Visualization of mass distribution within the nucleon and pion

Trace anomaly form factors from lattice QCD

Phys. Rev. D 109 (9), 094504 [arXiv:2401.05496]

US Lattice Quantum Chromodynamics

Bigeng Wang
Department of Physics and Astronomy, University of Kentucky
Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley

Collaborators: Fangcheng He, Gen Wang, Jian Liang, Terrence Draper, Keh-Fei Liu, Yi-Bo Yang (χQCD collaboration)

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The mass distribution is a fundamental property of a physical object.

Yet, while a lot of information is available about the charge distribution inside the proton, nothing is known at present (2021) about its mass radius……

"Mass radius of the proton", D. Kharzeev, Phys. Rev. D.104.054015

Because of the extreme weakness of the gravitational field created by a single proton, its direct measurement at short distances is clearly impossible. Likewise, a study of graviton–proton scattering is off limits for present experiments.

Does this mean that the mass radius of the proton cannot be measured?

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Does this mean that the mass radius of the proton cannot be measured?

"**the mass radius** of the proton can be rigorously defined through the **form factor** of **the trace** of **the energy-momentum tensor (EMT)** of **QCD** in the weak gravitational field approximation, as appropriate for this problem."

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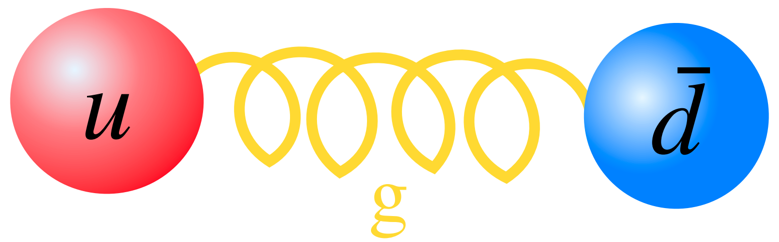
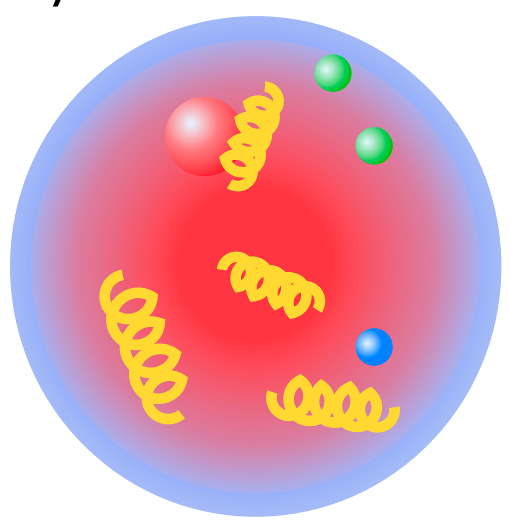
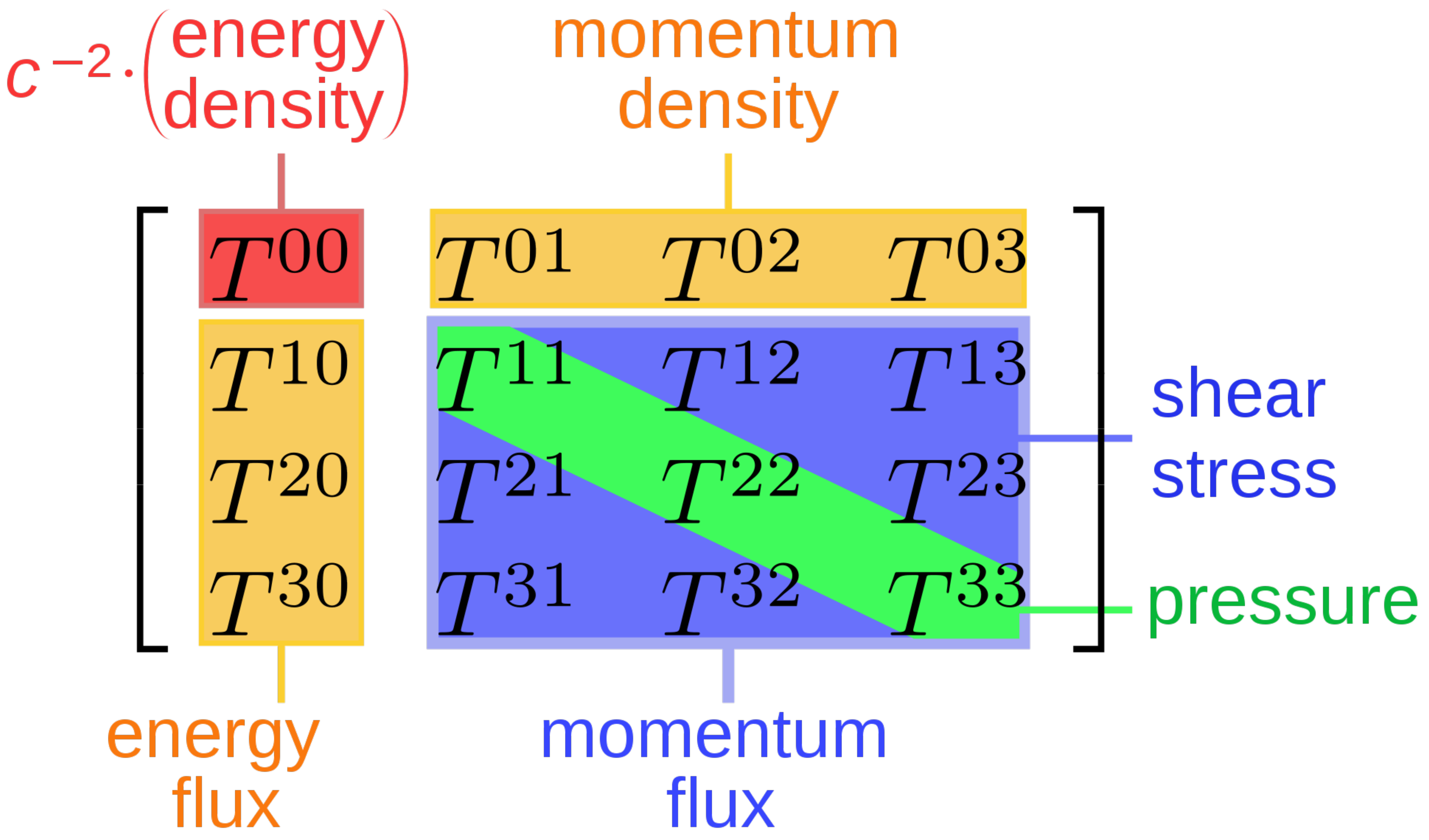
Hadron mass and the energy-momentum tensor

