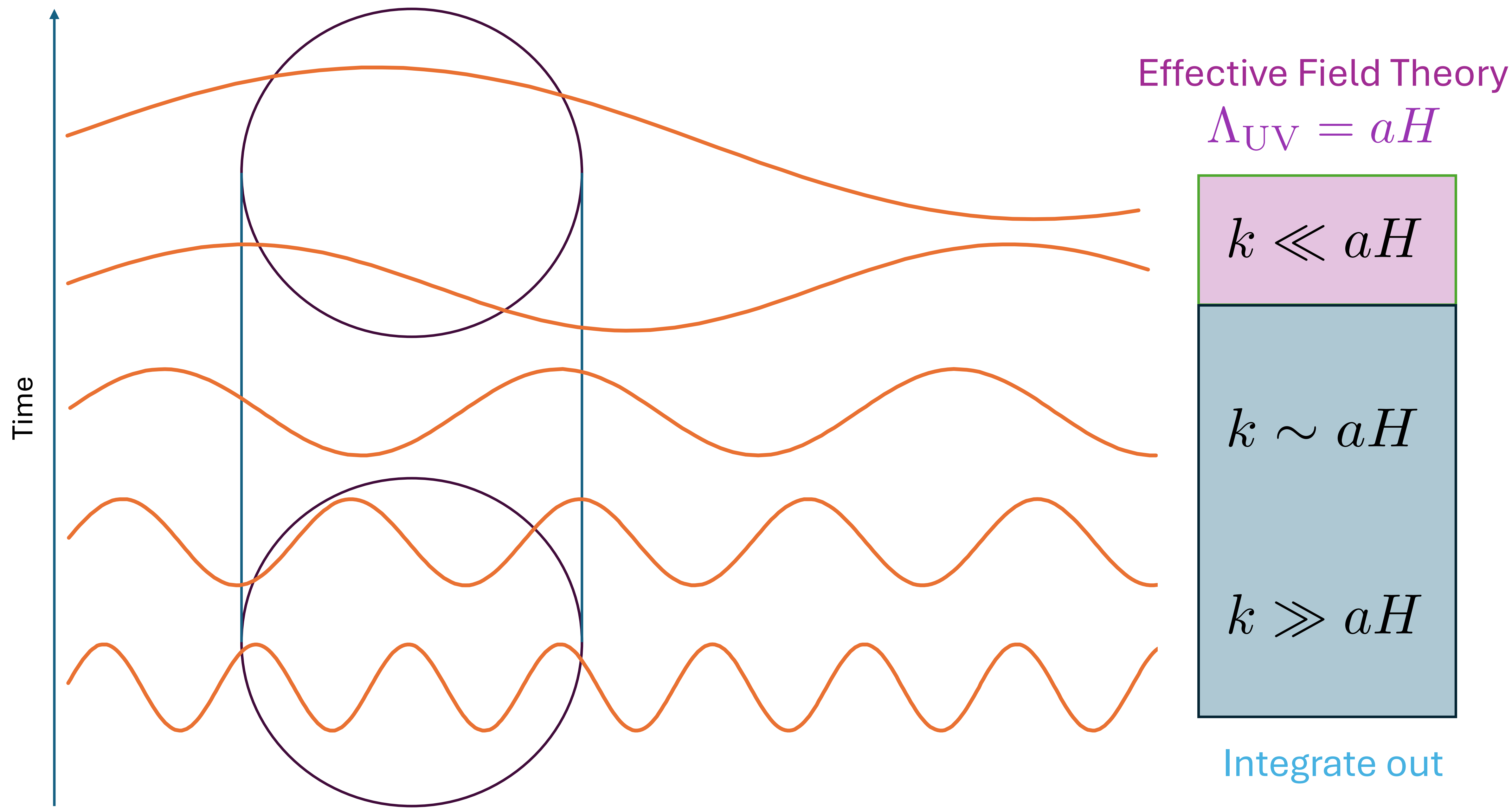
- We want to understand this as a statement similar to $\log p^2/\Lambda_{\rm UV}^2$ in flat spacetime.

- So, we want to construct an EFT with a (comoving) UV cutoff $\Lambda_{\rm UV} = a(t)H(t)$, thus capturing the effects of these logs.



Time
$k \ll aH$
$k \sim aH$
$k \gg aH$


Time
Full Calculation
(Runs over multiple scales)
$k \ll aH$
$k \sim aH$
$k \gg aH$


Time
Effective Field Theory
$\Lambda_{\text{UV}} = aH$
$k \ll aH$
$k \sim aH$
$k \gg aH$
Integrate out

What we want in our theory - 1

- We want to power count in $\lambda = \dfrac{k}{aH} = k\tau$, $\tau = (-aH)^{-1}$

- Accomplishes first goal - loop integrals are now nice scaleless power laws which can be regulated in a dim reg fashion

$$\int \frac{d^3k}{(2\pi)^3} k^n e^{ik\tau} \to \int \frac{d^3k}{(2\pi)^3} k^n \left(1 + ik\tau + \frac{(ik\tau)^2}{2} + \dots\right)$$

$$\to 0$$

What we want in our theory - 2

- We want to write the theory so that IR effects are obvious by looking at the action. Loop calculations only necessary to calculate factors of 2s and π s.

- The theory that does this is Soft de Sitter EFT (SdSET) Cohen and Green

SOFT de SITTER EFT

Identify degrees of freedom

- We start with the UV equations of motion

$$\partial_\tau^2\zeta - \frac{1}{\tau}(1+\epsilon)(2+\eta)\partial_\tau\zeta + k^2\zeta = 0$$

$$\tau = (-aH)^{-1}$$

ϵ, η slow roll parameters

- In the $k \to 0$ limit, we identify the soft degrees of freedom

$$\zeta \propto \tau^0 \qquad\qquad \zeta \propto \tau^{3+2\epsilon+\eta}$$

- We want to describe the fluctuations around these classical long wavelength solutions

Understand degrees of freedom from UV

- To see the structure of the EFT degrees of freedom, we take the solutions of the UV equations of motion

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- We take the superhorizon limit ⟹ Expand in $k\tau \ll 1$

$$\zeta = \left(1 + \frac{k^2\tau^2}{2} + \dots\right) - i\frac{k^3\tau^3}{3}(1 + \dots)$$

↑ ↑

Describe these two degrees of freedom

Understand degrees of freedom from UV

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$$\zeta = \left(1 + \frac{k^2\tau^2}{2} + \dots\right) - i\frac{k^3\tau^3}{3}(1 + \dots)$$

$$\zeta = \zeta_+ + \tau^3\zeta_-$$

Properties of ζ_+, ζ_-

- ζ_+, ζ_- act as scaling operators

$$x \to \lambda^{-1} x \qquad k \to \lambda k \qquad \zeta_+(\lambda k) \to \zeta_+(k) \qquad \zeta_-(\lambda k) \to \lambda^3 \zeta_-(k)$$

- ζ_+ scales as $[k]^0$, while ζ_- scales as $[k]^3$
- ζ_+ and ζ_- make explicit our intuition of power counting.
- To derive our EFT action, we can just expand in the soft limit. Just plug $\zeta = \zeta_+ + (aH)^{-3}\zeta_-$, and go.
- Correcting for slow roll factors, we substitute

$$\zeta = \zeta_+ + (aH)^{-\beta}\zeta_- \qquad\qquad \beta = 3 + 2\epsilon + \eta = 3 - (n_s - 1)$$

