Exploring Meson Structures from Lattice QCD

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Parton Distributions and Lattice Calculations (PDFLattice 2024) Nov 18-20, 2024

Outline

- Electromagnetic Form Factor (EMFF) of Pion and Kaon

  — based on Phys. Rev. Lett. **133**, 181902

- Pion Light-cone Generalized Parton Distribution (GPD)

  — based on arXiv: 2407.03516

Motivation

EPJA 48 (2012) 187 JPG 48 (2021) 075106 arXiv: 2102.09222

- Experiment: JLab, EIC, EicC …

Gao et al., PRD 96 (2017) 034024

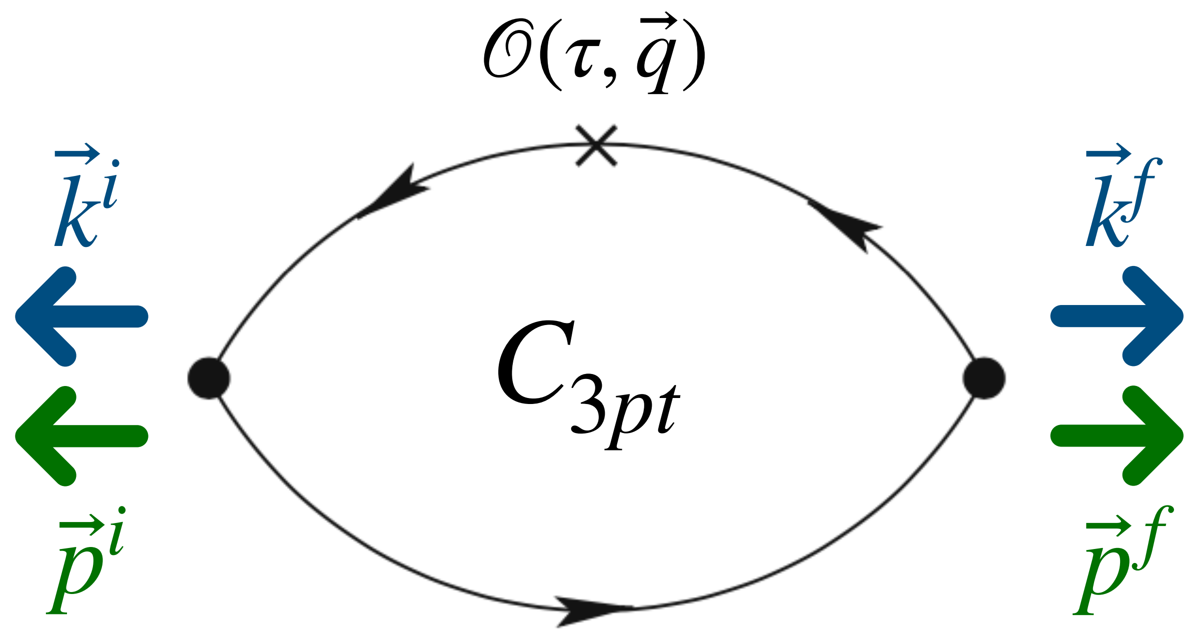
- Effective theory: QCD sum rules, DSE …

- Lattice QCD: first principle

PRD 96 (2017) 114509 ETMC, PRD 105 (2022) 054502

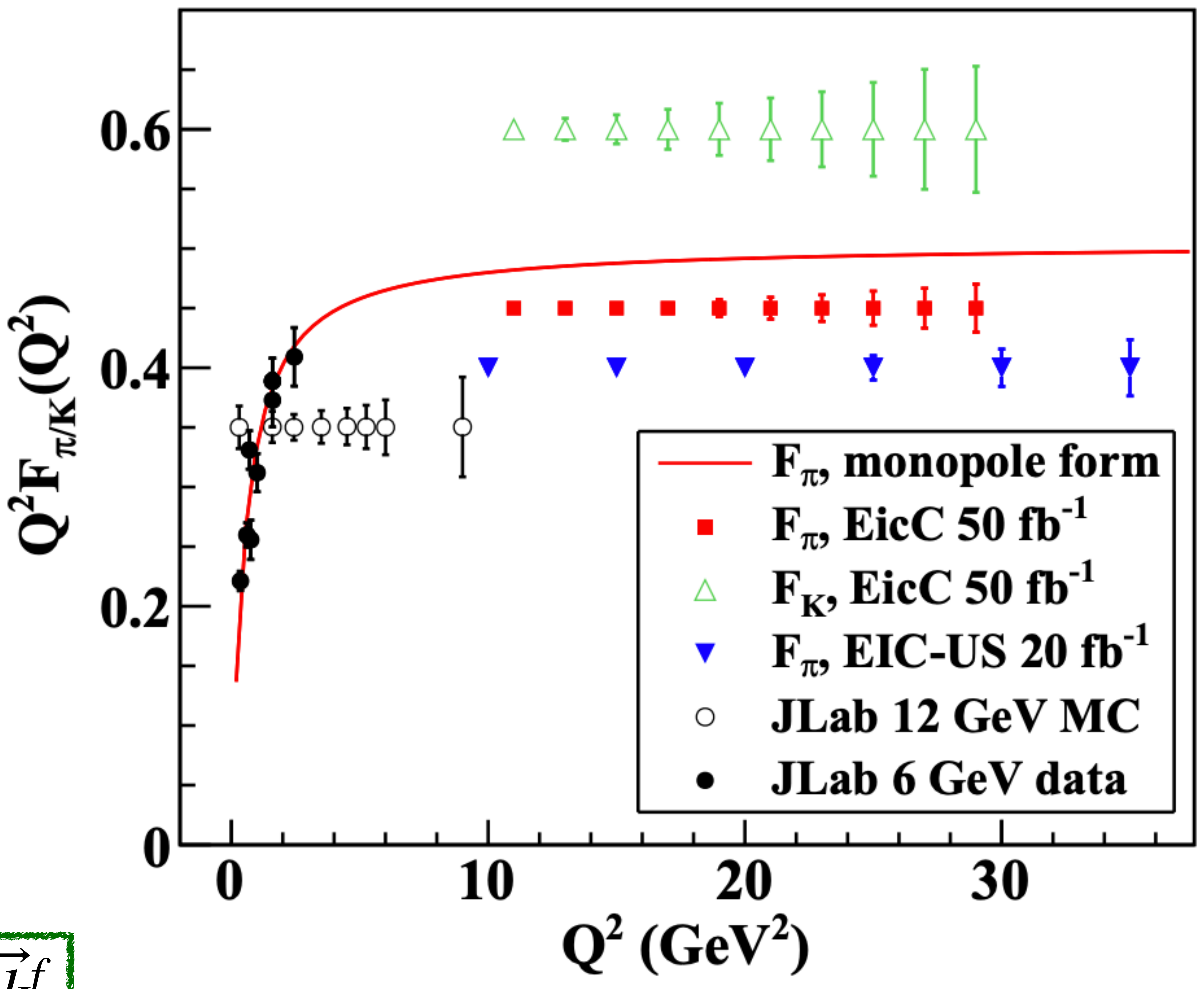
  ❍ State-of-the-art: $Q^2 \leq 6$ (pion), 3 (kaon) GeV^2

  ❍ This work: Q^2 up to 10, 28 GeV^2



Boost parameter $\zeta = \frac{\vec{k}^i}{\vec{p}^i} = \frac{\vec{k}^f}{\vec{p}^f}$

Front. Phys. 16 (2021) 64701



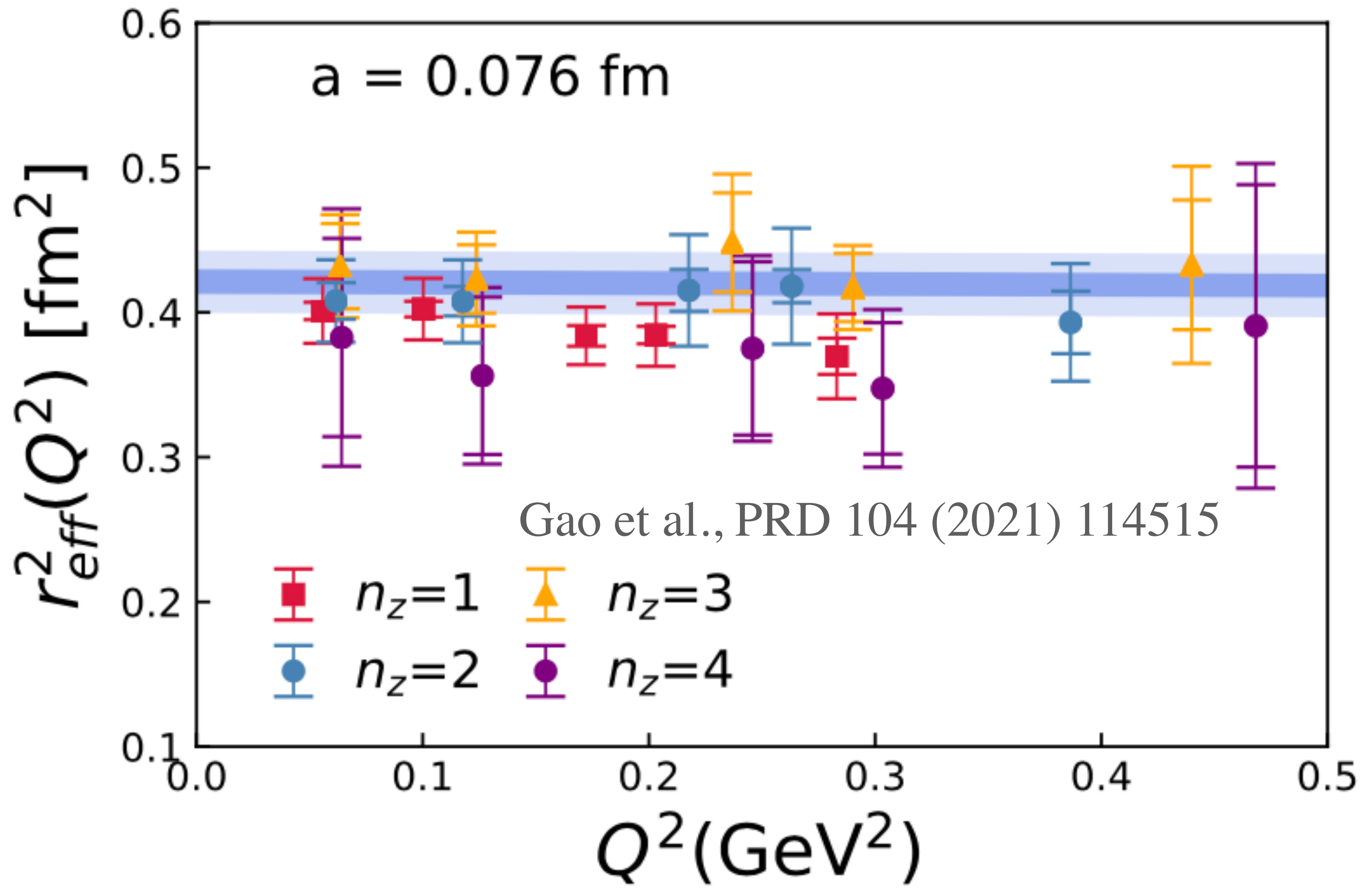
Momentum transfer $Q^2 = -t$

Motivation

Low Q^2: **V**ector **M**eson **D**ominance

$$r^2_{\mathrm{eff}}(Q^2) = 6[1/F_\pi(Q^2) - 1]/Q^2$$

$$\langle r^2_\pi \rangle = 0.42(2)\ \mathrm{fm}^2, \quad \langle r^2_\pi \rangle_{PDG} = 0.434(5)\ \mathrm{fm}^2$$