- The mass distribution of a hadron can be calculated through the form factor of the trace of the energy-momentum tensor (EMT) of QCD.
- In the forward limit ($Q^2 = 0$), we can obtain the hadron mass

$$m_{\mathrm{H}} = \langle T^{\mu}_{\mu} \rangle_{\mathrm{H}}$$
$$= \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\langle (T^{\mu}_{\mu})^{\mathrm{a}} \rangle \text{ trace anomaly} \neq 0}$$







$$\langle O \rangle_{\mathrm{H}} \equiv \langle \mathrm{H} | \int d^3x \, \gamma \, O(x) \, | \mathrm{H} \rangle / \langle \mathrm{H} | \mathrm{H} \rangle$$

Trace anomaly form factors and GFF

X. Ji, arXiv:2102.07830 [hep-ph]

K.-F. Liu, arXiv:2302.11600 [hep-ph]

Trace Anomaly Form Factors

$$T^{\mu}_{\mu} = \sum_f m_f \overline{\psi}_f \psi_f + \underbrace{\left[\sum_f m_f \gamma_m(g) \overline{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^2\right]}_{(T^{\mu}_{\mu})^{\mathrm{a}}\ trace\ anomaly,\ RG\ invariant}$$

Energy-momentum Tensor

$$T^{\mu\nu} = \hat{T}^{\mu\nu} + \bar{T}^{\mu\nu}$$

Trace part

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$$\langle r^2\rangle_m(\mathrm{H}) \sim \langle r^2\rangle_{ta} = -6\left(\frac{dA(Q^2)}{dQ^2} + \frac{3D(0)}{M^2} - \frac{d\mathscr{F}_\sigma(Q^2)}{dQ^2}\right)$$

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Drawing an analogy to the physics of the proton, the electron scattering experiments reveal the spatial distribution of quarks (matter visible to photons) but do not directly constrain **the spatial distribution of gluons—“dark matter of QCD”** that is not visible to photons.”

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Trace anomaly and hadron mass

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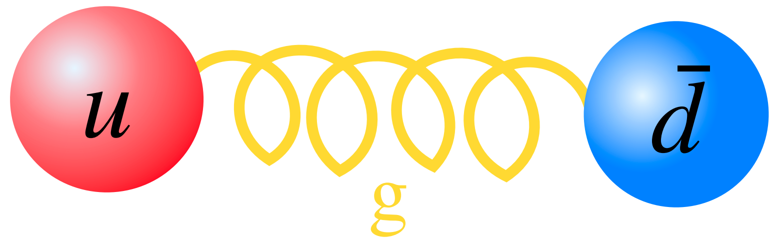

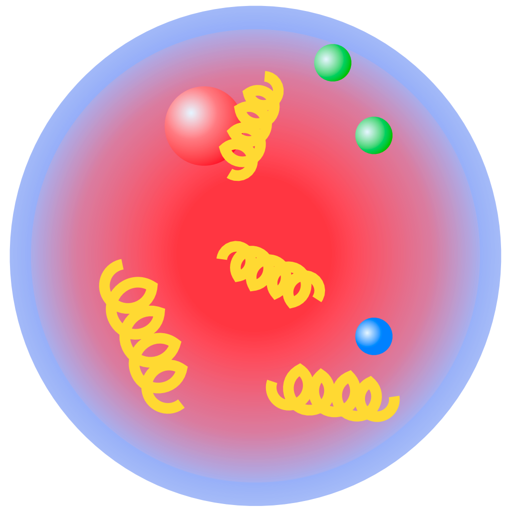

D. E. Kharzeev, Phys. Rev. D 104, 054015 (2021)

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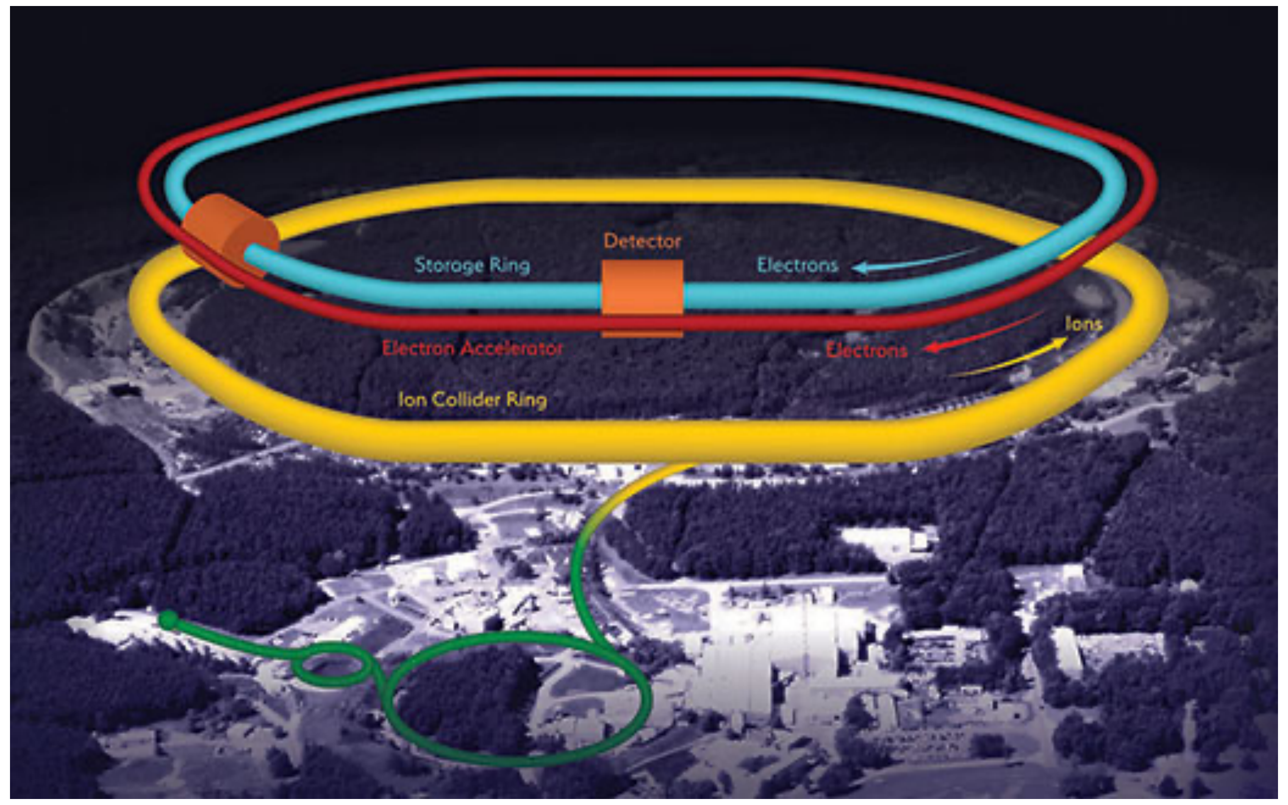

EIC at BNL(https://flic.kr/p/2ncjFe7)

