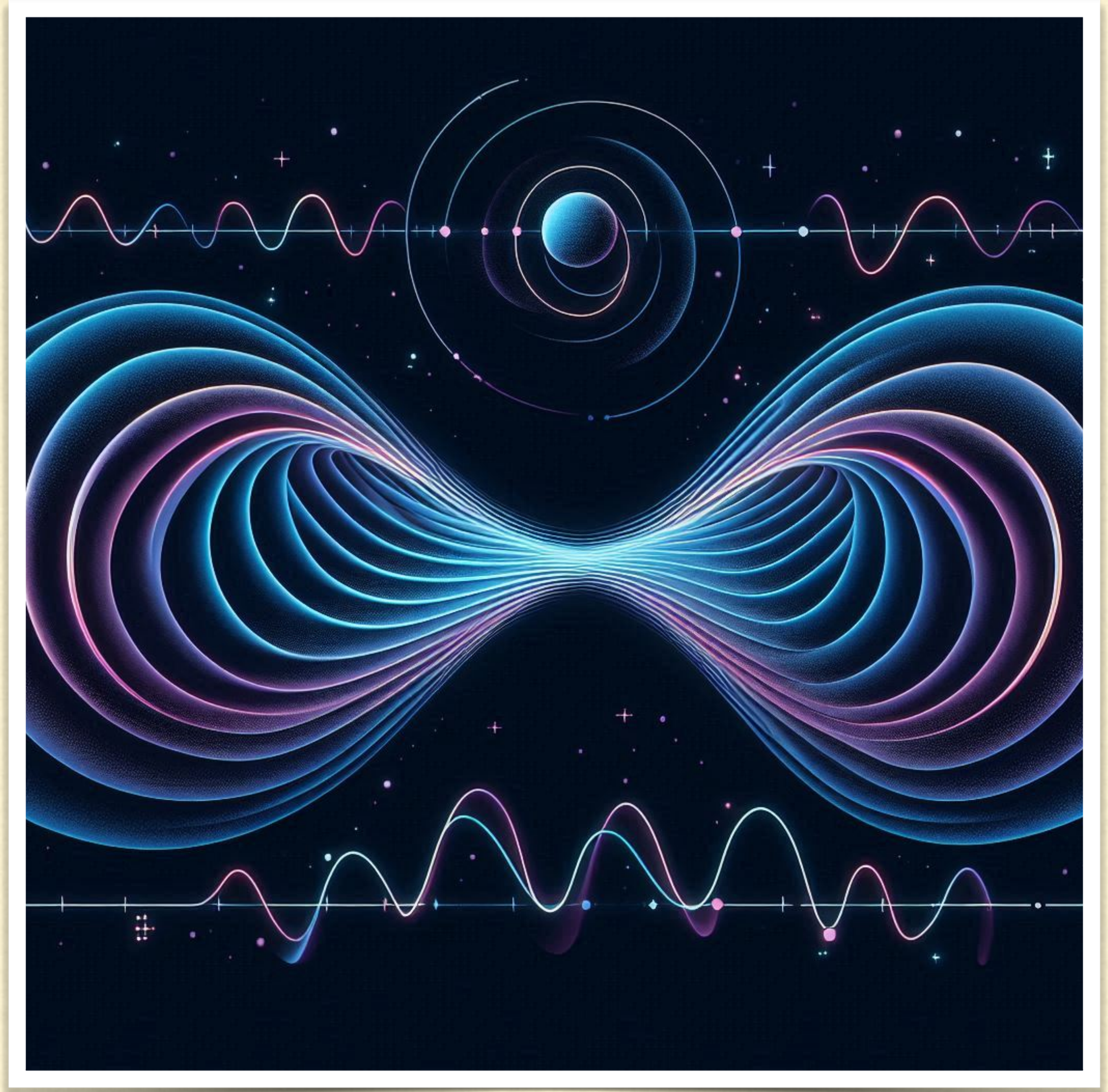
FINITE PARTS OF INFLATIONARY LOOPS



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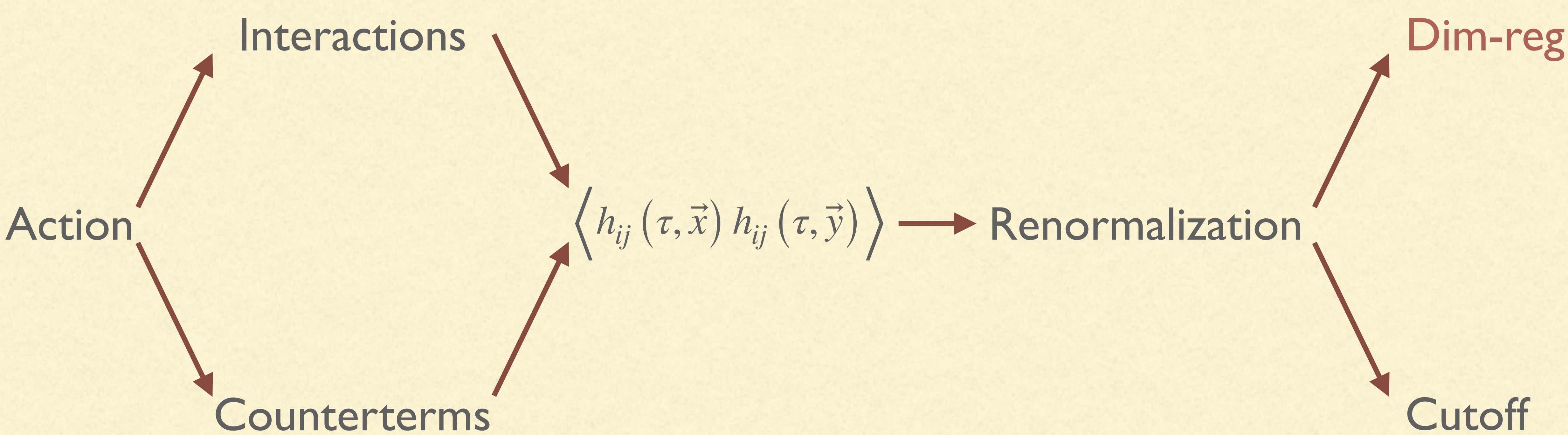
WORKSHOP "LOOPING IN THE PRIMORDIAL UNIVERSE"

GOAL & MAP OF THE TALK

🥅 Goal: present our tools and procedure to compute loop corrections in inflationary cosmologies through dimensional regularization

📱 Application: one-loop power spectrum of tensor modes in interaction with scalar modes on dS background



ACTION & VERTICES

Degrees of freedom ($\delta\phi$-gauge): $\delta\phi, h_{ij}$

$$S = S_h + S_{\delta\phi} + S_{\text{int}}$$