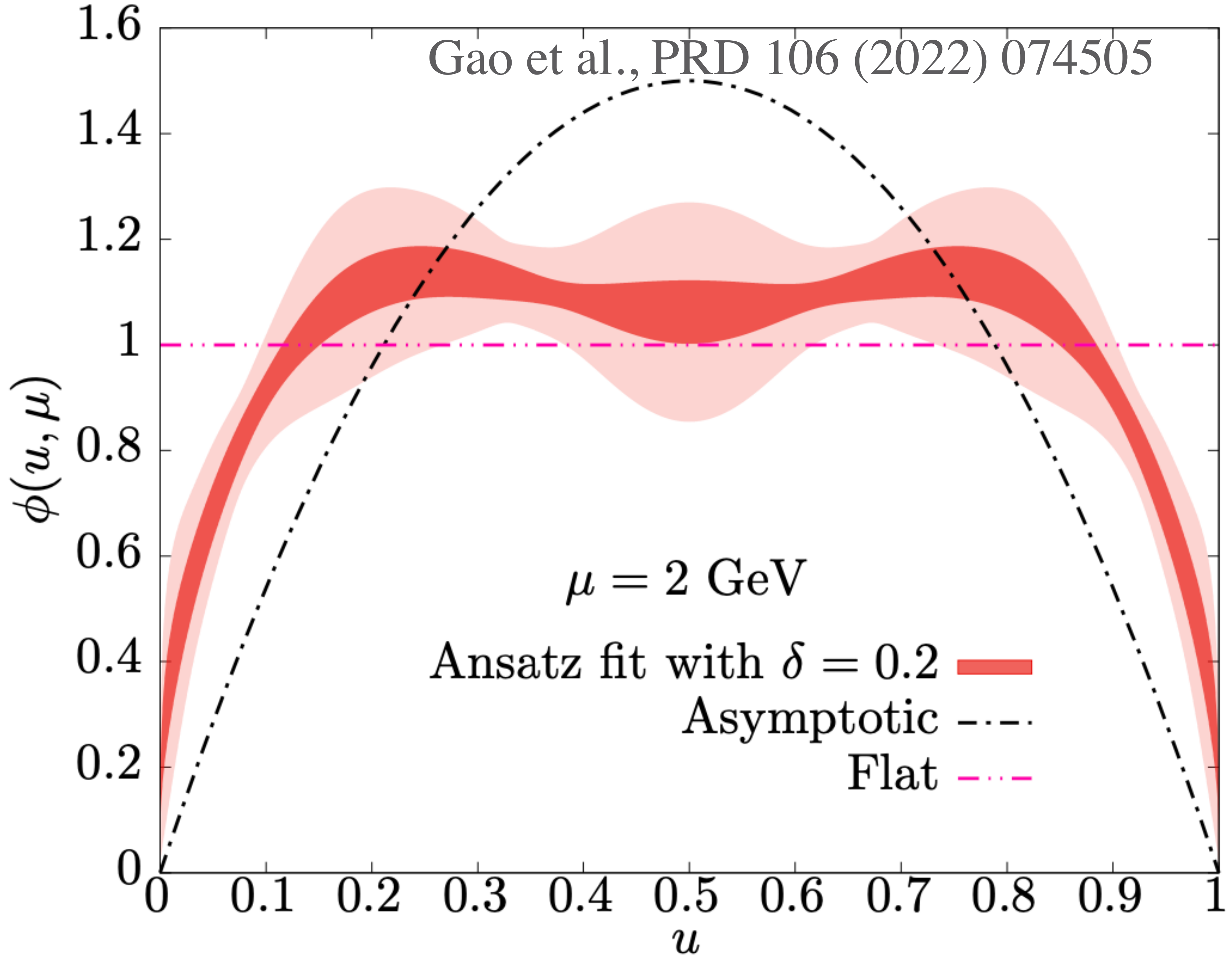
→ **Larger Q^2**

High Q^2: Factorization framework

Hard-process kernel

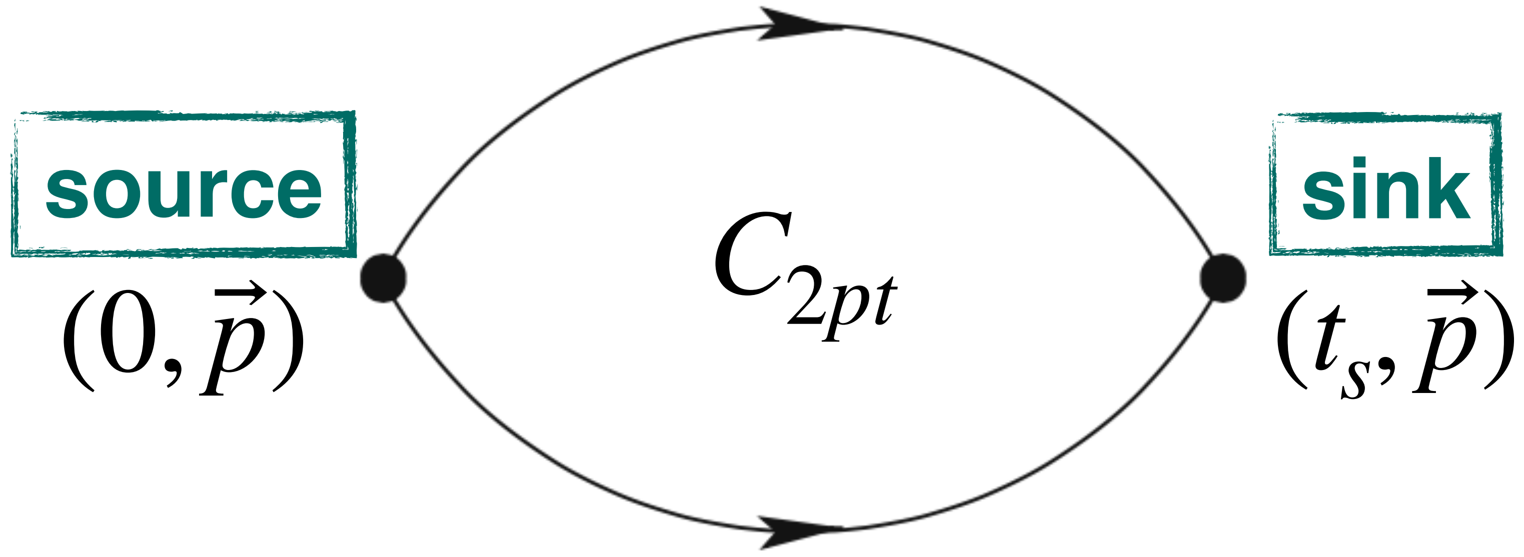
$$F(Q^2) = \int\int \mathrm{d}x\mathrm{d}y \Phi^*(y, \mu_F^2) T_H(x, y, Q^2, \mu_R^2, \mu_F^2) \Phi(x, \mu_F^2)$$

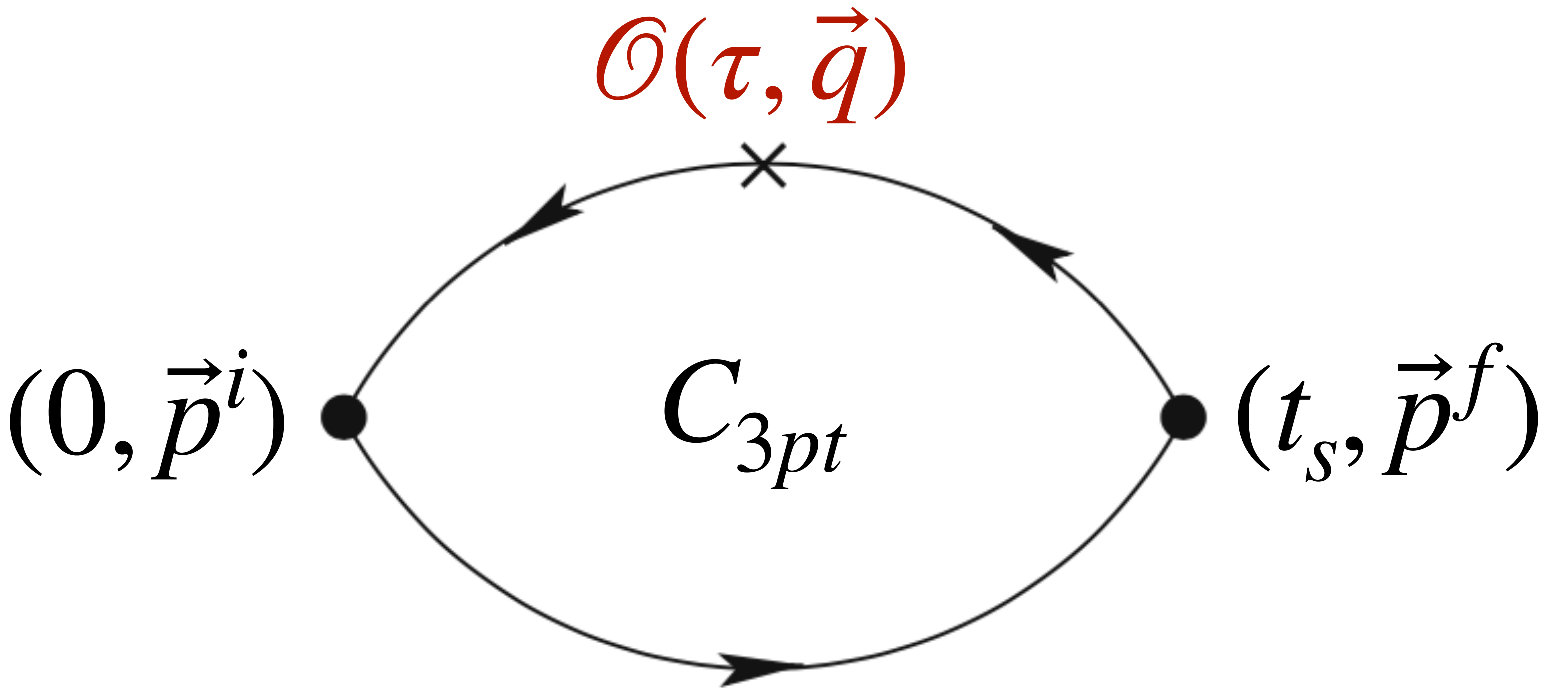
Distribution amplitude



→ **Test the factorization Q^2**

How to get the form factor on the lattice





$$C_{2pt}(t,\vec{p}) = \left\langle H(t_s,\vec{p})H^\dagger(0,\vec{p})\right\rangle$$

$$C_{3pt}(\tau,t_s;\vec{p}^i,\vec{p}^f) = \left\langle H(t_s,\vec{p}^f)\hat{\mathcal{O}}_{\gamma_\mu}(\tau,\vec{q})H^\dagger(0,\vec{p}^i)\right\rangle$$

