Perturbative Unitarity Bounds from Entanglement

Carlos Duaso Pueyo

Based on 2411.XXXXX, with
Harry Goodhew, Ciaran McCulloch & Enrico Pajer

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SUPERIORE

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Outline

Introducing the problem…

- **Perturbative unitarity bounds**

Proposing a solution…

- **Entanglement in QFT**
- **Computing the purity**

Reporting on the results…

- **Bounds in flat space**
- **Bounds in de Sitter space**

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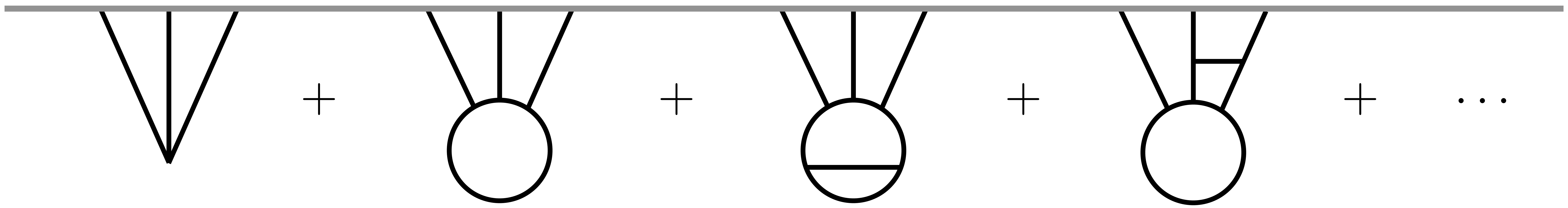
$$\langle \phi\phi\phi \rangle =$$



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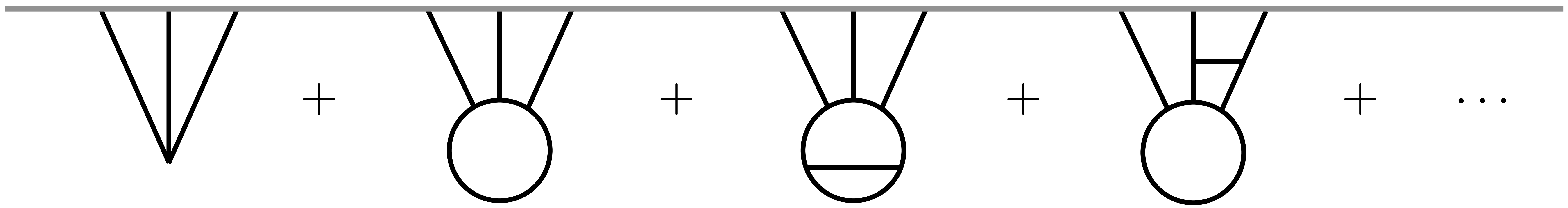

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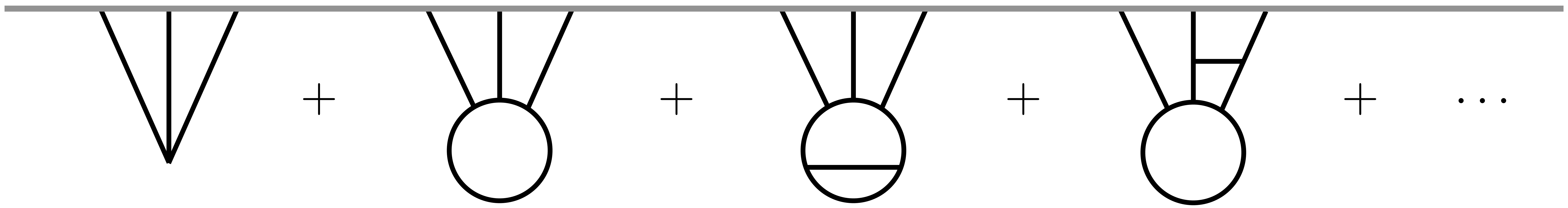

How can we diagnose the breakdown of the perturbative expansion?

Three ideas:

- Calculate next order
- Power counting
- Unitarity

Expanding the amplitude in partial waves,

$$\mathcal{A}_{2\to 2} = 16\pi \sum_{l=0}^{\infty} (2l+1)\, a_l \, P_l(\cos\theta)$$

Legendre polynomial

Partial wave coefficient

Unitarity requires:

$$|\mathrm{Re}\, a_l| \leq \frac{1}{2} \quad \forall \ l$$

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Partial wave unitarity bounds

Applied to WW scattering without the Higgs:

$$a_0 \sim \frac{s}{2400\,\mathrm{GeV}} \leq \frac{1}{2} \qquad \Rightarrow \qquad s \lesssim 1200\,\mathrm{GeV}$$

Lee, Quigg, Thacker '77

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Some proposals to use S-matrix unitarity bounds:

- Take the flat space limit of a dS theory

Baumann, Green, Lee, Porto ’15
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We need bounds that can be defined in any spacetime!

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It can be quantified by doing a bipartition between different sets of modes:

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e.g.

System: $\mathcal{H}_s = \mathcal{H}_{\vec{p}}$ Environment: $\mathcal{H}_\varepsilon = \bigotimes_{\vec{k}\neq\vec{p}} \mathcal{H}_{\vec{k}}$

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Then, the reduced density matrix is

$$\rho_s \equiv \mathrm{Tr}_\varepsilon\, \rho$$

Different quantities measure the entanglement between system and environment:

Entanglement entropy: $$S_E \equiv -\mathrm{Tr}_s(\rho_s \log \rho_s) \begin{cases} = 0 & \text{(no entanglement)} \\ > 0 & \text{(entanglement)} \end{cases}$$

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Some previous work on entanglement entropy in momentum space:

Balasubramanian, McDermott, Van Raamsdonk ’11
Nishioka ’18
Costa, van den Brink, Nogueira, Krein ‘22

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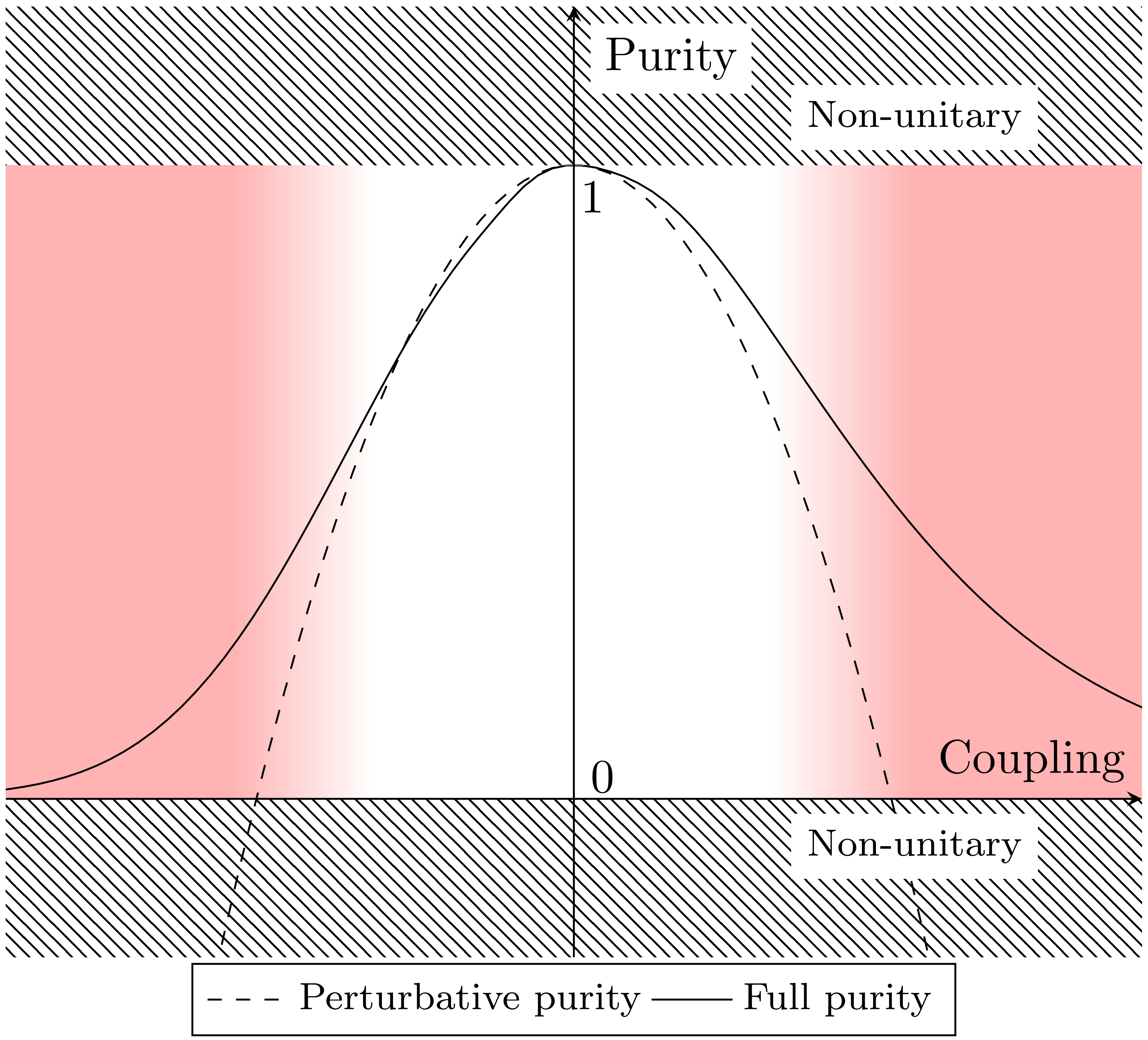
What about using the purity lower bound to diagnose the breakdown of perturbation theory?

