Rectilinear Cooling DA

International
MUON Collider
Collaboration

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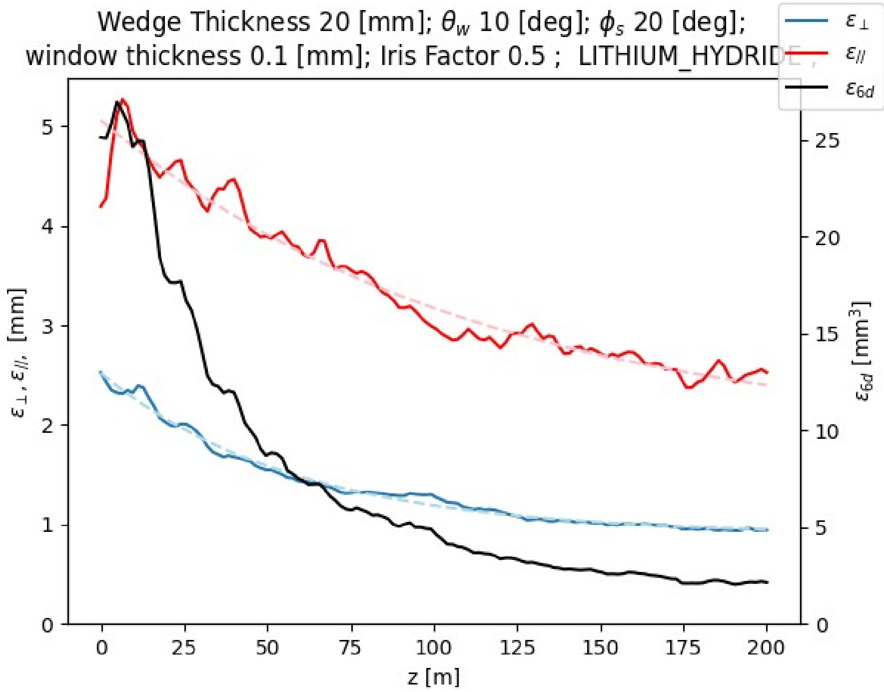
ISIS

DA and cooling optimisation

- Rectilinear cooling design is approaching a release version
  - Thanks to Ruihu!
  - Ruihu’s design now frozen → publication
- Performance is improved over MAP
  - Ongoing discussion about cost/etc
- **But** still want higher efficiency
  - Would like ~ factor 2 more muons from the production system
  - Needed to make baseline parameters
  - Luminosity goes with N^2
- Transmission of all rectilinear cooling system designs is rather low
  - Why?
  - Can we do better?

Ionisation cooling



- Emittance change goes as

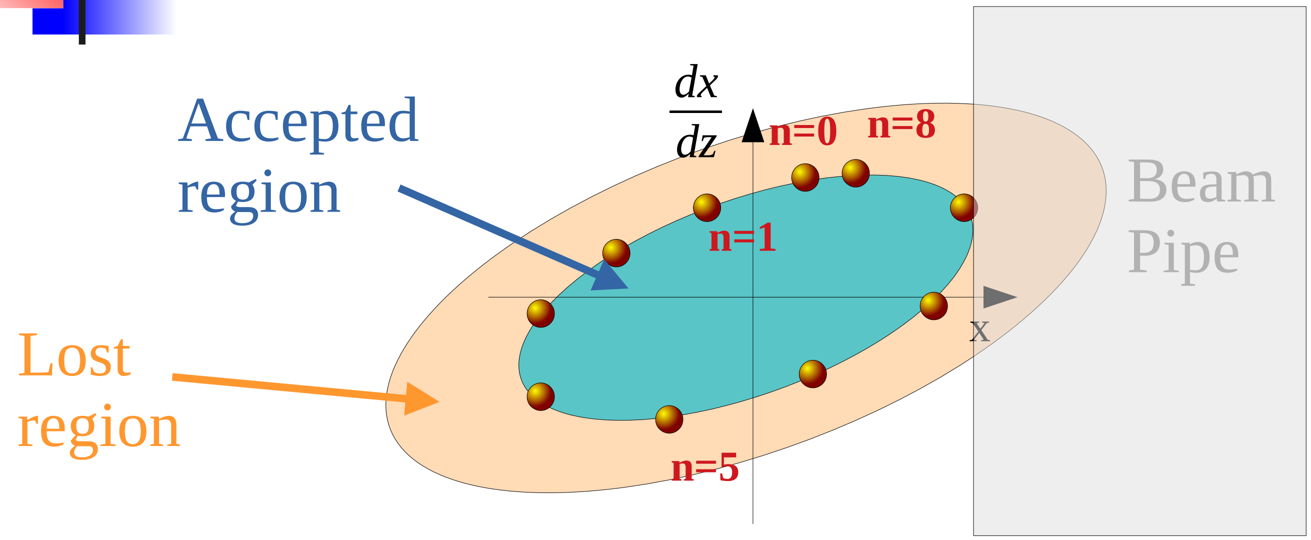
$$\varepsilon_n(z) = \left[\varepsilon_n(0) - \varepsilon_n^{eqm}\right] exp\left(\frac{-z}{L_{cool}}\right) + \varepsilon_n^{eqm}$$

- L_{cool} is cooling length, characteristic of the channel
- ε_n^{eqm} is equilibrium emittance, characteristic of the channel
- Quick (cheap) cooling → $\varepsilon_n(0) >> \varepsilon_n^{eqm}$

Acceptance

- We want $\varepsilon_n(0)$ as large as possible
- The maximum emittance is determined by the *acceptance* of the cooling system
  - Maximum emittance particle that makes it through the cooling cells
- Two classes of acceptance (aperture)
  - Physical acceptance – equipment intercepts the beam
  - Dynamical acceptance – aberrations in the focusing at large emittance cause particles to get lost

