Recent Lattice-QCD results for heavy flavors

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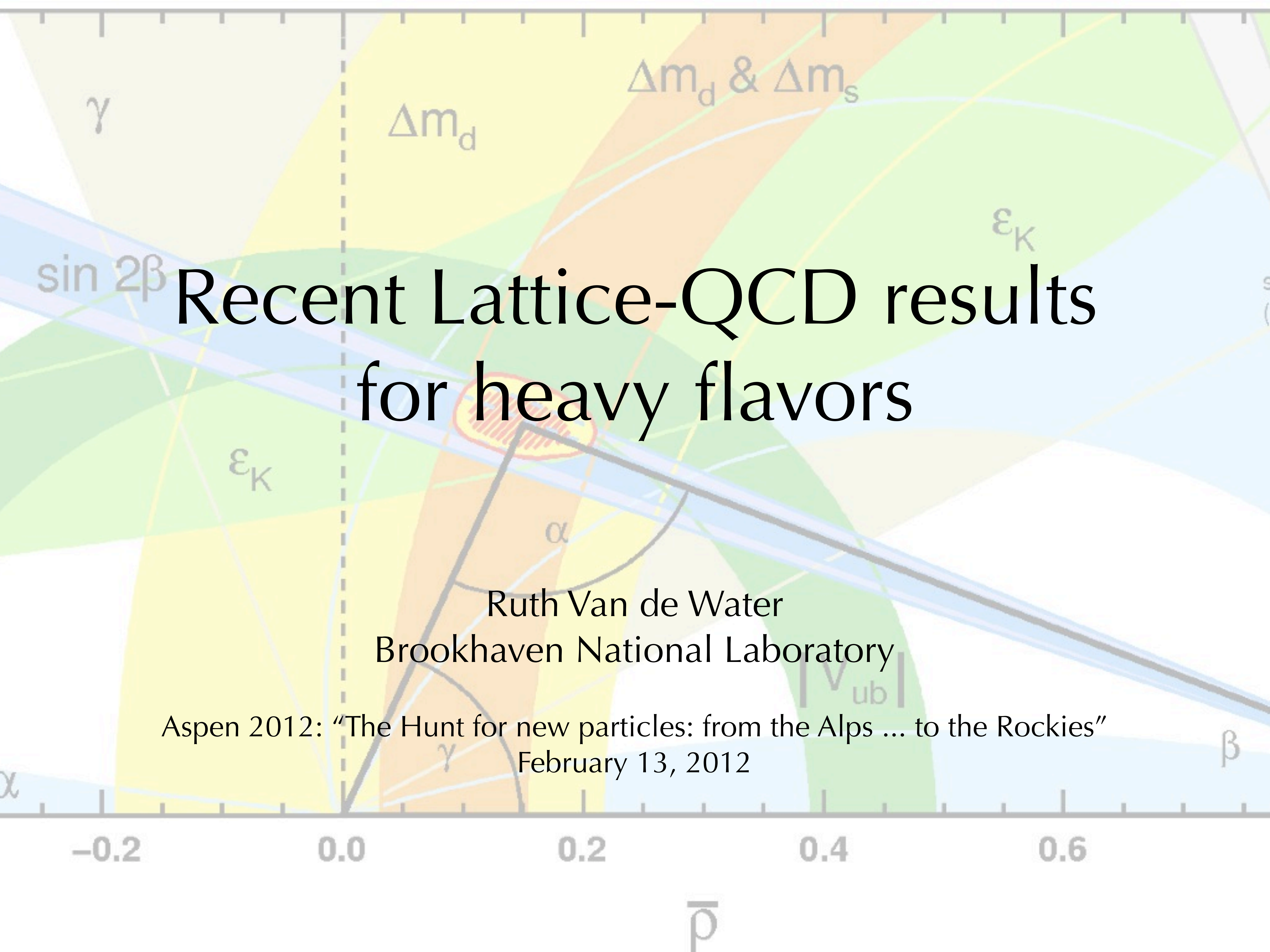
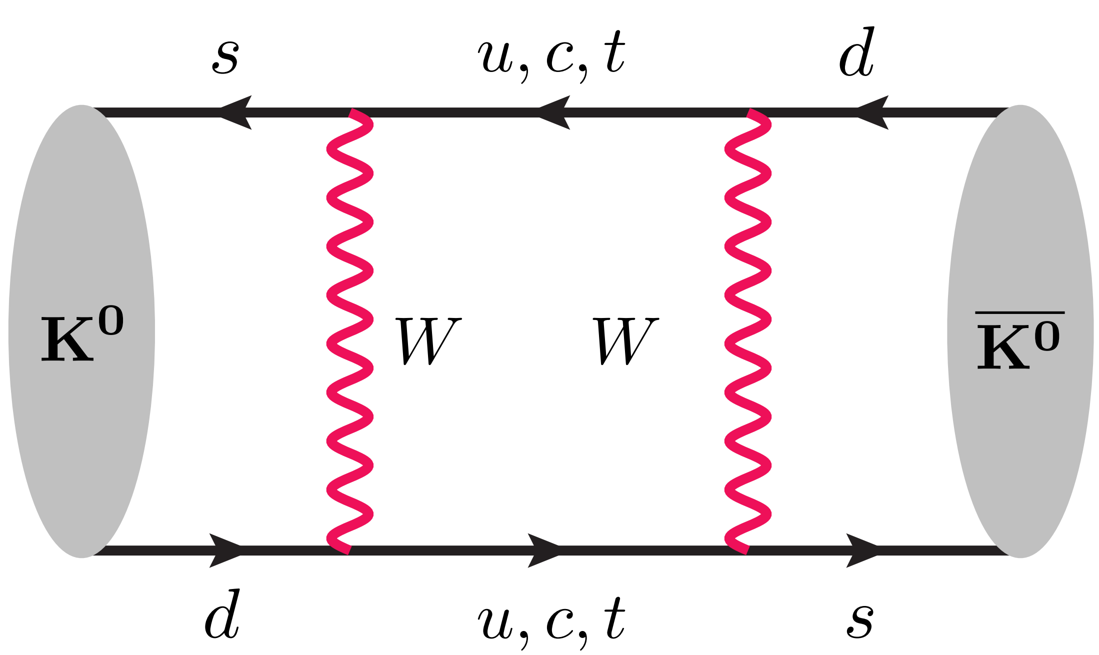
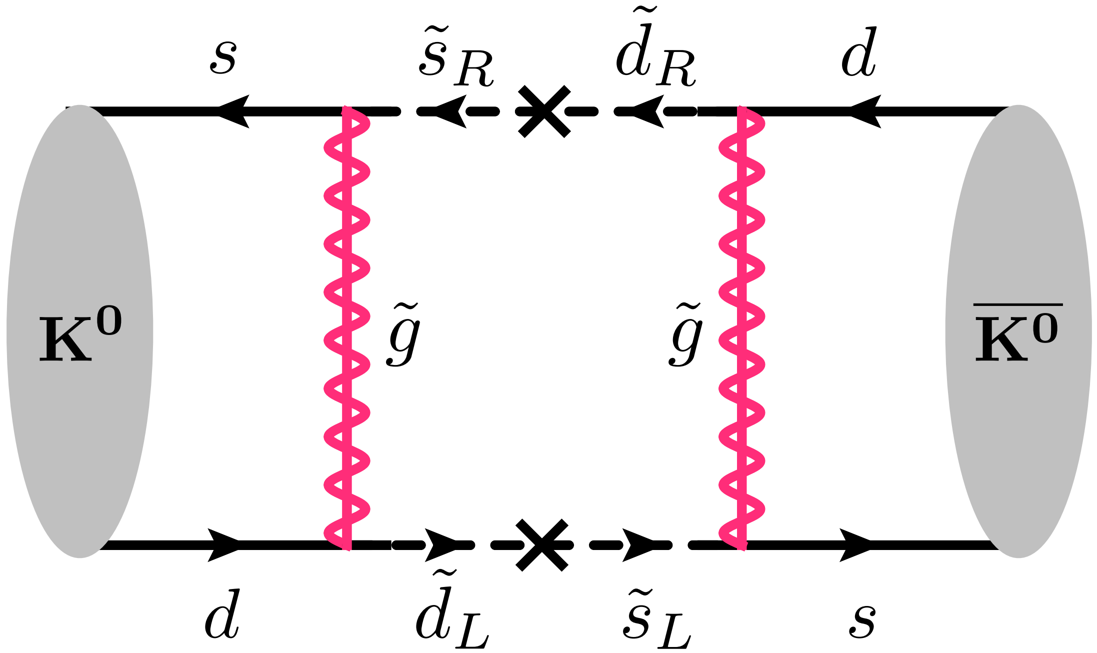
Aspen 2012: “The Hunt for new particles: from the Alps ... to the Rockies”
February 13, 2012



Why study flavor physics?

- Most Standard Model extensions contain new CP-violating phases and new quark-flavor changing interactions, so we expect new physics effects in the flavor sector
- New particles will typically appear in loop-level processes such as neutral kaon mixing:

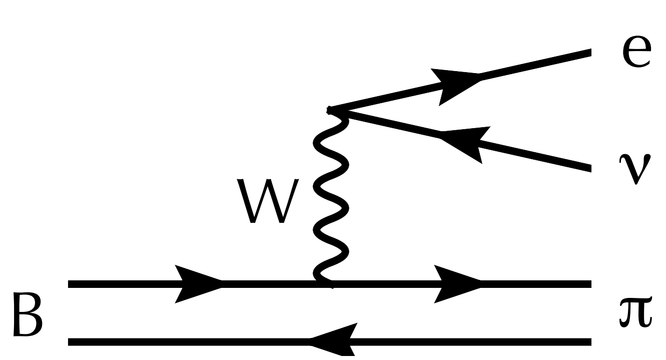


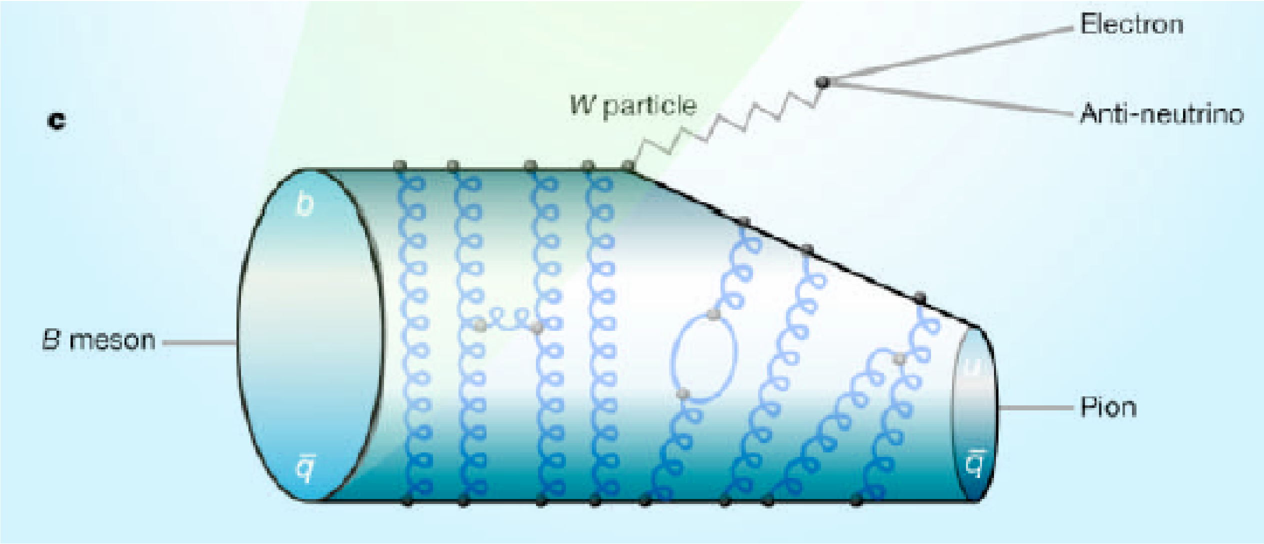


- Sensitive to physics at higher energy scales than those probed in high-p_T collider experiments, in some cases $\mathcal{O}$(1,000 - 10,000 TeV)
[Isidori, Nir, Perez, Ann.Rev.Nucl.Part.Sci. 60 (2010) 355]

Lattice QCD and precision flavor physics

- Flavor factories, Tevatron, and LHC have been pouring out data to pin down the CKM matrix and to measure decay rates for rare processes
  - ➡ Lattice-QCD calculations are needed to interpret may of their results
- To accurately describe weak interactions involving quarks, must include effects of confining quarks into hadrons:





- Absorb nonperturbative QCD effects into quantities such as decay constants, form factors, and bag-parameters
  - ➡ Only way to calculate hadronic weak matrix elements with all systematic uncertainties under control is numerically using lattice QCD

Lattice QCD & the CKM matrix

➡ To pin down the Standard-Model flavor couplings and search for new physics in the quark-flavor sector, need precise (few % or better) lattice-QCD calculations

✦ Gold-plated" processes are **most readily calculated with high precision in lattice QCD**

- ❖ One hadron in initial state and at most one hadron in final state
- ❖ Hadrons are stable under QCD (or narrow and far from threshold)
- ❖ Allow the determination of all CKM matrix elements except $|V_{tb}|$*

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

*Neutral kaon mixing also gold-plated and can be used to obtain the CKM phase (ρ, η)

(Typical) Systematics in lattice calculations

(1) Monte carlo statistics & fitting

(2) Tuning lattice spacing and quark masses

- Require that lattice results for a few quantities (*e.g.* m_π, m_K, m_{Ds}, m_{Bs}, f_π) agree with experiment

(3) Matching lattice gauge theory to continuum QCD

- Use fixed-order lattice perturbation theory, step-scaling, or other partly- or fully-nonperturbative methods

(4) Chiral extrapolation to physical up, down quark masses

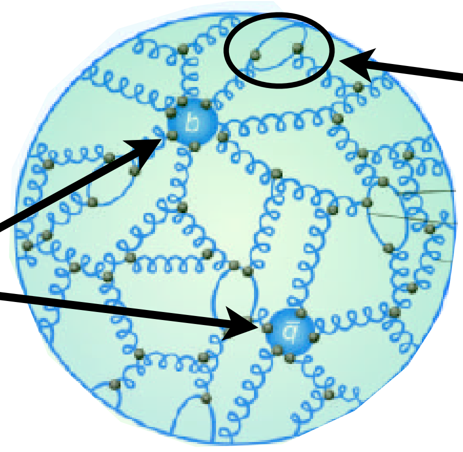
(5) Continuum extrapolation

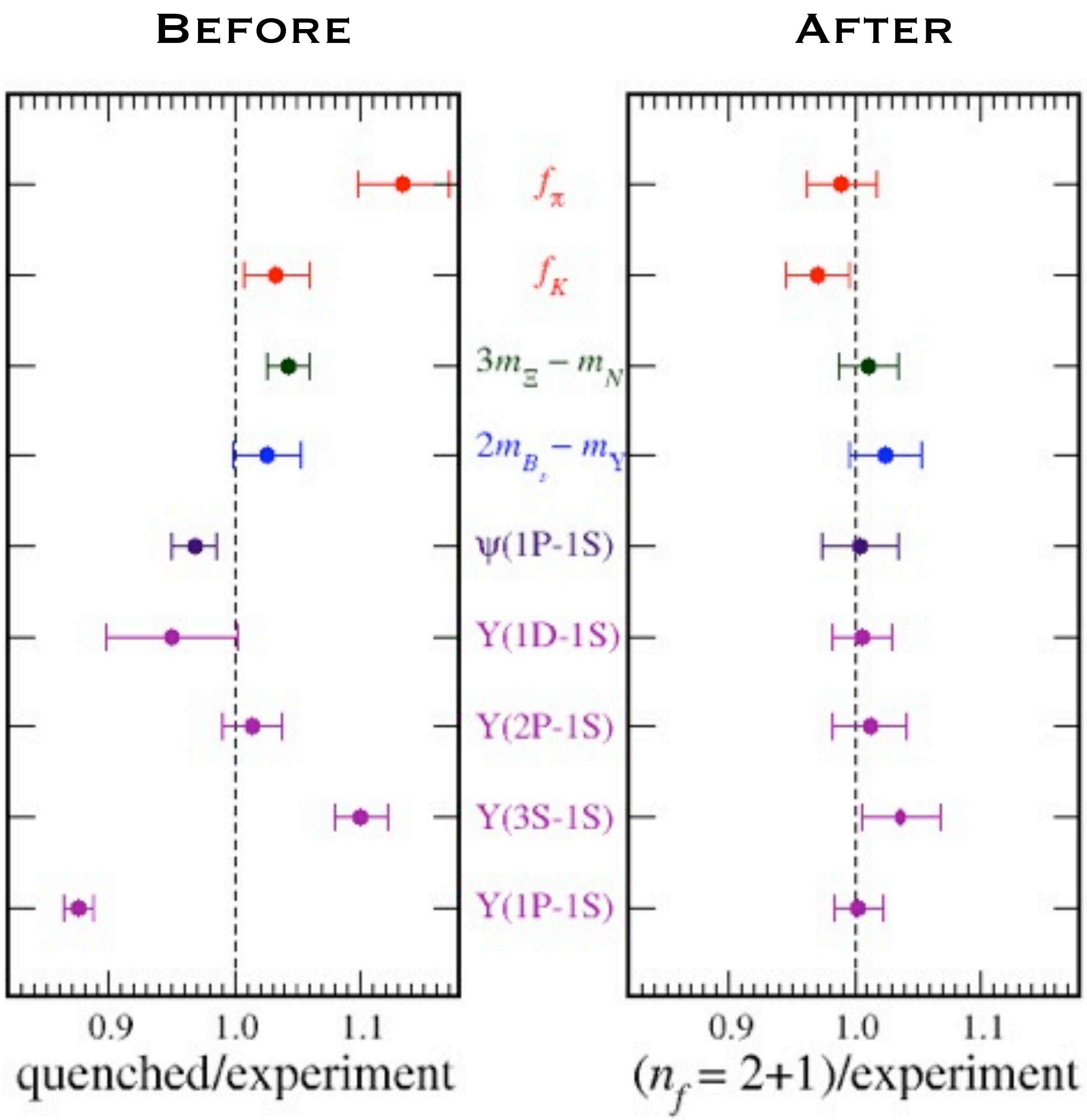
- Simulate at a sequence of quark masses & lattice spacings and extrapolate to $m_{lat} \rightarrow m_{phys}$ & $a \rightarrow 0$ using functional forms derived in chiral perturbation theory

✦ Verify understanding and control of systematic uncertainties in lattice calculations by COMPARING RESULTS FOR KNOWN QUANTITIES WITH EXPERIMENT

n_f=2+1 sea quarks

- ✦ Major breakthrough for lattice QCD
- ✦ **Realistic QCD calculations** that include the effects of the dynamical u, d, & s quarks in the vacuum





- ✦ Lattice QCD simulations now regularly include n_f=2+1 sea quarks

[HPQCD, MILC, & Fermilab Lattice Collaborations Phys.Rev.Lett.92:022001,2004]

n_f=2+1 sea quarks

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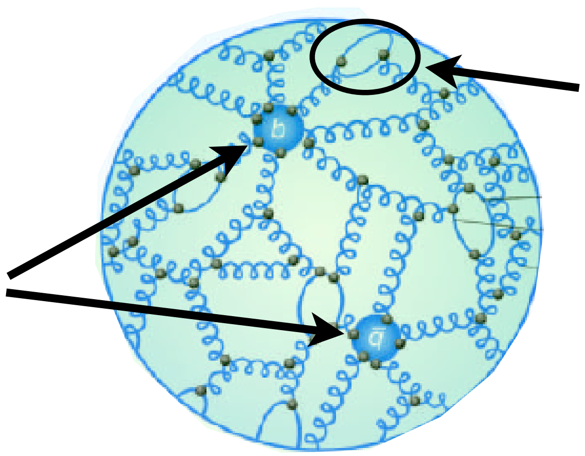
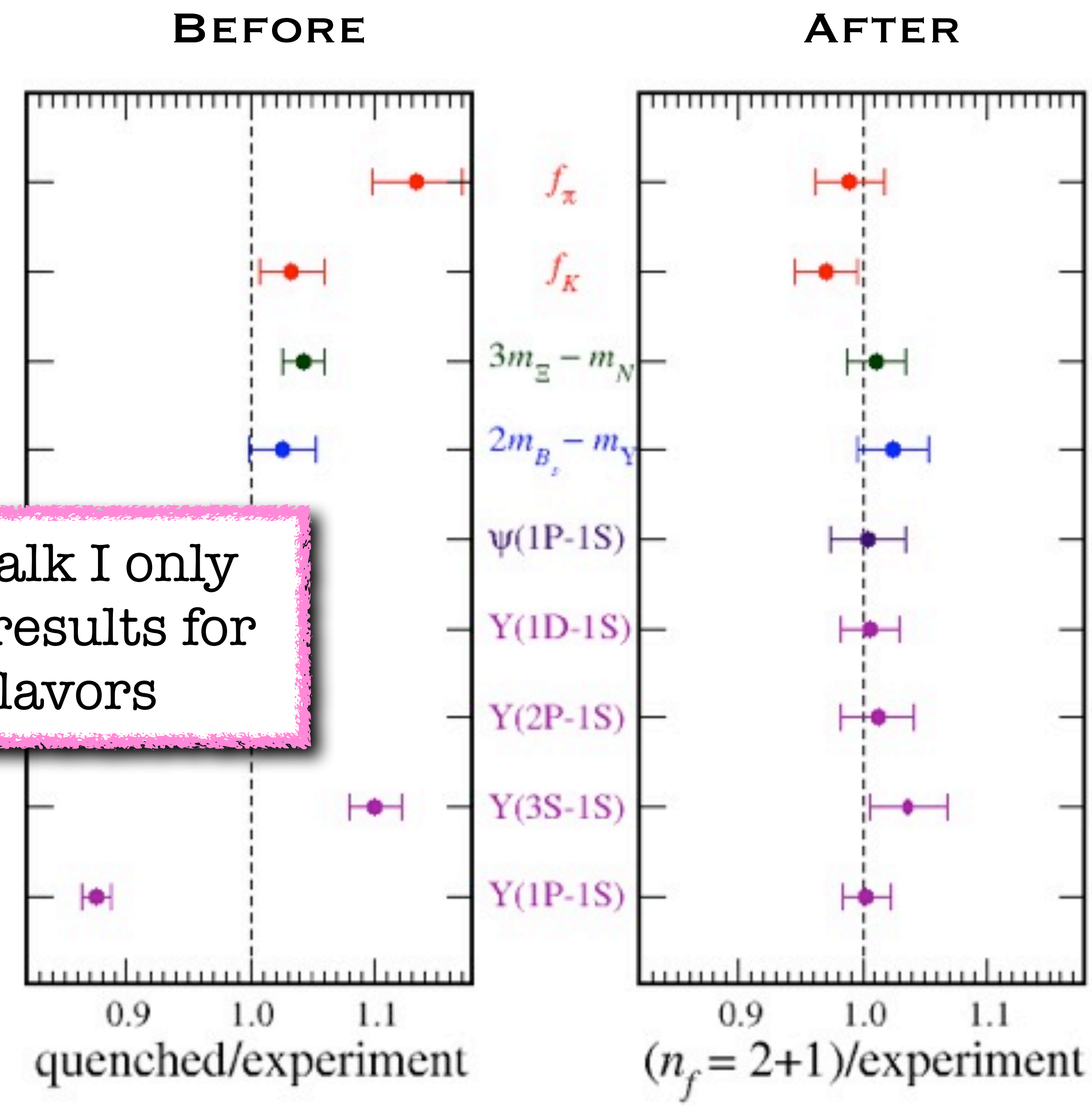
valence quarks



