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LHC EFT WG Note:
Basis for Anomalous Quartic Gauge Couplings

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Abstract

In this note, we give a definitive basis for the dimension-eight operators leading to quartic—but no cubic—interactions among electroweak gauge bosons. These are often called anomalous quartic gauge couplings, or aQGCs. We distinguish in particular the CP-even ones from their CP-odd counterparts.

1 Basis

We will denote operators that are CP-even as $\mathcal{O}_i$, and CP-odd ones as $\not{\mathcal{O}}_i$. The basis contains operators quartic in the Higgs (S-type), bi-quadratic in the Higgs and gauge field strengths (M-type), and quartic in the gauge field strengths (T-type). Throughout, we will use square brackets to indicate contraction of fundamental $\mathrm{SU}(2)$ indices. The full CP-even aQGC basis is given by the $\mathcal{O}_i^S$, $\mathcal{O}_i^M$, and $\mathcal{O}_i^T$ operators in Table 1. For completeness, the full basis of CP-odd terms is given by the $\not{\mathcal{O}}_i^M$ and $\not{\mathcal{O}}_i^T$ operators in Table 2. There are no S-type CP-odd terms.

2 Literature comparison

A brief comparison with the literature is useful. To our knowledge, no complete basis of aQGC operators with CP properties correctly identified has appeared in the literature. In this section, we will focus on the CP-even sector of the aQGC basis. The basis presented by Almeida, Éboli, Gonzalez-Garcia, and Mizukoshi in Refs. [1–3] lists operators that are both C-even and

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P-even and does not include two operators that are C-odd and P-odd, but CP-even. One is of $(DH)^2BW$ form and was identified in in Ref. [4]. Similarly, Ref. [3] contains only three $(DH)^2W^2$ operators, though in fact there are four independent such terms in the CP-even basis. (Ref. [1] contained a different fourth operator, but it was found to be redundant with the three others in Ref. [2].) The original basis of T-type CP-even operators, all of which are both C-even and P-even, presented in Refs. [1, 2] was incomplete. This was corrected first in Ref. [4], and subsequently in the updated basis of Ref. [3], which agree.

The counting of operators, of both CP-even and CP-odd type, agrees with the Hilbert series analysis of Ref. [5] by Kondo, Murayama, and Okabe. However, the identification of which specific operators are CP-even and -odd in that paper contains an error. While Ref. [5] identifies $\mathcal{O}_9^M$ as CP-odd and $\not{\mathcal{O}}_6^M$ as CP-even, Ref. [4] counts them both as CP-odd. Both being P-odd, in fact $\not{\mathcal{O}}_6^M$ is C-even, and $\mathcal{O}_9^M$ is C-odd. The result is that $\mathcal{O}_9^M$ is CP-even and $\not{\mathcal{O}}_6^M$ is CP-odd. Similarly, Ref. [4] misidentified $\not{\mathcal{O}}_3^M$ as CP-even and $\mathcal{O}_8^M$ as CP-odd. Thus, while Ref. [4] contained all the operators, it had two errors associated with the CP transformation properties. The presence of a single error was noted in Ref. [5], although the wrong operator was identified in that paper.

In Table 1, we further outline the correspondence of our basis with the lists of CP-even operators in Refs. [3], [4], and [6]; see also Ref. [7]. Where possible we follow the widely used basis of Ref. [3] in the notation we introduce. The correspondence in the CP-odd case is provided in Table 2 although here we do not follow Ref. [3] as that work did not consider CP-odd operators.

3 C and P

Given the subtleties associated with CP properties, we briefly review these details here.

Consider parity first. This is very straightforward. The Higgs is a (parity even) scalar, and derivatives transform as $f(\mu)$, where $f(\mu) = -1 + 2\delta_{\mu 0}$ equals 1 for $\mu = 0$ and -1 otherwise, so that gauge bosons transform as

$$\begin{aligned} \mathrm{P}: \;\; & W^I_{\mu\nu} \to f(\mu)f(\nu), \\ \mathrm{P}: \;\; & \widetilde{W}^I_{\mu\nu} \to -f(\mu)f(\nu), \end{aligned} \tag{1}$$

where the sign in the second case arises from $\mathrm{P}: \;\; \epsilon^{\mu\nu\rho\sigma} \to -1$ (although note ϵ^{IJK} will not flip sign, as parity is a spacetime transformation). An identical result holds for the hypercharge field strength. Accordingly, the parity transformation of an operator is simply controlled by the number of dual field strength tensors.

