

Three editions of the PDFLattice workshop: what we have learnt and what we would like to learn

Parton Distributions and Lattice Calculations (PDFLattice2024)

Emanuele R. Nocera

Università degli Studi di Torino and INFN, Torino

18 November 2024



UNIVERSITÀ
DI TORINO

Welcome to PDFLattice2024

First of all, THANK YOU

Thanks to Kimberly Sawyer and to David Dean

Thanks to the Organising Committee, to Jianwei and Thia, and to the Key Speakers

Thanks to all the speakers, poster presenters, and participants

Agenda and Logistics

A mixture of longer introductory/review talks and shorter focused talks
Ample time for discussions, as is customary with the PDFLattice workshop

Two sessions on Wednesday will be dedicated to discuss the White Paper

Take a moment to look at the Indico agenda [\[link\]](#)

Do not be afraid of asking questions and engaging in the discussions

Wednesday is a working day! We expect all participants to contribute!

Today, 3.50pm: Group Photo

Today, 6-8.30pm: Reception and Poster Session

Wednesday 4pm: JLab Theory Colloquium

Code of Conduct

Please take a moment to look at the dedicated Indico web page [\[link\]](#)

Three editions of the PDFLattice workshop

- 1 2017: Balliol College, Oxford, 22-24 March 2017 [[web page](#)]

FOCUS: the collinear structure of the proton



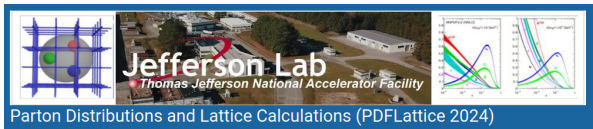
- 2 2019: Kellogg Biological Station, Hickory Corners, 25-27 September [[web page](#)]

FOCUS: towards the three-dimensional structure of the proton



- 3 2024: JLab Theory Center, 18-20 November 2024 [[web page](#)]

FOCUS: uncertainty quantification in parton distribution determination



Two White Papers

Progress in Particle and Nuclear Physics 100 (2018) 107–160

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics



Journal homepage: www.elsevier.com/locate/npnp



Review

Parton distributions and lattice QCD calculations: A community white paper

Huey-Wen Lin^{1,2}, Emanuele R. Nocera^{3,4}, Fred Olness⁵, Kostas Orginos^{6,7}, Juan Rojo^{8,9,10} (editors), Alberto Accardi^{11,10}, Constantia Alexandrou^{11,12}, Alessandro Bacchetta¹³, Giuseppe Bozzi¹³, Junn-Wei Chen¹⁴, Sara Collins¹⁵, Amanda Cooper-Sarkar¹⁶, Martha Constantinou¹⁷, Luigi Del Debbio¹⁸, Michael Engelhardt¹⁹, Jeremy Green¹⁹, Rajan Gupta²⁰, Lucian A. Harland-Lang²¹, Tomomi Ishikawa²², Aleksander Kusina²⁴, Keh-Fei Liu²⁵, Simonetta Liuti^{26,27}, Christopher Monahan²⁸, Pavel Nadolsky²⁹, Jian-Wei Qiu³⁰, Ingo Schienbein²¹, Gerrit Schierholz²⁰, Robert S. Thorne²¹, Werner Vogelsang³⁰, Hartmut Wittig²¹, C.-P. Yuan¹, James Zanotti²²

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[Prog.Part.Nucl.Phys. 100 (2018) 107]

Progress in Particle and Nuclear Physics 121 (2021) 103908

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics



Journal homepage: www.elsevier.com/locate/npnp



Review

Parton distributions and lattice-QCD calculations: Toward 3D structure

Martha Constantinou^{1,3}, Aureore Courtoy⁴, Markus A. Ebert⁵, Michael Engelhardt^{6,1}, Tommaso Giani¹, Tim Hobbs^{6,9,10}, Tie-Jun Hou¹¹, Aleksander Kusina¹², Krzysztof Kutak¹, Jian Liang¹, Huey-Wen Lin^{13,14,15}, Keh-Fei Liu¹⁶, Simonetta Liuti¹⁰, Cédric Mezrag¹⁷, Pavel Nadolsky¹⁸, Emanuele R. Nocera¹⁹, Fred Olness²⁰, Jian-Wei Qiu²¹, Marco Radici²², Anatoly Radyushkin²³, Abha Rajan¹, Ted Rogers²⁴, Juan Rojo²⁵, Gerrit Schierholz²⁶, C.-P. Yuan²⁷, Jian-Hui Zhang²⁸, Rui Zhang²⁹