EFT free action

- Goal now is to write down the action in terms of ζ_+, ζ_-

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$$S = -6M_{pl}^2 \int d^3x\, dt\, (H_*)^{-2} \epsilon_* (k_*)^{-2\epsilon-\eta} \left[\dot{\zeta}_+ \zeta_- \right]$$

EFT Properties and Inputs

- We see that ζ_+ and ζ_- are conjugate momenta of each other.

$$[\zeta_+(x), \zeta_-(y)] = -\frac{1}{6M_{pl}^2(H_*)^{-2}\epsilon_*(k_*)^{-2\epsilon-\eta}}\delta^3(x-y)$$

- The EFT is supplemented by initial conditions, which describe the statistics of ζ_+, ζ_- at horizon crossing.

$$\langle\zeta_+\zeta_+\rangle_{\rm IC} = \frac{H_*^4}{4M_{pl}^2\dot{H}_*}\frac{1}{k^3}\left(\frac{k_*}{k}\right)^{2\epsilon+\eta} \qquad \langle\zeta_-\zeta_-\rangle_{\rm IC} = \frac{H_*^4}{4M_{pl}^2\dot{H}_*}\frac{k^3}{9}\left(\frac{k}{k_*}\right)^{2\epsilon+\eta}$$

Time dependence $\longleftrightarrow$ RG

- Following the time dependence of ζ after horizon crossing is equivalent to following the time dependence of ζ_+ in the EFT.
- Time dependence is equivalent to RG. This is because our UV cutoff is $\Lambda_{\rm UV} = aH$. Hence, our RG equations are of the form

$$\Lambda_{\rm UV} \frac{d}{d\Lambda_{\rm UV}} \mathcal{O} = \frac{d}{d \log \Lambda_{\rm UV}} \mathcal{O} = 0$$

- Using $\Lambda_{\rm UV} = aH = e^{Ht}$, we see that the RG equations are written in the form

$$\frac{\partial}{\partial t} \mathcal{O} = \gamma_{\mathcal{O}} \mathcal{O}$$

Time dependence $\longleftrightarrow$ RG

- So, our goal is reduced to something simple. Write down the EFT, based on symmetries and power counting.

- Check whether EFT produces any marginal or relevant terms.

- If not, we can make an all order statements, including loops, about the time dependence of ζ_+

Interaction terms

- A generic interaction term can be written as

$$S \supset \int d^3x\,dt\,\frac{c_n(t)}{n!m!}(\Lambda)^{3-m\beta}(\zeta_+)^n(\zeta_-)^m$$

Properties

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Scales as $[k]^\beta$

Scales as $[k]^0$

- So, the scaling dimension of this operator is $m\beta$
- Measure scales as $[d^3x] = [k]^{-3}$
- Units made up by $\Lambda = a(t)H(t)$
- $c_n(t)$ captures slow roll effects, provides additional scaling like $(aH)^\epsilon, (aH)^\eta$.

How to power count operators

- A generic interaction term can be written as

$$S \supset \int d^3x\, dt\, \frac{c_n(t)}{n!m!}(\Lambda)^{3-m\beta}(\zeta_+)^n(\zeta_-)^m$$

- Power counting means that the term contributes

$$\left(\frac{k}{aH}\right)^{m\beta-3} = \left(\frac{k}{aH}\right)^{3(m-1)-3(n_s-1)}$$

$m=0$ relevant

$m=1$ marginal

$m>1$ irrelevant

$$\beta = 3-(n_s-1) \approx 3$$

How to power count operators

- A generic interaction term can be written as

$$S \supset \int d^3x\, dt\, \frac{c_n(t)}{n!m!} (\Lambda)^{3-m\beta} (\zeta_+)^n (\zeta_-)^m$$

- Moreover, any spatial derivative term power counts as

$$\frac{1}{a}\partial_i \to \frac{k}{aH}$$

- For time derivatives, we can use EOM. We have that both

$$[\dot{\zeta}_+] = \left(\frac{k}{aH}\right)^2 \qquad [\dot{\zeta}_-] = \left(\frac{k}{aH}\right)^2$$

Leading order Interaction

- So, leading order interaction will be

$$S \supset \int d^3x \, dt \, \frac{c_n(t)}{n!} (\Lambda)^3 (\zeta_+)^n \quad ?$$

Leading order Interaction

- So, leading order interaction will be

$$S \supset \int d^3x\, dt\, \frac{c_n(t)}{n!} (\Lambda)^3 (\zeta_+)^n \quad ?$$

- Turns out not. This can be removed by an appropriate field redefinition of ζ_-

$$\zeta_- \to \zeta_- + \frac{n c_n}{6(H_*)^{-2} \epsilon_* (k_*)^{-2\epsilon-\kappa} n!} (\Lambda)^3 (\zeta_+)^{n-1}$$

Leading order Interaction

- The leading order term actually becomes

$$S \supset \int d^3x\, dt\, \frac{c_n(t)}{n!} (\Lambda)^{3-\beta} (\zeta_+)^n \zeta_-$$

- As we see, even before imposing symmetries, we only have marginal terms – only logarithmic time dependence possible in the IR.
- This is Weinberg's classic result that divergences in the IR can be at most logarithmic.
- We have solved the power counting problem - Now let us impose our symmetries!

Symmetries of ζ_+, ζ_-

- The important symmetry for our purpose is the dilatation

$$\zeta(x) \to \zeta(xe^{\lambda}) - \lambda$$

- In the soft limit this breaks up as

$$\zeta_+(x) + (aH)^{-\beta}\zeta_-(x) \to (\zeta_+(xe^{\lambda}) - \lambda) + (aH)^{-\beta}\zeta_-(x)$$

- Thus, we have

$$\zeta_+(x) \to \zeta_+(xe^{\lambda}) - \lambda \qquad \zeta_-(x) \to \zeta_-(xe^{\lambda})$$

Symmetries of ζ_+, ζ_-

- Then, our action does not have any marginal term $\zeta_+^n \zeta_-$
- Rather, all terms should come with either spatial or temporal derivatives.
- Since we need 2 spatial derivatives, both of these give $\mathcal{O}\left(\frac{k}{aH}\right)^2$ suppression.

Action has only irrelevant terms?

- So, in the IR, all terms are suppressed by $\mathcal{O}(\lambda^2)$.

- The argument is not complete. Our action is gauge fixed. Thus, we can have non-local terms that are also allowed.

- We can put in terms of the form $a^2\partial^{-2}$, which seem to boost relevance of terms. For example, in the UV, we have the three point interaction :

$$\mathcal{L} \supset a^3(t)\frac{\epsilon^2}{c_s^4}\dot{\zeta}\left(\frac{\partial_i}{\partial^2}\dot{\zeta}\right)\partial^i\zeta$$

Maldacena

Non-local terms ARE suppressed

- The key idea is that dilatation implies that in the $q \to 0$ limit, we describe an unperturbed background.

Non-local terms ARE suppressed

- The key idea is that dilatation implies that in the $q \to 0$ limit, we describe an unperturbed background.

- So a term of the form $\mathcal{L} \supset a^{3+100}(t)\,\epsilon\,\zeta\,\partial^{-100}(\partial_i\zeta\partial_i\zeta)$ naively would not die away in the soft limit. Rather, this behaves naively as $q^2 \times q^{-100} = q^{-98}$ and hence grows in the $q \to 0$ limit.