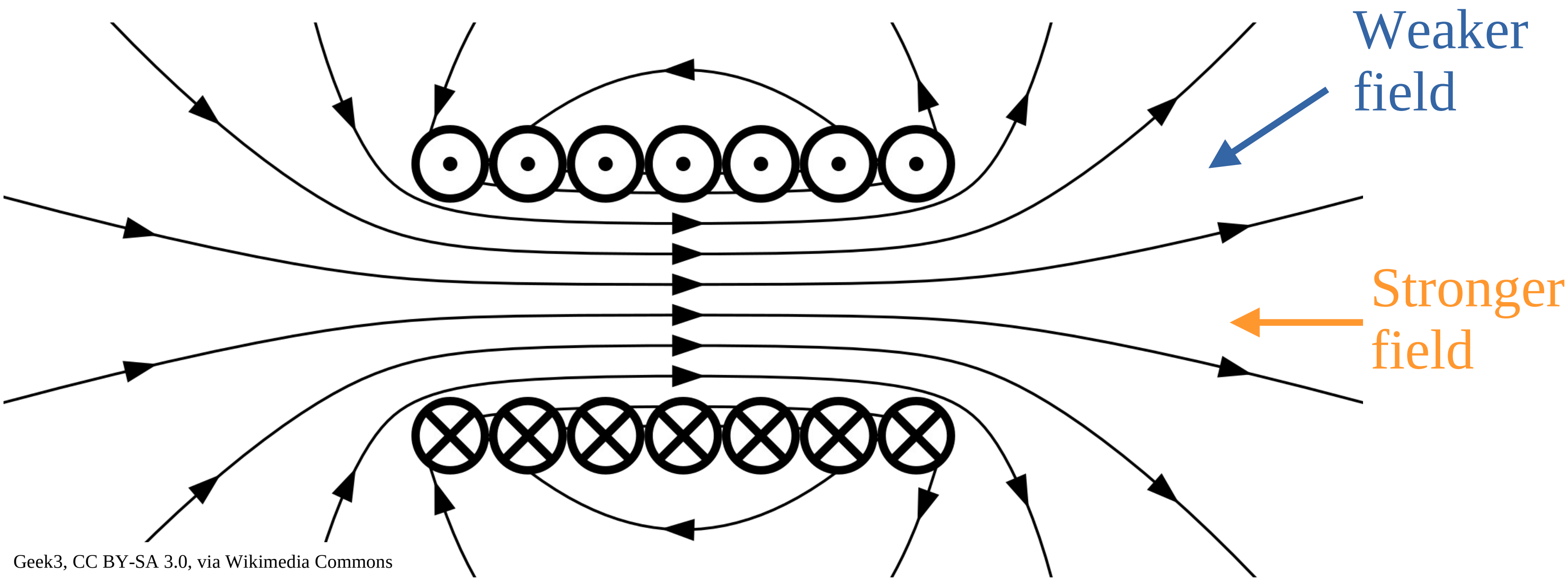
Physical Acceptance



- Particles undergoing focusing follow elliptical trajectories in (position, momentum) space
- Beam pipe/aperture → maximum position transmitted
- Trajectories that touch the beam pipe are lost
- Maximum emittance trajectories that do not touch the beam pipe → “Acceptance”

Dynamical Acceptance

- Focusing of solenoids varies across the magnet face



- Variation is only in the fringe of the solenoid
- Variation → determined purely in terms of the field on axis

Dynamical Acceptance

- Consider Maxwell’s equations

$$\nabla . B = 0$$

$$\nabla \times B = 0$$

- For a generalised solenoid symmetry

$$\begin{aligned}\nabla . B =& \frac{1}{r}\partial_r (r B_r) + \partial_z B_z \\ =& \partial_r B_r + \frac{B_r}{r} + \partial_z B_z \\ \nabla \times B =& (\partial_z B_r - \partial_r B_z)\hat{\phi} = 0\end{aligned}$$

- We can solve the PDE to get a generalised expression for B

Dynamical Acceptance

- Consider Maxwell's equations (magnetostatic)

$$\nabla . B = 0$$

$$\nabla \times B = 0$$

- For a generalised solenoid symmetry

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- We can solve the PDE to get a generalised expression for B

Dynamical Acceptance

- Assume generalised Taylor series for the field

$$B_r = \sum_{i=0} a_i(z) r^i$$

$$B_z = \sum_{i=0} b_i(z) r^i$$

- Substitute into Maxwell and solve

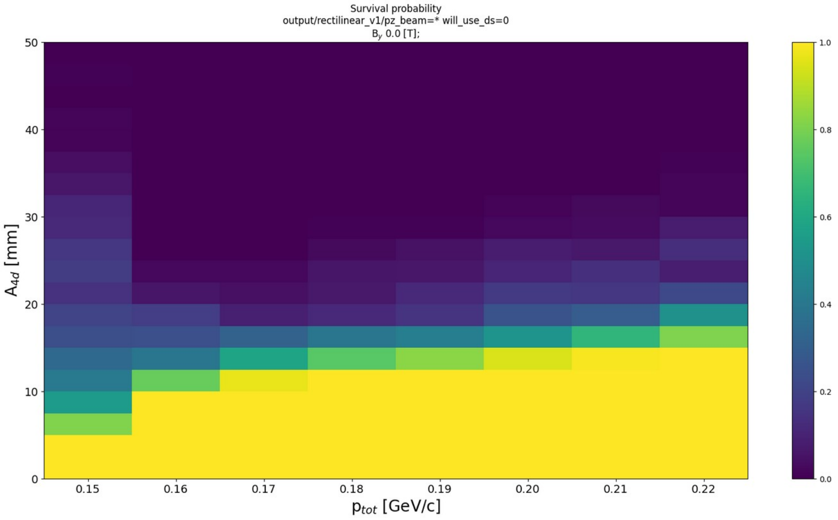
$$b_{i+2}(z) = -\frac{1}{(i+2)^2} \partial_z^2 b_i(z)$$

$$a_i = -\frac{1}{i+1} \partial_z b_{i-1}(z)$$

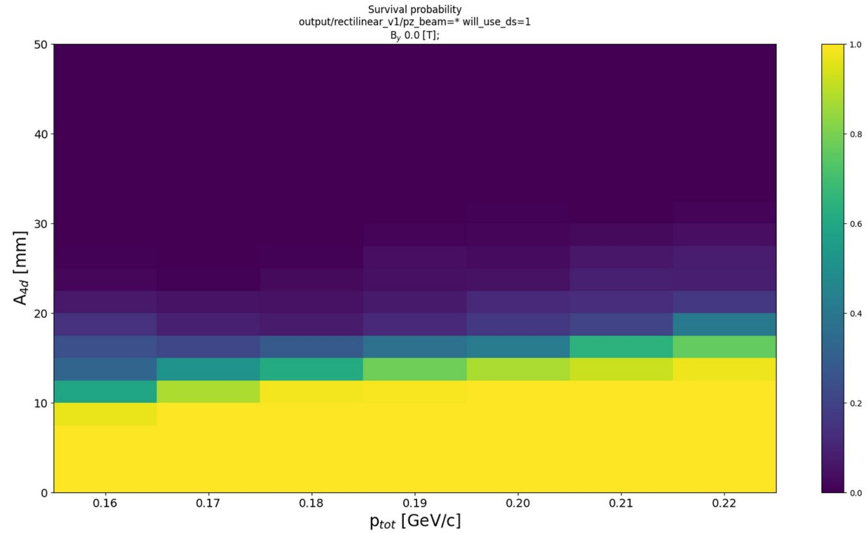
- **Off-axis behaviour of solenoid is unique function of the field on-axis**
  - Particular coil arrangement is **irrelevant** for beam dynamics
  - Of course it is important for many other reasons!

Example – Demo Lattice

- Consider Dynamic Aperture of demo lattice
  - Implementing “derivatives solenoid” into G4BL
  - Use B = h0 sin(kz) + h1 sin(2kz) + h2 sin(3kz)