Charge conjugation is more subtle. Our description of CP transformations will follow Ref. [5] (correcting minor sign issues noticed in Eqs. (4.1) and (4.3) of that work). Let us consider fields transforming under a SU(N) gauge group. The definition of charge conjugation then involves a matrix C acting on fundamental indices, which is unitary, $C^\dagger C = \mathbb{I}$, and satisfies $CC^* = \pm\mathbb{I}$. For N odd, only the plus sign is however allowed in the latter equality. A fundamental representation can then be defined to transform as C: $H \to CH^*$. For consistency, an adjoint representation (analogous to a $HH^\dagger$ combination of fundamentals) then transforms as C: $W \to -CW^TC^\dagger$, where the overall phase should be just a sign for a real representation and should be a minus sign to preserve the Lie algebra. In the Abelian case, the above gauge-field transformation reduces to C: $B \to -B$.

A combination like $H_a^\dagger W_1 ... W_n H_b$, which arises in M-type operators, then transforms as

	aQGC Operator Basis	C	P	Almeida, Éboli, Gonzalez-Garcia [3]	Remmen & Rodd [4]	Murphy [6]
$\mathcal{O}_0^S$	$[D_\mu H^\dagger D_\nu H][D^\mu H^\dagger D^\nu H]$	+	+	$\mathcal{O}_{S,0}$	$\mathcal{O}_2^{H^4}$	$Q_{H^4}^{(2)}$
$\mathcal{O}_1^S$	$[D^\mu H^\dagger D_\mu H][D^\nu H^\dagger D_\nu H]$	+	+	$\mathcal{O}_{S,1}$	$\mathcal{O}_3^{H^4}$	$Q_{H^4}^{(3)}$
$\mathcal{O}_2^S$	$[D_\mu H^\dagger D_\nu H][D^\nu H^\dagger D^\mu H]$	+	+	$\mathcal{O}_{S,2}$	$\mathcal{O}_1^{H^4}$	$Q_{H^4}^{(1)}$
$\mathcal{O}_0^M$	$\frac{1}{2}[D^\mu H^\dagger D_\mu H]W^I_{\nu\rho}W^{I\,\nu\rho}$	+	+	$\mathcal{O}_{M,0}$	$\frac{1}{2}\mathcal{O}_2^{H^2W^2}$	$\frac{1}{2}Q_{W^2H^2D^2}^{(2)}$
$\mathcal{O}_1^M$	$-\frac{1}{2}[D^\mu H^\dagger D^\nu H]W^I_{\mu\rho}W_\nu^{I\,\rho}$	+	+	$\mathcal{O}_{M,1}$	$-\frac{1}{2}\mathcal{O}_1^{H^2W^2}$	$-\frac{1}{2}Q_{W^2H^2D^2}^{(1)}$
$\mathcal{O}_2^M$	$[D^\mu H^\dagger D_\mu H]B_{\nu\rho}B^{\nu\rho}$	+	+	$\mathcal{O}_{M,2}$	$\mathcal{O}_2^{H^2B^2}$	$Q_{B^2H^2D^2}^{(2)}$
$\mathcal{O}_3^M$	$-[D^\mu H^\dagger D^\nu H]B_{\mu\rho}B_\nu{}^\rho$	+	+	$\mathcal{O}_{M,3}$	$-\mathcal{O}_1^{H^2B^2}$	$-Q_{B^2H^2D^2}^{(1)}$
$\mathcal{O}_4^M$	$[D^\mu H^\dagger \tau^I D_\mu H]B^{\nu\rho}W^I_{\nu\rho}$	+	+	$\mathcal{O}_{M,4}$	$\mathcal{O}_1^{H^2BW}$	$Q_{WBH^2D^2}^{(1)}$
$\mathcal{O}_5^M$	$[D^\mu H^\dagger \tau^I D^\nu H](B_\mu{}^\rho W^I_{\nu\rho} + B_\nu{}^\rho W^I_{\mu\rho})$	+	+	$\mathcal{O}_{M,5}$	$\mathcal{O}_3^{H^2BW}$	$Q_{WBH^2D^2}^{(4)}$
$\mathcal{O}_7^M$	$[D^\mu H^\dagger \tau^I \tau^J D^\nu H]W^J_{\mu\rho}W_\nu^{I\,\rho}$	+	+	$\mathcal{O}_{M,7}$	$\frac{1}{4}\mathcal{O}_1^{H^2W^2} - \frac{1}{2}\mathcal{O}_3^{H^2W^2}$	$\frac{1}{4}Q_{W^2H^2D^2}^{(1)} - \frac{1}{2}Q_{W^2H^2D^2}^{(4)}$