dS background

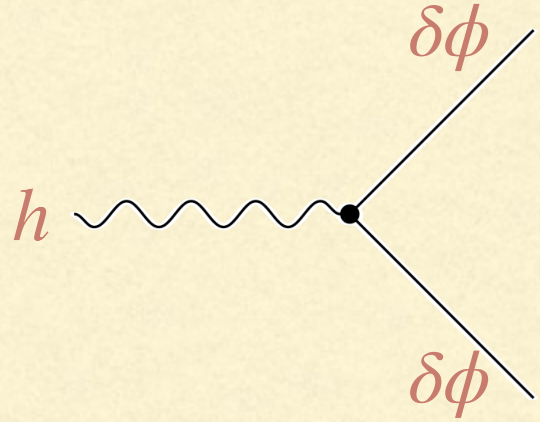
$$a(\tau) = -\frac{1}{H\tau}$$

$$S_h = \frac{M_P^2}{8}\int d\tau d^3x\, a^2(\tau)\left(h'_{ij}h'_{ij} - \partial_k h_{ij}\partial_k h_{ij}\right)$$

$$S_{\delta\phi} = \frac{1}{2}\int d\tau d^3x\, a^2(\tau)\left[\left(\delta\phi'\right)^2 - \partial_i\delta\phi\partial_i\delta\phi\right]$$

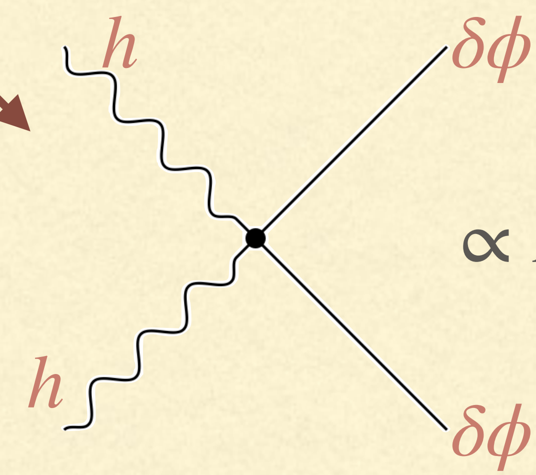
$$S_{\text{int}} = \int d\tau d^3x\, a^2(\tau)\left(\frac{1}{2}h_{ij}\partial_i\delta\phi\partial_j\delta\phi - \frac{1}{4}h_{ik}h_{kj}\partial_i\delta\phi\partial_j\delta\phi\right)$$