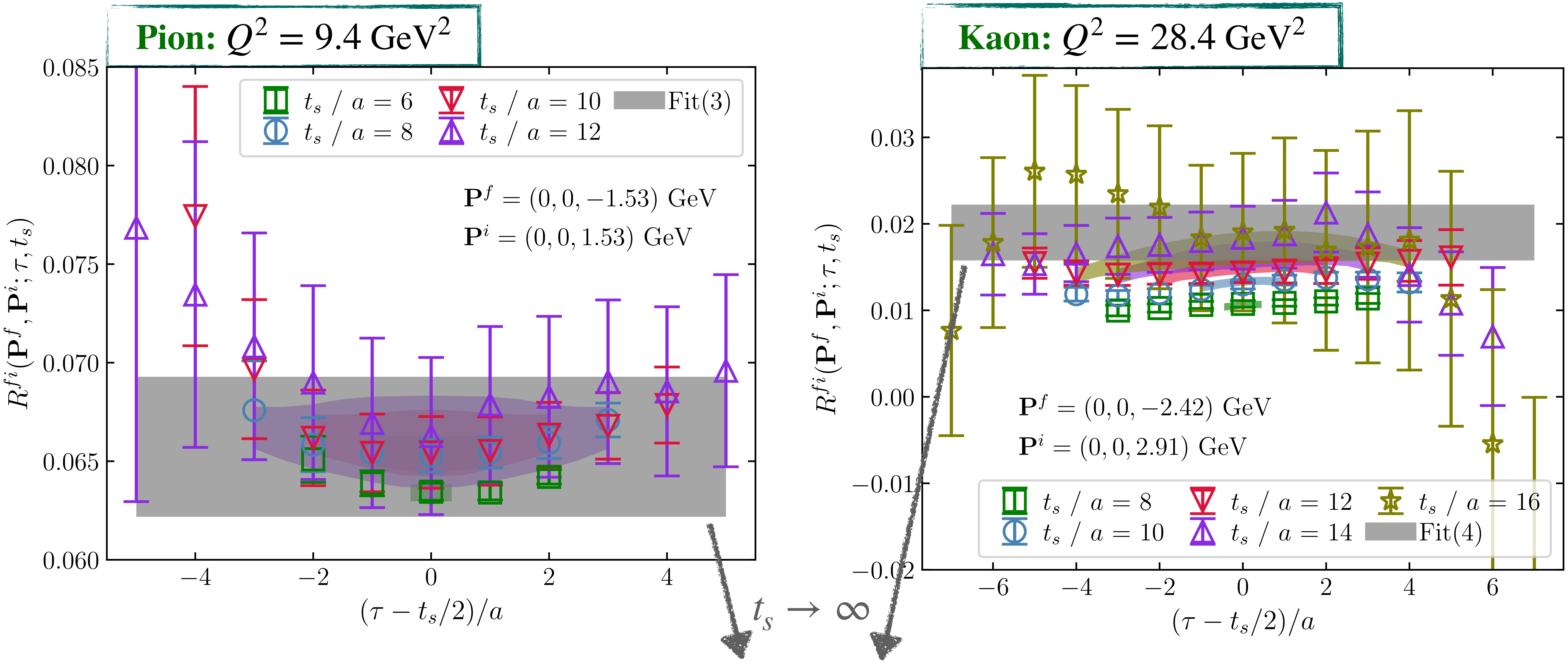
$$R^{fi} \sim C_{3pt} \,/\, C_{2pt} \xrightarrow{t_s\to\infty}$$

$$\longrightarrow \quad F^B = \left\langle E_0,\vec{p}^f \,|\, \hat{\mathcal{O}}_{\gamma^\mu}(\tau,\vec{q}) \,|\, E_0,\vec{p}^i \right\rangle \quad \longrightarrow \quad F(Q^2) = F^B \times Z_V^{-1}$$

$$(Q^2 = -t)$$

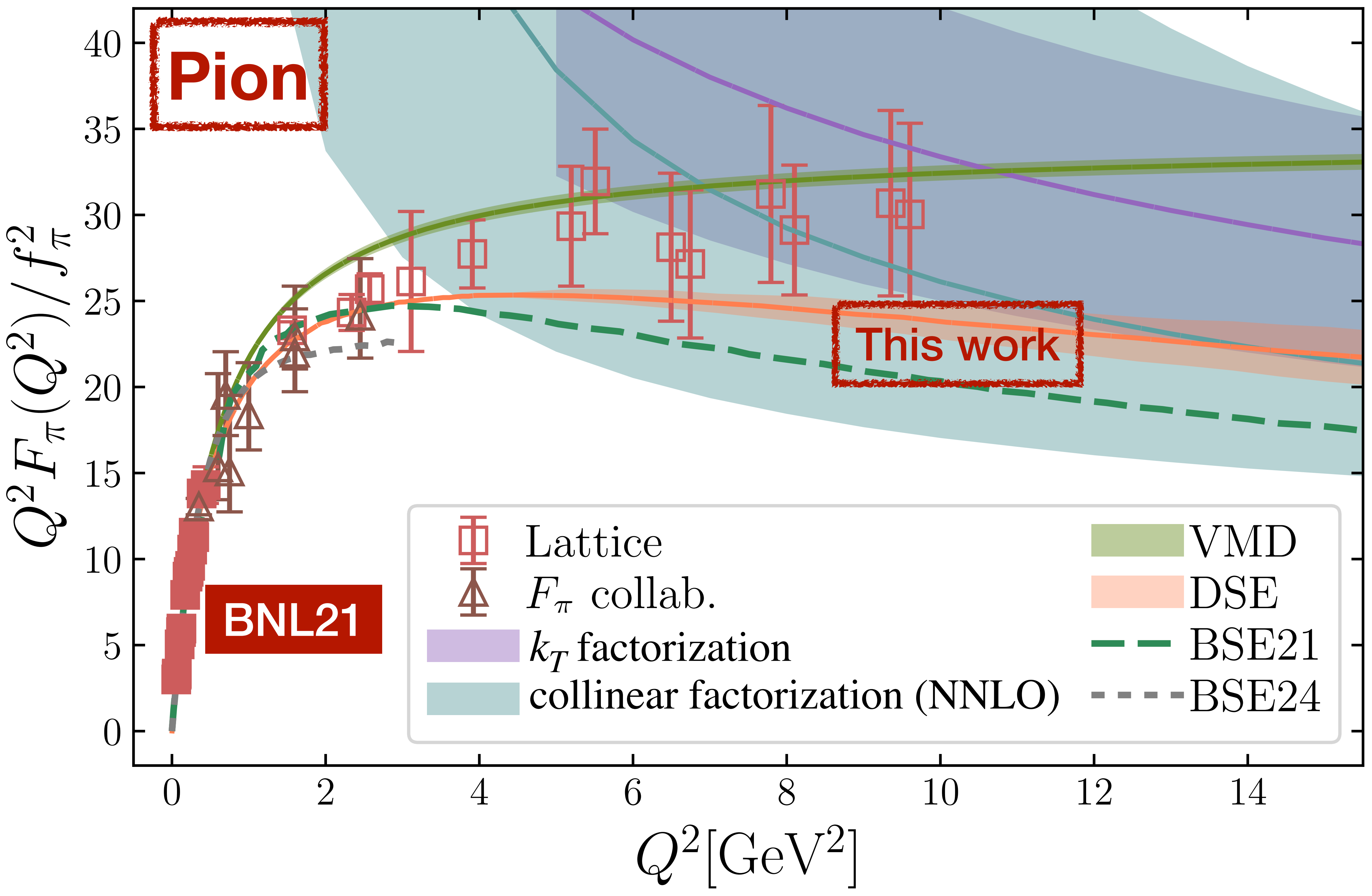
Bare Form factor

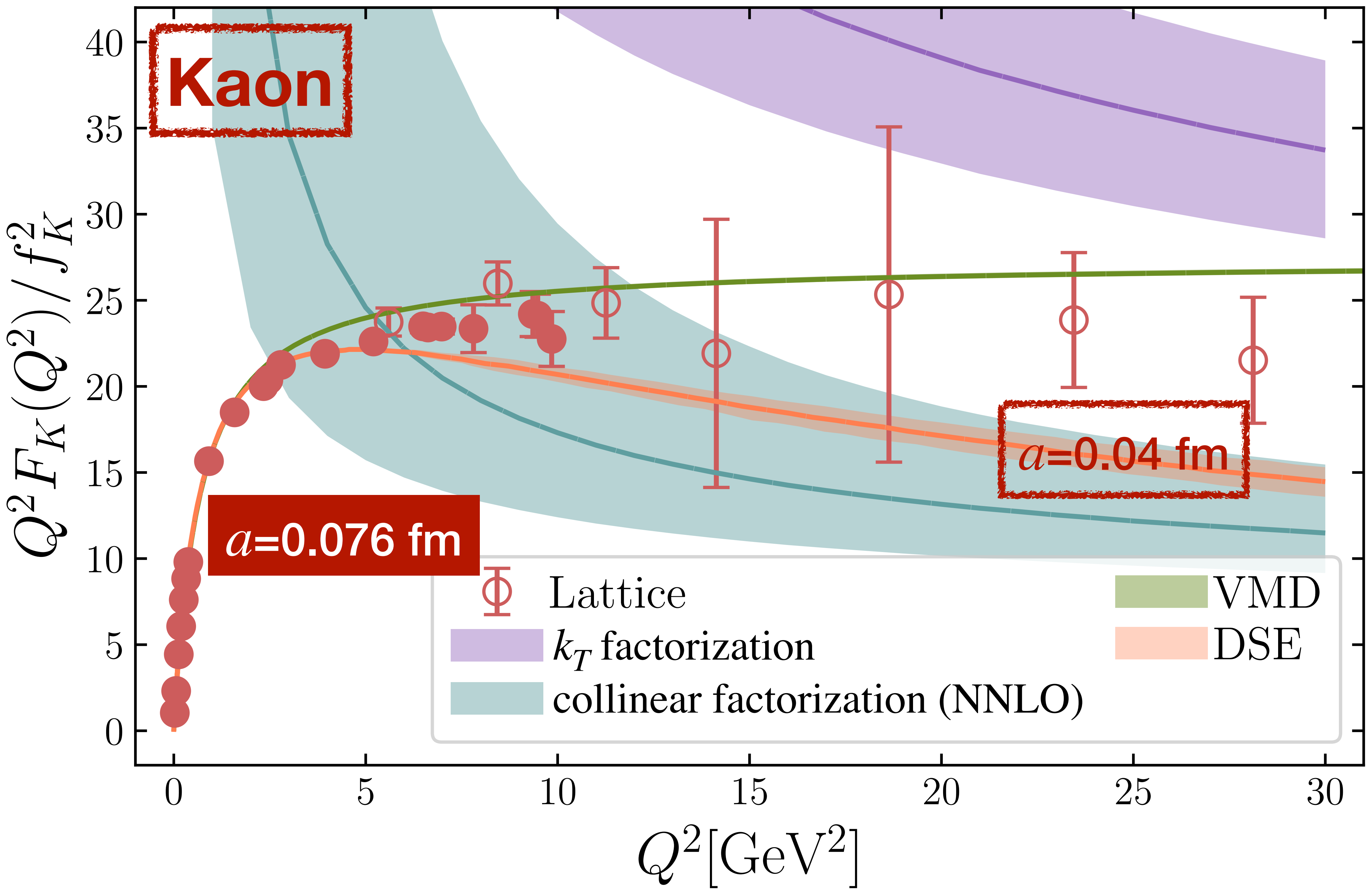
Bare Form Factor



Bare form factor $F^B \times Z_V^{-1} = F(Q^2)$

Electromagnetic Form Factor





Hard-process kernel

$$F(Q^2) = \iint \mathrm{d}x\mathrm{d}y\,\Phi^*(y,\mu_F^2)T_H(x,y,Q^2,\mu_R^2,\mu_F^2)\Phi(x,\mu_F^2)$$