In this talk I only present results for 2+1 flavors



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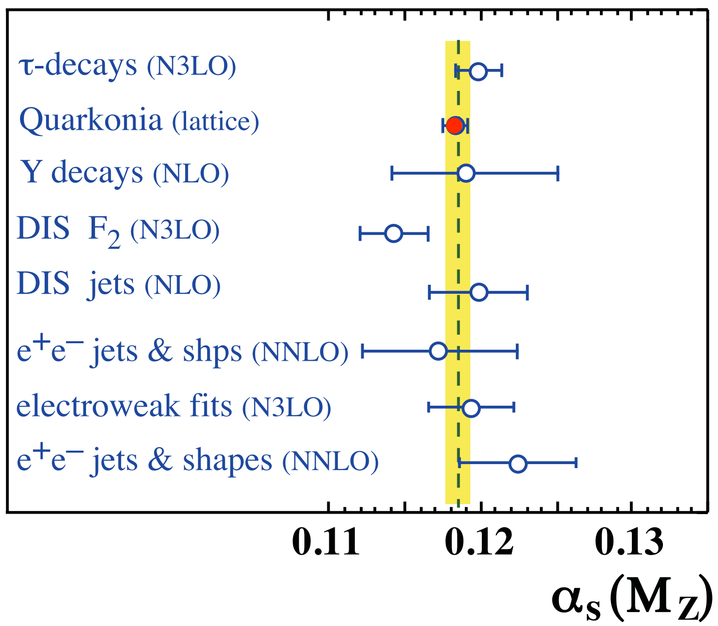
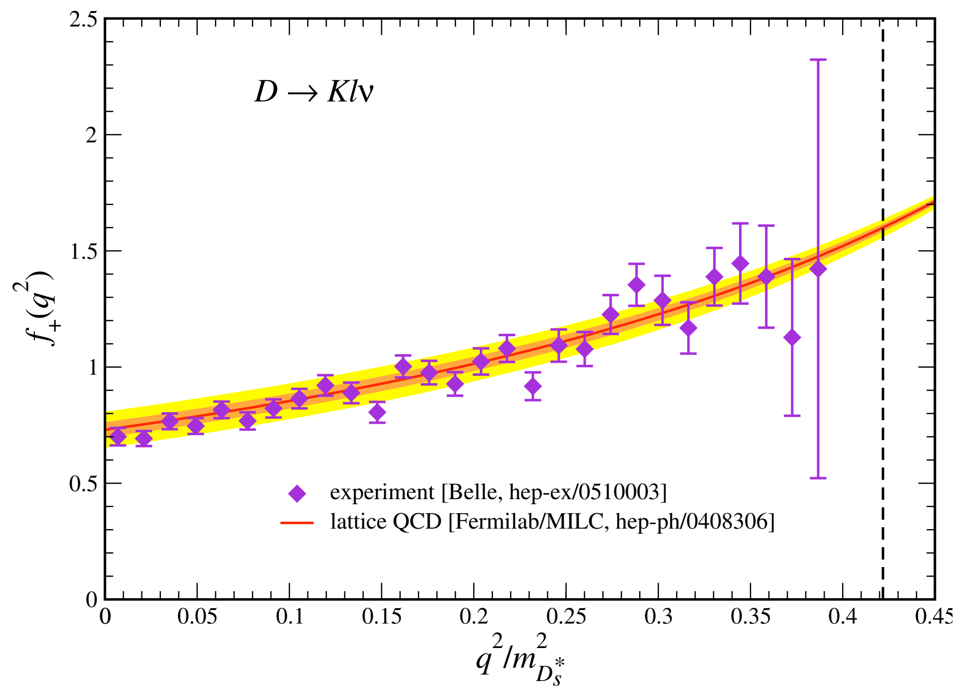
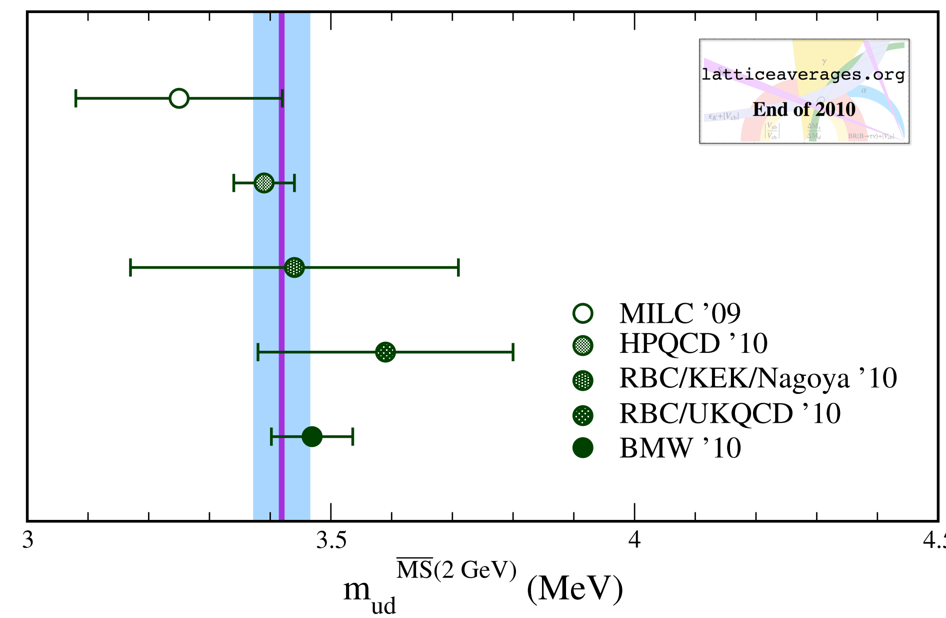
Successes of lattice QCD



[Bethke, Eur.Phys.J. C64 (2009)]



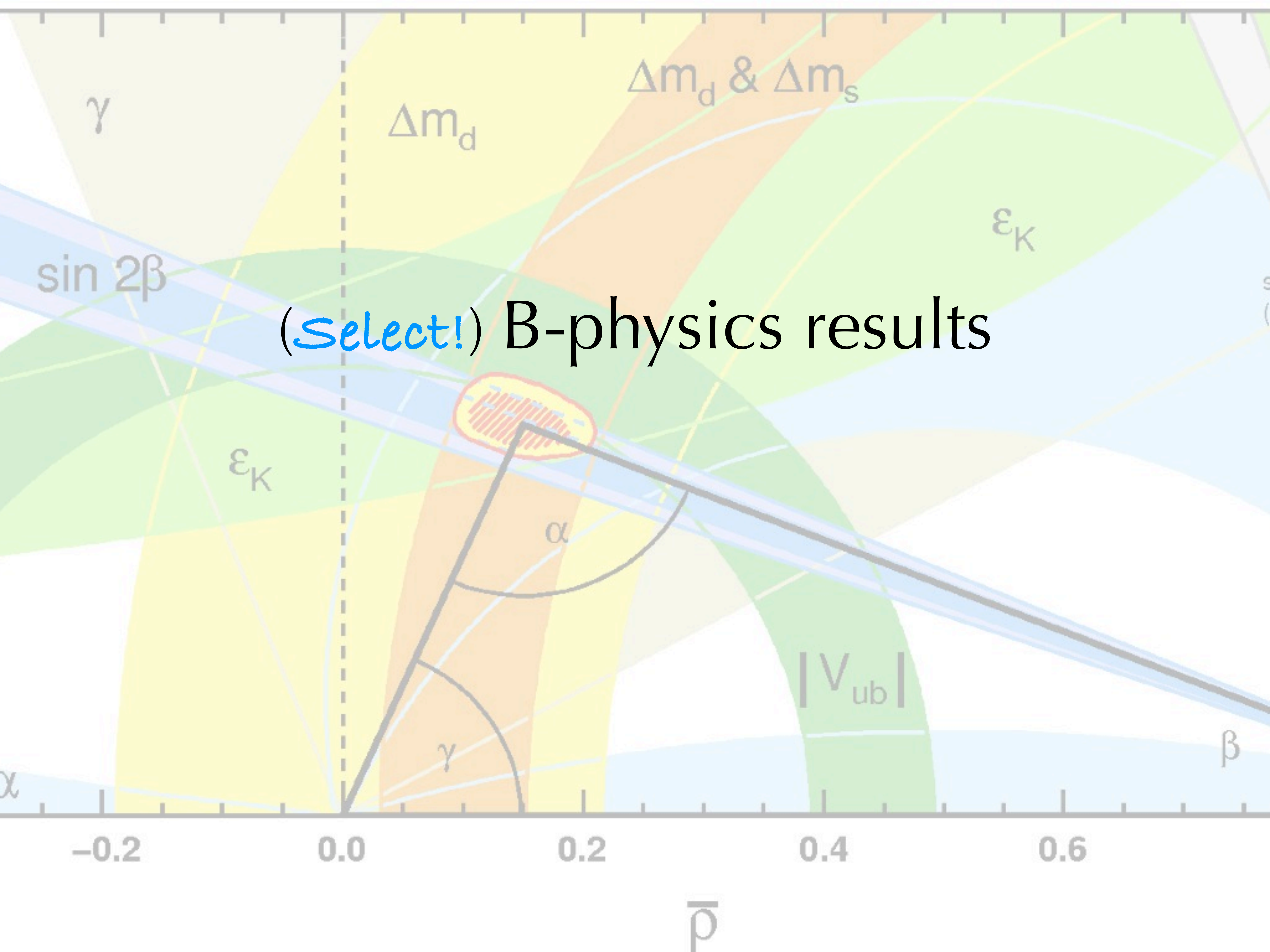
[Fermilab Lattice & MILC, Phys.Rev.Lett 94:011601,2005]



[Laiho, Lunghi, RV, Phys.Rev. D81 (2010) 034503 updates at www.latticeaverages.org]

- Lattice-QCD calculations now reproduce experimental results for a wide variety of hadron properties and provide the only *ab initio* QCD calculation of others, e.g.:
  - Most accurate determination of strong coupling constant
  - **Predictions** of B_c meson mass, decay constants f_D & f_{Ds}, and D→Klν form factor
  - Determinations of the light u, d, and s quark masses
- Demonstrate that **lattice-QCD calculations are reliable with controlled systematic errors**

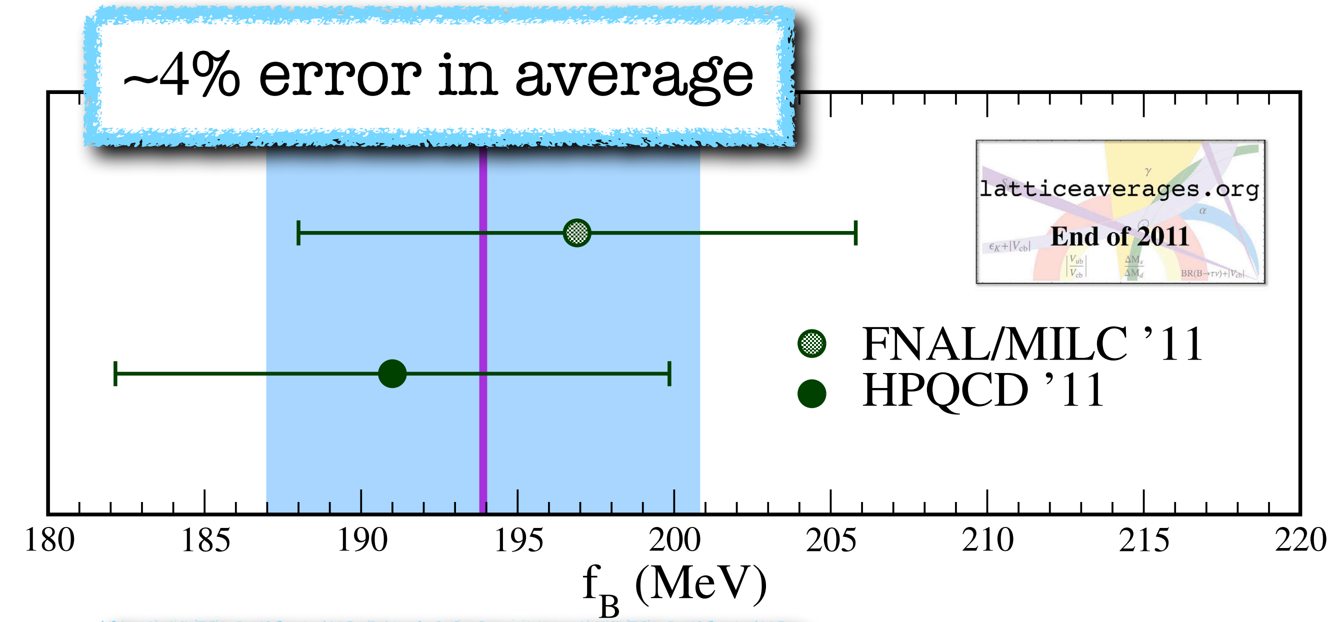
(Select!) B-physics results

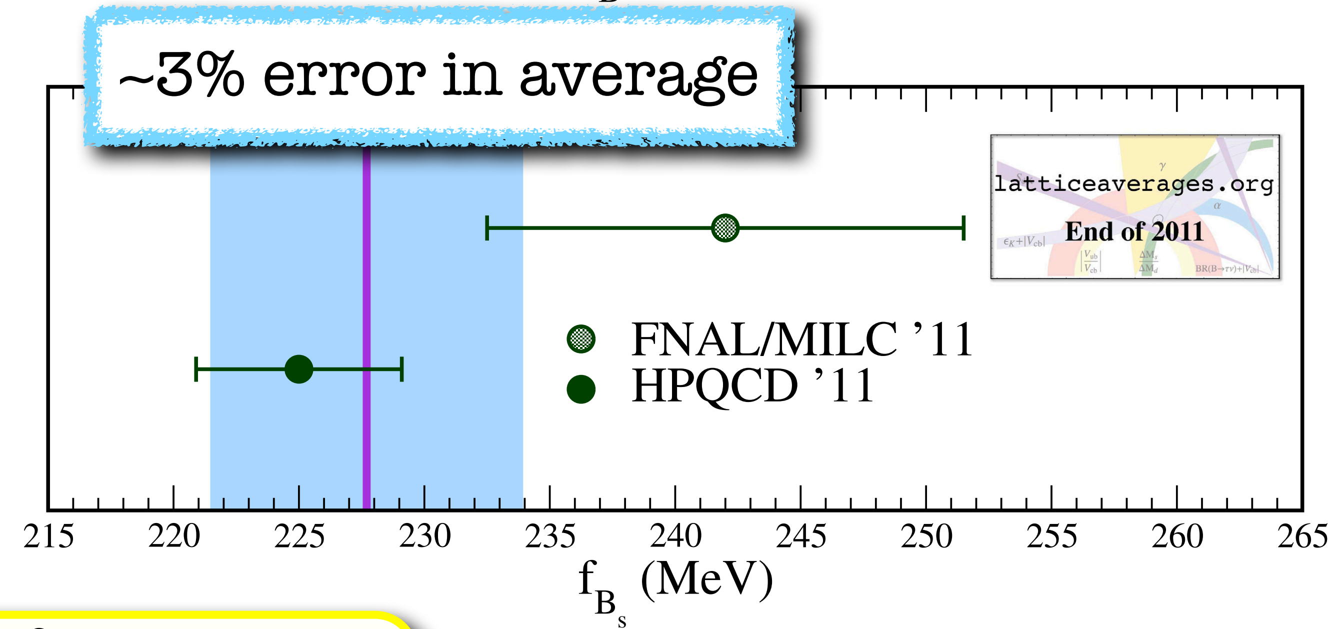


New result: f_B and f_{Bs}

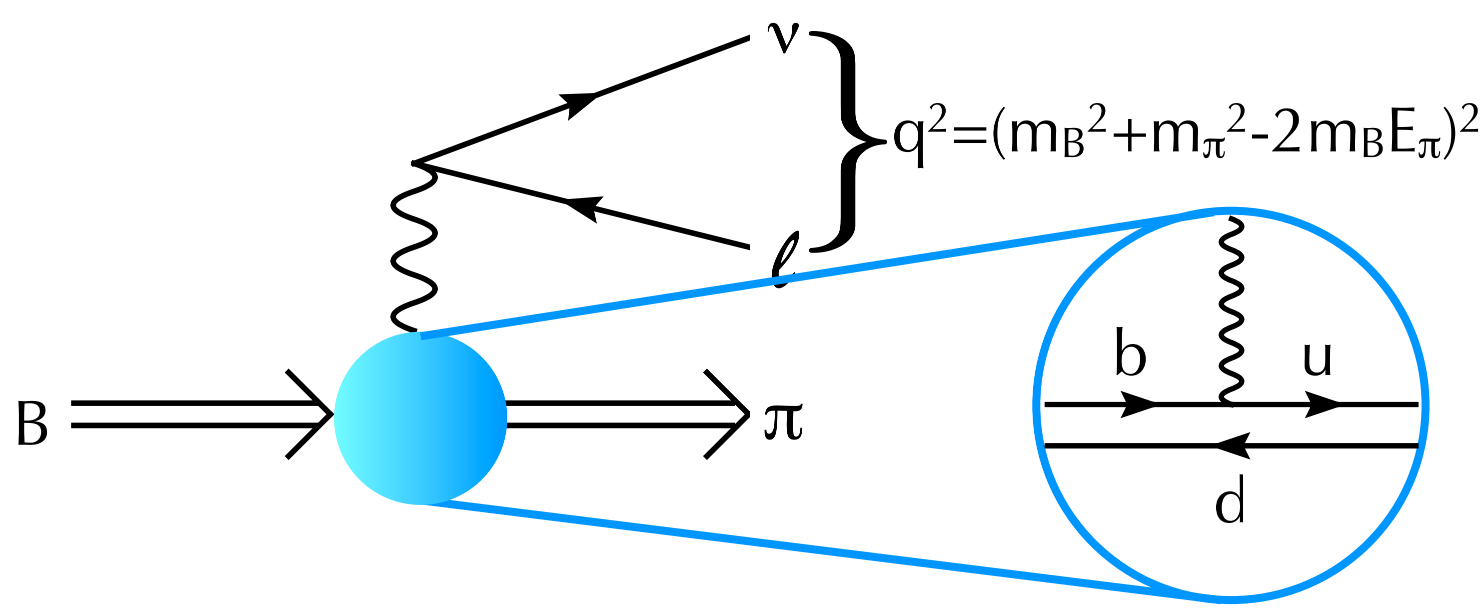
- Both Fermilab/MILC [1112.3051] and HPQCD [1110.5783, 1001.2023] recently presented new results for f_B and f_{Bs}
- Fermilab/MILC finalized work presented at conferences
- HPQCD presented preliminary f_{Bs} at Lattice 2011 using highly-improved staggered (HISQ) quarks:
  - Reduced error because no renormalization needed
  - Requires mass extrapolation up to physical b-quark
- f_B needed to obtain $|V_{ub}|$ from BR(B→τν):

$$\mathrm{BR}(B \to \tau\nu) = \frac{G_F^2 m_B m_\tau^2}{8\pi}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \mathbf{f_B^2}|\mathbf{V_{ub}}|^2 \tau_B$$