$\mathcal{O}_8^M$	$i[D^\mu H^\dagger \tau^I D^\nu H](B_\mu{}^\rho \widetilde{W}^I_{\nu\rho} - B_\nu{}^\rho \widetilde{W}^I_{\mu\rho})$	−	−	———	$\widetilde{\mathcal{O}}_2^{H^2BW}$	$Q_{WBH^2D^2}^{(5)}$
$\mathcal{O}_9^M$	$\epsilon^{IJK}[D^\mu H^\dagger \tau^I D^\nu H](W^J_{\mu\rho}\widetilde{W}_\nu^{K\,\rho} - \widetilde{W}^J_{\mu\rho}W_\nu^{K\,\rho})$	−	−	———	$\widetilde{\mathcal{O}}_2^{H^2W^2}$	$Q_{W^2H^2D^2}^{(5)}$
$\mathcal{O}_0^T$	$\frac{1}{4}W^I_{\mu\nu}W^{I\,\mu\nu}W^J_{\rho\sigma}W^{J\,\rho\sigma}$	+	+	$\mathcal{O}_{T,0}$	$\frac{1}{4}\mathcal{O}_1^{W^4}$	$\frac{1}{4}Q_{W^4}^{(1)}$
$\mathcal{O}_1^T$	$\frac{1}{4}W^I_{\mu\nu}W^{J\,\mu\nu}W^I_{\rho\sigma}W^{J\,\rho\sigma}$	+	+	$\mathcal{O}_{T,1}$	$\frac{1}{4}\mathcal{O}_3^{W^4}$	$\frac{1}{4}Q_{W^4}^{(3)}$
$\mathcal{O}_2^T$	$\frac{1}{4}W^I_{\mu\nu}W^{I\nu\alpha}W^J_{\alpha\beta}W^{J\beta\mu}$	+	+	$\mathcal{O}_{T,2}$	$\frac{1}{16}\mathcal{O}_1^{W^4} + \frac{1}{16}\mathcal{O}_3^{W^4} + \frac{1}{16}\mathcal{O}_4^{W^4}$	$\frac{1}{16}Q_{W^4}^{(1)} + \frac{1}{16}Q_{W^4}^{(3)} + \frac{1}{16}Q_{W^4}^{(4)}$
$\mathcal{O}_3^T$	$\frac{1}{4}W^I_{\mu\nu}W^{J\nu\alpha}W^I_{\alpha\beta}W^{J\beta\mu}$	+	+	$\mathcal{O}_{T,3}$	$\frac{1}{8}\mathcal{O}_3^{W^4} + \frac{1}{16}\mathcal{O}_2^{W^4}$	$\frac{1}{8}Q_{W^4}^{(3)} + \frac{1}{16}Q_{W^4}^{(2)}$
$\mathcal{O}_4^T$	$\frac{1}{2}W^I_{\mu\nu}B^{\nu\alpha}W^I_{\alpha\beta}B^{\beta\mu}$	+	+	$\mathcal{O}_{T,4}$	$\frac{1}{8}\mathcal{O}_2^{B^2W^2} + \frac{1}{4}\mathcal{O}_3^{B^2W^2}$	$\frac{1}{8}Q_{W^2B^2}^{(2)} + \frac{1}{4}Q_{W^2B^2}^{(3)}$
$\mathcal{O}_5^T$	$\frac{1}{2}B_{\mu\nu}B^{\mu\nu}W^I_{\rho\sigma}W^{I\,\rho\sigma}$	+	+	$\mathcal{O}_{T,5}$	$\frac{1}{2}\mathcal{O}_1^{B^2W^2}$	$\frac{1}{2}Q_{W^2B^2}^{(1)}$
$\mathcal{O}_6^T$	$\frac{1}{2}B_{\mu\nu}W^{I\,\mu\nu}B_{\rho\sigma}W^{I\,\rho\sigma}$	+	+	$\mathcal{O}_{T,6}$	$\frac{1}{2}\mathcal{O}_3^{B^2W^2}$	$\frac{1}{2}Q_{W^2B^2}^{(3)}$
$\mathcal{O}_7^T$	$\frac{1}{2}W^I_{\mu\nu}W^{I\nu\alpha}B_{\alpha\beta}B^{\beta\mu}$	+	+	$\mathcal{O}_{T,7}$	$\frac{1}{8}\mathcal{O}_1^{B^2W^2} + \frac{1}{8}\mathcal{O}_3^{B^2W^2} + \frac{1}{8}\mathcal{O}_4^{B^2W^2}$	$\frac{1}{8}Q_{W^2B^2}^{(1)} + \frac{1}{8}Q_{W^2B^2}^{(3)} + \frac{1}{8}Q_{W^2B^2}^{(4)}$
$\mathcal{O}_8^T$	$B_{\mu\nu}B^{\mu\nu}B_{\rho\sigma}B^{\rho\sigma}$	+	+	$\mathcal{O}_{T,8}$	$\mathcal{O}_1^{B^4}$	$Q_{B^4}^{(1)}$
$\mathcal{O}_9^T$	$B_{\mu\nu}B^{\nu\alpha}B_{\alpha\beta}B^{\beta\mu}$	+	+	$\mathcal{O}_{T,9}$	$\frac{1}{2}\mathcal{O}_1^{B^4} + \frac{1}{4}\mathcal{O}_2^{B^4}$	$\frac{1}{2}Q_{B^4}^{(1)} + \frac{1}{4}Q_{B^4}^{(2)}$