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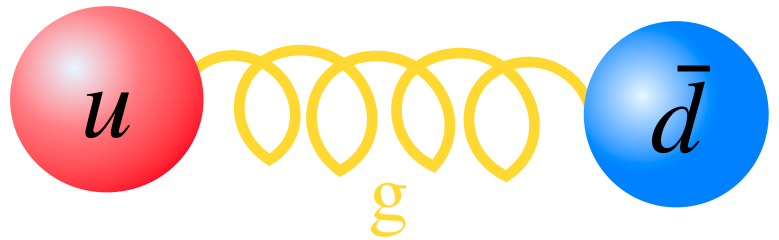

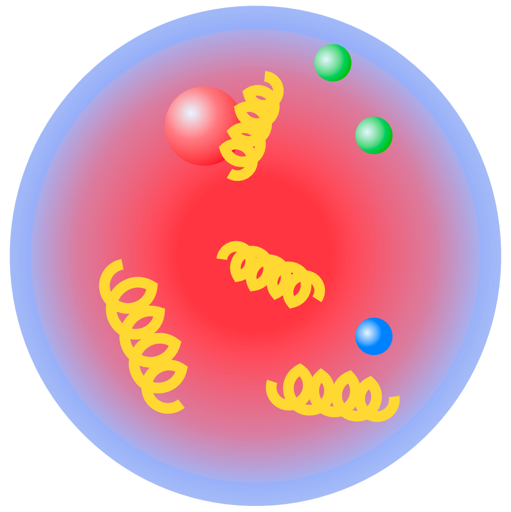

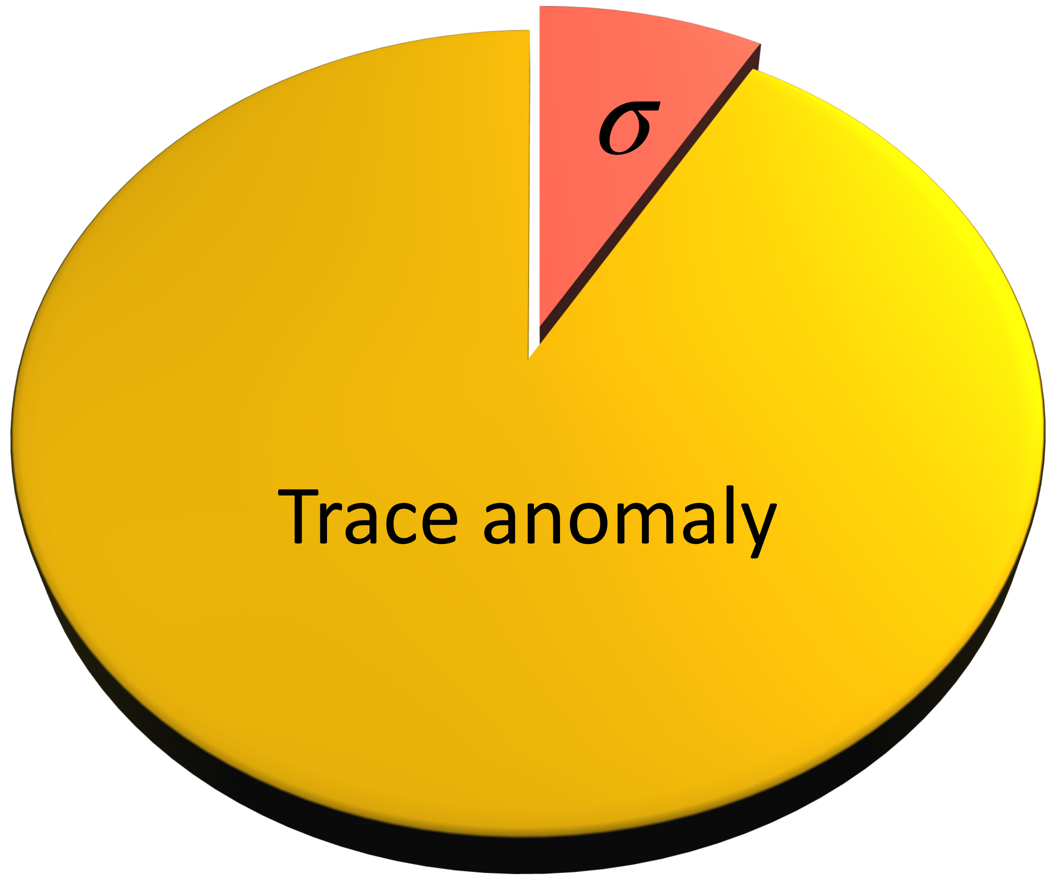

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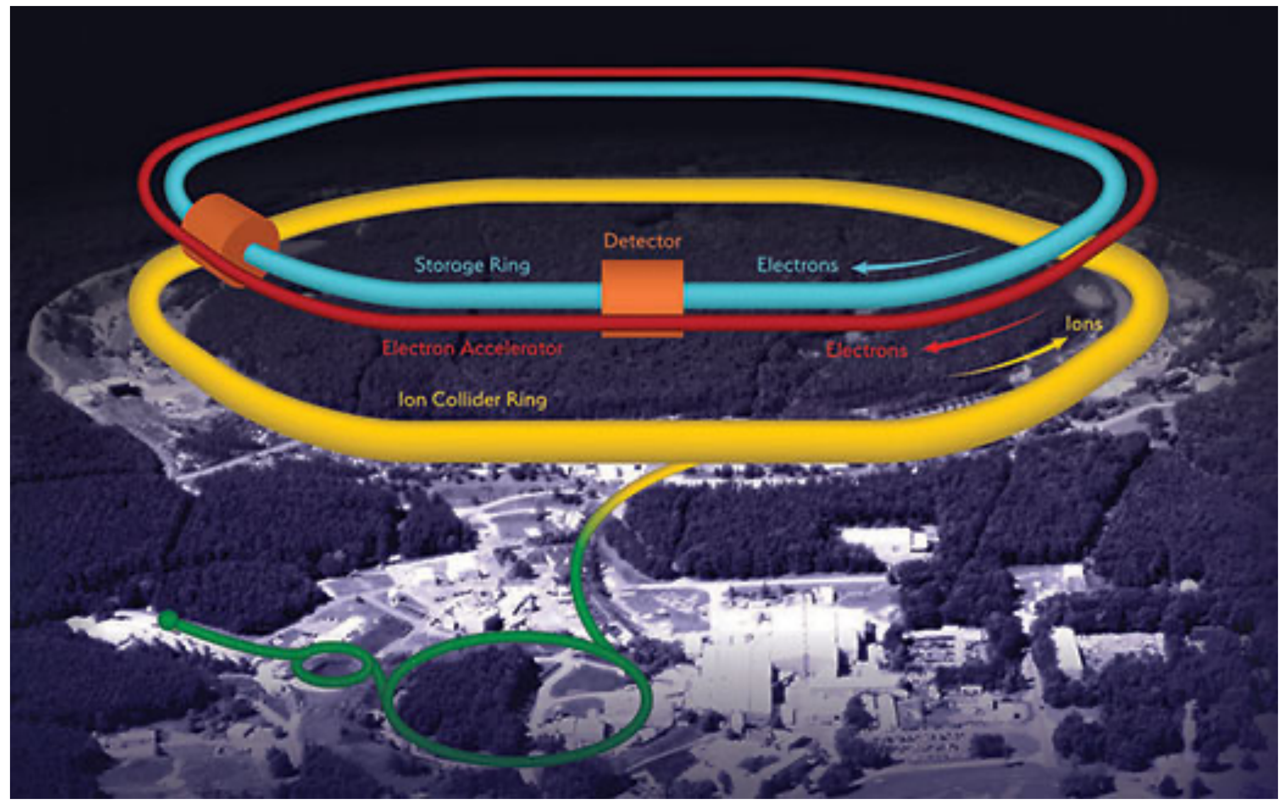

