Better Lattice Interpolators for PDF Calculations[1]

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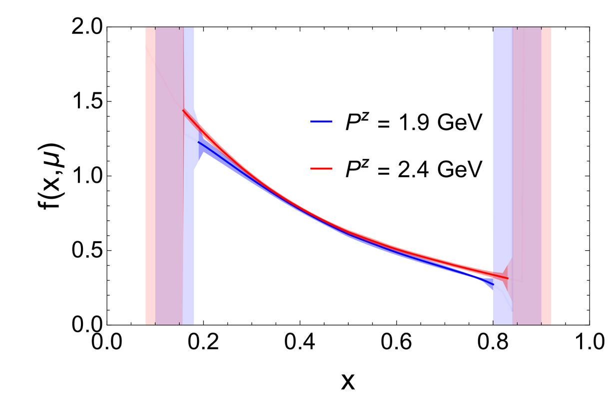
Introduction

Energetic hadrons are commonly required to study the partonic physics with near-lightcone approximations in lattice QCD. Measurements in hadron states with large hadron momentum allow us to suppress the power corrections and extend the calculable x-range in the large momentum expansion approach[2] and also extend the range of light-cone correlation distance at perturbative z in the short-distance factorization. Thus, they are important for precision PDF calculations on lattice. Recent studies have used boosted pions up to 2.4 GeV and kaons and baryons up to 3 GeV. However, the signal-to-noise (StN) ratio decreases rapidly when the hadron state is boosted to large momentum, which limits our calculations and tremendously increases the costs to get reliable physical results. Techniques to improve lattice calculation at large hadron momentum are thus extremely desirable.



Pion PDF[2] from different P_z.

Kinematically Enhanced Interpolators

Traditional hadron interpolators are usually obtained from minimal quark Fock states with the same quantum numbers as hadrons at rest, for example,

$$\chi^\pi = \bar{d}\gamma_5 u, \qquad \chi^N = \epsilon_{abc}(d_a^T C\gamma_5 u_b)u_c. \tag{1}$$

On the lattice, they correspond to specific representations of the cubic group O_h. On the other hand, it has been proposed that on the leading-twist Fock states of hadrons on the lightcone are made of the + component of the quark fields[3]. At finite momentum, these Fock states correspond to higher-spin interpolators with kinematically enhanced overlaps with ground-state hadrons.

For moving hadrons, the cubic group is broken, $O_h \to C_{4v}$, and the higher-spin operators with spin aligned to the momentum direction reduce to the little group representations, which mix with lower-spin states. Thus we can use new interpolators for pion and nucleons, with a kinematically enhanced signal,

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 u|\pi(P)\rangle \propto P_\mu, \qquad \langle 0|\epsilon_{abc}(d_a^T C\gamma_5\gamma_\mu u_b)u_c|N(P)\rangle \propto P_\mu u(P) \tag{2}$$

Using these interpolators, the signal (S) of the two-point correlators will be enhanced by a factor of $\mathcal{O}(100)$ for pions and $\mathcal{O}(10)$ for nucleons at $2 \sim 3$ GeV,

$$S(C_{2pt}(t)) \sim \sum_n P_\mu^2/M_n^2|c_n|^2 e^{-E_n t} \xrightarrow{x\to\infty} P_\mu^2/M_0^2|c_0|^2 e^{-E_0 t}. \tag{3}$$

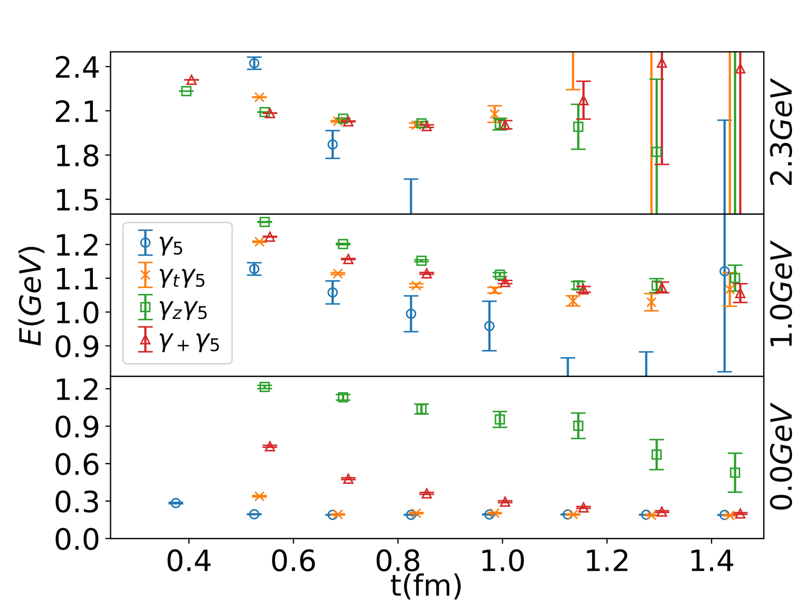
On the other hand, the noise scales as (with $l=1$ for pion, $l=3/2$ for nucleon)