B → πℓν form factor



- The B → πℓν form factor also allows the determination of $|V_{ub}|$ via:

$$\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3}\left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2\right]^{3/2} |\mathbf{V_{ub}}|^2 |\mathbf{f_+(q^2)}|^2$$

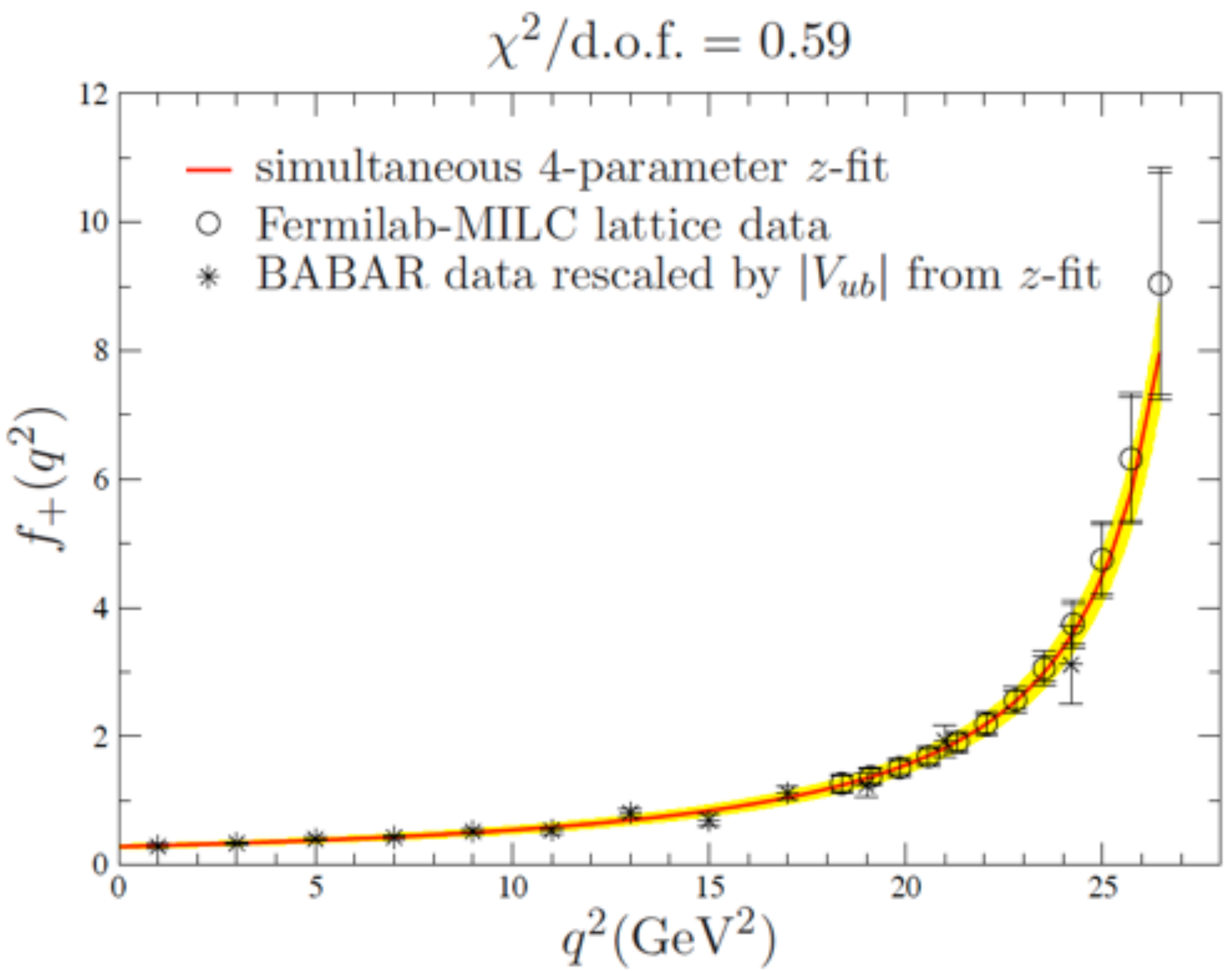
- **Few percent determination of exclusive $|V_{ub}|$ challenging**:
  - Lattice statistical errors grow with increasing pion momentum, so form factor determination best at large momentum-transfer (q^2), whereas errors in experimental branching fraction smallest at low q^2

Exclusive determination of $|V_{ub}|$

- Solution to fit perform a combined fit of the numerical lattice form factor data and experimentally-measured branching fraction data together to a **model-independent function based on analyticity, unitarity, and crossing-symmetry** leaving $|V_{ub}|$ as a free parameter [*c.f.* Arnesen *et. al.* Phys. Rev. Lett. 95, 071802 (2005)]

- Include full correlation matrices for lattice-QCD and experimental data

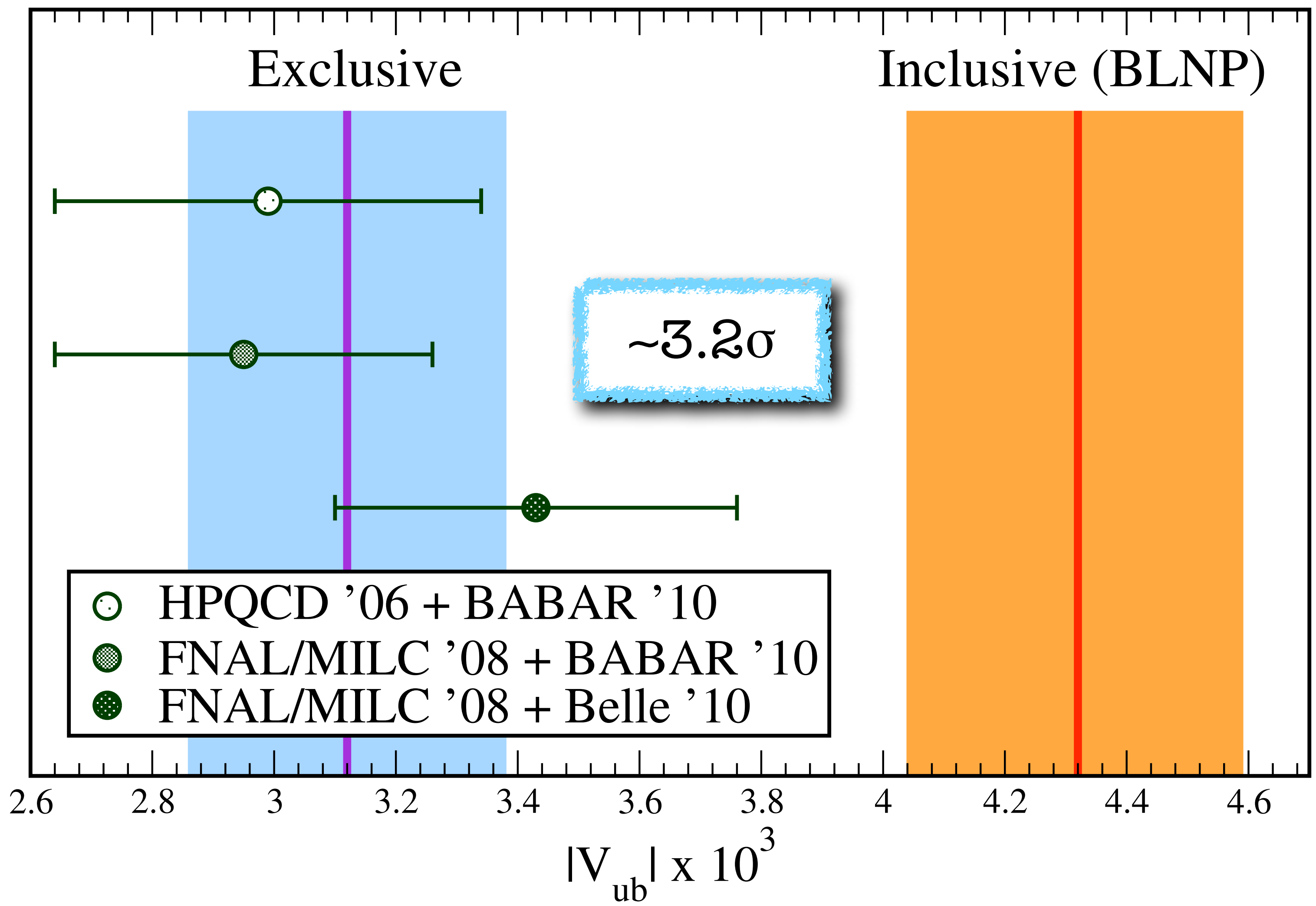
- Approach used by both BABAR [Phys.Rev.D83:032007,2011] and Belle [Phys.Rev.D83:071101,2011] and is being adopted by HFAG and the PDG



[Fermilab/MILC, Phys.Rev. D79 (2009) 054507]

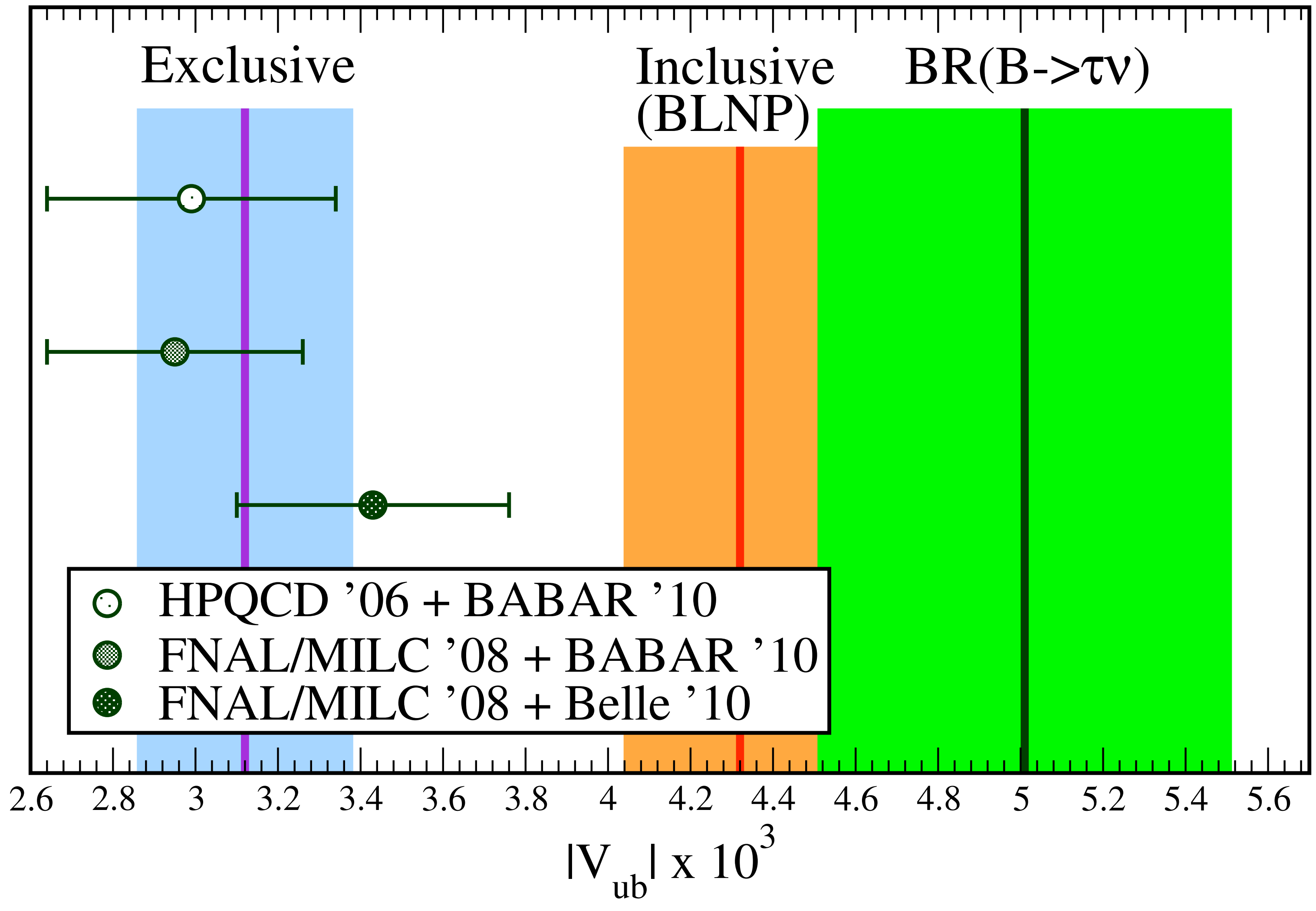
$|V_{ub}|$

- Long-standing puzzle is the **tension between inclusive and exclusive $|V_{ub}|$**



|V_{ub}|

- Long-standing puzzle is the **tension between inclusive and exclusive |V_{ub}|**
- Situation further muddled by BR($B \to \tau\nu$), which leads to even larger |V_{ub}|



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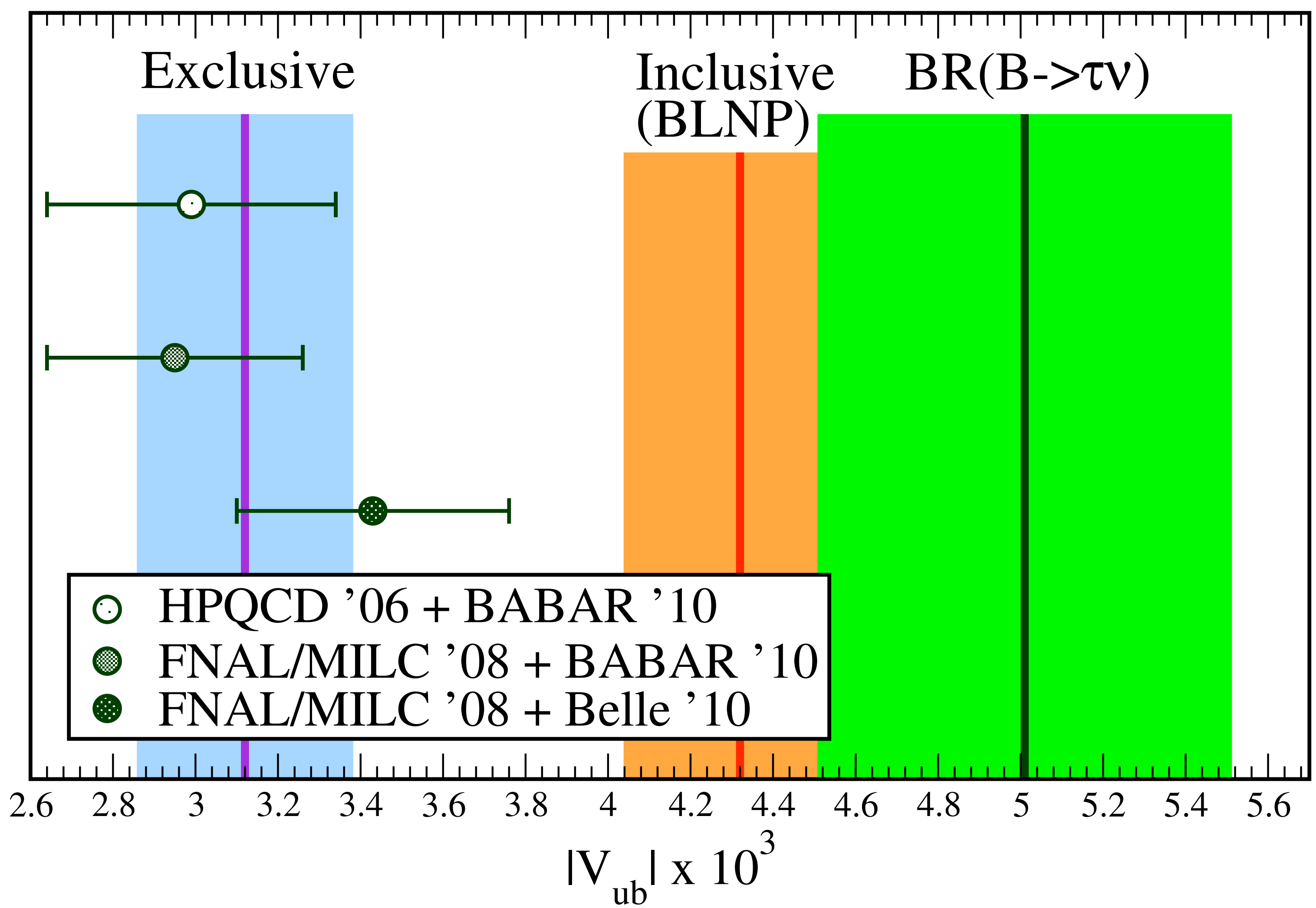
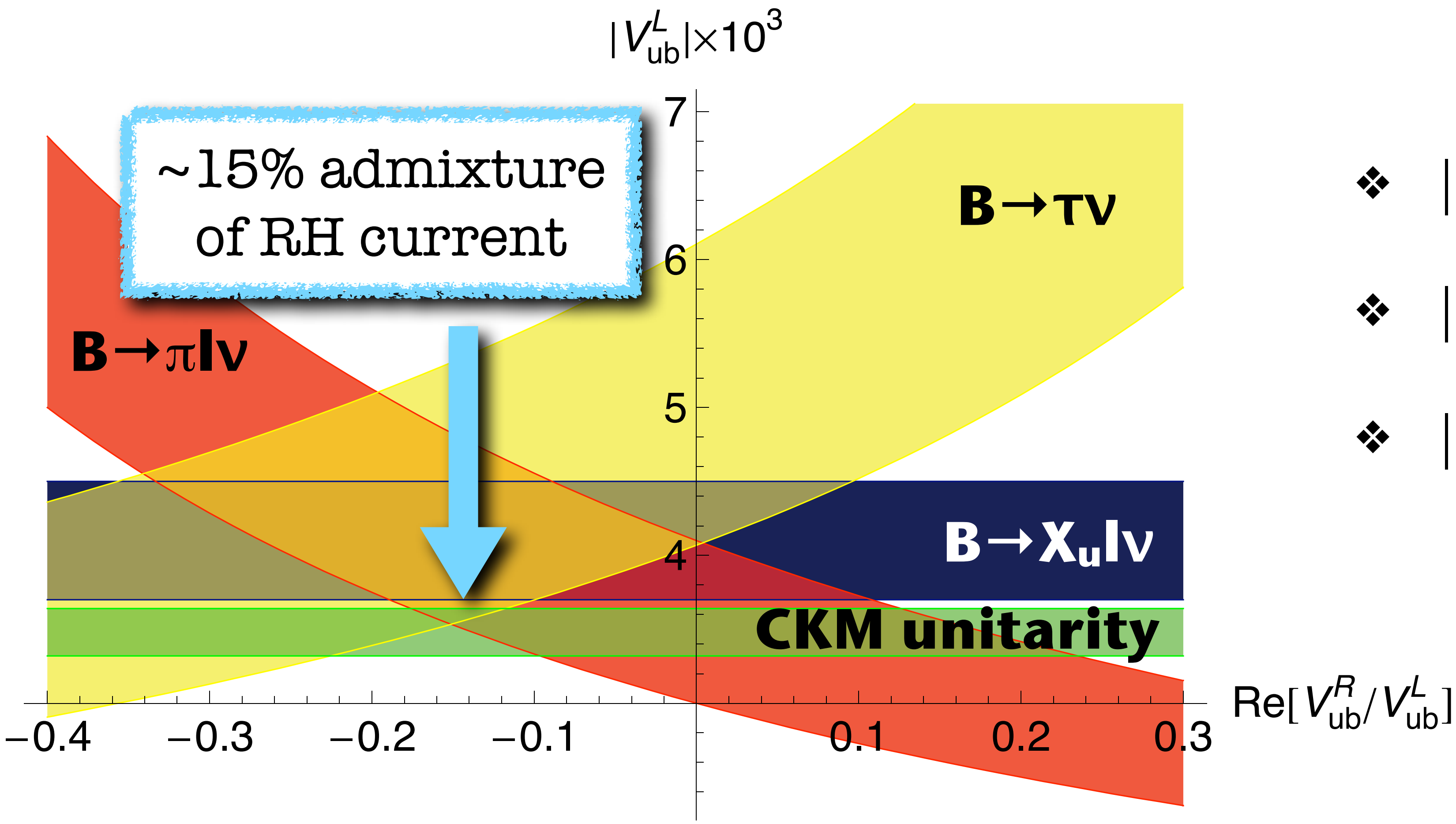


- Inclusive $|V_{ub}|$ varies depending upon theoretical framework and is highly sensitive to the input b-quark mass
- BR($B\to\tau\nu$) will be measured to greater precision at Belle II and super-B
- $|V_{ub}|$ can be obtained from other exclusive decay channels such as $B_s\to K\mu\nu$

Right-handed currents?

