Effects of threshold resummation for large-x PDF in large momentum effective theory

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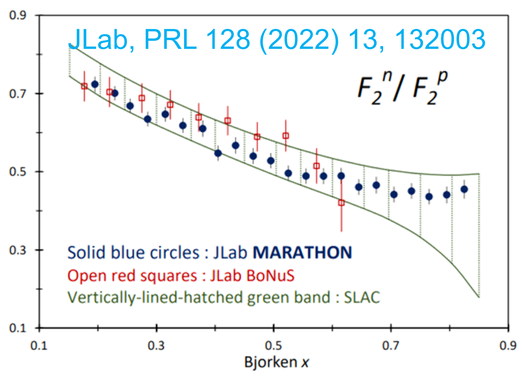
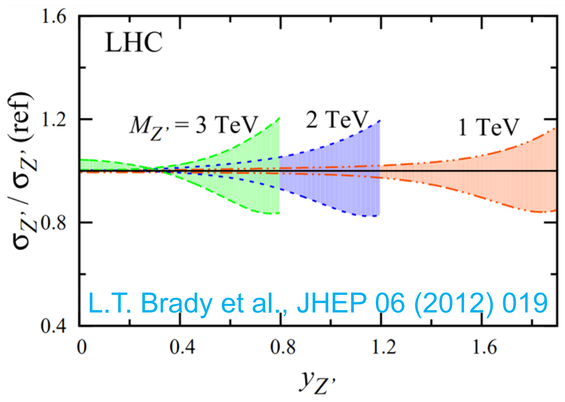
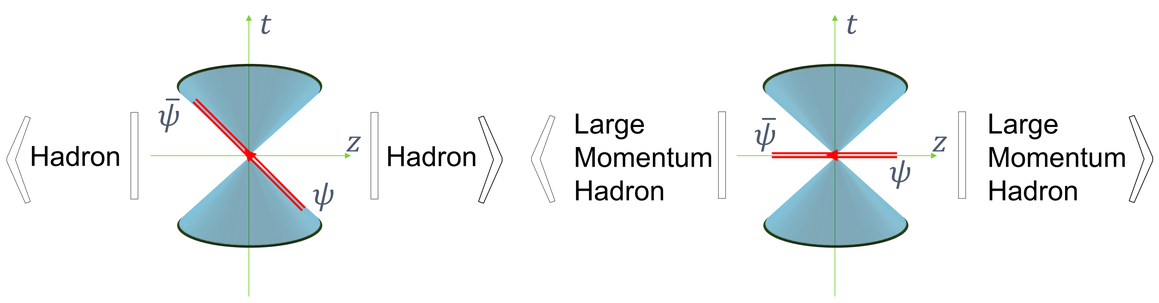
Introduction

Parton distribution functions (PDFs) at large x are essential for understanding hadron structure and searching for new physics beyond the Standard Model. However, it is challenging to extract them from experimental data due to low luminosity.





On the other hand, the large x PDFs can be calculated using Large Momentum Effective Theory (LaMET), which is a systematic approach to access the parton physics, defined as light-cone correlators, through large momentum expansion of Euclidean observables, calculable from the first principles of quantum chromodynamics (QCD), such as lattice QCD. Compared to global fits, LaMET utilizes the full range of data from lattice QCD, addressing the lack of experimental data at large x.



Methodology

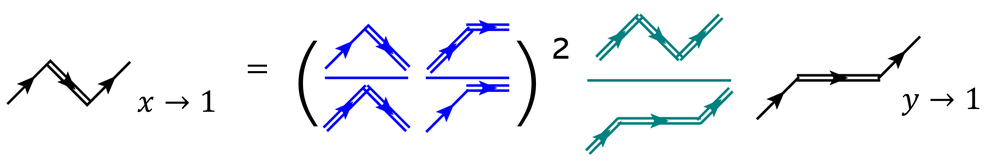
In this work, we study the large x light-cone PDF $f(x,\mu)$ under the large momentum P^z expansion of lattice quasi-PDF $\tilde{f}(y,P^z)$,

$$f(x,\mu)|_{x\to 1} = \int \frac{dy}{|y|} C_{\rm sg}\left(\frac{x}{y}, \frac{\mu}{y\,P^z}\right) \tilde{f}(y,P^z)|_{y\to 1} + \mathcal{O}\left[\frac{\Lambda^2_{\rm QCD}}{x^2 P_z^2}, \frac{\Lambda^2_{\rm QCD}}{(1-x)^2 P_z^2}\right].$$

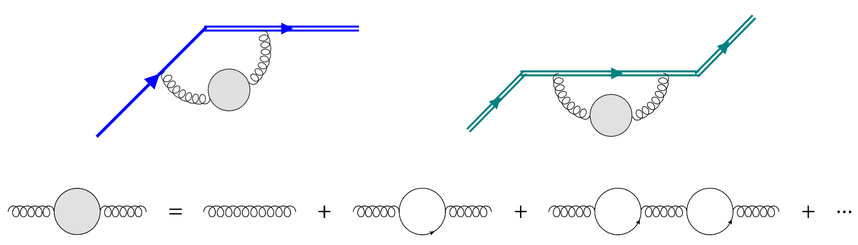
In the threshold limit that $x, y \to 1$, the perturbative matching kernel $C_{\rm sg}$ is factorized into the hard kernel H, related to the active quark momentum xP^z, and the threshold soft function S, associated with the spectator momentum $(1-x)P^z$,

$$C_{\rm sg} = S \otimes H \ .$$

The renormalization group equation of the perturbative soft function S enables the resummation of the threshold double logarithms $\alpha^n \ln^{2n}[2(1-x)P^z/\mu]$, which is crucial for a reliable and controllable calculation of large x PDFs.