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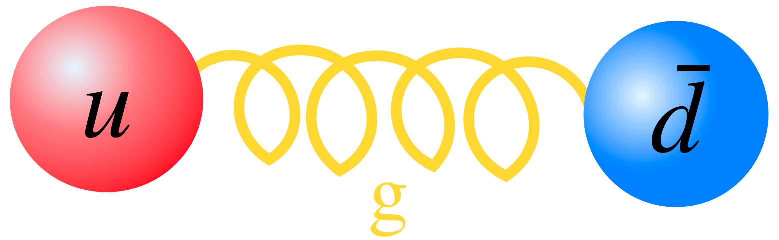

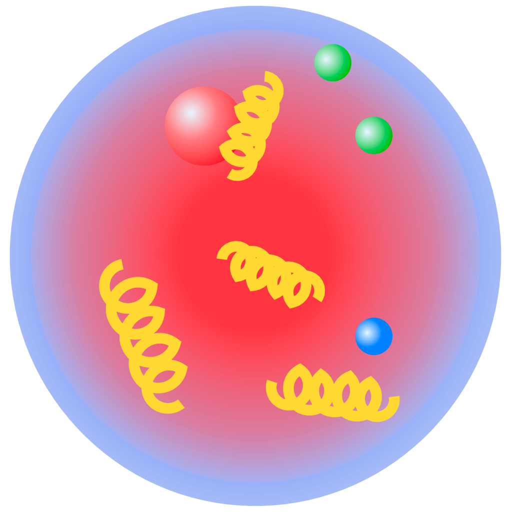

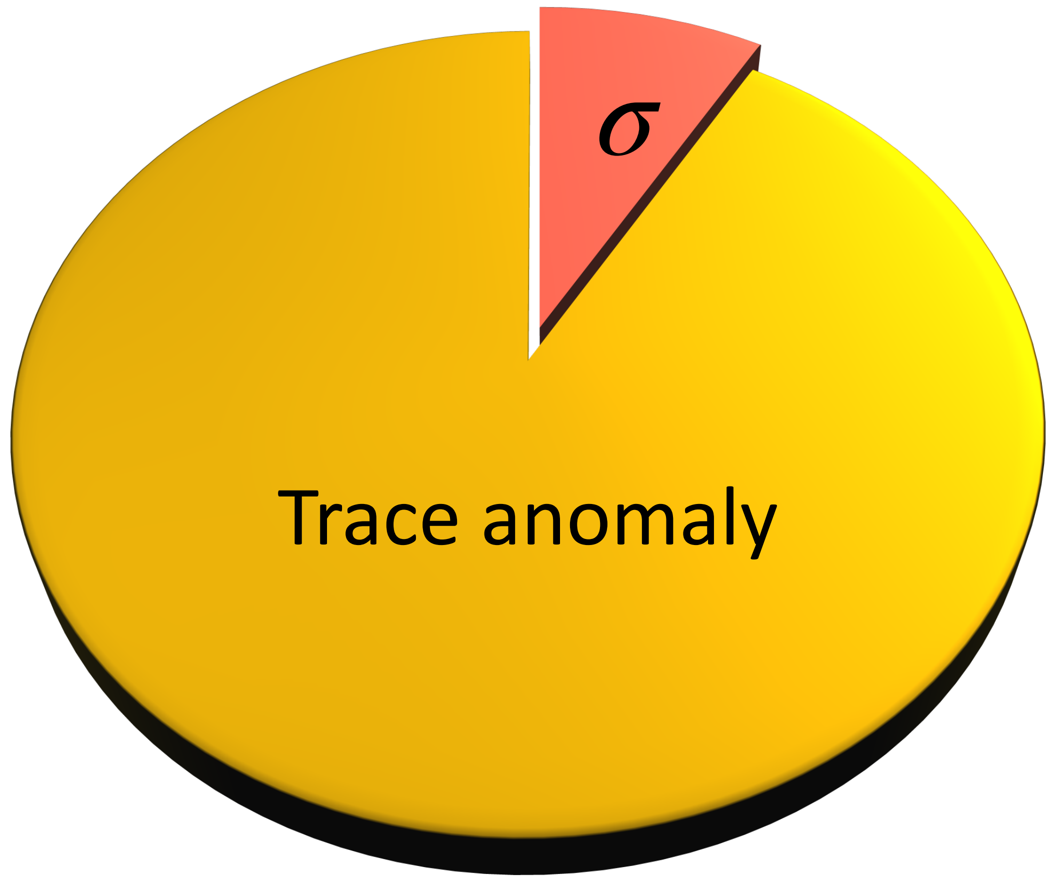

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Spatial distributions of mass?