✦ Elegant solution provided by a right-handed weak current with coupling V_{ub}^R [Crivellin, Phys.Rev. D81 (2010) 031301], which enters as:



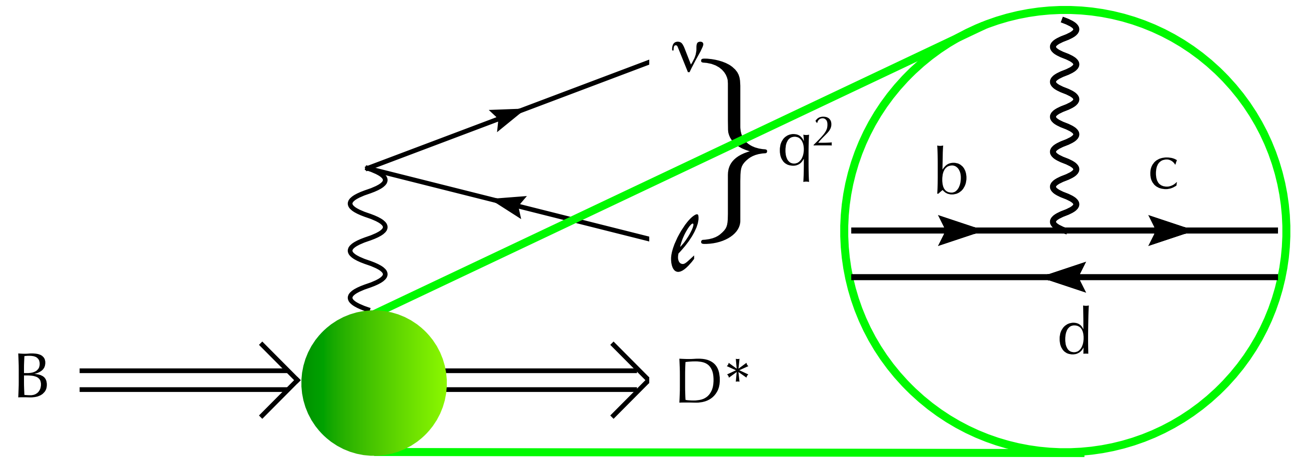
❖ $|V_{ub}^L+V_{ub}^R|^2$ in $B\to\pi l\nu$

❖ $|V_{ub}^L-V_{ub}^R|^2$ in $B\to\tau\nu$,

❖ $|V_{ub}^L|^2+|V_{ub}^R|^2$ in $B\to X_u l\nu$

✦ In practice, however, cannot realize this scenario in simplest left-right asymmetric models while satisfying other flavor constraints, especially from ϵ_K [Blanke, Buras, Gemmler, Heidsieck, arXiv:1111.5014 [hep-ph]]

$B \to D\ell\nu$ and $B \to D^*\ell\nu$ form factors



✦ The $B \to D\ell\nu$ and $B \to D^*\ell\nu$ semileptonic form factors allow determinations of $|V_{cb}|$ via

$$\frac{d\Gamma(B \to D^{(*)} l\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |\mathbf{V_{cb}}|^2 |\mathcal{F}_{\mathbf{B\to D^{(*)}}}(\mathbf{w})|^2$$

$w \equiv v_B \cdot v_D$

✦ Only need one normalization point from lattice-QCD, so choose zero recoil (w=1) because it can be computed most precisely ➡ **obtain results with ~2% errors:**

$$\mathcal{F}_{B\to D}(1) = 1.074(18)_{\text{stat}}(16)_{\text{sys}}$$

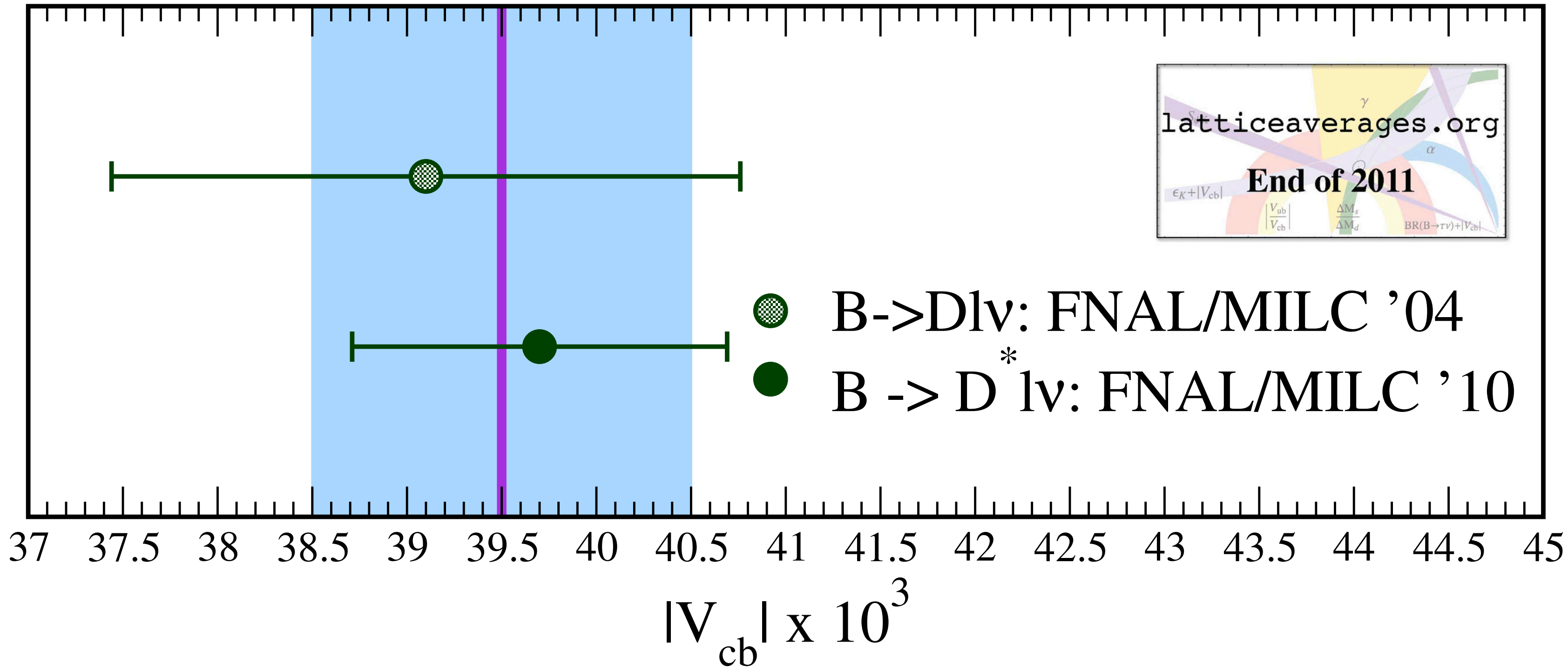
[Fermilab/MILC, Nucl. Phys. Proc. Suppl.140, 461 (2005)]

$$\mathcal{F}_{B\to D^*}(1) = 0.9077(51)_{\text{stat}}(158)_{\text{sys}}$$

[Fermilab/MILC, arXiv:1011.2166]

$|V_{cb}|$

~2.5% error in average

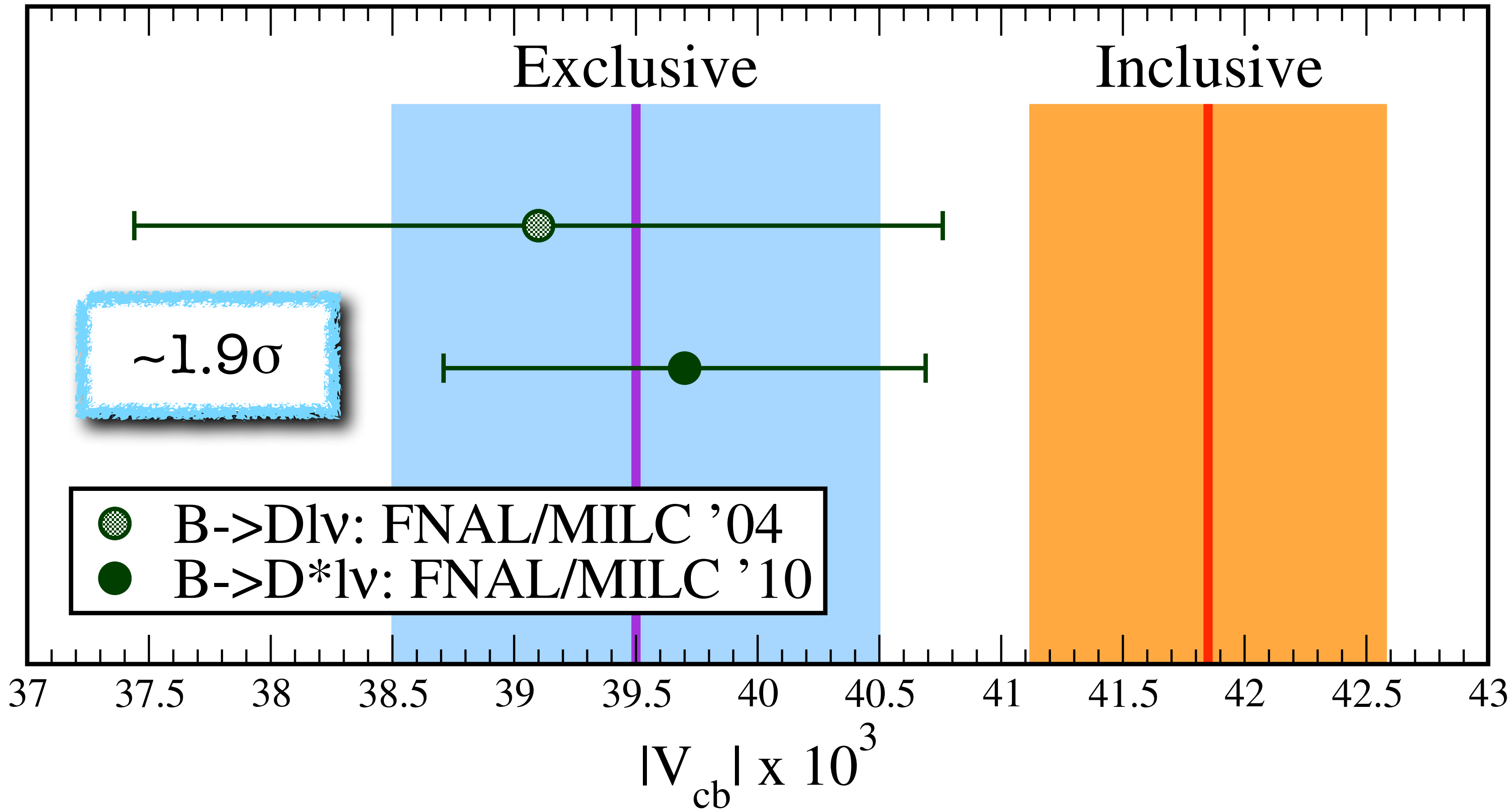


- Currently only two lattice-QCD results, but calculations by other collaborations in progress

- Extraction of $|V_{cb}|$ from BR($B \to D^* l\nu$) problematic because experiments are not consistent (confidence level of HFAG global fit is 2%)

- Errors in $|V_{cb}|_{excl}$ from $B \to Dl\nu$ may be reduced with lattice-QCD calculations of the form factor at nonzero recoil and with experimental untagged analysis on full BABAR/Belle datasets and new tagged analysis from BABAR [Lopes-Pegna CKM 2010]

$|V_{cb}|$



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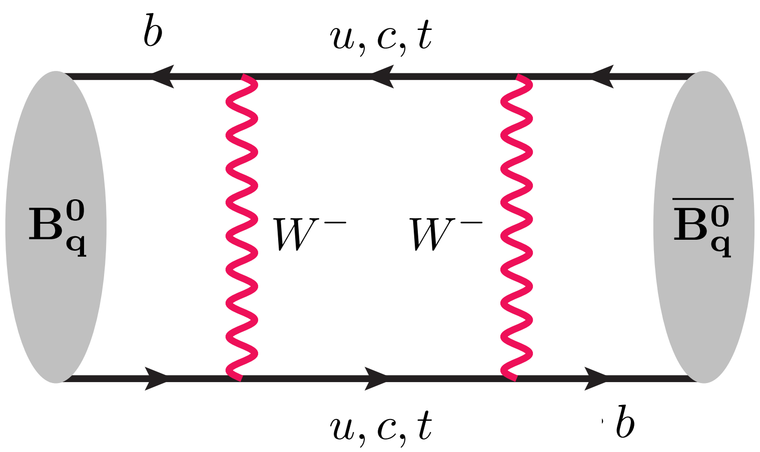
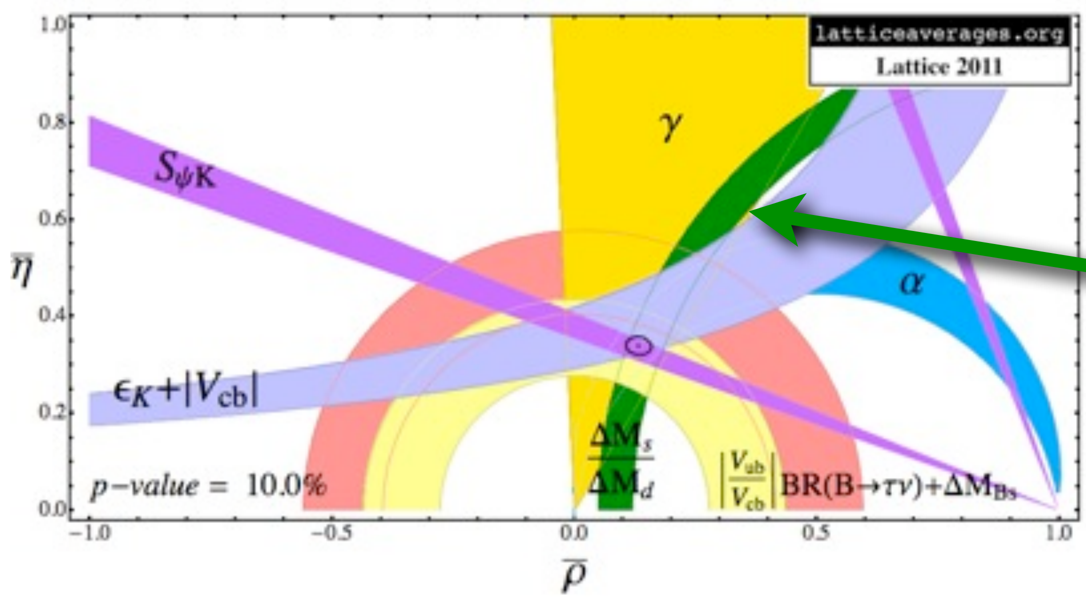
- Also persistent, but smaller, tension between inclusive and exclusive $|V_{cb}|$

Neutral $B_{(d,s)}$-meson mixing

- Sensitive to new heavy particles that can enter loops in the leading-order Standard-Model box diagram:
- Constrains scale of generic new physics to be 100 TeV [Isidori, Nir, Perez, Ann.Rev.Nucl.Part.Sci. 60 (2010) 355]





The ratio of B_d to B_s oscillation frequencies (Δm_q) constrains the apex of the CKM unitarity triangle

$$\frac{\Delta m_d}{\Delta m_s} = \left(\frac{f_{B_d}\sqrt{\hat{B}_{B_d}}}{f_{B_s}\sqrt{\hat{B}_{B_s}}}\right)^2 \frac{m_{B_d}}{m_{B_s}}\frac{|V_{td}|^2}{|V_{ts}|^2} = \xi^2 \frac{m_{B_d}}{m_{B_s}}\left(\frac{\lambda}{1-\lambda^2/2}\right)^2 \frac{((1-\bar{\rho})^2+\bar{\eta}^2)}{\left(1+\frac{\lambda^2}{1-\lambda^2/2}\bar{\rho}\right)+\lambda^4\bar{\eta}^2}$$

- Δm_q measured to better than 1% and $\lambda=|V_{us}|$ known to ~1%
- Dominant error currently from **uncertainty in lattice-QCD calculations of the ratio ξ**

New result: $B^0_{(d,s)}$-mixing matrix elements

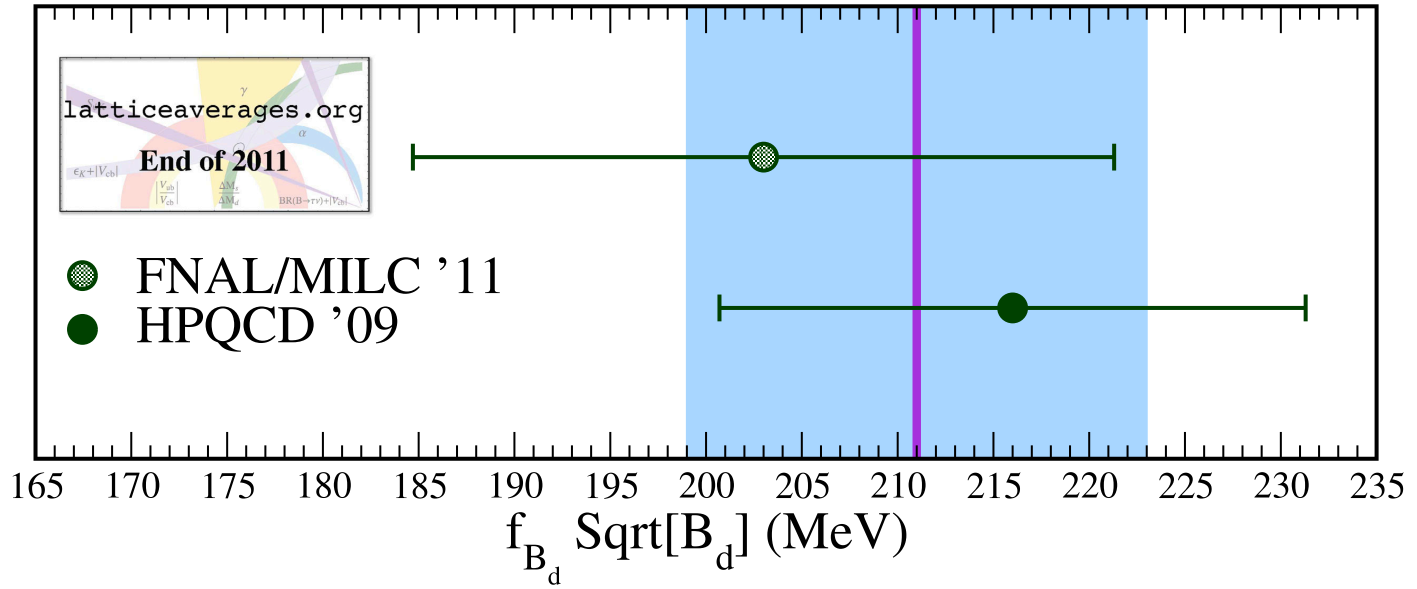
- Fermilab/MILC [Bouchard, Freeland, *et al.*,1112.5642] recently presented preliminary results for B-mixing matrix elements at Lattice 2011

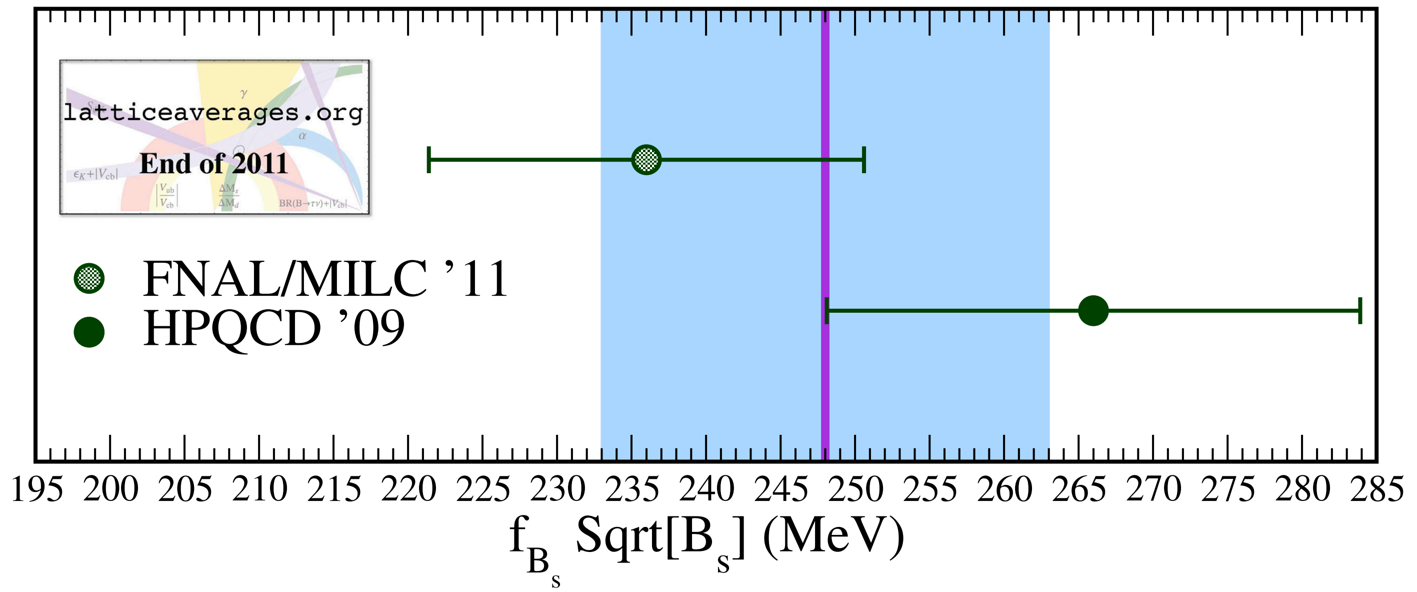
- Obtain first n_f=2+1 results for matrix elements for the full basis of ΔB=2 operators that appear in beyond-the Standard Model theories