$$\gamma(g) = 1 - \frac{g^2}{2} \left| \frac{\partial^2 \gamma}{\partial g^2} \right| + \mathcal{O}(g^3)$$



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We parameterise the vacuum wavefunction of a field theory as

$$\Psi[\phi; t_0] = \langle \phi; t_0 | \Omega \rangle = \exp\left[\sum_{n=2}^{\infty} \frac{1}{n!} \int_{\vec{k}_a} \psi_n(\vec{k}_a; t_0)\, \phi_{\vec{k}_1} \cdots \phi_{\vec{k}_n}\right]$$

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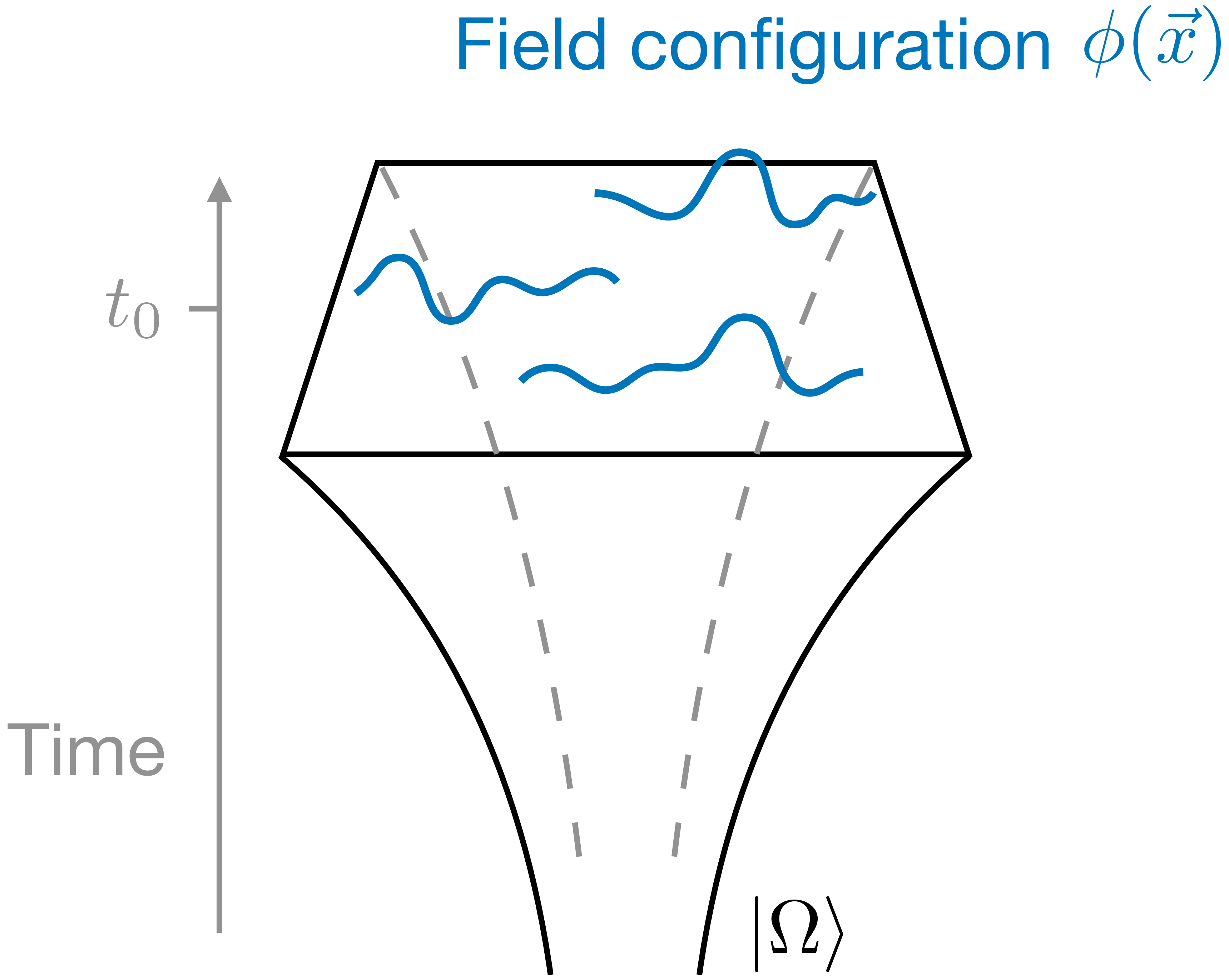
It gives the probability amplitude of finding some spatial field configuration at time t_0:



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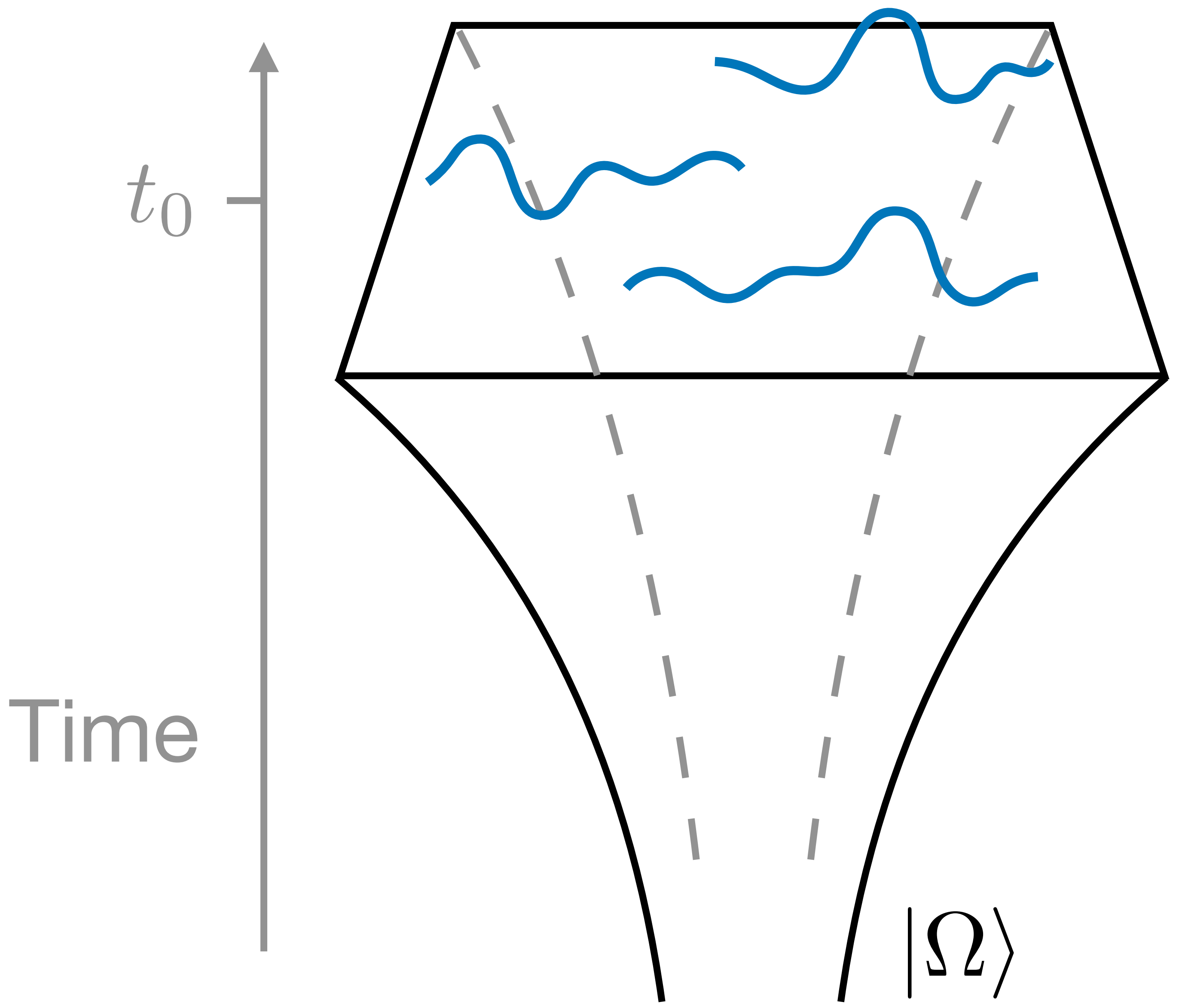
Field configuration $\phi(\vec{x})$