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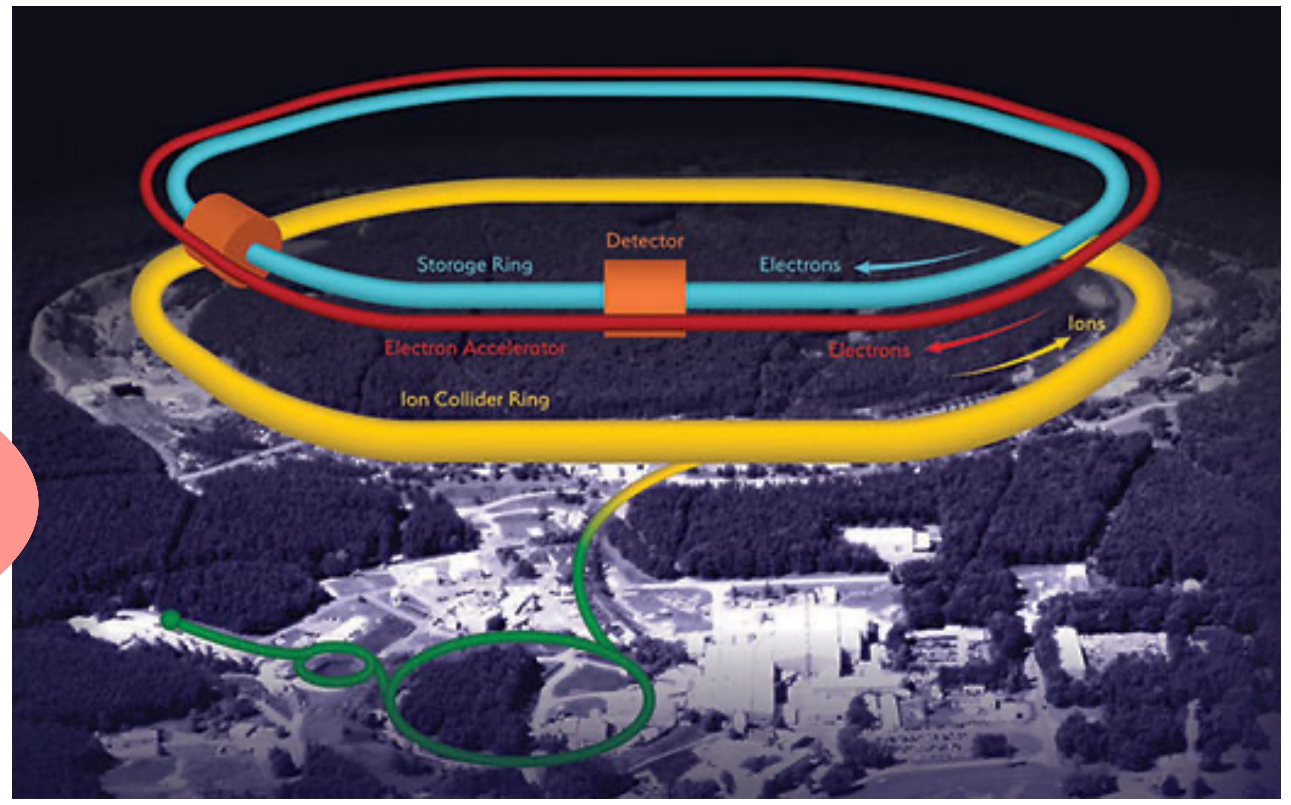

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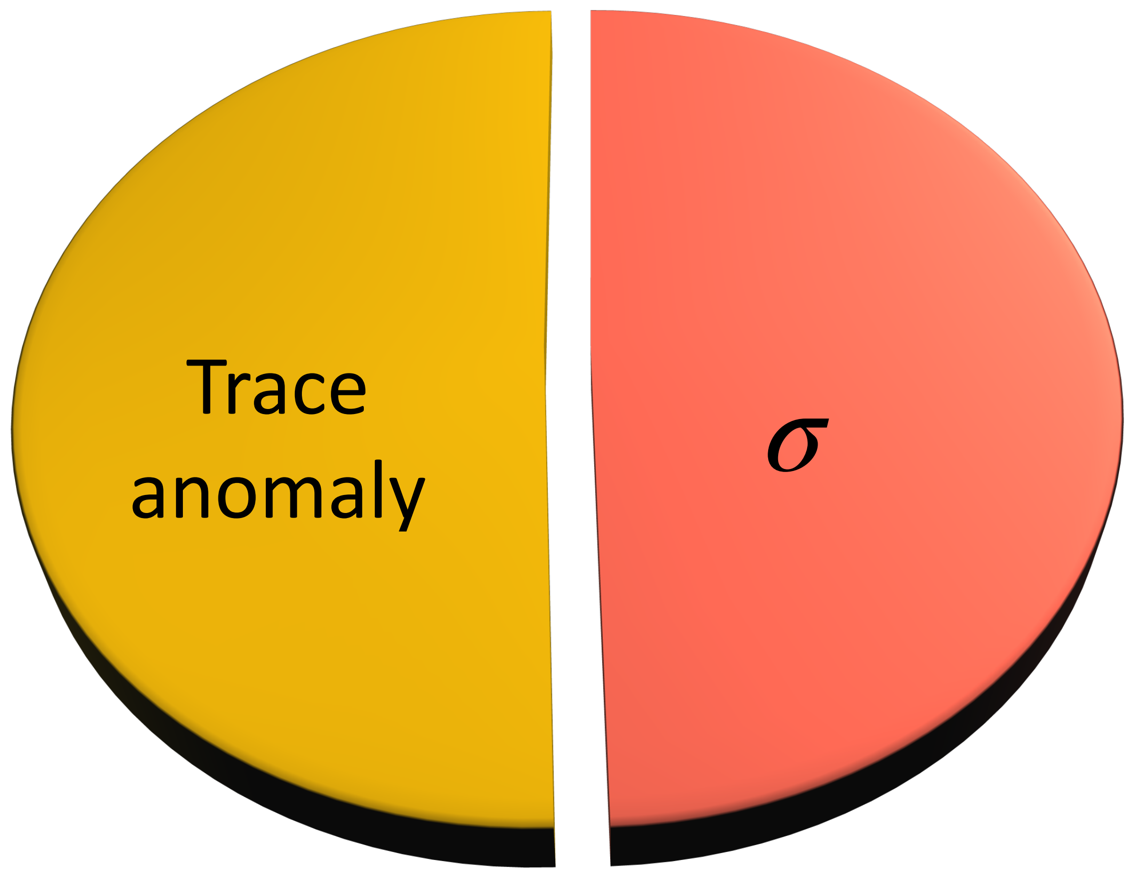
The "pion mass puzzle"

$$\underbrace{m_\pi}_{\propto \sqrt{m_f}} = \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_\pi}_{\text{the } \sigma \text{ term} \propto \sqrt{m_f}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_\pi}_{\langle (T^\mu_\mu)^{\mathrm{a}} \rangle \text{ trace anomaly} \propto \sqrt{m_f}}$$



