Non-perturbative Collins-Soper kernel from a Coulomb-gauge-fixed quasi-TMD

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Parton Distributions and Lattice Calculations (PDFLattice 2024) @ Jlab
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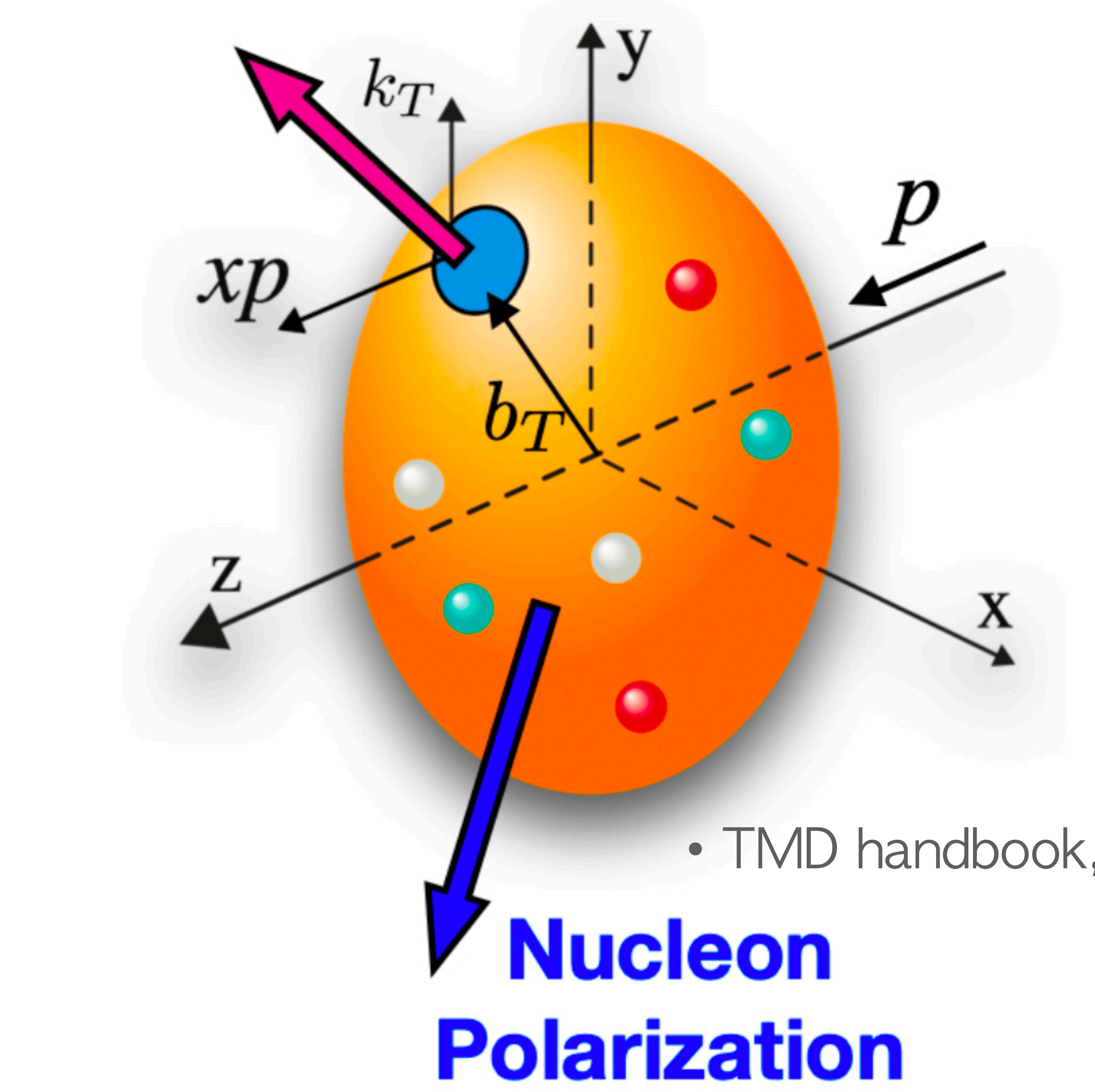
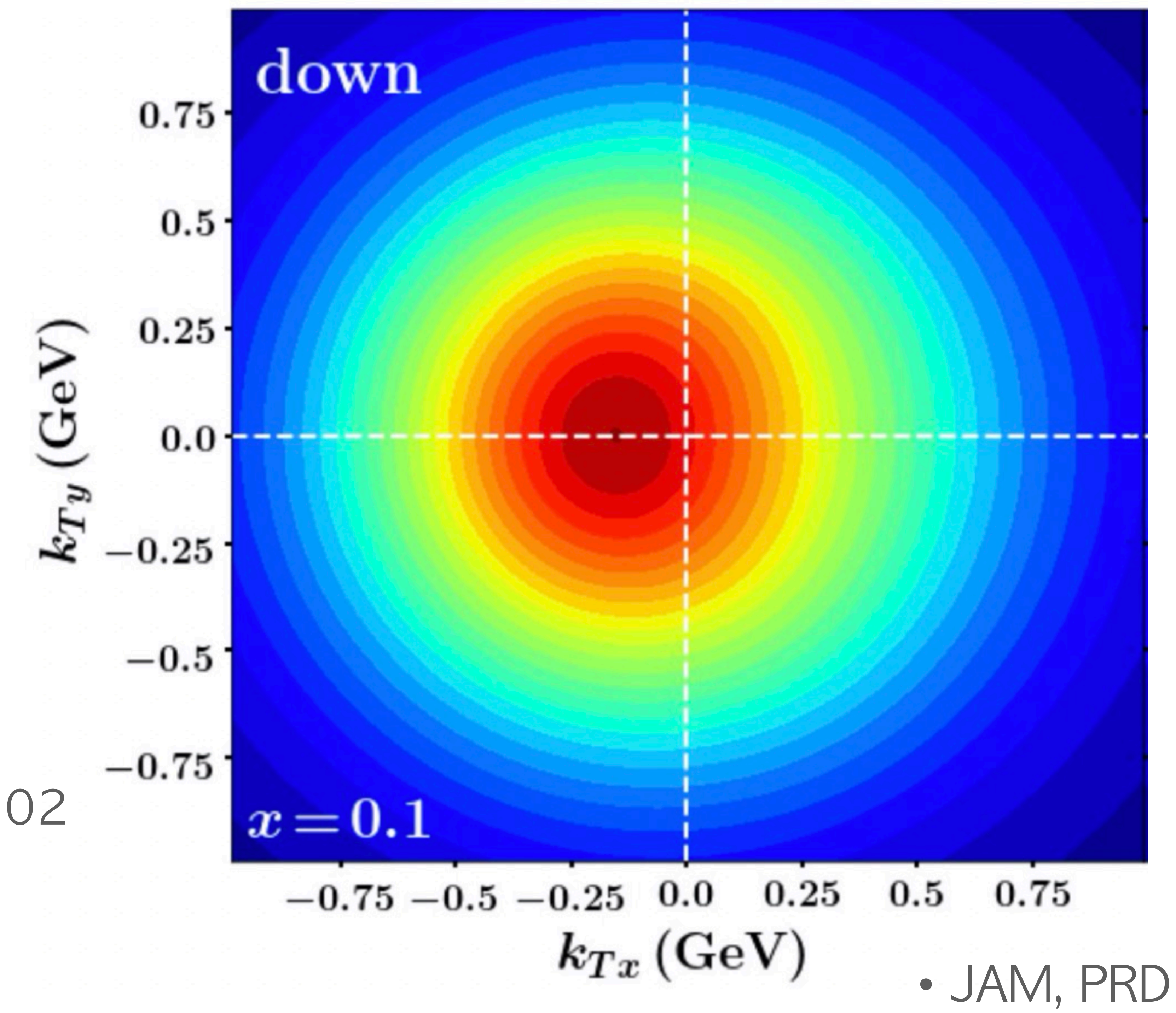
2 Transverse-momentum-dependent distributions

$f(x, \vec{k}_T)$



• TMD handbook, arXiv:2304.03302

Quark Sivers function



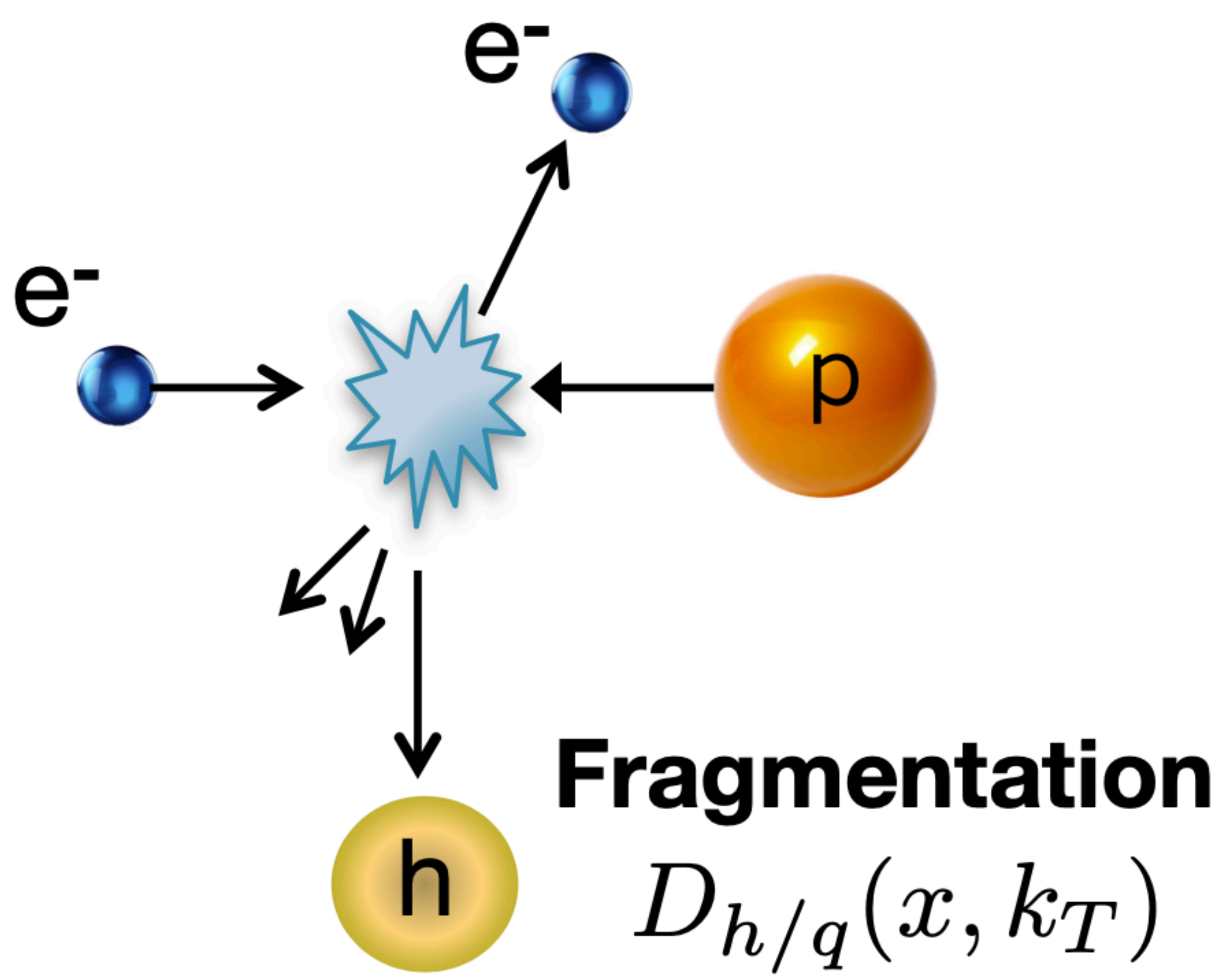
• JAM, PRD 102 (2020) 5, 054002

- 3D image: longitudinal momentum fraction x and confined motion $\vec{k}_T$.
- Nucleon spin structure: Spin-orbit correlations.

❸ TMDs from global analyses of experimental data

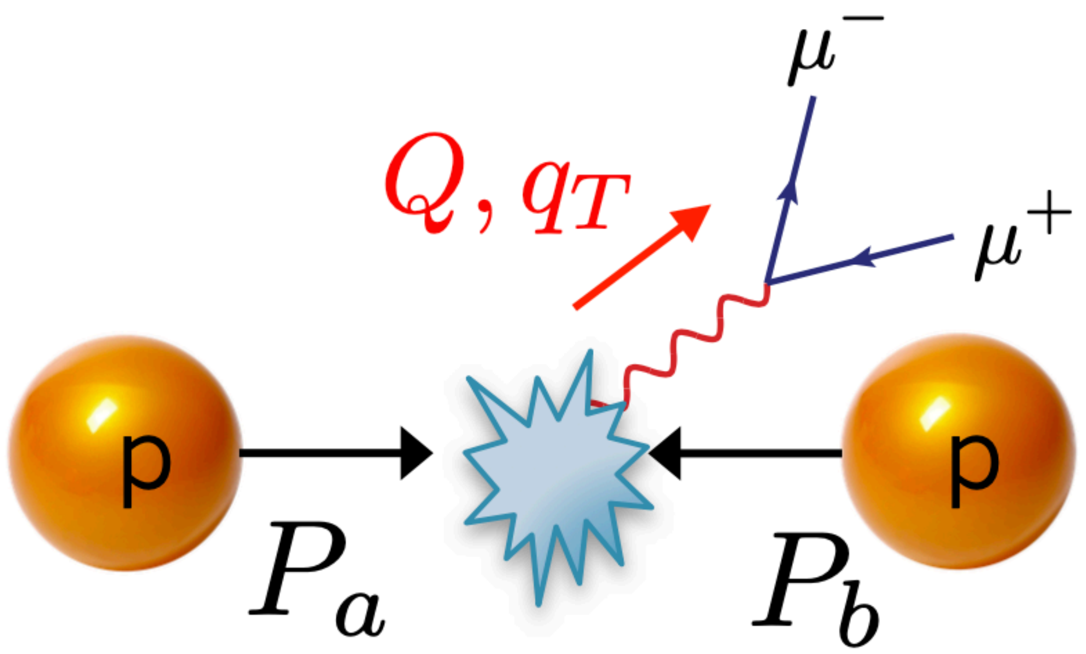
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



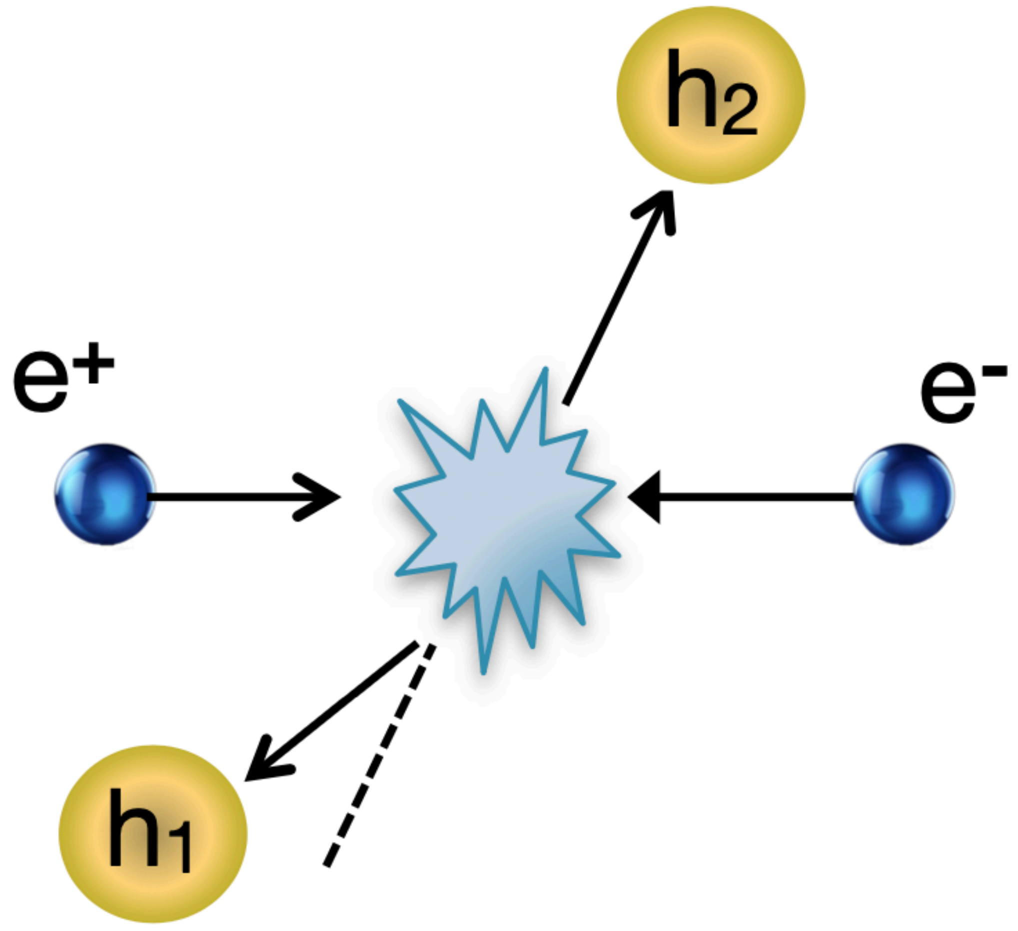
Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