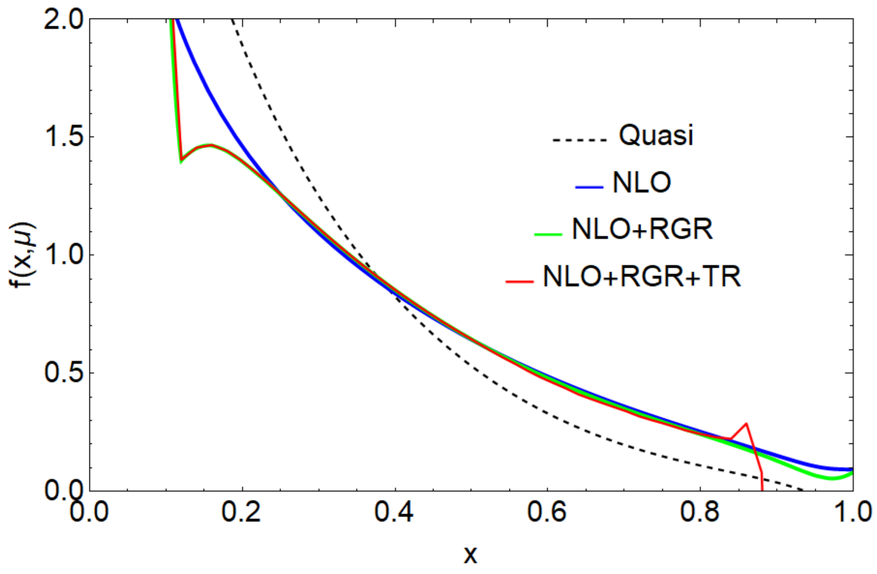


Additionally, numerically large coefficients $\sim n!\,\beta_0^n\,\alpha^{n+1}$, called leading renormalon terms, exist in the hard kernel H and soft function S, causing poor perturbative convergence. The factorial growth originates from the IR momenta k much smaller than P^z in the loop integral $\int d^dk \ln^n(-k^2)$, which is not the physics we seek. Therefore, we regulate and resum the leading renormalon series, which is expected to improve the perturbative convergence.

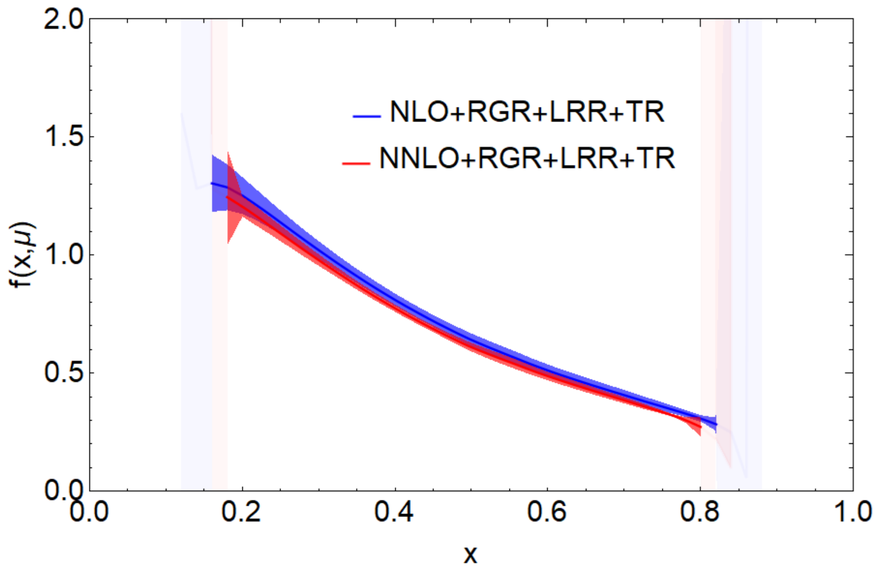


Results

We test the effects of threshold resummation with the pion valence PDF. The (ANL/BNL) lattice quasi PDF for $P^z = 1.9$ GeV is shown as the dashed curve. Various perturbative matching kernels are implemented on the quasi-PDF to obtain the light-cone PDF. NLO denotes a fixed order matching. NLO+RGR includes the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) log resummation. NLO+RGR+TR considers both DGLAP and threshold logs resummation, which indicates that perturbative matching breaks down when the spectator momentum $(1-x)P^z$ approaches $\Lambda_{\rm QCD}$, but remains valid when both xP^z and $(1-x)P^z$ are much larger than $\Lambda_{\rm QCD}$.



Moreover, we incorporate leading renormalon resummation (LRR) within the threshold framework, demonstrating good perturbative convergence in the region where both spectator and active quark momenta are perturbative scales. In the following plot, the bands indicate the uncertainties from the scale variations of RG resummation.



Conclusion

We implement threshold resummation with leading renormalon resummation for the pion valence PDF case under LaMET. The results indicate the breakdown of perturbative matching near the end-point region (e.g. $2|1-x|P^z < 0.8$ GeV) and demonstrate good perturbative convergence in the moderate x range (e.g. $2|x|P^z > 1$ GeV and $2|1-x|P^z > 1$ GeV). As P^z increases in the future, LaMET has the potential to probe larger x region.

Importantly, because the threshold resummation for the matching kernel of the quark PDF is independent of the specific gamma structures or external states, the formalism and methods developed here are broadly applicable. They can be generalized to various quark polarizations in any large-momentum hadron states, expanding the utility of these techniques across a range of hadronic systems in lattice QCD.

References

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