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Generalized Parton Distributions (GPD)

ABSTRACT

The strong force which binds hadrons is described by the theory of quantum chromodynamics (QCD). Determining the character and manifestations of QCD is one of the most important and challenging outstanding issues necessary for a comprehensive understanding of the structure of hadrons. Within the context of the QCD parton picture, the parton distribution functions (PDFs) have been remarkably successful in describing a wide variety of processes. However, these PDFs have generally been confined to the description of collinear partons within the hadron. New experiments and facilities provide the opportunity to additionally explore the transverse structure of hadrons which

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[Prog.Part.Nucl.Phys. 121 (2021) 103908]

Collectively, more than 450 citations on iNSPIRE HEP as of today

PDF determination in global QCD analyses

Inverse problem

Given a set of data D , determine $p(f|D)$ in the space of functions $f : [0, 1] \rightarrow \mathbb{R}$

Solution: parametric regression

Approximate $p(f|D)$ with its projection in the space of parameters $p(\theta|D)$

$$x f_i(x, Q_0^2) = A_{f_i} x^{a_{f_i}} (1-x)^{b_{f_i}} \mathcal{F}(x, \{c_{f_i}\})$$

Determine $p(\theta|D) \propto p(D|\theta)p(\theta)$ as MAP $\theta^* = \arg \max_{\theta} p(\theta|D)$

$$\chi^2 = \sum_{i,j}^{N_{\text{dat}}} [T_i[\theta] - D_i] (\text{cov}^{-1})_{ij} [T_j[\theta] - D_j]$$

Use a prescription to compute expectation values and uncertainties of observables

$$E[\mathcal{O}] = \int \mathcal{D}f \mathcal{P}(f|D) \mathcal{O}(f) \quad V[\mathcal{O}] = \int \mathcal{D}f \mathcal{P}(f|D) [\mathcal{O}(f) - E[\mathcal{O}]]^2$$

Monte Carlo: $\mathcal{P}(f|D) \rightarrow \{f_k\}$

Maximum likelihood: $\mathcal{P}(f|D) \rightarrow f_0$

$$E[\mathcal{O}] \approx \frac{1}{N} \sum_k \mathcal{O}(f_k)$$

$$E[\mathcal{O}] \approx \mathcal{O}(f_0)$$

$$V[\mathcal{O}] \approx \frac{1}{N} \sum_k [\mathcal{O}(f_k) - E[\mathcal{O}]]^2$$

$$V[\mathcal{O}] \approx \text{Hessian}, \Delta\chi^2 \text{ envelope}, \dots$$

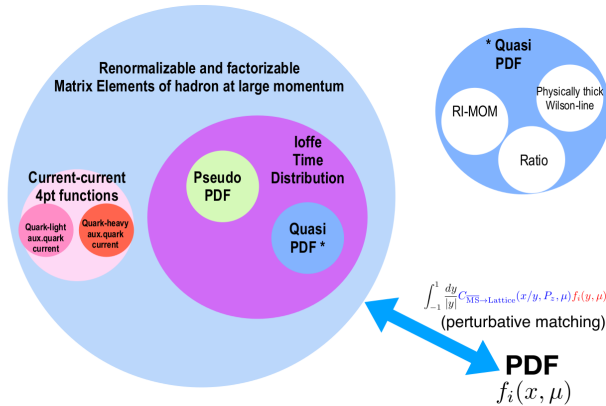
[For details, see e.g. *Ann.Rev.Nucl.Part.Sci.* **70** (2020) 43; *Rev.Mod.Phys.* **92** (2020) 045003]