- Expect further error reduction before publication due to inclusion of data at finer lattice spacings and a more rigorous estimate of some systematics

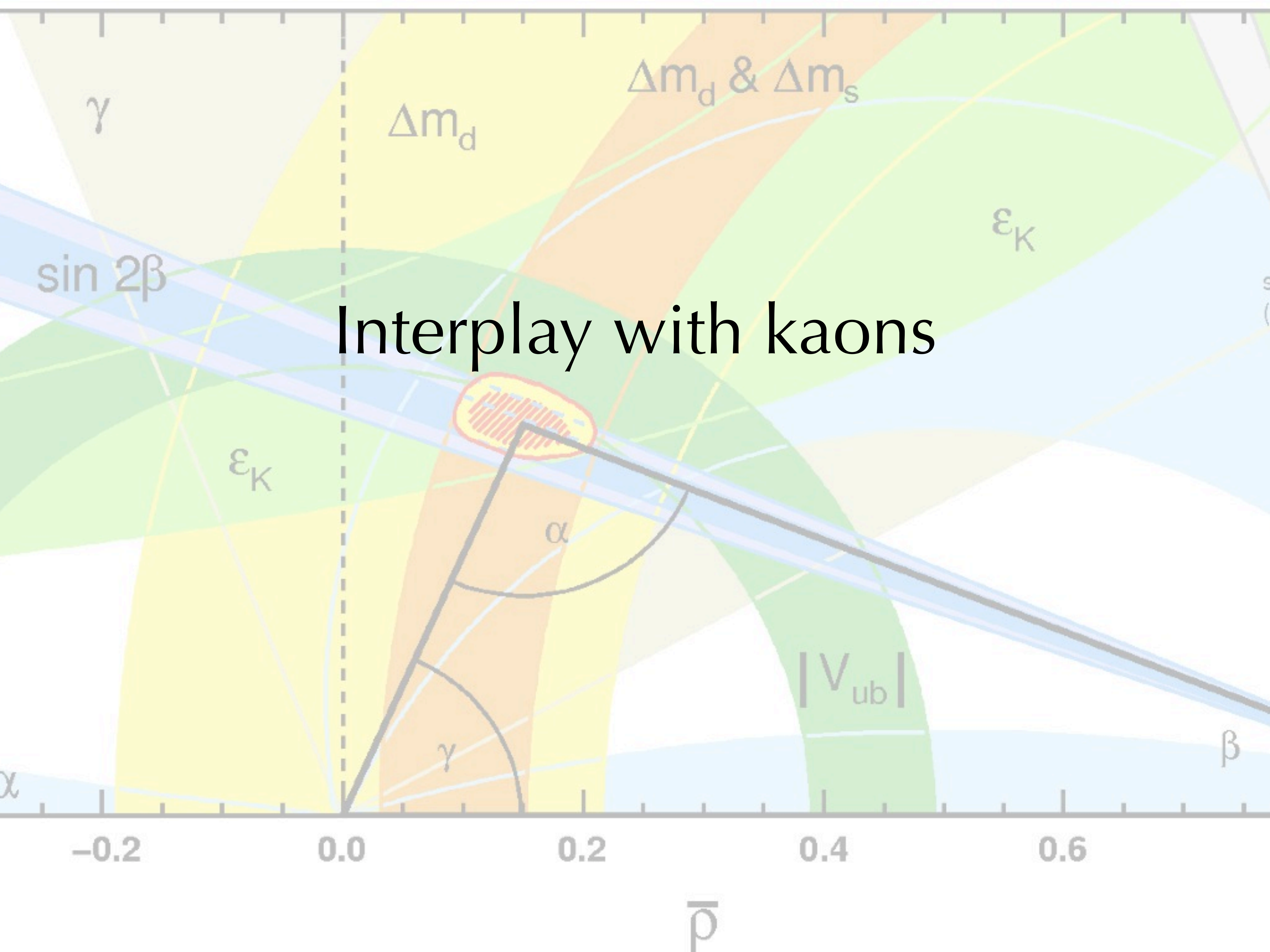
- **Hadronic uncertainty still much larger than experimental errors**

~6% error in averages





Interplay with kaons



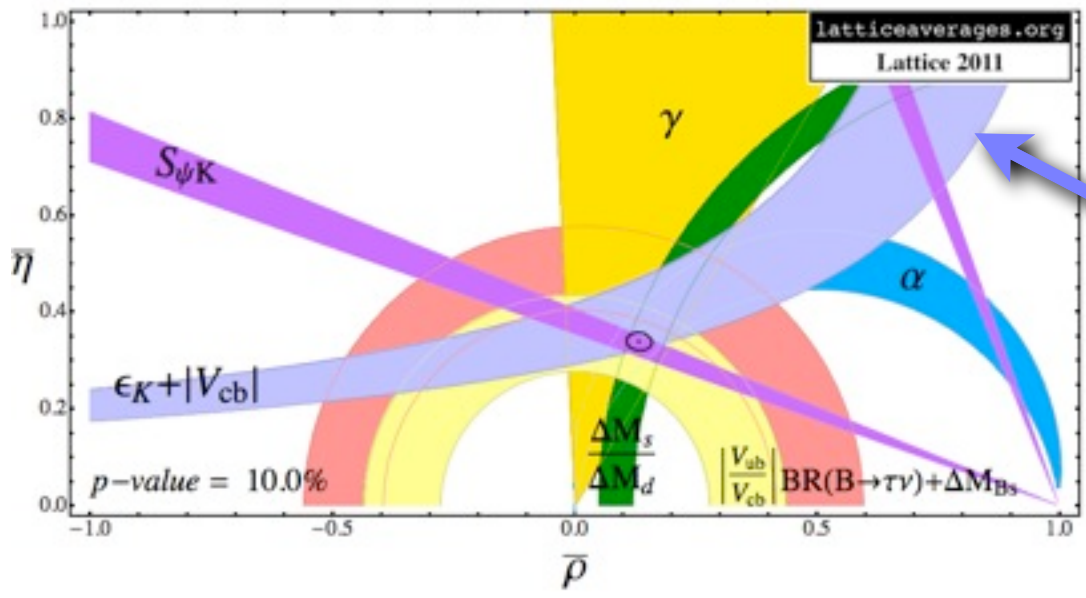
Neutral kaon mixing

- Standard-Model flavor-changing effects suppressed most strongly in the kaon sector due to hierarchical structure of CKM matrix:

$$\underbrace{V_{ts}^{*}V_{td}}_{\text{K system}} \sim 5\cdot 10^{-4} \ll \underbrace{V_{tb}^{*}V_{td}}_{B_d \text{ system}} \sim 10^{-2} \ll \underbrace{V_{tb}^{*}V_{ts}}_{B_s \text{ system}} \sim 4\cdot 10^{-2}$$

➡ Kaon system exhibits the highest new physics sensitivity

➡ Conversely, indirect CP-violation in the neutral kaon system (ϵ_K) currently places the single most stringent constraint on the scale of generic new physics $\gtrsim$10,000 TeV [Isidori, Nir, Perez, Ann.Rev.Nucl.Part.Sci. 60 (2010) 355]



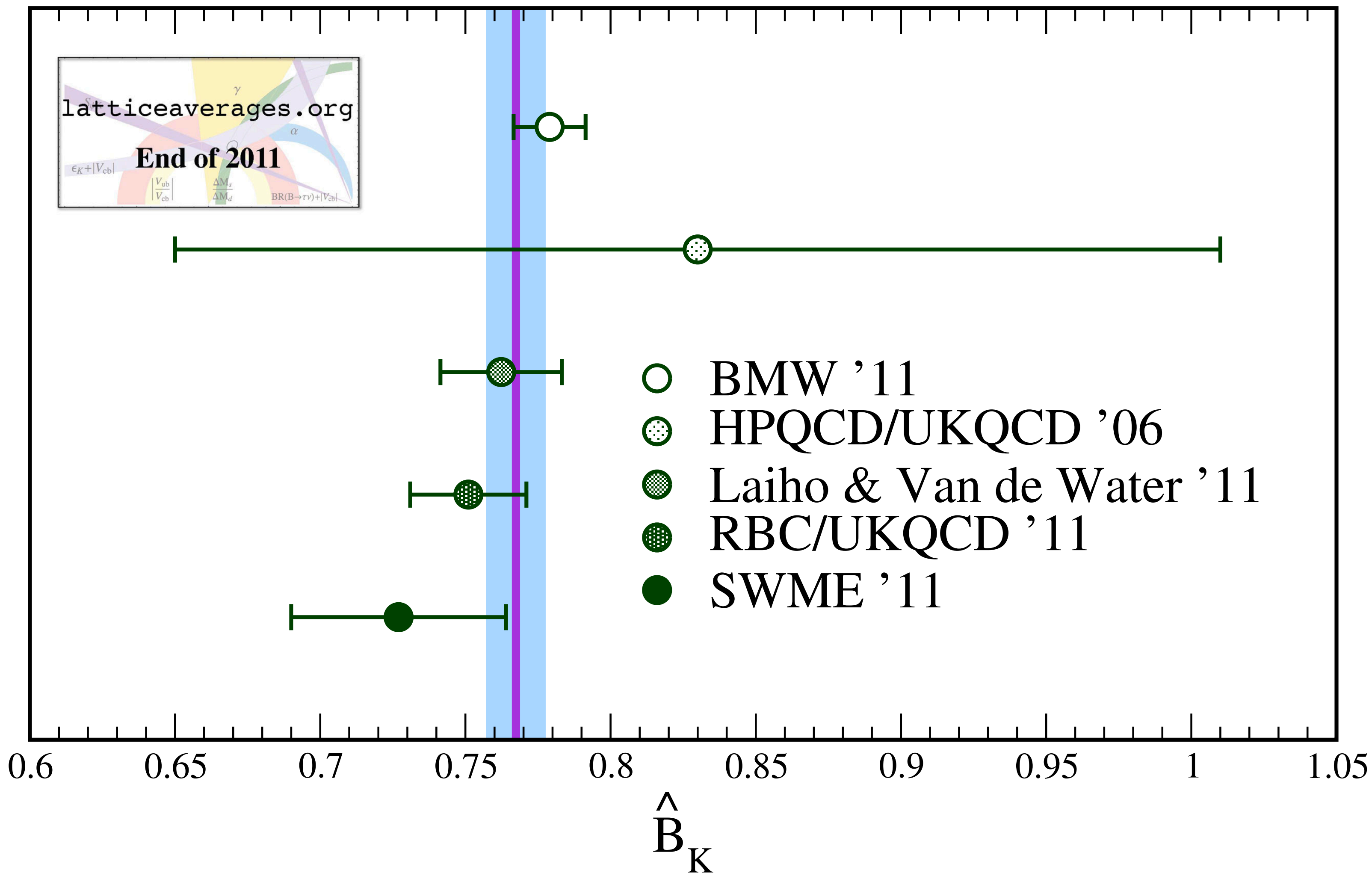
ϵ_K measured experimentally to sub-percent accuracy and places a key constraint on the apex of the CKM unitarity triangle

New result: B_K

- Until recently, the unitarity-triangle constraint from ϵ_K was limited by the large uncertainty in the hadronic matrix element B_K

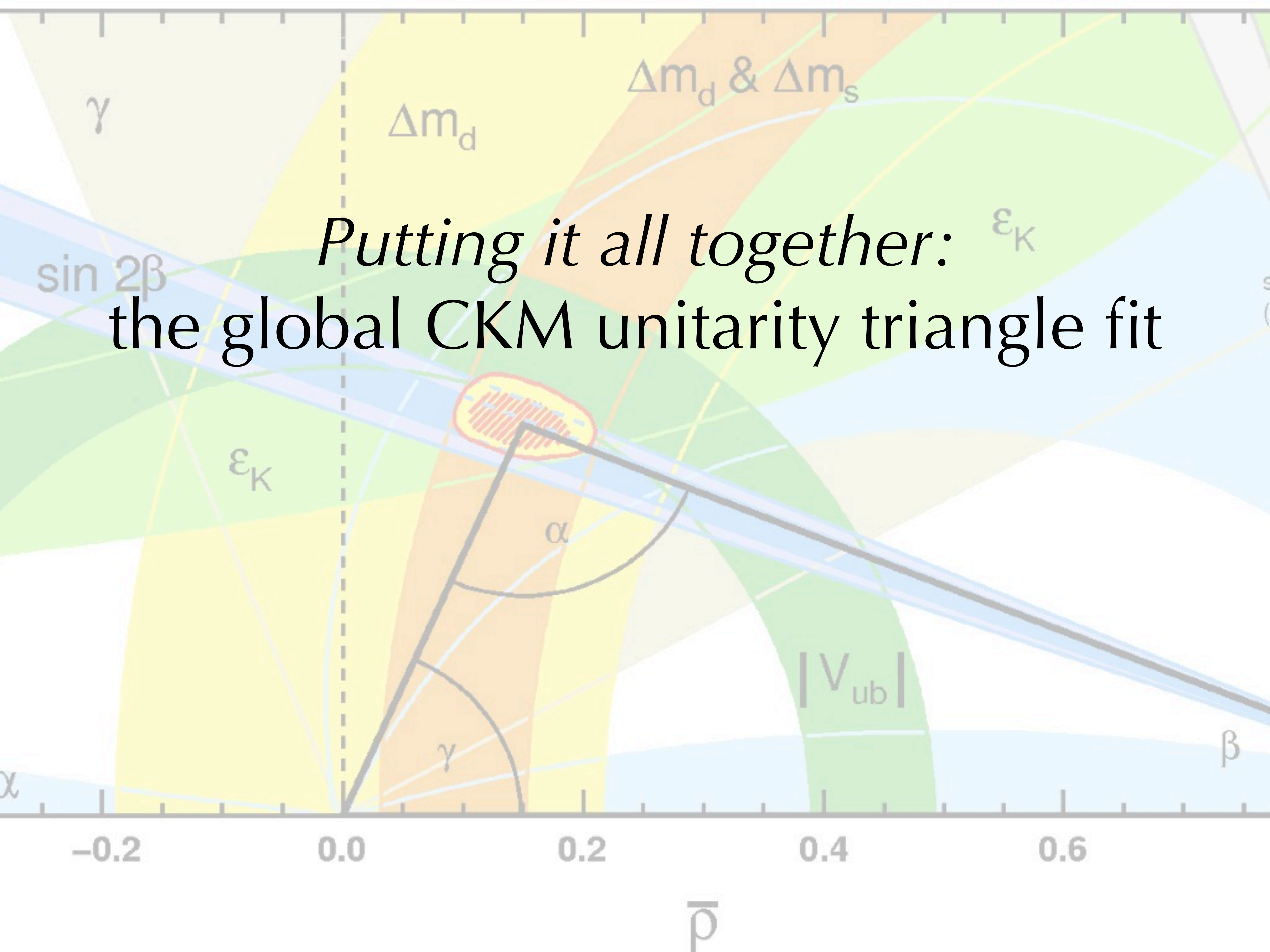
- Now several independent lattice-QCD calculations in good agreement

~1.3% error in average



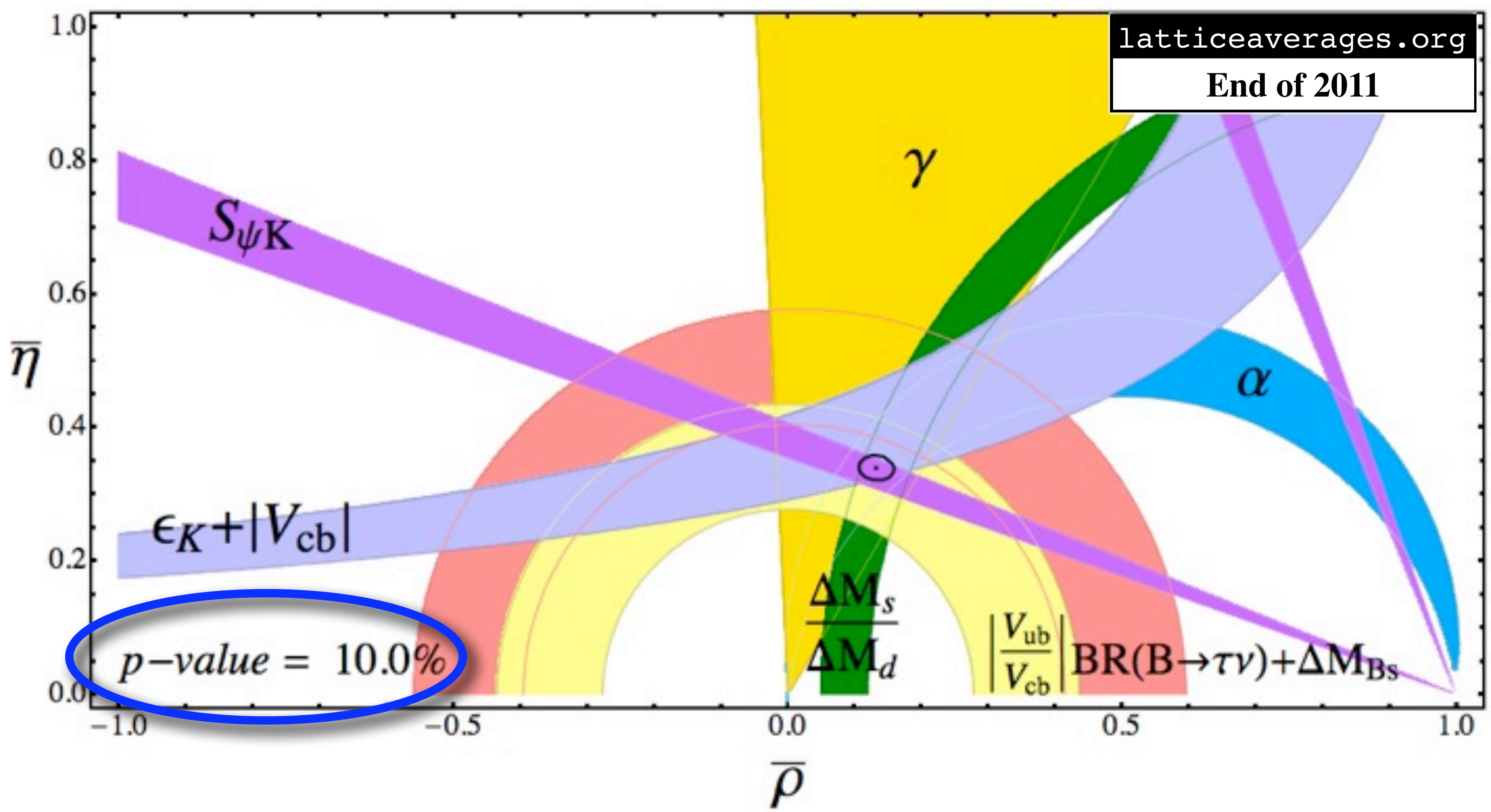
- B_K only the 4th largest uncertainty in the Standard-Model prediction for ϵ_K:
  - Below perturbative-QCD errors from short-distance coefficients [Brod & Gorbahn]
  - Largest uncertainty in the ϵ_K band is now from the ~10% parametric error in $A^4 \propto |V_{cb}|^4$

Putting it all together:
the global CKM unitarity triangle fit



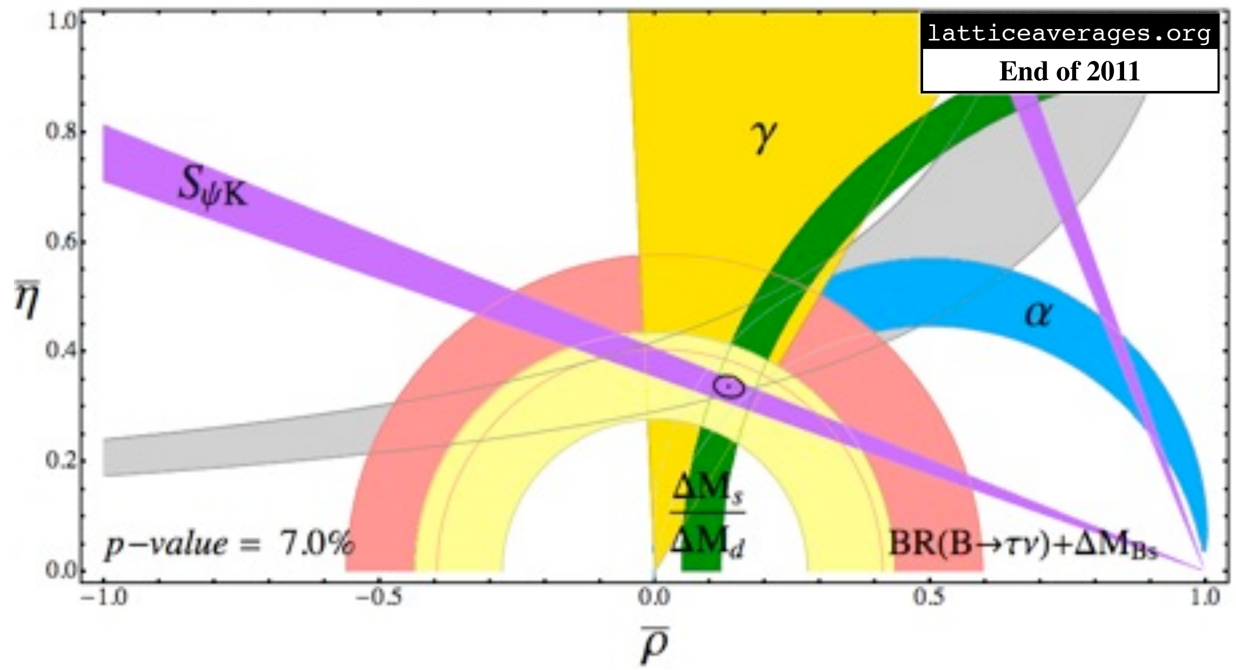
Tension in the CKM unitarity triangle?