Table 1: The basis of CP-even aQGC operators and their C & P transformation properties. We further outline the map between our basis and different conventions used in the literature. As seen, our basis is chosen to closely align with Ref. [3] where possible, and we write ——— for the two CP-even operators that were excluded from that work.

	aQGC Operator Basis	C	P	AEG [3]	RR [4]	M [6]
$\not{\mathcal{O}}^M_1$	$[D^\mu H^\dagger D_\mu H] B_{\nu\rho} \widetilde{B}^{\nu\rho}$	+	−	N/A	$\widetilde{\mathcal{O}}_1^{H^2B^2}$	$Q^{(3)}_{B^2H^2D^2}$
$\not{\mathcal{O}}^M_2$	$[D^\mu H^\dagger \tau^I D_\mu H] B_{\nu\rho}\widetilde{W}^{I\,\nu\rho}$	+	−	N/A	$\widetilde{\mathcal{O}}_1^{H^2BW}$	$Q^{(2)}_{WBH^2D^2}$
$\not{\mathcal{O}}^M_3$	$i[D^\mu H^\dagger\tau^I D^\nu H](B_{\mu\rho}W^{I\,\rho}_\nu - B_{\nu\rho}W^{I\,\rho}_\mu)$	−	+	N/A	$\mathcal{O}_2^{H^2BW}$	$Q^{(3)}_{WBH^2D^2}$
$\not{\mathcal{O}}^M_4$	$[D^\mu H^\dagger\tau^I D^\nu H](B_{\mu\rho}\widetilde{W}^{I\,\rho}_\nu + B_{\nu\rho}\widetilde{W}^{I\,\rho}_\mu)$	+	−	N/A	$\widetilde{\mathcal{O}}_3^{H^2BW}$	$Q^{(6)}_{WBH^2D^2}$
$\not{\mathcal{O}}^M_5$	$[D^\mu H^\dagger D_\mu H] W^I_{\nu\rho}\widetilde{W}^{I\,\nu\rho}$	+	−	N/A	$\widetilde{\mathcal{O}}_1^{H^2W^2}$	$Q^{(3)}_{W^2H^2D^2}$
$\not{\mathcal{O}}^M_6$	$i\,\epsilon^{IJK}[D^\mu H^\dagger \tau^I D^\nu H](W^J_{\mu\rho}\widetilde{W}^{K\,\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\,\rho}_\nu)$	+	−	N/A	$\widetilde{\mathcal{O}}_3^{H^2W^2}$	$Q^{(6)}_{W^2H^2D^2}$
$\not{\mathcal{O}}^T_1$	$B_{\mu\nu}B^{\mu\nu}B_{\rho\sigma}\widetilde{B}^{\rho\sigma}$	+	−	N/A	$\widetilde{\mathcal{O}}_1^{B^4}$	$Q^{(3)}_{B^4}$
$\not{\mathcal{O}}^T_2$	$B_{\mu\nu}\widetilde{B}^{\mu\nu}W^I_{\rho\sigma}W^{I\,\rho\sigma}$	+	−	N/A	$\widetilde{\mathcal{O}}_1^{B^2W^2}$	$Q^{(5)}_{W^2B^2}$
$\not{\mathcal{O}}^T_3$	$B_{\mu\nu}B^{\mu\nu}W^I_{\rho\sigma}\widetilde{W}^{I\,\rho\sigma}$	+	−	N/A	$\widetilde{\mathcal{O}}_2^{B^2W^2}$	$Q^{(6)}_{W^2B^2}$
$\not{\mathcal{O}}^T_4$	$B_{\mu\nu}W^{I\,\mu\nu}B_{\rho\sigma}\widetilde{W}^{I\,\rho\sigma}$	+	−	N/A	$\widetilde{\mathcal{O}}_3^{B^2W^2}$	$Q^{(7)}_{W^2B^2}$
$\not{\mathcal{O}}^T_5$	$W^I_{\mu\nu}W^{I\,\mu\nu}W^J_{\rho\sigma}\widetilde{W}^{J\,\rho\sigma}$	+	−	N/A	$\widetilde{\mathcal{O}}_1^{W^4}$	$Q^{(5)}_{W^4}$
$\not{\mathcal{O}}^T_6$	$W^I_{\mu\nu}W^{J\,\mu\nu}W^I_{\rho\sigma}\widetilde{W}^{J\,\rho\sigma}$	+	−	N/A	$\widetilde{\mathcal{O}}_2^{W^4}$	$Q^{(6)}_{W^4}$