Dihadron in e+e-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$



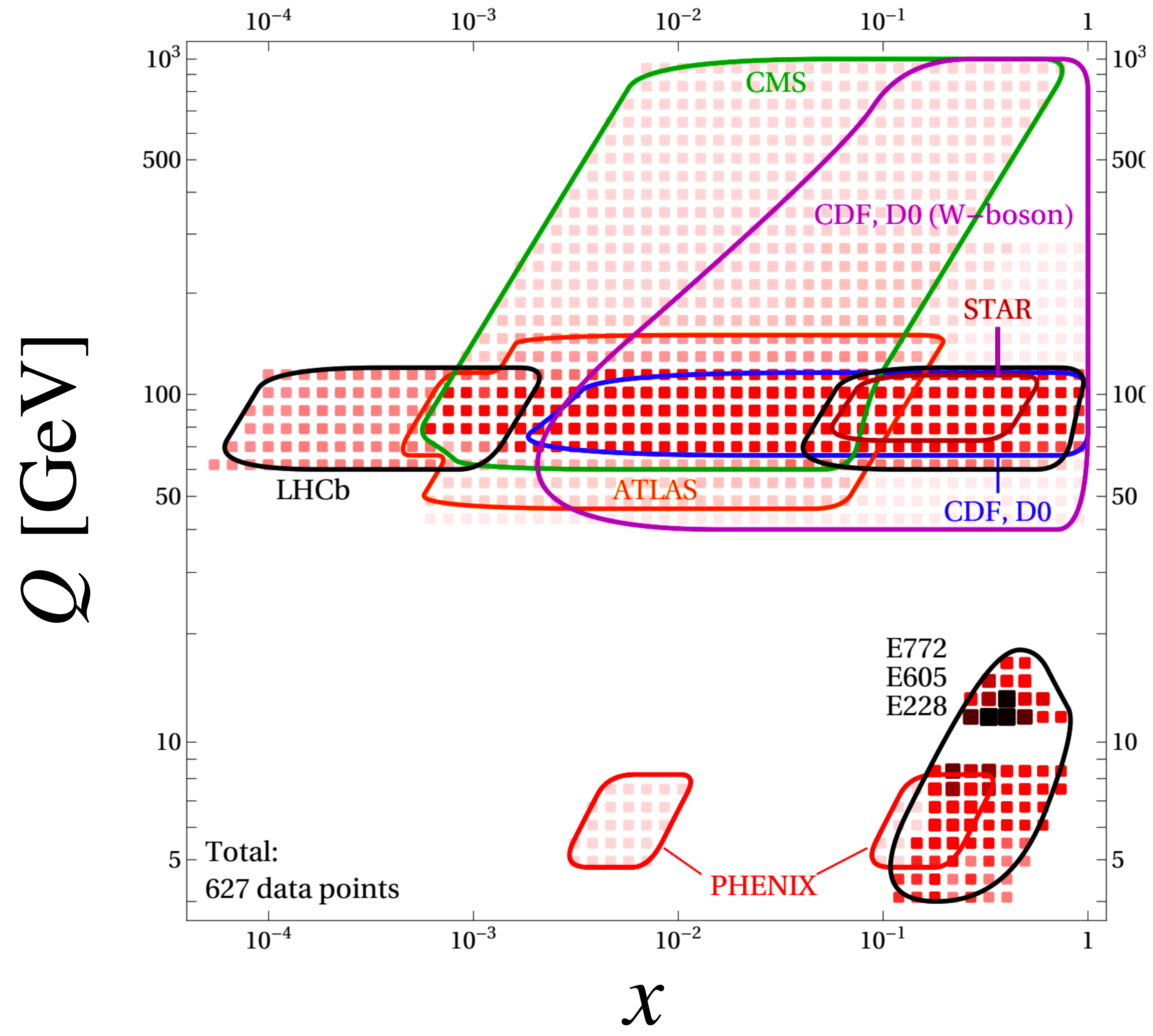
$$\frac{d\sigma_{DY}}{dQ\,dY\,dq_T^2} = H(Q,\mu) \int d^2\vec{b}_T e^{i\vec{q}_T\cdot\vec{b}_T} f_q(x_a, \vec{b}_T, \mu, \zeta_a)\, f_q(x_b, \vec{b}_T, \mu, \zeta_b)[1 + \mathcal{O}(\frac{q_T^2}{Q^2})]$$

Perturbative hard kernels

Nonperturbative TMDs

$$q_T^2 \ll Q^2$$

TMDs from global analyses of experimental data

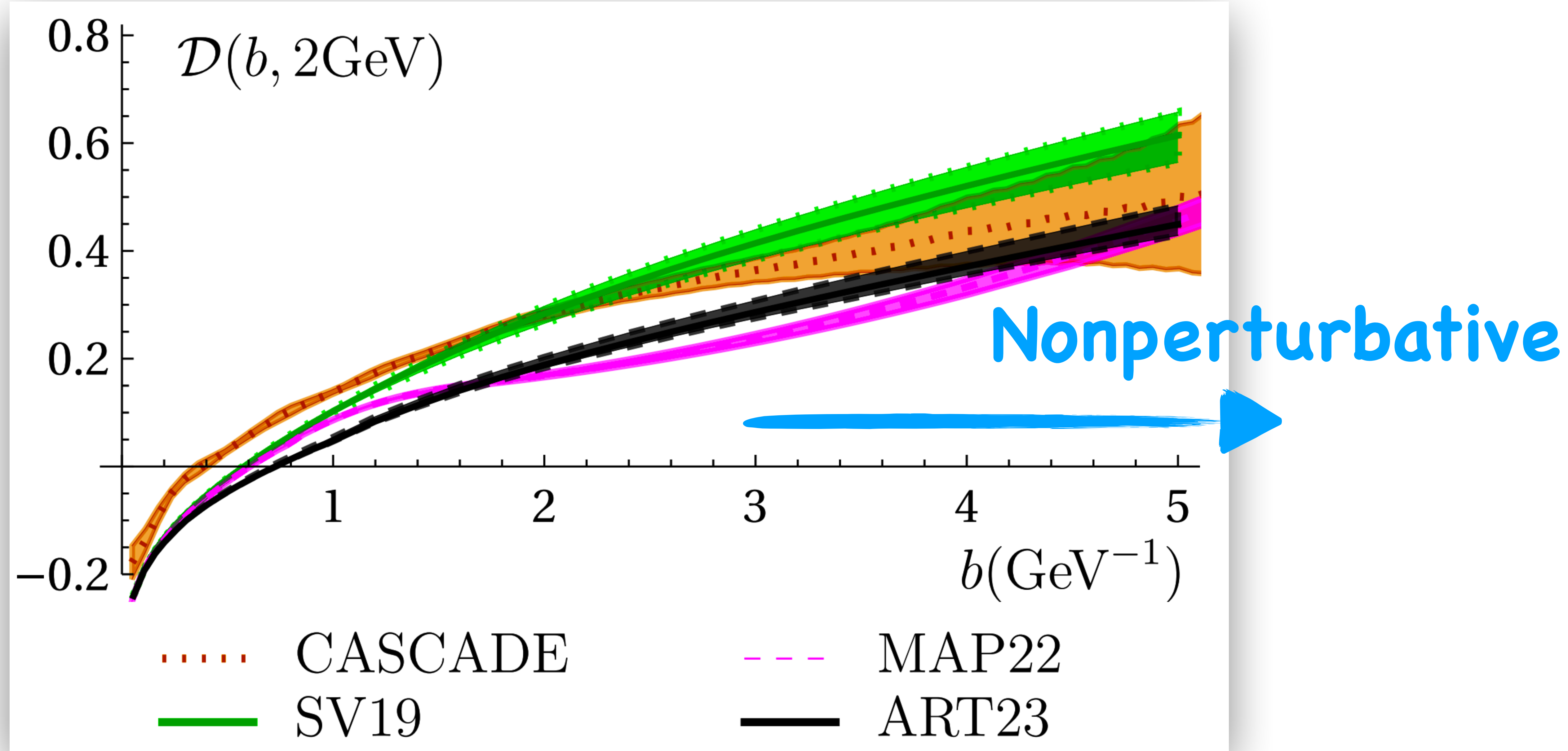


• V. Moos, et. al. (ART23), JHEP 05 (2024) 036

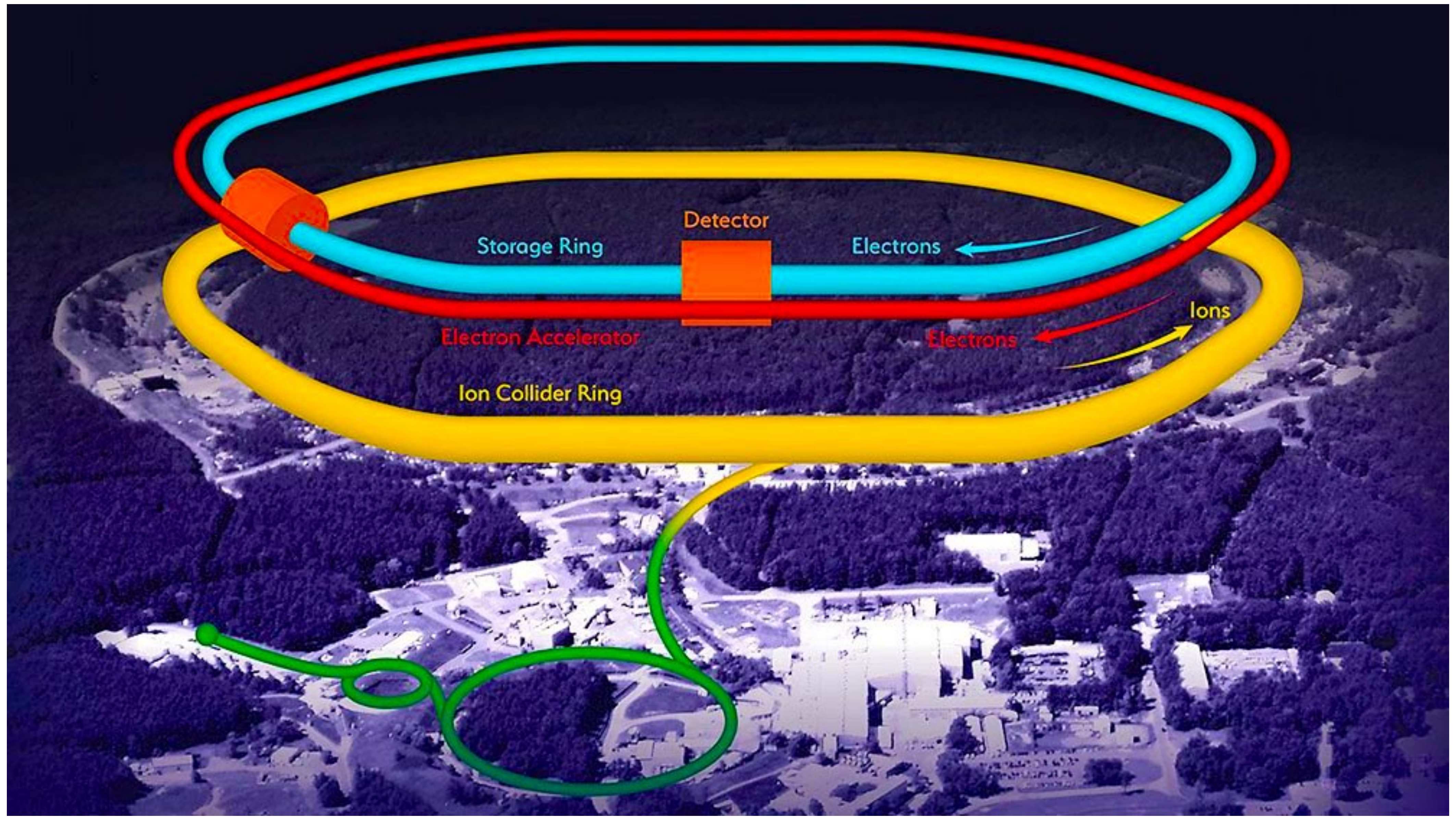
- **Relate TMDs at different energy scales**

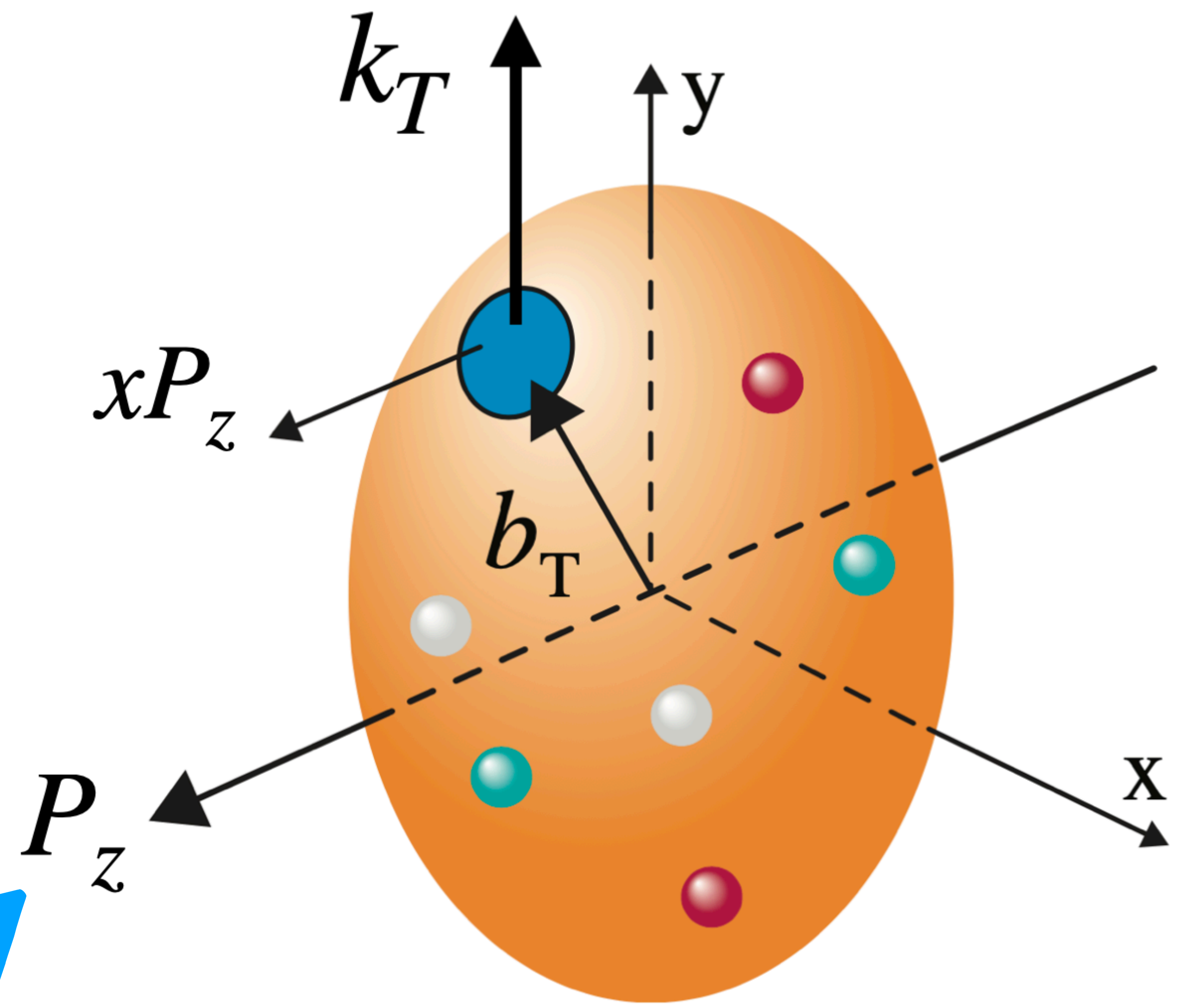
$$\left.\begin{aligned}\mu\frac{d}{d\mu}\ln f_q(x,\vec{b}_T,\mu,\zeta) &= \gamma^q_\mu(\mu,\zeta)\\ \zeta\frac{d}{d\zeta}\ln f_q(x,\vec{b}_T,\mu,\zeta) &= \gamma^q_\zeta(\mu,b_T)\end{aligned}\right\}$$