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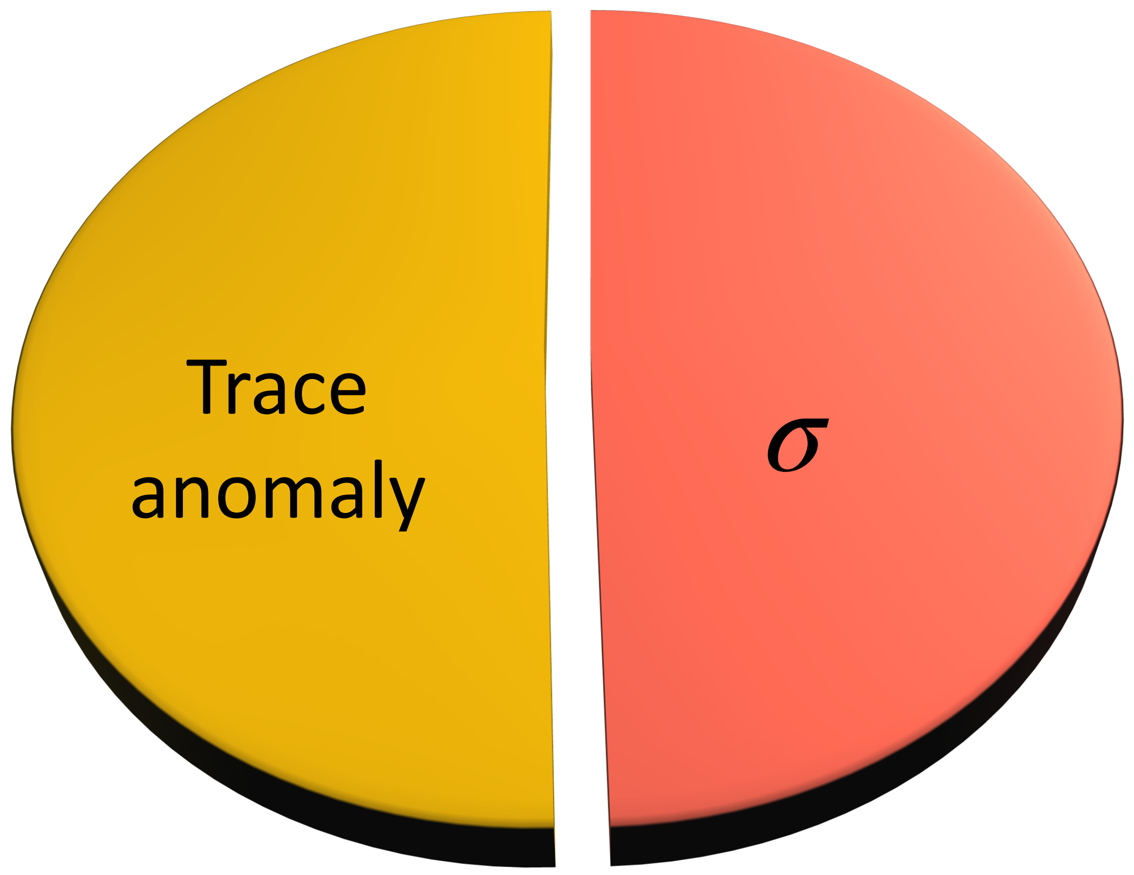

- Why does the trace anomaly(conformal symmetry breaking) have a chiral-symmetry-related behavior? **K.-F. Liu, Phys. Rev. D 104, 076010 (2021)**

- **What kind of structure** change can facilitate this attribute when approaching the chiral limit? **K.-F. Liu, Phys. Lett. B 849, 138418 (2024) arXiv:2302.11600 [hep-ph]**

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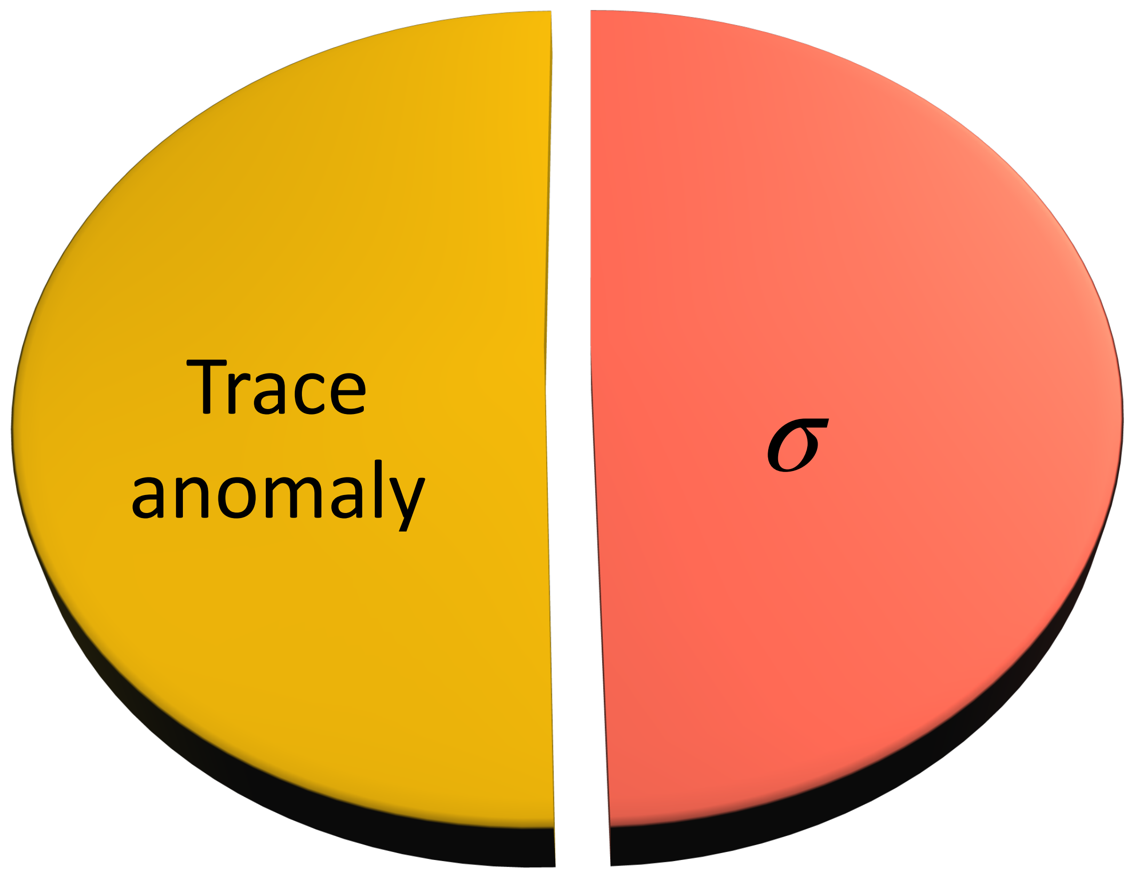