Distribution amplitude

$$F(Q^2 \to \infty) = 8\pi\alpha_s(Q^2)f^2/Q^2, \quad Q^2F/f^2 \sim \text{Constant}$$

Lattice (BNL21): Gao et al., PRD 104 (2021) 114515

F^π collaboration: Huber et al., PRC 78 (2008) 045203

DSE (Dyson-Schwinger equation): Yao et al., PLB 855 (2024) 138823

BSE21 (Bethe-Salpeter equation): Ydrefors et al., PLB 820 (2021) 136494

BSE24: Jia and Cloët, arXiv:2402.00285

k_T factorization: Cheng, PRD 100 (2019) 013007

pion: Chai et al., EPJC 83 (2023) 556

kaon: in preparation

T_H in the collinear factorization: Chen et al., PRL 132 (2024) 201901

DA in the collinear factorization: Cloët et al., arXiv:2407.00206

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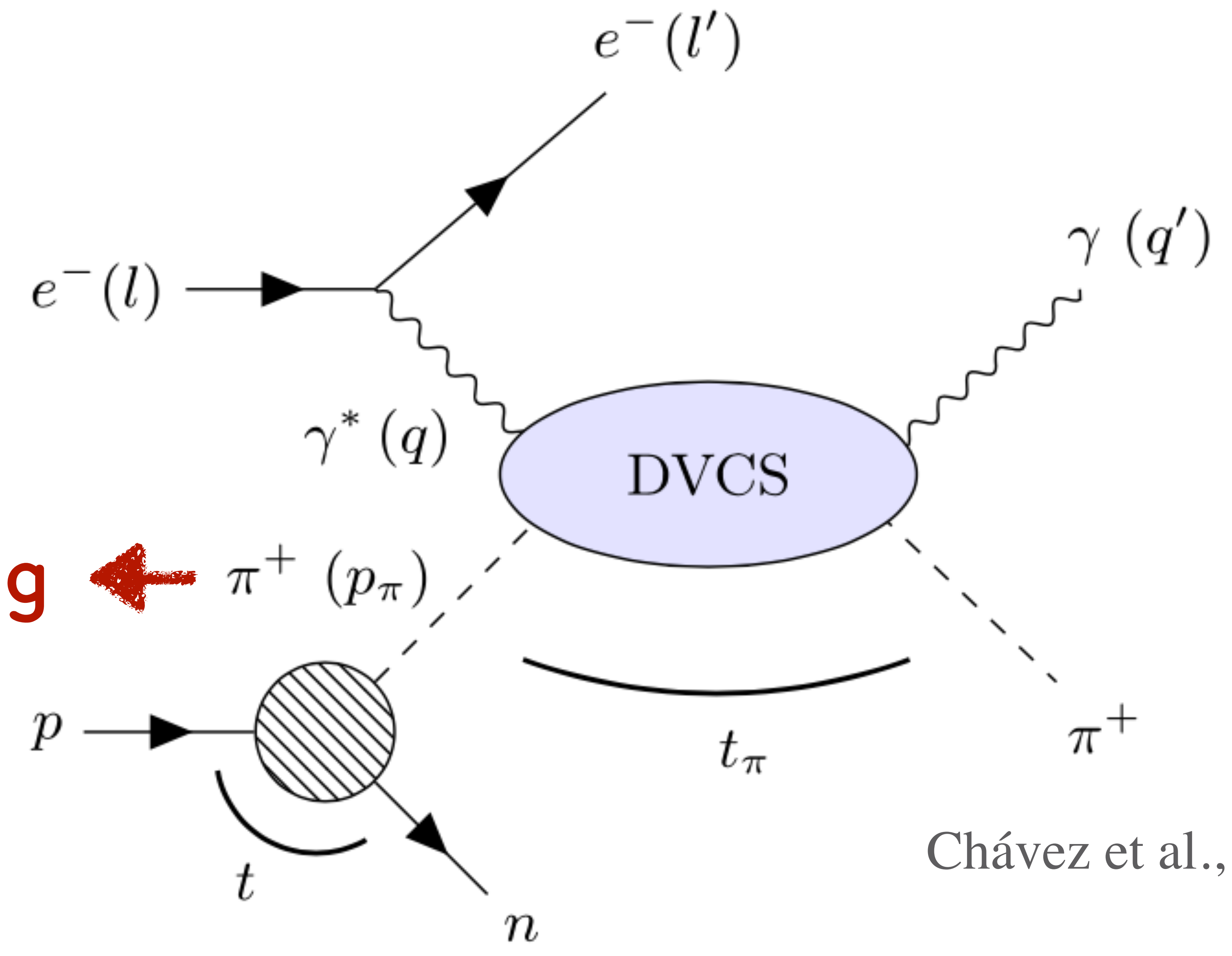
Motivation

1D

Form Factor (pion-electron scattering)

Parton Distribution Functions (Drell-Yan process)

} Generalized Parton Distributions **3D**



More challenging ←

Chávez et al., PRL 128 (2022) 202501

+ Models

Lattice QCD: from first principle

Frame-independent approach

Bhattacharya et al., PRD 106 (2022) 114512

Calculate in the asymmetric frame

Save computational cost

- Lorentz-invariant amplitudes A_i's (frame-independent)

$$M^\mu(P^m u, z^\mu, \Delta^\mu) = \bar{P}^\mu A_1 + m^2 z^\mu A_2 + \Delta^\mu A_3, \quad \bar{P}^\mu = (p_f^\mu + p_i^\mu)/2, \quad \Delta^\mu = p_f^\mu - p_i^\mu .$$

- Lorentz-invariant quasi-GPD $\tilde{H}_{\rm LI}$

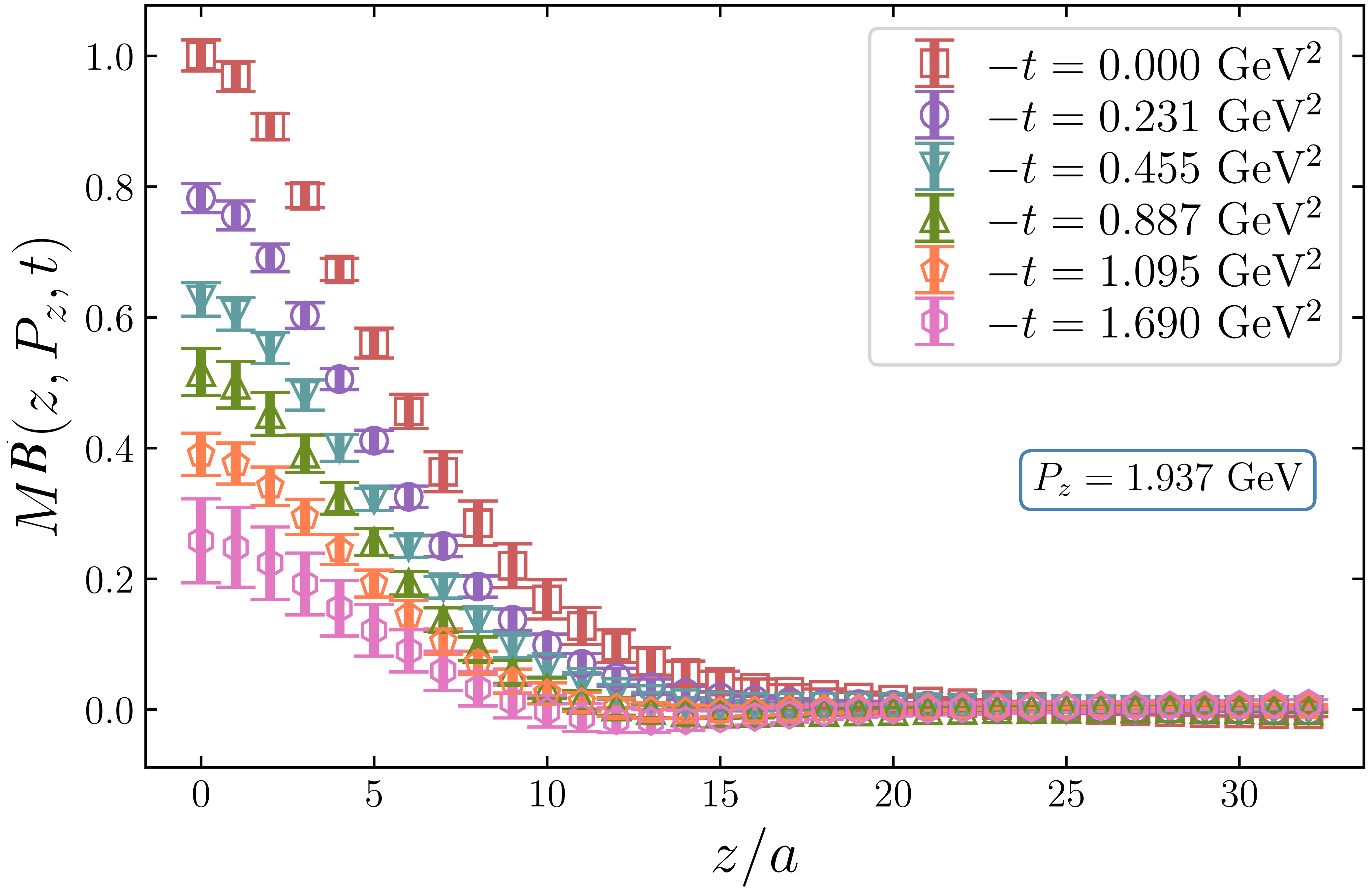
$$\tilde{H}_{\rm LI}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_1 + \frac{z\cdot\Delta}{z\cdot\bar{P}} A_3$$

$$A_3(-z\cdot\Delta) = -A_3(z\cdot\Delta) \longrightarrow A_3(z\cdot\Delta = 0) = 0$$