2024-05-24-release
Solenoids only



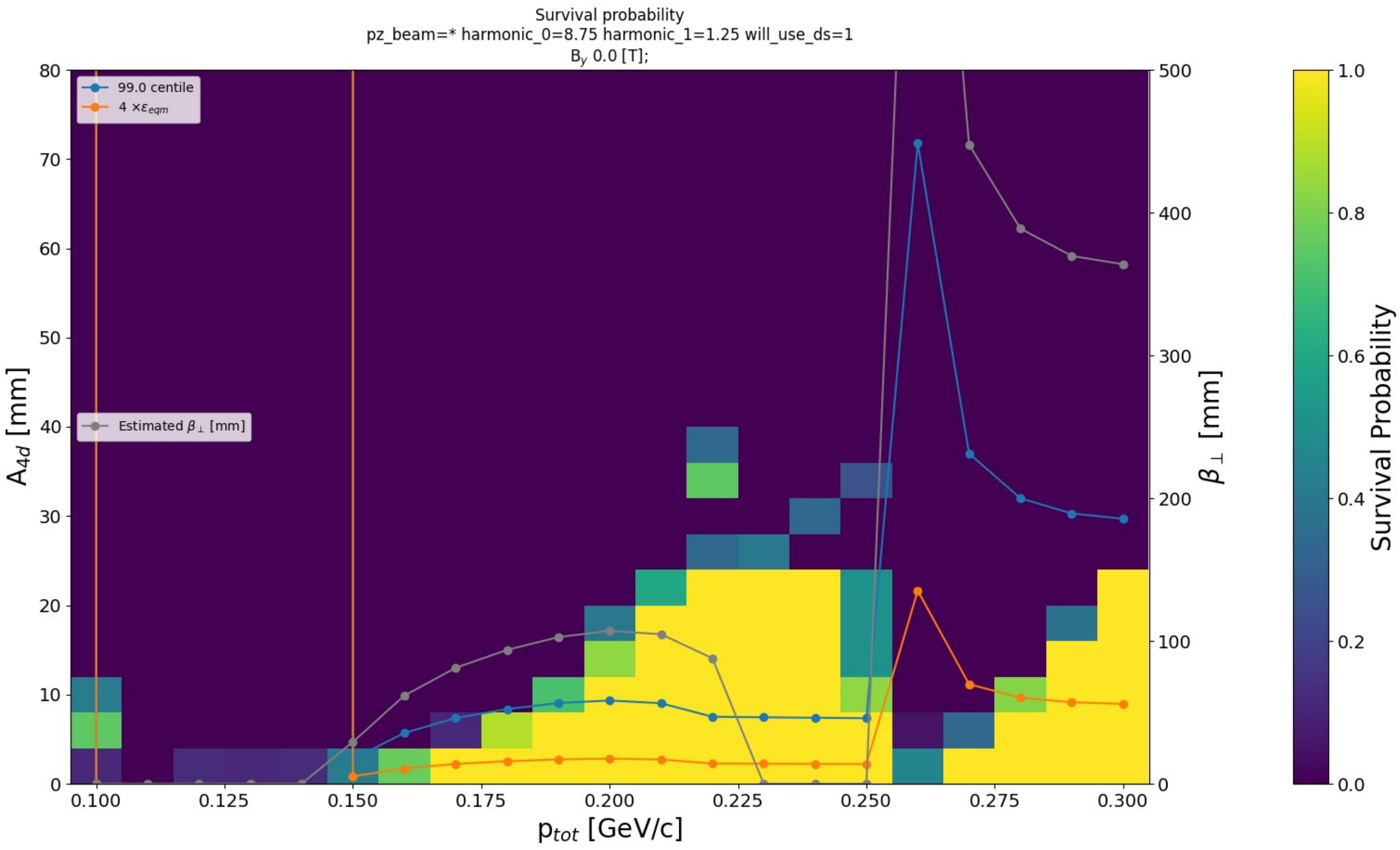
Field expansion
h0 = 8.75 T
h1 = 1.25 T
Truncate at r^9

Nb Amplitude distribution

- For a beam that follows a gaussian distribution…
- 4D Amplitude follows a chi^2 distribution with 4 degrees of freedom
- By comparing DA with emittance we can estimate number of muons outside acceptance
  - Assuming gaussian beam
  - (For ionisation cooling, expect a bit more tail than in a Gaussian)
- By comparing DA with equilibrium emittance, we can develop “figure of merit” for cooling lattice
  - “Dynamic range” of a given cooling lattice

Demo – zoom out

- Interesting features if we zoom out
  - Acceptance even in the stop band
  - Here I assume beta for 200 MeV/c (reference) trajectory to calculate amplitude



Back to cooling...

- So what about cooling?

Stratakis et al, PRAB 18 031003, 2015

Stage	ε_T^{sim} [mm]	ε_L^{sim} [mm]	P_z^{sim} [MeV/c]	T [%]
Begin	17.00	46.00	255	
A1	6.28	14.48	238	70.6
A2	3.40	4.64	229	87.5
A3	2.07	2.60	220	88.8
A4	1.48	2.35	215	94.6

51.9 %

Zhu et al, in progress

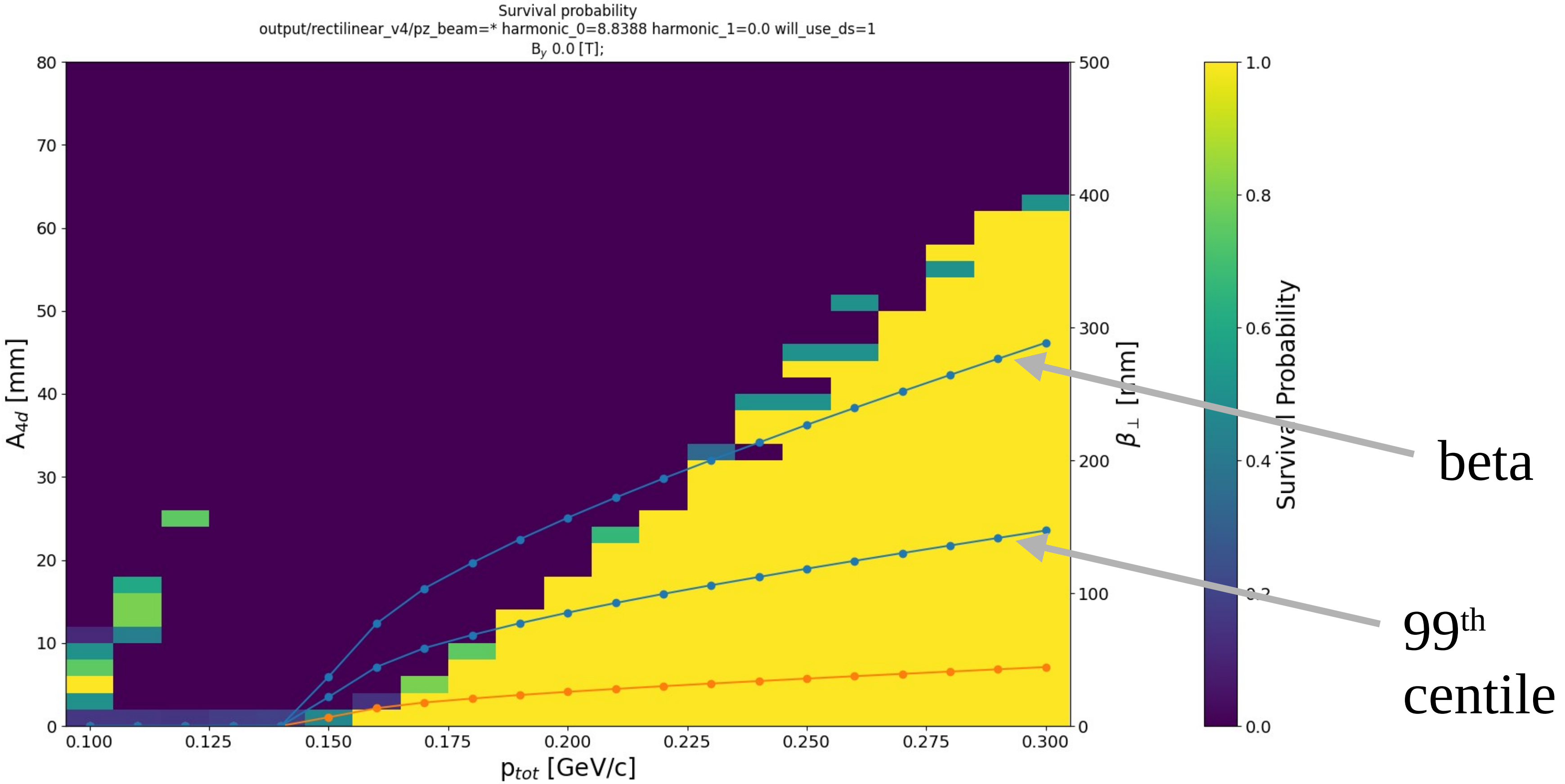
	$\varepsilon_{T,sim}$ (mm)	$\varepsilon_{L,sim}$ (mm)	$\varepsilon_{6D,sim}$ (mm^3)	Transmission
Start	16.96	45.53	13500	
A-Stage 1	5.165	18.31	492.6	75.2%
A-Stage 2	2.473	7.113	44.03	84.4%
A-Stage 3	1.556	3.880	9.594	85.6%
A-Stage 4	1.239	1.741	2.861	91.3%