Non-local terms ARE suppressed

- Thus, we need more factors of momenta in numerator of any operator. Thus, in soft limit the operators are guaranteed to be of order

$$\left(\frac{k}{aH}\right)^{n; n>0}$$

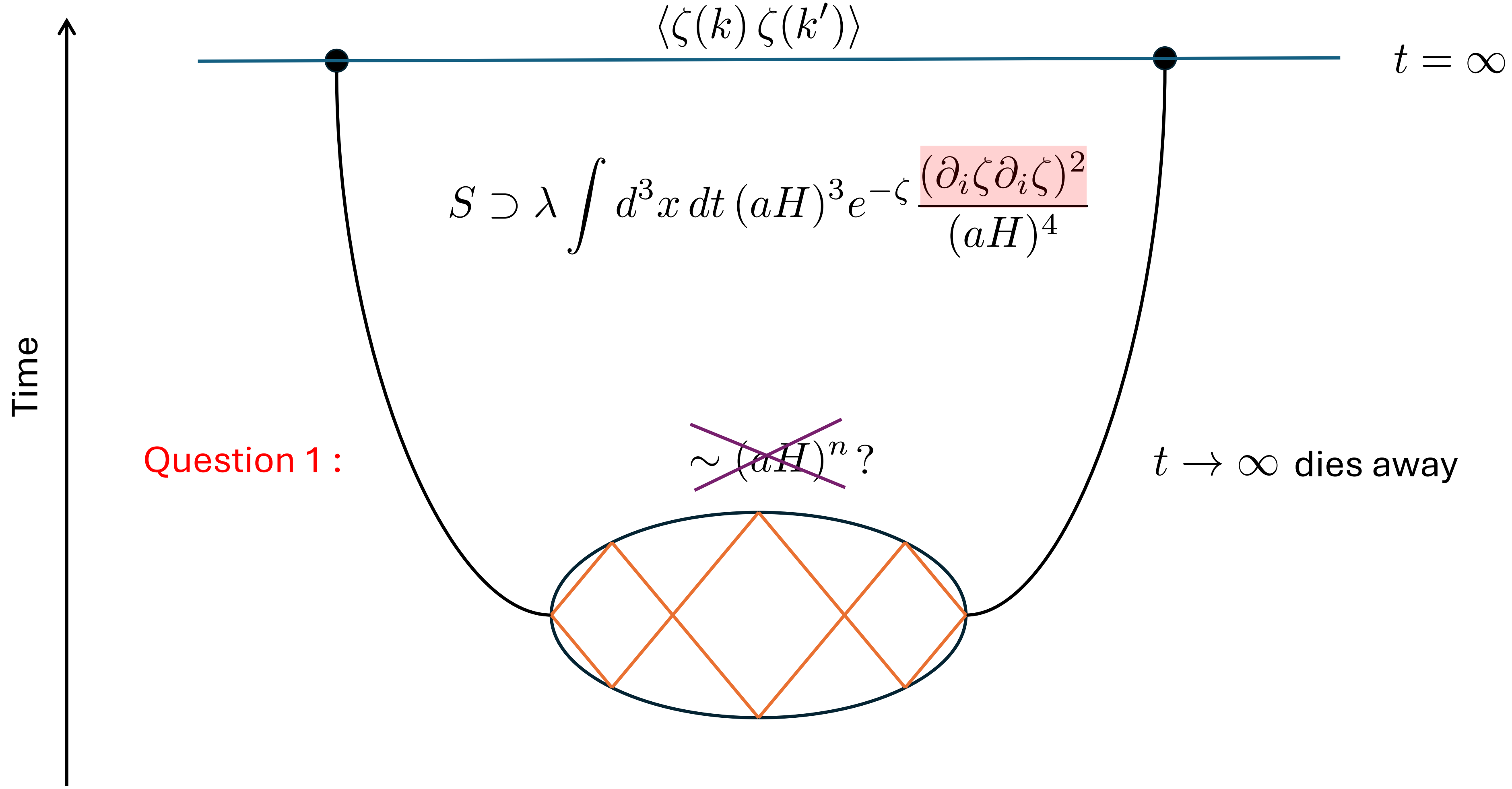
- A more formal argument can be given by constructing the charge operator, but the physics is the same.

Action has only irrelevant terms

- With this, we have shown that any interaction term in the theory is irrelevant. Hence, ζ_+ does not generate time dependence in the theory.

- So, is the IR theory completely trivial, with no interesting behavior? What does the effective theory predict?

The worry



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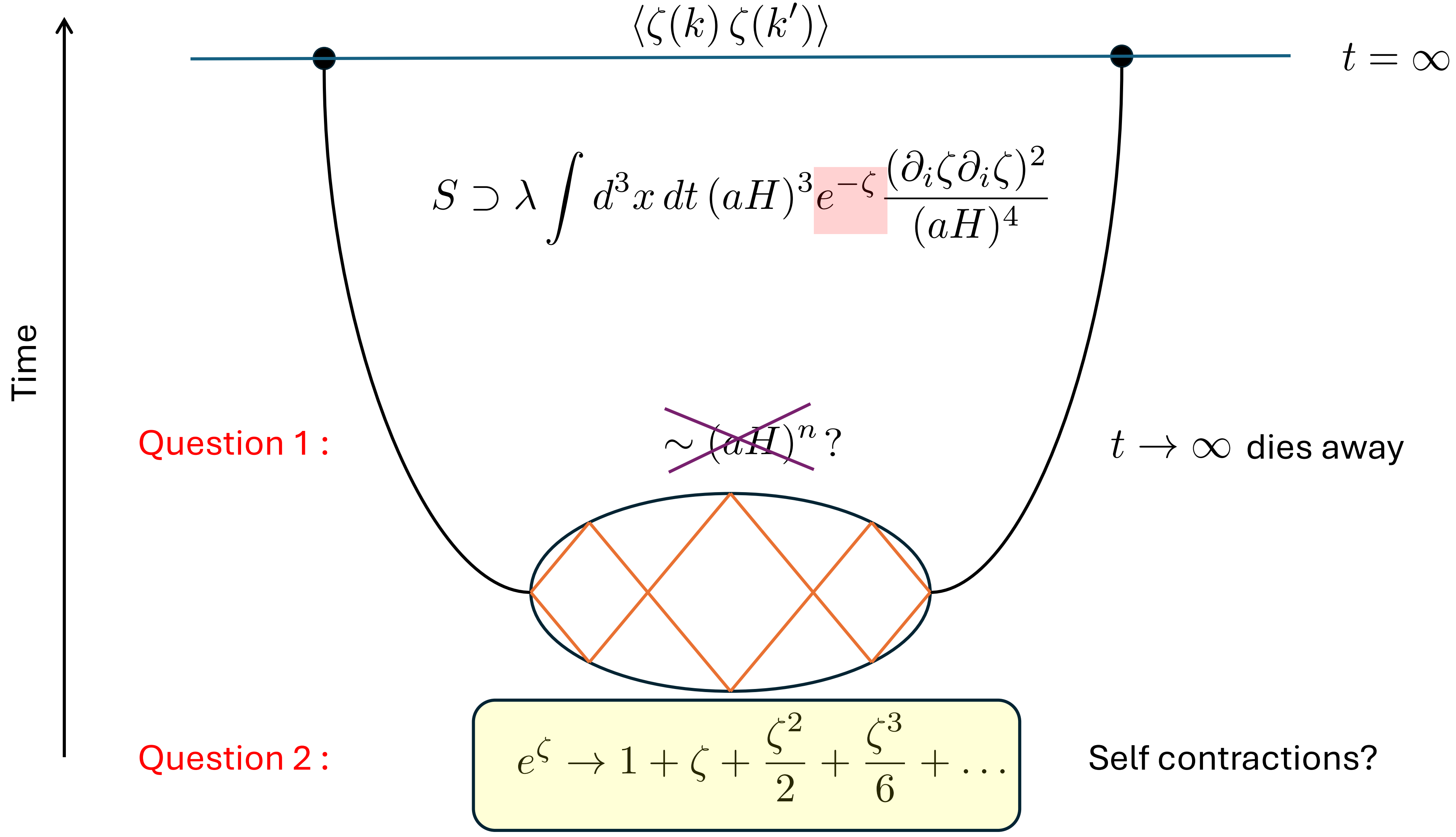
Question 1 :
$\sim (aH)^n$?
$t \to \infty$ dies away

The worry



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Question 2 :
$e^{\zeta} \to 1 + \zeta + \frac{\zeta^2}{2} + \frac{\zeta^3}{6} + \ldots$
Self contractions?