Correlation functions are then:

$$\langle \phi_{\vec{k}_1} \cdots \phi_{\vec{k}_n} \rangle(t_0) = \int \mathcal{D}\phi\, \phi_{\vec{k}_1} \cdots \phi_{\vec{k}_n} \, |\Psi[\phi; t_0]|^2$$

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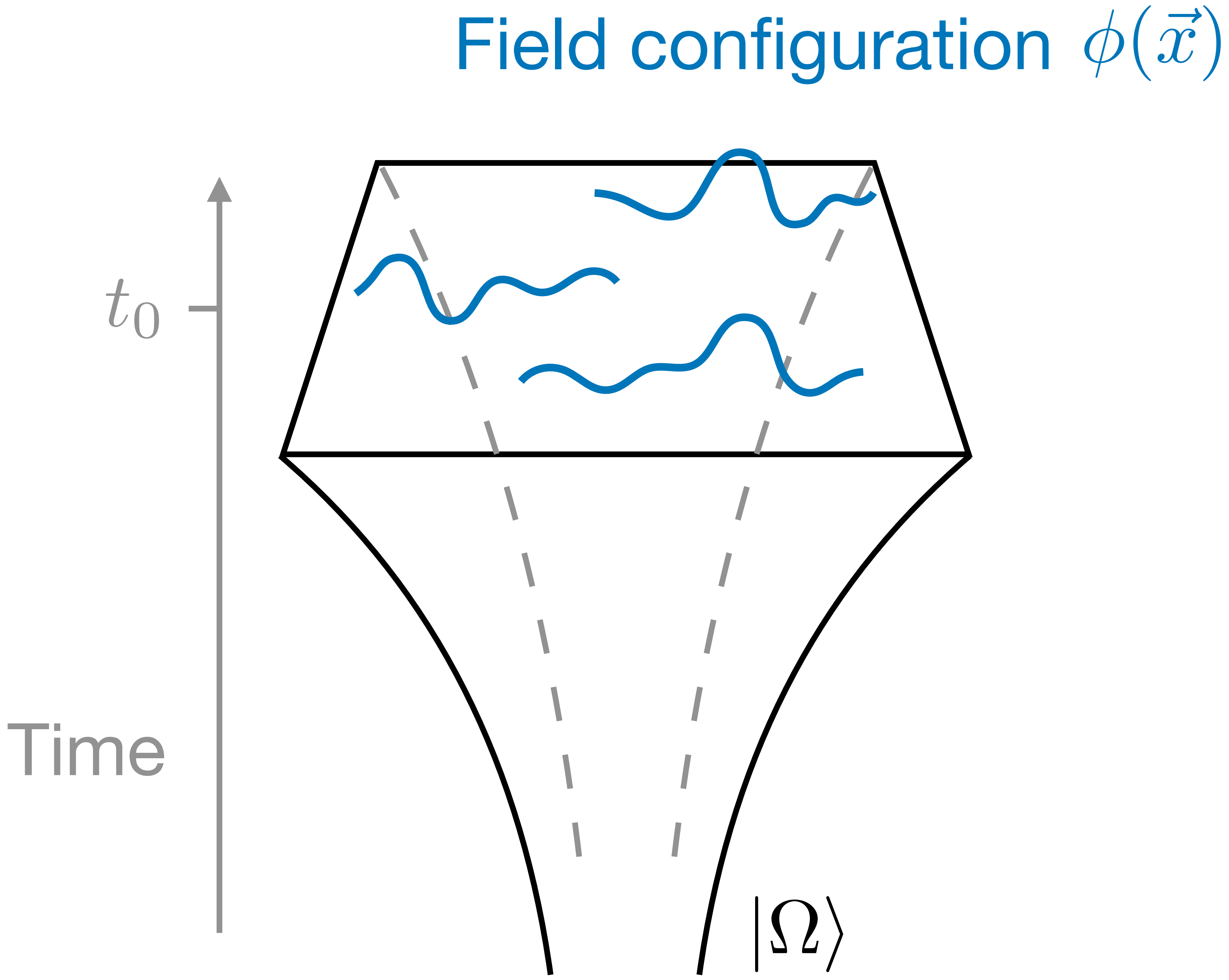

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We will separate the dependence on system and environment:

$$\Psi[\phi] = \Psi[\phi_s, \phi_\varepsilon]$$

$$\rho_s \equiv \mathrm{Tr}_\varepsilon\, \rho$$

$$\Downarrow$$

$$(\rho_s)_{\phi_s \bar{\phi}_s} = \int \mathcal{D}\phi_\varepsilon\ (\rho)_{\phi\bar{\phi}}\Big|_{\phi_\varepsilon = \bar{\phi}_\varepsilon} = \int \mathcal{D}\phi_\varepsilon\, \Psi[\phi_\varepsilon, \phi_s]\Psi[\phi_\varepsilon, \bar{\phi}_s]^*$$

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We…

- …worked at finite spatial volume, then took the infinite limit
- …developed some diagrammatic rules to streamline the computation

With the help of diagrams we actually computed the N-th traces:

$$\frac{\mathrm{Tr}\,\rho_{\mathcal{S}}^N}{(\mathrm{Tr}\,\rho_{\mathcal{S}})^N} = \exp(-ND) \qquad \forall\ N \geq 2$$

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N-th Renyi entropy: $S_N \equiv \frac{1}{1-N} \log \left[\frac{\mathrm{Tr}\,\rho_{\mathcal{S}}^N}{(\mathrm{Tr}\,\rho_{\mathcal{S}})^N} \right]$

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The purity is then

$$\gamma \equiv \frac{\mathrm{Tr}\,\rho_{\mathcal{S}}^2}{(\mathrm{Tr}\,\rho_{\mathcal{S}})^2} = \exp(-2D)$$

Let us focus on theories with just a cubic interaction: $g\phi^3, \; g\phi(\partial\phi)^2, \ldots$

$$\psi_3 \sim \mathcal{O}(g) \qquad ; \qquad \psi_{n\geq 4} \sim \mathcal{O}(g^2)$$

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At leading order in the coupling, the purity is

$$\gamma = 1 - 2\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \frac{|\psi_3(\vec{p},\vec{k},-\vec{p}-\vec{k})|^2}{2\mathrm{Re}\,\psi_2(\vec{p})\,2\mathrm{Re}\,\psi_2(\vec{k})\,2\mathrm{Re}\,\psi_2(\vec{p}+\vec{k})} = 1 - g^2 I$$

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Hence

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For a theory with just a quartic interaction, and at leading order:

$$\gamma = 1 - \frac{1}{3}\int \frac{\mathrm{d}^3\vec{k}_1}{(2\pi)^3}\frac{\mathrm{d}^3\vec{k}_2}{(2\pi)^3} \frac{|\psi_4(\vec{p},\vec{k}_1,\vec{k}_2,-\vec{p}-\vec{k}_1-\vec{k}_2)|^2 + |\psi_4(-\vec{p},-\vec{k}_1,-\vec{k}_2,\vec{p}+\vec{k}_1+\vec{k}_2)|^2}{2\mathrm{Re}\,\psi_2(\vec{p})\,2\mathrm{Re}\,\psi_2(\vec{k}_1)\,2\mathrm{Re}\,\psi_2(\vec{k}_2)\,2\mathrm{Re}\,\psi_2(\vec{p}+\vec{k}_1+\vec{k}_2)}$$