Bare matrix element

$$\tilde{H}_{\rm LI} = A_1 = KM^t \equiv M^B$$

K: normalization factor



$z \uparrow$, $-t \uparrow$ } $M^B \downarrow$

Renormalization ⟶ **Quasi-GPDs**

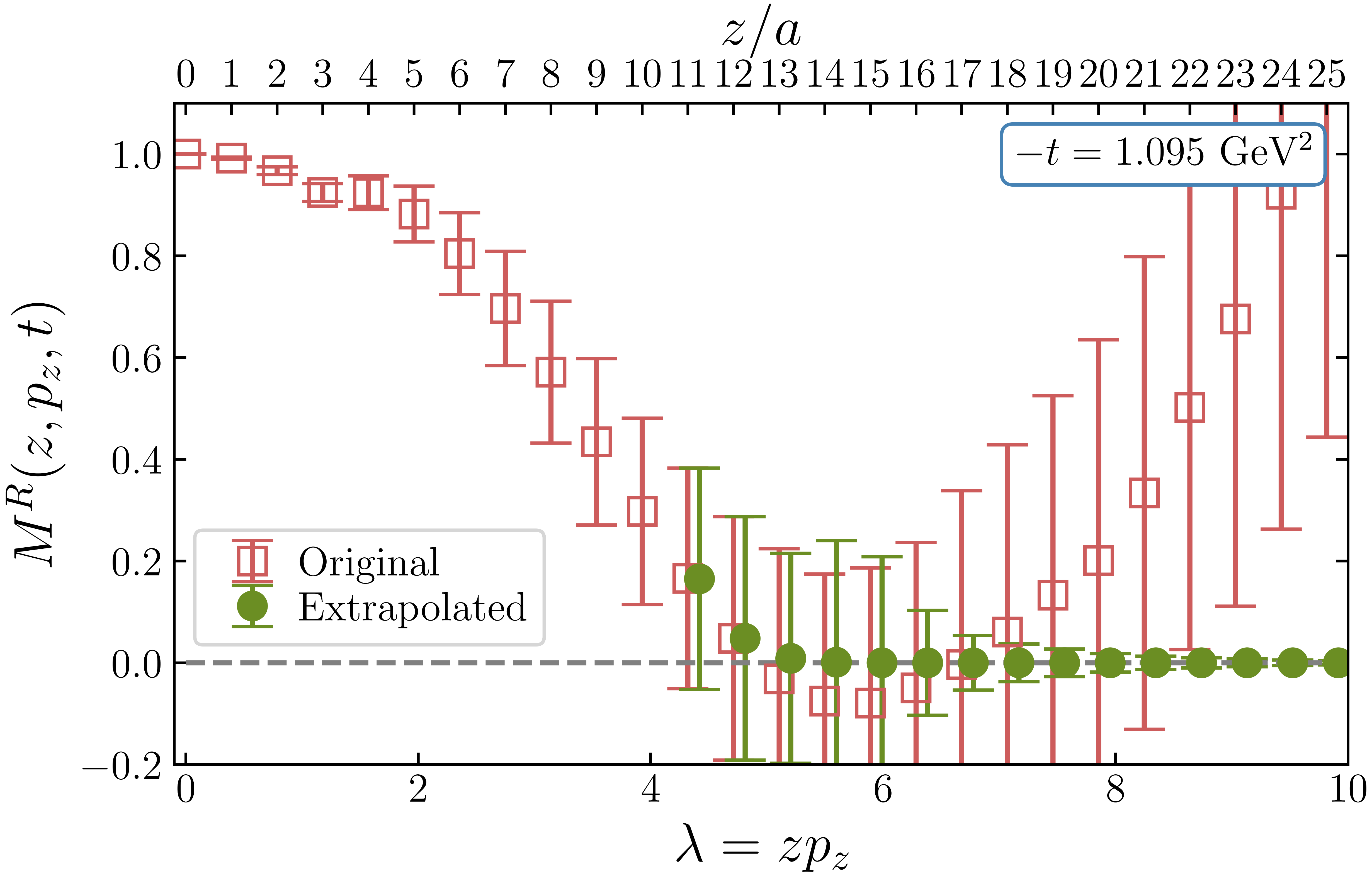
Renormalization & Extrapolation

Ji et al., NPB 964 (2021) 115311

Hybrid scheme,

$$\begin{cases} z \le z_S: & \dfrac{M^R(z,\vec{p},\vec{q})}{M^R(z,0,0)} = \dfrac{M^B(z,\vec{p},\vec{q})}{M^B(z,0,0)}, \\ z \ge z_S: & \dfrac{M^R(z,\vec{p},\vec{q})}{M^R(z_S,0,0)} = e^{(\delta m + \bar{m}_0)|z-z_S|}\dfrac{M^B(z,\vec{p},\vec{q})}{M^B(z_S,0,0)}. \end{cases}$$

Ratio scheme



Renormalized (Original)

Lattice artifacts ↓

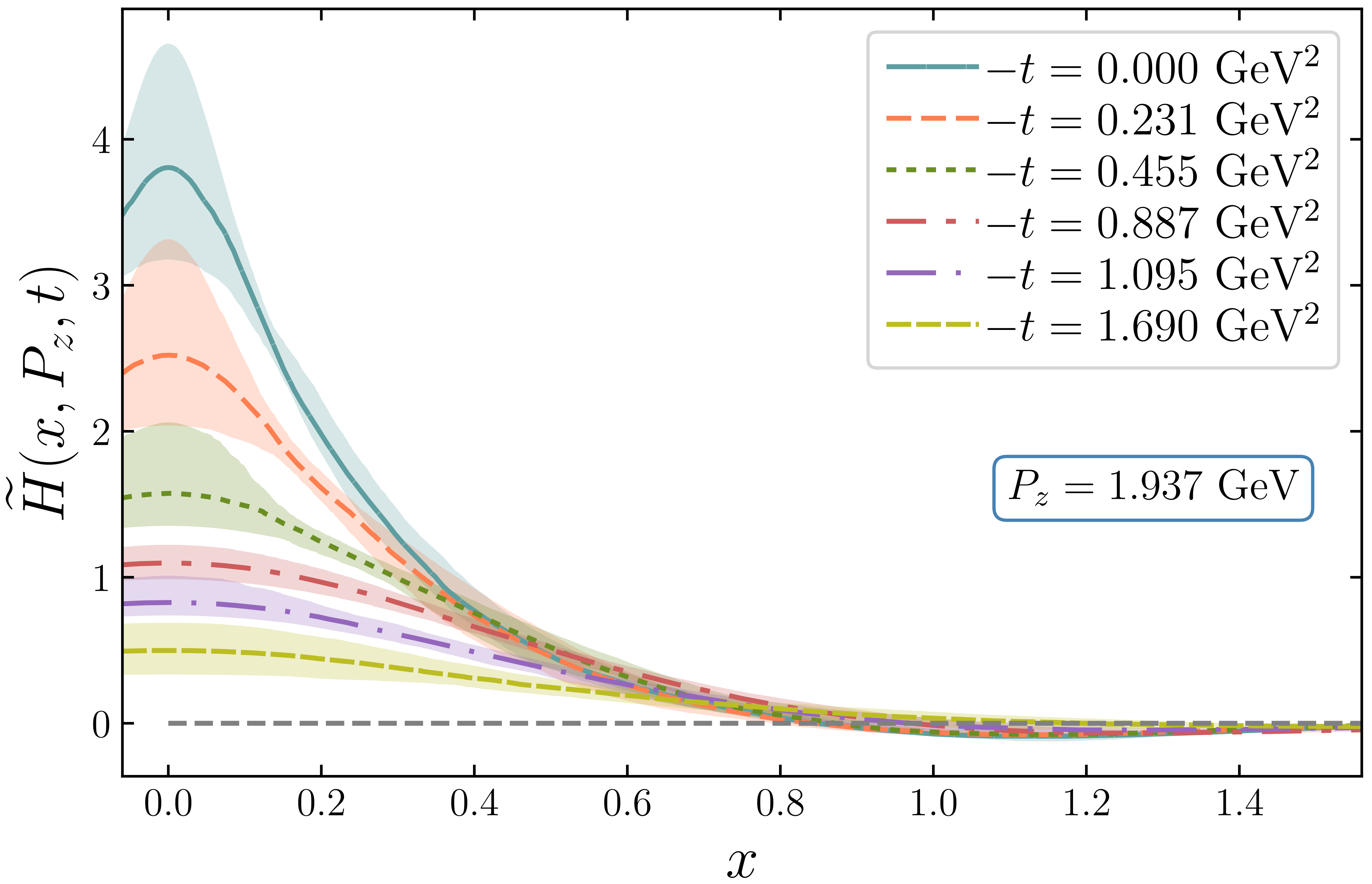
$$M^R = A\frac{e^{-mz}}{\lambda^d}$$

Remove the unphysical ↓ oscillations in quasi-GPDs

Extrapolated

Quasi-GPDs

$$\tilde{H}(x, P_z, t) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} M^R(z, P_z, t)$$