Time dependence in composite operators

- Actually, composite operators do generate time dependence. Operator mixing is $\zeta_+^n \to \mathbb{1}$.

- ζ_+ has scaling dimension 0. ζ_- has scaling dimension 3. So only powers of ζ_+ can mix.

- This tells us that $e^{\zeta} \to e^{\zeta_+}$ can generate time dependence.

e^{ζ} in the IR

- We have that $e^{\zeta_+} = 1 + \zeta_+ + \frac{\zeta_+^2}{2} + \frac{\zeta_+^3}{6} + \ldots$
- Now, we can consider various self contractions, and consider divergences due to all of them.
- What we get is a nice result – The various Wick contractions of e^{ζ_+} actually give a renormalized e^{ζ_+}, that is, e^{ζ_+} is an eigenvector under RG, and acquires an anomalous scaling dimension :

$$e^{\zeta_+} = Z(t_*)e^{\zeta_+} \qquad\qquad \frac{d}{dt^*}e^{\zeta_+} = e^{\zeta_+}\sum_{n\geq 2}\frac{\gamma_n}{n!}$$

where t_* is some fixed reference time, equivalent to μ in MS bar.

Loops build up probability distribution

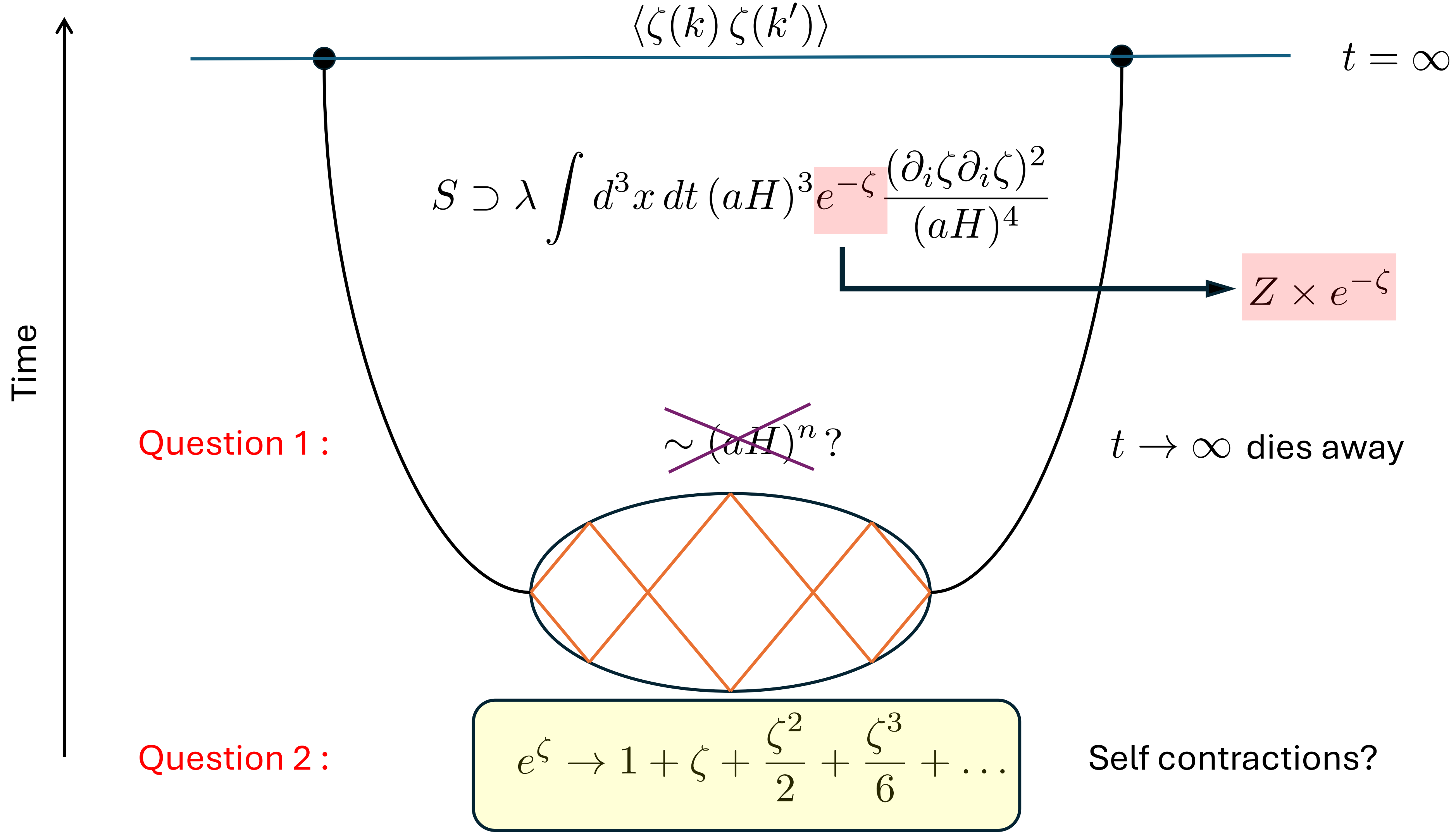
$$\frac{d}{dt^*}e^{\zeta_+} = e^{\zeta_+}\sum_{n\geq 2}\frac{\gamma_n}{n!}$$

- The coefficients γ_n are the cumulants of the probability distribution of ζ at any point x.

- $\gamma_2 = \Delta_\zeta = \dfrac{H(t_*)^4}{4\pi^2 M_{pl}^2 \dot{H}(t_*)}$ encodes the standard deviation.

- The higher γ_n encode the higher moments of the probability distribution.

The worry



$\langle \zeta(k)\,\zeta(k')\rangle$
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Question 1 :
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$e^{\zeta} \to 1 + \zeta + \frac{\zeta^2}{2} + \frac{\zeta^3}{6} + \dots$
Self contractions?

ζ retains symmetries to all orders

- The first nice thing this tells us that after taking loops into account e^{ζ_+} renormalizes into itself. Hence, ζ_+ and thus ζ still retains all its non-linear symmetries in the IR.

- This is another way to see quantum loops cannot change the time independence of ζ .

- Moreover, the time dependence of e^{ζ_+} encodes the time dependence of the volume of the Universe at the end of inflation.

Volume fluctuations in the IR

- In our gauge, we have chosen inflation to end at the same time everywhere. The inflaton is given by $\phi = \phi(t)$ with no perturbations.
- We have $g_{ij} = a^2(t) e^{2\zeta} \delta_{ij}$
- Thus, time dependence of e^{ζ} is just telling us that $a(t)$, and hence the volume of the Universe, gets an additional statistical time dependence due to random walk of ζ.
- This fact encodes that the quantum nature of the perturbations give rise to statistical nature of the volume of reheating surface.

EFT accomplishments, and matching onto UV

- So, we have used EFT techniques to understand the effect of loops in the IR and understood how to regulate loops in the soft limit.