Just decays: 80 %

49.6 %

A-type Acceptance

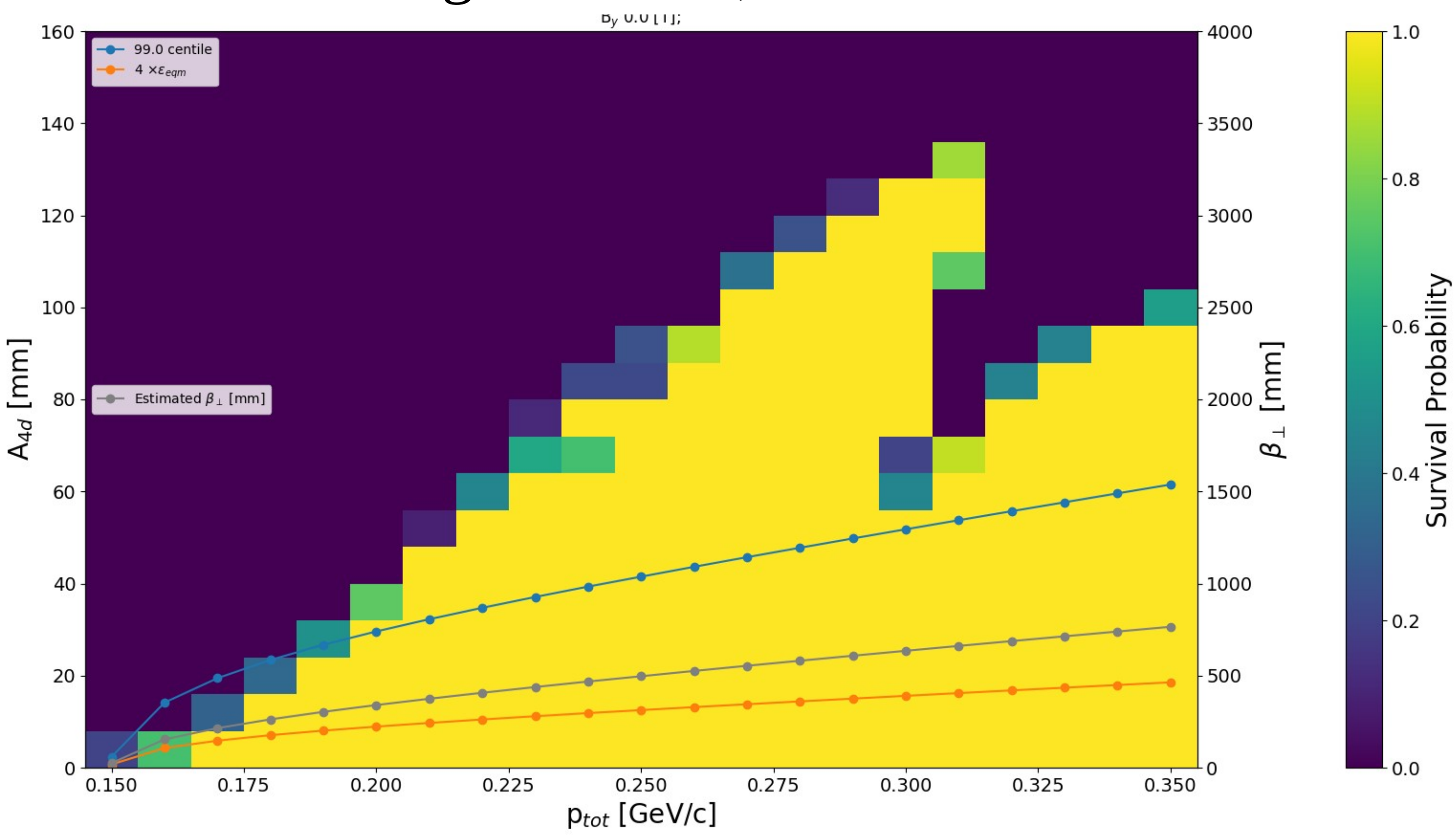
- A-type lattices
  - Set H_1 to 0 and adjust H_0 so that integral B_z^2 dz is constant
    - Average focusing strength goes with integral B_z^2 dz
  - Field is approximately sine wave – stop band disappears