In general, an EFT is valid in some range of scales:

$$\Lambda_{\rm IR} \leq E \leq \Lambda_{\rm UV}$$

so we should be more careful with our definition of the environment:

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The purity integral is then a function of the cutoffs:

$$\gamma = 1 - 2 \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \frac{|\psi_3(\vec{p}, \vec{k}, -\vec{p} - \vec{k})|^2}{2\mathrm{Re}\,\psi_2(\vec{p})\, 2\mathrm{Re}\,\psi_2(\vec{k})\, 2\mathrm{Re}\,\psi_2(\vec{p} + \vec{k})} = 1 - g^2 I(p, \Lambda_{\mathrm{IR}}, \Lambda_{\mathrm{UV}})$$

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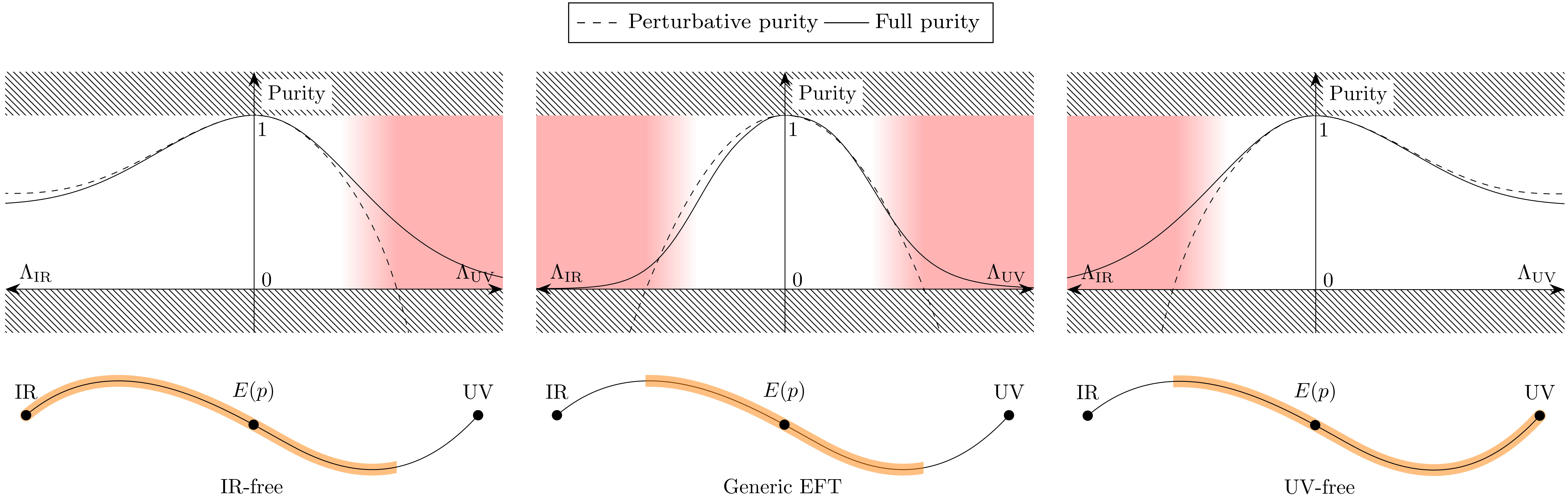
We get bounds on the EFT validity regime!

$$\gamma = 1 - g^2\, I(p, \Lambda_{\mathrm{IR}}, \Lambda_{\mathrm{UV}})$$

We can fix the coupling and vary the cutoffs:



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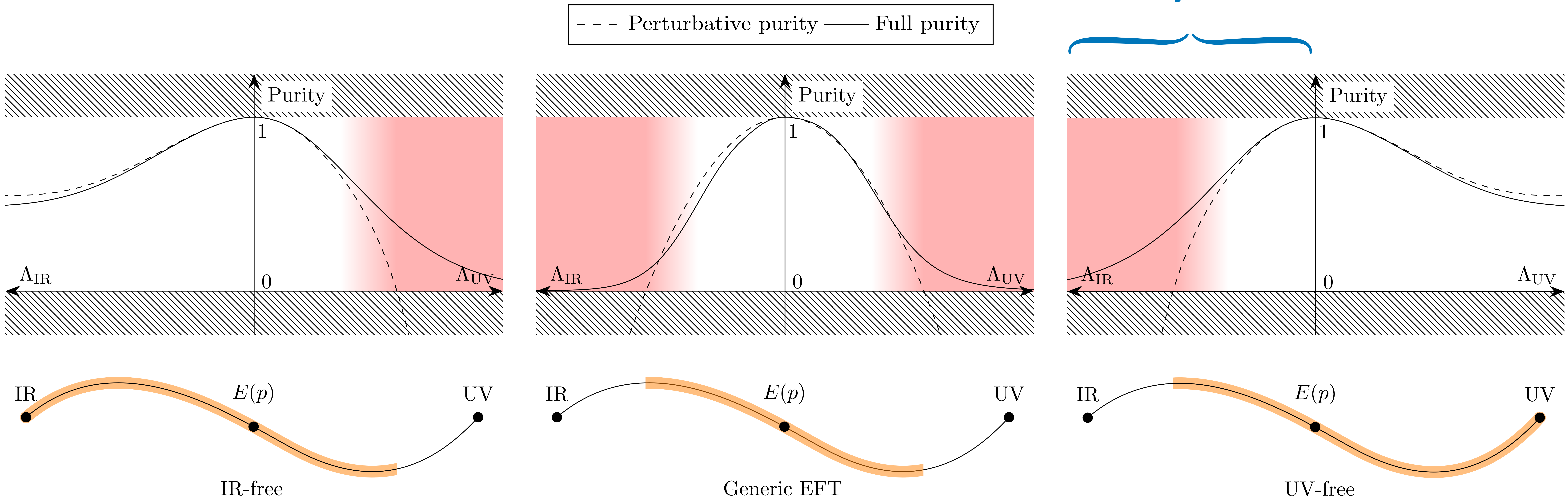
Fix Λ_{UV}

Vary Λ_{IR}



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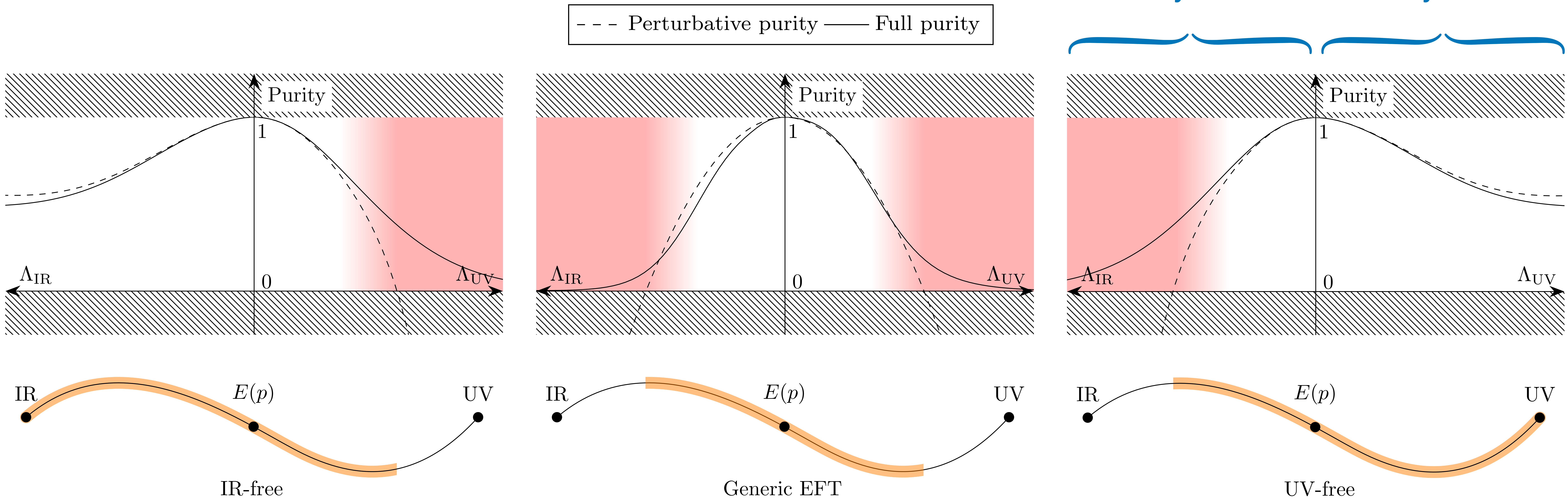

