Leonardo Senatore (ETH)

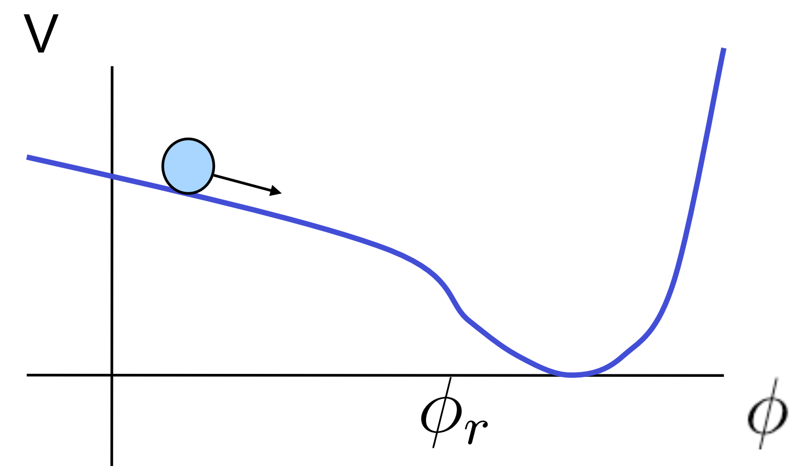
On Loops in Inflation

with M. Zaldarriaga **JHEP 2010**
JHEP 2012
JHEP 2013
with Pimentel and Zaldarriaga **JHEP 2013**
with Gorbenko **2019**

Outline
On Loops in Inflation

- Introduction
  - Learning to compute quantum corrections in Inflation
  - Some have IR divergencies
  - Eternal Inflation
- IR effects in Single Field Inflation
  - Log running $\log(H/\mu)$
  - No effect from $\log(kL)$
  - ζ is time in-dependent in standard slow roll
  - One true physical IR effect (already resumed)
- IR effects in multifield inflation

Who cares?



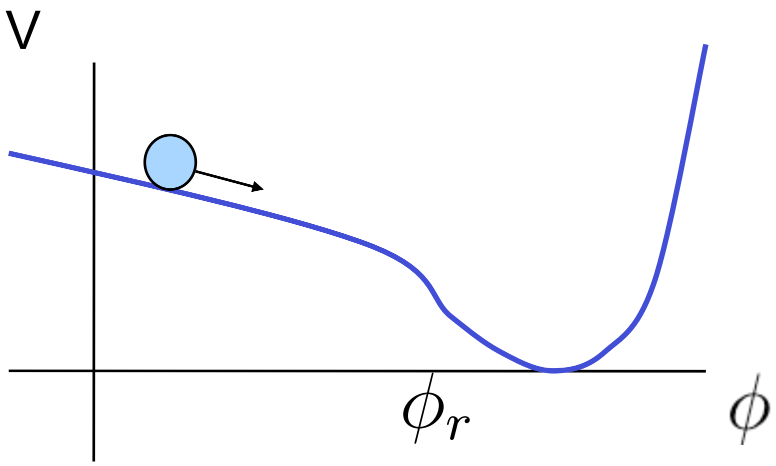
- Tiny Effect

$$\langle \delta\phi_k^2 \rangle_{\rm tree} \sim \frac{H^2}{k^3}$$

$$\langle \zeta_k^2 \rangle_{\rm tree} \sim \frac{H^2}{\epsilon M_{\rm Pl}^2} \frac{1}{k^3} \sim 10^{-10} \quad \Rightarrow \quad \langle \delta\phi_k^2 \rangle_{\rm 1-loop} \sim \frac{H^2}{k^3} \frac{H^2}{M_{\rm Pl}^2} \sim 10^{-10} \, \langle \delta\phi_k^2 \rangle_{\rm tree}$$

- We have more interacting theories (large non-Gaussianities! but still small)
- Weinberg cares: understand prediction of your theory S.Weinberg **2005**
  - –These are the quantum corrections to the predictions of Inflation.
  - –This has been *followed by lots of `amplitude', `bootstrap' activity.*
- *For ultra slow roll, recent claims suggest the effect is large*.
- dS is a puzzling spacetime (again, amplitudehidron, etc.), and inflation is a regularization
- Let us elaborate on this...

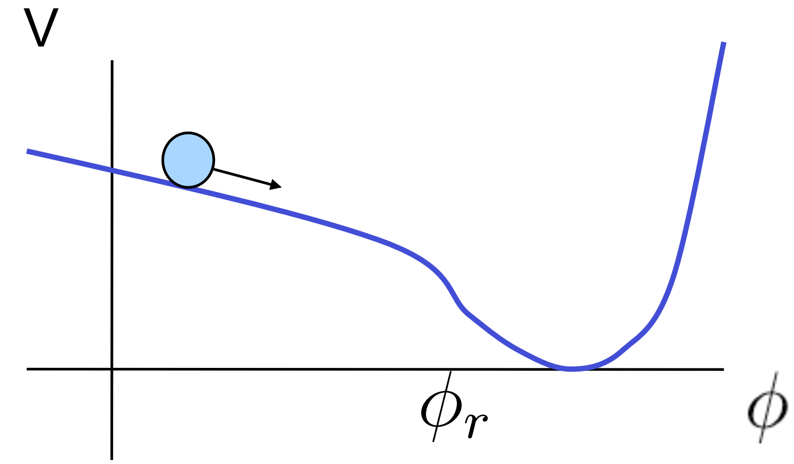
Who cares?



- Tiny Effect

$$10^{-10}$$

Who cares?



- One Loop in Quantum Gravity

$$S_4 = \frac{1}{2}\int dt d^3x a^3 \Big[-a^{-2}\epsilon\zeta^2(\partial\zeta)^2 + \alpha_1^4\Big(2\Sigma + 9\lambda + \frac{10}{3}\Pi\Big) - 6\zeta\alpha_1^3(\Sigma + 2\lambda) + 9\zeta^2\alpha_1^2\Sigma - 2\alpha_2^2(\Sigma - 3H^2)$$

$$+ a^{-4}\Big(\frac{\zeta^2}{2} + \zeta\alpha_1 + \alpha_1^2\Big)(\partial^k N_j^{(1)}\partial_k N^{j(1)} - \partial^j N_j^{(1)}\partial^k N_k^{(1)}) - 2a^{-4}(\zeta + \alpha_1)\partial^k N^{j(1)}(\partial_k N_j^{(2)} - \delta_{kj}\partial^n N_n^{(2)} - 2\partial_j\zeta$$

$$+ a^{-4}(-4\partial_k N_j^{(1)}\partial^j\zeta N^{k(2)} + 2N_k^{(1)}\partial_j\zeta N^{k(1)}\partial^j\zeta) + \frac{a^{-4}}{2}\partial_k\tilde{N}_j^{(2)}\partial^k\tilde{N}^{j(2)} - 2a^{-4}\partial_k N_j^{(2)}(\partial^j\zeta N^{k(1)} + \partial^k\zeta N^{j(1)})\Big].$$

$$\begin{aligned}\frac{4H}{a^2}\partial^2\psi_2 = {}& -2a^{-2}\partial_i\zeta(\partial^i\zeta + 2H\partial^i\psi_1) \\ & - 4\alpha_1(a^{-2}\partial_i\partial^i\zeta - 2\Sigma\zeta) - 2\alpha_1^2(\Sigma + 6\lambda) \\ & - a^{-4}(\partial^i\partial_k\psi_1\partial_i\partial^k\psi_1 - \partial^2\psi_1\partial^2\psi_1) \\ & + 4\alpha_2(\Sigma - 3H^2),\end{aligned}$$

What is Eternal Inflation?

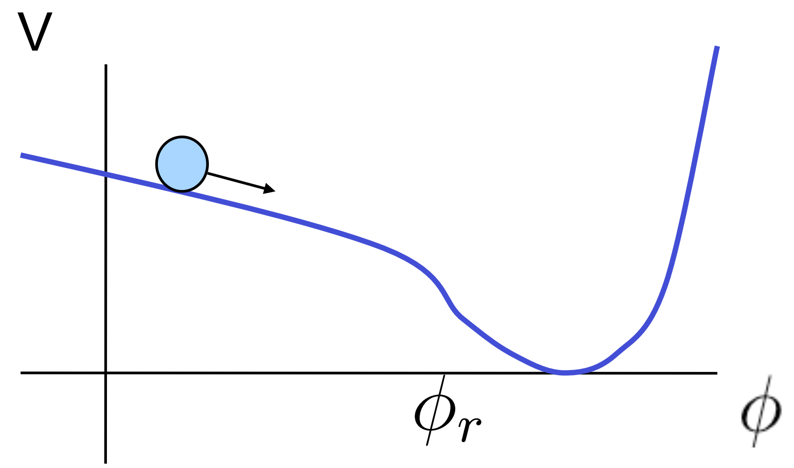
with P. Creminelli, S. Dubovsky, A. Nicolis, M. Zaldarriaga, **JHEP 2008**

with S. Dubovsky and G. Villadoro **JEHP 2009** **11.2011 [hep-th]**

- What is Inflation?

$$a \sim e^{Ht}$$

$$\dot{\phi} \sim \frac{V'}{H}$$



- What is Eternal Inflation?

Classical Motion Vs Quantum Motion

$$\Delta\phi_{\rm Cl} \sim \dot{\phi}\, H^{-1} \quad \text{Vs} \quad \Delta\phi_{\rm Q} \sim H$$

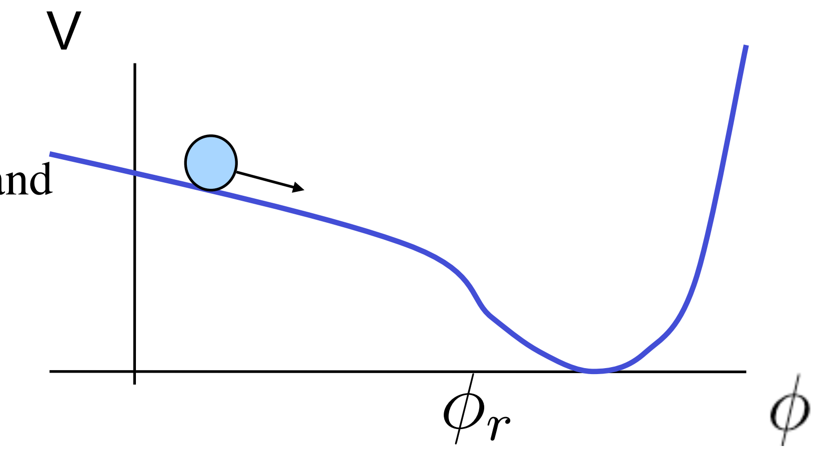
Reproduction of space

Quantum dominates for $\frac{\dot{\phi}}{H^2} \lesssim 1 \Longrightarrow$ Slow Roll Eternal Inflation

Eternal Inflation

- If $\langle \delta\phi_k^2 \rangle \sim \dfrac{H^2}{k^3} \quad \Rightarrow$

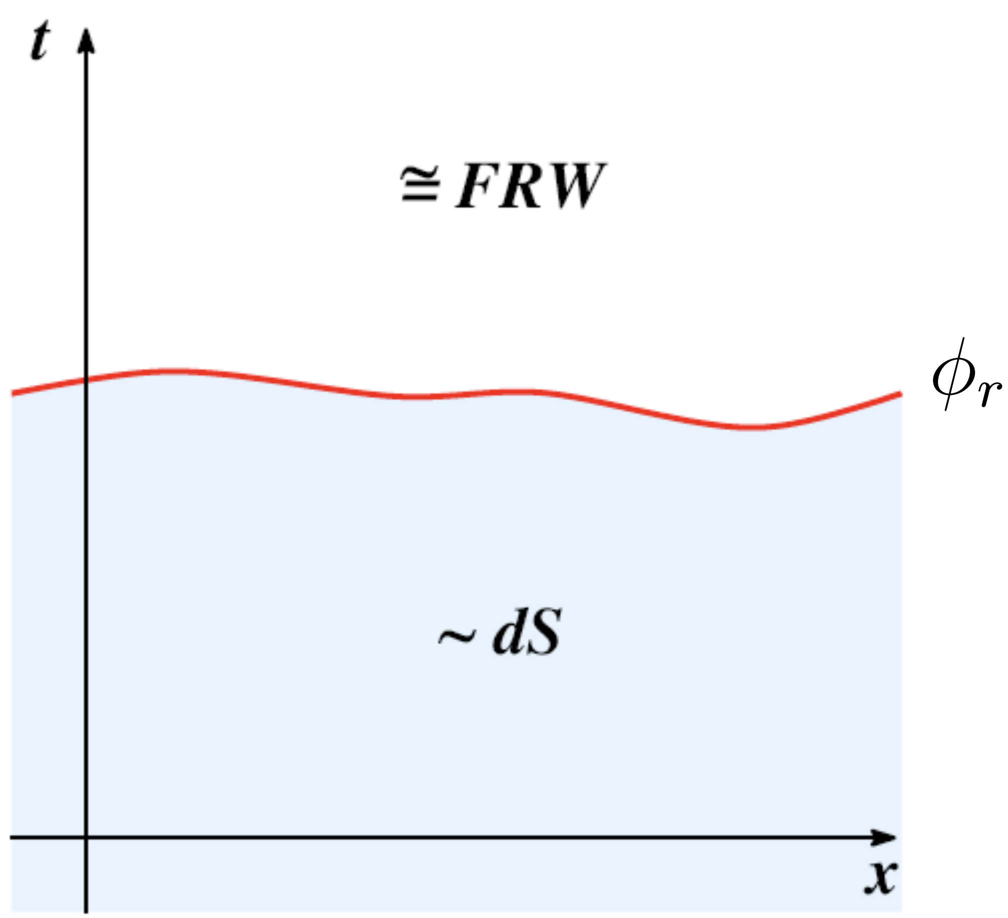
With Creminelli, Dubovsky, Nicolis and Zaldarriaga **JHEP 2008**
With Gorbenko **2019**



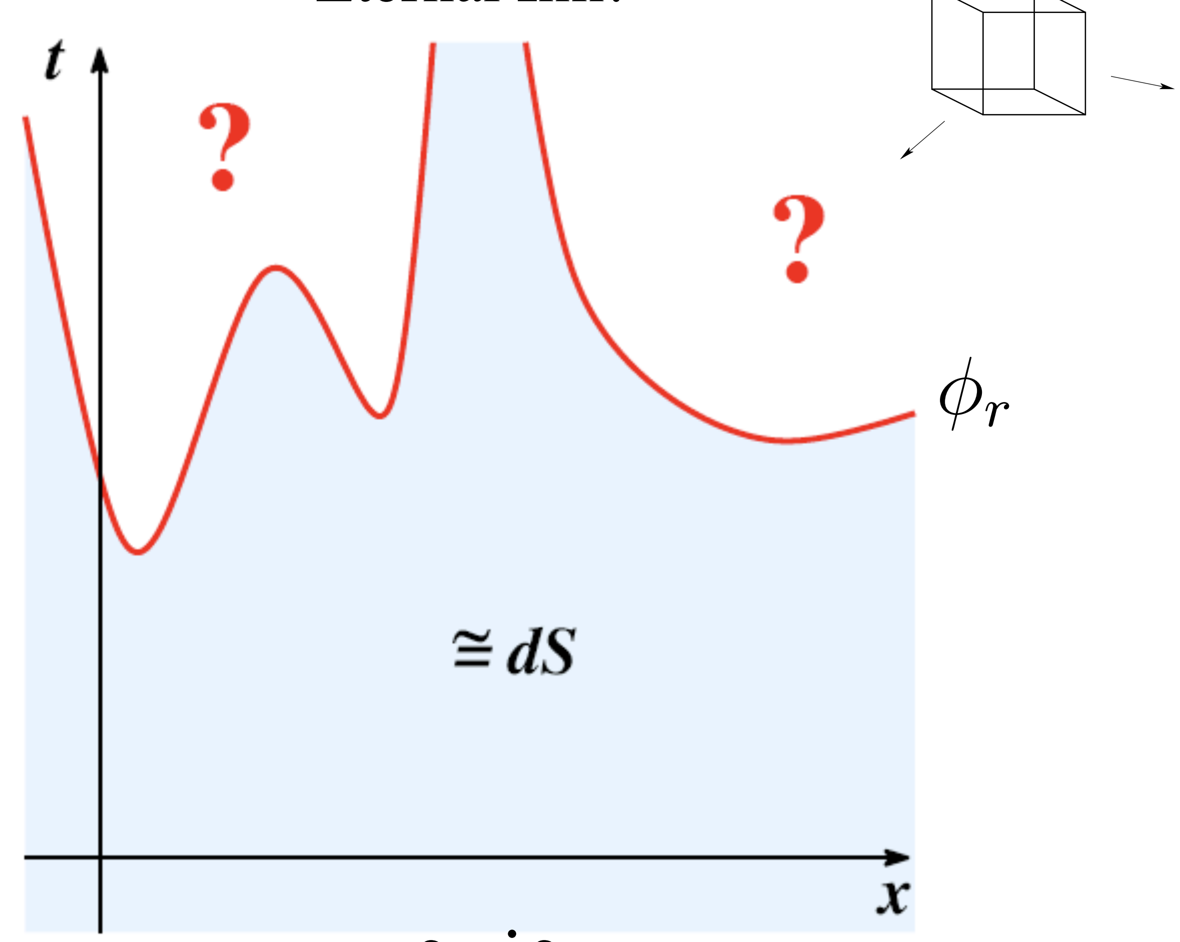
$$\langle \delta\phi(x,t)^2 \rangle = \int^{\Lambda a[t]} d^3k \frac{H^2}{k^3} \sim H^2 \log(a) \sim H^3 t + \text{ const.} \ ,$$

- With this you can prove that slow roll eternal inflation exists

Standard Infl.

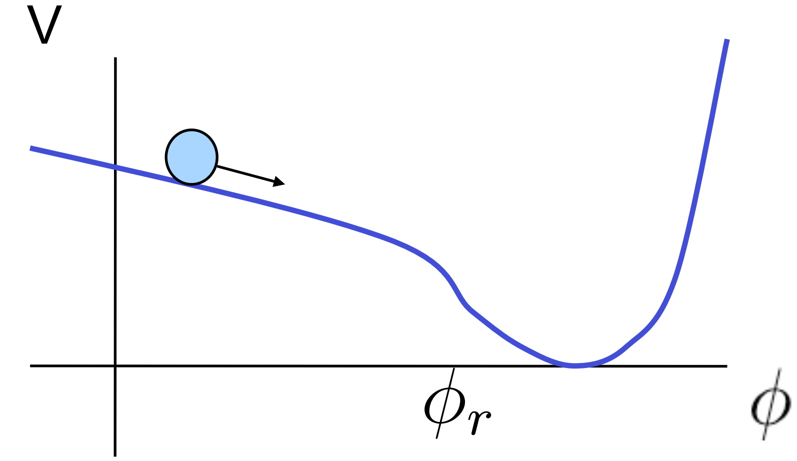


Eternal Infl.