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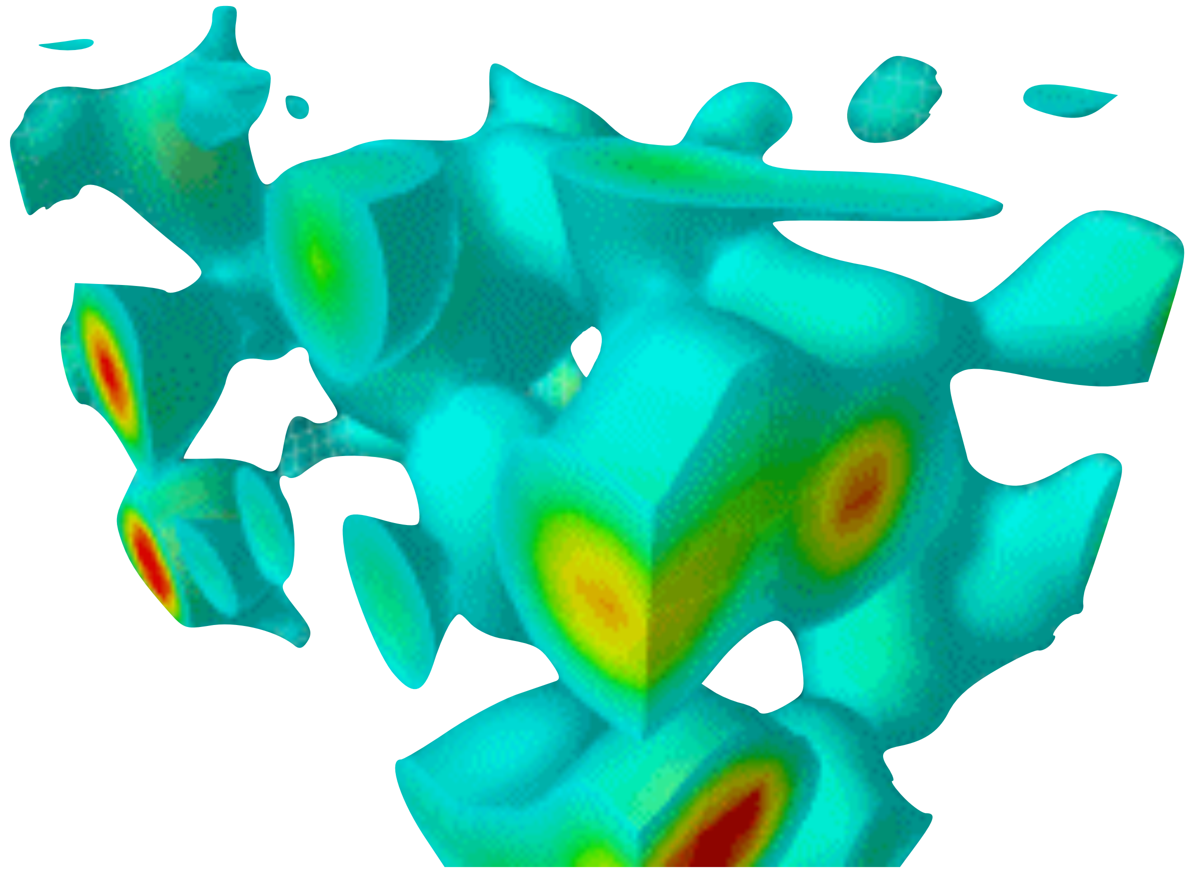
$\langle p' | T^\mu_\mu | p \rangle$ → $\mathscr{F}_{\mathrm{m,H}}(Q^2)$

Matrix elements → Form factors → Spatial distributions of mass?

Trace anomaly form factors from lattice QCD

Trace anomaly form factors from lattice QCD

QCD lattice ensembles with chiral fermion actions (overlap on DWF)



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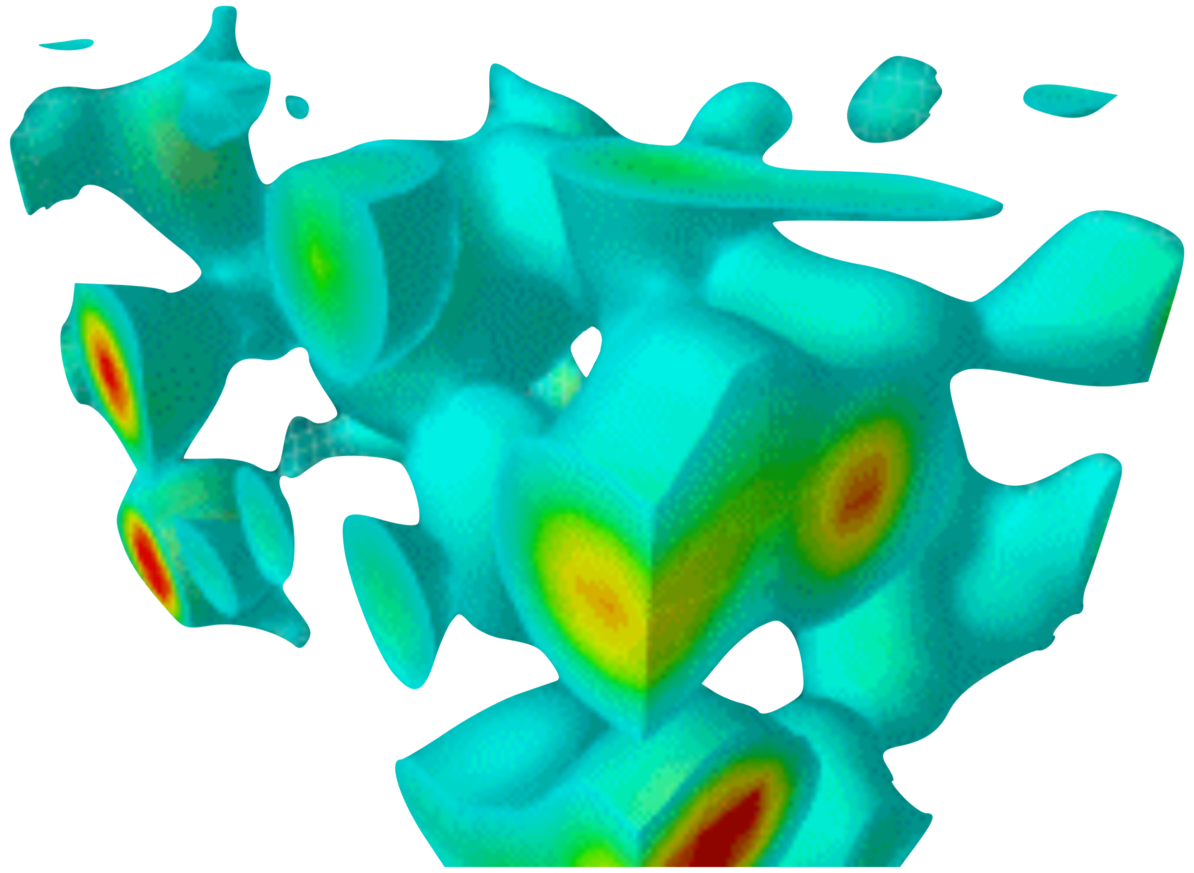