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Purity bound:

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$$|a_0| \leq \frac{1}{2} \qquad \Rightarrow \qquad \frac{|g|}{m} \leq \frac{12\pi}{5} \sim 3$$

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The bounds are qualitatively similar

$$\boxed{\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 - \frac{g}{2}\phi(\partial_\mu \phi)^2}$$

Under a field redefinition, this theory becomes free:

$$\phi \quad \longrightarrow \quad \phi(\varphi) = \frac{1}{g}\left(1 + \frac{3g}{2}\varphi\right)^{2/3} - \frac{1}{g}$$

$$\mathcal{L}(\phi) \quad \longrightarrow \quad \mathcal{L}\Big[\phi(\varphi)\Big] = -\frac{1}{2}(\partial_\mu \varphi)^2$$

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Purity bounds exist even in absence of partial wave bounds

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{g}{2}\phi(\partial_\mu \phi)^2$$

Partial wave bound:

$$|a_0| \leq \frac{1}{2} \qquad \Rightarrow \qquad g^2 m^2 \leq \frac{32\pi}{19} \sim 5$$

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Under the previous field redefinition:

$$\mathcal{L}(\phi) \quad \longrightarrow \quad \mathcal{L}\Big[\phi(\varphi)\Big] = -\frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^2 + \frac{g m^2}{2}\varphi^3 - \frac{19 g^2 m^2}{96}\varphi^4 + \dots$$

Then:

$$\gamma \geq 0 \qquad \Rightarrow \qquad g^2 m^2 \lesssim 64$$

Outline

Introducing the problem…

- **Perturbative unitarity bounds**

Proposing a solution…

- **Entanglement in QFT**
- **Computing the purity**

Reporting on the results…

- **Bounds in flat space**
- **Bounds in de Sitter space**

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2a^2}(\partial_i\phi)^2 + \frac{g_1}{3!}\dot{\phi}^3 + \frac{g_2}{2a^2}\dot{\phi}(\partial_i\phi)^2 + \ldots$$

$$\gamma = 1 - \left(\frac{H^2}{80\pi}\right)^2 \left(\frac{331}{18}g_1^2 + \frac{22959}{2}g_2^2 - 879 g_1 g_2\right)$$

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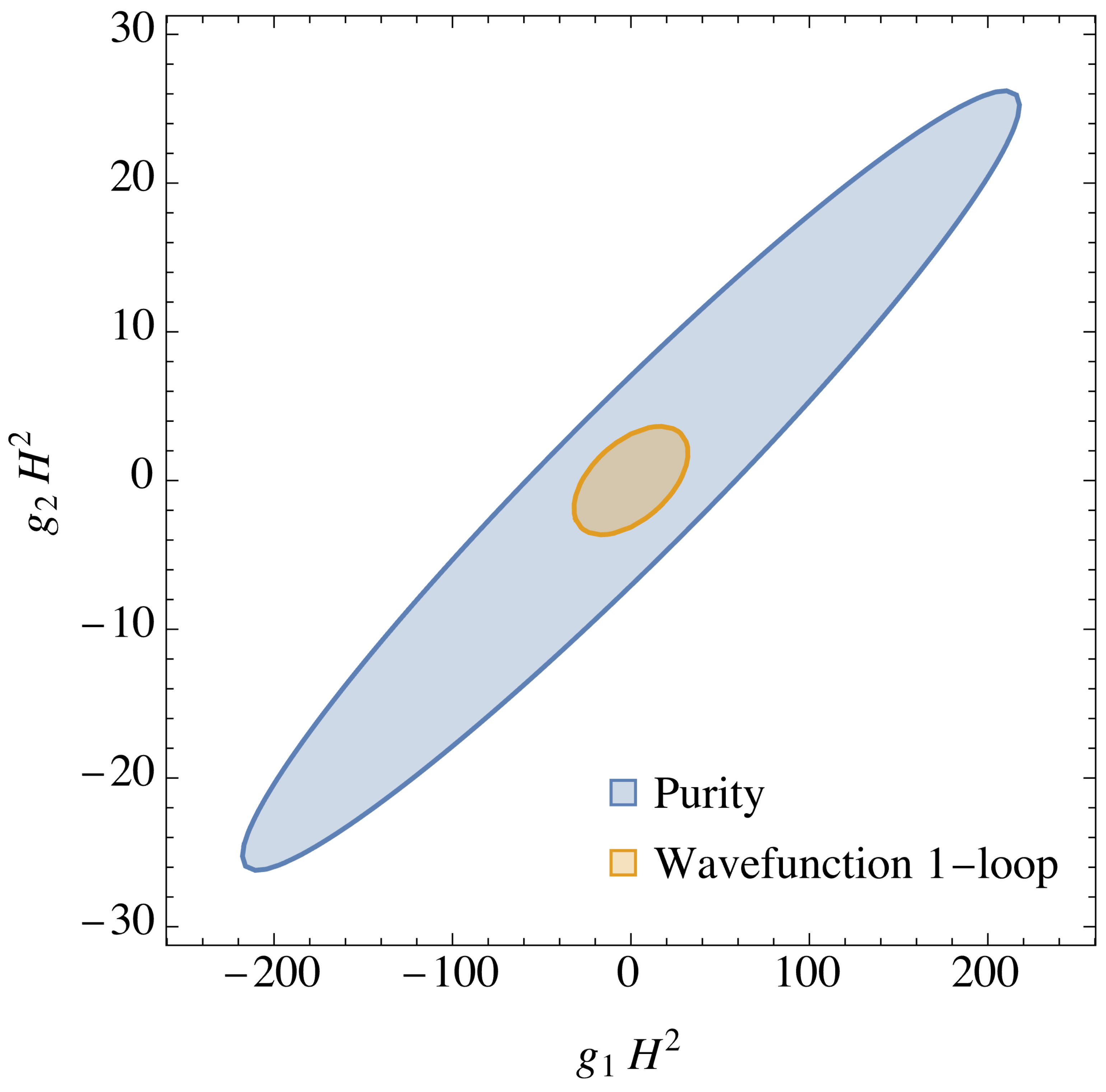