Collins-Soper kernel

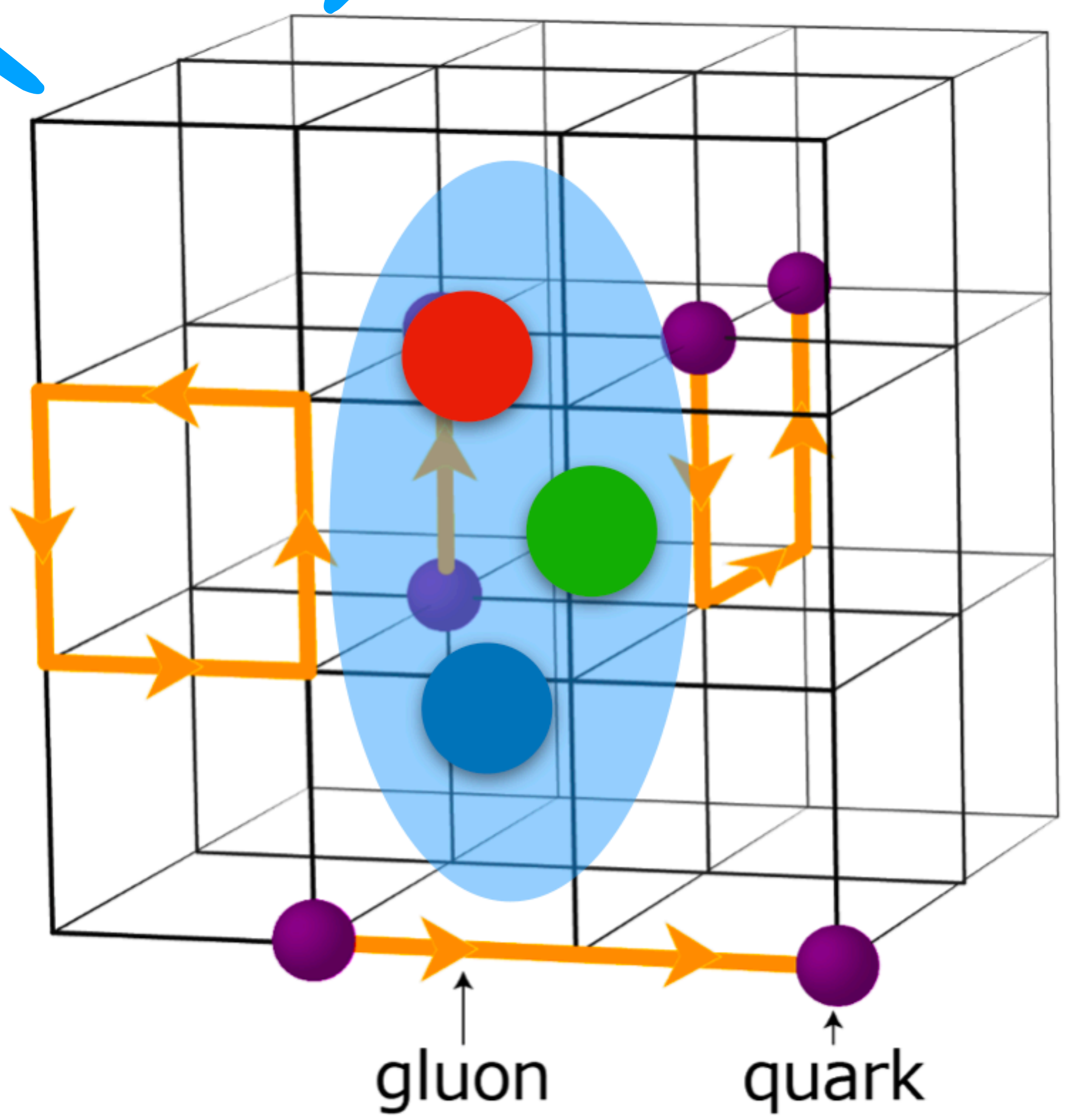


Determination of TMDs





- Global analysis of experimental data.
- Complementary knowledge from lattice QCD is essential.

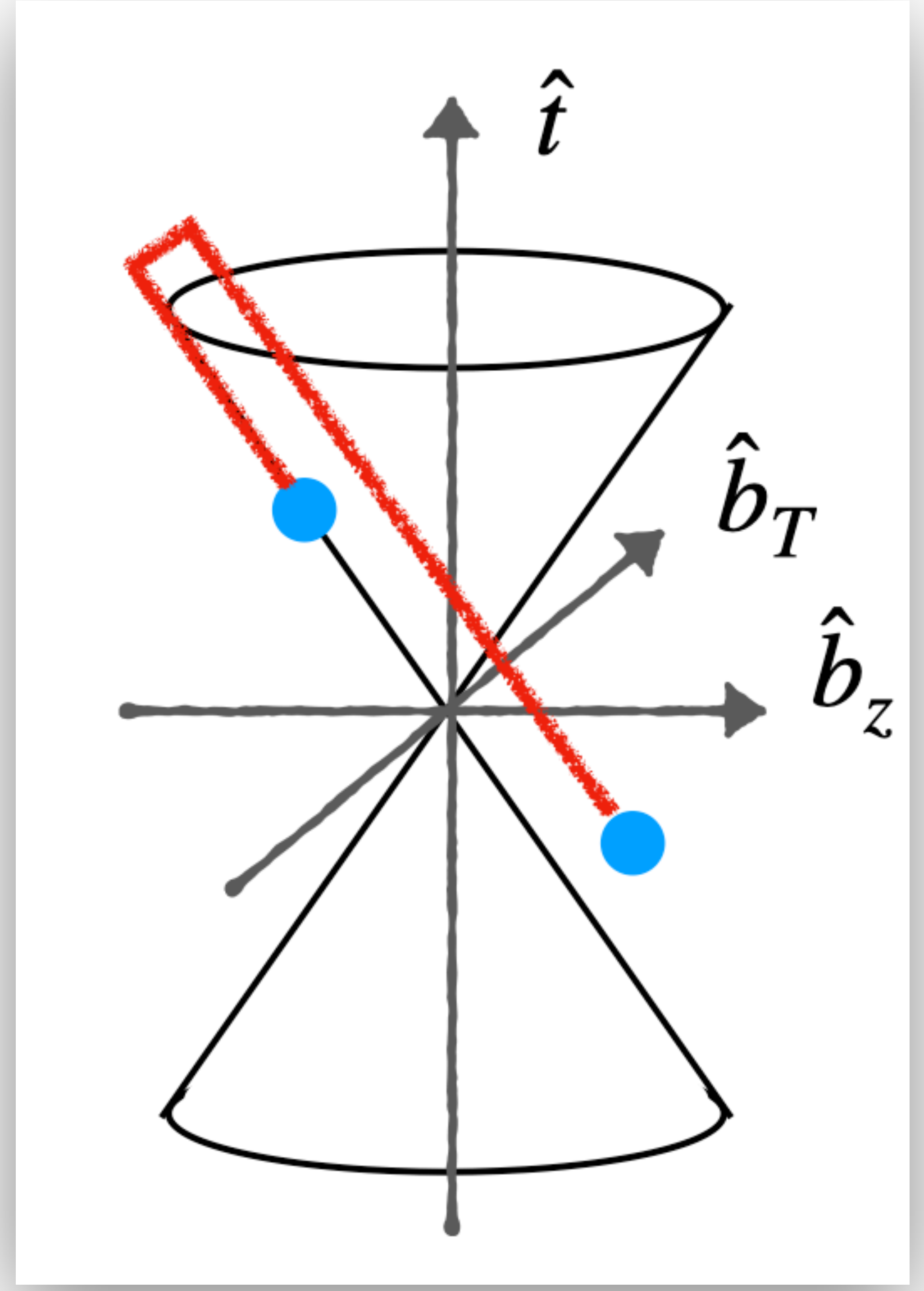


The definition of TMDs

Beam function

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{UV}}(\epsilon, \mu, \zeta) \lim_{\tau \to 0} \frac{B_q(x, \vec{b}_T, \epsilon, \tau, \zeta)}{\sqrt{S_q(\vec{b}_T, \epsilon, \tau)}}$$

UV regulator   Rapidity regulator   Soft function



$$\langle P | \overline{\psi}(\frac{b^+}{2}, b_\perp) \Gamma W_{\sqsupset^+} \psi(-\frac{b^+}{2}, 0) | P \rangle$$

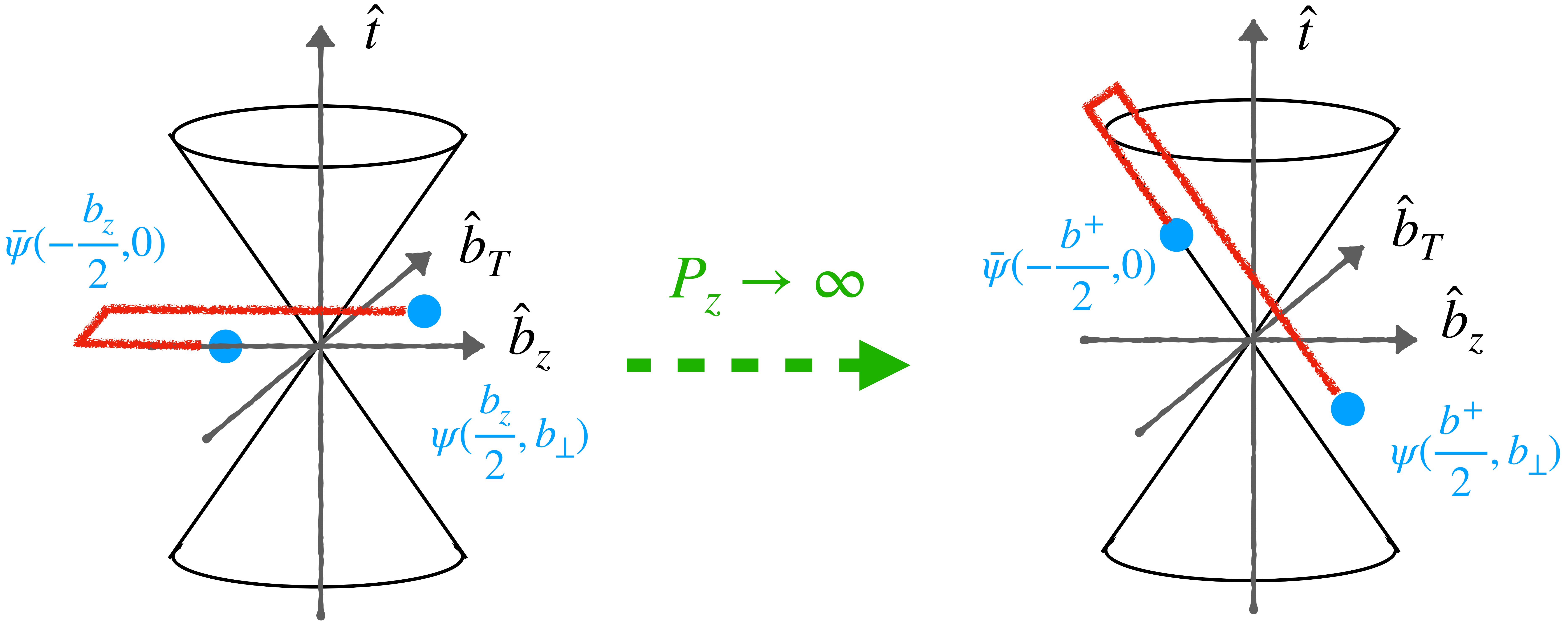
✗ Light-cone correlations: forbidden on Euclidean lattice

7 TMDs from lattice: quasi TMDs

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- A. Vladimirov, A. Schäfer Phys.Rev.D 101 (2020), 074517
- I. Stewart, Y. Zhao et al., JHEP 09 (2020) 099
- X. Ji et al., Phys.Rev.D 103 (2021) 7, 074005
- I. Stewart, Y. Zhao et al., JHEP 08 (2022) 084

Quasi-TMDs from equal-time correlators:

- Computable from Lattice QCD.



Quasi TMD

$$\langle P|\overline{\psi}(\frac{b_z}{2},b_\perp)\Gamma W_{\sqsupset^z}\psi(-\frac{b_z}{2},0)|P\rangle$$

Light-cone TMD

$$\langle P|\overline{\psi}(\frac{b^+}{2},b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2},0)|P\rangle$$

Large P_z expansion and perturbative matching

Quasi TMDs **Collins-Soper kernel**

$$\frac{\tilde{\phi}(x,\vec{b}_T,\mu,P_z)}{\sqrt{S_r(\vec{b}_T,\mu)}} = C(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda^2_{\rm QCD}}{(xP_z)^2}]\}$$

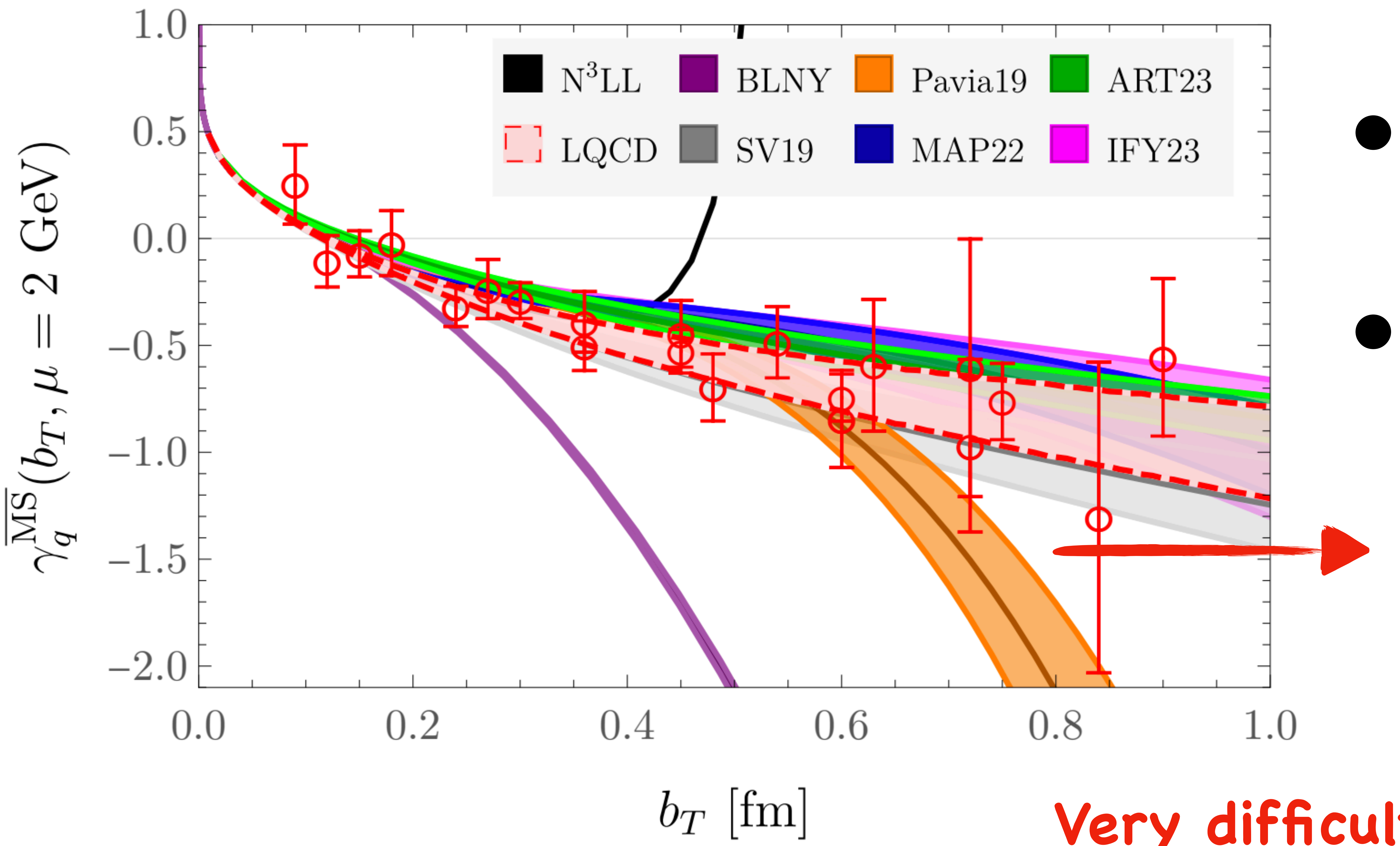
Physical TMD

$P_z < a^{-1}$

- Have same IR physics as light-cone TMDs: large momentum expansion.
- Quasi TMDs differ from the light-cone TMDs (Collins scheme) by order of P_z (or rapidity $\rm y_B$) $\to \infty$ and a (or ϵ) $\to 0$ limit, inducing a **perturbative matching** $C(\mu, xP_z)$.