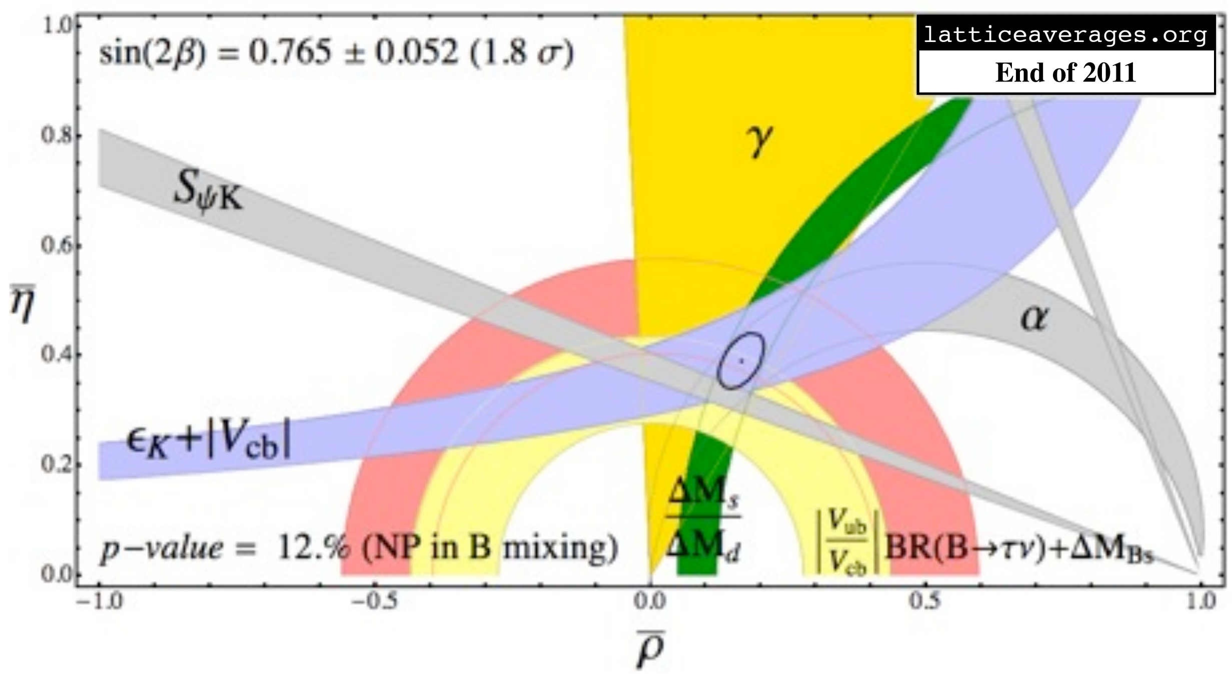
- Improved lattice matrix element QCD calculations have shrunk substantially the allowed region of parameter space in the $\rho-\eta$ plane and revealed a ~3σ tension in the CKM unitarity triangle [CKMfitter; Laiho, Lunghi, RV; Lunghi & Soni; UTFit]
  - Tension remains significant even omitting more problematic inputs $|V_{ub}|$ and $|V_{cb}|$

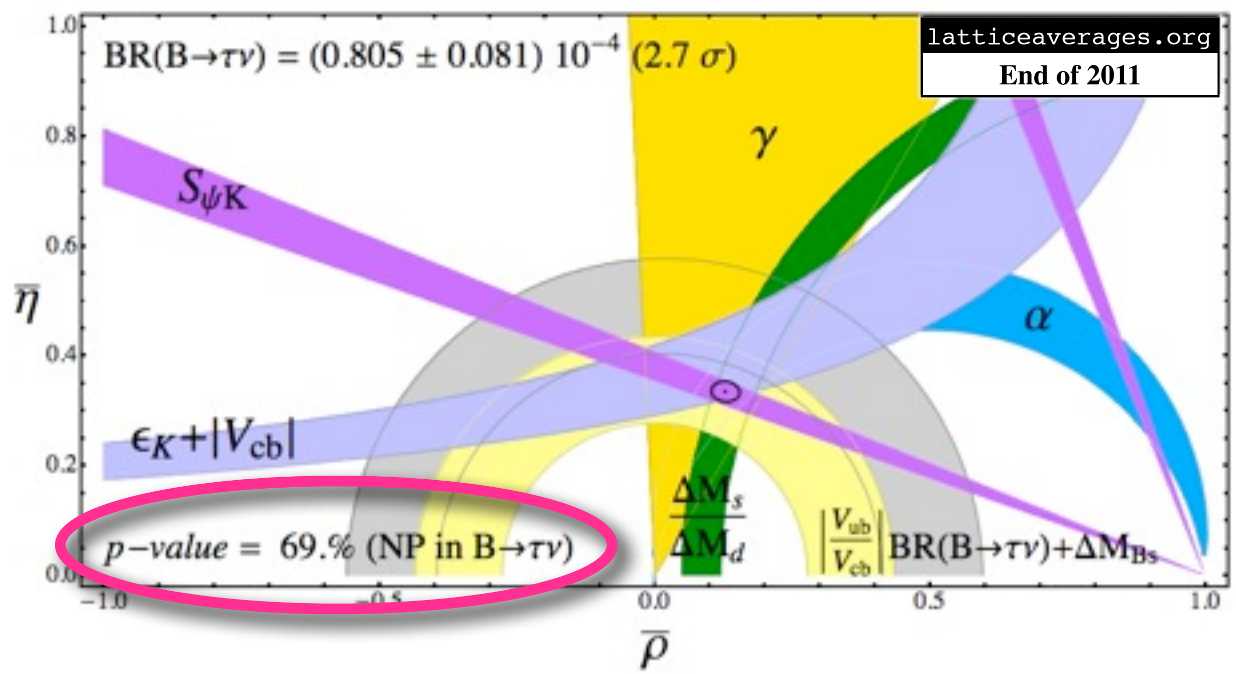


New-physics scenarios

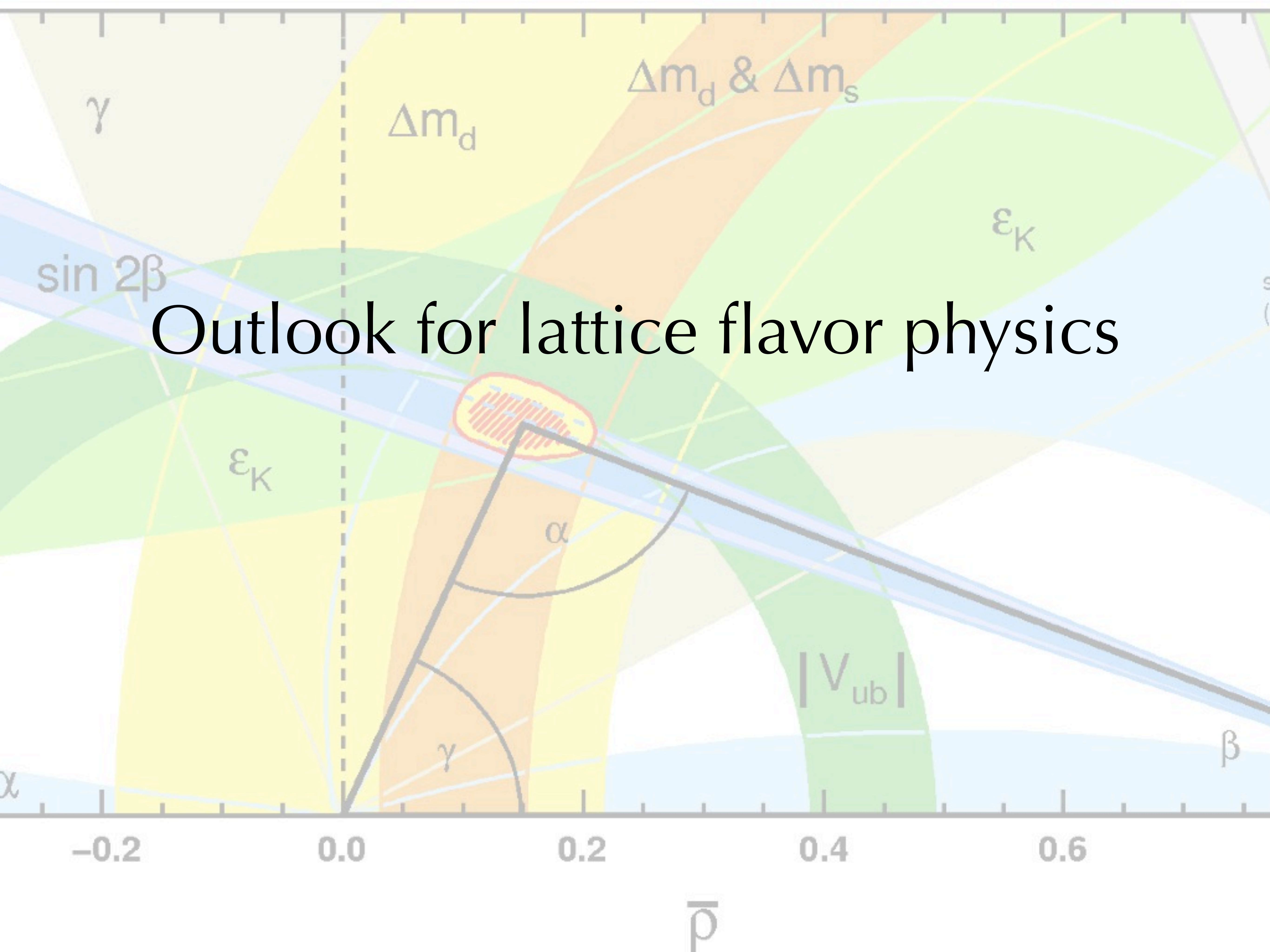
- Consider scenarios in which new heavy particles modify Standard-Model predictions for kaon mixing, B_d-mixing, or $B \to \tau\nu$ decay:
  - Data prefers hypothesis of new physics in $B \to \tau\nu$







Outlook for lattice flavor physics



Forecasts and plans

(1) Still work needed to obtain precision comparable to experiment for many quantities

- Future increases in computing power will help most sources of uncertainty, either directly or indirectly
- Improved algorithms and analysis methods being pursued, but difficult to predict

Quantity	CKM element	present expt. error	present lattice error	2014 lattice error	2020 lattice error	error from non-lattice method
f_K/f_π	V_{us}	0.2%	0.6%	0.3%	0.1%	–
$f_{K\pi}(0)$	V_{us}	0.2%	0.5%	0.2%	0.1%	1% (ChPT)
$D\to\pi\ell\nu$	V_{cd}	2.6%	10.5%	4%	1%	–
$D\to K\ell\nu$	V_{cs}	1.1%	2.5%	2%	$<1\%$	5% (ν scatt.)
$B\to D^*\ell\nu$	V_{cb}	1.8%	1.8%	0.8%	$<0.5\%$	$<2\%$ (Incl. $b\to c$)
$B\to\pi\ell\nu$	V_{ub}	4.1%	8.7%	4%	2%	10% (Incl. $b\to u$)
$B\to\tau\nu$	V_{ub}	21%	6.4%	2%	$<1\%$	–
ξ	V_{ts}/V_{td}	1.0%	2.5%	1.5%	$<1\%$	–

Forecasts and plans

(1) Still work needed to obtain precision comparable to experiment for many quantities

- ❖ Future increases in computing power will help most sources of uncertainty, either directly or indirectly
- ❖ Improved algorithms and analysis methods being pursued, but difficult to predict

(2) Given success with simplest quantities, expanding repertoire of calculations, e.g.:

- ❖ Rare B-decays such as $B \to K^{(*)} l^+ l^-$ and $B \to K^* \gamma$
- ❖ $K \to \pi\pi$ decays ($\Delta I = 1/2$ rule and ϵ'/ϵ)

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- Isospin breaking in sea sector (*in progress*)
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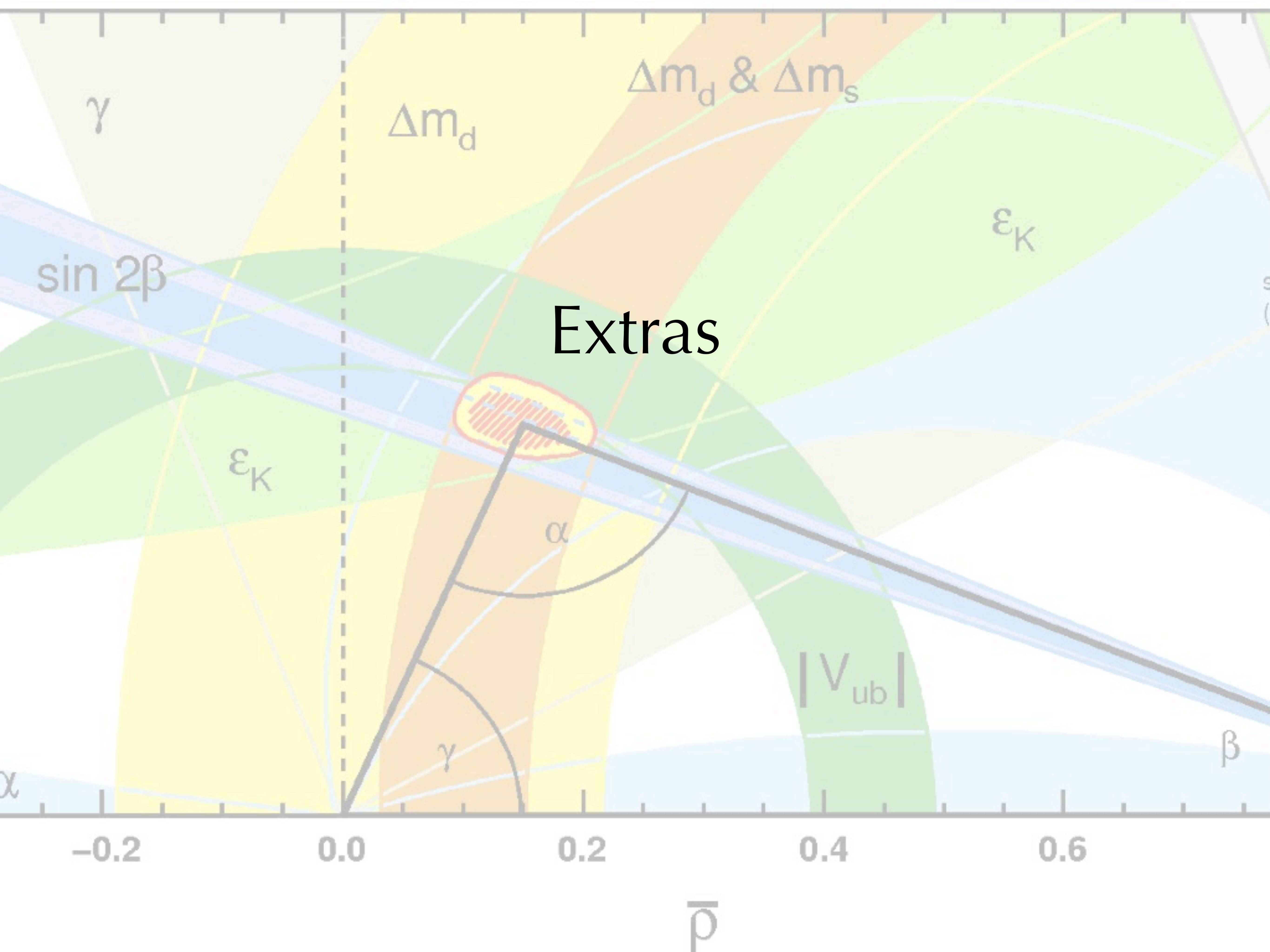
See USQCD Collaboration white paper at
http://www.usqcd.org/documents/HiIntensityFlavor.pdf
for further details

Summary

- Lattice QCD can reliably compute weak matrix elements needed to extract CKM matrix elements and test the Standard Model in the quark-flavor sector
  - Experimental observations are consistent with the Standard Model CKM framework at the ~10% level, but there is now a ~3σ tension in the CKM unitarity triangle
- Lattice matrix element calculations are essential to maximize the impact of LHCb, Super-B factories, and other heavy-flavor experiments
  - Given the expected algorithmic improvements and increase in computing power, **lattice QCD will continue to systematically and steadily reduce the uncertainties in the needed hadronic parameters over the next several years**
- With improved experimental and theoretical precision, heavy-flavor measurements can be a powerful diagnostic tool to reveal the underlying nature of new physics discovered at the LHC or elsewhere

Extras



Flavor-physics sensitivity to NP

	LHT	RSc	4G	2HDM	RHMFV
$D^0 - \bar{D}^0$ (CPV)	★★★	★★★	★★	★★	
ϵ_K	★★	★★★	★★	★★	★★
$S_{\psi\phi}$	★★★	★★★	★★★	★★★	★★★
$S_{\phi K_S}$	★	★	★★		
$A_{\rm CP}\,(B \to X_s\gamma)$	★		★		
$A_{7,8}(K^*\mu^+\mu^-)$	★★	★	★★		
$B_s \to \mu^+\mu^-$	★	★	★★★	★★★	★★
$K^+ \to \pi^+\nu\bar{\nu}$	★★★	★★★	★★★		★★
$K_L \to \pi^0\nu\bar{\nu}$	★★★	★★★	★★★		★★
$\mu \to e\gamma$	★★★	★★★	★★★		
$\tau \to \mu\gamma$	★★★	★★★	★★★		
$\mu + N \to e + N$	★★★	★★★	★★★		
d_n	★	★★★	★	★★★	
d_e	★	★★★	★	★★★	
$(g-2)_\mu$	★	★★	★		

Table 3. "DNA" of flavour physics effects for the most interesting observables in a selection of non-SUSY models. ★★★ signals large NP effects, ★★ moderate to small NP effects and ★ implies that the given model does not predict visible NP effects in that observable. Empty spaces reflect my present ignorance about the given entry.

Lattice actions and parameters: B & D physics

- Fewer collaborations have obtained "2+1" flavor results for B- and D-meson quantities than in the light-quark sector

Collaboration	light-quark action	c-/b-quark action	a (fm)	$m_\pi^{\rm min}$ (MeV) sea/val.**
Fermilab/MILC	Asqtad staggered	"Fermilab" Clover	0.06–0.15	330/250
HPQCD	MILC	HISQ staggered/NRQCD	0.045–0.12	340/270
RBC/UKQCD	domain-wall	static b	0.11	430/430

  - Multiple lattice spacings and light pion masses enable control of systematic errors

- The most advanced calculations by Fermilab/MILC and HPQCD both use the publicly available MILC gauge configurations, but different heavy-quark formulations

  - Additional independent calculations with different light-quark formulations are in progress by RBC/UKQCD and other collaborations

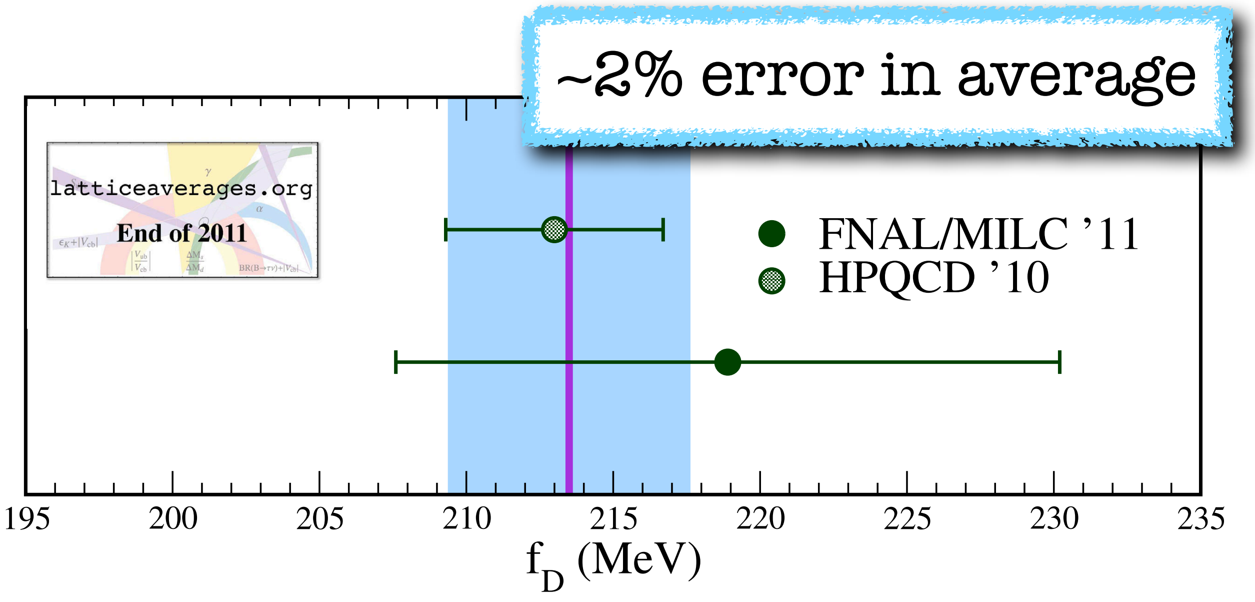
** For dynamical staggered simulations, the RMS sea pion mass is given.
For staggered valence quarks, the lightest Goldstone pion mass is given.