cubic



$\propto M_P^{-1}$

quartic



$\propto M_P^{-2}$

CORRELATION FUNCTION & POWER SPECTRUM

in-in

$$\left\langle h_{ij}\left(\tau,\vec{x}\right)h_{kl}\left(\tau,\vec{y}\right)\right\rangle = \langle 0|F^{-1}(\tau,-\infty_+)h^I_{ij}\left(\tau,\vec{x}\right)h^I_{kl}\left(\tau,\vec{y}\right)F(\tau,-\infty_-)|0\rangle\Big|_{\text{no bubbles}}$$

$$F(\tau,\tau_0) = T\exp\left[-i\int_{\tau_0}^{\tau} d\tau' H_I(\tau')\right]$$

fields in the interaction picture

$$\delta\phi^I(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}}\left[\delta\phi_k(\tau)a_{\vec{k}} + \delta\phi^*_k(\tau)a^\dagger_{-\vec{k}}\right]$$

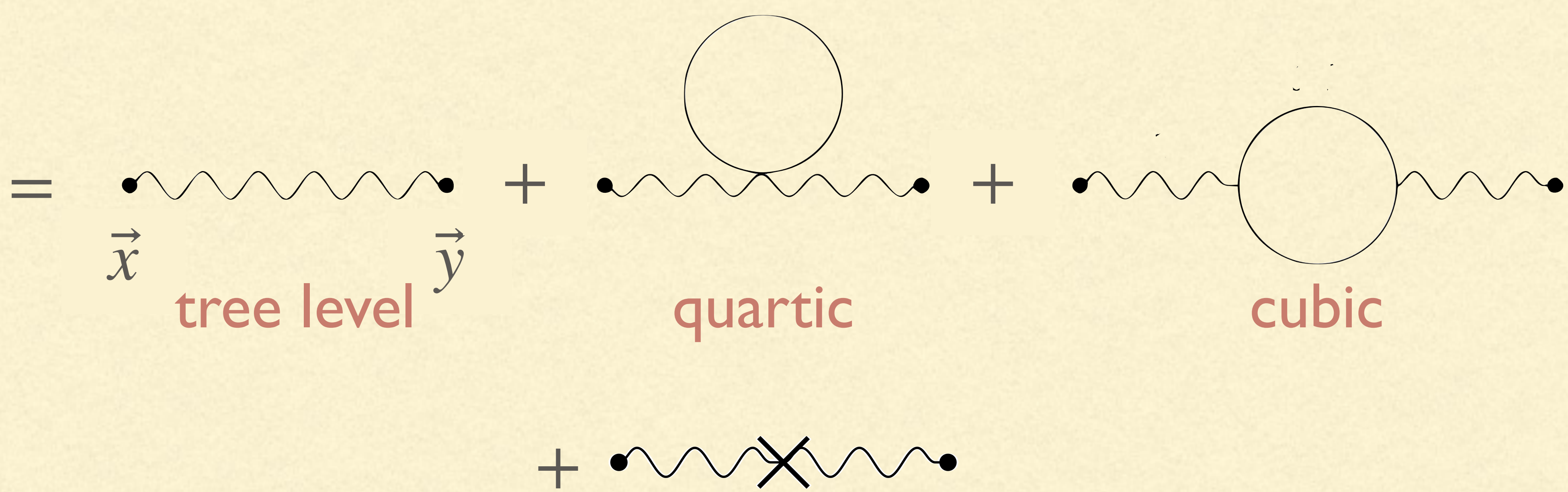
$$h^I_{ij}(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \sum_{\lambda=+,\times} \epsilon^\lambda_{ij}(\vec{k})\left[h_k(\tau)a_{\lambda,\vec{k}} + h^*_k(\tau)a^\dagger_{\lambda,-\vec{k}}\right]$$

CORRELATION FUNCTION & POWER SPECTRUM

power spectrum

$$\left\langle h_{ij}\left(\tau,\vec{x}\right) h_{ij}\left(\tau,\vec{y}\right)\right\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \frac{2\pi^2}{k^3} 2\mathscr{P}_h(\tau,k)$$

polarizations

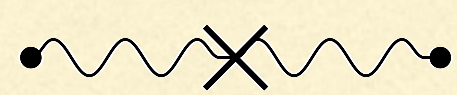


how do counterterms look like?