Momentum range

- Can scale the momentum by scaling B_z

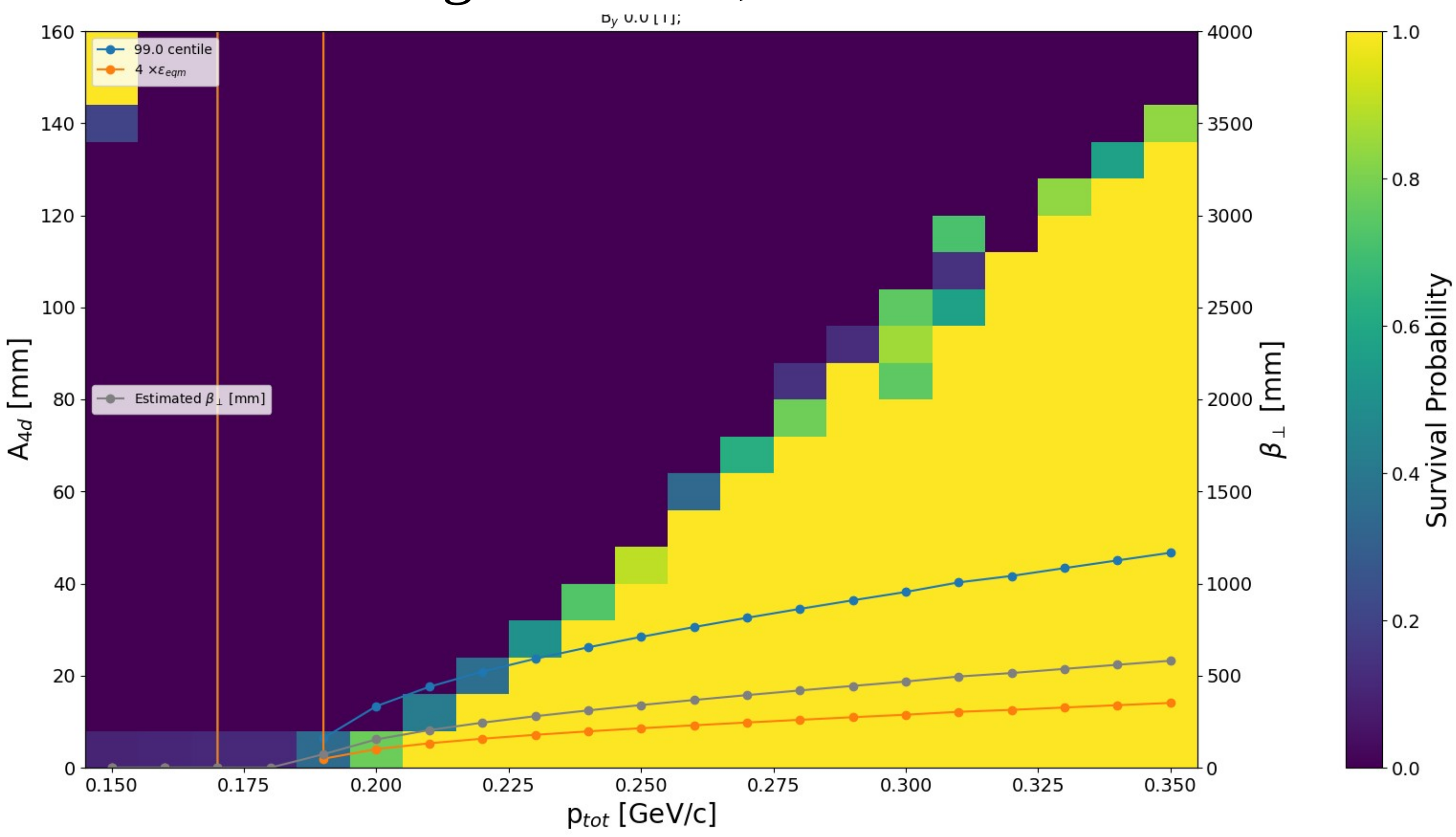
Cell length = 1.8 m; h0 = 4.0



Momentum range

- Can scale the momentum by scaling B_z

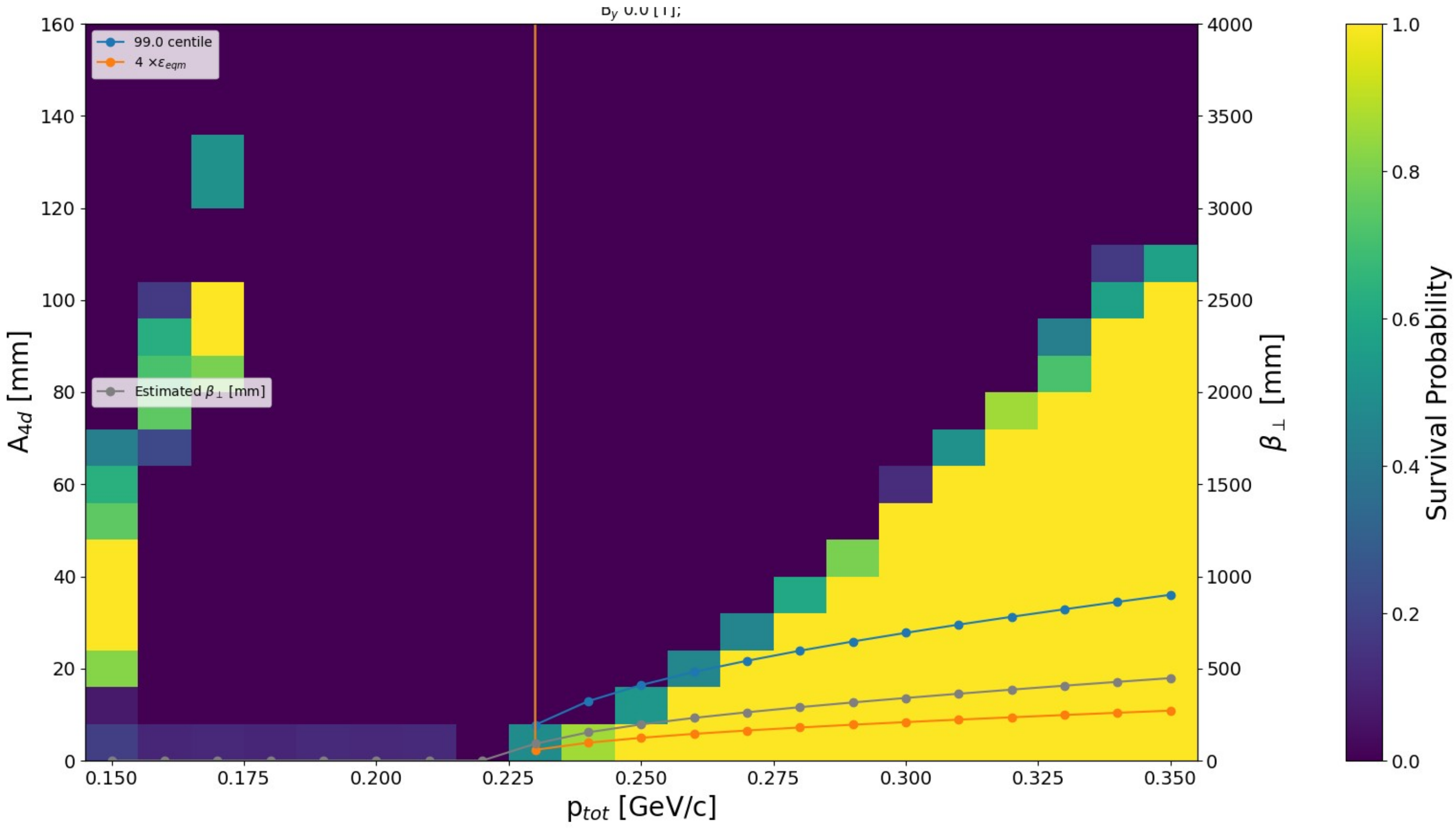
Cell length = 1.8 m; h0 = 5.0



Momentum range

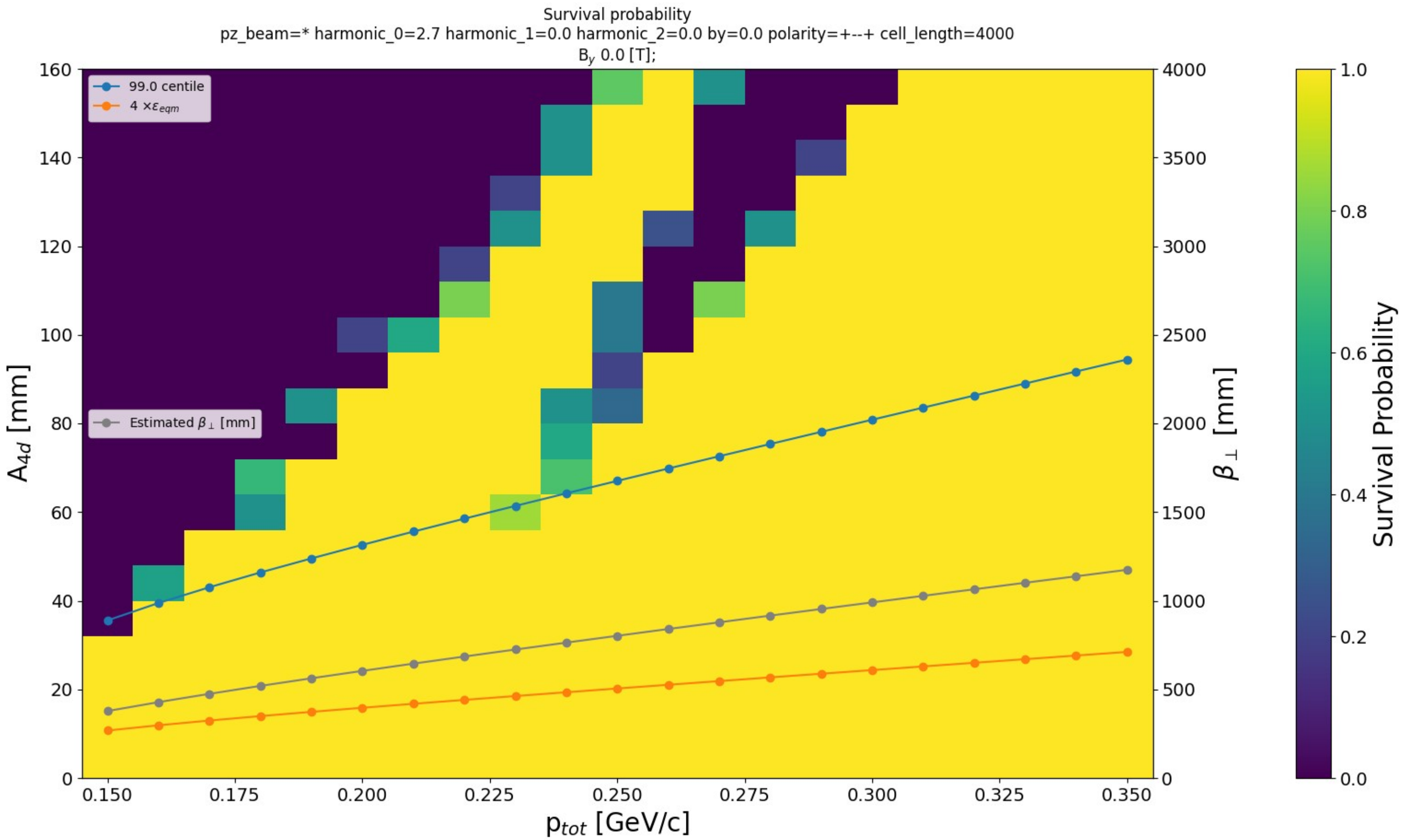
- Can scale the momentum by scaling B_z

Cell length = 1.8 m; h0 = 6.0



Beta scaling & acceptance

- Can scale the beta function by scaling B_z and cell length
  - Keep (cell length) * B_z constant