- Sharp phase transition: $P(V = \infty) \neq 0$ for $\Omega \equiv \dfrac{2\pi^2}{3} \dfrac{\dot{\phi}^2}{H^4} < 1$

Eternal Inflation: the Universal Volume Bound



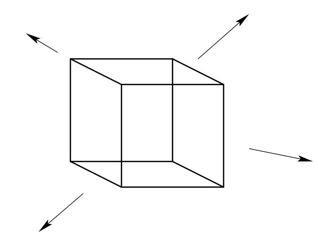
- With quite more work:

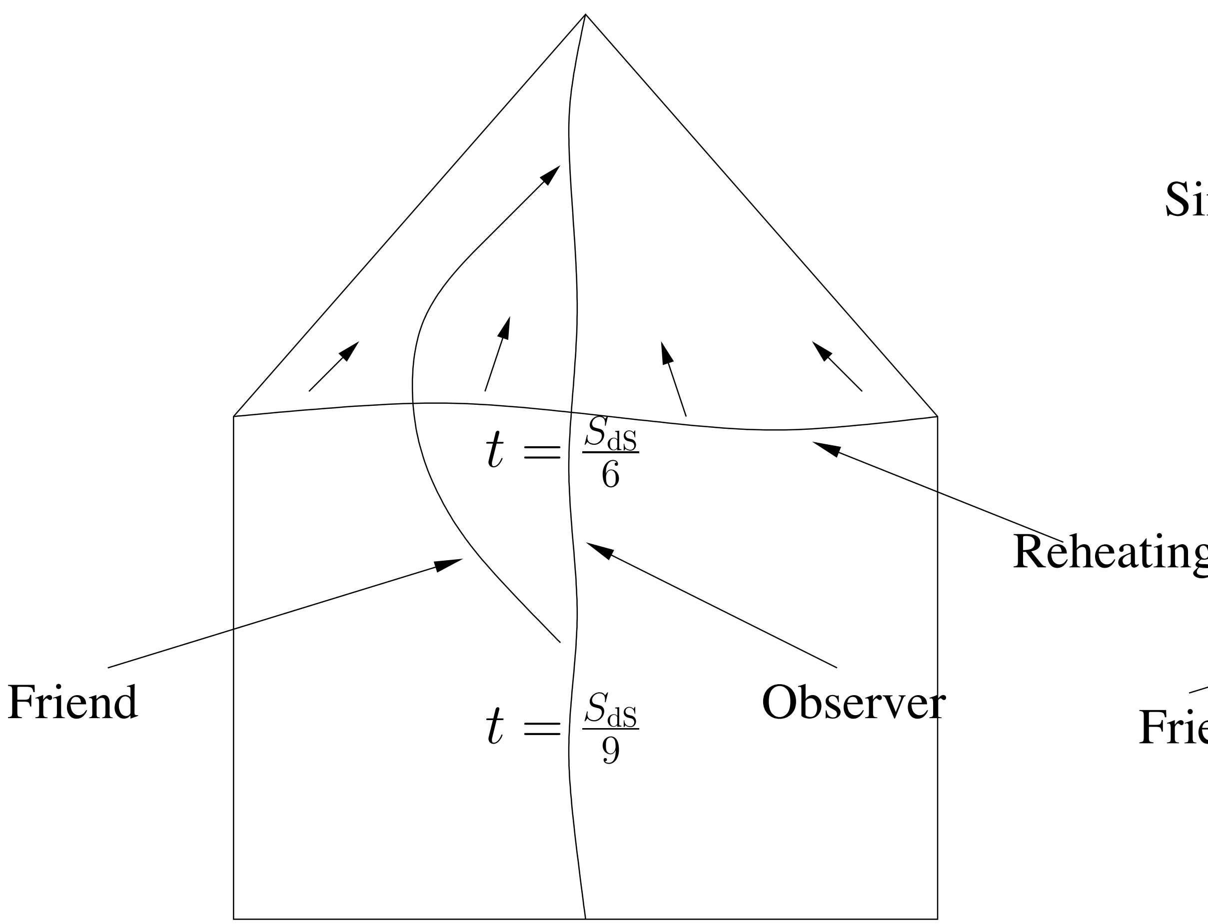
With Dubovsky and Villadoro
JHEP 2009, JHEP 2012
generalization of
Arkani-Hamed *et al*.
JHEP 2007

$$P(V > e^{\frac{S_{ds}}{2}}) < e^{-\alpha S_{ds}}$$

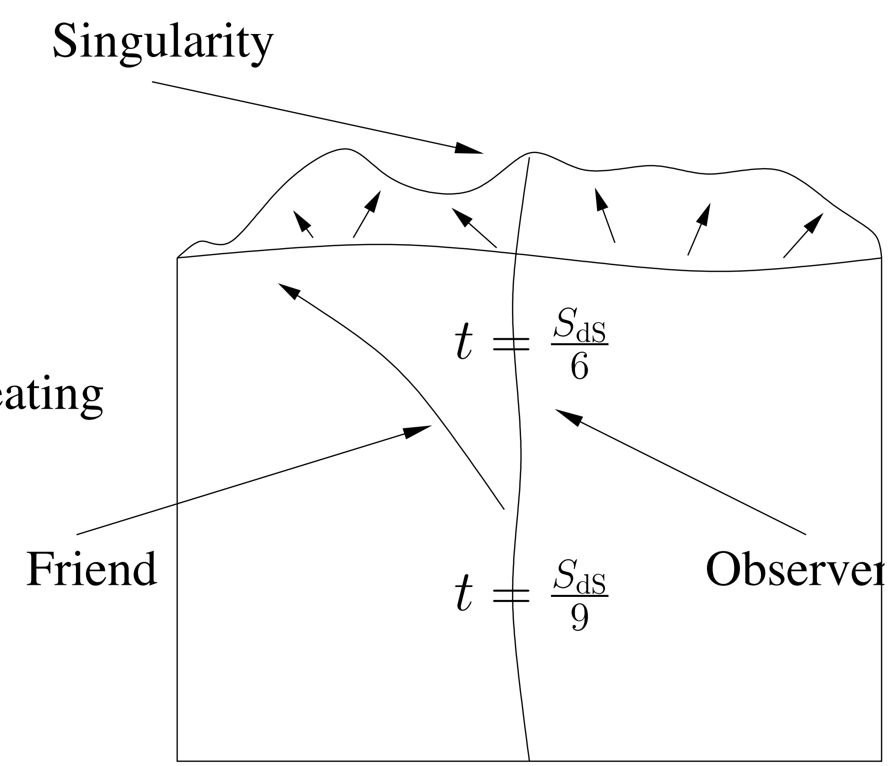
$$V_{\text{Finite Realization}} < e^{\frac{S_{ds}}{2}}$$

- A consistency check for Holography, a possible xerox paradox



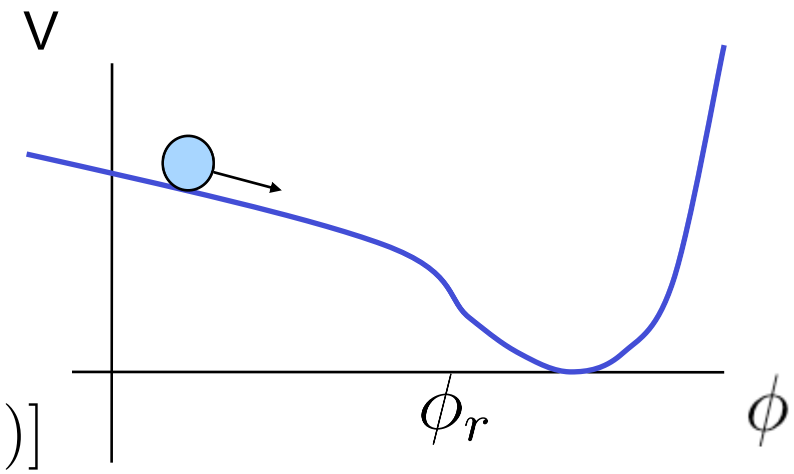


Xerox Paradox



No Xerox Paradox

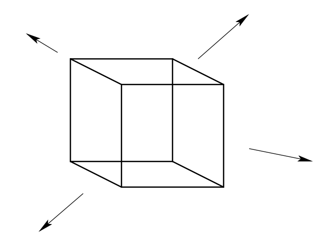
Eternal Inflation

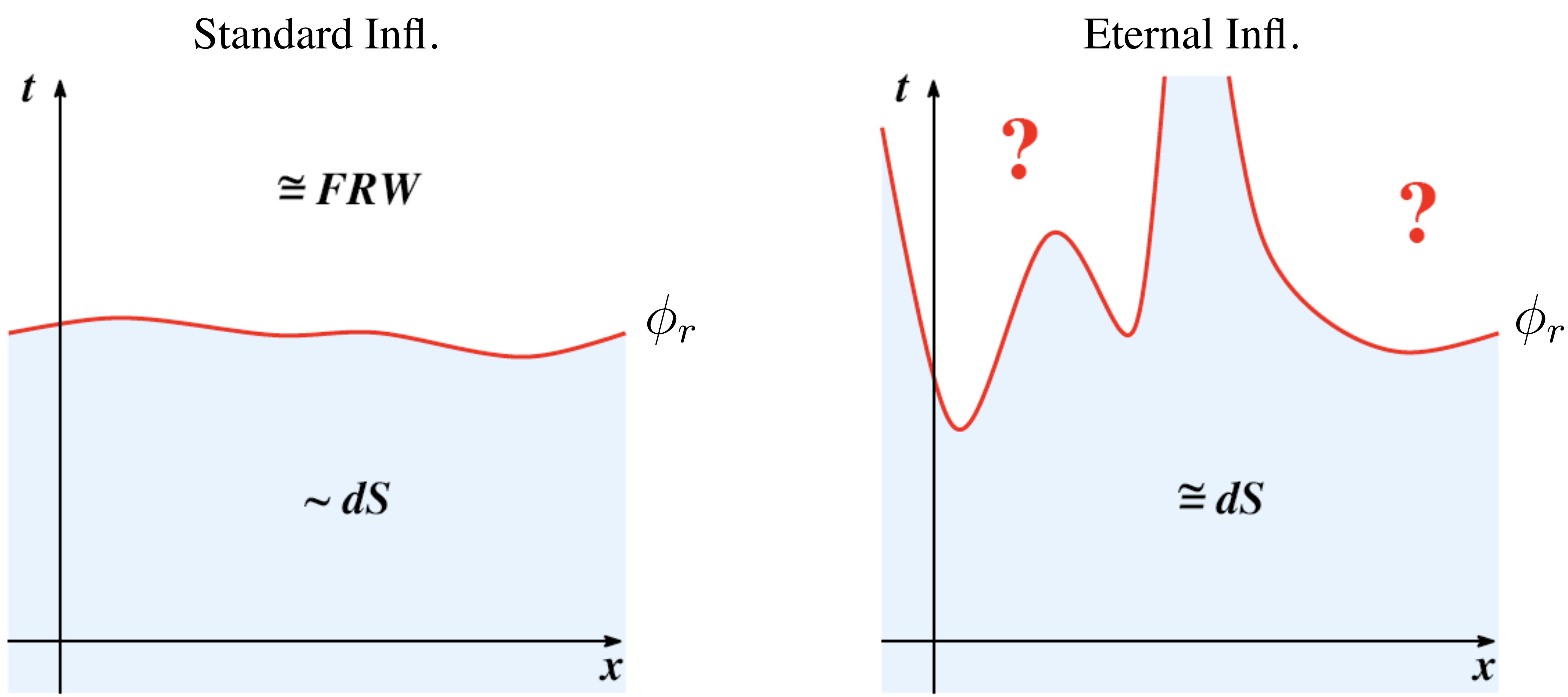


- All of this fails if

$$\langle \delta\phi_k^2 \rangle_{1-\text{loop}} \sim \frac{H^2}{k^3}\frac{H^2}{M_{\text{Pl}}^2} \times [\log(k) \ \text{or} \ \log(a(t))]$$

$$\langle \delta\phi^2(x,t) \rangle_{1-\text{loop}} \sim t^2$$

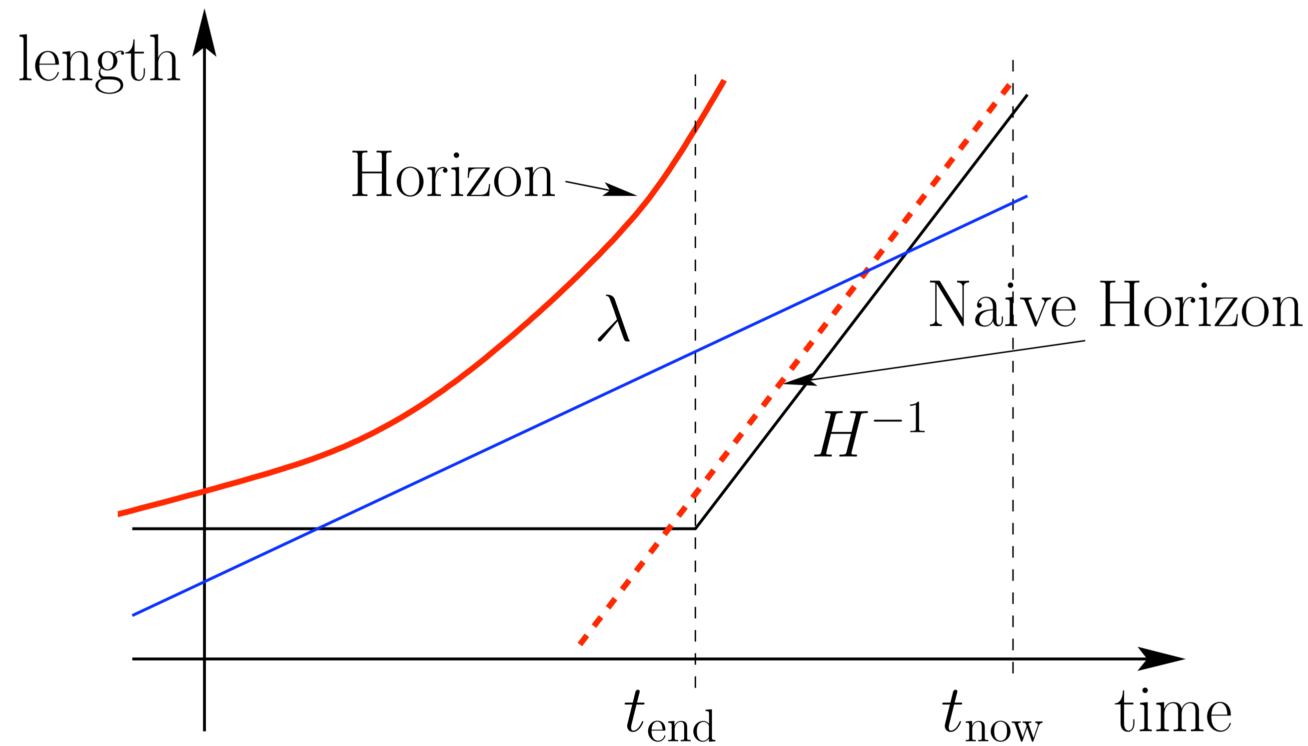




Predictivity of Inflation

- If $$\langle\zeta_k^2\rangle_{1-\text{loop}} \sim \langle\zeta_k^2\rangle_{\text{tree}}\frac{H^4}{\dot{H}M_{\text{Pl}}^2}\log\left(a(t)\right)$$ when $k/a \ll H$

  – if ζ is time in-dependent: ok



  – if ζ is time-dependent, we need to know the history.

  – this is a weakly coupled version of what could happen at other epochs

claims by **Woodard** *et al*.
Phys. Lett. B (2010)
several recent authors

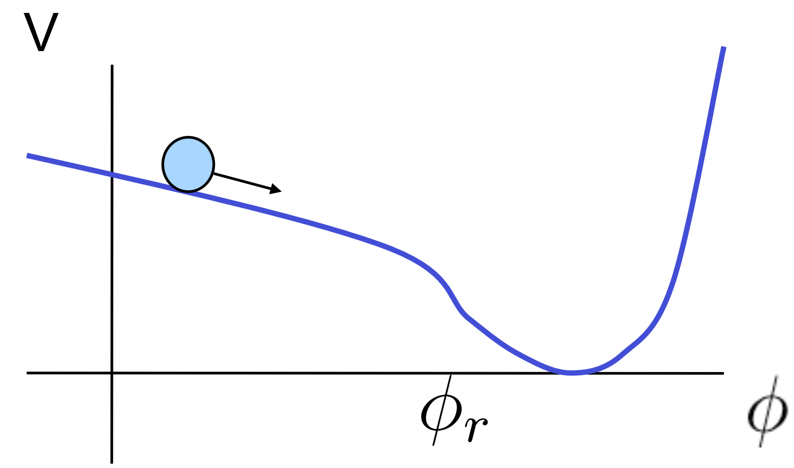
What does it mean to compute Loop Corrections?