- To understand the effects of loops in the UV, as well as use a convenient regulator to match the EFT to the IR, we will use a convenient Mellin representation.

- Mellin has the advantage of doing the full theory loops in manifestly dim reg way.

MELLIN REPRESENTATION

Introduction

- The Mellin representation allows us to write down Hankel functions as power series in the argument :

$$i\pi e^{i\pi\nu/2} H_\nu^{(1)}(z) = \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \Gamma\left(s+\frac{\nu}{2}\right)\Gamma\left(s-\frac{\nu}{2}\right)\left(-\frac{iz}{2}\right)^{-2s}$$

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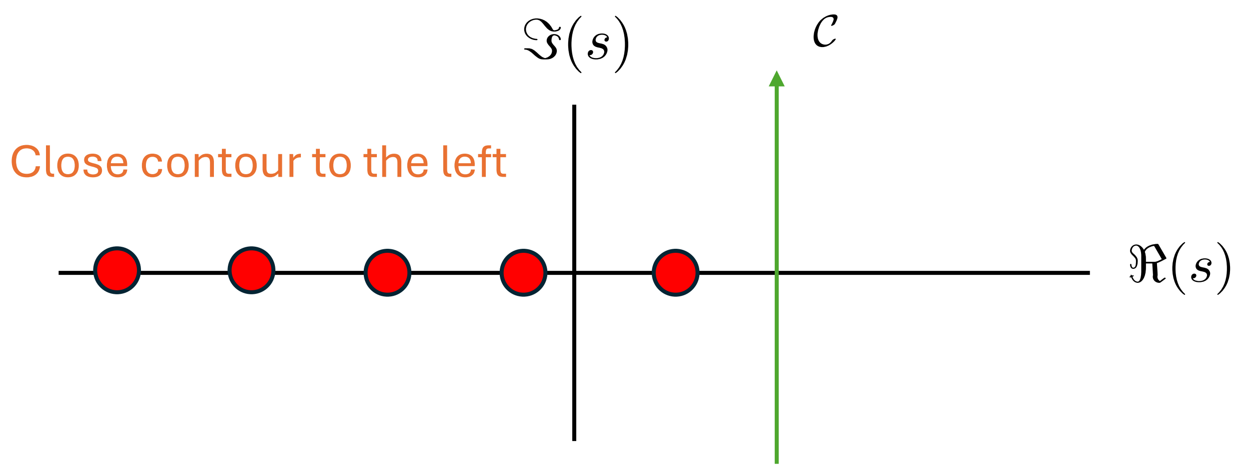
- The integral is essential a sum over all the poles of the Γ function :

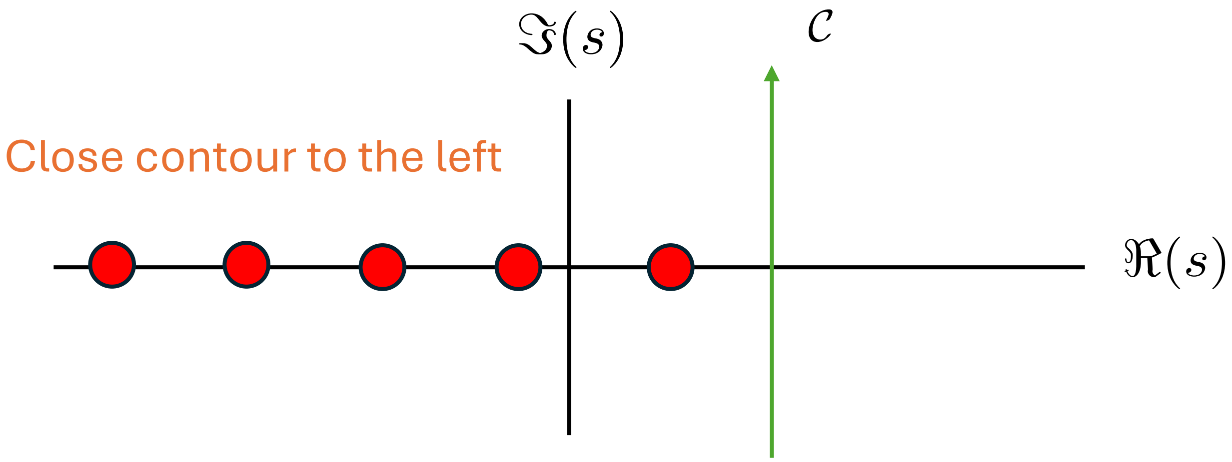


Introduction

- We turned the Hankel function into a sum of powers! This will accomplish goal of doing integrals of power laws.

$$i\pi e^{i\pi\nu/2} H_\nu^{(1)}(z) = \sum \mathrm{Res}\left[\Gamma\left(s+\frac{\nu}{2}\right)\Gamma\left(s-\frac{\nu}{2}\right)\left(-\frac{iz}{2}\right)^{-2s}\right]$$

- We always close contours to the left!



Connection to inflation

- Useful because $\zeta(k\tau) \propto \tau^{\nu} \mathrm{H}_{\nu}^{(1)}(-k\tau)$, $\nu = \frac{3}{2} + \frac{(n_s - 1)}{2}$. Mellin gives

$$\zeta(k\tau) = -\frac{1}{2\sqrt{\pi}}(-\tau)^{\frac{3+n_s-1}{2}} \int \frac{ds}{2\pi i}\Gamma\left(s + \frac{3+(n_s-1)}{4}\right)\Gamma\left(s - \frac{3+(n_s-1)}{4}\right)\left(-\frac{ik\tau}{2}\right)^{-2s}$$

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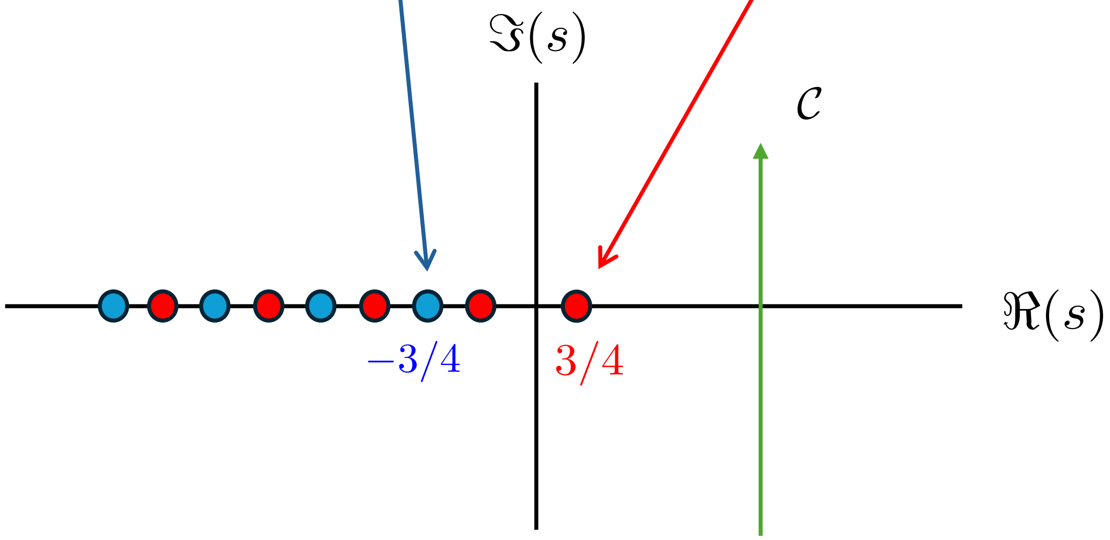
- The Γ function pole structure becomes