COUNTERTERMS

$$S_{\text{HDO}} = \int d^4x\sqrt{-g}\,\left(A_1R^2 + A_2R_{\mu\nu}R^{\mu\nu} + A_3R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + A_4\Box R\right)$$

$$S_{cts} = -\int d\tau d^3x\, a^2\left[C_1H^2h'_{ij}h'_{ij} + C_2H^2\partial_k h_{ij}\partial_k h_{ij} + \frac{C_3}{a^2}\left(\partial_k h'_{ij}\partial_k h'_{ij} - \partial^2 h_{ij}\partial^2 h_{ij}\right)\right]$$



Other operators break gauge invariance
(discuss them with cutoff)

RENORMALIZATION

Regularization

Redefinition

Subtraction

Regularization $\to 0$

DIMENSIONAL REGULARIZATION: MODE FUNCTIONS

$d = 3 + \delta$

Not present on flat space!

Equations of motion (scalar)

$$\begin{cases} \delta\phi_k'' + (2+\delta)aH\,\delta\phi_k' + k^2\delta\phi_k = 0 \\ \delta\phi_k(\tau) \xrightarrow{k\tau\to-\infty} \dfrac{1}{a^{1+\delta/2}(\tau)\mu^{\delta/2}}\dfrac{e^{-ik\tau}}{\sqrt{2k}} \end{cases}$$

∃ analytical solution, but we solve it perturbatively:

$$\delta\phi_k(\tau) = \delta\phi_k^{(0)}(\tau) + \delta\,\delta\phi_k^{(1)}(\tau) + \mathcal{O}\left(\delta^2\right)$$

$$\delta\phi_k(\tau) = i\frac{H}{\sqrt{2k^3}}e^{-ik\tau}(1+ik\tau)\left\{1 + \frac{\delta}{2}\left[\frac{1-ik\tau}{1+ik\tau}e^{2ik\tau}(i\pi - \mathrm{Ei}(-2ik\tau)) + \frac{2}{1+ik\tau} + \log\left(-\frac{H\tau}{\mu}\right)\right] + \mathcal{O}\left(\delta^2\right)\right\}$$

DIMENSIONAL REGULARIZATION: INTEGRALS

$$I(\delta) = \int_0^\infty dp\, p^\delta f(\delta, p) = \int_0^L dp\, p^\delta f(\delta, p) + \int_L^\infty dp\, p^\delta f(\delta, p)$$

from the d-dimensional measure

$$\int_0^L dp\, f(0,p) + \mathcal{O}(\delta)$$

$$\sum_{n=-\infty}^{N} c_n(\delta) p^n$$

Logarithmic divergence as single pole to subtract

$$I(\delta) = -\frac{c_{-1}(0)}{\delta} - \left.\frac{dc_{-1}}{d\delta}\right|_{\delta=0} + \lim_{L\to\infty}\left[\int_0^L dp\, f(0,p) - c_{-1}(0)\log L - \sum_{n=0}^{N}\frac{L^{n+1}}{n+1}c_n(0)\right] + \mathcal{O}(\delta)$$

Only here we need $\mathcal{O}(\delta)$ corrections to the mode functions

RENORMALIZED TENSOR POWER SPECTRUM

$$\mathscr{P}_h(\tau, k)\Big|_{k\tau=y} = \alpha(1+y^2) + \frac{3}{80}\alpha^2 \left\{ c_1 + c_2 y^2 + c_3 y^4 + (1+y^2)\log\left(-\frac{H}{\mu}y\right) + \mathrm{Re}\left[e^{2iy}(i+y)^2\left(\mathrm{Ei}(-2iy) - i\pi\right)\right] \right\} + \mathcal{O}\left(\alpha^3\right)$$