Table 2: As in Table 1 but for the CP odd operators. We write "N/A" for the conversion to Ref. [3] as that work did not consider CP-odd operators.

follows:

$$\mathrm{C}:\; H_a^\dagger W_1 ... W_n H_b \to (H_a^T C^\dagger)(-CW_1^T C^\dagger)...(-CW_n^T C^\dagger)(CH_b^*) = (-1)^n H_b^\dagger W_n ... W_1 H_a. \tag{2}$$

We thus see that charge conjugation reverses the order of the fields and introduces an extra minus sign for odd numbers of adjoint representations. For instance, the $\mathcal{O}^M_9$ and $\not{\mathcal{O}}^M_6$ operators involve two terms of the form $(D_\mu H)^\dagger [W_{\mu\rho}, \widetilde{W}_{\nu\rho}](D_\nu H)$, which transform under charge conjugation as

$$\mathrm{C}:\; (D_\mu H)^\dagger [W_{\mu\rho}, \widetilde{W}_{\nu\rho}](D_\nu H) \to (D_\nu H)^\dagger [\widetilde{W}_{\nu\rho}, W_{\mu\rho}](D_\mu H) = (D_\mu H)^\dagger [\widetilde{W}_{\mu\rho}, W_{\nu\rho}](D_\nu H), \tag{3}$$

where the last equality only involves a relabeling of the Lorentz indices. In such a combination of fields, the adjoint field strength and its dual are therefore just exchanged (leaving all indices untouched). Since $\mathcal{O}^M_9$ is by construction odd under this exchange, it is therefore odd under charge conjugation. Given that $\mathcal{O}^M_9$ is also odd under parity (because of the dual field strength), it is actually CP-even. On the contrary, $\not{\mathcal{O}}^M_6$ is even under the exchange of the field strength and its dual. It is therefore even under charge conjugation, while also being parity odd, so it is CP-odd altogether.

There are two specific realizations of the C matrix often employed in the literature (see again Ref. [5]): one symmetric $C_S = \mathbb{I}$, and one skew $C_A = i\sigma_2$. The latter satisfies $C_A C_A^* = -\mathbb{I}$ and is allowed for SU(N) with $N = 2$ even as in the electroweak sector of the SM. With these specific representations, the transformation of various field components and combinations are the following:

$$\begin{aligned}
&C_S\colon H \to H^*, && C_A\colon H \to (i\sigma_2)H^*,\\
&C_S\colon B_{\mu\nu} \to -B_{\mu\nu}, && C_A\colon B_{\mu\nu} \to -B_{\mu\nu},\\
&C_S\colon W^I_{\mu\nu} \to f(1-\delta_{I2})W^I_{\mu\nu}, && C_A\colon W^I_{\mu\nu} \to +W^I_{\mu\nu},\\
&C_S\colon (D^\mu H^\dagger \tau^I D^\nu H) \to f(\delta_{I2})(D^\nu H^\dagger \tau^I D^\mu H), && C_A\colon (D^\mu H^\dagger \tau^I D^\nu H) \to -(D^\nu H^\dagger \tau^I D^\mu H),
\end{aligned} \tag{4}$$

where the final result on the last line can be derived from that of the first line. Note this final expression involves an interchange $\mu \leftrightarrow \nu$ in addition to the prefactor (which cancels out in both cases in a $(D^\mu H^\dagger \tau^I D^\nu H) W^I_{\rho\sigma}$ combination).