Inflationary density fluctuations

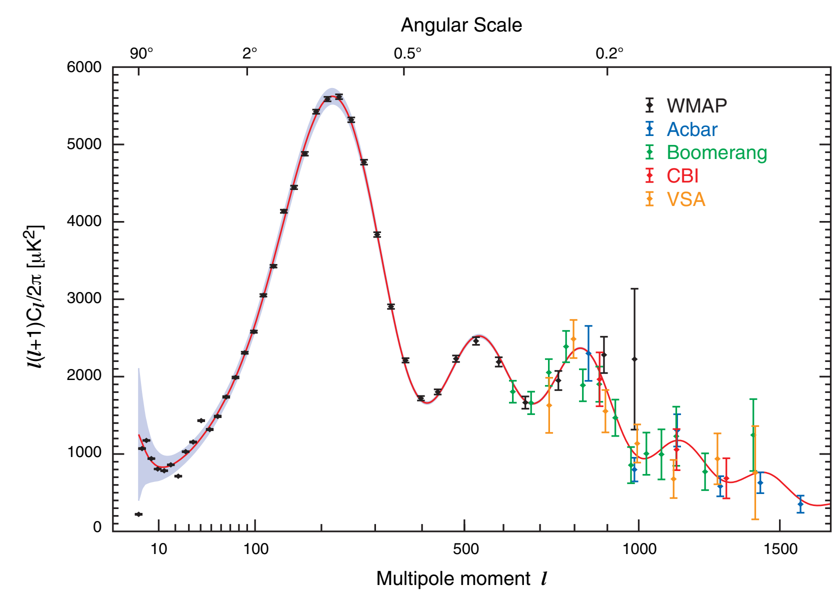
- Expand fluctuations and choose gauge

$$ds^2 = -N^2 dt^2 + \delta_{ij} a(t)^2 e^{2\zeta} \left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right) \ ,$$

$$\delta\phi = 0$$



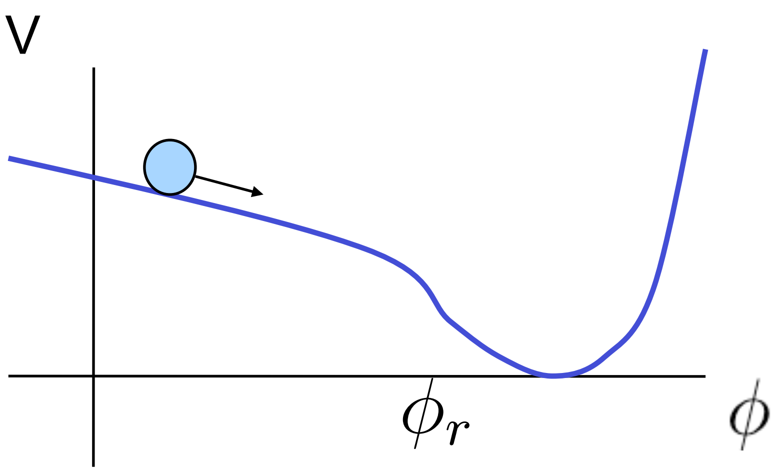
- Action $\quad S = \int d^4x \, a^3 \, \frac{\dot{H} M_{\rm Pl}^2}{H^2} \, (\partial\zeta)^2 + \dots$



- Quantize $\quad \hat{\zeta} = \zeta_{cl}(k,t) a_k + \zeta_{cl,}(k,t)^\star a_k^\dagger$
- Solve equations $\quad \langle \zeta_k \zeta_{k'} \rangle \sim \delta^{(3)}(k+k') \frac{1}{k^3} \, \frac{H^4}{\dot{H} M_{\rm Pl}^2}$
- Non-linear terms $\Rightarrow$ Non-Gaussianities
- $\Rightarrow$ Loop corrections

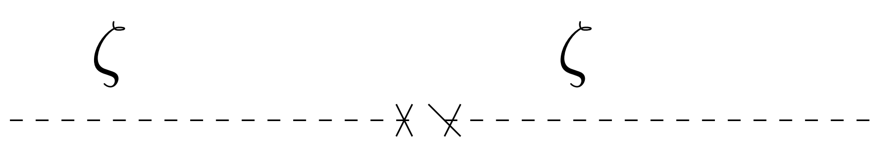
Rule of Thumb

- Single Field Slow-Roll Inflation
- $\langle \delta\phi_k^2 \rangle \sim H^2\Big|_{t_{h.c.}}$ where $\frac{k}{a(t_{h.c})} \sim H(t_{h.c.})$



$$\langle \zeta_k^2 \rangle^{1/2} \sim H\delta t_k \sim H\frac{\delta\phi_k}{\dot\phi} \sim \left(\frac{H}{\dot H M_{\rm Pl}^2}\right)^{1/2}\Bigg|_{t_{h.c.}}$$

- Only horizon-crossing time matters.
- Is this true at quantum level?



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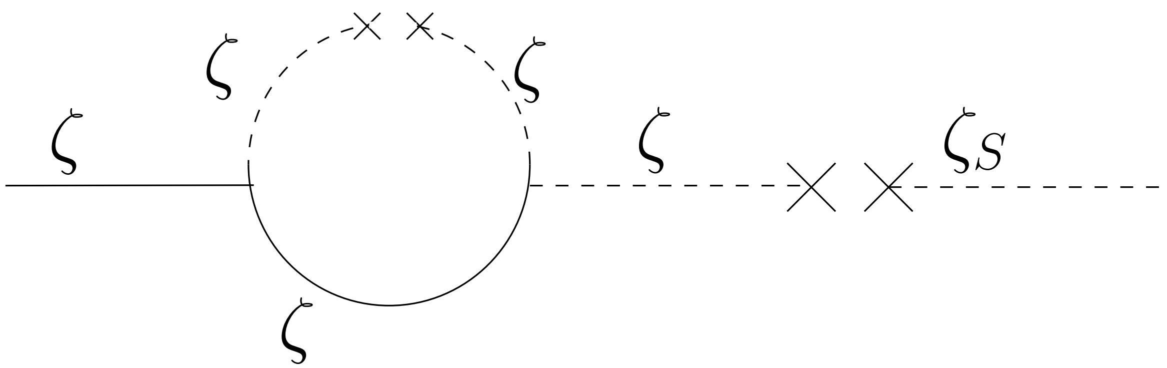

- ...Yes, but with care...

Physical Organization of Diagrams

What we compute

- We want

$$\langle\Omega|\zeta(t)\zeta(t)|\Omega\rangle = \langle 0|U_{int}(t,-\infty_+)^\dagger \zeta_I(t)\zeta_I(t)U_{int}(t,-\infty_+)|0\rangle$$

- with

$$U_{int}(t,-\infty_+) = Te^{-i\int_{-\infty_+}^{t} dt'\, H_{int}(t')}\ ,$$

- At one loop:

$$\langle\Omega|\zeta(t)\zeta(t)|\Omega\rangle = -2\ \mathrm{Re}\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \langle [H_3(t_2), [\ H_3(t_1), \zeta_k(t)]]\ \zeta_k(t)\rangle$$

$$+2\ \mathrm{Re}\langle \left[i\int_{-\infty}^{t} dt_1 H_4(t_1), \zeta(t)\right]_k \zeta_k(t)\rangle$$

$$-\langle \left[\int_{-\infty}^{t} dt_1\ H_3(t_1), \zeta_k(t)\right] \left[\int_{-\infty}^{t} dt_2\ H_3(t_2), \zeta_k(t)\right]\rangle$$

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$$U_{int}(t,-\infty_+) = Te^{-i\int_{-\infty_+}^{t} dt'\, H_{int}(t')} \;,$$

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What we compute

- .

$$\langle \zeta^{(3)} \zeta^{(1)} \rangle$$

- At one loop:

$$\langle \Omega | \zeta(t)\zeta(t) | \Omega \rangle = -2 \,\mathrm{Re} \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \langle [H_3(t_2), [\, H_3(t_1), \zeta_k(t)]]\, \zeta_k(t) \rangle$$

$$+2 \,\mathrm{Re} \langle \left[i \int_{-\infty}^{t} dt_1 H_4(t_1), \zeta(t) \right]_k \zeta_k(t) \rangle$$

$$- \langle \left[\int_{-\infty}^{t} dt_1 \; H_3(t_1), \zeta_k(t) \right] \left[\int_{-\infty}^{t} dt_2 \; H_3(t_2), \zeta_k(t) \right] \rangle$$

What we compute

- .

$$\text{Cut In the Side} = \text{CIS} = \left\langle \zeta^{(3)} \zeta^{(1)} \right\rangle$$

- At one loop:

$$\langle \Omega | \zeta(t)\zeta(t) | \Omega \rangle = -2\,\text{Re} \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \langle [H_3(t_2), [\, H_3(t_1), \zeta_k(t)]]\, \zeta_k(t) \rangle$$

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What we compute

- .

Cut In the Side = CIS = $\left\langle \zeta^{(3)} \zeta^{(1)} \right\rangle$

- At one loop:

$$\langle \Omega | \zeta(t) \zeta(t) | \Omega \rangle = -2 \text{ Re} \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \langle [H_3(t_2), [\, H_3(t_1), \zeta_k(t)]] \, \zeta_k(t) \rangle$$

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What we compute

- .

Cut In the Middle = CIM = $\left\langle \zeta^{(2)} \zeta^{(2)} \right\rangle$

- At one loop:

$$\langle \Omega | \zeta(t) \zeta(t) | \Omega \rangle = -2\ \mathrm{Re} \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \langle [H_3(t_2), [H_3(t_1), \zeta_k(t)]]\, \zeta_k(t) \rangle$$

$$+2\ \mathrm{Re} \langle \left[i \int_{-\infty}^{t} dt_1 H_4(t_1), \zeta(t) \right]_k \zeta_k(t) \rangle$$

$$- \langle \left[\int_{-\infty}^{t} dt_1\ H_3(t_1), \zeta_k(t) \right] \left[\int_{-\infty}^{t} dt_2\ H_3(t_2), \zeta_k(t) \right] \rangle$$

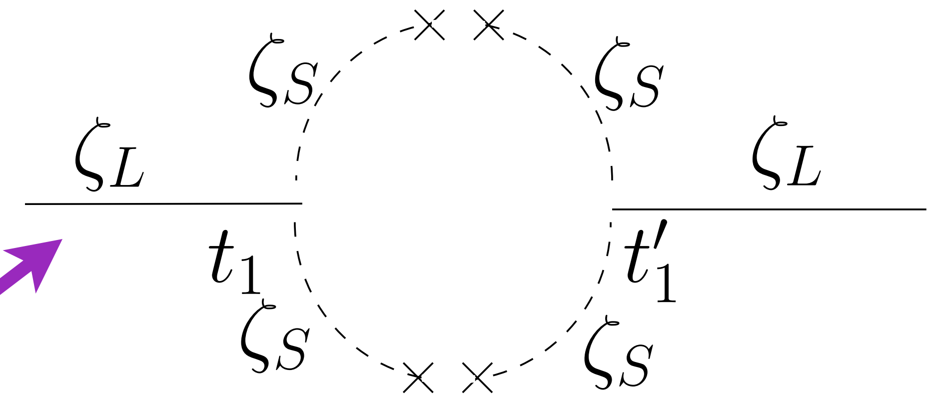
Diagrammatic Representation

- $\partial_t^2 \zeta \simeq \zeta^2 + \zeta^3 + \dots \qquad \Rightarrow \qquad \zeta^{(2)}(t) \sim \int G(t-t')\zeta^{(1)}(t')^2$

- Green's Function

$$G^R_\zeta(x,x') = i\theta(t-t')\,[\zeta(x),\zeta(x')]$$

- At one loop:



Green's function

Correlation function $\langle \zeta^{(2)} \zeta^{(2)} \rangle$

$$-\left\langle \left[\int_{-\infty}^{t} dt_1\; H_3(t_1), \zeta_k(t)\right] \left[\int_{-\infty}^{t} dt_2\; H_3(t_2), \zeta_k(t)\right] \right\rangle$$

Diagrammatic Representation

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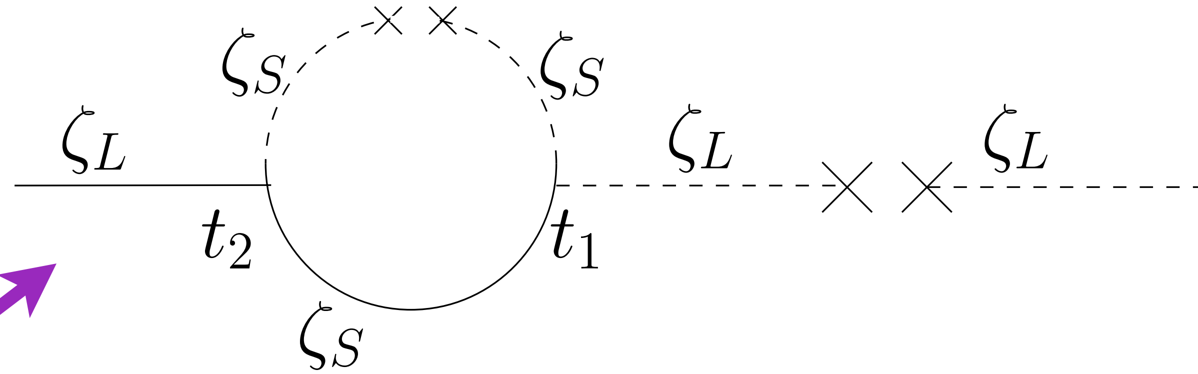


- At one loop:

Green's function

Correlation function $\langle \zeta^{(3)} \zeta^{(1)} \rangle$

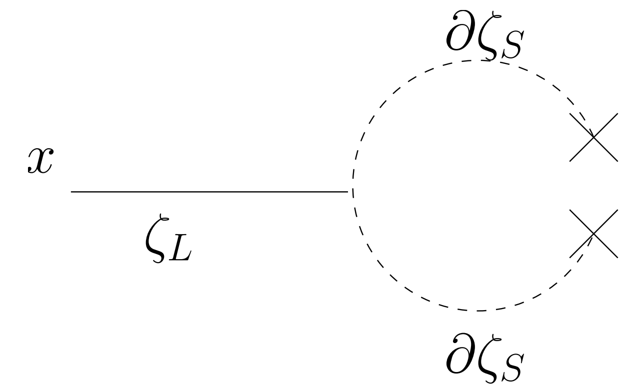
$$-2\,\mathrm{Re}\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \langle [H_3(t_2), [\,H_3(t_1), \zeta_k(t)]]\,\zeta_k(t)\rangle$$

1-point Function
Tadpole Diagrams

Tadpole Diagrams

- .

with Zaldarriaga **0912:2734** [hep-th]



- You need to renormalize the history
- and define ζ accordingly

$$ds^2 = -dt^2 + a(t)_B^2 e^{2\zeta} dx^2$$

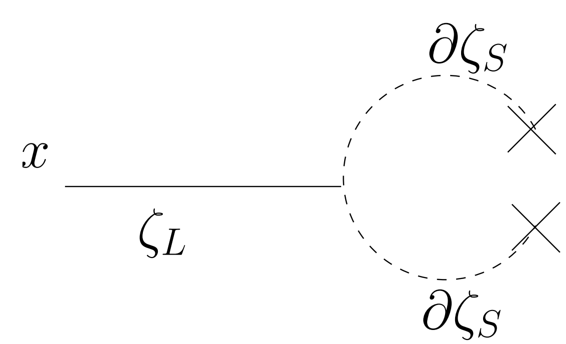
- Add counterterms:

$$\mathcal{S}_{tad} = \int d^4\sqrt{-g}\,\left[\sqrt{-g}g^{00}\left(M_{\rm Pl}^2\dot{H} + \delta M^4\right) - \sqrt{-g}M_{\rm Pl}^2\left(\left(3H^2 + \dot{H}\right) + \delta\Lambda\right)\right]$$

$$\delta M^4 \sim \langle\dot{\zeta}^2\rangle + \ldots \qquad \delta\Lambda \sim \langle\dot{\zeta}^2\rangle + \langle(\partial_i\zeta)^2\rangle + \ldots$$

Based on EFT of Inflation
with C. Cheung, P. Creminelli,
L. Fitzpatrick, J. Kaplan
JHEP 2008

- Tadpole cancellation:



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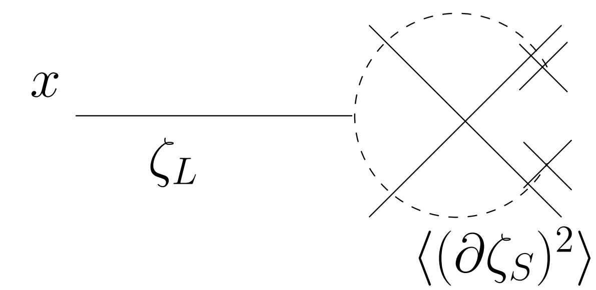


=0

- Define ζ here.

2-Point Function

Log Running

Log Running

- Weinberg's result $\langle \zeta_k^2 \rangle_{1-\text{loop}} \sim \langle \zeta_k^2 \rangle_{\text{tree}} \log\left(k/\mu\right)$

S.Weinberg **PRD2005**
and others thereafter

- Gives you all these troubles (Eternal Inflation, Predictivity of Inflation)
- But problem with gauge symmetry $a \to \lambda\, a\ , \qquad x \to x/\lambda\ , \qquad k \to \lambda\, k$
- Study simplest possible theory

$$S = \int d^4x\ a^3 \left[-\dot{H} M_{\rm Pl}^2 \left(\dot{\pi}^2 - \frac{1}{a^2}(\partial_i \pi)^2 \right) + \frac{2}{3} c_3 M^4 \left(2\dot{\pi}^3 + 3\dot{\pi}^4 - 3\frac{1}{a^2}\dot{\pi}^2 (\partial_i \pi)^2 \right) \right]$$

Based on EFT of Inflation
with C. Cheung, P. Creminelli,
L. Fitzpatrick, J. Kaplan
JHEP 2008

- Technical problem in implementing the regularization
- . $\langle \zeta_k^2 \rangle_{1-\text{loop}} \propto H^6 \log\left(H/\mu\right)$

with Zaldarriaga **JHEP 2010**

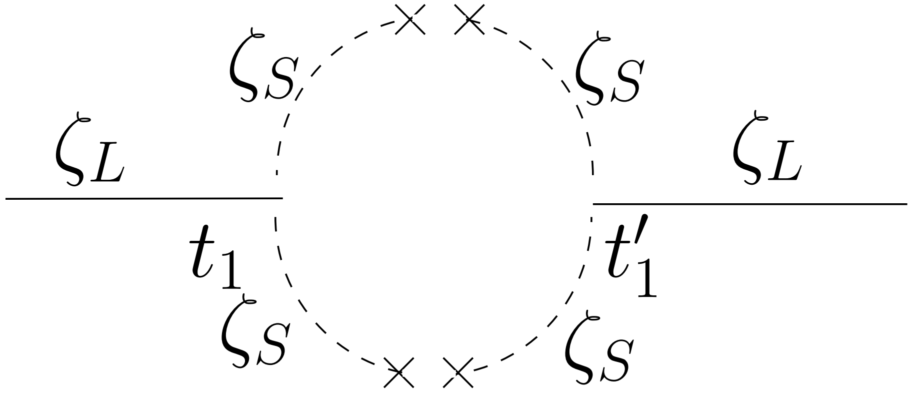
- Effect in the IR much larger than in Minkowski space

$$\langle \zeta_k^2 \rangle_{1-\text{loop}} \propto k^6 \log\left(k/\mu\right)$$

- Analogy with particle physics: small logs for $\mu \sim H$

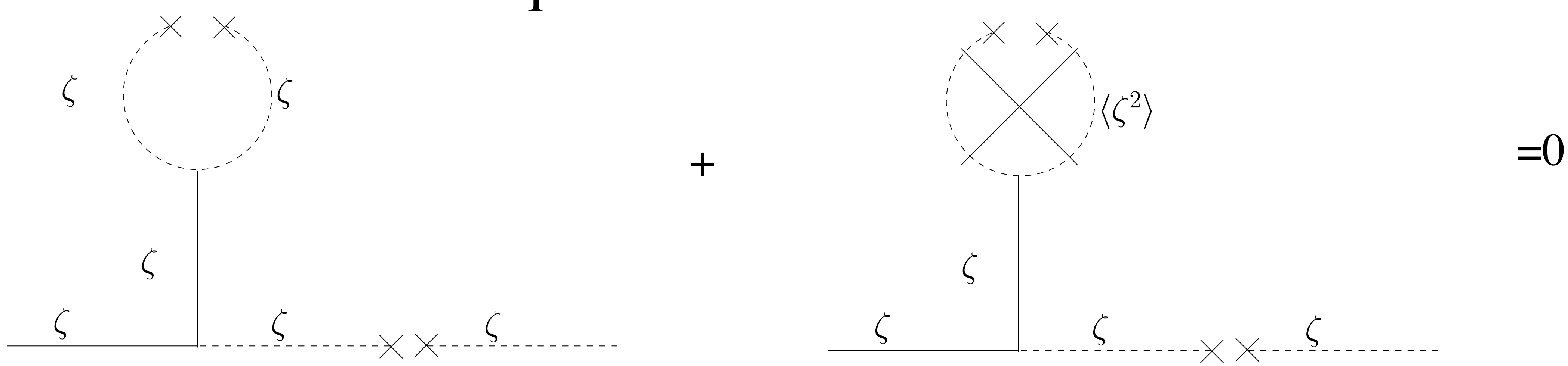
IR logs: just projection effects

Effects of shorter modes:
no induced ζ time dependence



Summary part I

- Short modes in the loop do not lead to a time-dependence for standard slow roll
- Subtle cancellations
  - Renormalization of the background
    - Cancellation between quartic and non-1PI