Connection to inflation

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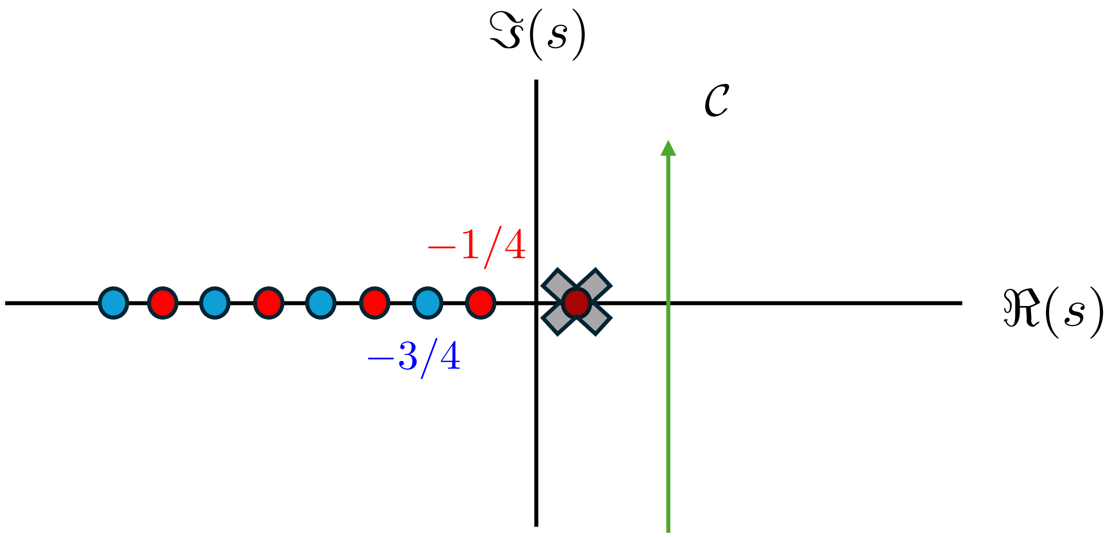

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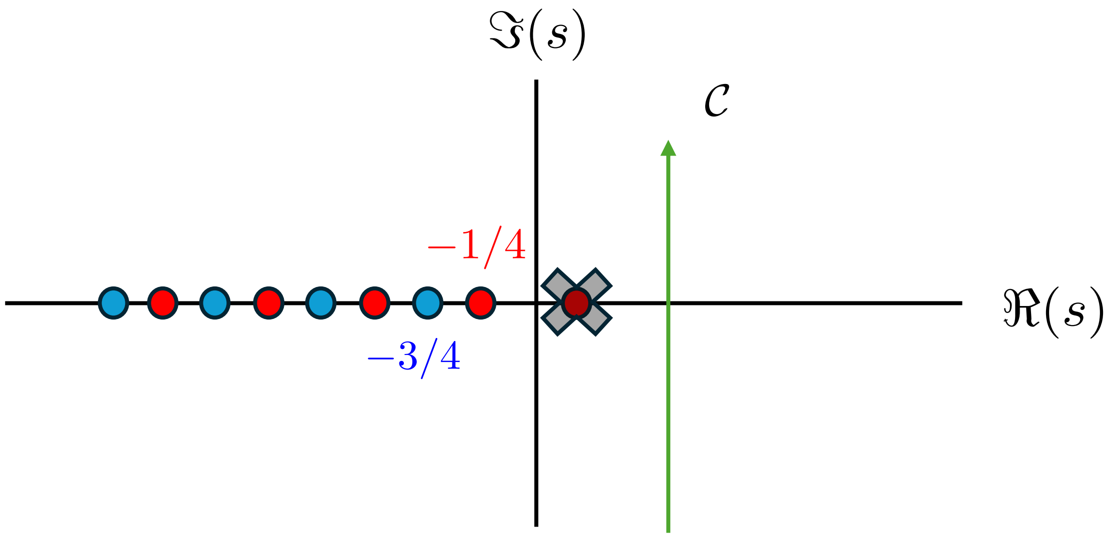
In the $\tau \to 0$ limit, we have that $\dot{\zeta} \to 0$



Divergences in Mellin

- In generic loop momenta, we have a bunch of ζ floating around. Each ζ comes with its own poles.

- Now, when we do loops, we enforce constraints on the poles. This comes from the momenta integrals, which is of the form

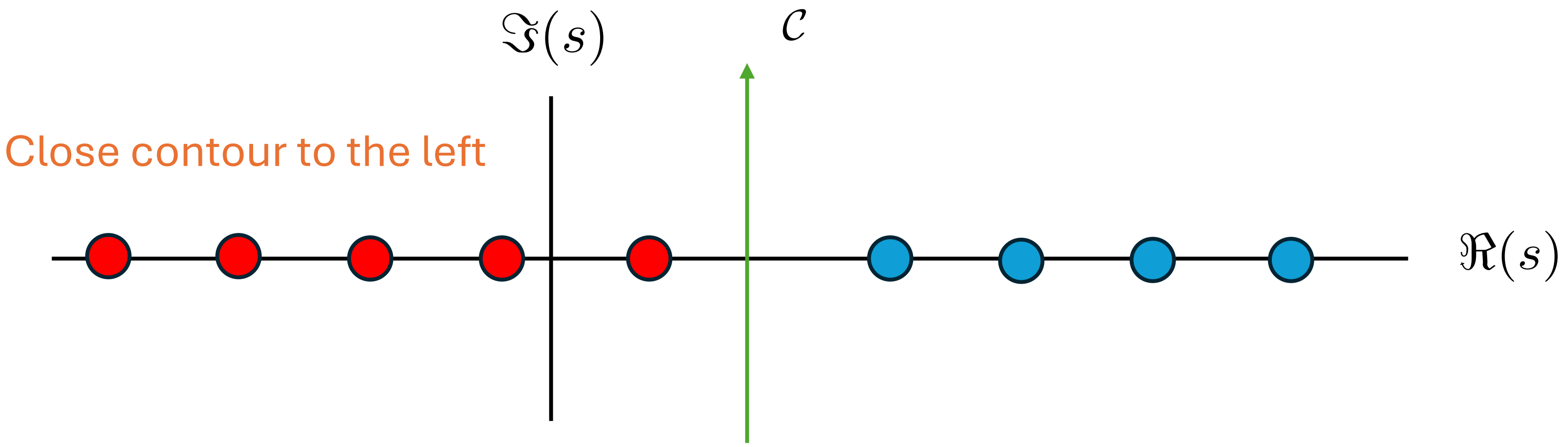
$$\int \frac{d^3p}{(2\pi)^3} p^{-2(s_3+s_4+s_5+s_6)} \to -\frac{i}{2\pi}\delta\left(\frac{3}{2} - (s_3+s_4+s_5+s_6)\right)$$

“Only $\int \frac{d^3p}{(2\pi)^3}\frac{1}{p^3}$ should survive”

Divergences in Mellin

- This shifts some of the poles to the right!

- So generically, we get left poles and right poles. Right poles not an issue, because our prescription doesn't pick them up.