4 Monte Carlo implementation

An implementation of the C-even and P-even operators of Refs. [1–3] in a UFO model enabling event generation in various Monte Carlo simulation tools is available at https://feynrules.irmp.ucl.ac.be/wiki/AnomalousGaugeCoupling. The code provided by the authors of Ref. [8] at https://www.fuw.edu.pl/smeft also allows one to generate a UFO model implementation of the aQGC operators following the conventions of Ref. [6] (see also the conversion between the two bases provided at https://www.fuw.edu.pl/smeft/Validation.pdf). We provide an implementation of all the dimension-eight aQGC operators listed in Table 1 and Table 2 at https://github.com/gdurieux/aqgc. Adopting the same parameter naming as in `AnomalousGaugeCoupling` whenever possible, the coefficients of the implemented operators satisfy the following relations, deriving directly from the map between operators provided in Table 1:

$$
\begin{array}{l}
\texttt{FS0} = \texttt{cS2}\\
\texttt{FS1} = \texttt{cS3}\\
\texttt{FS2} = \texttt{cS1}
\end{array}
\qquad
\begin{array}{l}
\texttt{FM0} = 2\,\texttt{cM7}\\
\texttt{FM1} = -2\,\texttt{cM6} - \texttt{cM8}\\
\texttt{FM2} = \texttt{cM2}\\
\texttt{FM3} = -\texttt{cM1}\\
\texttt{FM4} = \texttt{cM3}\\
\texttt{FM5} = \texttt{cM5}\\
\texttt{FM7} = -2\,\texttt{cM8}\\
\texttt{FM8} = \texttt{cM4}\\
\texttt{FM9} = \texttt{cM9}
\end{array}
\qquad
\begin{array}{l}
\texttt{FT0} = 4\,\texttt{cT7} - 4\,\texttt{cT10}\\
\texttt{FT1} = -8\,\texttt{cT8} + 4\,\texttt{cT9} - 4\,\texttt{cT10}\\
\texttt{FT2} = 16\,\texttt{cT10}\\
\texttt{FT3} = 16\,\texttt{cT8}\\
\texttt{FT4} = 8\,\texttt{cT4}\\
\texttt{FT5} = 2\,\texttt{cT3} - 2\,\texttt{cT6}\\
\texttt{FT6} = -4\,\texttt{cT4} + 2\,\texttt{cT5} - 2\,\texttt{cT6}\\
\texttt{FT7} = 8\,\texttt{cT6}\\
\texttt{FT8} = \texttt{cT1} - 2\,\texttt{cT2}\\
\texttt{FT9} = 4\,\texttt{cT2}
\end{array}
, \qquad (5)
$$

where, for convenience, we introduced the following shorthand notation for the coefficients of the operators defined in Refs. [4, 6]:

$$
\begin{array}{l}
c^S_1 \equiv c^{H^4}_1 = c^{(1)}_{H^4}\\
c^S_2 \equiv c^{H^4}_2 = c^{(2)}_{H^4}\\
c^S_3 \equiv c^{H^4}_3 = c^{(3)}_{H^4}
\end{array}
\qquad
\begin{array}{l}
c^M_1 \equiv c^{H^2B^2}_1 = c^{(1)}_{B^2H^2D^2}\\
c^M_2 \equiv c^{H^2B^2}_2 = c^{(2)}_{B^2H^2D^2}\\
c^M_3 \equiv c^{H^2BW}_1 = c^{(1)}_{WBH^2D^2}\\
c^M_4 \equiv \tilde{c}^{H^2BW}_2 = c^{(5)}_{WBH^2D^2}\\
c^M_5 \equiv c^{H^2BW}_3 = c^{(4)}_{WBH^2D^2}\\
c^M_6 \equiv c^{H^2W^2}_1 = c^{(1)}_{W^2H^2D^2}\\
c^M_7 \equiv c^{H^2W^2}_2 = c^{(2)}_{W^2H^2D^2}\\
c^M_8 \equiv c^{H^2W^2}_3 = c^{(4)}_{W^2H^2D^2}\\
c^M_9 \equiv \tilde{c}^{H^2W^2}_2 = c^{(5)}_{W^2H^2D^2}
\end{array}
\qquad
\begin{array}{l}
c^T_1 \equiv c^{B^4}_1 = c^{(1)}_{B^4}\\
c^T_2 \equiv c^{B^4}_2 = c^{(2)}_{B^4}\\
c^T_3 \equiv c^{B^2W^2}_1 = c^{(1)}_{W^2B^2}\\
c^T_4 \equiv c^{B^2W^2}_2 = c^{(2)}_{W^2B^2}\\
c^T_5 \equiv c^{B^2W^2}_3 = c^{(3)}_{W^2B^2}\\
c^T_6 \equiv c^{B^2W^2}_4 = c^{(4)}_{W^2B^2}\\
c^T_7 \equiv c^{W^4}_1 = c^{(1)}_{W^4}\\
c^T_8 \equiv c^{W^4}_2 = c^{(2)}_{W^4}\\
c^T_9 \equiv c^{W^4}_3 = c^{(3)}_{W^4}\\
c^T_{10} \equiv c^{W^4}_4 = c^{(4)}_{W^4}
\end{array}
. \qquad (6)
$$