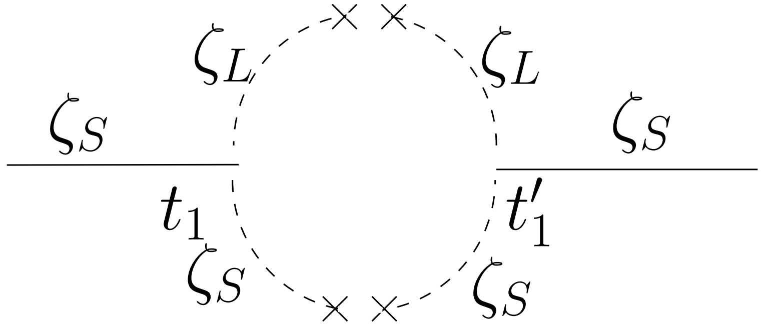
  - Consistency condition (as inflation is an attractor)
    - Locally a long-wavelength inflaton mode is unobservable

$$\frac{\partial}{\partial\zeta}\langle T_{\sigma,\zeta;\mu\nu}(k,t')\rangle\Big|_{\zeta=0} \to 0 \quad \text{as} \quad k\eta \to 0$$

- Huge, but quite physical, calculation

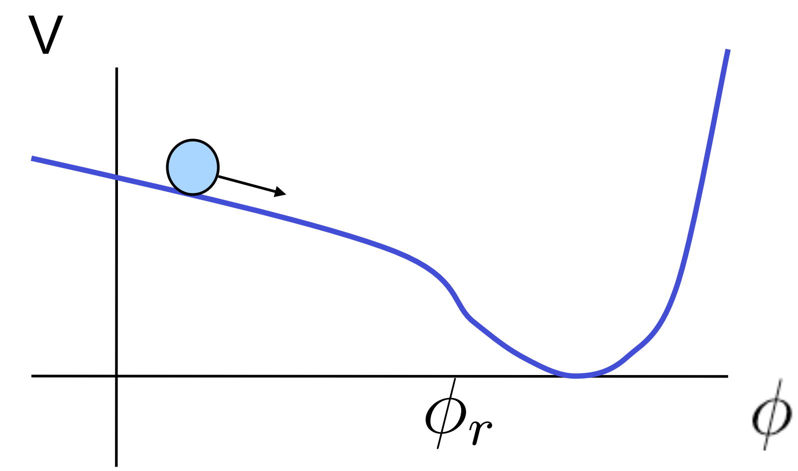
Effect of longer modes:

$\log(kL)$ IR logs



Large IR logs $\log(kL)$

- Single Field Slow-Roll Inflation (assumption on dynamics)

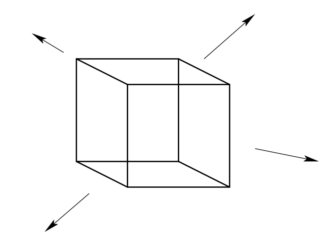


$$\langle \zeta_k^2 \rangle \sim \frac{H^4}{\dot\phi^2}\bigg|_{t_{h.c.}} \quad \text{where} \quad \frac{k}{a(t_{h.c})} \sim H(t_{h.c.})$$

- Possible Infrared Effects:

–Modes emitted earlier can change the position on the potential at horizon crossing

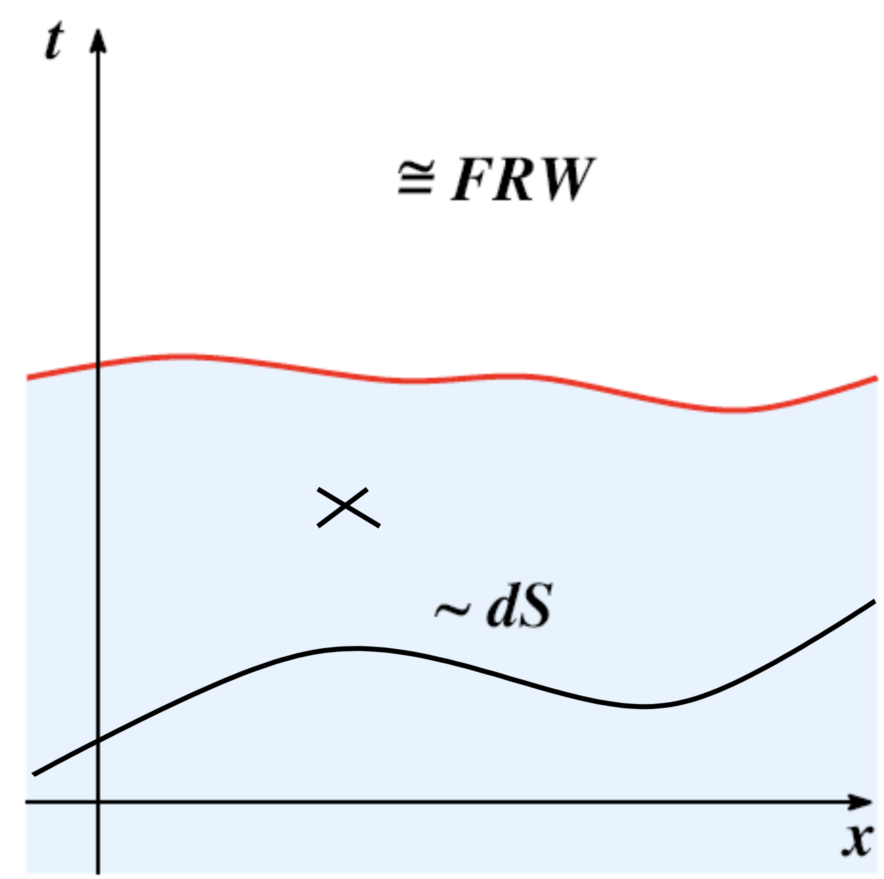
$$\langle \delta\phi(\vec{x},t)^2 \rangle \sim H^3 t \sim H^2 N_{beginning}$$



–But this is all of its effect on the dynamics as:

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta_B} dx^2$$

$$\langle \zeta(\vec{x}_1,t)\zeta(\vec{x}_2,t)\rangle_B = \langle \zeta(e^{-\zeta_B}\vec{x}_1,t)\zeta(e^{-\zeta_B}\vec{x}_2,t)\rangle_0$$



–By Taylor expanding

$$\langle \zeta_k \rangle_B = \langle \zeta_B^2 \rangle \frac{\partial^2 \left[k^3 \langle \zeta_k^2 \rangle\right]}{\partial \log(k)^2} = \widetilde{\langle \zeta \rangle}^2 N_{\rm beginning} \left((n_s-1)^2 + \alpha\right) \langle \zeta_k \rangle^2$$

$$\widetilde{\langle \zeta^2 \rangle} \sim 10^{-10} \qquad N_{beginning} \sim \log(kL)$$

–Very big effect

Giddings and Sloth **2010,11**, Hebecker **2010**

$\log(kL)$

How to check for this effect?

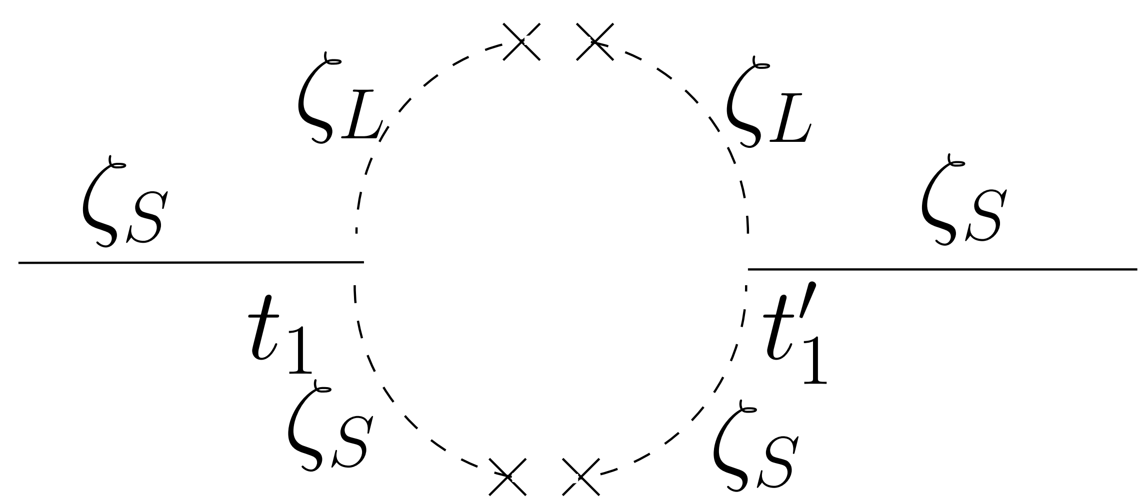
–Solve perturb. equations:

$$\partial_t^2\zeta \simeq \zeta^2 + \zeta^3 + \ldots \qquad \Rightarrow \qquad \zeta^{(2)}(t) \sim \int G(t-t')\zeta^{(1)}(t')^2$$

–Cut-In-the-Middle diagrams: Each mode interacts once

$$\langle \zeta^{(2)}\zeta^{(2)}\rangle_{1-\mathrm{loop}}$$

$$\zeta^{(2)}(t) = \int G(t-t')\zeta^{(1)}(t')^2$$

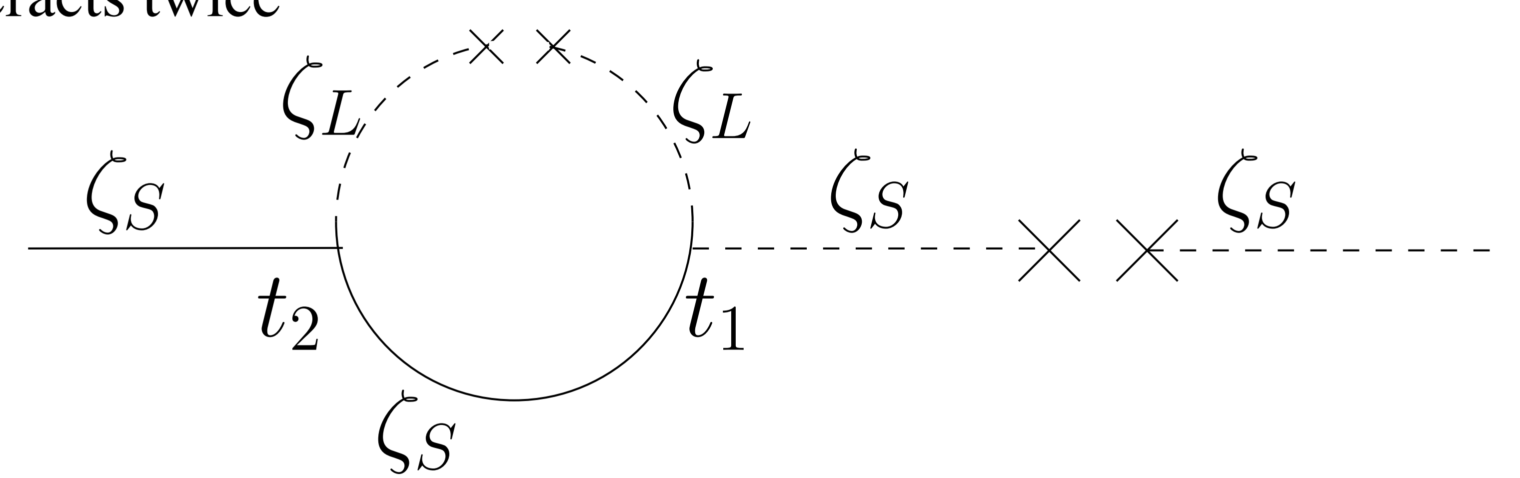


- This gives the tilt squared $(n_s-1)^2$

–Cut-In-the-Side diagrams: One mode interacts twice

$$\langle \zeta^{(3)}\zeta^{(1)}\rangle_{1-\mathrm{loop}}$$

$$\zeta^{(3)}(t) = \int G(t-t')\zeta^{(1)}(t')\zeta^{(2)}(t')$$

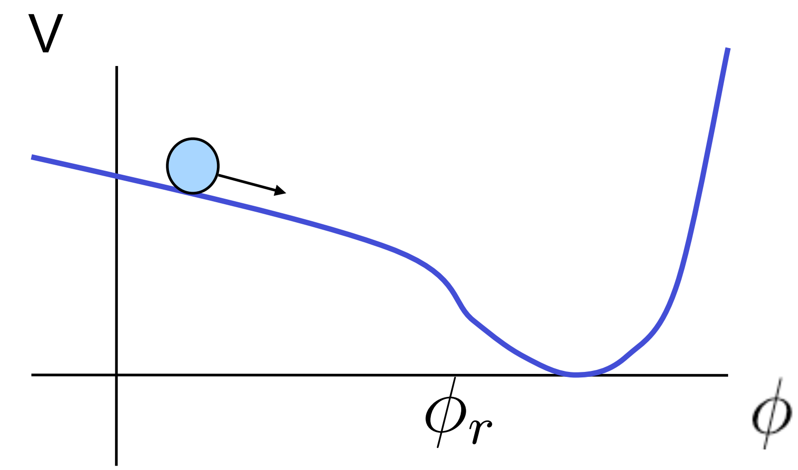


- This gives the running α

$$\langle\zeta_k\rangle_B = \langle\zeta_B^2\rangle\frac{\partial^2\left[k^3\langle\zeta_k^2\rangle\right]}{\partial\log(k)^2} = \widetilde{\langle\zeta\rangle}^2 N_{\mathrm{beginning}}\left((n_s-1)^2+\alpha\right)\langle\zeta_k\rangle^2$$

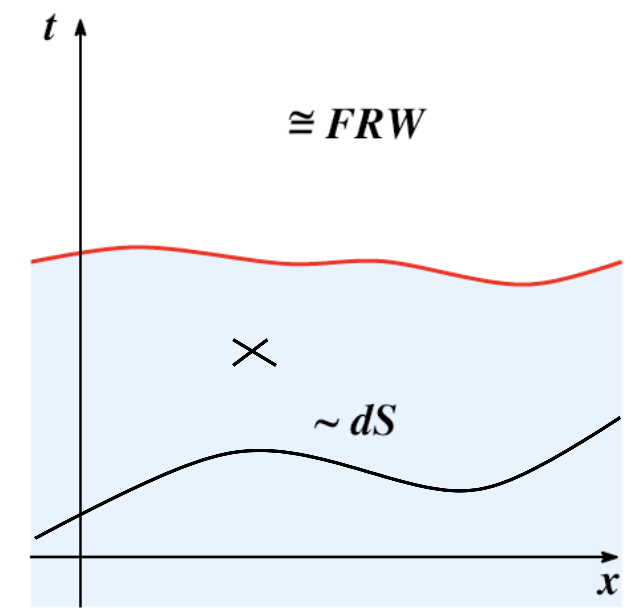
Large IR logs $\log(kL)$ =0 physically

- Single Field Slow-Roll Inflation: no dynamical effect

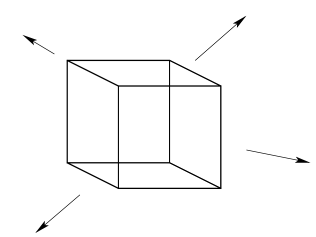


$$\langle\zeta_k^2\rangle \sim \left.\frac{H^4}{\dot\phi^2}\right|_{t_{h.c.}} \quad \text{where} \quad \frac{k}{a(t_{h.c})} \sim H(t_{h.c.})$$

$$\Rightarrow \qquad \langle\zeta_k\rangle_B = \langle\zeta_B^2\rangle \frac{\partial^2\left[k^3\langle\zeta_k^2\rangle\right]}{\partial\log(k)^2} = \widetilde{\langle\zeta\rangle}^2 N_{\text{beginning}}\left((n_s-1)^2+\alpha\right)\langle\zeta_k\rangle^2$$



- Let us concentrate on what an observer measures:



  – measures a distance $\Delta r(t_{\rm rh})$

  – ask when mode came out of the horizon:

    - Since a background mode is equal to a rescaling of scale factor

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta_B} dx^2$$

    - We have

$$\Delta r(t) = \frac{e^{\zeta(x,t)}a(t)}{e^{\zeta(x,t_{\rm rh})}a(t_{\rm rh})}\Delta r(t_{\rm rh}) = e^{\zeta(x,t)-\zeta(x,t_{\rm rh})}\frac{a(t)}{a(t_{\rm rh})}\Delta r(t_{\rm rh}) \simeq H^{-1}$$

    - Longer modes cancel exactly.

$$\zeta(x,t_{rh}) = \int d^3k\ \zeta_k(t_{rh})$$

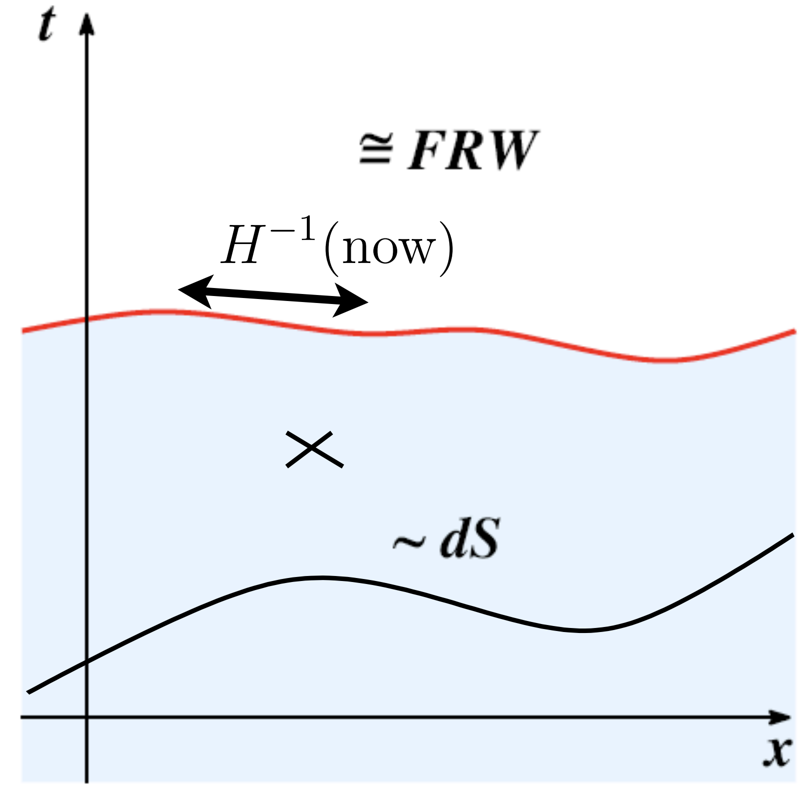
    - In every realization (no average needed).