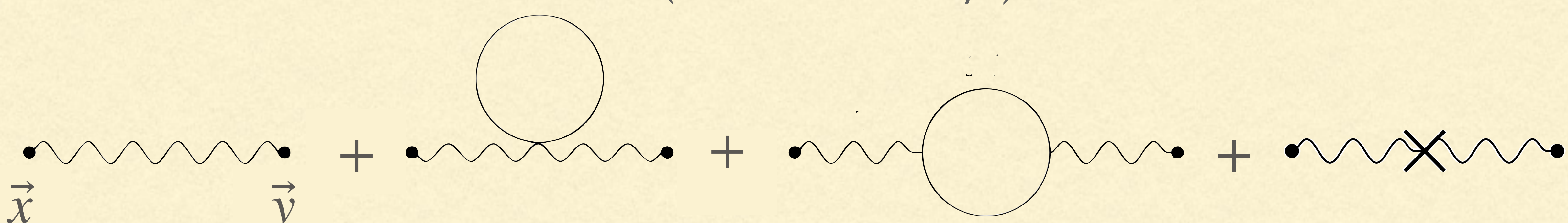
tree level

expansion parameter

$$\alpha = \left(\frac{H}{\pi M_P}\right)^2$$

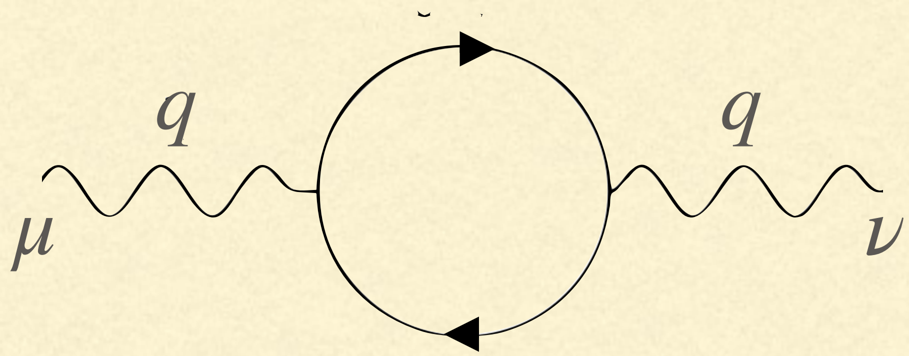
$y \to 0$

$$\mathscr{P}_h(0,k) = \alpha + \frac{3}{80}\alpha^2\left(c_1 - \gamma_E + \log\frac{H}{2\mu}\right) + \mathcal{O}\left(\alpha^3\right)$$



CUTOFF: QED

WT

$$\Pi^{\mu\nu}(q) =$$ 

$$= \Pi_2\left(q^2\right)\left(q^\mu q^\nu - q^2\eta^{\mu\nu}\right)$$

$$= \Pi_2\left(q^2\right)\left(q^\mu q^\nu - q^2\eta^{\mu\nu}\right) + A(q^2)\eta^{\mu\nu}$$

cutoff

~~WT~~

~~$U(1)$~~

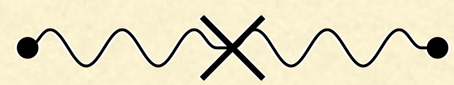
dim-reg $\mathscr{L}_{cts} = -\frac{1}{4}\left(Z_3 - 1\right) F_{\mu\nu}F^{\mu\nu}$

cutoff $\mathscr{L}_{cts} = \frac{1}{2}\left(M^2 A_\mu A^\mu + c\partial_\mu A_\nu \partial^\mu A^\nu + c'\partial_\mu A_\nu \partial^\nu A^\mu\right)$

CUTOFF: TENSOR POWER SPECTRUM

More freedom in the counterterms

$$S_{\text{cts}} = \int d\tau d^3x\, a^4 \left[C_1 H^4 h_{ij} h_{ij} + \frac{H^2}{a^2} \left(C_2 h'_{ij} h'_{ij} + C_3 \partial_k h_{ij} \partial_k h_{ij} \right) + \frac{1}{a^4} \left(C_4 \partial^2 h_{ij} \partial^2 h_{ij} + C_5 \partial_k h'_{ij} \partial_k h'_{ij} \right) \right]$$



$$\mathcal{P}_h(\tau, k)\Big|_{k\tau=y} = \alpha(1+y^2) + \frac{3}{80}\alpha^2 \Bigg\{ c_1 + c_2 y^2 + c_3 y^4 + (1+y^2)\log\left(-\frac{H}{\mu}y\right) + c_4 \mathrm{Re}\left[e^{2iy}(i+y)^2 \left(\mathrm{Ei}(-2iy) - i\pi \right) \right] \Bigg\} + \mathcal{O}\left(\alpha^3\right)$$

You must impose by hand the cancellation of the late-time divergence choosing $c_4 = 1$

$$\alpha = \left(\frac{H}{\pi M_P} \right)^2$$

CONCLUSIONS

- Importance of loops in inflation ⇒ Renormalization
- Procedure for dimensional regularization $I(\delta)$
- Cutoff: Analogy with QED (HIC SUNT LEONES)
- GWs in inflation: computation of the one-loop PS, familiar $\propto \log\left(\frac{H}{\mu}\right)$, computation up to finite parts

Same as Senatore & Zaldarriaga JHEP 1012