Uncertainties come from data, theory, and methodology

PDF determination from Lattice QCD

Hadronic tensor [PRL 72 (1994) 1790]
 Auxiliary scalar quarks [PLB 441 (1998) 371]
 Fictitious heavy quark [PRD 73 (2006) 014501]
 Higher moments [PRD 86 (2012) 054505]

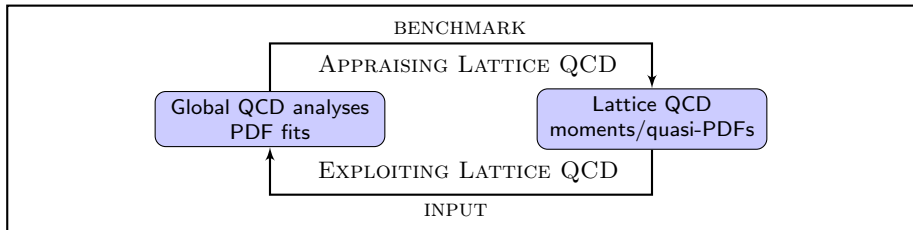
Quasi-PDFs (LaMET) [PRL 110 (2013) 262002]
 Good Cross Sections [PRL 120 (2018) 022003]
 Compton Amplitudes [PRL 118 (2017) 242001]
 Pseudo-PDFs [PRD 96 (2017) 034025]



[Figure by Nikhil Karthik, PDFLattice2019; for details, see e.g. Adv.High Energy Phys. 2019 (2019) 3036904]

Each strategy is associated to systematic uncertainties and theoretical challenges

Connecting two facets of the same world



What we have learnt so far

Define a mutually agreed conventional notation
for relevant PDF-related quantities, such as PDF moments
Assess the sources of systematic uncertainties in lattice-QCD calculations

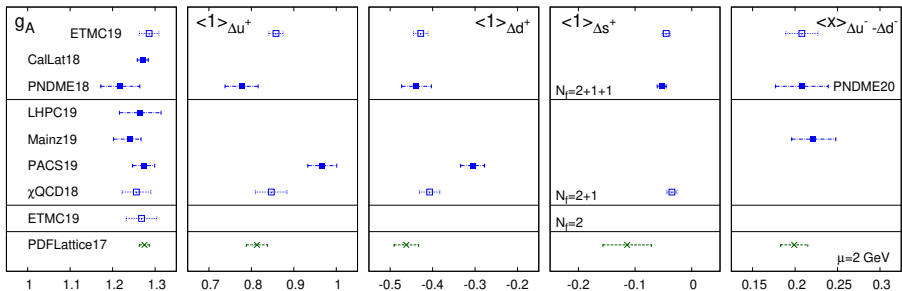
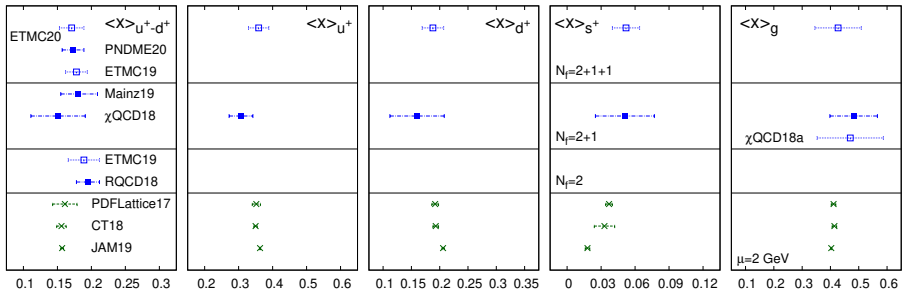
Identify a best-set of quantities
to benchmark lattice-QCD calculations against global-fit determinations

Set precision targets for lattice-QCD calculations
with respect to global-fit determinations

Assess the impact of lattice-QCD calculations
on global-fit determinations within their current/projected precision