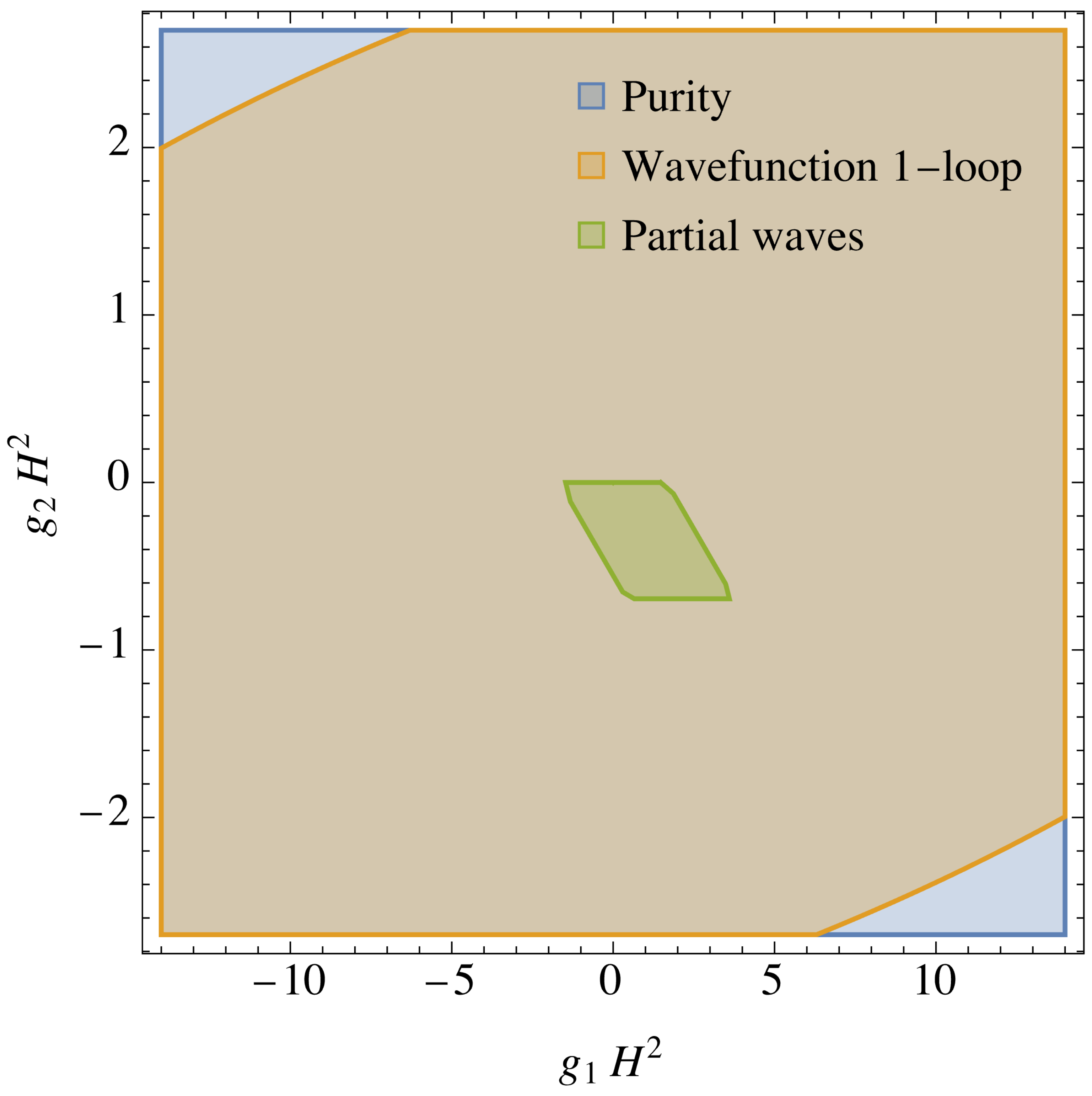

We compare with:

- $\psi_2^{(1-\text{loop})} \leq \psi_2^{(\text{tree})}$

  Pajer, Melville '21

- Partial wave bounds

  Grall, Melville '20

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 - \frac{g}{2}\phi\dot{\phi}^2$$

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The integral diverges due to the squeezed configurations:

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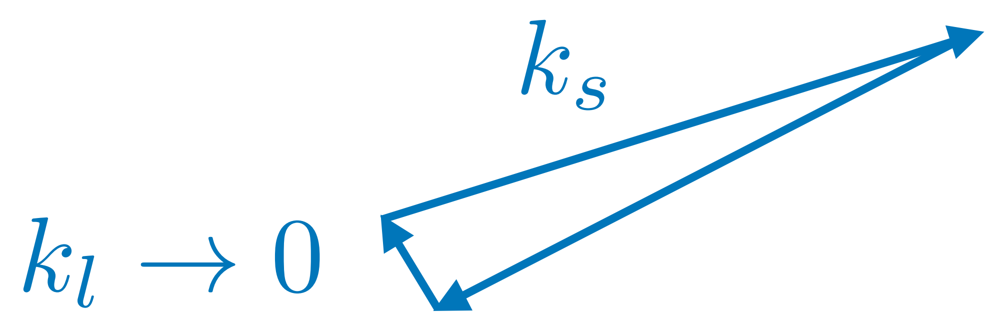

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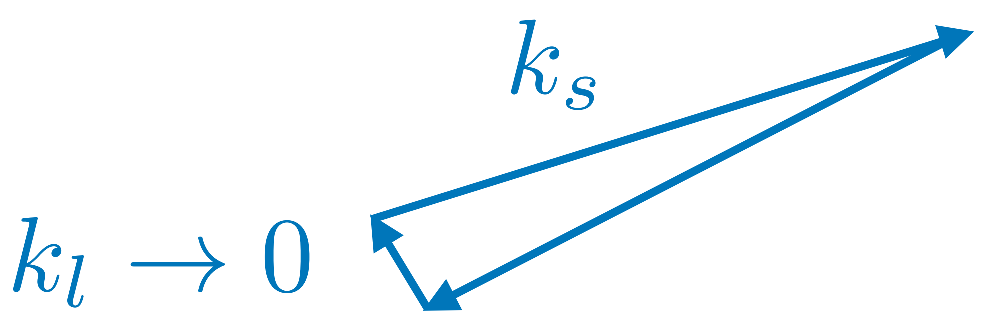

We use a cutoff on k_s/k_l and get

$$\gamma \geq 0 \qquad \Rightarrow \qquad \left(\frac{k_s}{k_l}\right)^3 \leq 6\left(\frac{4\pi}{gH}\right)^2$$

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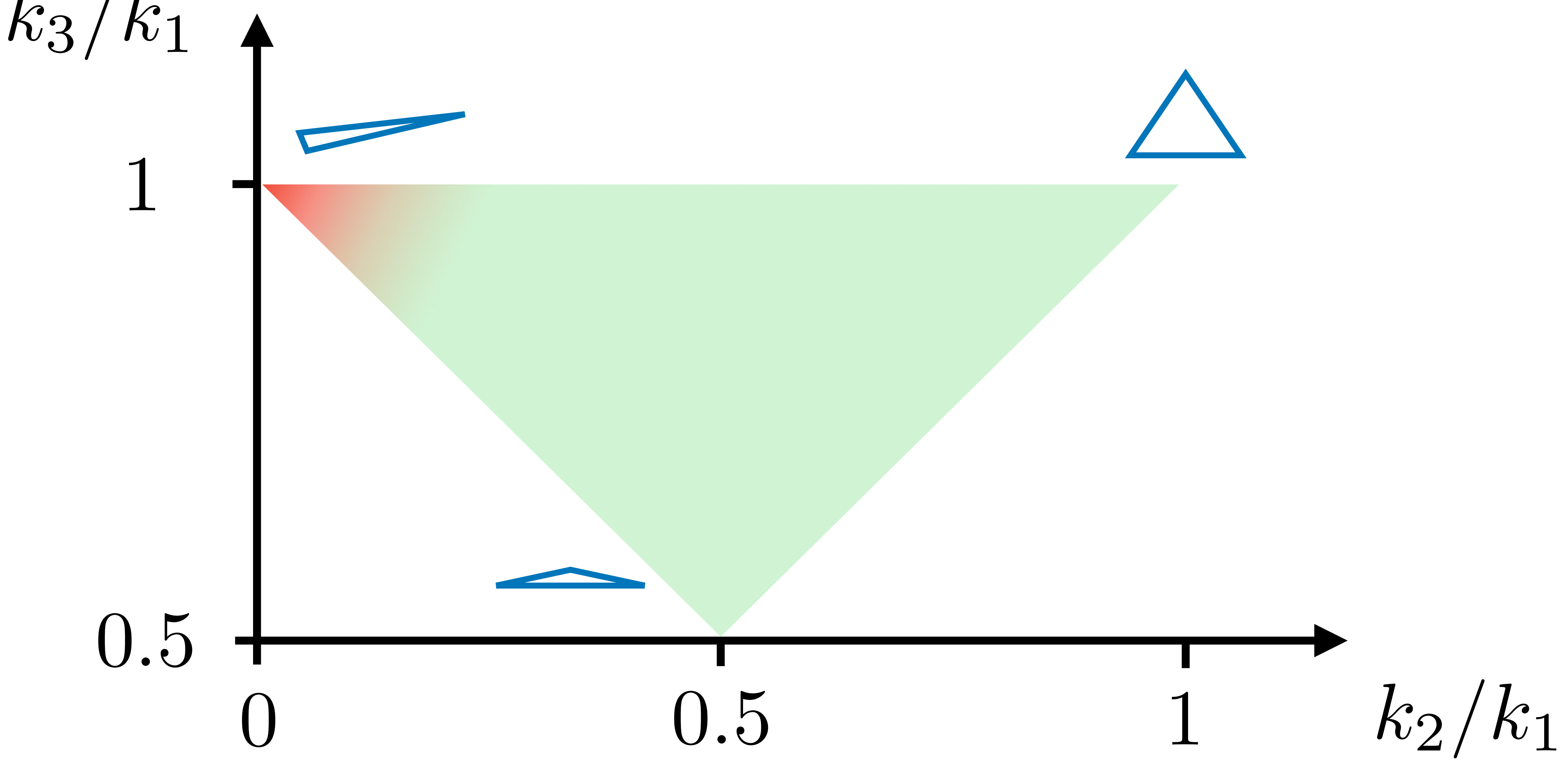