Second projection effect:
a Physical IR one

A Physical IR effect

- If we do not see the gradients of ζ_B, we do not observe ζ_B

$$\langle \zeta_k^2 \rangle \sim \left.\frac{H^4}{\dot\phi^2}\right|_{t_{h.c.}} \qquad \text{where} \qquad \frac{k}{a(t_{h.c})} \sim H(t_{h.c.})$$



- Since $\quad k^{us}_{phys.} = \dfrac{k}{a(t_{reh.}) e^{\zeta_B(t_{reh})}}$

$$\frac{k}{a(t_{h.c.}) e^{\zeta_B(t_{h.c.})}} \cdot \left[a(t_{reh.}) e^{\zeta_B(t_{reh.})}\right] \sim H(t_{h.c.})$$

- Let us massage the new time of horizon-crossing

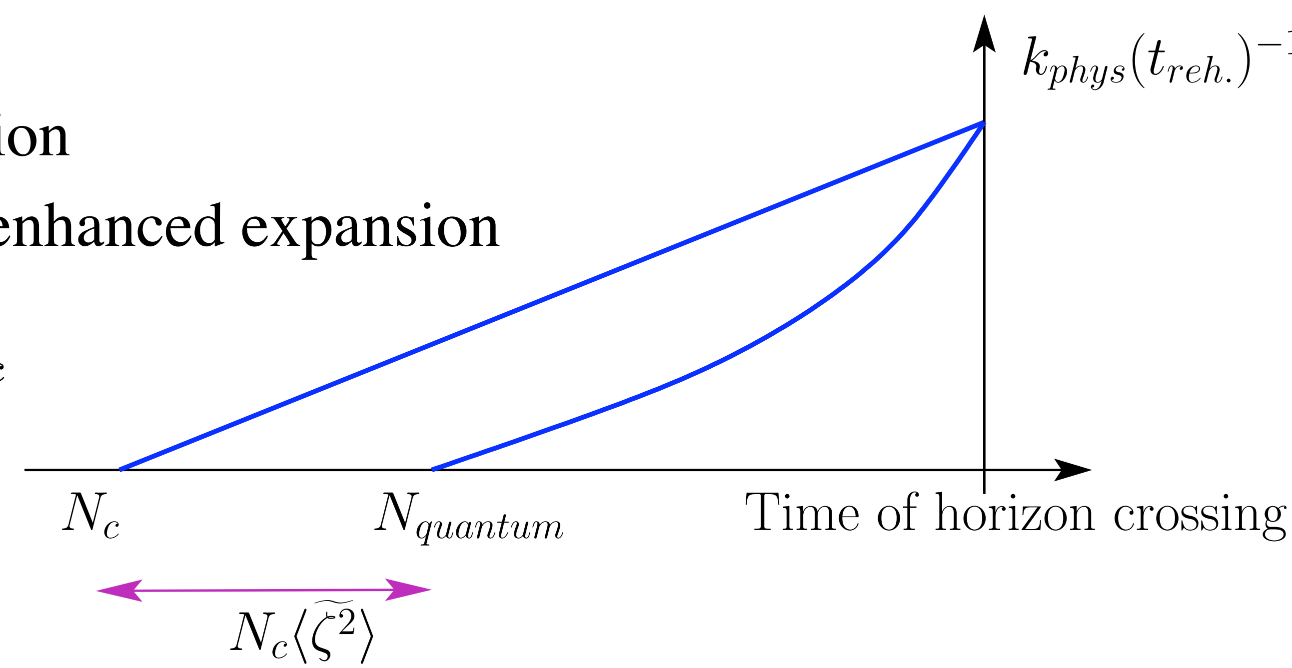
$$\frac{a(t_{(reh.)})}{a(t_{h.c.})} \sim e^{N_c}\ , \qquad N_c \sim 60$$

classical expansion

enhanced expansion

$$\langle e^{\zeta_B(t_{reh.}) - \zeta_B(t_{h.c.})} \rangle \sim 1 + \langle \zeta(x)^2 \rangle^{t_{h.c.}}_{t_{reh.}} \sim 1 + \widetilde{\langle \zeta^2 \rangle} N_c$$

$$\widetilde{\langle \zeta^2 \rangle} \sim 10^{-10}$$



- We have

$$k\ e^{N_c(1 + \widetilde{\langle \zeta^2 \rangle})} = H(t_{h.c.}) \quad \Rightarrow \quad \delta N \sim \widetilde{\langle \zeta^2 \rangle} N_c$$

- True IR (tiny) effect:

$$\langle \zeta_k^2 \rangle \sim \left.\frac{H^4}{\dot\phi^2}\right|_{t_c} \left(1 + (n_s - 1) N_c \widetilde{\langle \zeta^2 \rangle}\right)$$

Different tilt and N dependence

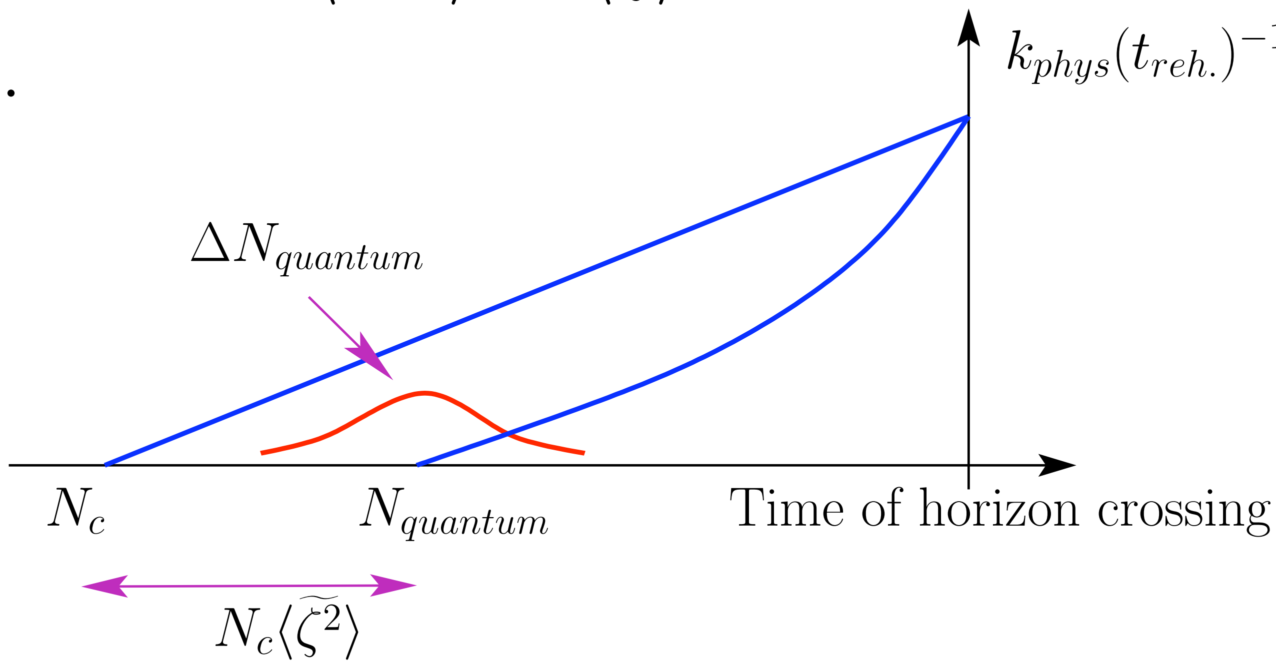
First issue

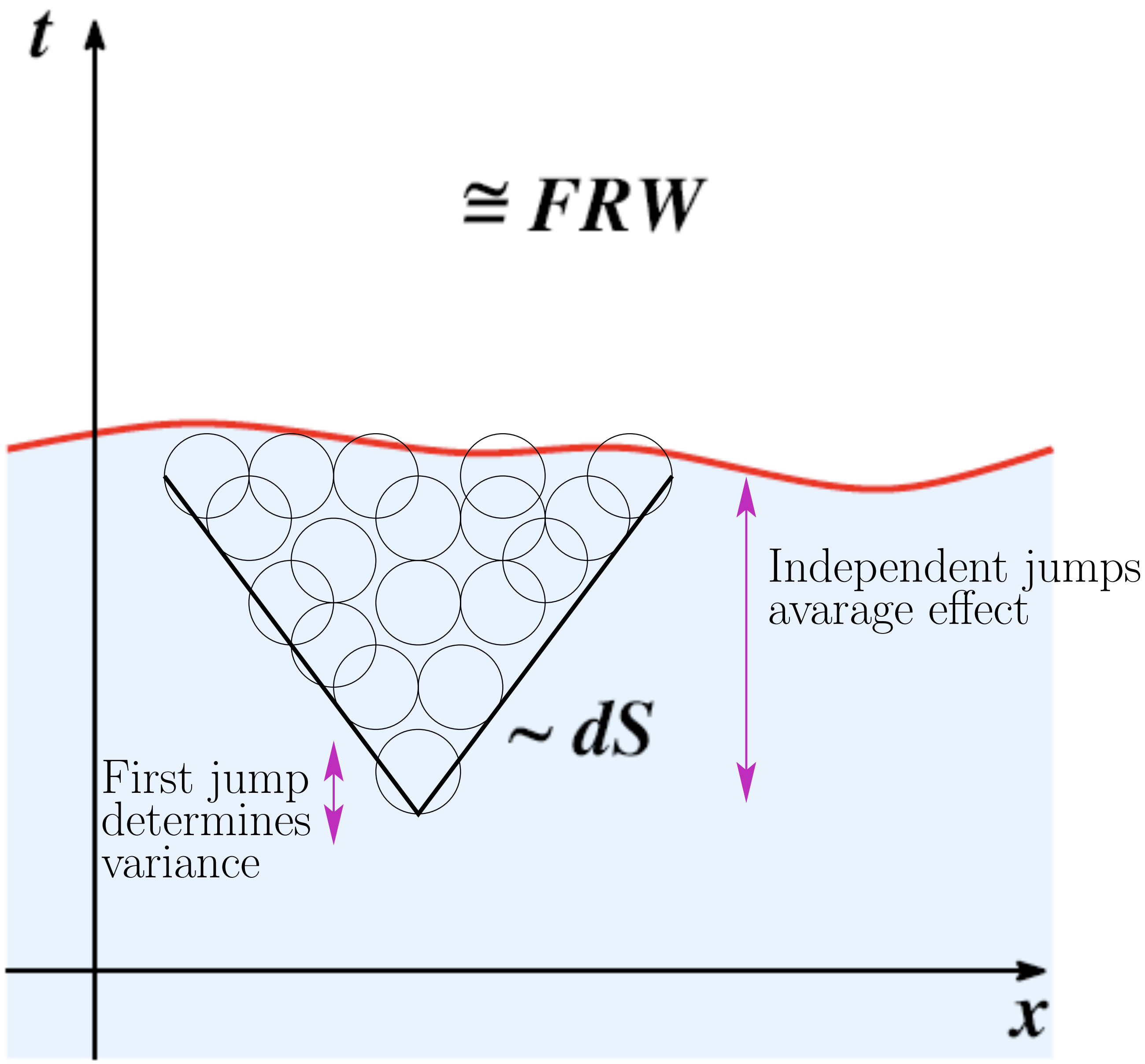
- Why did we took the average of the enhanced expansion?

$$\langle \delta N \rangle \sim \widetilde{\langle \zeta \rangle} N_c$$

 –Small variance:

$$\langle \Delta N^2_{quantum} \rangle^{1/2} \sim \widetilde{\langle \zeta^2 \rangle}^{1/2}$$





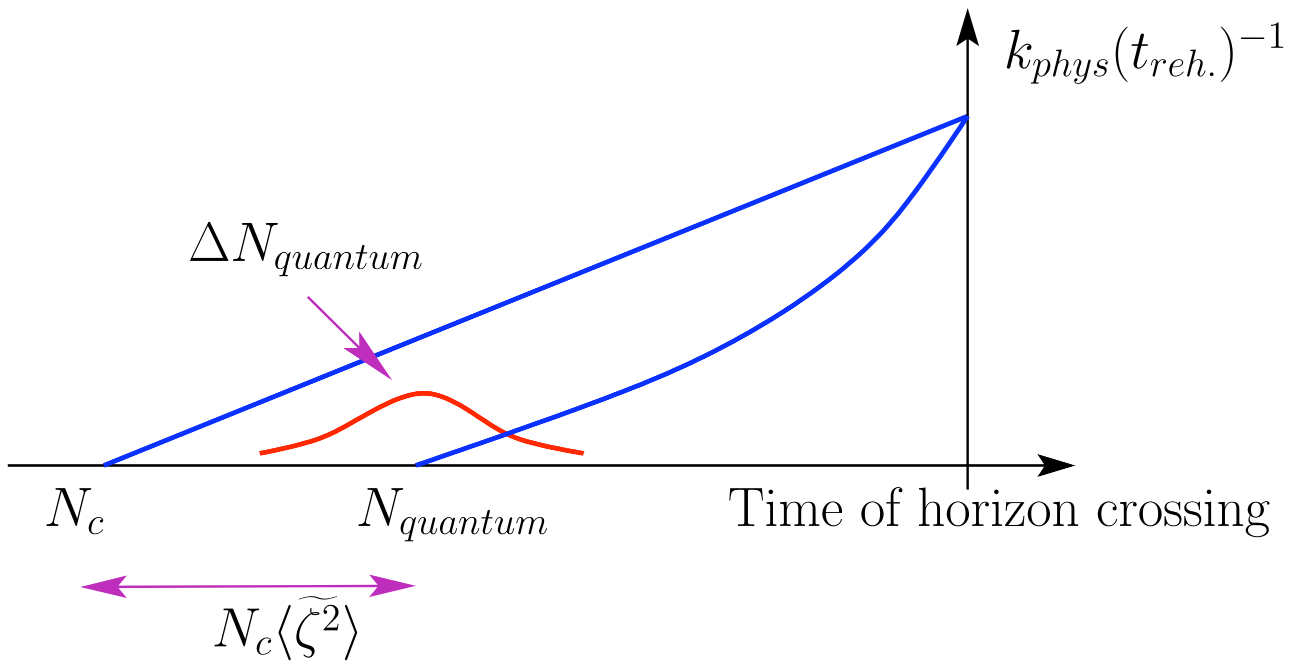
Second issue

- Enhanced expansion $\delta N_{quantum} \sim N_c \widetilde{\langle\zeta^2\rangle}$
  - What happens for $\widetilde{\langle\zeta^2\rangle} \sim 1$ (Close to Eternal Inflation)? or very large N_c ?

- Non-perturbative treatment necessary
  - Already done! in With Dubovsky and Villadoro **JHEP2009,JHEP2011** with Gorbenko **2019**

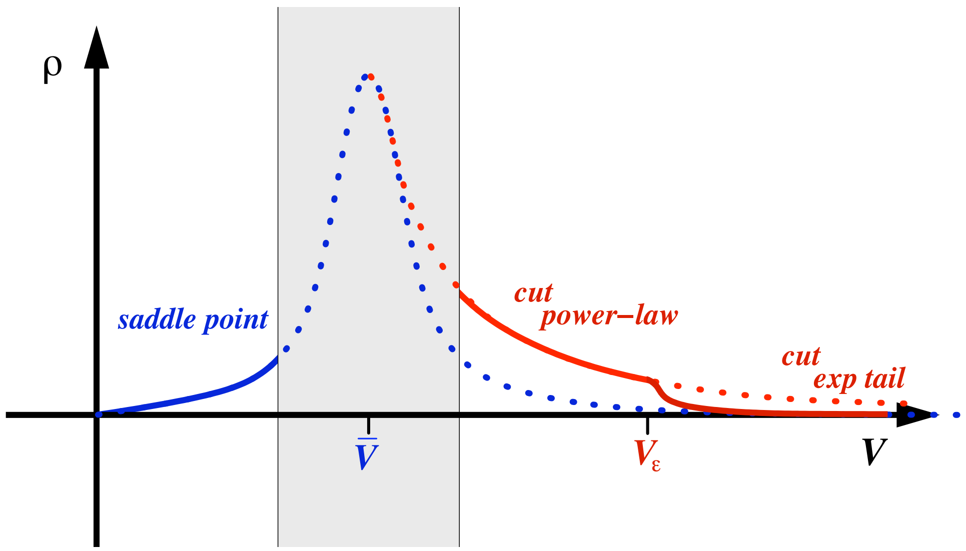
$$\langle V\rangle = \langle e^{3\zeta_B}\rangle \simeq e^{3N_c \frac{2}{1+\sqrt{1-1/\Omega}}}$$

$$\Omega \equiv \frac{2\pi^2}{3}\frac{\dot\phi^2}{H^4} \sim 1/\widetilde{\langle\zeta^2\rangle}$$

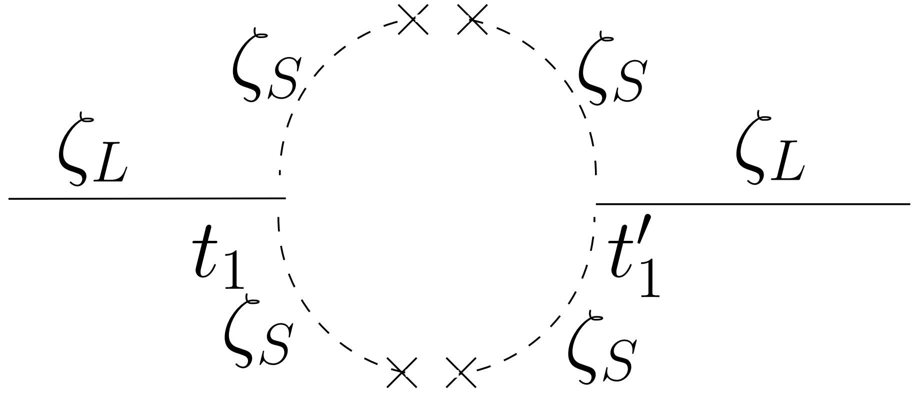


  - Maximum enhancement: $\delta N^{max}_{quantum} \simeq N_c$,
  - Small variance $\langle\Delta N^2_{quantum}\rangle^{1/2} \sim \widetilde{\langle\zeta^2\rangle}^{1/2}$
  - Probability distribution known (even within eternal inflation)

$$\rho(V,\tau) \approx \mathcal{N} e^{-\Omega\left[\frac{3N}{2}\left(1+\sqrt{1-\frac{1}{\Omega}}\right)-3N_c\right]^2}$$