$$N(C_{2pt}(t)) \sim |c_0'|^2 e^{-lM_\pi t} + \sum_n P_\mu^2/M_n^2|c_n'|^2 e^{-E_n t} \xrightarrow{x\to\infty} |c_0'|^2 e^{-lM_\pi t}, \tag{4}$$

where the first term from multi-pion state dominates at large Euclidean time. Asymptotically, the StN are kinematically enhanced for the new interpolators,

$$StN(C_{2pt}(t\to\infty)) \propto P_\mu^2/M^2 e^{-(E_0 - lM_\pi)t}. \tag{5}$$

Pion Results

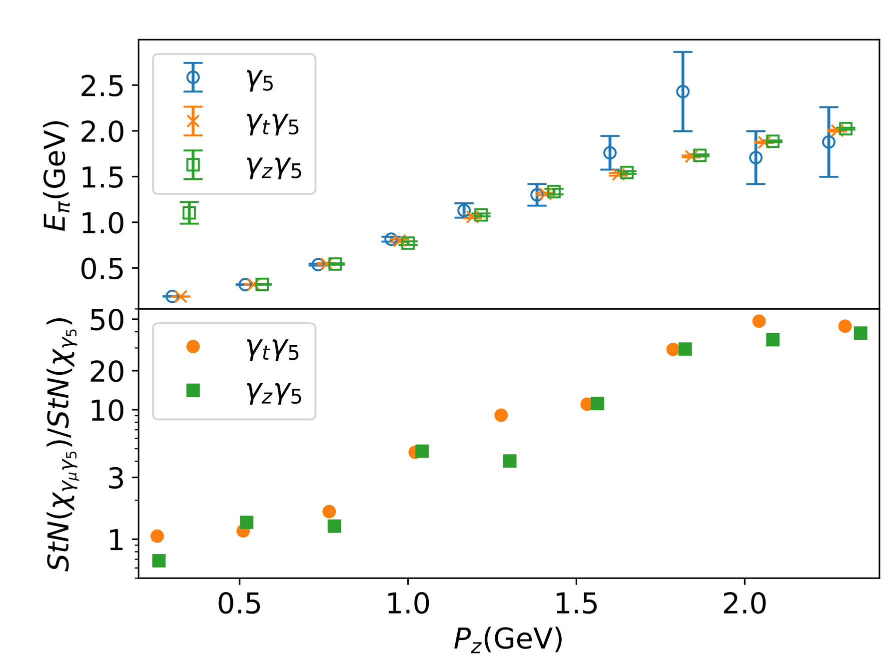


Effective mass for pion.

Spacing a	Volume	m_π^{sea}(MeV)	m_π^{val}(MeV)	P^π(GeV)	P^N(GeV)	n_{stat}^π	n_{stat}^N	$n_{\text{stat}}^{\text{3pt}}$
0.15 fm	$32^3\times 48$	135	{190, 400}	[0, 2.3]	[0, 3.1]	21,376	6,688	109

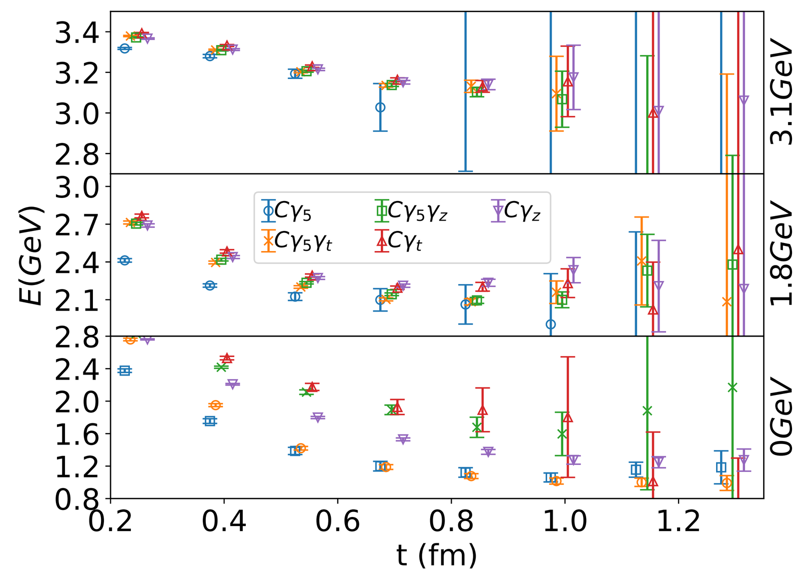
We test the new interpolators on a gauge ensemble produced by the MILC collaboration, with clover-on-HISQ action, 2 steps of HYP smearing, and momentum smearing of $k = 1.55$ GeV. The measurements are performed with the QUDA software and GLU.