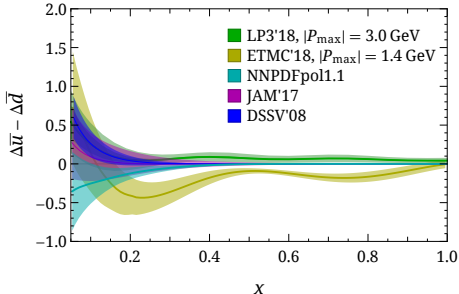
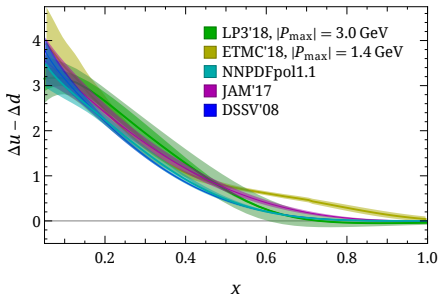
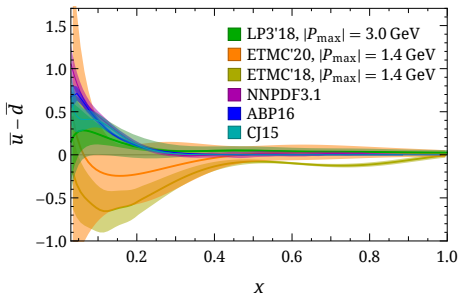
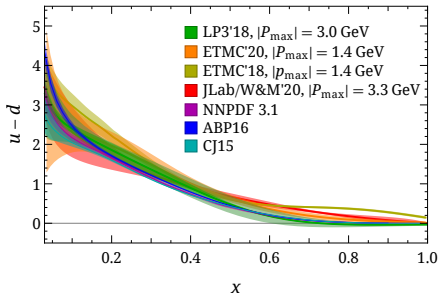
Appraising Lattice QCD: benchmark of PDF moments

From PDFLattice2019



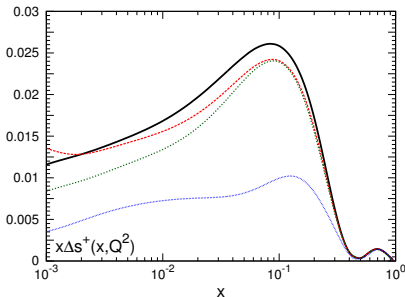
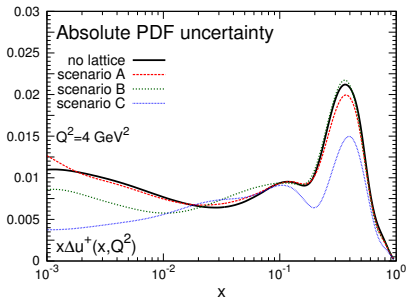
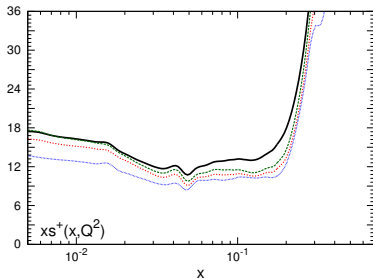
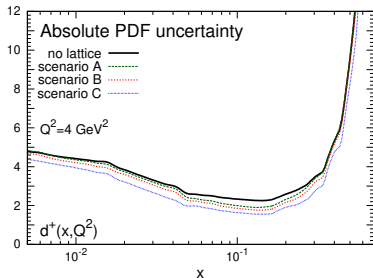
Appraising Lattice QCD: benchmark of PDFs