Effects of shorter modes: no induced ζ time dependence

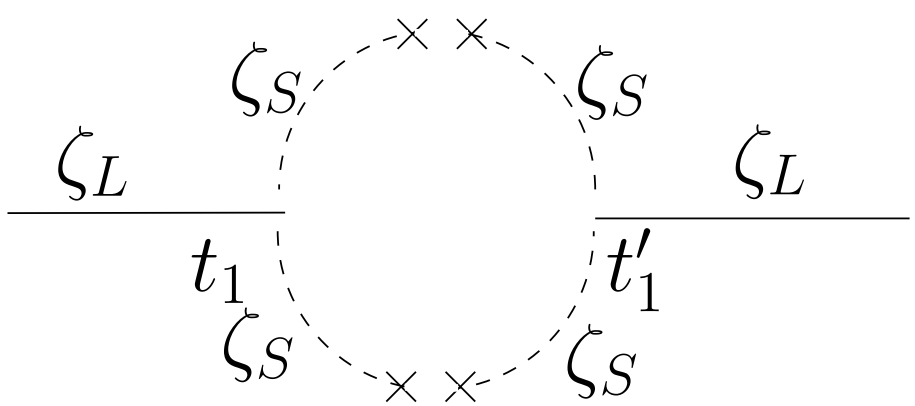


Two kinds of diagrams: CIM

$\langle \zeta^{(2)} \zeta^{(2)} \rangle_{1-\text{loop}}$

$$\partial_t^2 \zeta \simeq \zeta^2 + \zeta^3 + \ldots \qquad \Rightarrow \qquad \zeta^{(2)}(t) \sim \int G(t-t')\zeta^{(1)}(t')^2$$

- Modes shorter than ours

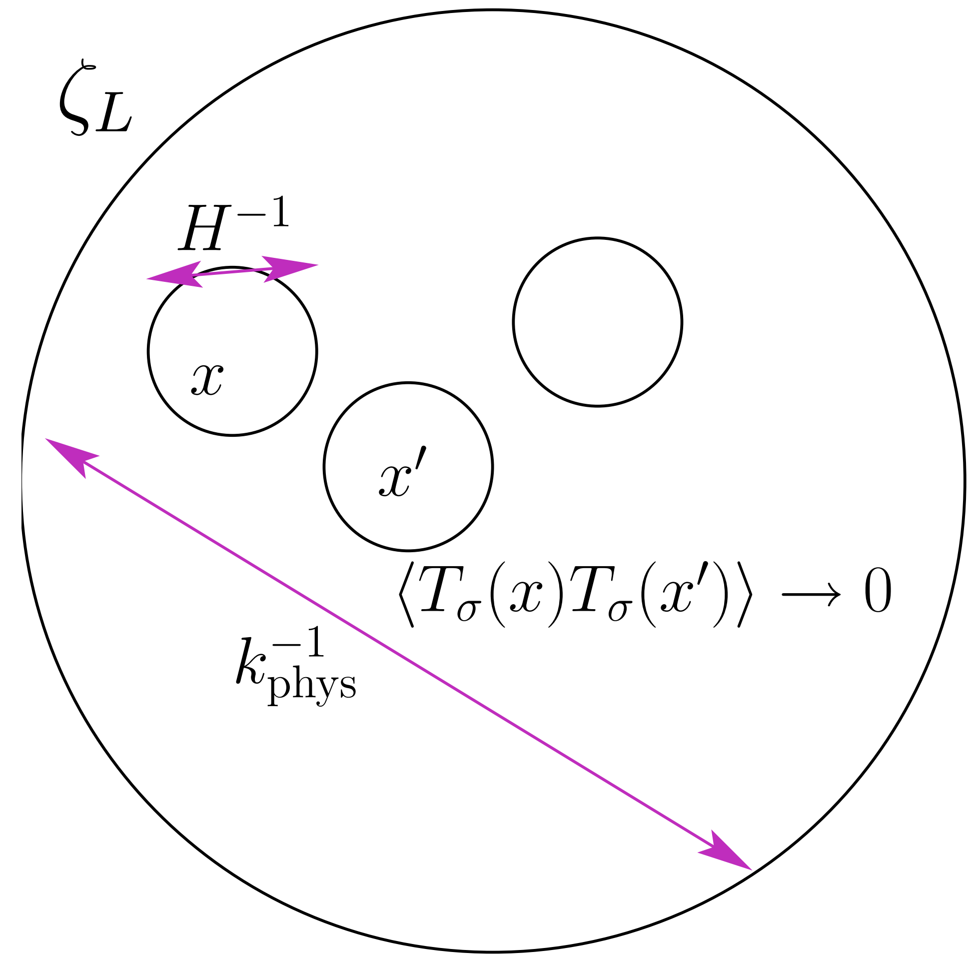


- CIM cut-in-the-middle: ζ_S vacuum fluctuations sourcing

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta} dx_i^2$$

$$\dot{\zeta} \sim \delta H$$

- The ζ_S have a derivative acting on them
- Get uncorrelated on distances larger than Hubble
- Source only for $\frac{k}{a(t)} \gtrsim H$
- They shut down as $k/a(t) \to 0$

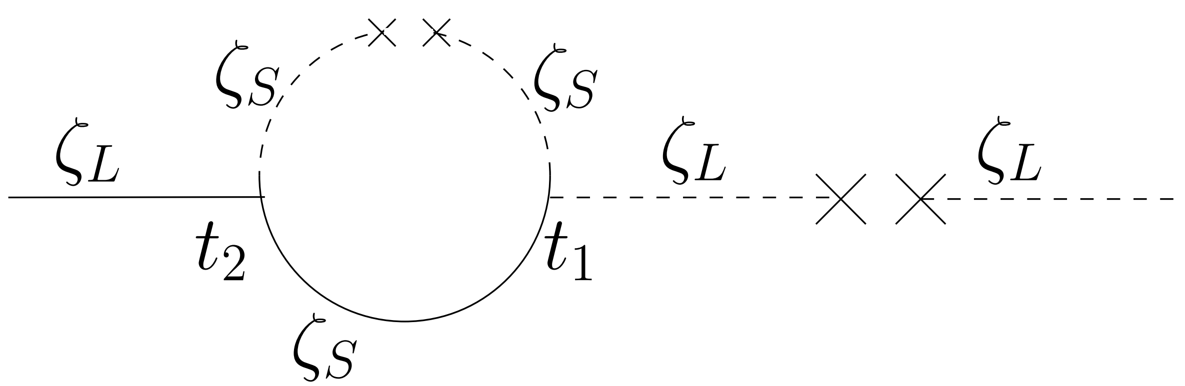


Two kinds of diagrams: CIS

$\langle \zeta^{(3)} \zeta^{(1)} \rangle_{1-\text{loop}}$

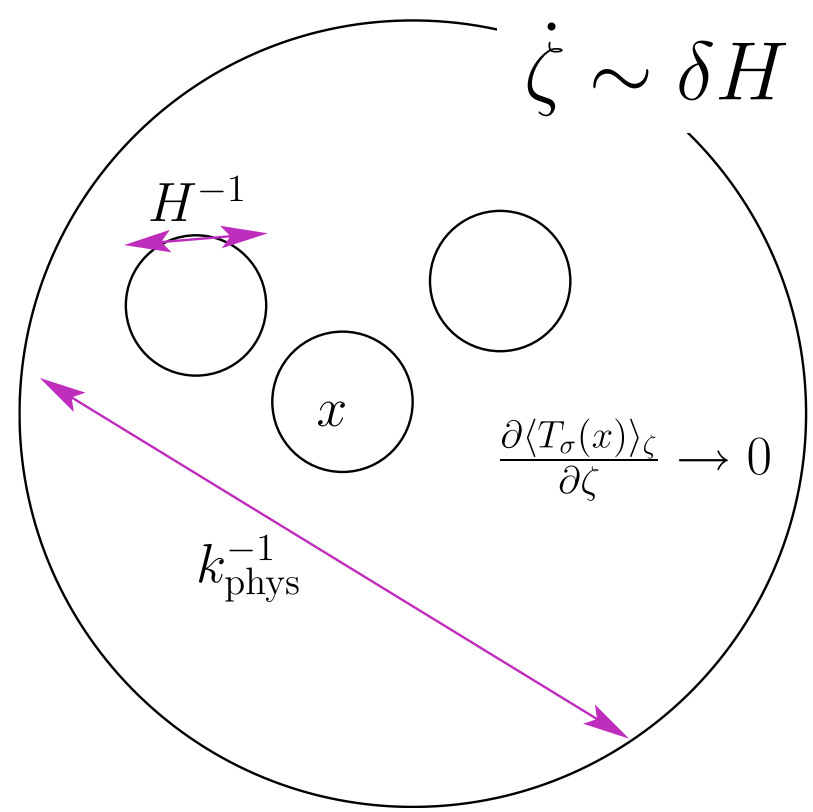
$$\zeta^{(3)}(t) = \int G(t-t')\zeta^{(1)}(t')\zeta^{(2)}(t')$$

- Modes shorter than ours



- CIS cut-in-the-side: ζ_L affecting ζ_S in tidal way and this affecting ζ_L back
- Since the freely evolved initial ζ becomes constant and unobservable out of the horizon
- $$ds^2 = -dt^2 + a(t)^2 e^{2\zeta} dx_i^2$$

$$\Rightarrow \qquad \frac{\partial}{\partial \zeta} \langle T_{\sigma,\zeta;\mu\nu}(k,t') \rangle \Big|_{\zeta=0} \to 0 \quad \text{as} \quad k\eta \to 0$$

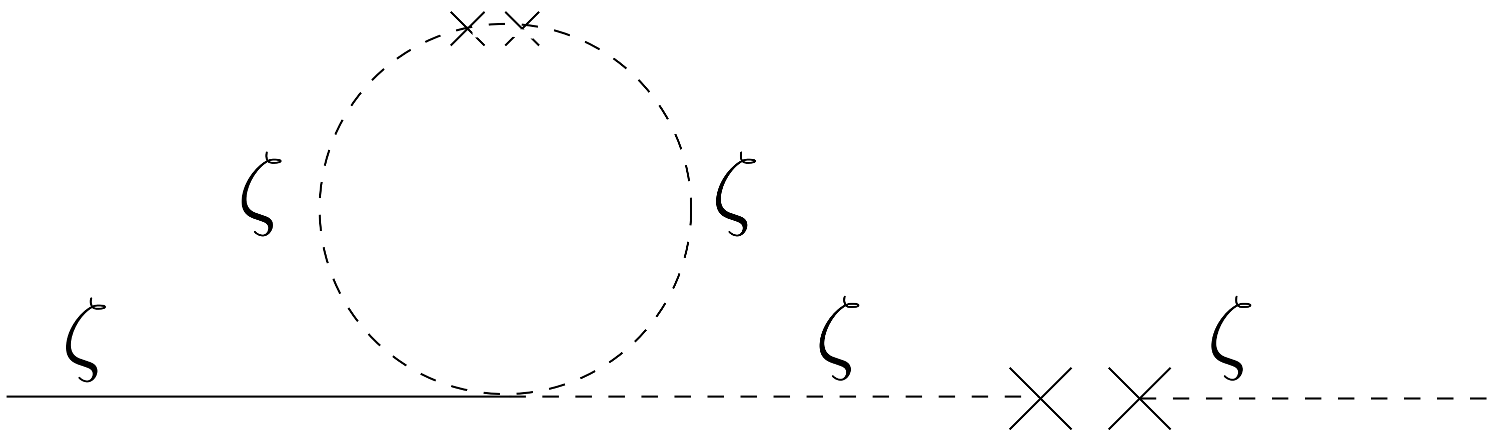


- No time dependence of ζ_L . Only $t_{h.c.}$ counts for dynamics.

The Role of the Quartics

$\langle \zeta^{(3)} \zeta^{(1)} \rangle_{1-\text{loop}}$

$$H_4 \supset -\mathcal{L}_4$$



- It does not represent a tidal effect
- And it gives IR effects $\langle \zeta^2 \rangle \sim Ht$

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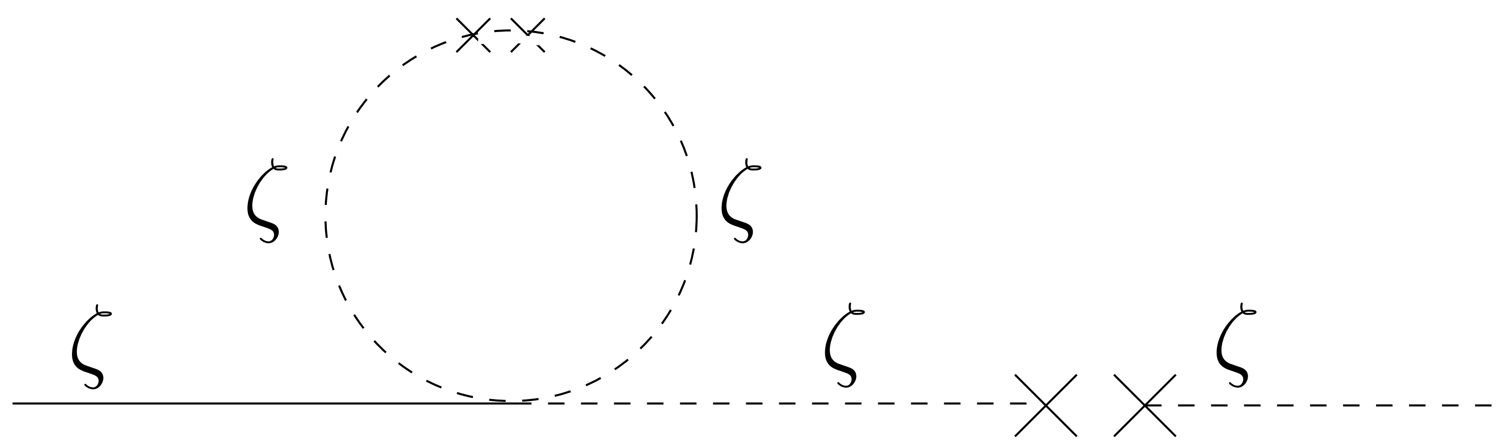
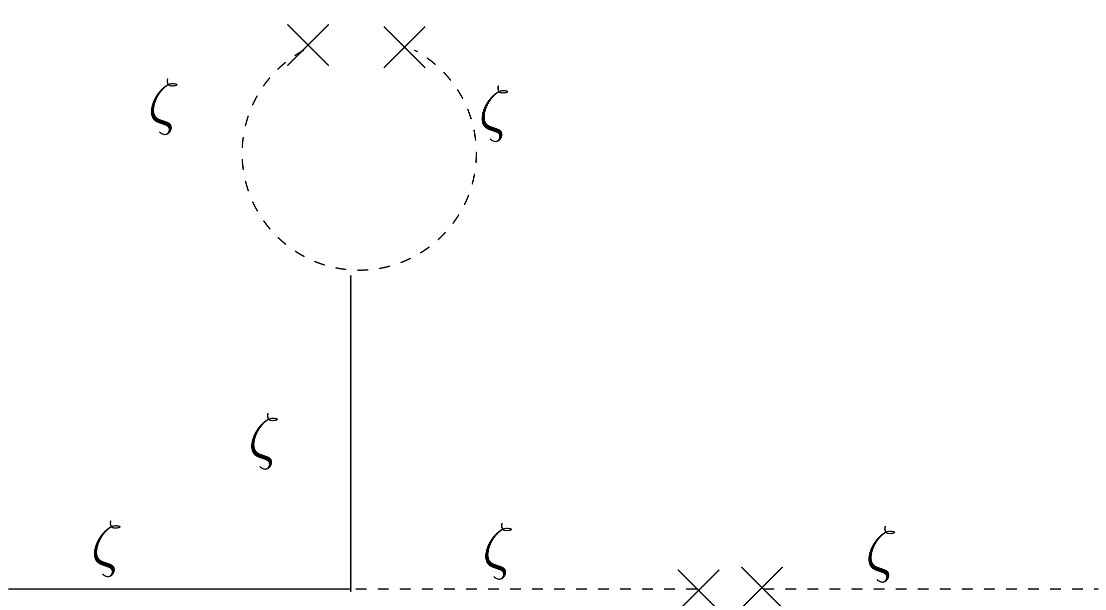


- It does not represent a tidal effect
- And it gives IR effects $\langle \zeta^2 \rangle \sim Ht$
- We need to compute everything



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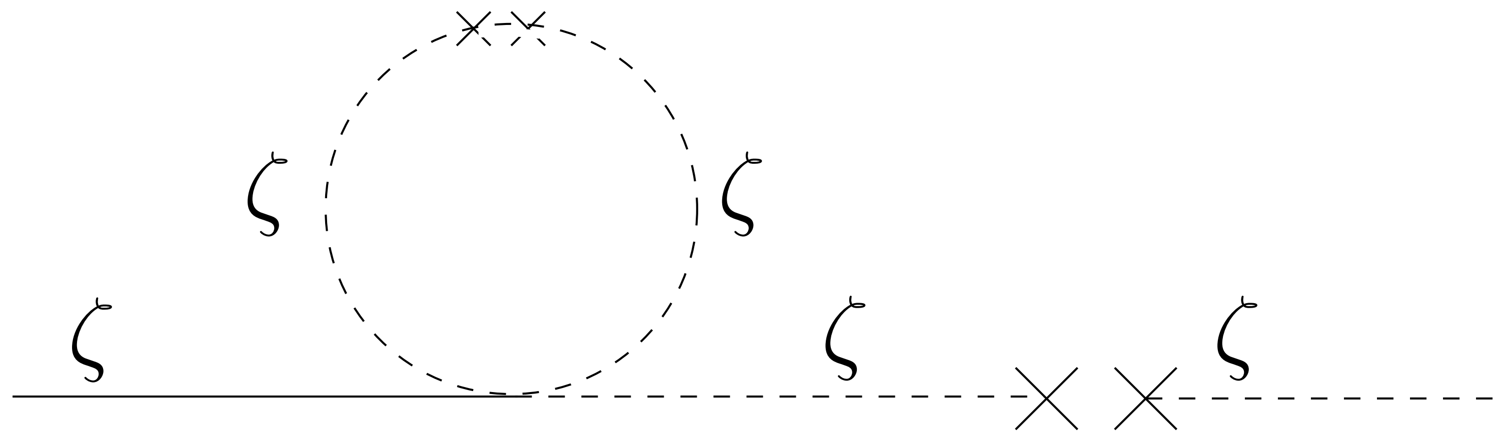