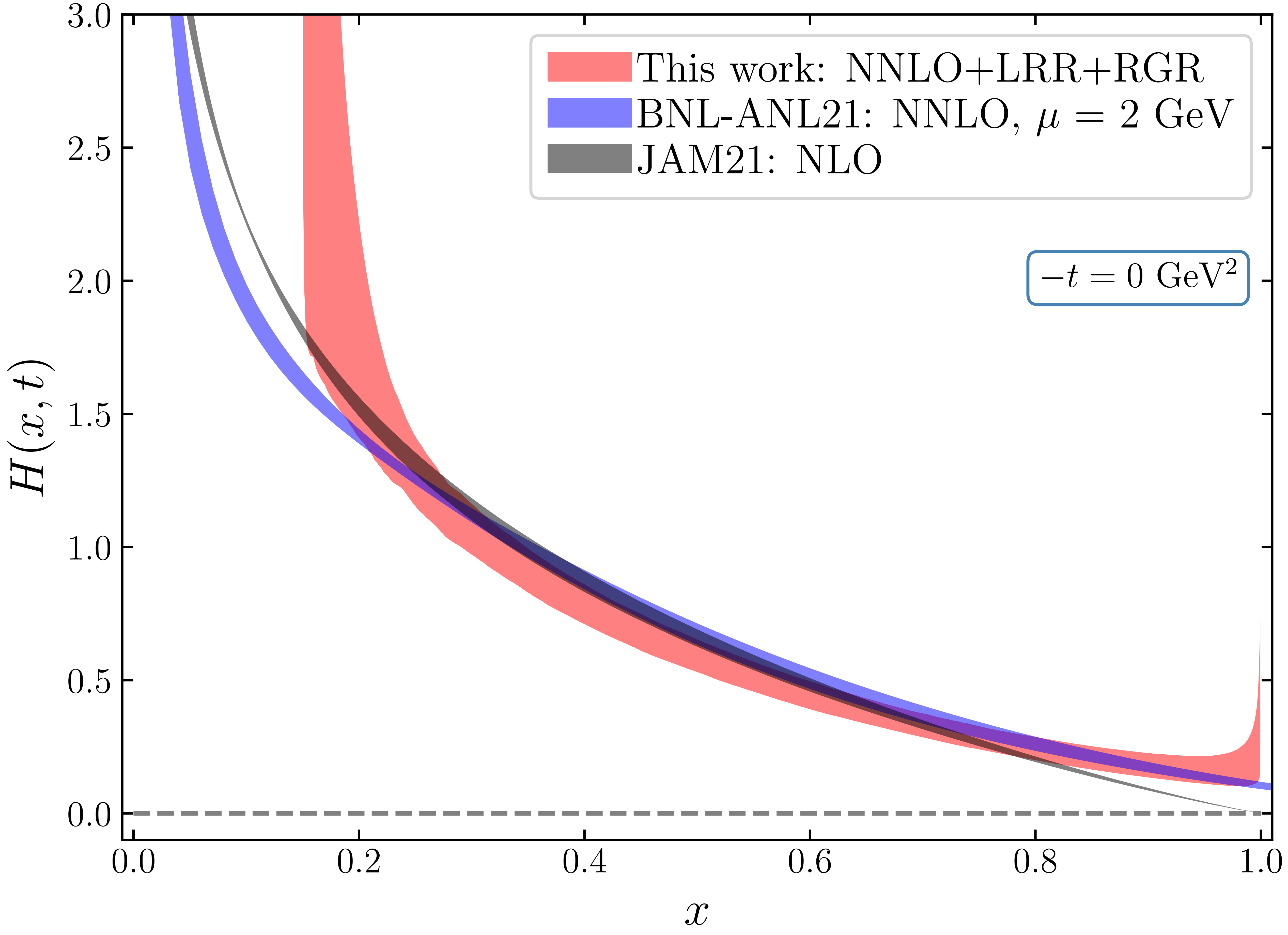
LC GPDs

NNLO+LRR+RGR ($\kappa = [1, \sqrt{2}, 2]$)

quasi-GPDs

$$H(x,t) = \int \frac{dk}{|k|} \int \frac{dy}{|y|} C_{\text{evo}}^{-1}\left(\frac{x}{k}, \frac{\mu}{\mu_0}\right) C^{-1}\left(\frac{k}{y}, \frac{\mu_0}{yP_z}, |y|\lambda_S\right) \tilde{H}(y, P_z, t, z_S, \mu_0) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{(xP_z)^2}, \frac{\Lambda_{\text{QCD}}^2}{[(1-x)P_z]^2}\right)$$

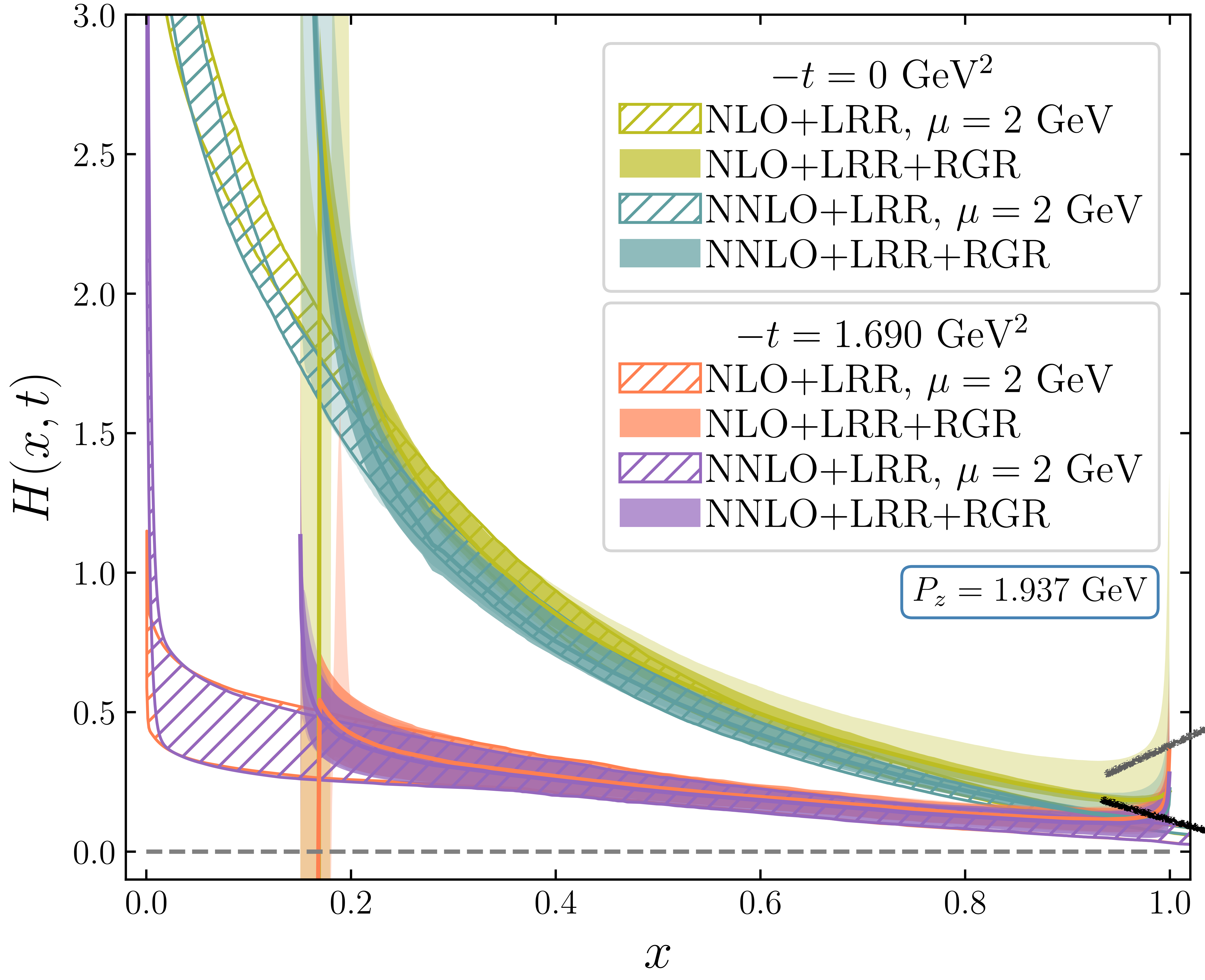
Light-cone GPDs



- **GPDs at $-t = 0$ GeV^2 —— PDFs**

  agree well with BNL-ANL21 and JAM21

Light-cone GPDs

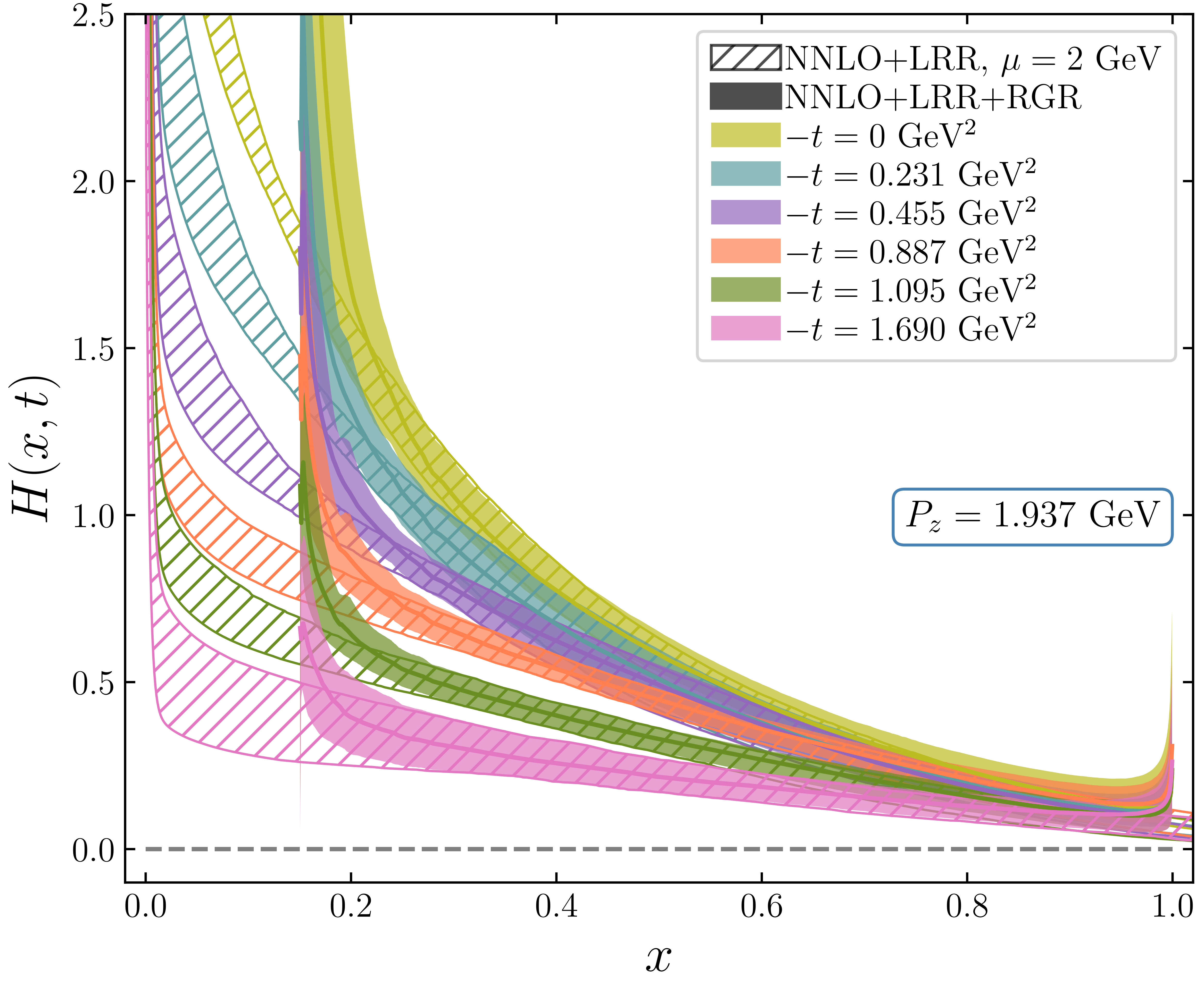


- Scale variation
  - More significant at small and large x
  - Smaller at higher order
  - Smaller at larger $-t$
- Good convergence: NLO & NNLO