9 The Collins-Soper kernel from quasi-TMDs

Collins-Soper kernel

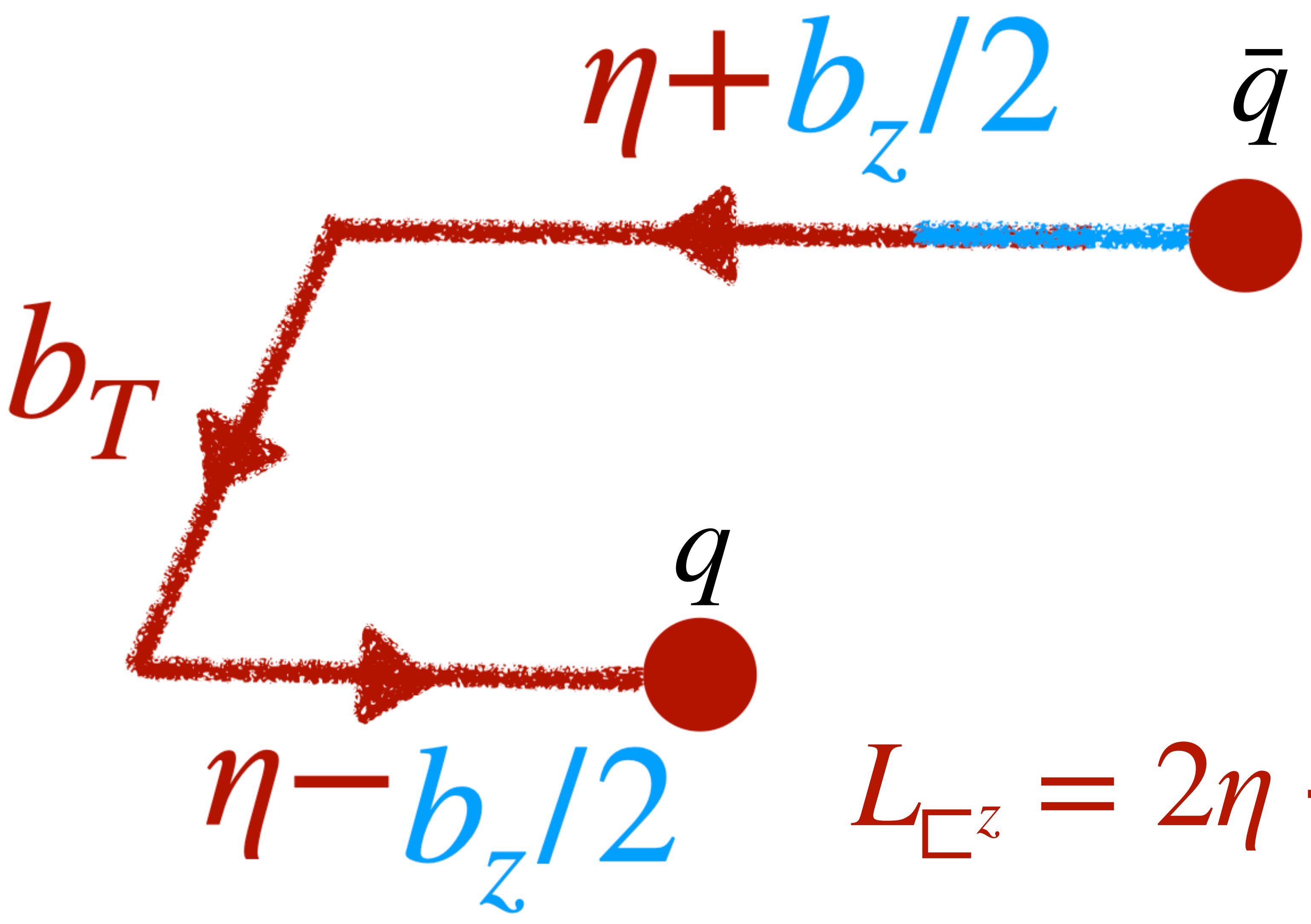


- Significant progress has been reported recently.
- Can lattice QCD push further with good precision?

Very difficult: errors grow rapidly!

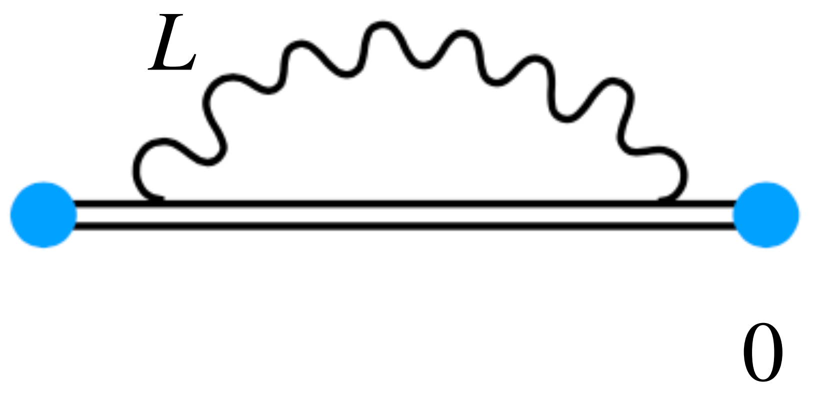
• A. Avkhadiev, P. Shanahan, M. Wagman, Y. Zhao, PRL 132 (2024) 23, 231901

10 Difficulties in the conventional quasi-TMDs



$L_{\sqsubset z} = 2\eta + b_T$

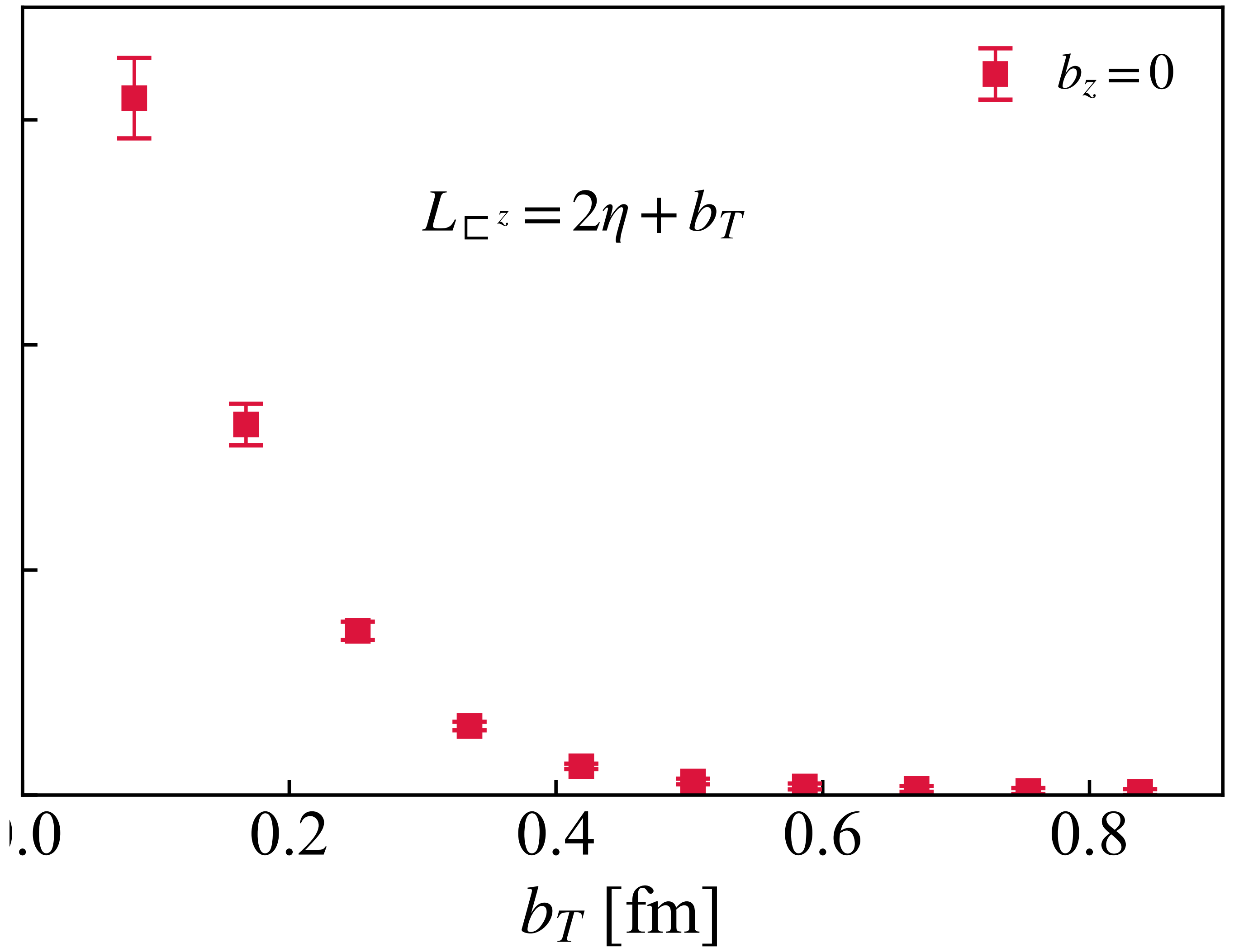
- Exponential decaying signal and complicated renormalization due to the Wilson line artifacts.



$\sim e^{-\delta m(a) \cdot L_{\sqsubset z}}$

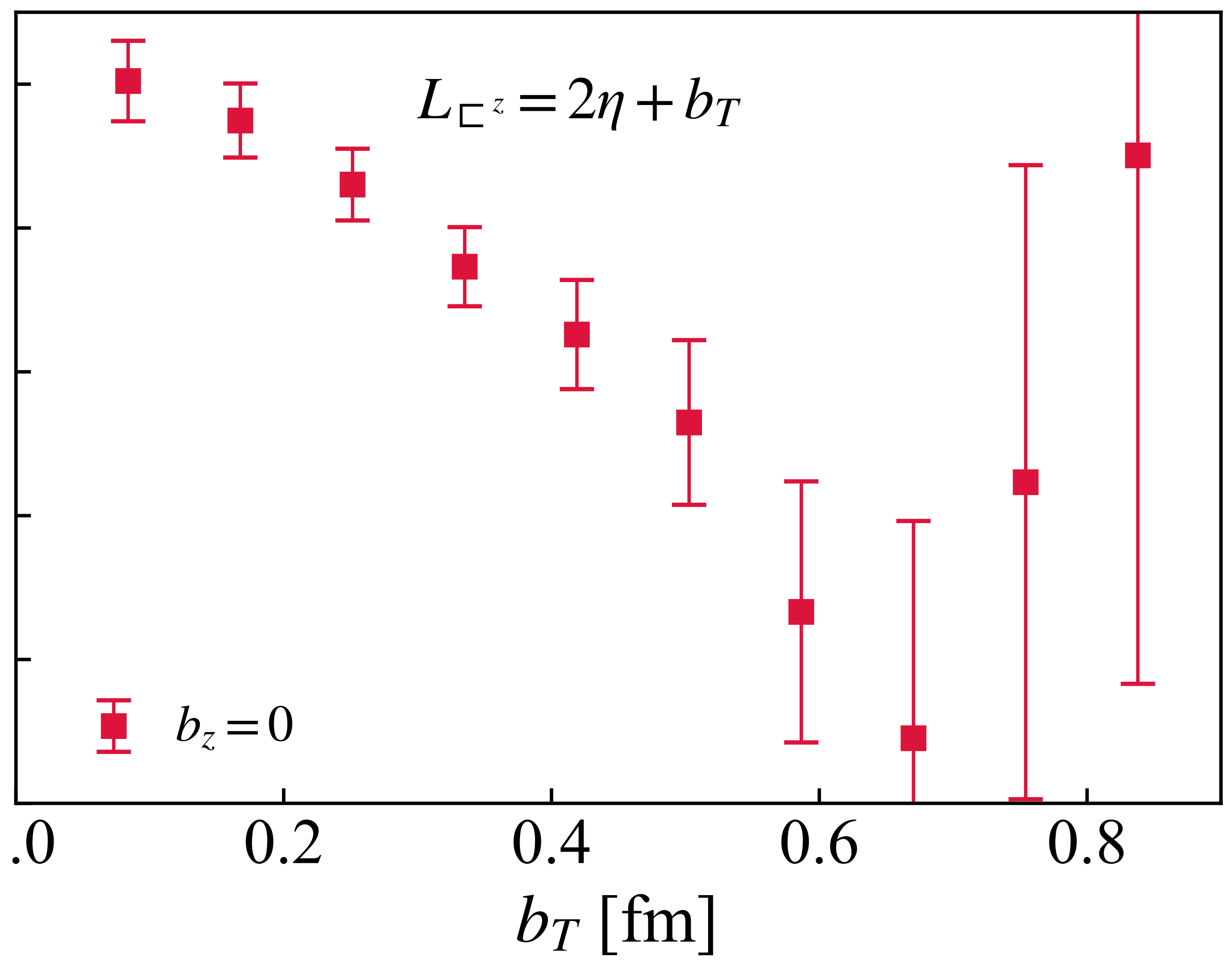
Linear divergence from Wilson line self energy

Bare matrix elements



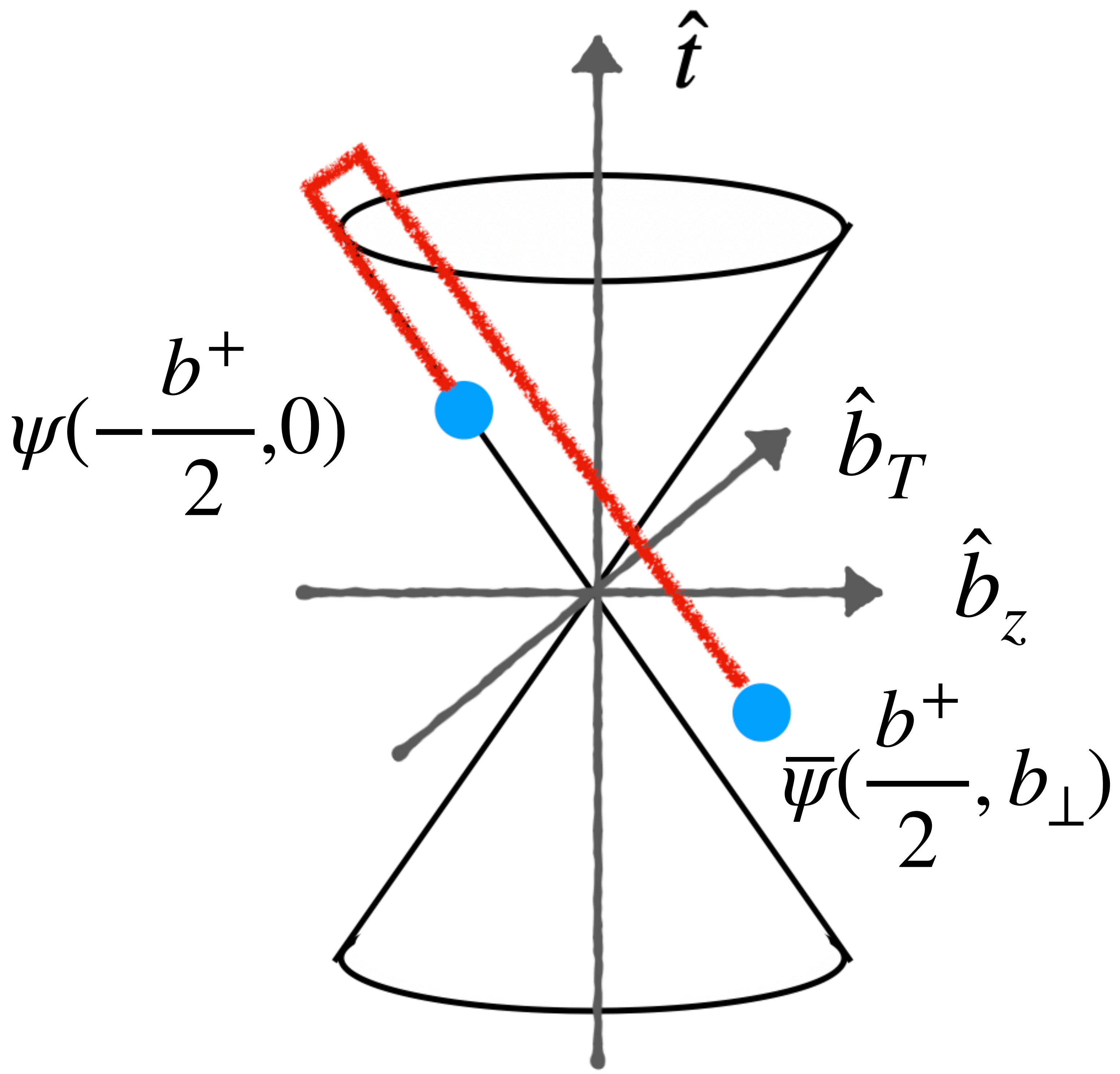
$\times\, e^{\delta m(a) \cdot L_{\sqsubset z}}$