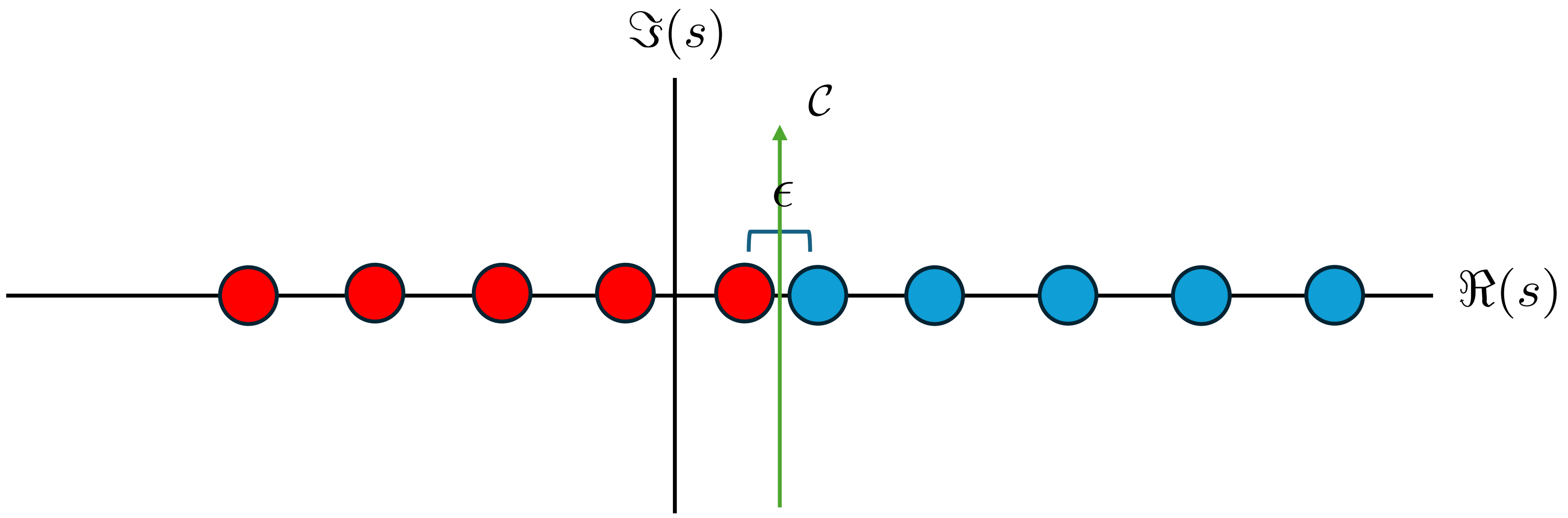
Divergences in Mellin

- Unless … These right poles overlap with the left poles.

Divergences in Mellin

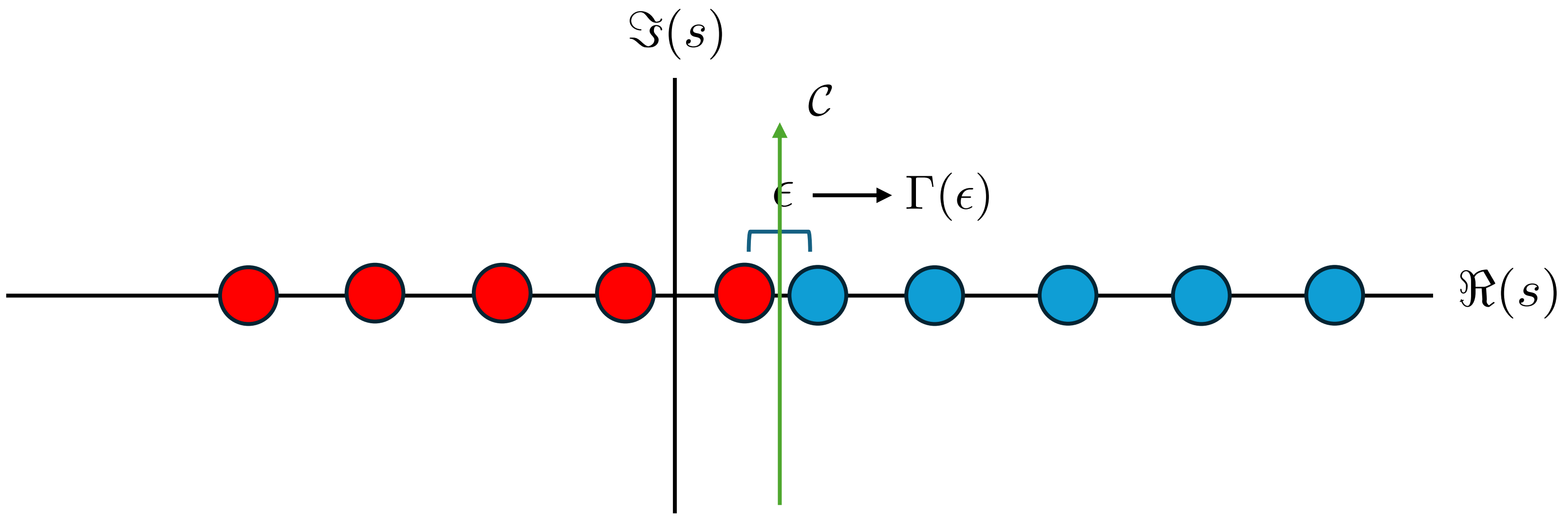
- Unless … These right poles overlap with the left poles.

- When we have colliding poles, divergences start showing up, because we cannot separate the left and the right poles.



Divergences in Mellin

- Since these poles are due to Γ functions, Cauchy's theorem gives us that evaluating residue of one pole gives us a $\Gamma(\epsilon)$ pole.

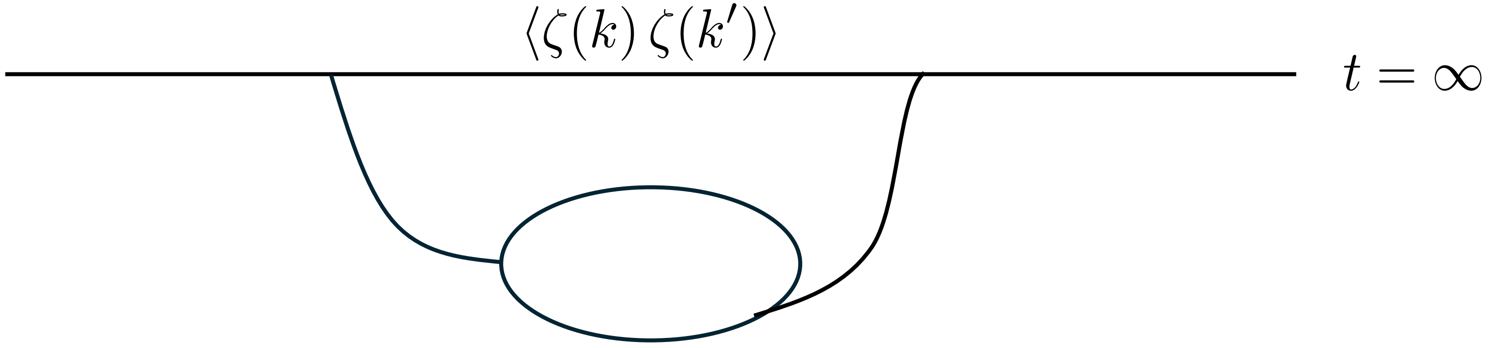


Calculating loops in Mellin

- So in Mellin, divergences becomes a statement of colliding poles.