Systematic error: $1 < \kappa < 2$

Statistical error: $\kappa = \sqrt{2}$

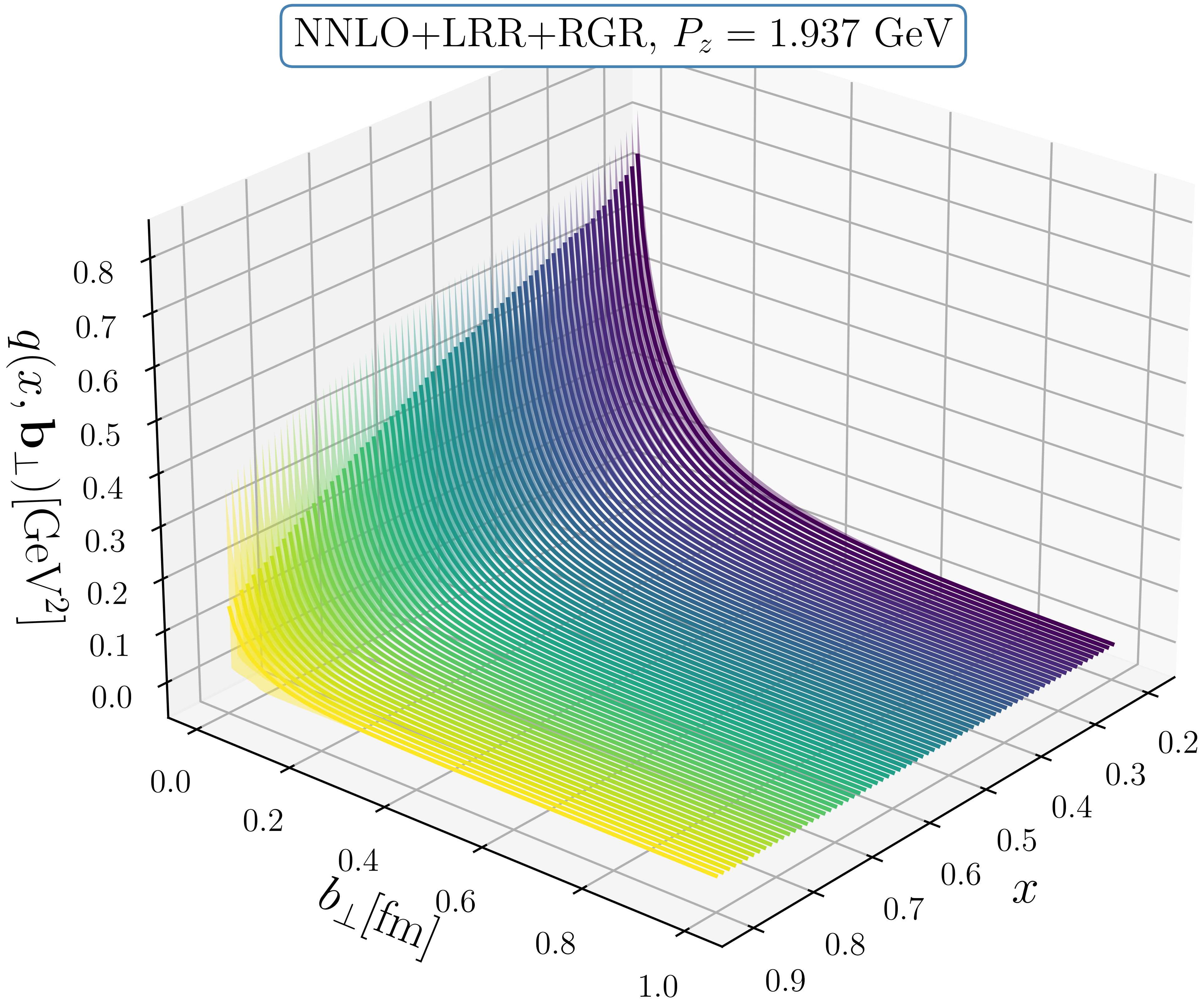
Light-cone GPDs: t-dependence



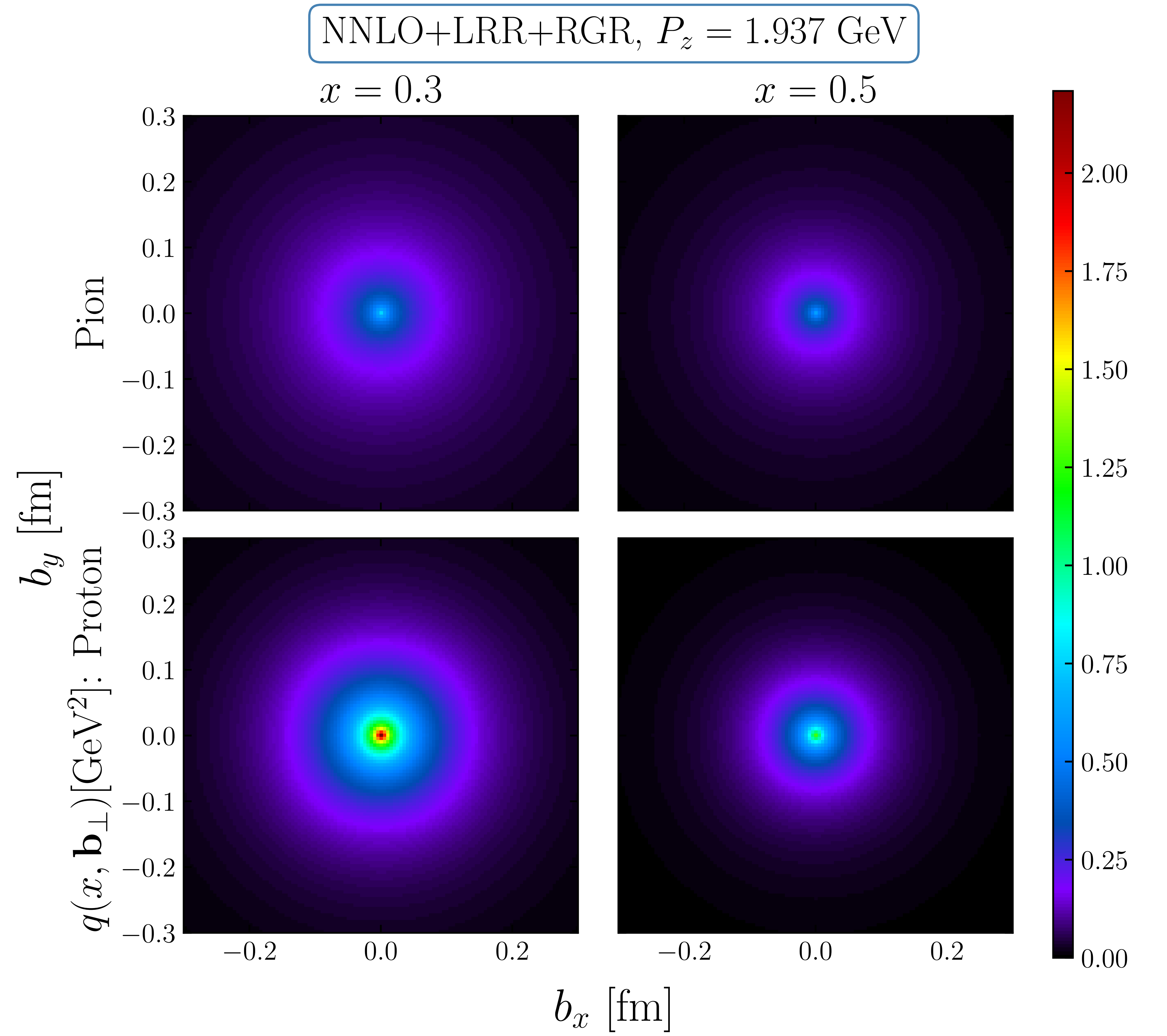
- At fixed x, H decreases as $-t$ increases
- The decrease of H along x is slower at larger $-t$
- RGR: logarithms diverge,
  perturbation theory breaks down

Impact-parameter-space parton distributions (IPDs)

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, \xi = 0, \mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$



Quarks with higher x are more concentrated



LC Proton GPDs: Cichy et al., arXiv:2304.14970

- Distributions are more concentrated at larger x
- Proton is more broader than pion

Summary

- **Pion and kaon EMFF at the physical point**
  - $-t$ up to 10 and 28 GeV^2 for the pion and kaon
  - Consistent with the existing experimental and the collinear factorization results
  - Serve as benchmark QCD predictions for model-based studies and the future experimental measurements
- **Pion LC GPD in the asymmetric frame**
  - Hybrid-scheme renormalization
  - Matching with NNLO + LRR + RGR
  - t-, x-dependence of the LC GPDs, IPDs
  - Provide a comprehensive three-dimensional imaging of the pion structure

Thanks for your attention!

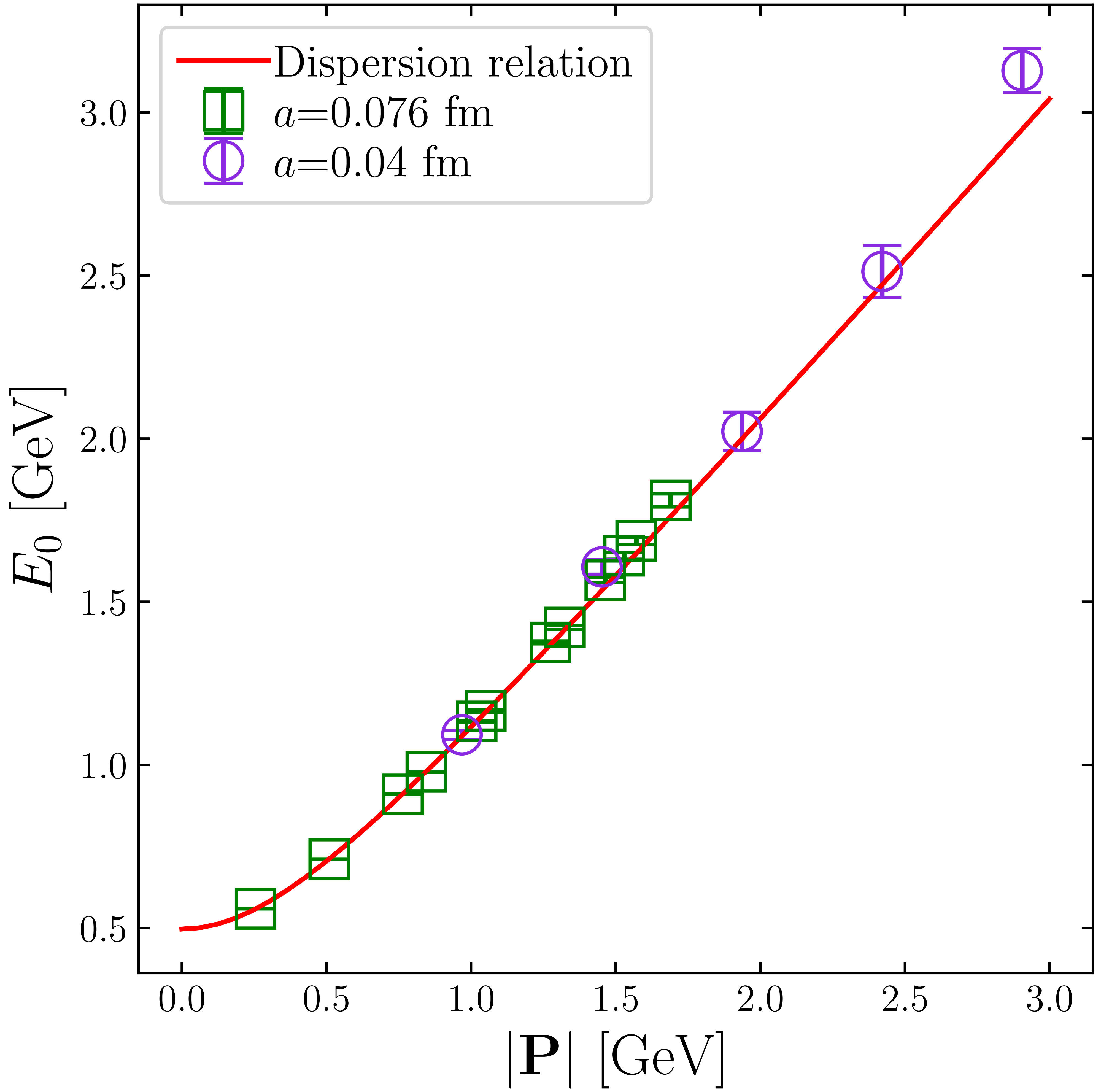
Backup

Lattice Setup

- $N_s^3 \times N_t = 64^3 \times 64$
- Pion / Kaon: $a = 0.076$ fm / $a = 0.04, 0.076$ fm
- HISQ action + Wilson-Clover action, **at the physical point**