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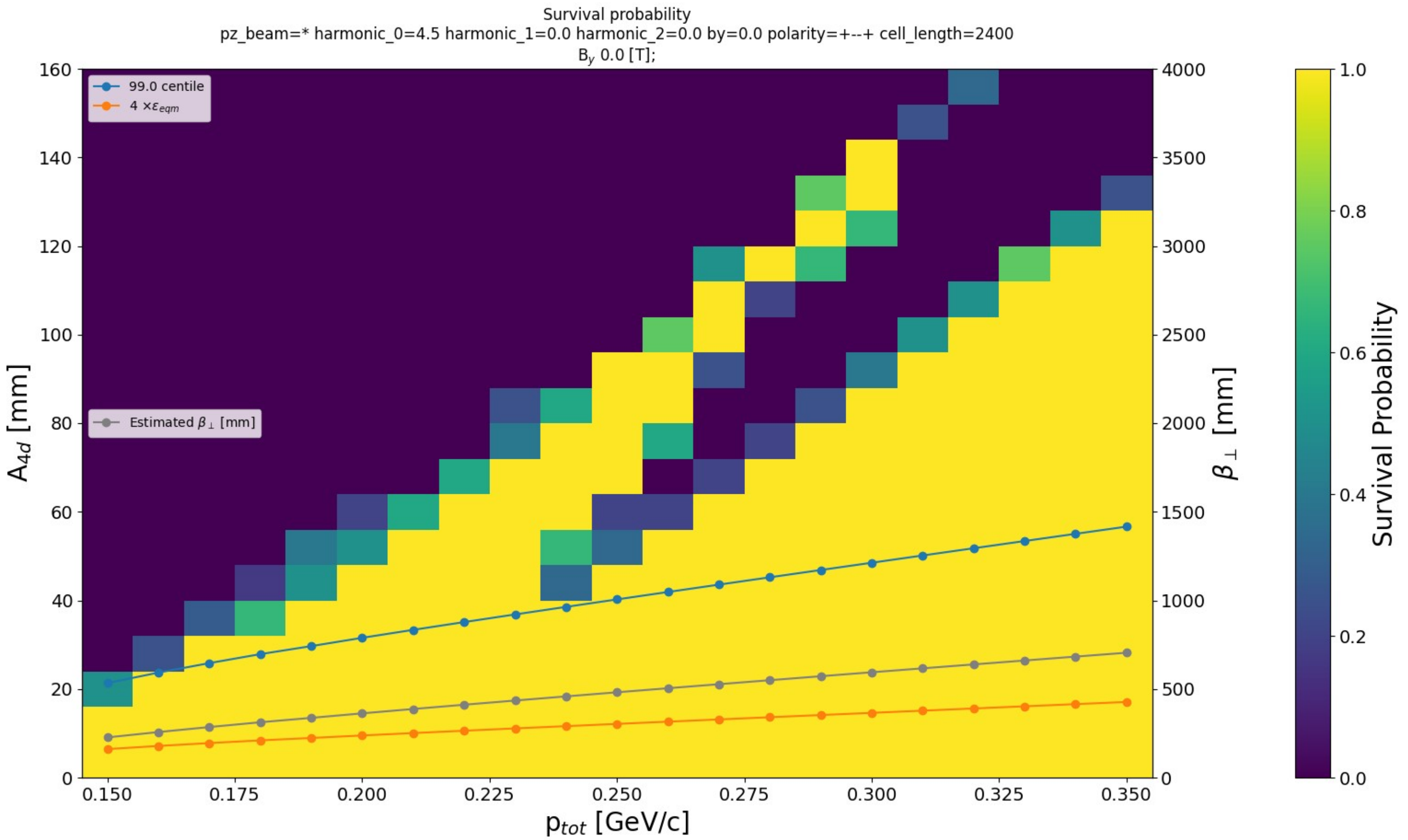

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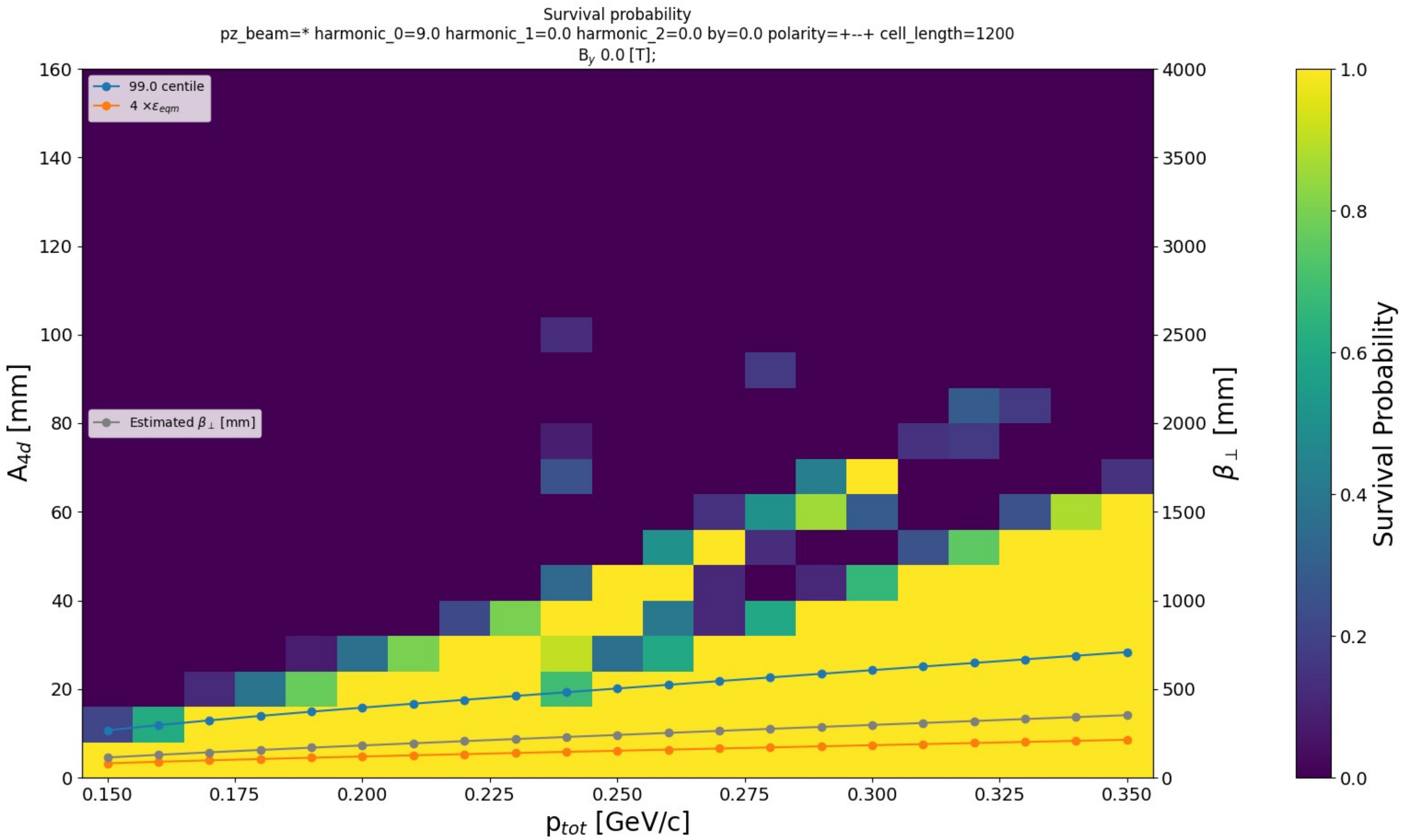
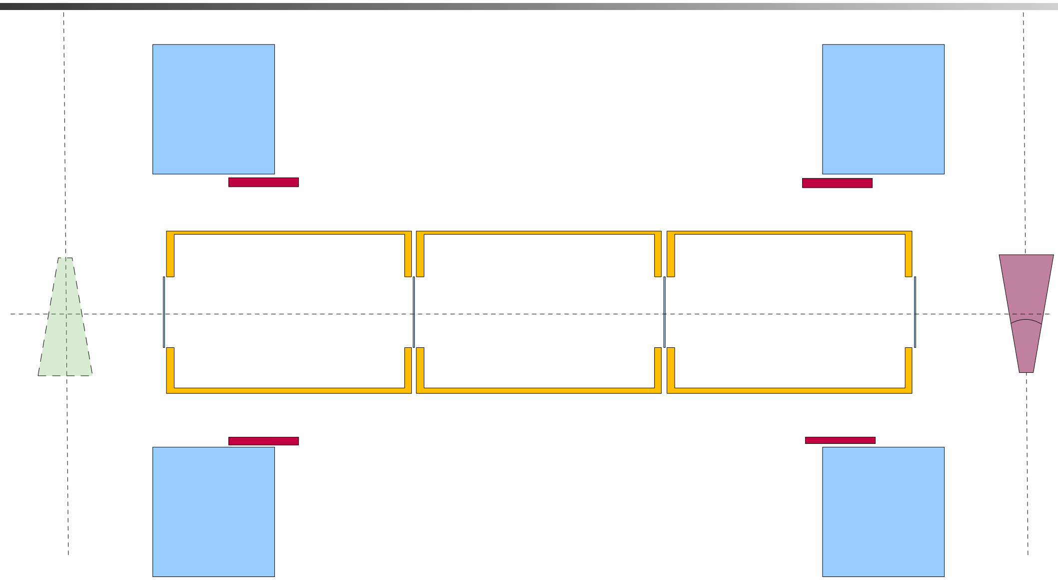