particles	gauge structure	kinematics	C	P	coefficient
$H_1^* H_2^* H_3 H_4$	$\delta_1^3\delta_2^4 + \delta_1^4\delta_2^3$	s^2	$+$	$+$	$\frac{i}{2} c_2^S$
	$\delta_1^3\delta_2^4 + \delta_1^4\delta_2^3$	$t^2 + u^2$	$+$	$+$	$\frac{i}{4}(c_1^S + c_3^S)$
	$\delta_1^3\delta_2^4 - \delta_1^4\delta_2^3$	$t^2 - u^2$	$+$	$+$	$-\frac{i}{4}(c_1^S - c_3^S)$
$B_1 B_2 H_3^* H_4$		$([12]^2 + \langle 12\rangle^2)\, s$	$+$	$+$	$\frac{i}{4}(c_1^M + 4c_2^M)$
		$([12]^2 - \langle 12\rangle^2)\, s$	$+$	$-$	$-\slashed{c}_1^M$
		$[1(3-4)2\rangle^2 + \langle 1(3-4)2]^2$	$+$	$+$	$-\frac{i}{8} c_1^M$
$W_1 B_2 H_3^* H_4$	$[\tau^1]^4{}_3$	$([12]^2 + \langle 12\rangle^2)\, s$	$+$	$+$	$\frac{i}{4}(2c_3^M + c_5^M)$
	$[\tau^1]^4{}_3$	$([12]^2 - \langle 12\rangle^2)\, s$	$+$	$-$	$-\frac{1}{4}(2\slashed{c}_2^M + \slashed{c}_4^M)$
	$[\tau^1]^4{}_3$	$([12]^2 + \langle 12\rangle^2)\,(t-u)$	$-$	$+$	$-\frac{1}{4}\slashed{c}_3^M$
	$[\tau^1]^4{}_3$	$([12]^2 - \langle 12\rangle^2)\,(t-u)$	$-$	$-$	$-\frac{i}{4} c_4^M$
	$[\tau^1]^4{}_3$	$[1(3-4)2\rangle^2 + \langle 1(3-4)2]^2$	$+$	$+$	$-\frac{i}{8} c_5^M$
	$[\tau^1]^4{}_3$	$[1(3-4)2\rangle^2 - \langle 1(3-4)2]^2$	$+$	$-$	$\frac{1}{8}\slashed{c}_4^M$
$W_1 W_2 H_3^* H_4$	$\delta^{12}\delta_3^4$	$([12]^2 + \langle 12\rangle^2)\, s$	$+$	$+$	$\frac{i}{4}(c_6^M + 4c_7^M)$
	$\delta^{12}\delta_3^4$	$([12]^2 - \langle 12\rangle^2)\, s$	$+$	$-$	$-\slashed{c}_5^M$
	$[\tau^1, \tau^2]^4{}_3$	$([12]^2 + \langle 12\rangle^2)\,(t-u)$	$+$	$+$	$\frac{i}{4} c_8^M$
	$[\tau^1, \tau^2]^4{}_3$	$([12]^2 - \langle 12\rangle^2)\,(t-u)$	$+$	$-$	$-\frac{1}{2}\slashed{c}_6^M$
	$\delta^{12}\delta_3^4$	$[1(3-4)2\rangle^2 + \langle 1(3-4)2]^2$	$+$	$+$	$-\frac{i}{8} c_6^M$
	$[\tau^1, \tau^2]^4{}_3$	$[1(3-4)2\rangle^2 - \langle 1(3-4)2]^2$	$-$	$-$	$\frac{i}{4} c_9^M$
$B_1 B_2 B_3 B_4$		$([12]^2[34]^2 + \langle 12\rangle^2\langle 34\rangle^2) + \text{perm.}$	$+$	$+$	$8i(c_1^T - c_2^T)$
		$([12]^2[34]^2 - \langle 12\rangle^2\langle 34\rangle^2) + \text{perm.}$	$+$	$-$	$-8\slashed{c}_1^T$
		$([12]^2\langle 34\rangle^2 + \langle 12\rangle^2[34]^2) + \text{perm.}$	$+$	$+$	$8i(c_1^T + c_2^T)$
$B_1 B_2 W_3 W_4$	δ^{34}	$[12]^2[34]^2 + \langle 12\rangle^2\langle 34\rangle^2$	$+$	$+$	$4i(c_3^T - c_4^T)$
	δ^{34}	$[12]^2[34]^2 - \langle 12\rangle^2\langle 34\rangle^2$	$+$	$-$	$-4(c_2^T + c_3^T)$
	δ^{34}	$([13]^2[24]^2 + [14]^2[23]^2) + (\langle 13\rangle^2\langle 24\rangle^2 + \langle 14\rangle^2\langle 23\rangle^2)$	$+$	$+$	$2i(c_5^T - c_6^T)$
	δ^{34}	$([13]^2[24]^2 + [14]^2[23]^2) - (\langle 13\rangle^2\langle 24\rangle^2 + \langle 14\rangle^2\langle 23\rangle^2)$	$+$	$-$	$-2\slashed{c}_4^T$