- Now, we take a one loop correction to the power spectrum. Let us take the sample interaction term

$$H_{int} = -\lambda a^3 H^{-3} \dot{\zeta}^3$$



Calculating loops in Mellin

- Then we have that loop corrections to momenta is given by

$$\langle\zeta(k)\zeta(k')\rangle = -\lambda^2 \int_{-\infty}^{\tau} d\tau_1 a^4(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^4(\tau_2) \int \frac{d^3k}{(2\pi)^3} \times$$

$$\bar{\zeta}(k,\tau)\bar{\zeta}(k',\tau)\dot{\bar{\zeta}}^*(k,\tau_1)\dot{\bar{\zeta}}^*(k',\tau_2)\dot{\bar{\zeta}}(p,\tau_1)\dot{\bar{\zeta}}^*(p,\tau_2)\dot{\bar{\zeta}}(p-k,\tau_1)\dot{\bar{\zeta}}^*(k-p,\tau_2)$$

$$+\, 3 \text{ other terms}$$

- We can see that in Mellin, we will have power law integrals over both p, τ

$$\zeta = -\frac{1}{2\sqrt{\pi}}(-\tau)^{\frac{3+n_s-1}{2}} \int \frac{ds}{2\pi i}\Gamma\left(s + \frac{3+(n_s-1)}{4}\right)\Gamma\left(s - \frac{3+(n_s-1)}{4}\right)\left(-\frac{ik\tau}{2}\right)^{-2s}$$

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- Substituting, we get

$$\langle\zeta(k)\zeta(k')\rangle = -\lambda^2\zeta(k,\tau)\zeta(k',\tau) \int_{-\infty}^{\tau} d\tau_1\, a^4(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2\, a^4(\tau_2) \int \frac{d^3p}{(2\pi)^3}$$

$$\left(\prod_{i=1}^{6} \int \frac{ds_i}{(2\pi i)} \Gamma\left(s_i + \frac{3+(n_s-1)}{4}\right) \Gamma\left(s_i + \frac{1+(n_s-1)}{4}\right) \left(-\frac{ip_i\tau_i}{2}\right)^{-2s_i}\right)$$

$$\times \frac{1}{\pi^3} (-\tau_1)^{\frac{9+3(n_s-1)}{2}} (-\tau_2)^{\frac{9+3(n_s-1)}{2}} (-1)^{-2s_1-2s_2-2s_4-2s_6}$$

Momenta Loop integral

- Calculating the loop integrals is now easy, since we get power laws. The internal momenta integral is of the form

$$\int \frac{d^3p}{(2\pi)^3} p^{-2(s_3+s_4)} (k-p)^{-2(s_5+s_6)}$$

- Taking internal momenta to be large, $p \gg k$,

$$\int \frac{d^3p}{(2\pi)^3} p^{-2(s_3+s_4+s_5+s_6)} \to -\frac{i}{2\pi} \delta \left(\frac{3}{2} - (s_3+s_4+s_5+s_6) \right)$$

- We check whether poles collide or not. In our case, it turns out to not.

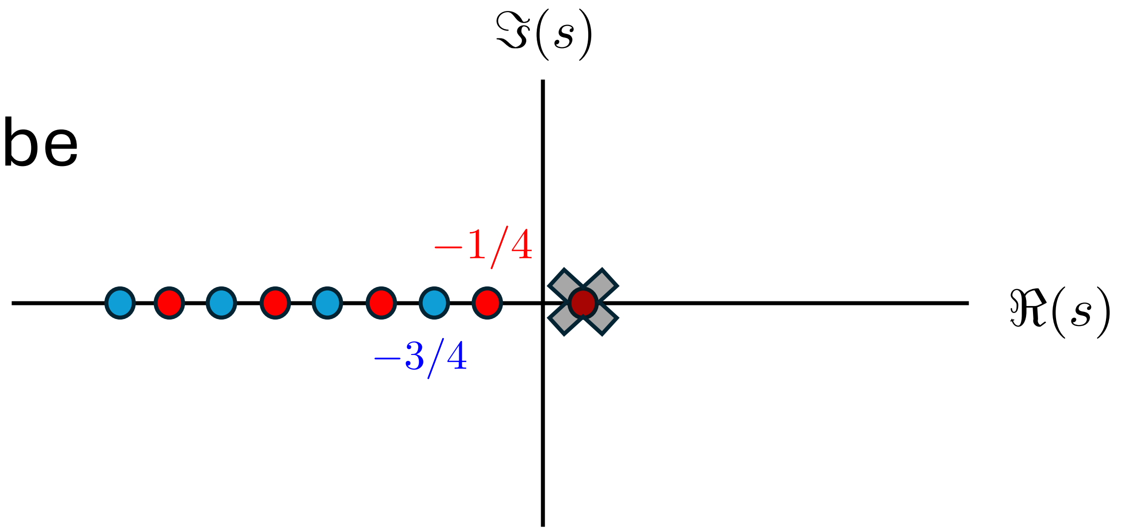
Time integral

- Looking at the time integrals, we see that we get

$$\langle \zeta(k,\tau)\zeta(k',\tau)\rangle \propto \frac{(k\tau)^{-2(s_1+s_2)}}{s_1+s_2}$$

- Now, remember that every pole for $\dot{\zeta}$ was on the negative real axis.

- So the τ dependence is supposed to be a positive power law, and hence dies in the $\tau \to 0$ limit.



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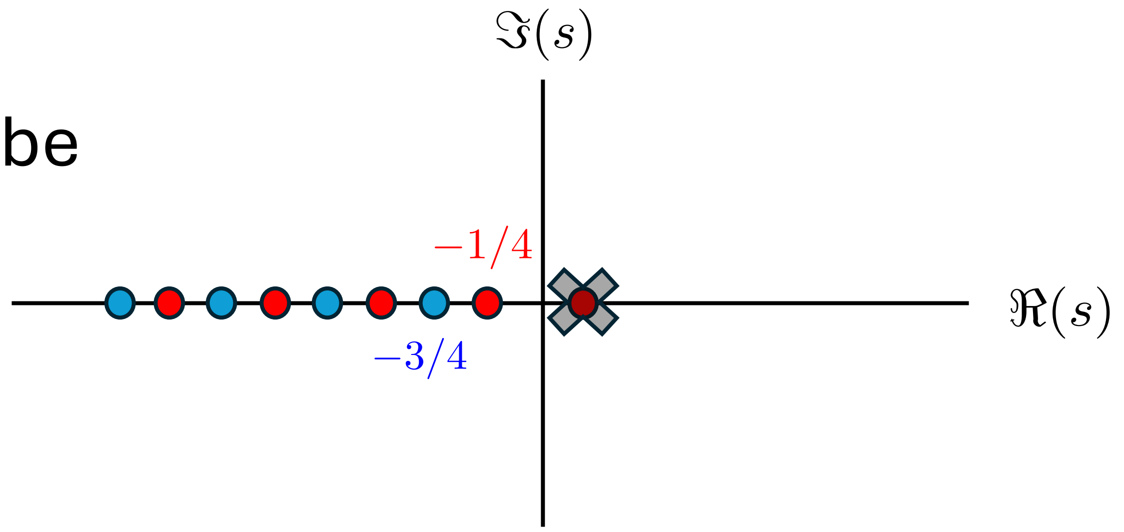
- So, Mellin sets this integral to 0!



We do not get divergences

- Thus, what we learn is that this term does not contribute to RG, and hence cannot give time dependence in the IR.

- This is what we learnt from the EFT method as well.

- Moreover, Mellin sets terms to 0 in a similar way that dim reg sets terms to 0 in flat spacetime.

Conclusion

- We use EFT to show that using symmetries and power counting, we can show that ζ is time independent to all orders.
- We study the IR effects of loops and show that the natural quantity that does get modified through loops is the spacetime volume.
- We introduce the Mellin representation to regulate loops in the UV. We use Mellin to show ζ time independence, and elucidate how it reduces the problem to calculating scaleless integrals.

THANK YOU FOR YOUR TIME!! ☺