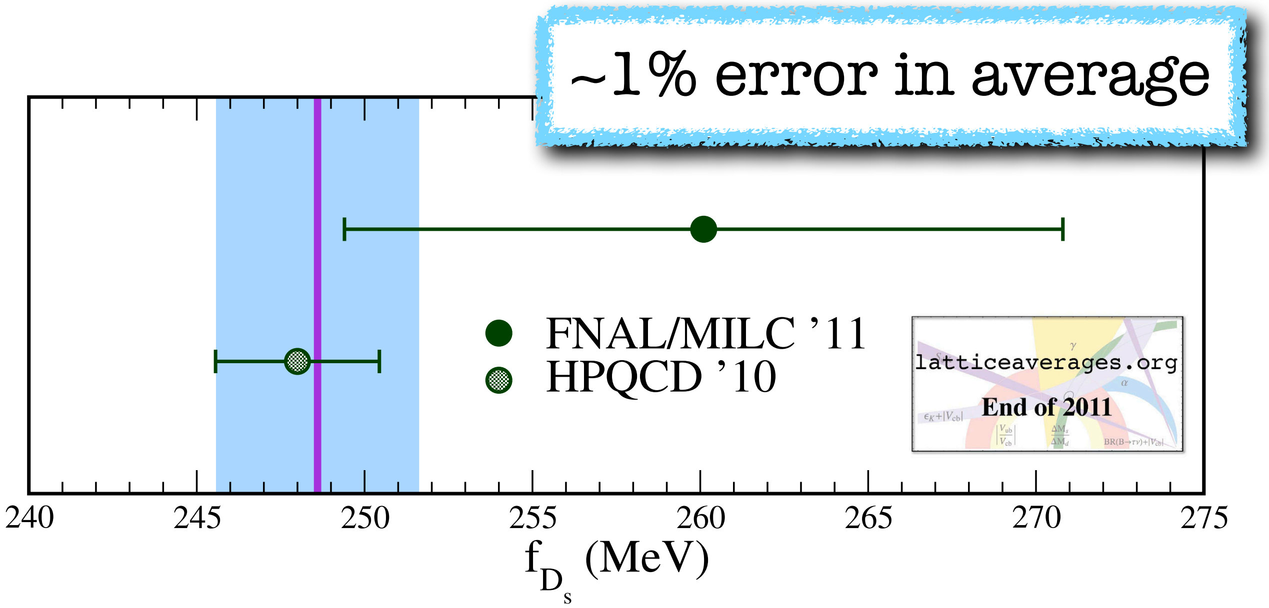
f_D and f_{D_s}

- f_D and f_{D_s} can be used to obtain $|V_{cd}|$ and $|V_{cs}|$ via:

$$\Gamma(D_{(s)} \to \ell\nu) = \frac{G_F^2}{8\pi} \mathbf{f^2_{D_{(s)}}} M_{D_{(s)}} \left(1 - \frac{m_\ell^2}{M_{D_{(s)}}^2}\right)^2 |\mathbf{V_{cd(s)}}|^2$$

- Currently only two 2+1 flavor lattice-QCD results f_D and f_{D_s}
- HPQCD Collaboration results more precise because they use the highly-improved staggered quark (HISQ) action for the u,d,s, and c quarks
  - Small discretization errors; axial current does not get renormalized





f_D and f_{Ds}

- f_D and f_{Ds} can be used to obtain $|V_{cd}|$ and $|V_{cs}|$ via:

$$\Gamma(D_{(s)} \to \ell\nu) = \frac{G_F^2}{8\pi} \mathbf{f^2_{D_{(s)}}} M_{D_{(s)}} \left(1 - \frac{m_\ell^2}{M^2_{D_{(s)}}}\right)^2 |\mathbf{V_{cd(s)}}|^\mathbf{2}$$

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- HPQCD Collaboration results more precise because they use the highly-improved staggered quark (HISQ) action for the u,d,s, and c quarks
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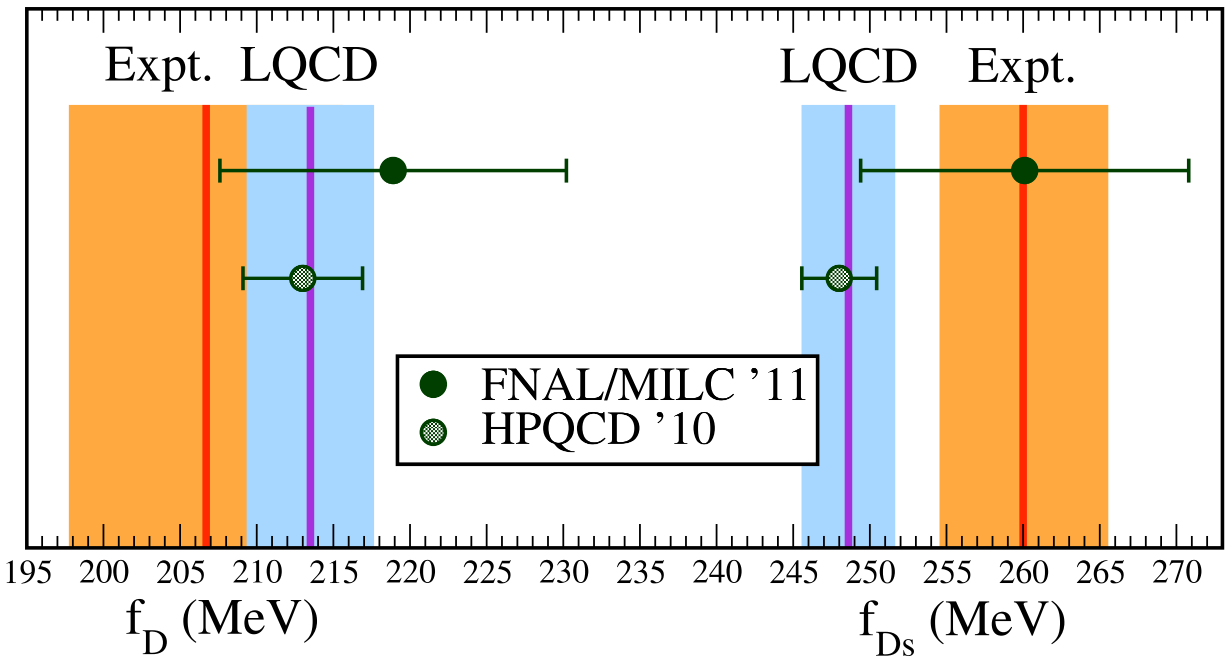
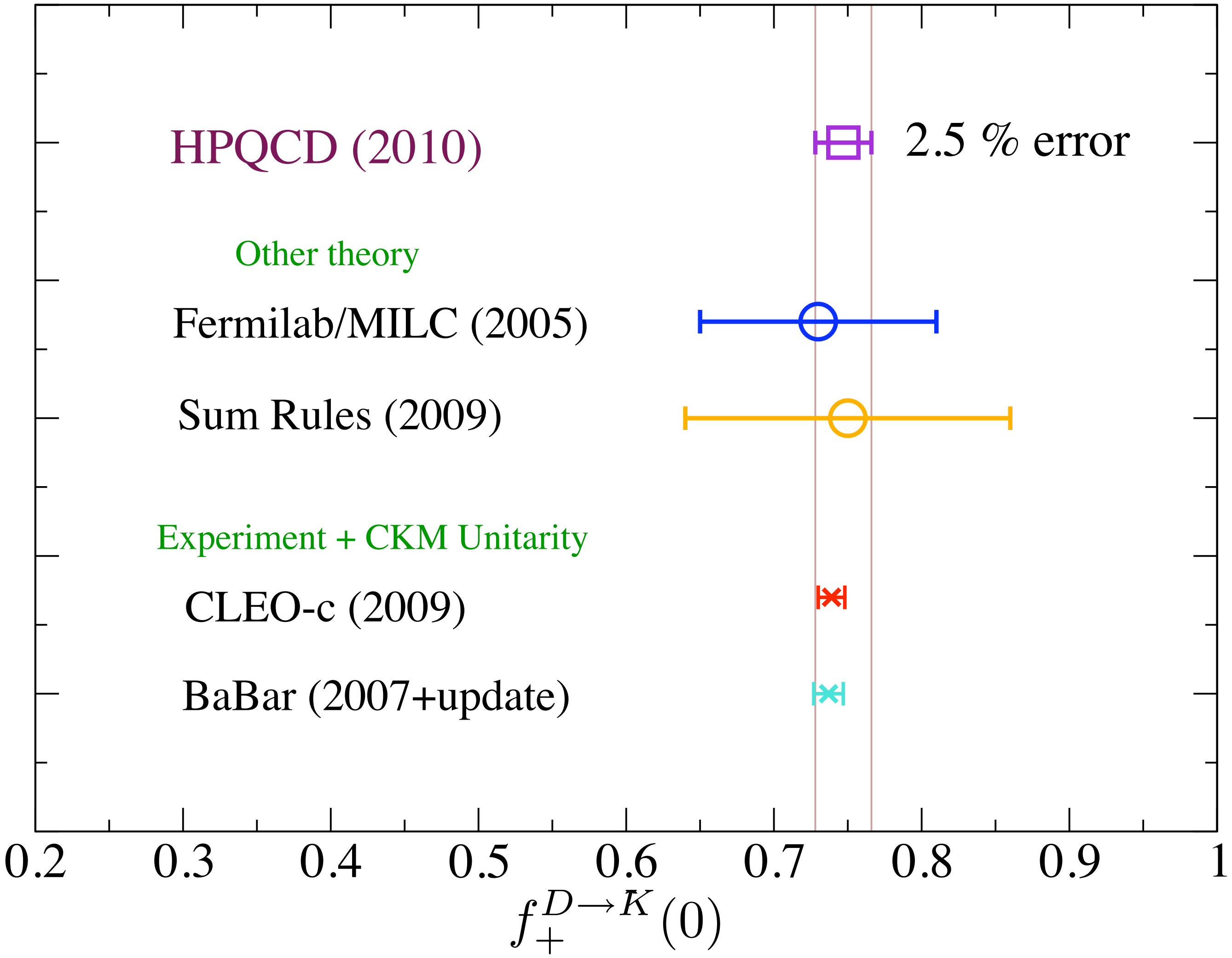


- Conversely, can use $|V_{cd}|$ and $|V_{cs}|$ from CKM unitarity to obtain "experimental" values for f_D and f_{Ds} [Rosner & Stone, arXiv:1201.2401]
  - ➡ Lattice-QCD and experimental determinations agree within 2σ

New result: $D \to \pi \ell \nu$ & $D \to K \ell \nu$ form factors

- HPQCD Collaboration developed a new method for obtaining the form factor at zero momentum transfer ($q^2=0$) [Na et al., PRD 82 (2010) 114506, PRD 84 (2011) 114505]
  - Compute scalar form factor $f_0(q^2)$ and exploit kinematic identity $f_+(0) = f_0(0)$
  - Matrix element of scalar current does not get renormalized
  - Use highly-improved staggered quark (HISQ) action for u,d,s, and c quarks



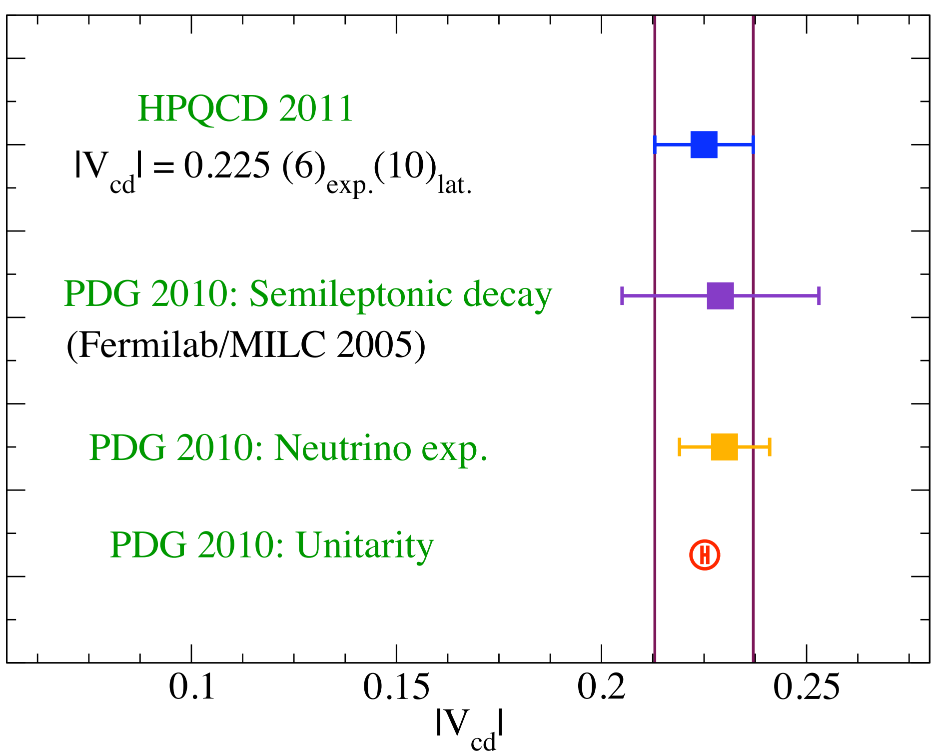
- Reduced lattice-QCD errors significantly:

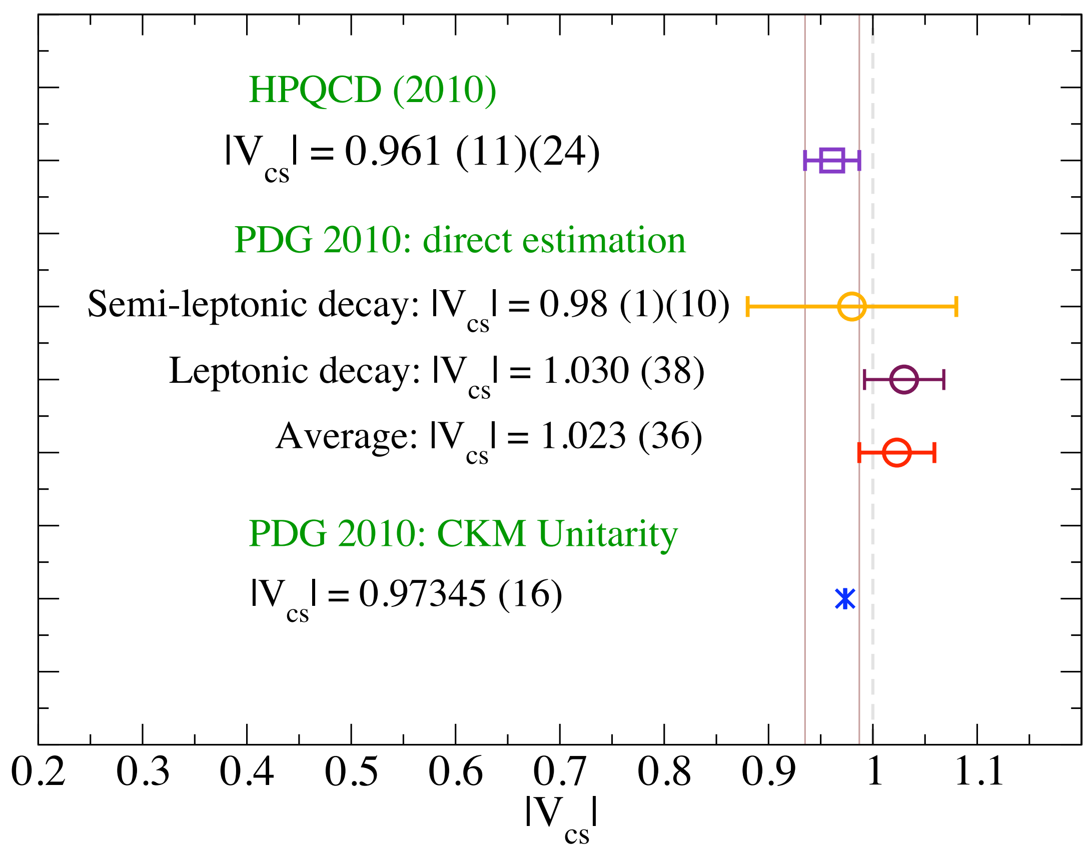
$$f_+^{D\to\pi}(0) = 0.666(20)_{\text{stat.}}(21)_{\text{sys.}} \qquad f_+^{D\to K}(0) = 0.747(11)_{\text{stat.}}(15)_{\text{sys.}}$$

$|V_{cd}|$, $|V_{cs}|$, and 2nd row unitarity

- D→πℓν and D→Kℓν form factors factors can be combined with experimentally-measured branching fractions to obtain $|V_{cd}|$ and $|V_{cs}|$ in the Standard Model

$$\frac{d\Gamma(D \to K(\pi)\ell\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p}_{K(\pi)}|^3 |\mathbf{V_{cs(d)}}|^2 |\mathbf{f_+^{D\to K(\pi)}(q^2)}|^2$$



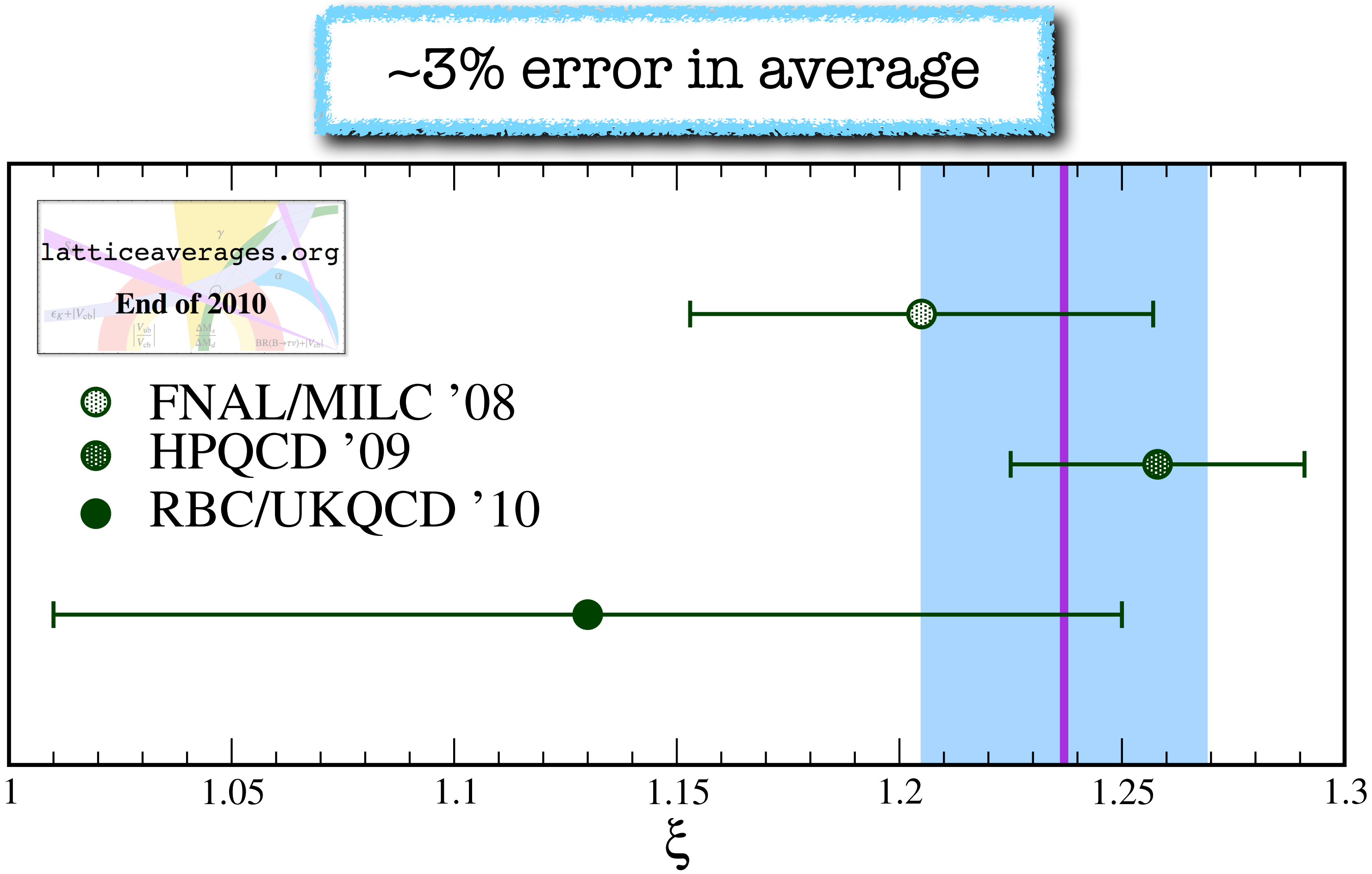


- **Results enable percent-level test of unitarity of 2nd row of the CKM matrix:**

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.976(50)$$

Lattice results for SU(3)-breaking ratio ξ

- Can be obtained precisely on the lattice because statistical fluctuations and some systematic uncertainties largely cancel in the ratio



- Improved calculations with better actions (RBC/UKQCD) and lighter pions and finer lattice spacings (Fermilab/MILC) are in progress, so expect further reduction in errors soon

Lattice actions and parameters: π & K physics

- Several collaborations have now obtained three-flavor results for quantities such as f_π, the $K \to \pi l \nu$ form factor, and the kaon mixing parameter B_K

Collaboration	action	a (fm)	$m_\pi L$	$m_\pi^{\min}$ (MeV) sea/val. **
BMW	Clover	0.054–0.125 fm	≥ 4	120/120
HPQCD	HISQ (staggered) on MILC	0.045–0.15 fm	≥ 3.7	340/270
Laiho & Van de Water	DW on MILC	0.06 –0.12 fm	> 3.5	260/210
MILC	Asqtad staggered	0.045–0.12 fm	> 4	260/180
RBC/UKQCD	Domain Wall	0.085–0.11 fm	> 4	290/220
SWME	HYP-staggered on MILC	0.06–0.12 fm	> 2.7	330/200

- Multiple lattice spacings, light pion masses, and large volumes enable control of systematic errors
- Different lattice actions and analysis methods provide independent checks

** For dynamical staggered simulations, the RMS sea pion mass is given.
For staggered valence quarks, the lightest Goldstone pion mass is given.