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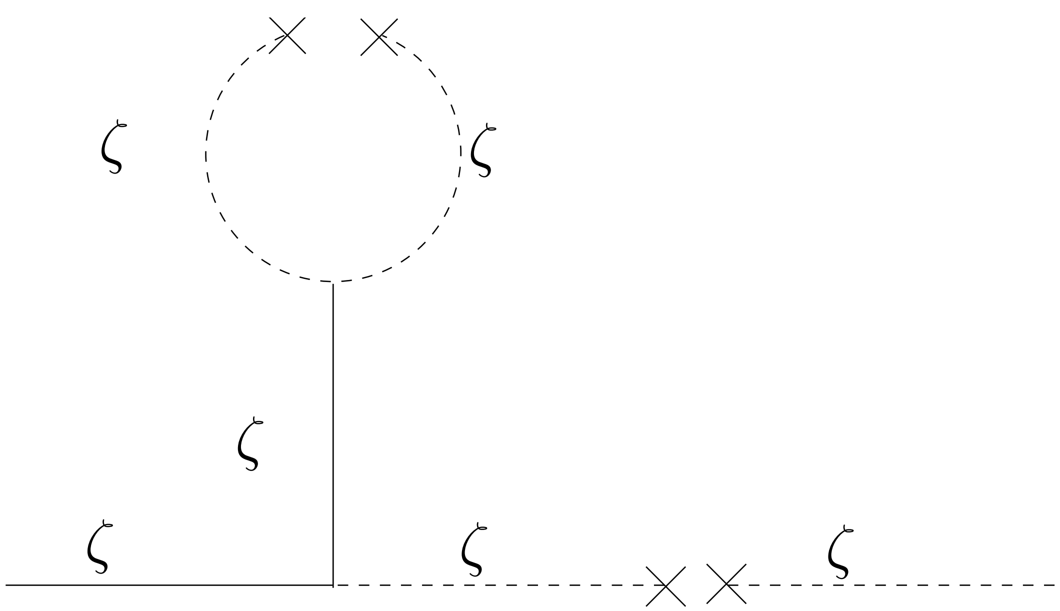

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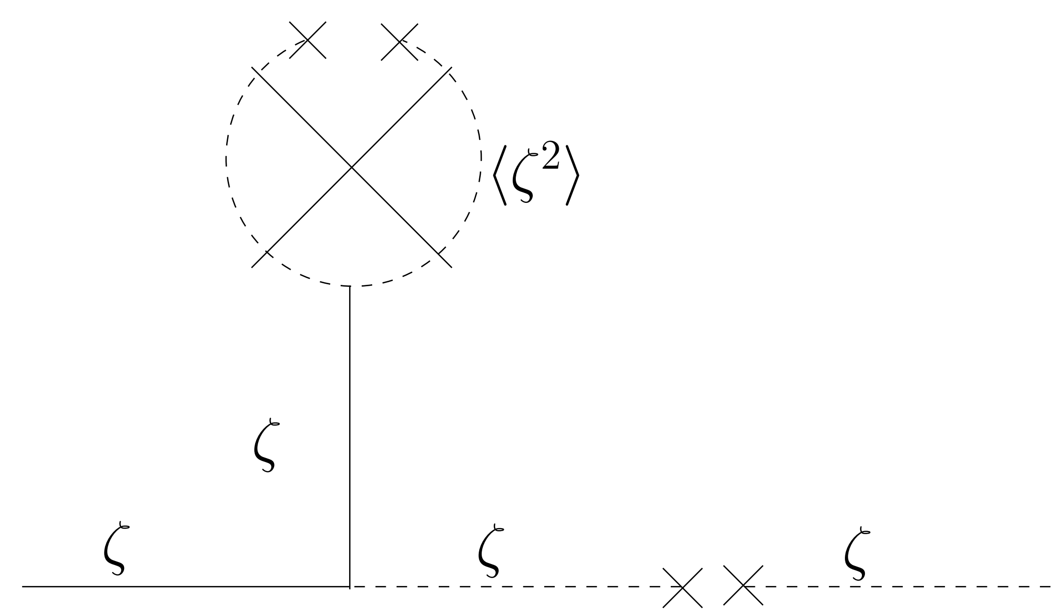

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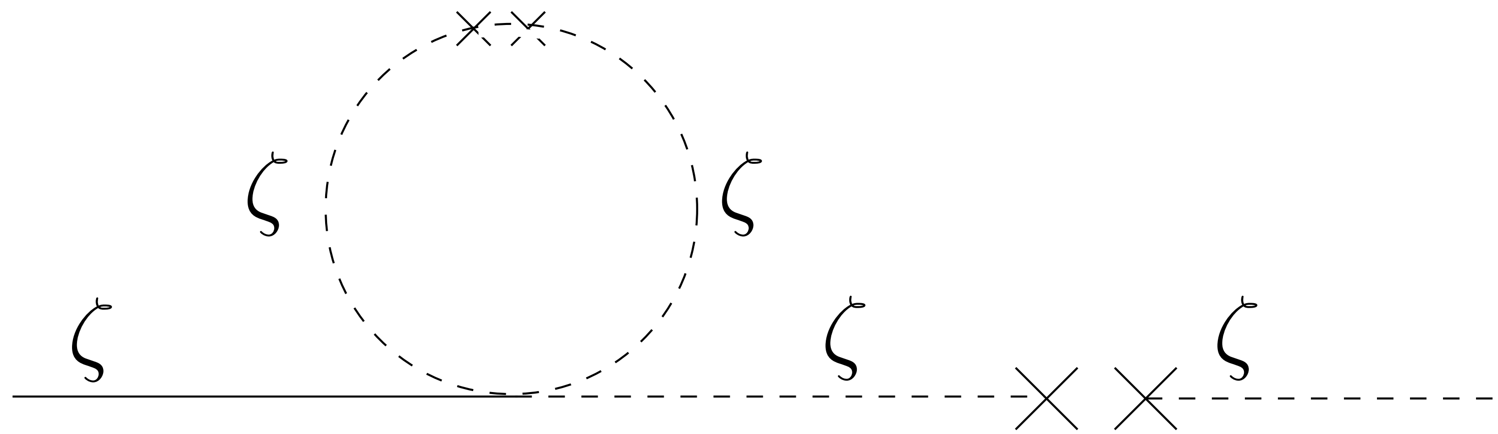
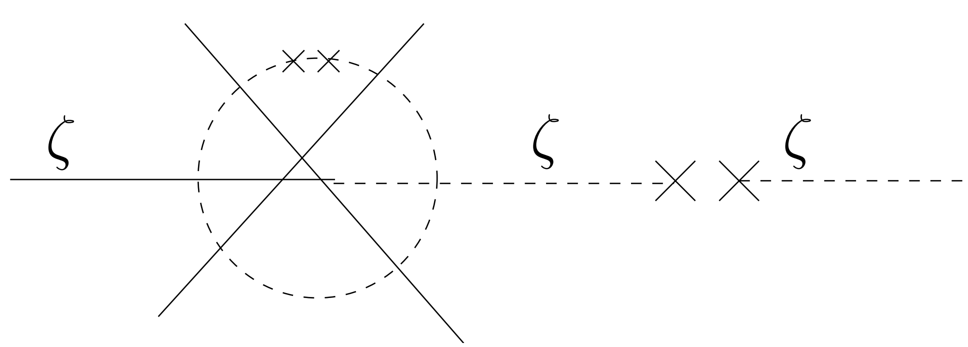


- It does not represent a tidal effect
- And it gives IR effects $\langle \zeta^2 \rangle \sim Ht$
- We need to compute everything
- What is left?

$$\mathcal{S}_{tad} = \int d^4\sqrt{-g}\ \left[\sqrt{-g}g^{00}\left(M_{\rm Pl}^2\dot{H} + \delta M^4\right) - \sqrt{-g}M_{\rm Pl}^2\left(\left(3H^2 + \dot{H}\right) + \delta\Lambda\right)\right]$$

- Induced quadratic terms from tadpoles (diff. invariace)



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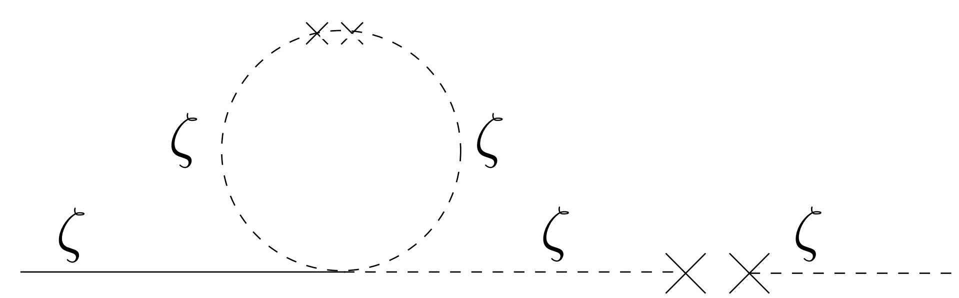


= 0

Example from EFT of Inflation $\langle\zeta^{(3)}\zeta^{(1)}\rangle_{1-\text{loop}}$

- Let us consider

$$\mathcal{S}=\int d^4x\sqrt{-g}\ \Big[\sqrt{-g}(1+\dot\pi+(\partial\pi)^2)\left(M_{\rm Pl}^2\dot H+\delta M^4(t+\pi)\right)-\sqrt{-g}M_{\rm Pl}^2\left(\left(3H^2+\dot H\right)+\delta\Lambda\right)+ M_3^4(t+\pi)\dot\pi^3\Big]$$

- Dangerous term:

$$\delta\mathcal{S}=\int d^4x\sqrt{-g}\ \left[3\dot M_3^4\pi\dot\pi\langle\dot\pi^2\rangle\right]$$

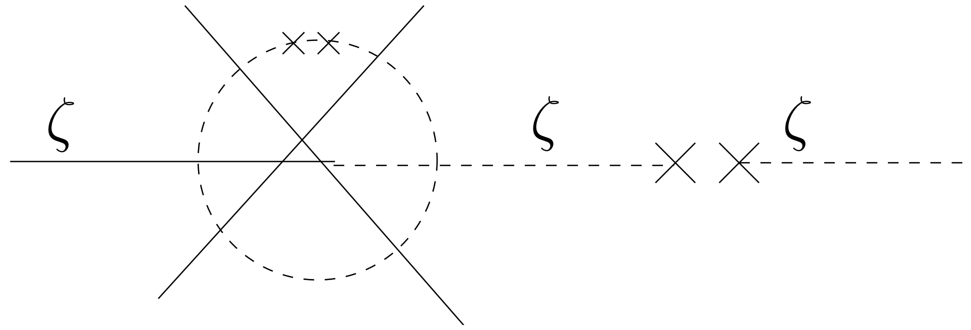
- Tadpole cancellation:

$$\delta\mathcal{S}_{Tad,M_3}=\int d^4x\sqrt{-g}\left[3M_3^4(t+\pi)\delta g^{00}\langle(\delta g^{00})^2\rangle\right]\quad\Rightarrow\quad \delta M(t+\pi)^4=-3M_3^4(t+\pi)\langle(\delta g^{00})^2\rangle$$

- Quadratic term from tadpole

$$\mathcal{S}_{tad}\supset\int d^4\sqrt{-g}\ \left[-\sqrt{-g}\cdot\delta g^{00}3M_3^4(t+\pi)\langle(\delta g^{00})^2\rangle\right]$$

$$=\int d^4x\sqrt{-g}\ \left[-\dot\pi 3M_3^4(t+\pi)\langle\dot\pi^2\rangle\right]\supset\int d^4x\sqrt{-g}\ \left[-\dot\pi 3\dot M_3^4(t)\pi\langle\dot\pi^2\rangle\right]$$



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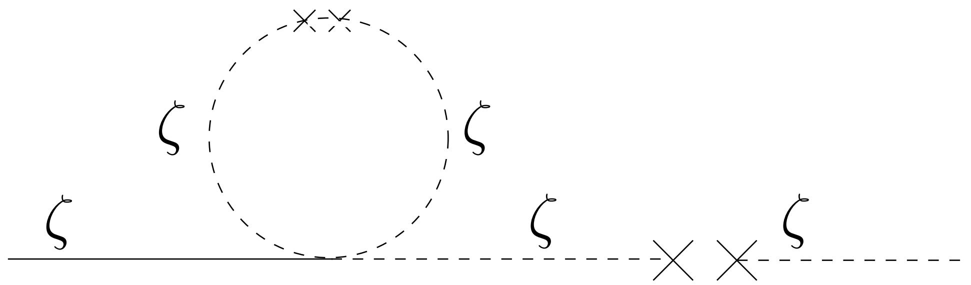

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Example from EFT of Inflation $\langle \zeta^{(3)} \zeta^{(1)} \rangle_{1-\text{loop}}$

- Let us consider

$$\mathcal{S} = \int d^4x \sqrt{-g}\ \left[\sqrt{-g}(1+\dot\pi+(\partial\pi)^2)\left(M_{\rm Pl}^2 \dot H + \delta M^4(t+\pi)\right) - \sqrt{-g} M_{\rm Pl}^2\left(\left(3H^2+\dot H\right)+\delta\Lambda\right) + M_3^4(t+\pi)\dot\pi^3\right]$$

- Dangerous term:

$$\delta\mathcal{S} = \int d^4x \sqrt{-g}\ \left[3\dot M_3^4 \pi\dot\pi\langle\dot\pi^2\rangle\right]$$

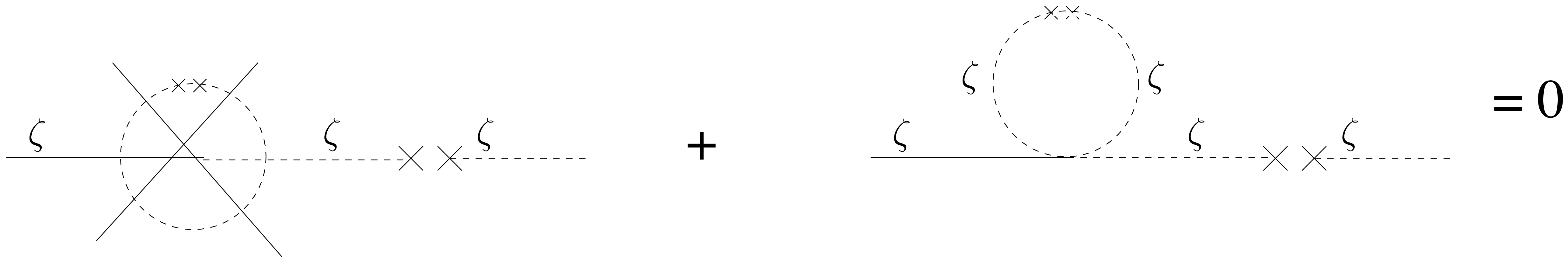
- Tadpole cancellation:

$$\delta\mathcal{S}_{Tad,M_3} = \int d^4x\sqrt{-g}\left[3M_3^4(t+\pi)\delta g^{00}\langle(\delta g^{00})^2\rangle\right] \quad \Rightarrow \quad \delta M(t+\pi)^4 = -3M_3^4(t+\pi)\langle(\delta g^{00})^2\rangle$$

- Quadratic term from tadpole

$$\mathcal{S}_{tad} \supset \int d^4\sqrt{-g}\ \left[-\sqrt{-g}\cdot\delta g^{00} 3M_3^4(t+\pi)\langle(\delta g^{00})^2\rangle\right]$$

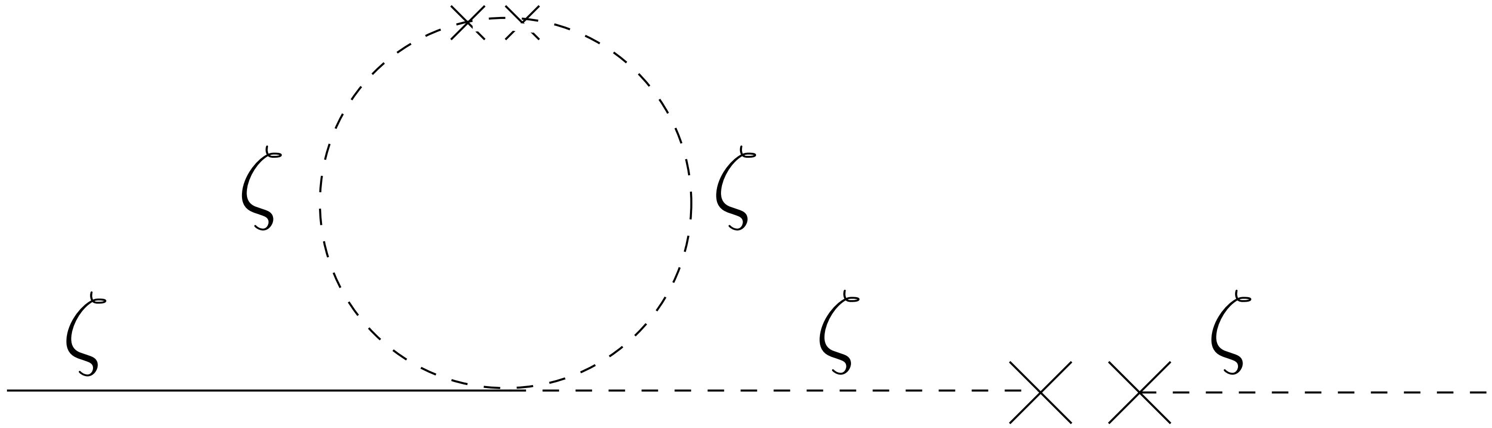
$$= \int d^4x\sqrt{-g}\ \left[-\dot\pi 3M_3^4(t+\pi)\langle\dot\pi^2\rangle\right] \supset \int d^4x\sqrt{-g}\ \left[-\dot\pi 3\dot M_3^4(t)\pi\langle\dot\pi^2\rangle\right]$$



The Role of the Additional Quartics

$\langle \dot{\zeta}^{(3)} \dot{\zeta}^{(1)} \rangle_{1-\text{loop}}$

$$H_4 \supset -\mathcal{L}_4$$



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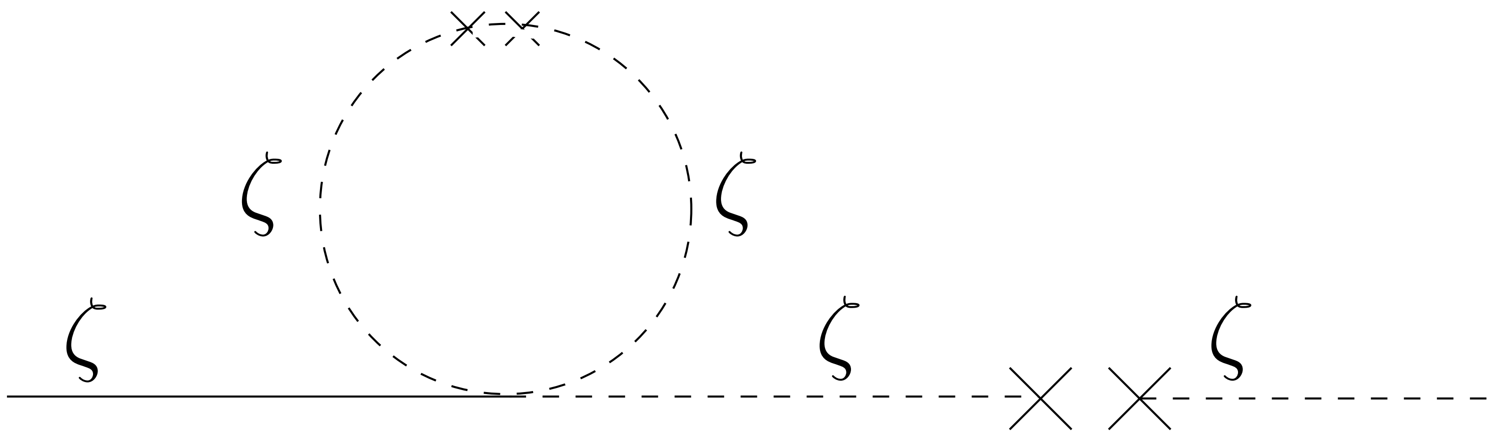
$$H_4 \supset \left(\frac{\delta \mathcal{L}_3}{\delta \dot{\zeta}}\right)^2 + \zeta \partial_i \zeta \frac{\delta \mathcal{L}_3}{\delta \partial_i \zeta}$$