Acceptance

- We can generate *any* acceptance we like
- Dynamic range is conserved
- High acceptance is *easier* to generate than low acceptance
- Note also that dynamic range is better at high momentum

- So why do we have a bad transmission for A1+???

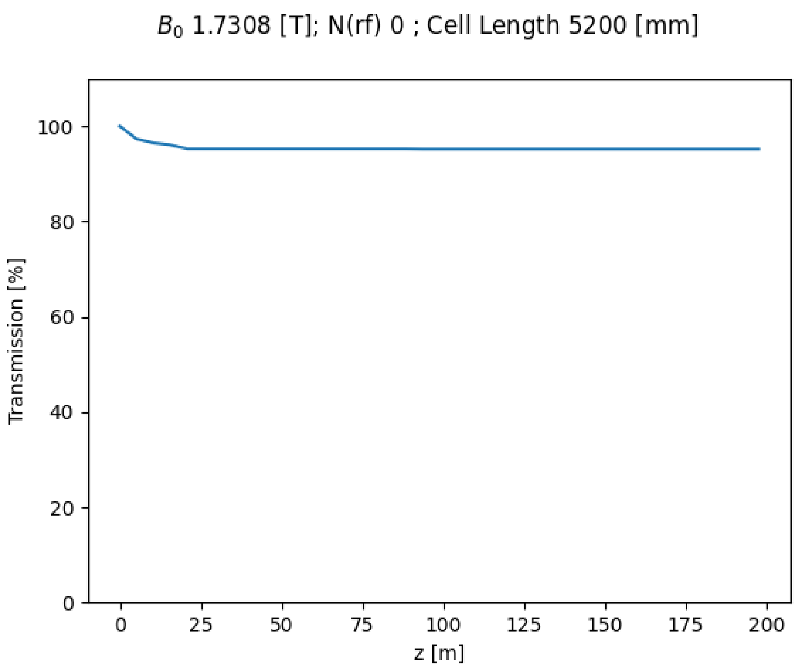
Physical Acceptance

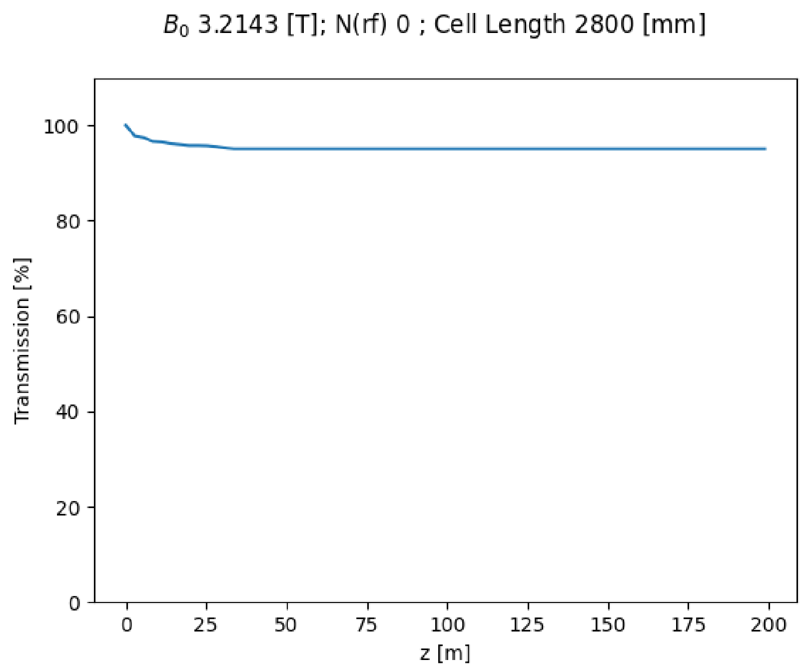


- Physical acceptance driven by RF cavity iris radius
- RF team advice: (iris radius) = 0.5*(ideal cavity radius)
- Define “iris factor” = (iris radius)/(ideal cavity radius)