Renormalized matrix elements



Overcoming difficulties

Light-cone TMD



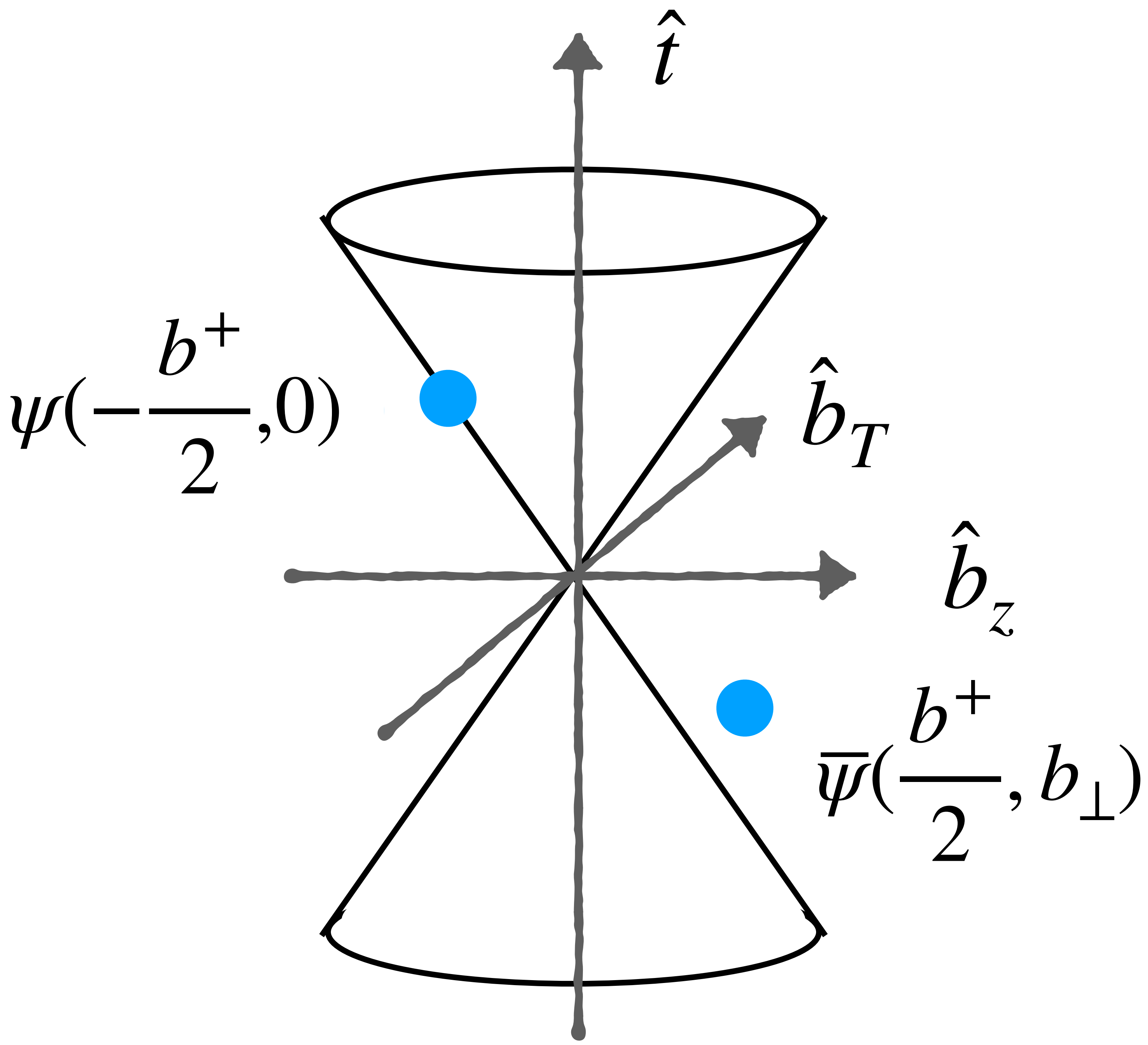
$$\overline{\psi}(\frac{b^+}{2},b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2},0)$$

Equivalent

=

TMD in light gauge

$A^+ = 0$



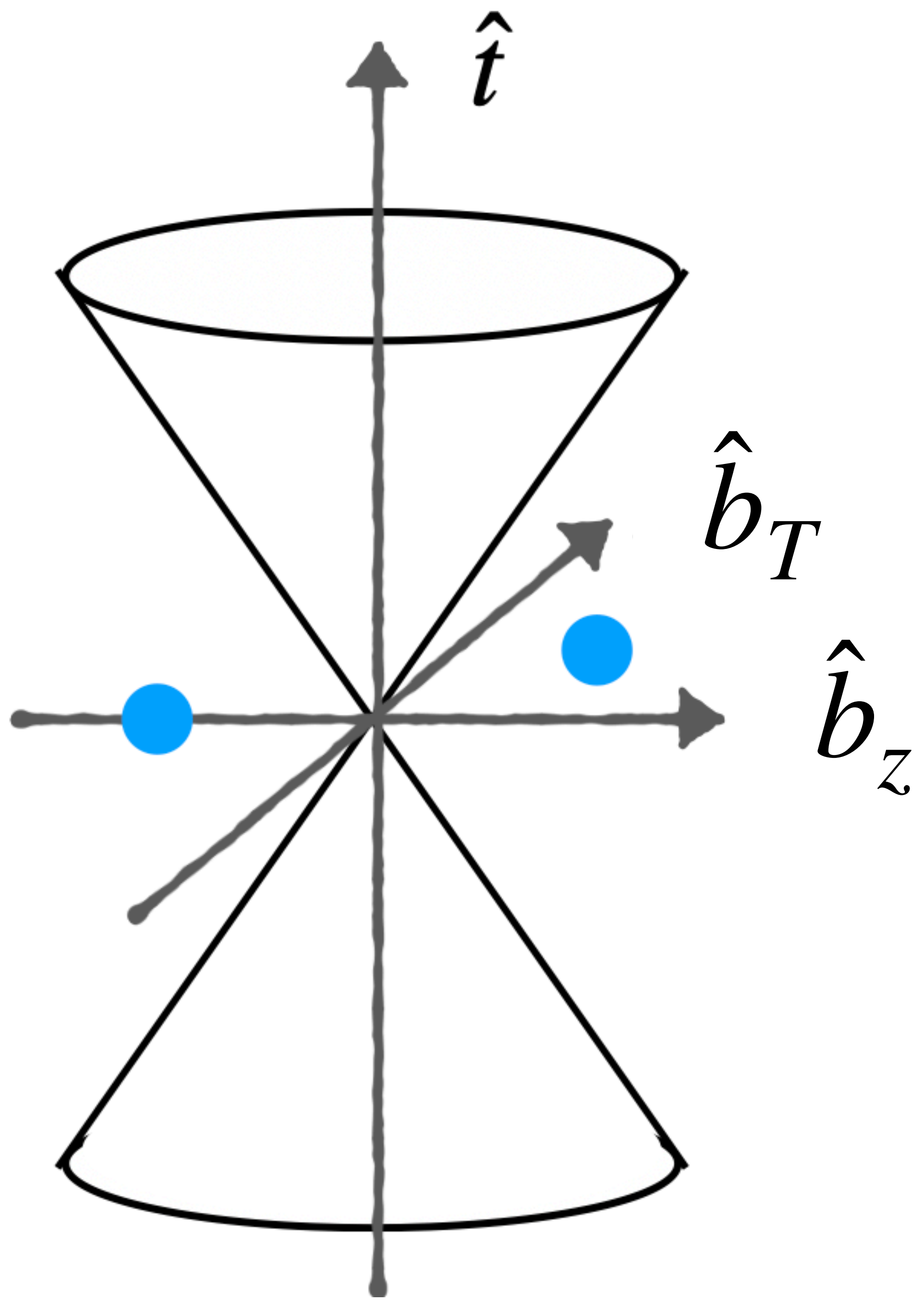
$$\overline{\psi}(\frac{b^+}{2},b_\perp)\Gamma\psi(-\frac{b^+}{2},0)\,|_{A^+=0}$$

Overcoming difficulties: Coulomb-gauge qTMDs

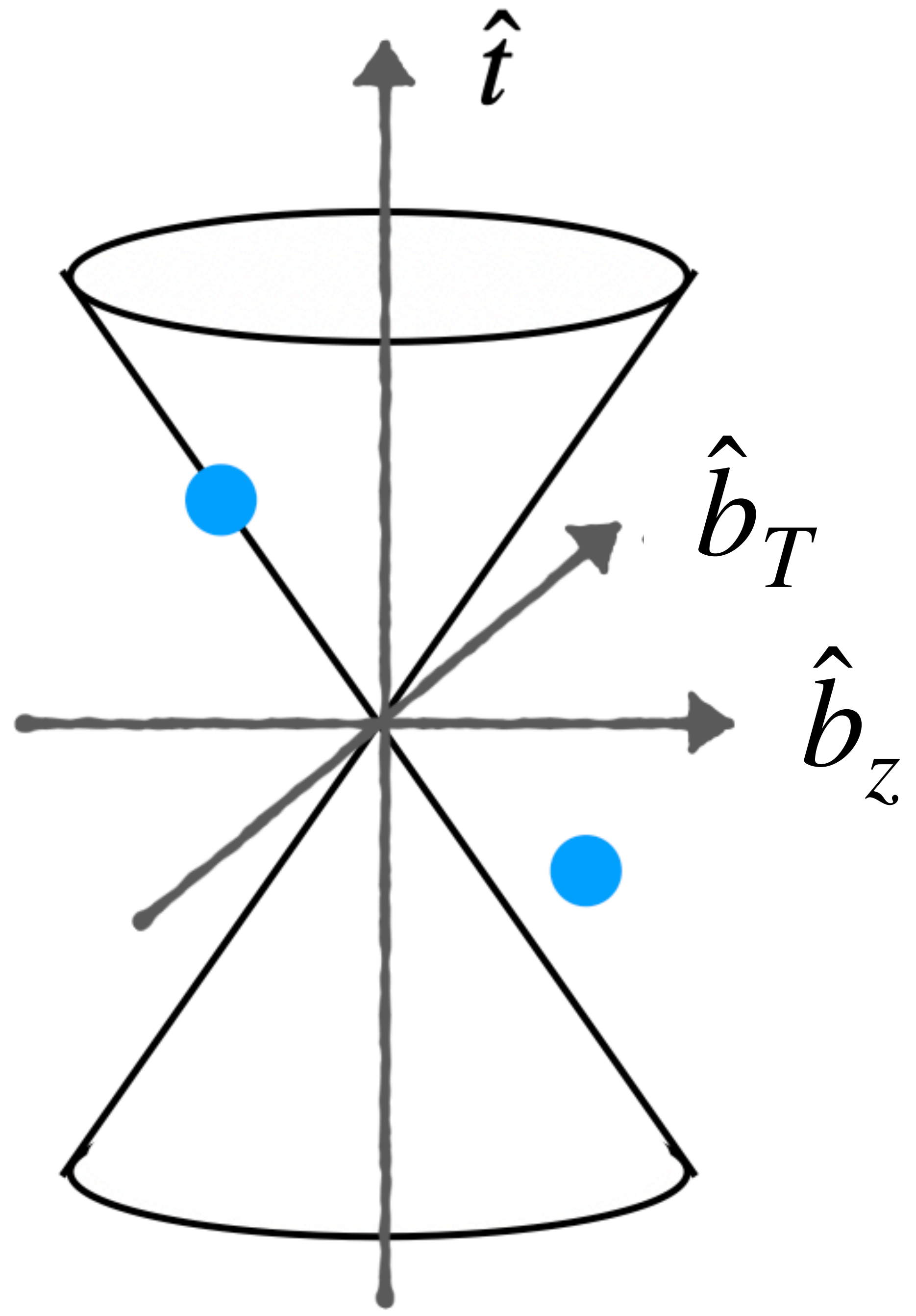
Quasi-TMD in physical gauge

$P_z \to \infty$

TMD in light gauge $A^+ = 0$



$P_z \to \infty$



$$\overline{\psi}(\frac{b^z}{2}, b_\perp)\Gamma\psi(-\frac{b^z}{2}, 0)\,|_{\vec{\nabla}\cdot\vec{A}=0}$$

$$\overline{\psi}(\frac{b^+}{2}, b_\perp)\Gamma\psi(-\frac{b^+}{2}, 0)\,|_{A^+=0}$$

- **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506
- Y. Zhao, arXiv: 2311.01391, accepted by PRL.

CG quasi distribution without Wilson lines

- **The quark field in the Coulomb gauge**

$$\psi_C(z) = U_C(z)\psi(z)$$

satisfying,

$$\vec{\nabla}\cdot\left[U_C\vec{A}U_C^{-1} + \frac{i}{g}U_C\vec{\nabla}U_C^{-1}\right] = 0$$

order by order in g, the solution:

$$U_C = \sum_{n=0}^{\infty}\frac{(ig)^n}{n!}\omega_n$$

$$\omega_1 = -\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A},$$

$$\omega_2 = \frac{1}{\nabla^2}\left(\vec{\nabla}\cdot(\omega_1^\dagger\vec{\nabla}\omega_1) - [\vec{\nabla}\omega_1, \vec{A}]\right)$$

$$\ldots$$

▸ $P \to \infty$ **limit boost**

$$-\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A} = i\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot z}\frac{1}{k_z^2+k_\perp^2}[k_zA_z(k) + k_\perp A_\perp(k)]$$

$$\approx i\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot z}\frac{k^+}{(k^+)^2+\epsilon^2}A^+(k)$$

$$= \frac{1}{2}\left[\int_{-\infty^-}^{z^-} + \int_{+\infty^-}^{z^-}\right]d\eta^- A^+ \equiv \frac{1}{\partial^+_{\mathrm{P.V.}}}A^+(z)$$

Principle value prescription (P.V.) averaging over past and future.

Path-ordered integral

$$\frac{\omega_n}{n!} \to \left(\ldots\left(\frac{1}{\partial^+_{\mathrm{P.V.}}}\left(\left(\frac{1}{\partial^+_{\mathrm{P.V.}}}A^+\right)A^+\right)A^+\right)\ldots A^+\right)$$

$$U_C \to \mathcal{P}\exp\left[-ig\int_{z^-}^{\mp\infty^-}dzA^+(z)\right] \equiv W(z^-, \mp\infty^-)$$