Meson	a[fm]	$\mathbf{n}_\mathbf{P}^f = (n_{P_1}^f, n_{P_2}^f, n_{P_3}^f)$	$n_{k_3}^f$	$\mathbf{n}_\mathbf{P}^i = (n_{P_1}^i, n_{P_2}^i, n_{P_3}^i)$	$n_{k_3}^i$	Q^2[GeV^2]	(#ex, #sl)
Pion	0.076	(0, 0, -3)	-2	(0,0,2)	2	1.56	(3, 96)
				(0,0,3)		2.34	
				(0,0,4)		3.12	
				(2,0,3)		2.58	
		(0, 0, -5)	-4	(0,0,3)	4	3.90	(7, 224)
				(0,0,4)		5.20	
				(0,0,5)		6.50	
				(2,0,4)		5.50	
				(2,0,5)		6.75	
		(0, 0, -6)	-5	(0,0,5)	5	7.80	(18, 576)
				(0,0,6)		9.35	
				(2,0,5)		8.10	
				(2,0,6)		9.61	

Extract Energy and Amplitude

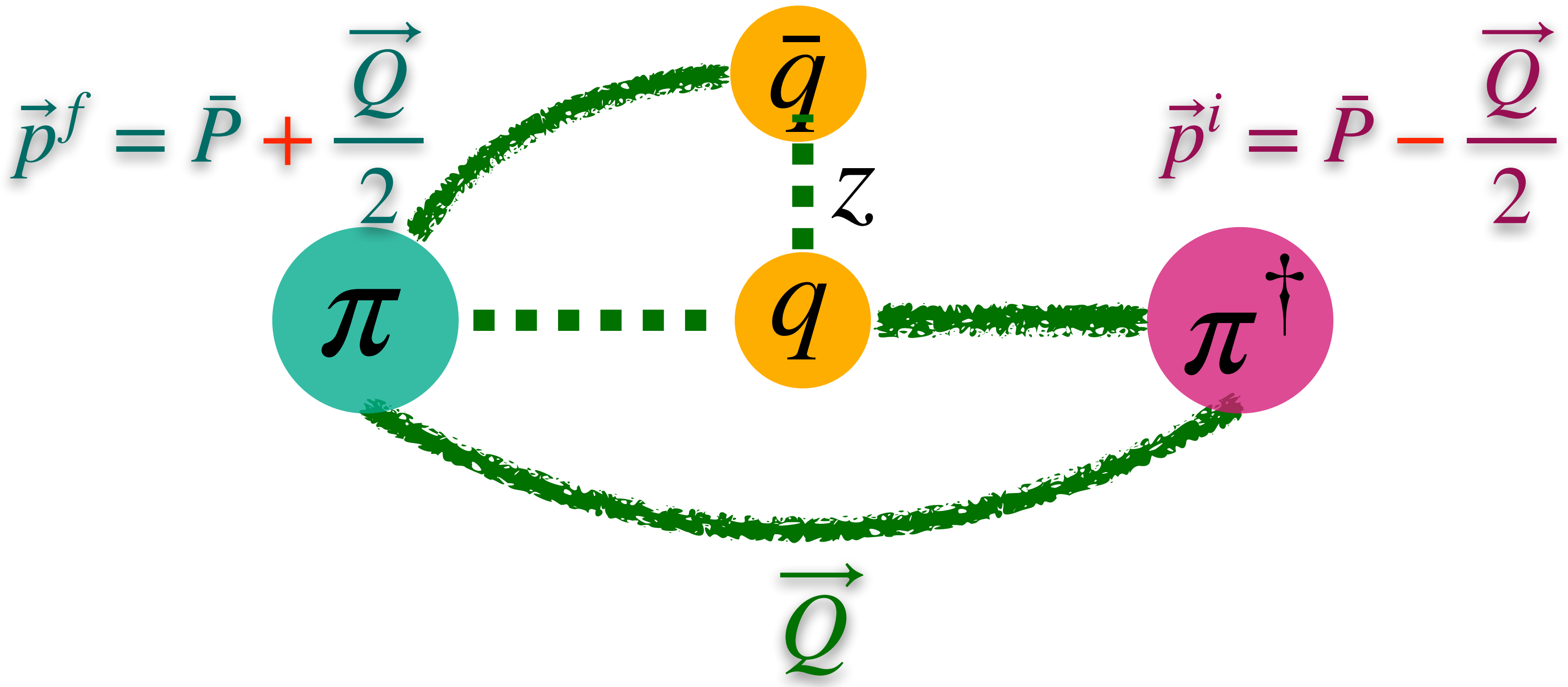


Frame-independent approach

$N_{\text{color}} \otimes N_{\text{flavor}} \otimes N_{\text{spin}} \otimes N^3_{\text{space}} \otimes N_{\text{time}} \gtrsim 10^9$

Lattice: each $\vec{p}^f$ requires a seperate calculation -> Goal: reduce the number of $\vec{p}^f$

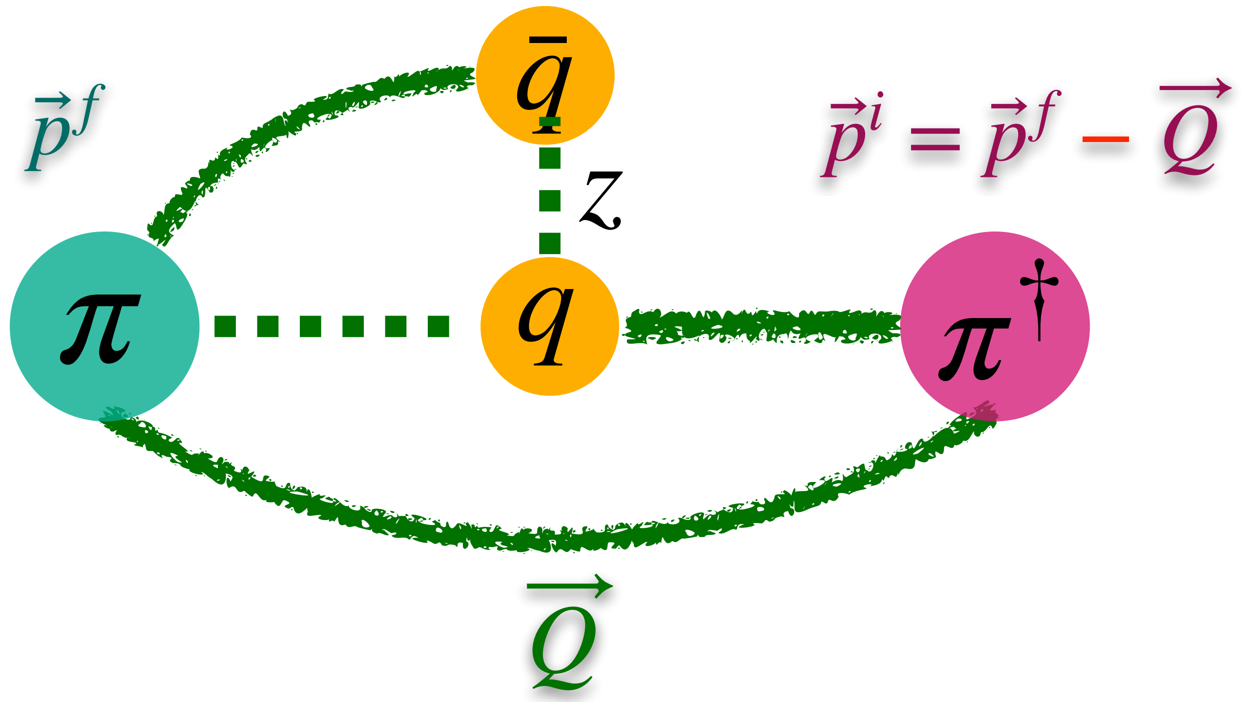
Traditional: Symmetric



- One $\vec{p}^f$ — only one $Q^2 = -t$ is useful
- Each Q^2 requires a seperate calculation

Newly proposed: Asymmetric

Bhattacharya, Constantinou et al., PRD 106 (2022)



- One $\vec{p}^f$ — several Q^2 are useful

Fix $\vec{p}^f$, vary Q^2 in one calc

Vary $\vec{p}^f$ in several calcs

Computational cost

Lattice Setup

- $N_s^3 \times N_t = 64^3 \times 64, \qquad a = 0.04$ fm
- HISQ action + Wilson-Clover action ⇒ $m_\pi^{\text{val}} = 0.3$ GeV
- Using boost smearing to enhance the signal
- Momentum transfer Q^2:   0 ~ 1.7 GeV^2

Frame	t_s/a	$\mathbf{n}^f = (n_x^f, n_y^f, n_z^f)$	m_z	P_z[GeV]	$\mathbf{n}^\Delta = (n_x^\Delta, n_y^\Delta, n_z^\Delta)$	$-t$[GeV^2]	#cfgs	(#ex, #sl)
Breit	9,12,15,18	(1, 0, 2)	2	0.968	(2, 0, 0)	0.938	115	(1, 32)
non-Breit	9,12,15,18	(0,0,0)	0	0	(0,0,0)	0	314	(3, 96)
	9,12,15,18	(0,0,2)	2	0.968	(1,2,0)	0.952	314	(4, 128)
	9,12,15	(0,0,3)	2	1.453	[(0,0,0), (1,0,0) (1,1,0), (2,0,0) (2,1,0), (2,2,0)]	[0, 0.229, 0.446, 0.855, 1.048, 1.589]	314	(4, 128)
	9,12,15	(0,0,4)	3	1.937		[0, 0.231, 0.455, 0.887, 1.095, 1.690]	564	(4, 128)