	δ^{34}	$[12]^2\langle 34\rangle^2 + \langle 12\rangle^2[34]^2$	$+$	$+$	$4i(c_3^T + c_4^T)$
	δ^{34}	$[12]^2\langle 34\rangle^2 - \langle 12\rangle^2[34]^2$	$+$	$-$	$-4(\slashed{c}_2^T - \slashed{c}_3^T)$
	δ^{34}	$[13]^2\langle 24\rangle^2 + [14]^2\langle 23\rangle + \langle 13\rangle^2[24]^2 + \langle 14\rangle^2[23]^2$	$+$	$+$	$2i(c_5^T + c_6^T)$
$W_1 W_2 W_3 W_4$	$\delta^{12}\delta^{34} + \text{perm.}$	$([12]^2[34]^2 + \langle 12\rangle^2\langle 34\rangle^2) + \text{perm.}$	$+$	$+$	$4i(c_9^T - c_{10}^T)$
	$\delta^{12}\delta^{34} + \text{perm.}$	$([12]^2[34]^2 - \langle 12\rangle^2\langle 34\rangle^2) + \text{perm.}$	$+$	$-$	$-4\slashed{c}_6^T$
	$\{\delta^{12}\delta^{34}$	$([12]^2[34]^2 + \langle 12\rangle^2\langle 34\rangle^2)\} + \text{perm.}$	$+$	$+$	$4i(2c_7^T - 2c_8^T - c_9^T + c_{10}^T)$
	$\{\delta^{12}\delta^{34}$	$([12]^2[34]^2 - \langle 12\rangle^2\langle 34\rangle^2)\} + \text{perm.}$	$+$	$-$	$-4(2\slashed{c}_5^T - \slashed{c}_6^T)$
	$\delta^{12}\delta^{34} + \text{perm.}$	$([12]^2\langle 34\rangle^2 + \langle 12\rangle^2[34]^2) + \text{perm.}$	$+$	$+$	$4i(c_9^T + c_{10}^T)$
	$\{\delta^{12}\delta^{34}$	$([12]^2\langle 34\rangle^2 + \langle 12\rangle^2[34]^2)\} + \text{perm.}$	$+$	$+$	$4(2c_7^T + 2c_8^T - c_9^T - c_{10}^T)$

Table 3: Independent linear combinations of massless dimension-eight amplitudes involving four electroweak bosons. The CP-even coefficients are given in terms of the shorthand notation of Eq. (6).

5 Massless amplitudes

Considering massless amplitudes instead of operators, one can readily form linear combinations with definite C and P transformation properties. This provides an alternative view on the independent aQGC gauge and kinematic structures. Table 3 lists these independent linear combinations. They are symmetric under the exchange of identical bosons as required by Bose statistics. Momenta and gauge indices are all substituted by the corresponding particle labels. Parity just exchanges square and angle spinors, which is equivalent to a complex conjugation of the kinematic structures. Charge conjugation effectively acts as a complex conjugation of the gauge structure, in combination with an exchange of conjugate particle labels. It therefore exchanges the t and u Mandelstam invariants in various amplitudes of Table 3. The only gauge structure that is effectively C-odd is the anticommutator $[\tau^1, \tau^2]^4{}_3$, appearing in $W_1 W_2 H_3^* H_4$ amplitudes, that is sent to its Hermitian conjugate according to Eq. (2).

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