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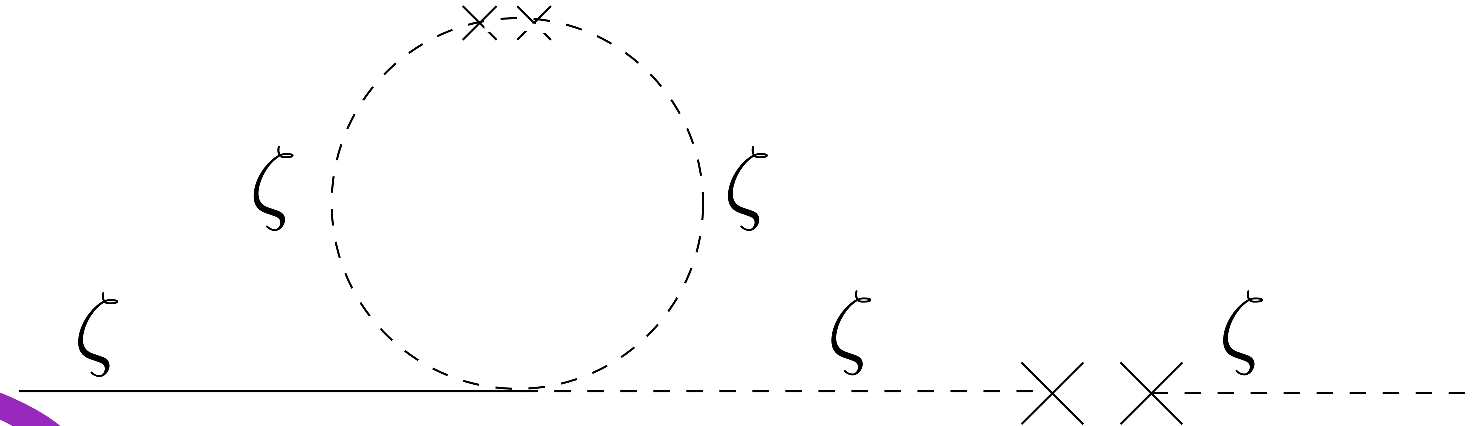

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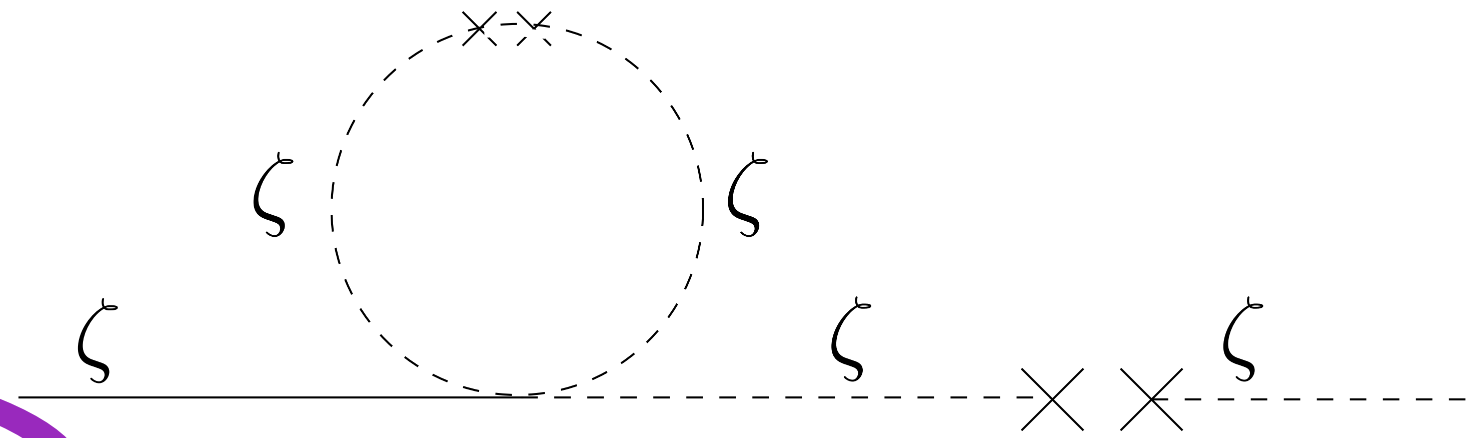
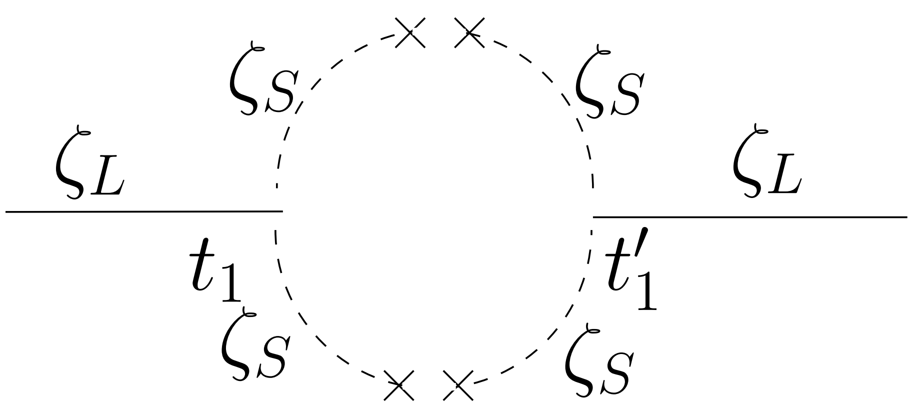


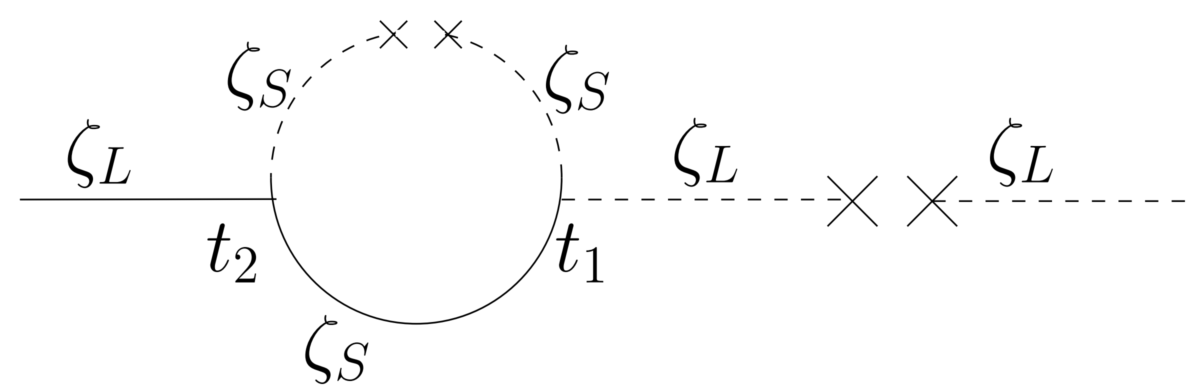
$$H_4 \supset -\mathcal{L}_4$$

$$H_4 \supset \left(\frac{\delta \mathcal{L}_3}{\delta \dot{\zeta}} \right)^2 + \zeta \partial_i \zeta \frac{\delta \mathcal{L}_3}{\delta \partial_i \zeta}$$

- Master Formula:

$$\langle \zeta_k \zeta_k \rangle_{CIS+CIM+Quartic_3} \sim \int_{-\infty}^{t} dt_1 \; Green(t, t_1) \;\; \times \;\; \langle \partial \zeta_S \partial \zeta_S \zeta_L \rangle (t_1)$$





The Role of the Additional Quartics

- Master Formula:

$$\langle\zeta_k\zeta_k\rangle_{CIS+CIM+Quartic_3} \sim \int_{-\infty}^{t} dt_1 \; Green(t,t_1) \;\times\; \langle\partial\zeta_S\partial\zeta_S\zeta_L\rangle(t_1)$$

$$\sim \int^{\eta} d\eta_1 \left(\frac{1}{\eta_1}\right)^4 \left(\eta^3-\eta_1^3\right) \langle\partial\zeta_S\partial\zeta_S\zeta_k(\eta_1)\rangle$$

- Maldacena Consistency condition $\quad \langle\partial\zeta_S\partial\zeta_S\zeta_k(\eta_1)\rangle \simeq \frac{1}{q^{3+\delta}}\frac{\partial\langle\left[q^{3+\delta}\partial\zeta_S\partial\zeta_S\right]_q\rangle}{\partial\log q}\langle\zeta_k(t_1)^2\rangle$

$$\Rightarrow \int d^{3+\delta}q\langle\partial\zeta_S\partial\zeta_S\zeta_k(\eta_1)\rangle \simeq \int d^{3+\delta}q\,\frac{1}{q^{3+\delta}}\frac{\partial\langle\left[q^{3+\delta}\partial\zeta_S\partial\zeta_S\right]_q\rangle}{\partial\log q} = \frac{1}{q^{3+\delta}}\frac{\partial\langle\left[q^{3+\delta}\partial\zeta_S\partial\zeta_S\right]_q\rangle}{\partial\log q}\Bigg|_{q=k} \quad\rightarrow\quad 0$$

- $\Rightarrow \quad \langle\zeta_k\zeta_k\rangle \simeq \int_{\eta_{k_{out}}}^{\eta} d\eta_1 \left(\frac{1}{\eta_1}\right)^{4+\delta}\left(\eta^3-\eta_1^3\right)\frac{1}{q^{3+\delta}}\frac{\partial\langle\left[q^{3+\delta}\partial\zeta_S\partial\zeta_S\right]_q\rangle}{\partial\log q}\Bigg|_{q=k}\langle\zeta_k(t)^2\rangle \quad\rightarrow\quad 0$

- Only effect comes if there is time dependence of long mode due to absence of attractor
- End of Effects from Short Modes

for non perturbative proofs, see
with Zaldarriaga **2013**
Baumann and Green **2013**

The Role of the Additional Quartics

- Master Formula:

$$\langle\zeta_k\zeta_k\rangle_{CIS+CIM+Quartic_3} \sim \int_{-\infty}^{t} dt_1\ Green(t,t_1)\ \times\ \langle\partial\zeta_S\partial\zeta_S\zeta_L\rangle(t_1)$$

$$\sim \int^{\eta} d\eta_1\ \left(\frac{1}{\eta_1}\right)^4 (\eta^3-\eta_1^3)\ \langle\partial\zeta_S\partial\zeta_S\zeta_k(\eta_1))\rangle$$

IR Problematic

- Maldacena Consistency condition

$$\langle\partial\zeta_S\partial\zeta_S\zeta_k(\eta_1))\rangle \simeq \frac{1}{q^{3+\delta}}\frac{\partial\langle\left[q^{3+\delta}\partial\zeta_S\partial\zeta_S\right]_q\rangle}{\partial\log q}\langle\zeta_k(t_1)^2\rangle$$

$$\Rightarrow \int d^{3+\delta}q\langle\partial\zeta_S\partial\zeta_S\zeta_k(\eta_1))\rangle \simeq \int d^{3+\delta}q\ \frac{1}{q^{3+\delta}}\frac{\partial\langle\left[q^{3+\delta}\partial\zeta_S\partial\zeta_S\right]_q\rangle}{\partial\log q} = \frac{1}{q^{3+\delta}}\frac{\partial\langle\left[q^{3+\delta}\partial\zeta_S\partial\zeta_S\right]_q\rangle}{\partial\log q}\Bigg|_{q=k} \quad\to\quad 0$$

- $$\Rightarrow \qquad \langle\zeta_k\zeta_k\rangle \simeq \int_{\eta_{k_{out}}}^{\eta} d\eta_1\ \left(\frac{1}{\eta_1}\right)^{4+\delta}(\eta^3-\eta_1^3)\ \frac{1}{q^{3+\delta}}\frac{\partial\langle\left[q^{3+\delta}\partial\zeta_S\partial\zeta_S\right]_q\rangle}{\partial\log q}\Bigg|_{q=k}\langle\zeta_k(t)^2\rangle \quad\to\quad 0$$

- Only effect comes if there is time dependence of long mode due to absence of attractor.
- End of Effects from Short Modes

for non perturbative proofs, see
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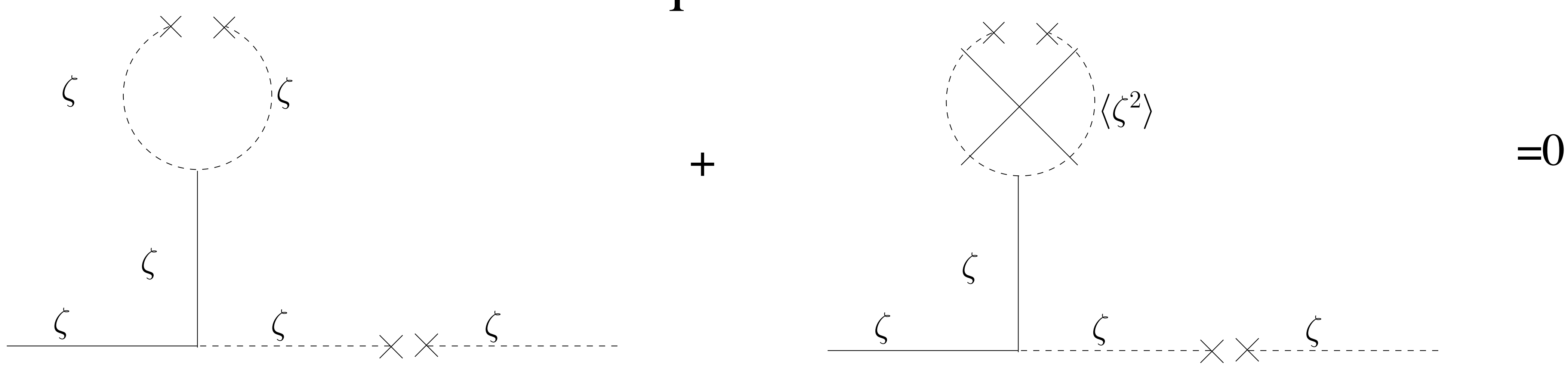
Summary part I

- Short modes in the loop do not lead to a time-dependence for standard slow roll
- Subtle cancellations
  - Renormalization of the background
    - Cancellation between some quartics and non-1PI



  - All other diagrams: consistency condition (as inflation is an attractor)
    - Locally a long-wavelength inflaton mode is unobservable

$$\frac{\partial}{\partial\zeta}\langle T_{\sigma,\zeta;\mu\nu}(k,t')\rangle\Big|_{\zeta=0} \rightarrow 0 \quad \text{as} \quad k\eta \rightarrow 0$$

- Huge, but quite physical, calculation

Multi-field inflation

Muultifield case

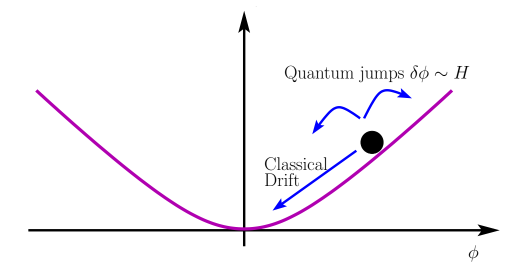
with Gorbenko **2019**

- Both
  - Massless $\lambda\,\phi^4$ in dS or multifield inflation
  - Slow-Roll eternal inflation
- have IR divergencies and so are non-perturbative phenomena
- The Stochastic equation $\frac{\partial}{\partial t}P(\phi(\vec{x})) = H^2\frac{\partial^2}{\partial\phi(\vec{x})^2}P(\phi(\vec{x})) + \frac{\partial}{\partial\phi(\vec{x})}\left(V'(\phi(\vec{x}))P(\phi(\vec{x}))\right)$
  - we show it provides a way to solve for them
  - providing a systematic derivation and proof of accuracy

- We proved that that equation is the leading-order truncation of a generalized equation, from which we can derive arbitrary accurate results.
  - Proving the existence slow-roll eternal inflation
  - Solving $\lambda\,\phi^4$ in rigid de Sitter in a systematic expansion

Effective Probability for long modes

– To all orders in λ & ϵ and leading in δ , we obtain the following effective equation. It is Fokker-Planck-like, but it has differences



$$\frac{\partial P_\ell[\phi_\ell]}{\partial t} = \text{Drift} + \text{Diff.}$$

$$\text{Drift} = \frac{\delta}{\delta\phi_\ell(\vec{x})}\left(\left\langle \left[\text{Re}\left(\frac{\Pi(\phi(\vec{x}))}{a^3}\right)\right]_\Lambda \right\rangle_{\phi_\ell} P_\ell[\phi_\ell]\right)$$

$$\text{Diff.} = \frac{\delta}{\delta\phi_\ell(\vec{x})}\left(\left\langle\left(-\frac{\partial}{\partial t}\Delta\phi(\vec{x})\right)\right\rangle_{\phi_\ell} P_\ell[\phi_\ell, t]\right)$$

$$+\frac{\delta^2}{\delta\phi_\ell(\vec{x})\delta\phi_{\ell,b}(\vec{x}')}\left(\left\langle\frac{\partial}{\partial t}\left(\Delta\phi(\vec{x})\Delta\phi(\vec{x}')\right)\right\rangle_{\phi_\ell} P_\ell[\phi_\ell, t]\right)$$

– tadpole-diffusion term.

– Strategy: compute these expectation values for the short modes in perturbation theory with a given background for the long modes in expansion in $\lambda t \sim \lambda \log\epsilon \ll 1, \sqrt{\lambda} \ll 1$, and solve this functional Fokker-Planck-like equation containing only long modes in $\sqrt{\lambda} \ll 1$

Summary

with Zaldarriaga, **2009**
2012
2012
with Pimentel and Zaldarriaga **2012**
with Gorbenko **2019**

On Loops In Inflation

- Log Running
  - The logarithmic running is of the form $\log\left(\frac{H}{\mu}\right)$
- IR divergency in Single Field Inflation:
  - need to take into account projection effects:
    - there is no time-dependence for standard slow roll
    - and no non-trivial scale dependence $\log(kL)$
    - There is a physical enhanced expansion $\delta N_{quantum} \sim N_c \widetilde{\langle\zeta^2\rangle}$
  - The predictivity of inflation is fine
  - Eternal Inflation is in better shape
- Multifield Inflation
  - Rigorous derivation and systematization of stochastic approach