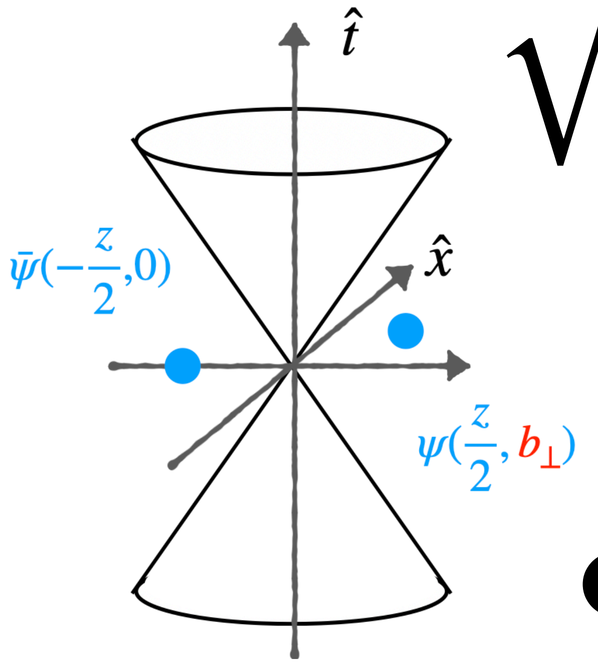
Infinite light-cone Wilson link

CG quasi-TMDs without Wilson lines

Quasi TMDs in CG

Physical TMD

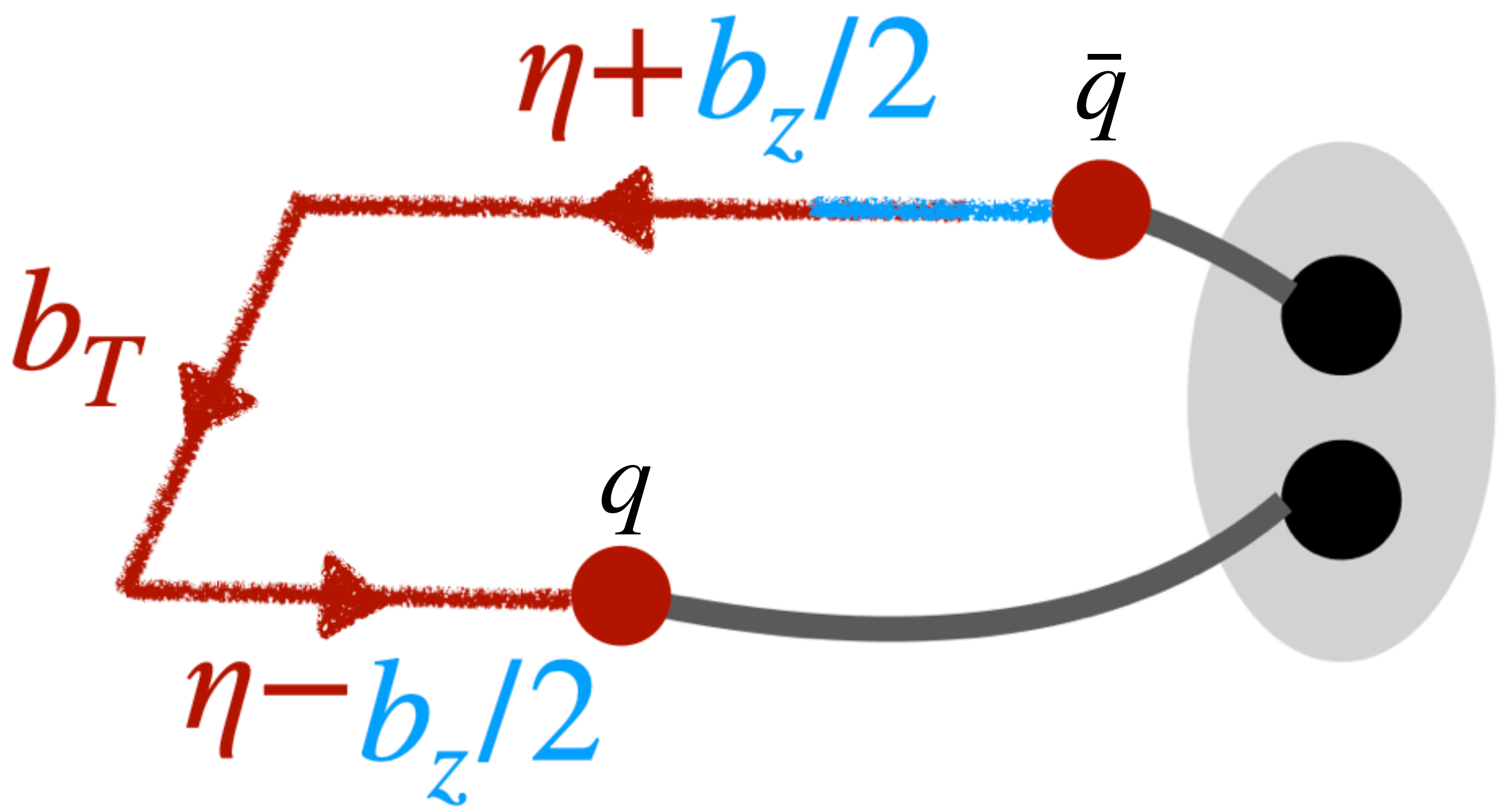
$$\frac{\tilde{f}_C(x,\vec{b}_T,\mu,P_z)}{\sqrt{S_C(\vec{b}_T,\mu)}} = C(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda^2_{\rm QCD}}{(xP_z)^2}]\}$$

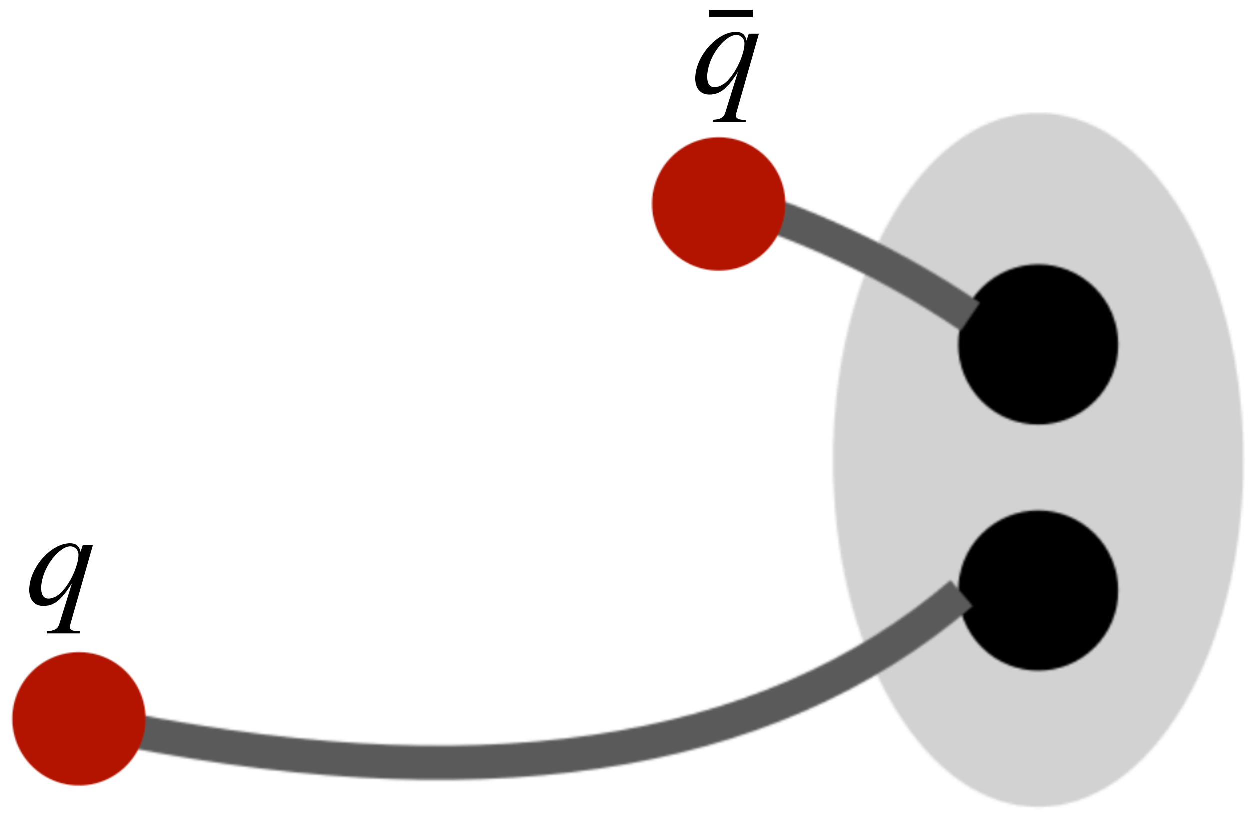


- Y. Zhao, arXiv: 2311.01391, accepted by PRL.
- Y.-Z. Liu, Y.-S. Su., JHEP 02 (2024) 204

- The **same form of factorization formula** as the conventional gauge invariant (GI) method (verified through SCET), with different perturbative and power corrections.
- **Can the CG quasi-TMDs avoid the complexity from Wilson line?**

Quasi-TMDs in the Coulomb gauge





$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp)\Gamma W_{\sqsupset^z}\psi(-\frac{b_z}{2},0) | \pi^+, P_z\rangle$$

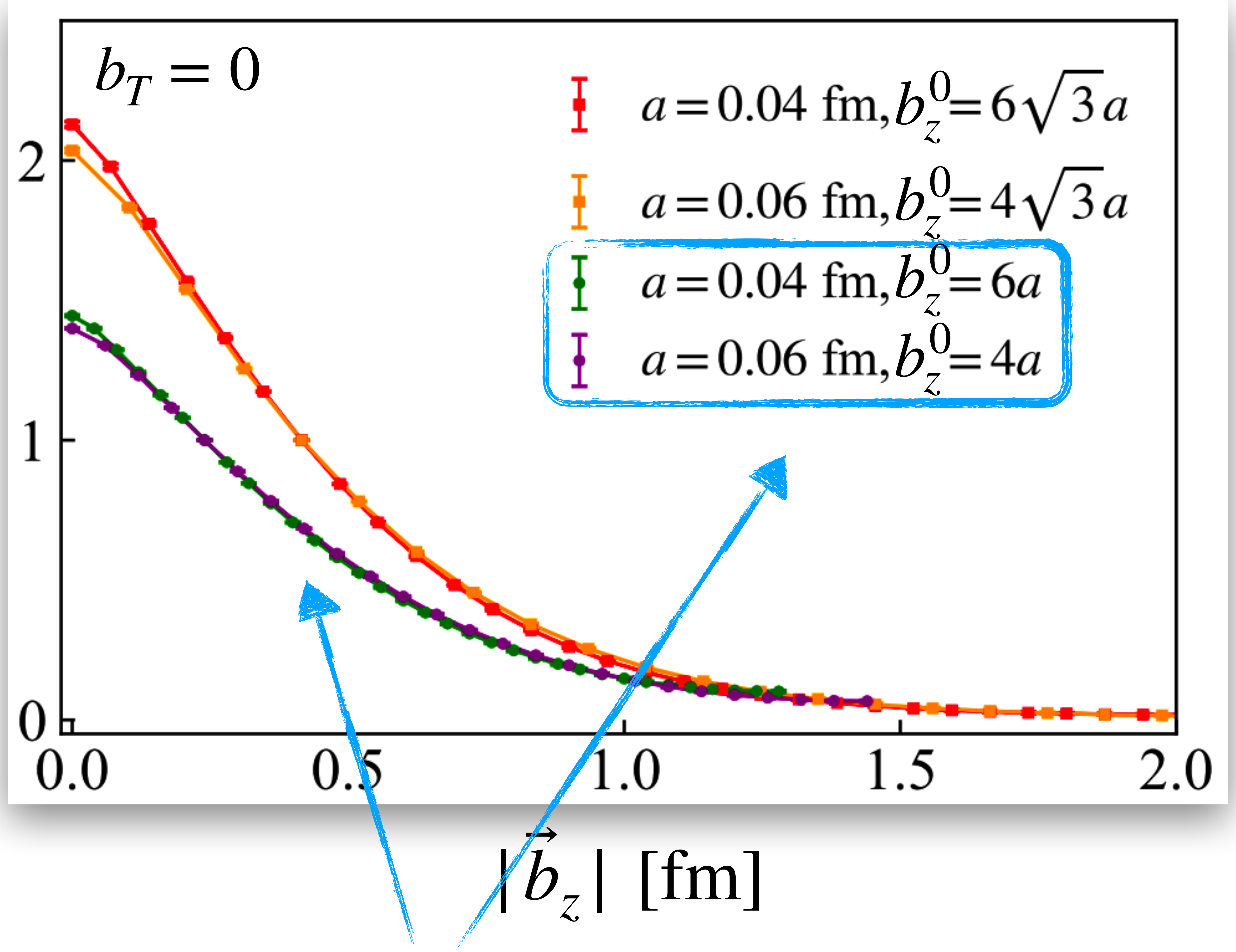
Gauge-invariant (GI)
quasi-TMDWF

$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp)\Gamma \psi(-\frac{b_z}{2},0) |_{\vec{\nabla}\cdot\vec{A}=0} | \pi^+, P_z\rangle$$

Coulomb gauge (CG)
quasi-TMDWF

CG quasi-TMDs: simplified renormalization

Renormalized matrix elements



Two lattice spacings: excellent continuum limit!

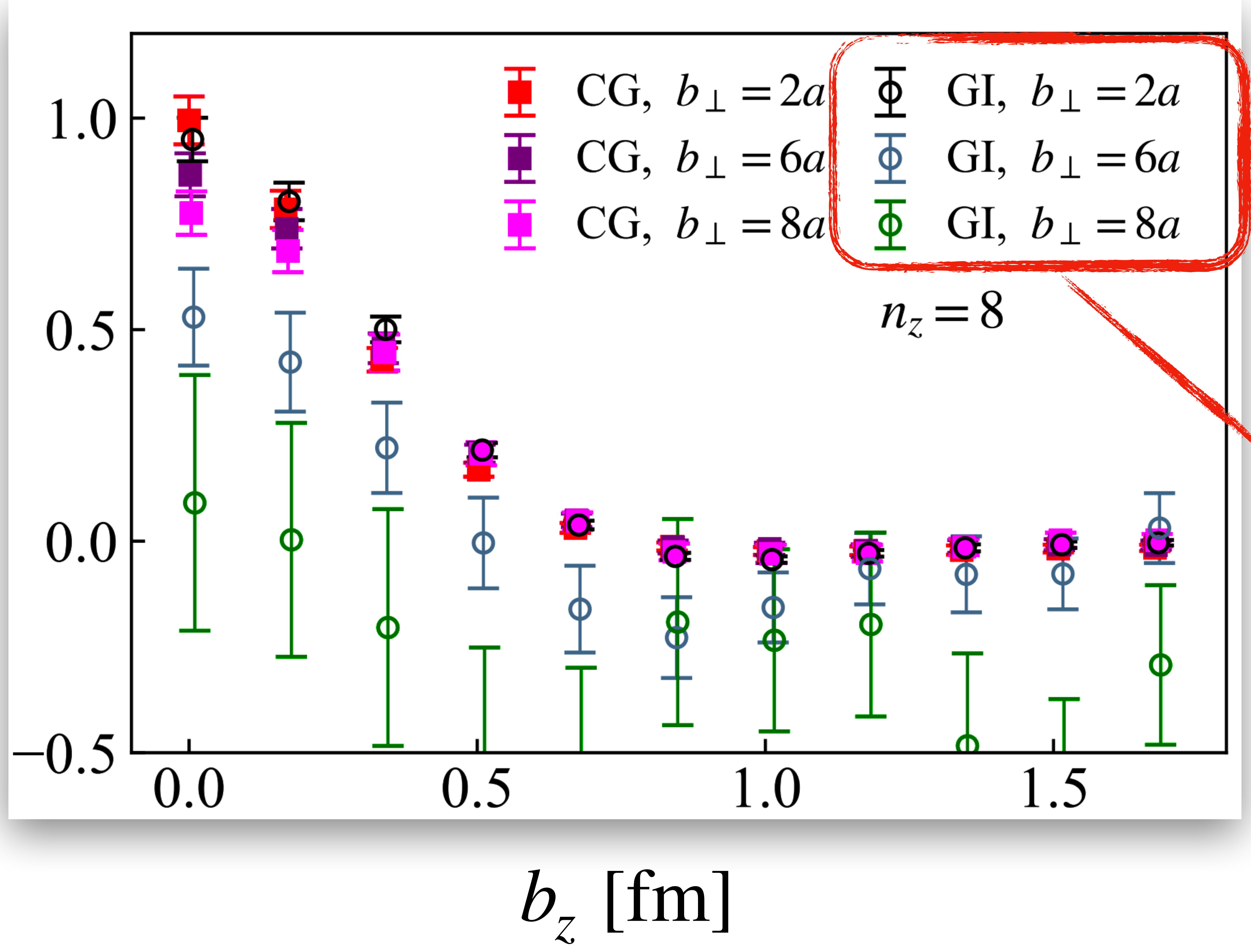
- No linear divergence: the renormalization is an overall constant.