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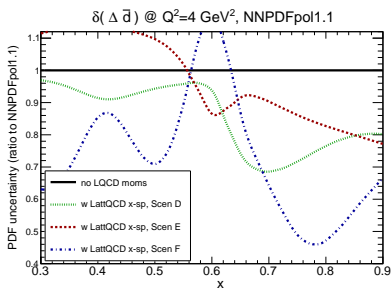
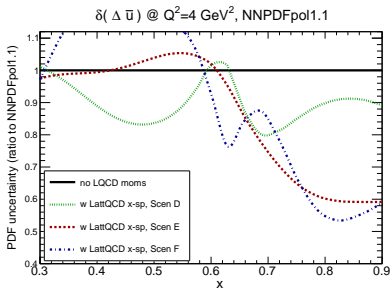
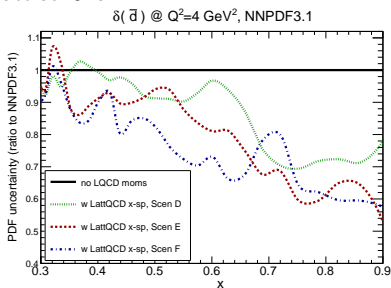
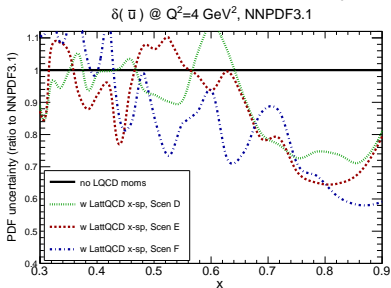
Exploiting Lattice QCD: impact of moments

From PDFLattice2017



Exploiting Lattice QCD: impact of x -space PDFs

From PDFLattice2017



A plethora of papers since PDFLattice2019

PHYSICAL REVIEW LETTERS **120**, 152502 (2018)

First Monte Carlo Global Analysis of Nucleon Transversity with Lattice QCD Constraints

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 (Received 27 October 2017; published 11 April 2018)

PHYSICAL REVIEW D **109**, 036031 (2024)

Gluon helicity from global analysis of experimental data and lattice QCD Ioffe time distributions

J. Karpie¹, R. M. Whitehill², W. Melnitchouk¹, C. Monahan^{1,3}, K. Orginos,^{1,3}
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PHYSICAL REVIEW D **107**, 076018 (2023)

Impact of lattice strangeness asymmetry data in the CTEQ-TEA global analysis

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Parton distributions from lattice data: the nonsinglet case

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Notes on lattice observables for parton distributions: nongauge theories

Luigi Del Debbio,^a Tommaso Gianì^b and Christopher J. Monahan^{c,d}



PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: October 26, 2019

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ACCEPTED: January 6, 2020

PUBLISHED: February 16, 2021

Neural-network analysis of Parton Distribution Functions from Ioffe-time pseudodistributions

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What is this workshop about?

The focus of this workshop is on Uncertainty Quantification
A **list of Key Questions** is attached to the Indico page of the workshop

- 1 Accessing PDFs: global analyses and lattice computations
→ How does PDF determination work in global analyses and lattice QCD?
- 2 Global QCD analyses: inverse problem and objective functions
→ How is the inverse problem entailed by PDF determination addressed?
- 3 Lattice QCD: considerations on the validity of the perturbative matching
→ How is the equivalence between zP_z and ξ^-P^+ defined?
- 4 Setting up a common language: definitions and benchmarks
→ How to benchmark lattice moments and quasi-/pseudo-PDFs with global analyses?
- 5 Combining lattice and experimental data to determine PDFs
→ What are the efforts/limitations to incorporate lattice data in PDF determinations?
- 6 Uncertainty quantification and bias/variance trade-off
→ How are aleatoric and epistemic uncertainties combined? How is a model chosen?

Final remarks (yes, this is the same as the first slide)

Finally, again, THANK YOU

Thanks to Kimberly Sawyer and to David Dean

Thanks to the Organising Committee, to Jianwei and Thia, and to the Key Speakers

Thanks to all the speakers, poster presenters, and participants

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A mixture of longer introductory/review talks and shorter focused talks

Ample time for discussions, as is customary with the PDF Lattice workshop

Two sessions on Wednesday will be dedicated to discuss the White Paper

Take a moment to look at the Indico agenda [\[link\]](#)

Do not be afraid of asking questions and engaging in the discussions

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Enjoy PDFLattice2024!

Questions?