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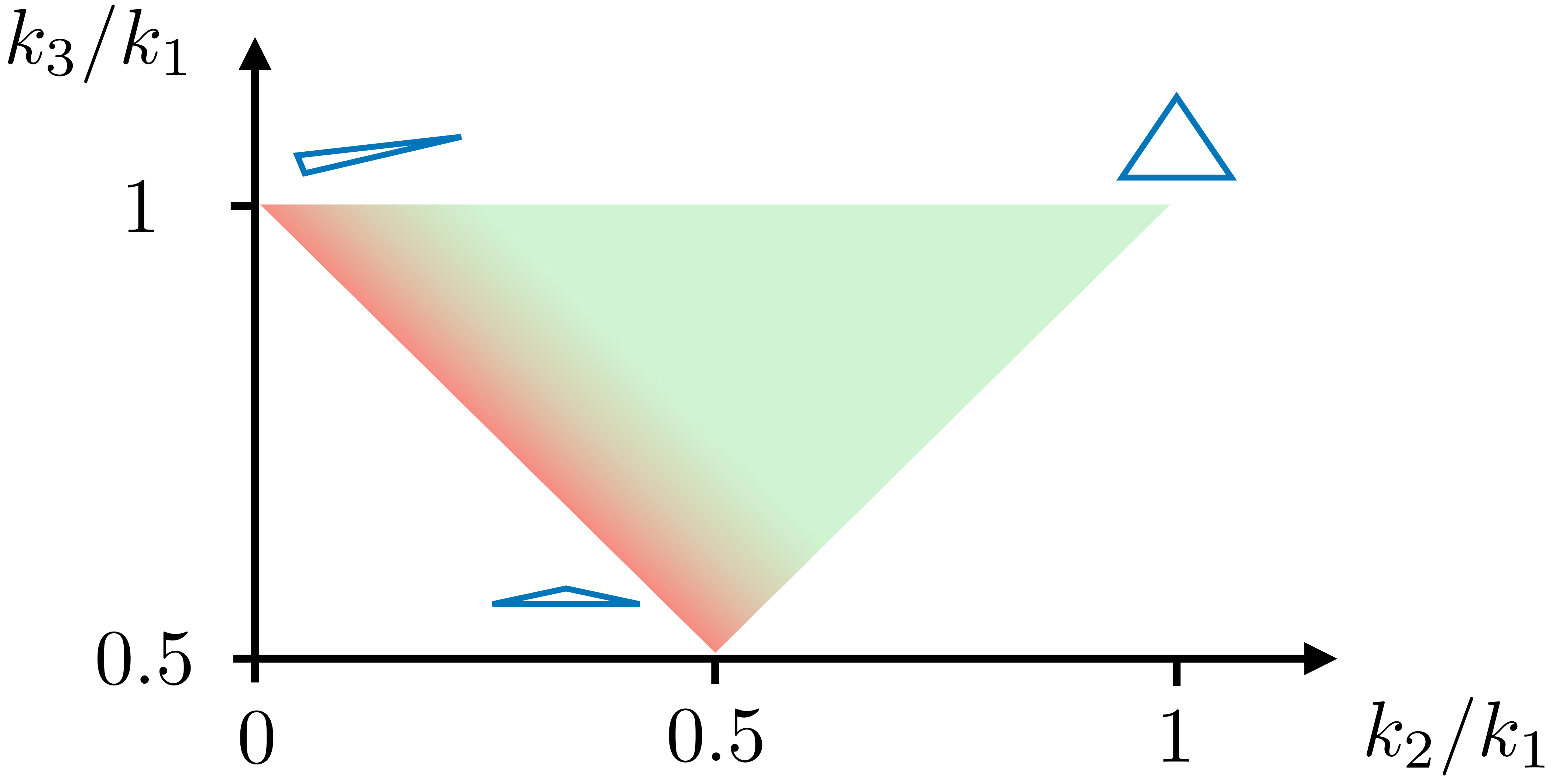

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EFTs in de Sitter fail to describe large energy hierarchies and folded momentum configurations

High-dimension operators

$$\frac{\lambda}{\Lambda^{\Delta-4}} \partial^{\Delta-3} \phi^3 \quad \Rightarrow \quad \psi_3 \sim \begin{cases} \int \mathrm{d}t \, e^{ikt} \sim \frac{\lambda}{\Lambda^{\Delta-4}} \cdot \frac{f(k'\mathrm{s})}{k} & \text{in Minkowski} \\ \int \mathrm{d}\eta \, \eta^{\Delta-4} \, e^{ik\eta} \sim \frac{\lambda}{\Lambda^{\Delta-4}} \cdot \Gamma(\Delta-3) \cdot \frac{f(k'\mathrm{s}, H)}{k^{\Delta-3}} & \text{in dS} \end{cases}$$

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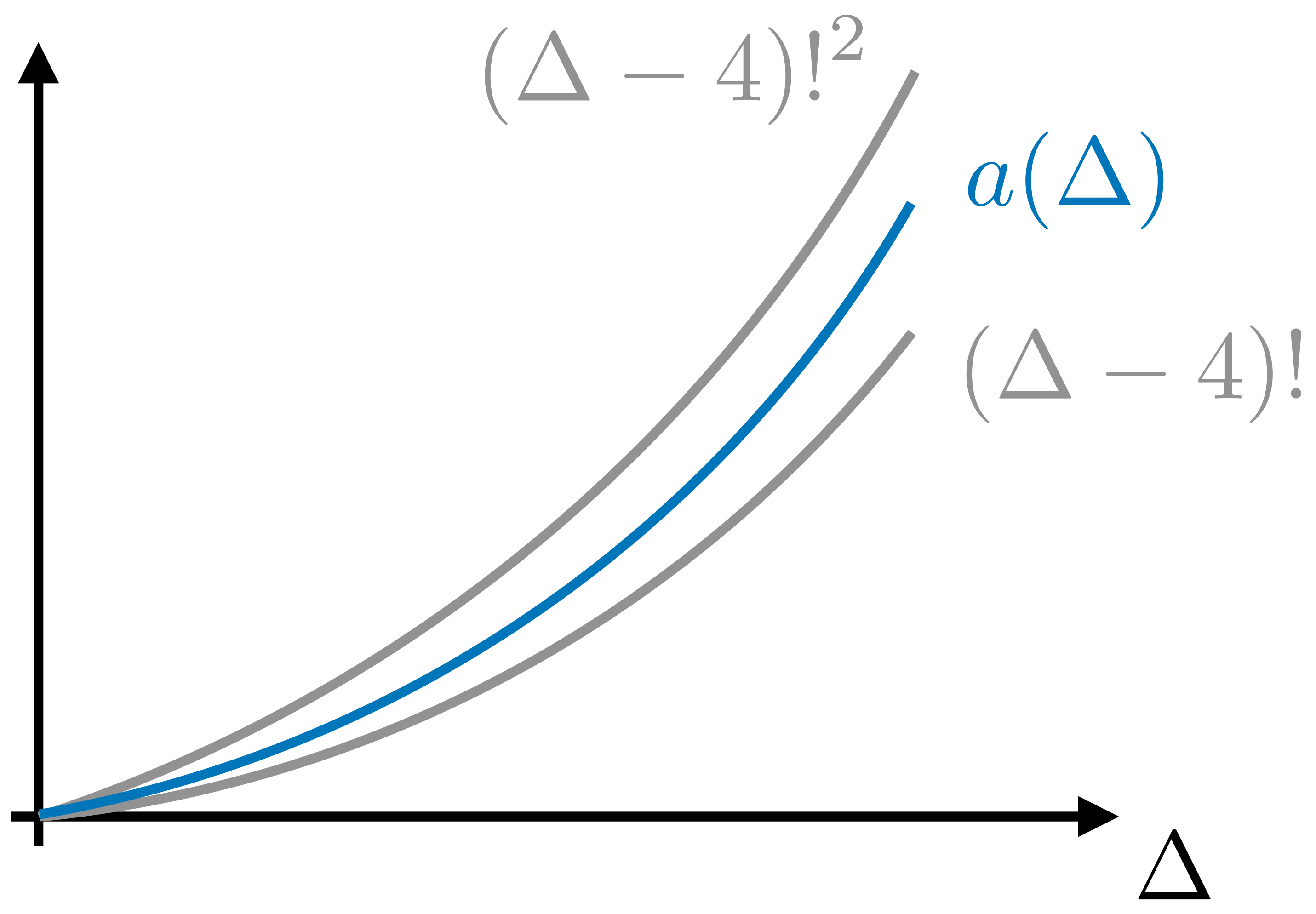