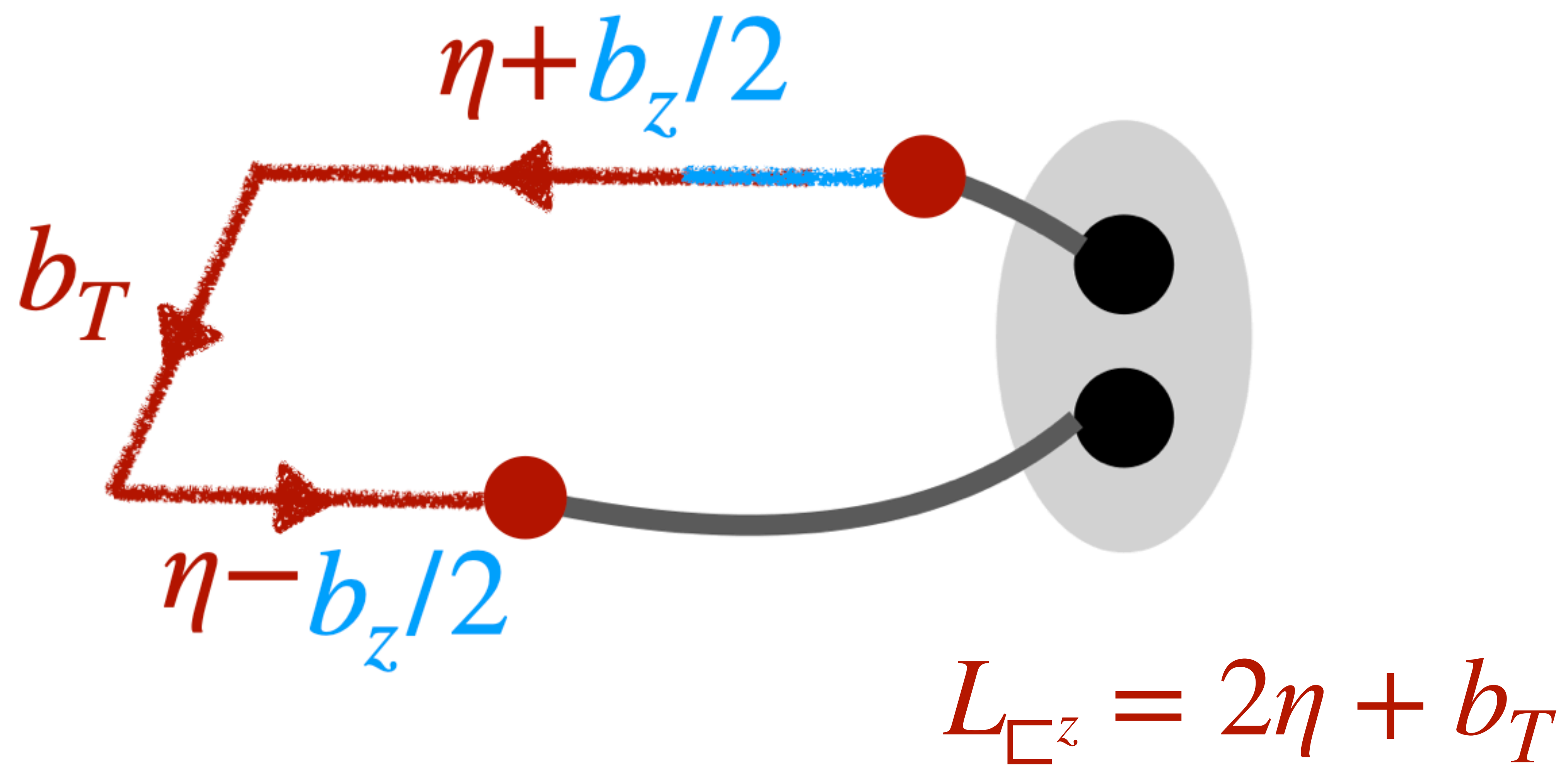
$$[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_B = Z_\psi(a)[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_R$$

- **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506

CG quasi-TMDs: enhanced long-range precision

Renormalized matrix elements





b_T: $2a \to 6a \to 8a$

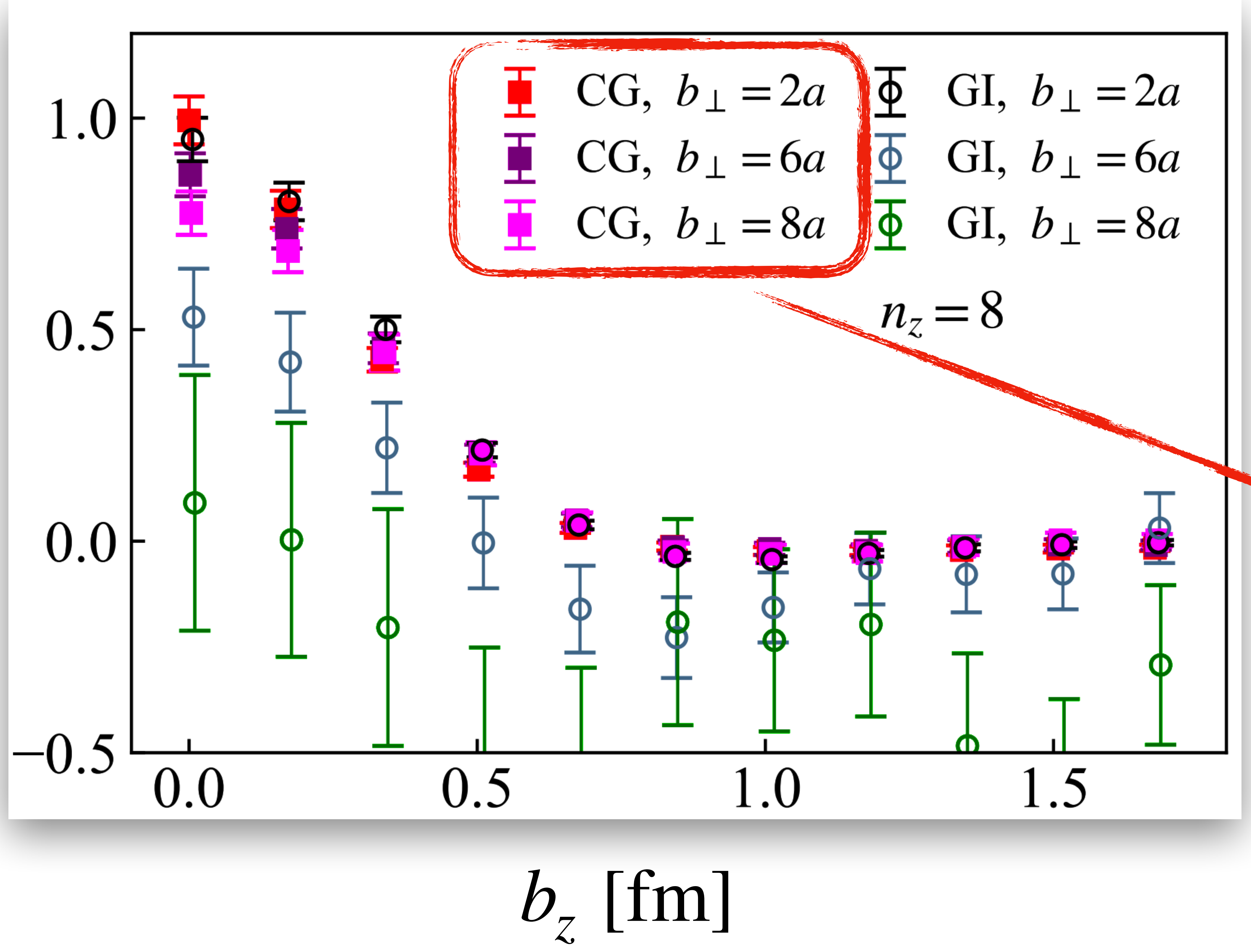
- GI shows rapid signal decay.

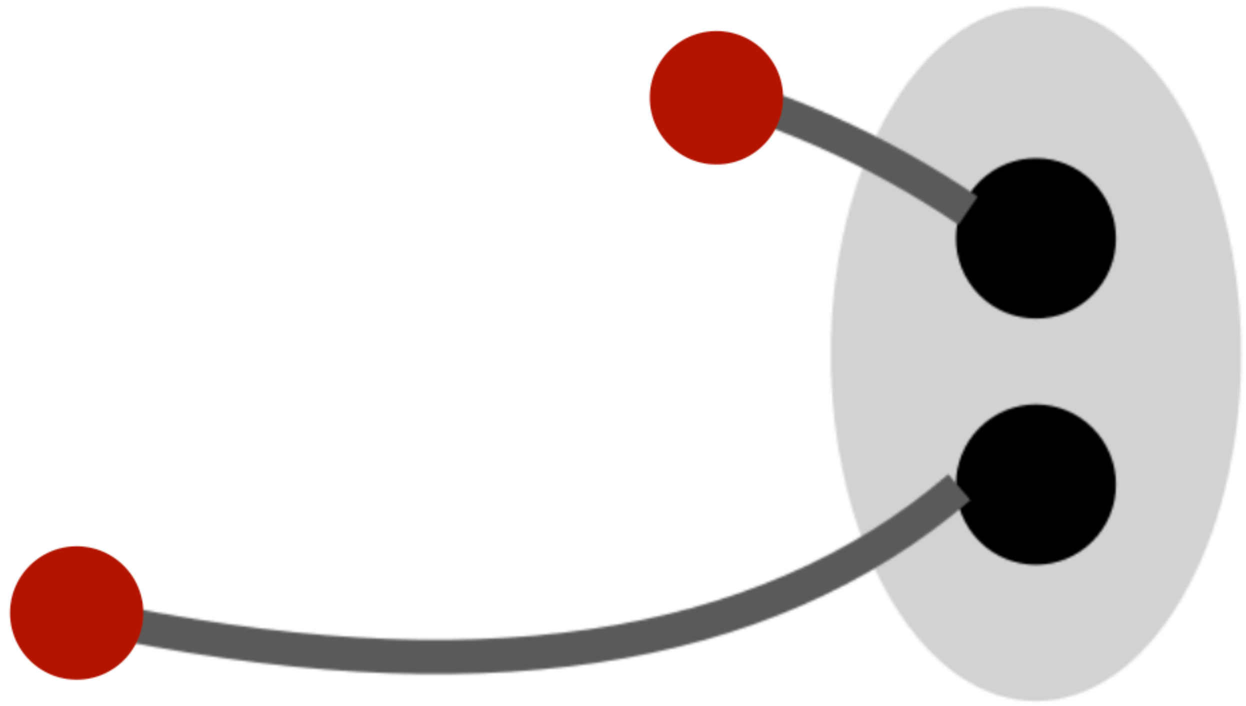
Domain wall fermion, physical quark masses
$64^3 \times 128$, a = 0.084 fm

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

CG quasi-TMDs: enhanced long-range precision

Renormalized matrix elements





b_T: $2a \rightarrow 6a \rightarrow 8a$

- CG shows much slower signal decay compared to the GI cases.

Domain wall fermion, physical quark masses
$64^3 \times 128$, a = 0.084 fm

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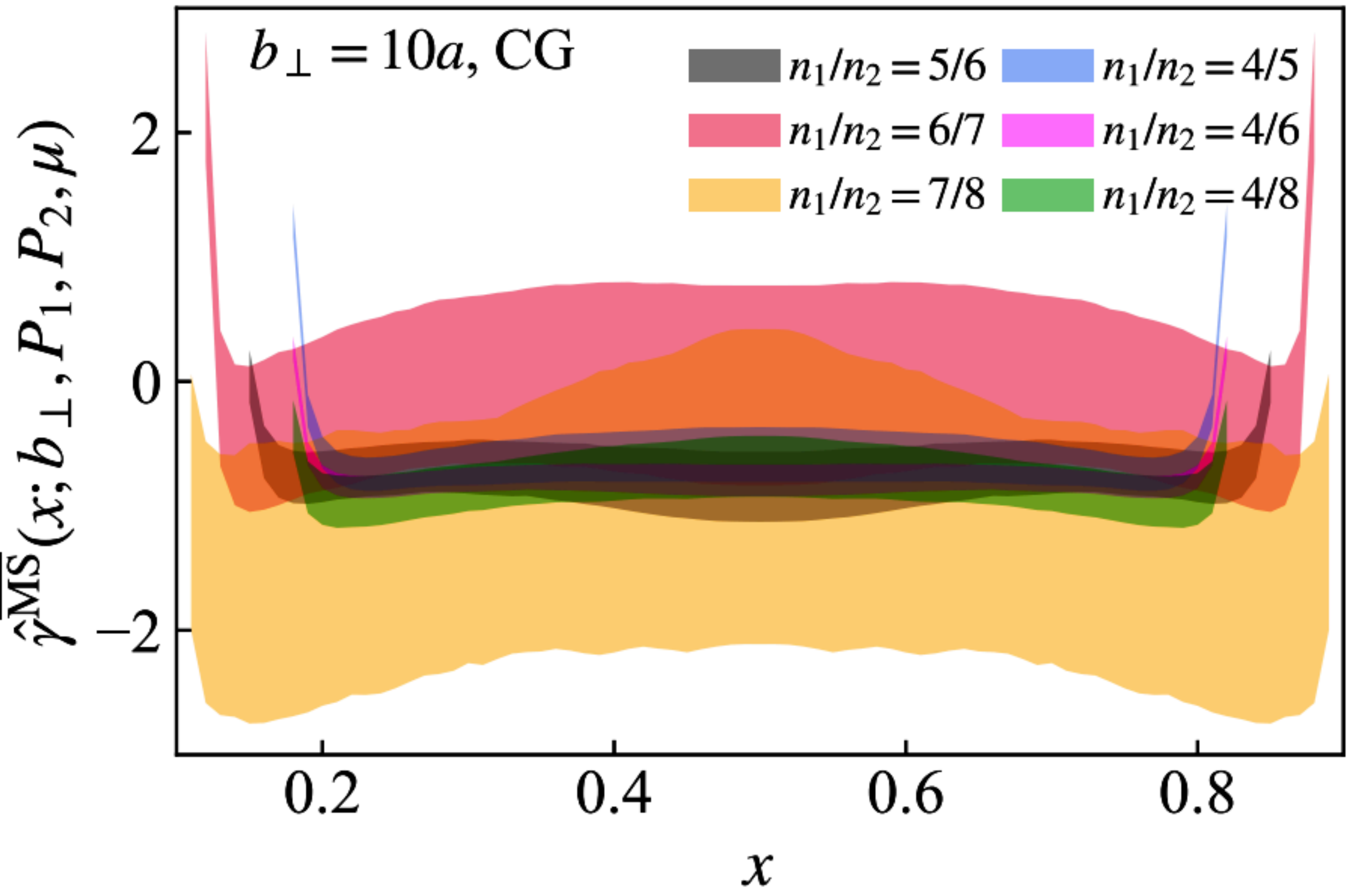
The Collins-Soper kernel from CG quasi-TMDWF

Perturbative correction

$$\gamma^{\overline{\rm MS}}(b_\perp,\mu) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\phi}(x,b_\perp,P_2,\mu)}{\tilde{\phi}(x,b_\perp,P_1,\mu)}\right] + \delta\gamma^{\overline{\rm MS}}(x,\mu,P_1,P_2) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{1}{(b_\perp(xP_z))^2}\right)$$

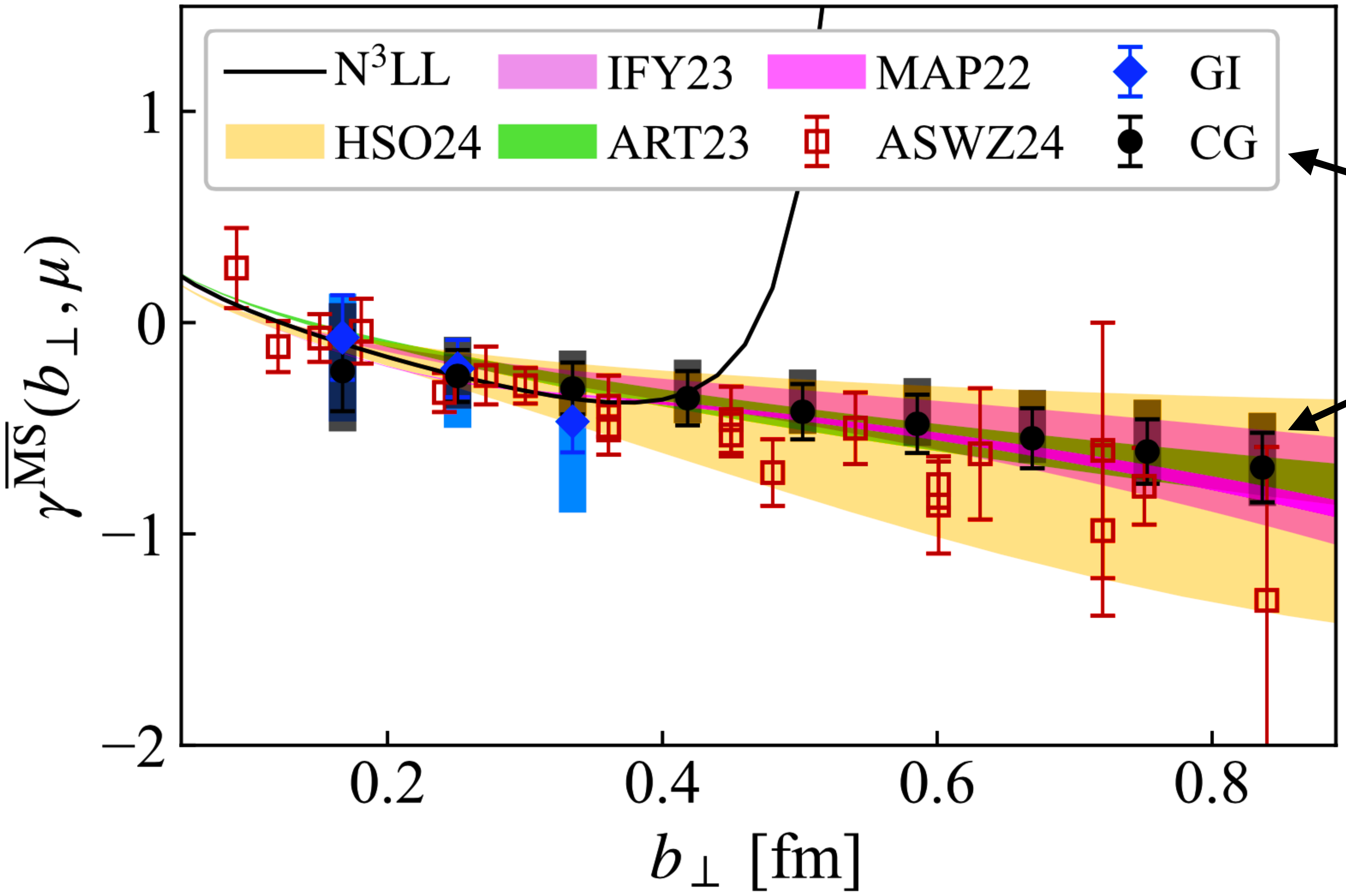
Evolution of quasi-TMDWFs: $P_1 \to P_2$

$a = 0.084$ fm, $\mathrm{P_z} = \mathrm{n_z} \cdot 0.23$ GeV



- The CS kernel $\gamma^{\overline{\rm MS}}(b_\perp,\mu)$ is **independent (universal) of P_z and x**, up to higher-order and power corrections.