$K \to \pi\ell\nu$ form factor

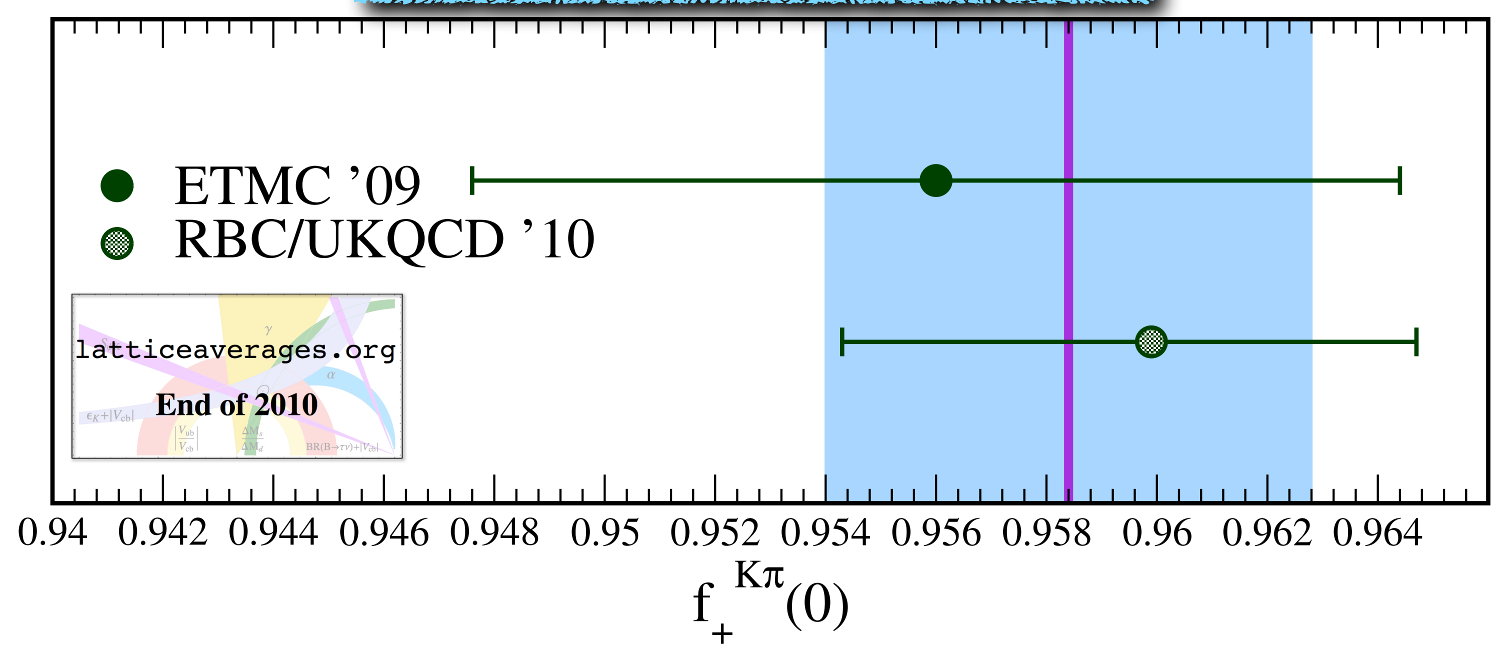
- The $K \to \pi\ell\nu$ semileptonic form factor allows a determination of $|V_{us}|$:

$$\Gamma(K \to \pi\ell\nu) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)^2$$

- The zero-recoil form factor $f_+(0)$ can be computed to high precision in lattice QCD because it is highly constrained by $SU(3)_f$ and chiral symmetry
  - $f_+(0) = 1$ in the SU(3) limit $m_s = m_{ud}$, and leading-order correction to 1 (f_2) is a known function of $\{m_\pi, m_K, f_\pi\}$ [Leutwyler & Roos]
  - Ademollo-Gatto theorem ensures that corrections to 1 are second-order in $(m_K^2 - m_\pi^2)$, so $f_2 = -0.023$ is small

- Lattice calculations by other collaborations are in progress (e.g. PACS-CS & Fermilab/MILC), so **expect further error reduction soon**

~0.5% error in average



Status of the $|\epsilon_K|$ band

- Recent calculation by Brod & Gorbahn [Phys.Rev. D82 (2010) 094026] gives the following error breakdown for $|\epsilon_K|$ in the Standard Model:

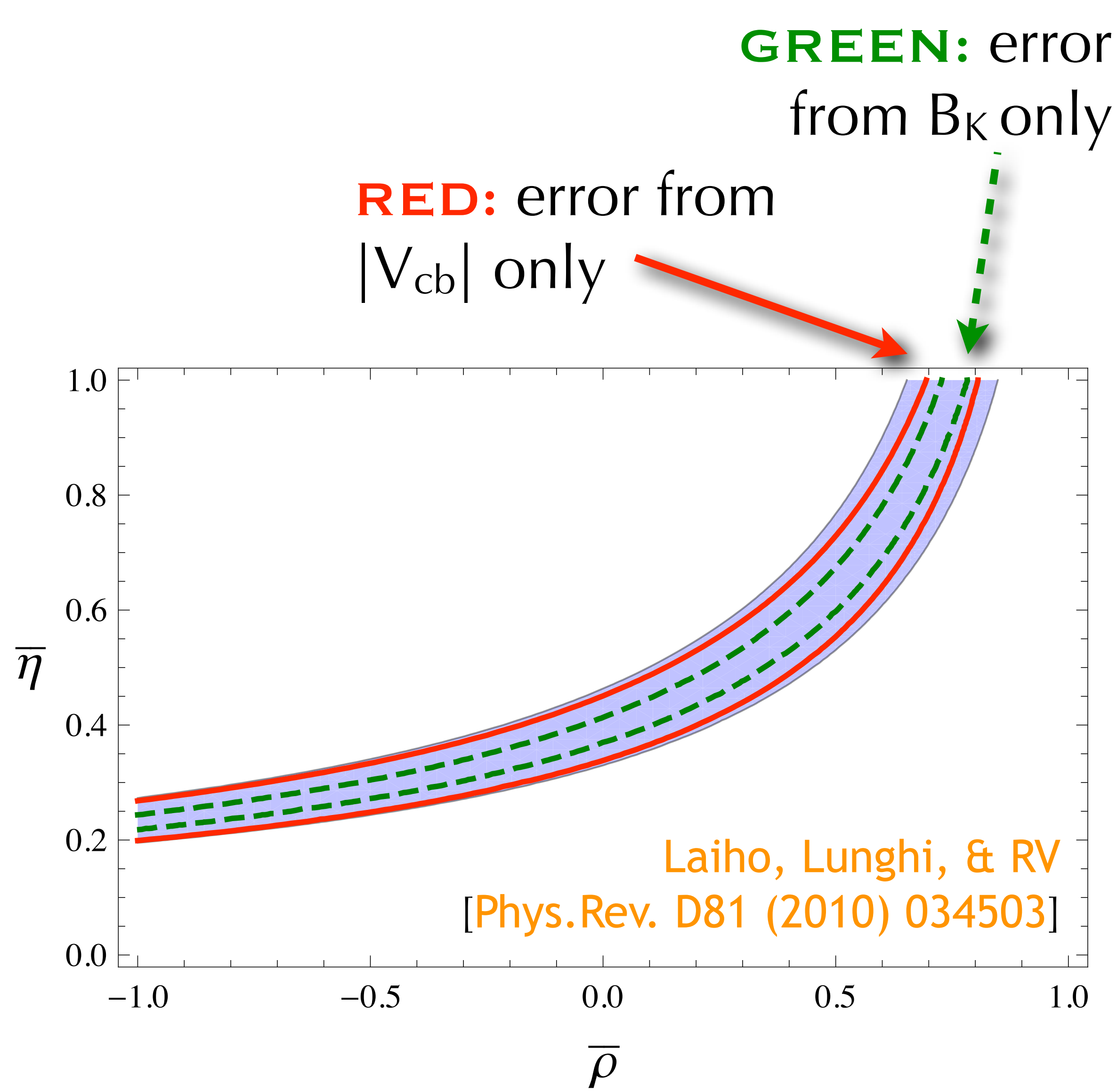
$$|\epsilon_K| = (1.90 \pm 0.04_{\eta_{cc}} \pm 0.02_{\eta_{tt}} \pm 0.07_{\eta_{ct}} \pm 0.11_{\mathrm{LD}} \pm 0.22_{\mathrm{parametric}}) \times 10^{-3}$$

(1) Largest ~10% uncertainty is from parametric error in $A^4 \propto |V_{cb}|^4$

(2) Next-largest error is ~4% uncertainty from η_{ct}, which was just computed to 3-loops (NNLO)

(3) Error from B_K is #3

(4) Other individual error contributions are 2% or less



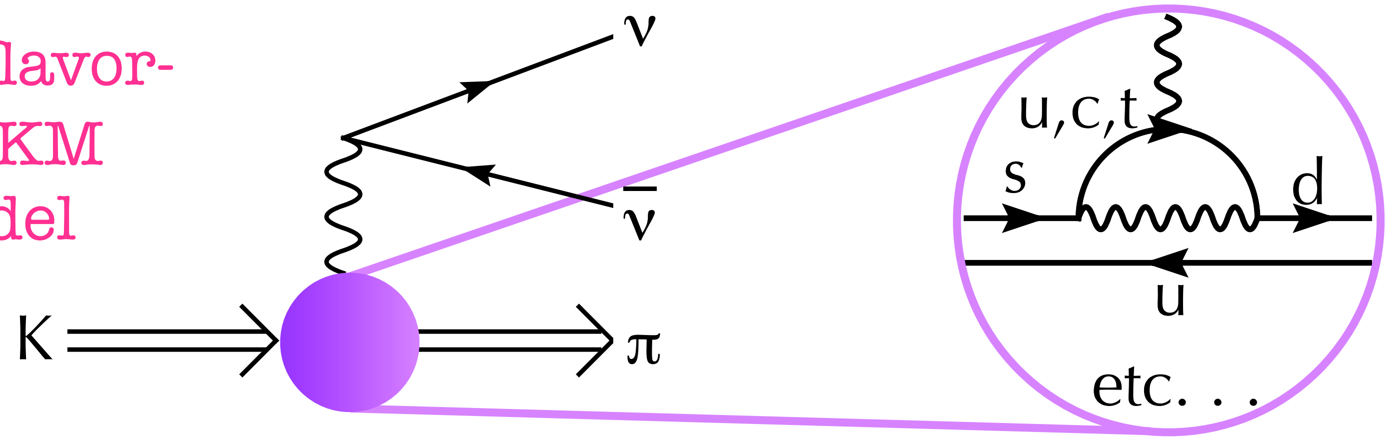
- Time to move on to other more challenging kaon physics quantities ...

Lattice QCD progress on ϵ'_K/ϵ_K

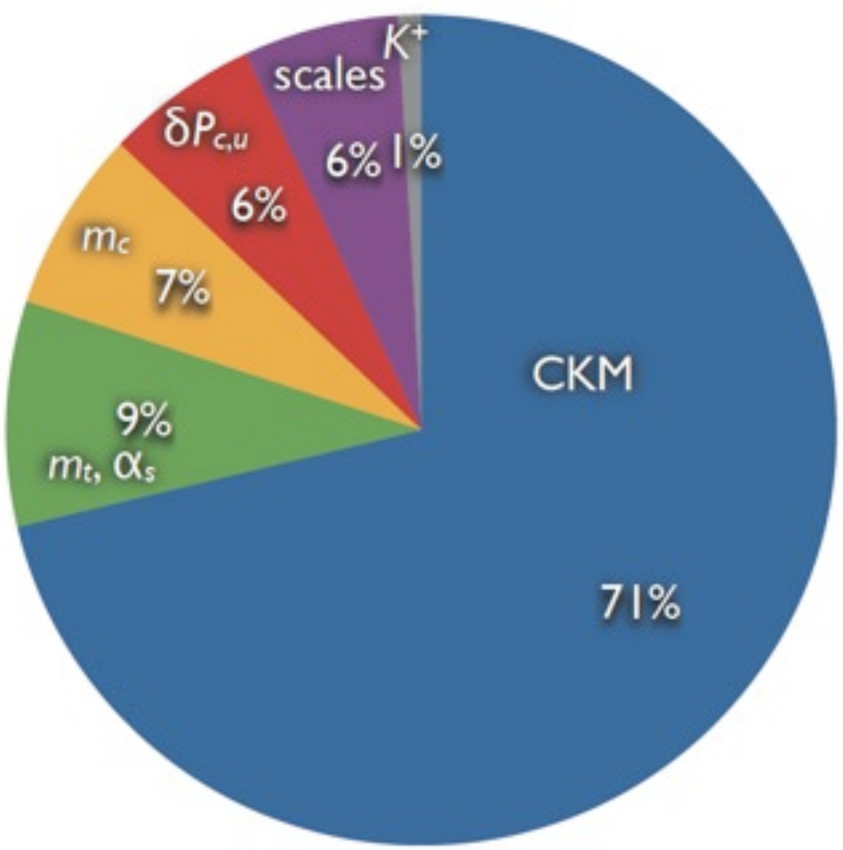
- RBC/UKQCD have resolved the outstanding theoretical issues associated with the "direct" Lellouch-Lüscher approach [Christ, "Lattice QCD Meets Experiment" 2010]
  - Computed $\Delta I = 3/2$ matrix elements with nearly physical pion and kaon masses, and obtained $\mathrm{Re}(A_2)$ & $\mathrm{Im}(A_2)$ with ~20% errors [arXiv:1111.1699]
  - Studied $\Delta I = 1/2$ matrix elements with unphysically-heavy ~330 MeV pions, demonstrating ability to perform power-divergent subtractions and tackle expensive disconnected diagrams [arXiv:1106.2714]
  - Installation of BlueGene/Q at BNL will allow a realistic calculation of with larger volumes and lighter pions
- Laiho & RV developed an alternate method for obtaining $K \to \pi\pi$ matrix elements from lattice simulations that utilizes chiral perturbation theory and is less computationally costly than the direct approach, but is expected to achieve comparable errors
  - Demonstrated approach with $\Delta I = 3/2$ channel [PoS LATTICE2010 (2010) 312]
- Expect 20% result for $\Delta I=1/2$ rule and $\varepsilon'_K/\varepsilon_K$ in one or two years!

Rare $K \to \pi\nu\bar{\nu}$ decays

- $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ are both flavor-changing neutral current and CKM suppressed in the Standard Model



BR($K^+ \to \pi^+\nu\bar{\nu}$)



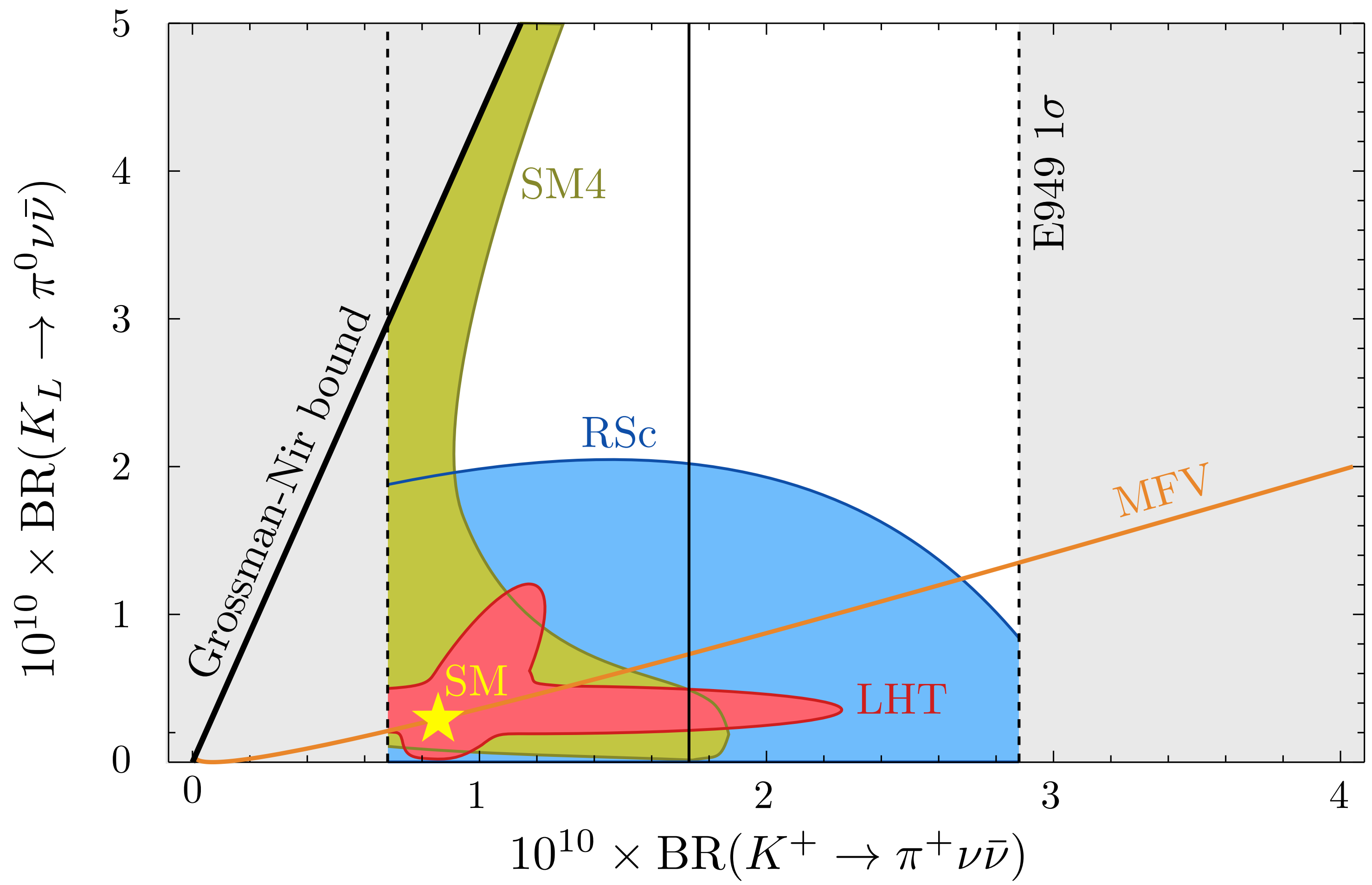
[Brod & Gorbahn Phys.Rev. D83 (2011) 034030]

- Often called **"GOLDEN" MODES** because SM branching ratios known to a precision unmatched by any other quark FCNC processes
  - Hadronic form factor can be obtained precisely using experimental $K \to \pi\ell\nu$ data combined with chiral perturbation theory [Mescia & Smith, arXiv: 0705.2025]

➡ Limited by ~10% parametric uncertainty in $A^4 \propto |V_{cb}|^4$

Room for new physics

- Sensitive to Little Higgs models, warped extra dimensions, and 4th generation [Buras, Acta Phys.Polon.B41:2487-2561,2010]



[D. Straub, arXiv:1012.3893 (CKM 2010)]

- **Spectacular deviations from the Standard Model are possible in many new physics scenarios**

- Correlations between the two channels can help distinguish between models

Future prospects

- In the next several years, NA62 at the CERN SPS will measure $\mathcal{O}(100)$ K^+ events (assuming the SM), while KOTO at J-PARC will collect the first K^0_L events
- Subsequently, ORKA (Fermilab P1021) proposes to collect $\mathcal{O}(1000)$ K^+ events in a main injector experiment, while a Project X experiment could collect $\mathcal{O}(1000)$ K^0_L events

➡ Expect to obtain few-percent errors in the branching fractions

- By 2014, expect to halve error on $|V_{cb}|$ from lattice-QCD calculations of $B \to D^{(*)} \ell \nu$, corresponding to an error in the Standard Model branching fractions of ~6%

➡ Theory error in Standard-Model predictions will be commensurate with expected experimental errors

Rare kaon decays will constrain models of TeV-scale physics, and may even reveal new physics . . .