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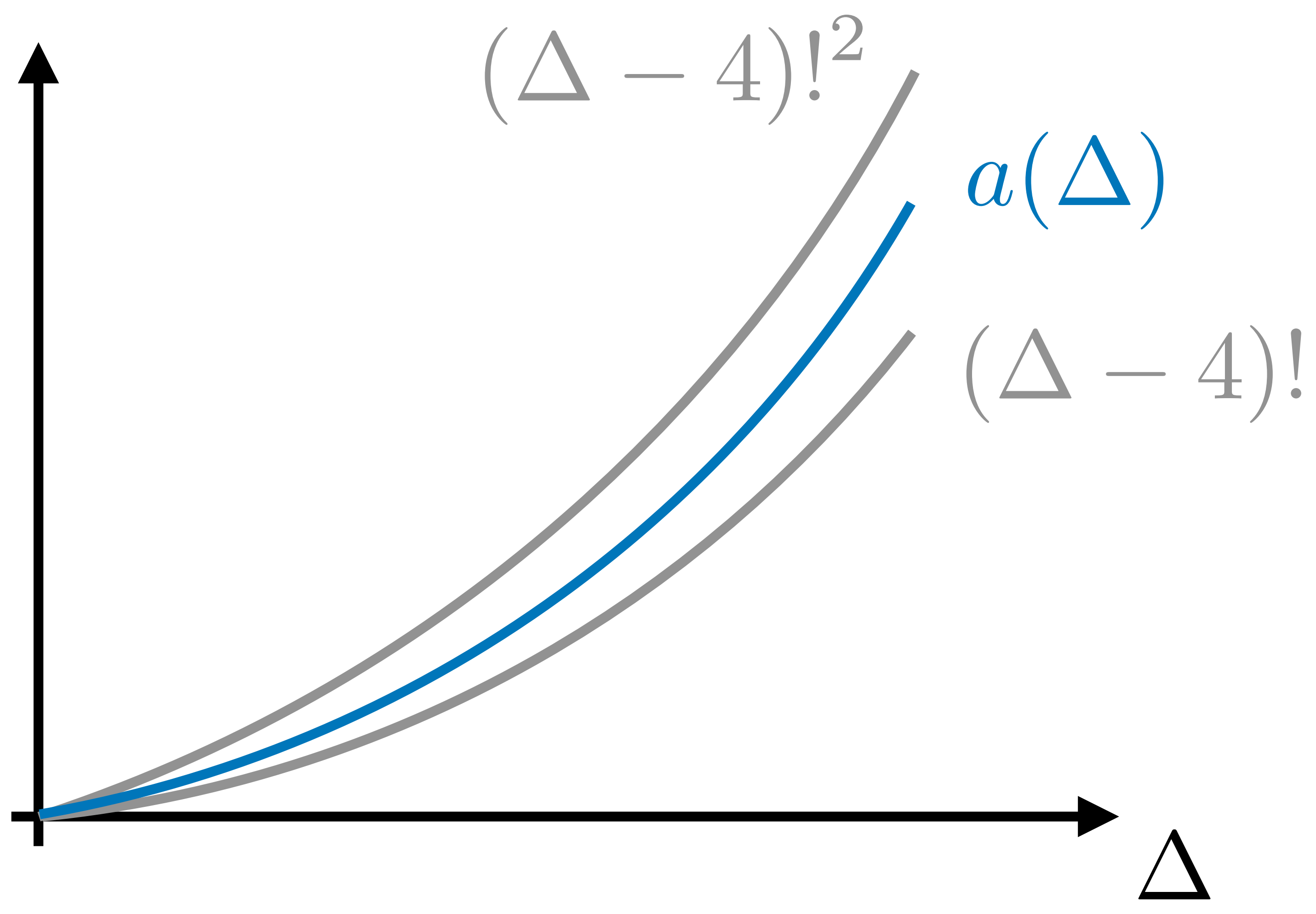

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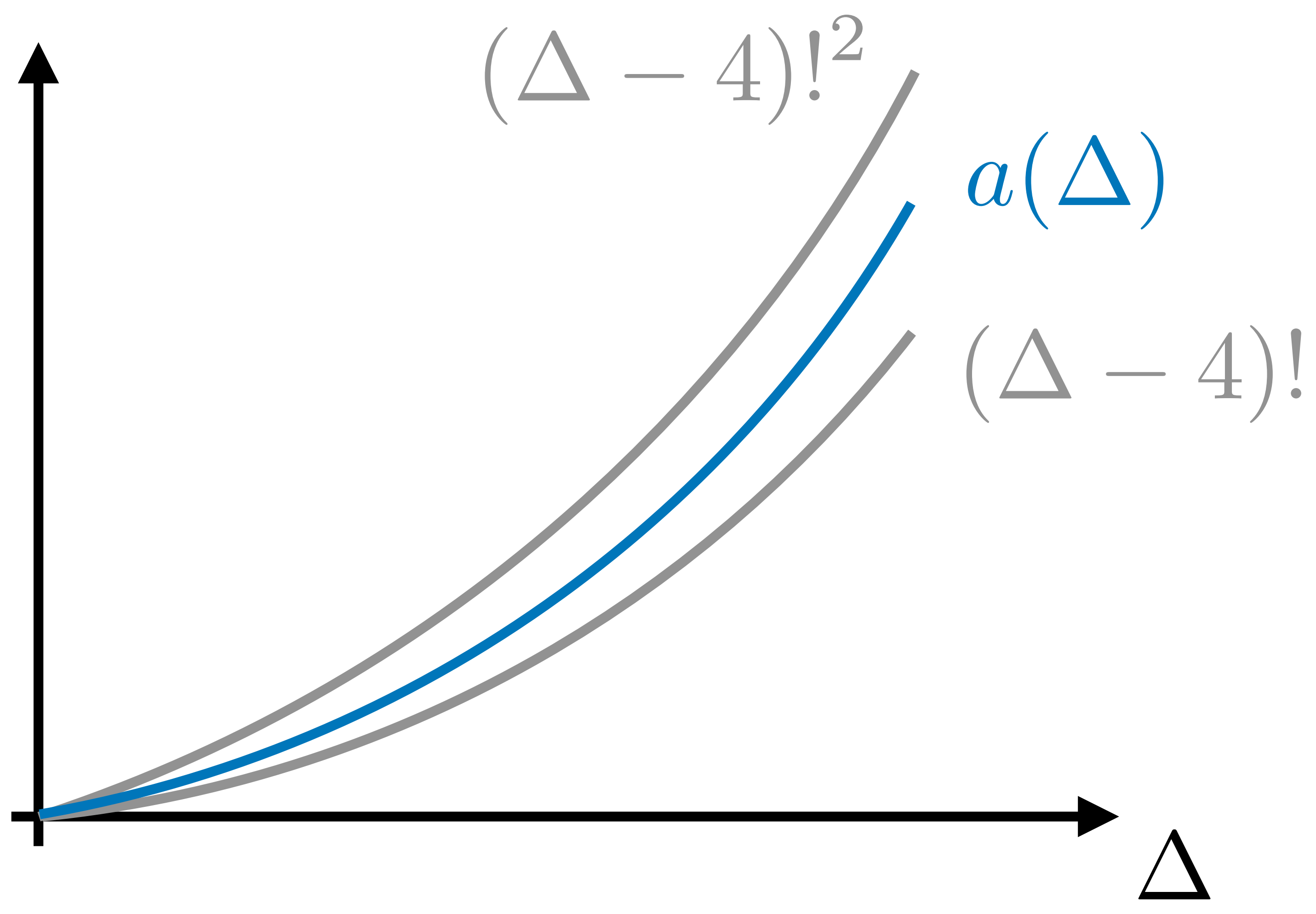

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The dS power-counting scheme is different!

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A naive application of our purity bounds to three-graviton interaction in flat space yields

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- Local non-Gaussianity

For a dS theory with only a 3-point wavefunction coefficient corresponding to local NG:

$$\left|f_{\rm NL}^{\rm (loc)}\right| \lesssim \frac{5\pi}{6\sqrt{A}}\left(\frac{k_{\rm min}}{k_{\rm max}}\right)^{3/2} \sim 0.8$$

Inflationary models with local NG avoid this problem by working non-perturbatively

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Thank you!