There is a significant improvement for the pion at large momentum. New excited state contamination from the axial vector meson is introduced at small momentum but is kinematically suppressed at large momentum. We use the Lanczos method[4,5] to analyze the two-point correlators and extract the ground-state energy. The enhancement with new interpolators clearly increases with the momentum and can reach a factor of 40 for $P > 2$ GeV, corresponding to 1000-2000 times improvement in statistics.



Lanczos result and StN improvement.

Nucleon Results

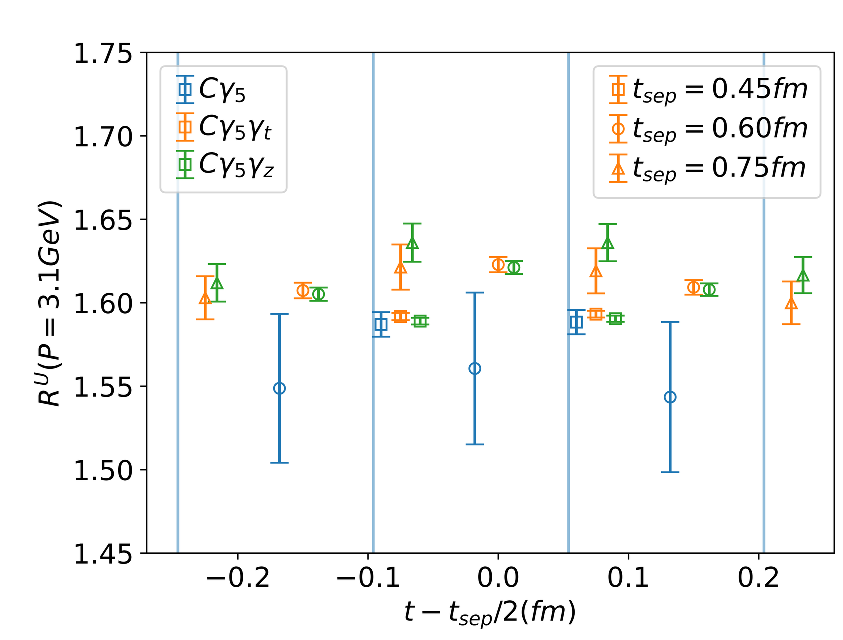


Effective mass for nucleon.

We include two sets of interpolators that overlap with the ground-state nucleon at large momentum,

$$\chi_{\gamma_5\gamma_\mu} = \epsilon_{abc}(d_a^T C\gamma_5\gamma_\mu u_b)u_c, \qquad \chi_{\gamma_\mu} = \epsilon_{abc}(d_a^T C\gamma_\mu u_b)u_c. \tag{6}$$

All these interpolators suggest a similar enhancement at large momentum, but they have different excited state contamination. An excited state contamination from the spin-$\frac{3}{2}$ baryons is introduced at small momentum but is kinematically suppressed at large momentum. The enhancement is up to $\mathcal{O}(10)$ in the nucleon interpolator at $P_z > 3$ GeV. The same enhancement applies to the three-point correlators for the unpolarized quark PDF matrix element. The consistency and flatness for $t \geq 0.6$ fm indicates suppressed excited state contamination. The enhancement is larger at large t_{sep} where the ground state dominates.



$R^U = C_{\text{3pt}}^U / C_{\text{2pt}}$ for up quark PDF.

Conclusion

We propose new interpolators that enhance the StN by a factor of 40 for pions boosted to $P > 2$ GeV and $\mathcal{O}(10)$ for nucleons boosted to $P > 3$ GeV. The enhancement applies to both two- and three-point correlators, and the excited states are suppressed at large momentum. They will significantly improve the precision for lattice calculation of partonic observables, form factors at large Q^2, heavy meson decays with energetic pion in the final state, and $\pi - \pi$ scattering with large invariant mass.

References

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