Dynamic Acceptance

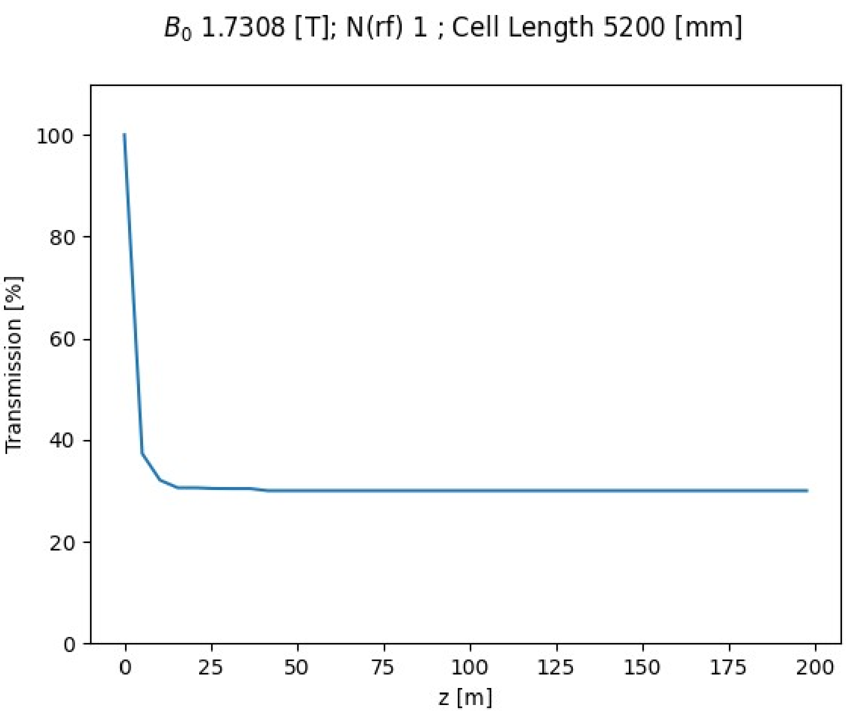
- Consider the beam used by Ruihu at A1
- Scan DA
- Transmission is high
- No dependence on DA

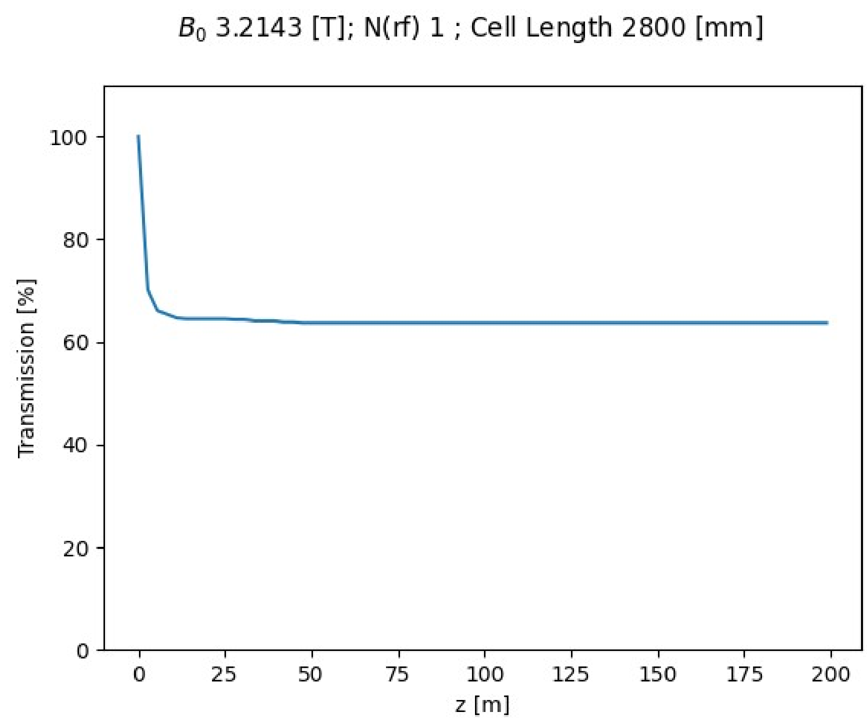




Physical Acceptance

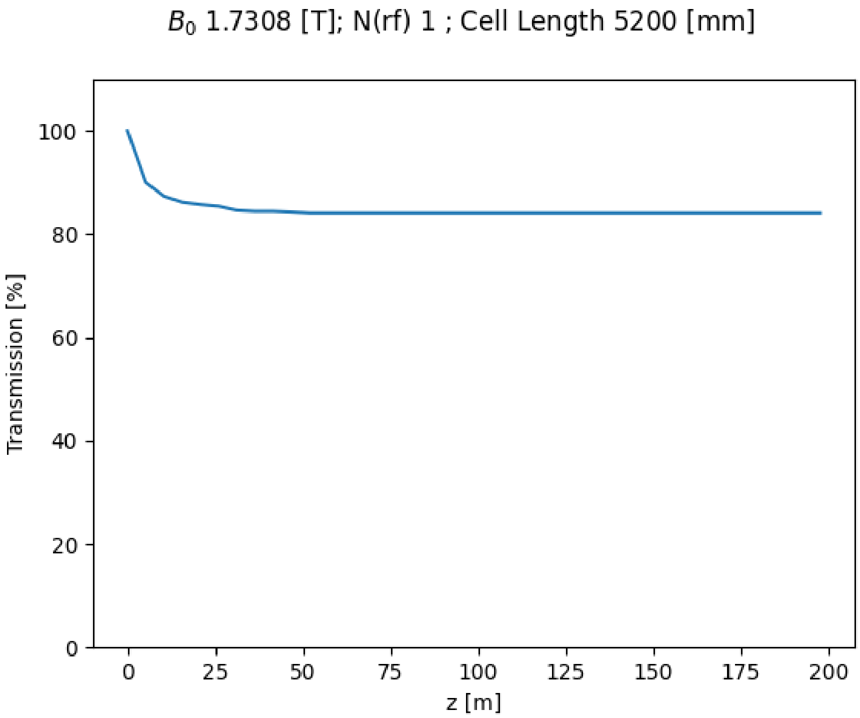
- Consider the beam used by Ruihu at A1
- Add a single RF cavity @ 352 MHz (iris factor 0.5)
- Transmission is terrible

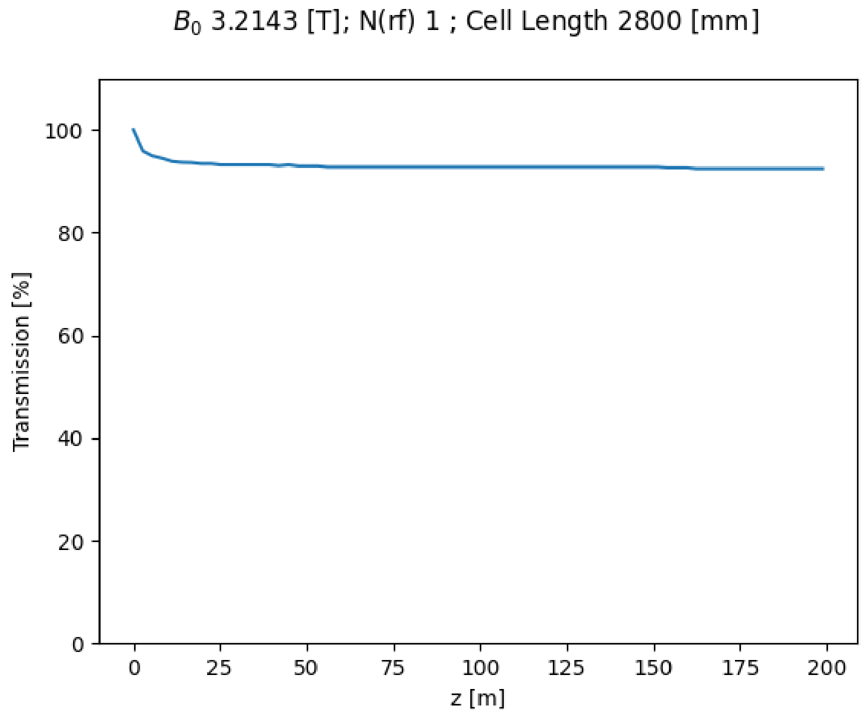




RF frequency

- Higher radius → lower frequency (176 MHz)





Conclusions

- Physical acceptance is limiting performance of A-type lattices
- Improved physical acceptance → lower frequency
- Need to consider 176 MHz RF for A-type lattice

- We can generate a good dynamic range for the A-type lattices
  - Should be possible to make a good cooling performance with decay-dominated transmission

- Addendum… can we capture into 176 MHz RF?