Nonperturbative Collins-Soper kernel



- First-principle determination with no model dependence.
- Consistent with most recent global fits and lattice results from GI operators.
- Showing a near-linear dependence on b_T.

Domain wall fermion, physical quark masses
$64^3 \times 128$, a = 0.084 fm

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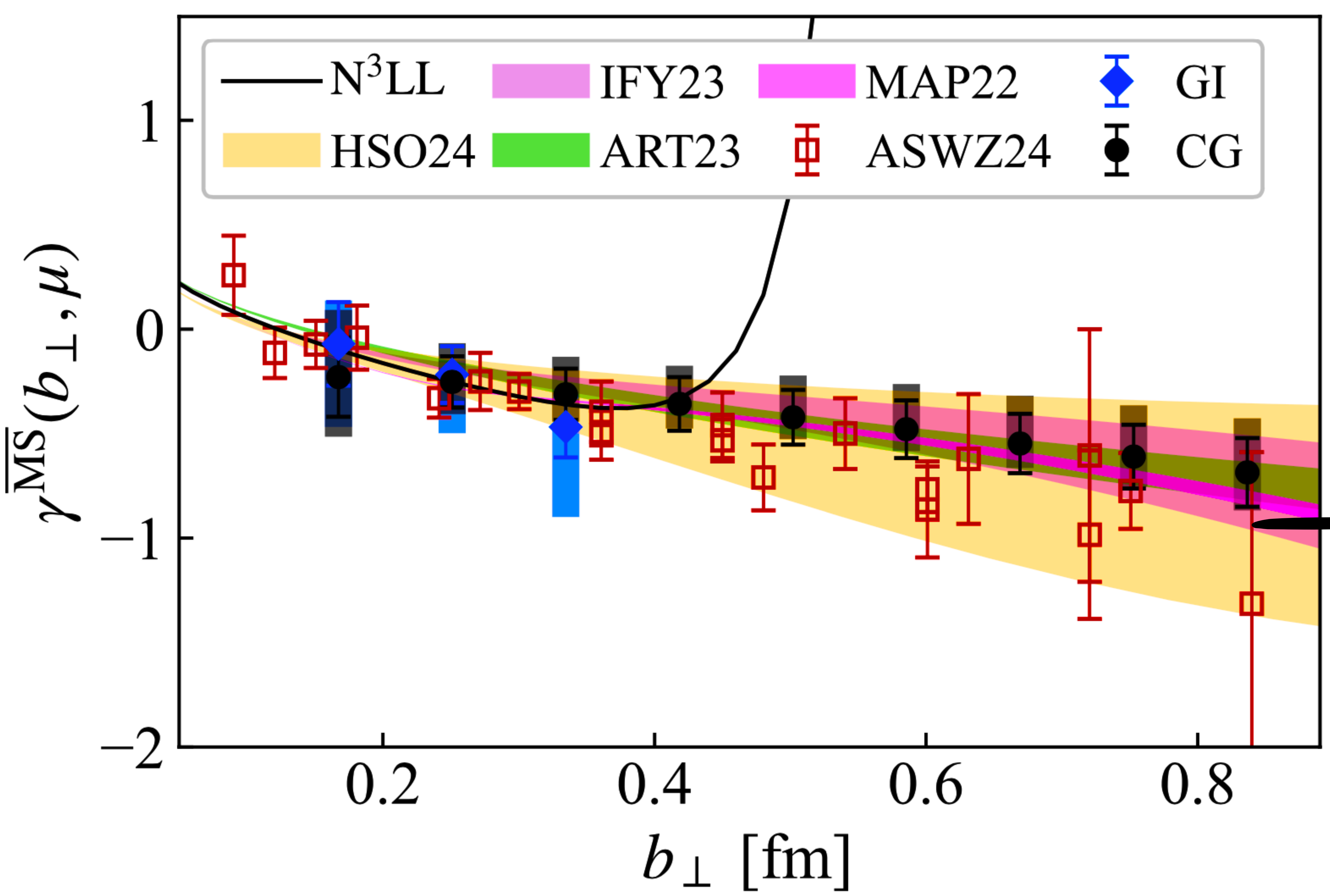
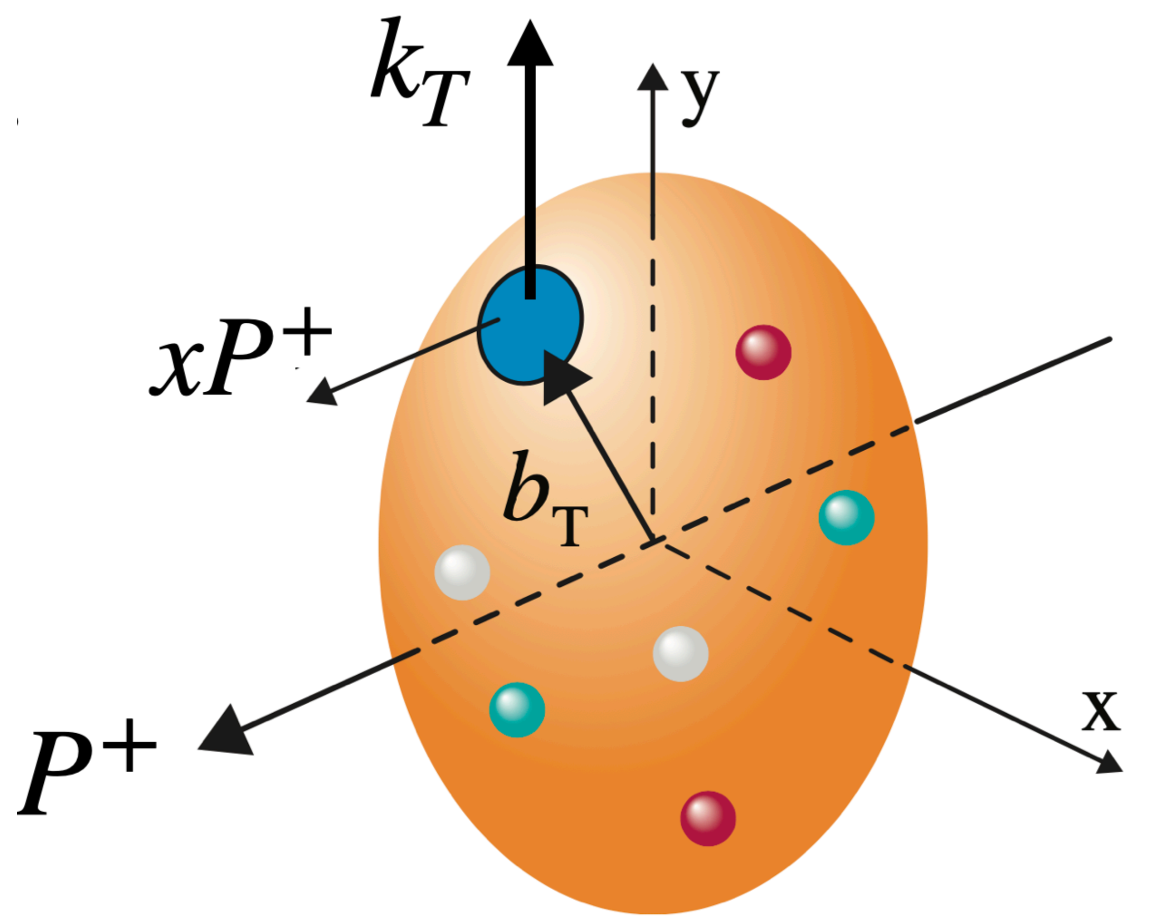
Nonperturbative Collins-Soper kernel





- CG approach greatly improve the efficiency/precision: broader use in the nonperturbative regime of TMDs.

- D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

Summary

- The TMDs can be extracted from quasi-TMD correlators. The novel CG quasi-TMDs have several advantages with an emphasize of the enhanced long-range precision.
- We computed the quasi-TMD wave functions in the CG using a chiral lattice discretization at the physical pion mass. The extracted non-perturbative CS kernel appears to be consistent with recent parametrization of experimental data.
- The CG methods could have broader use in the future particularly in the precision calculation of non-perturbative regime of TMD physics, including the gluons and the Wigner distributions.

Thanks for your attention!

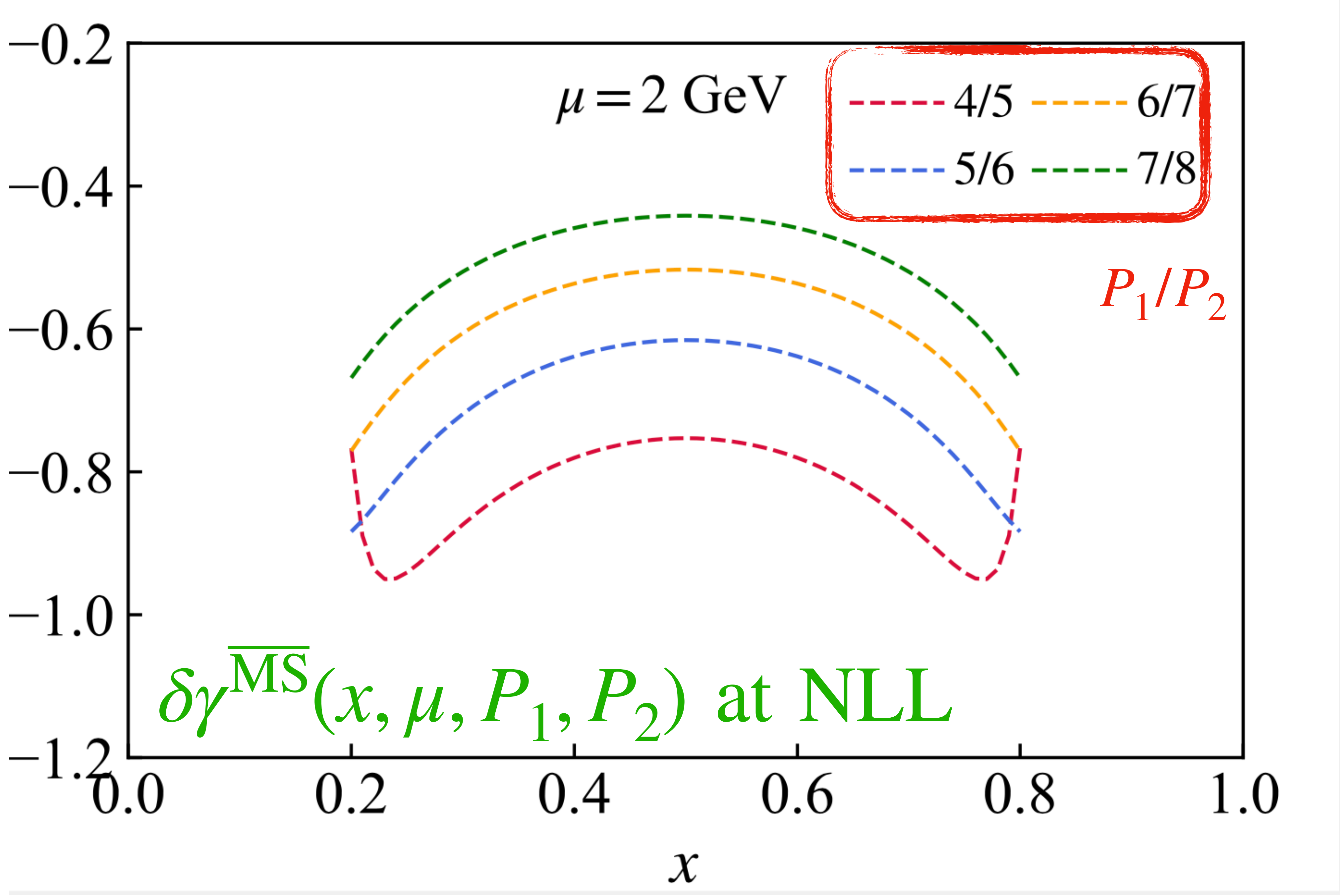
Back up

The Collins-Soper kernel from CG quasi-TMDWF

Perturbative correction

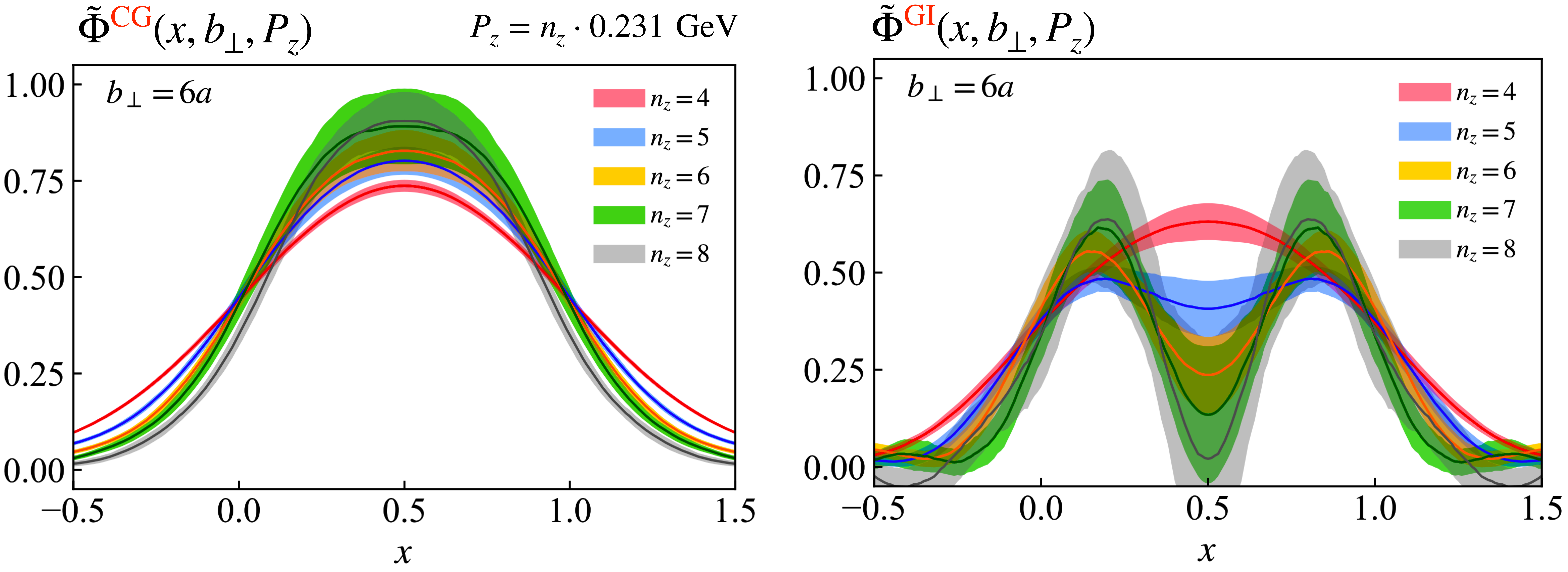
$$\gamma^{\overline{\rm MS}}(b_\perp,\mu) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\phi}(x,b_\perp,P_2,\mu)}{\tilde{\phi}(x,b_\perp,P_1,\mu)}\right] + \delta\gamma^{\overline{\rm MS}}(x,\mu,P_1,P_2) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{1}{(b_\perp(xP_z))^2}\right)$$

Ratio of quasi-TMDWFs



- The CS kernel $\gamma^{\overline{\rm MS}}(b_\perp,\mu)$ is **independent (universal) of P_z and x**, up to higher-order and power corrections.

Quasi-TMD wave functions after F.T.



- The CG quasi-TMD wave functions are more stable and show better signal.