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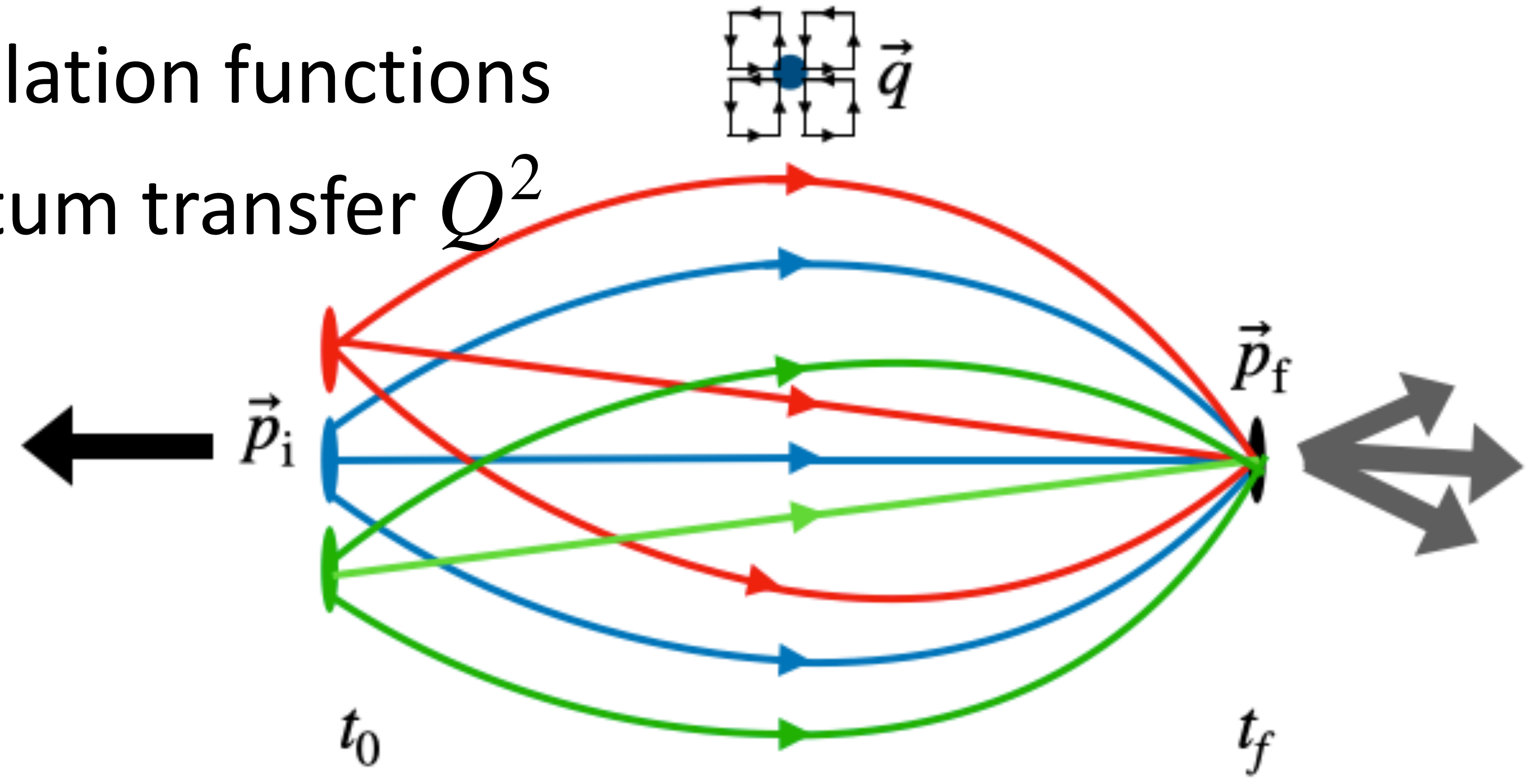
3pt/2pt correlation functions with momentum transfer Q^2



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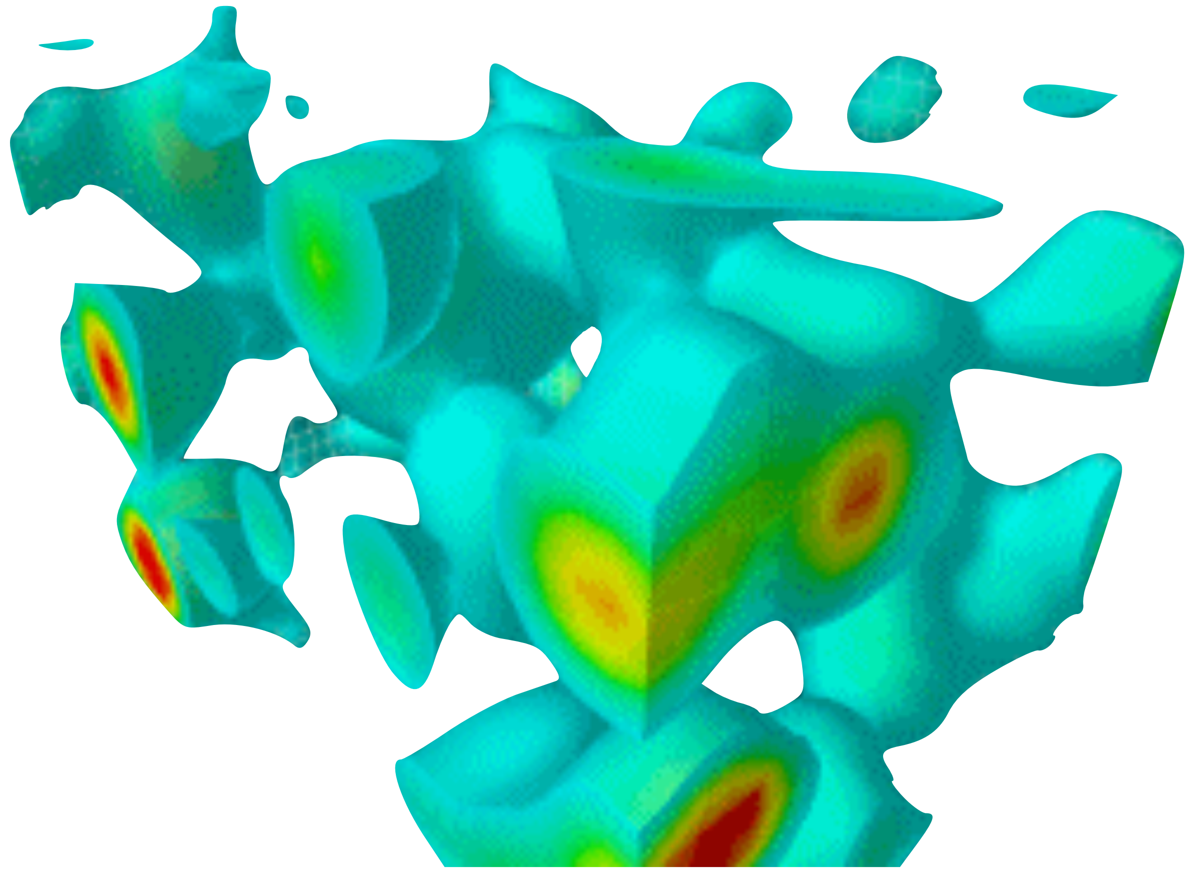

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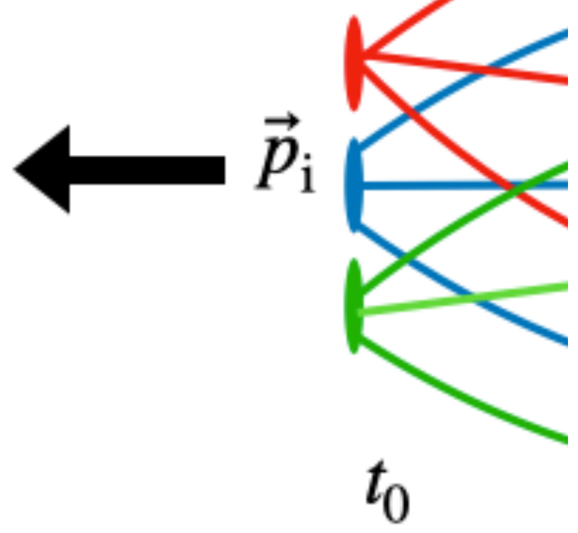

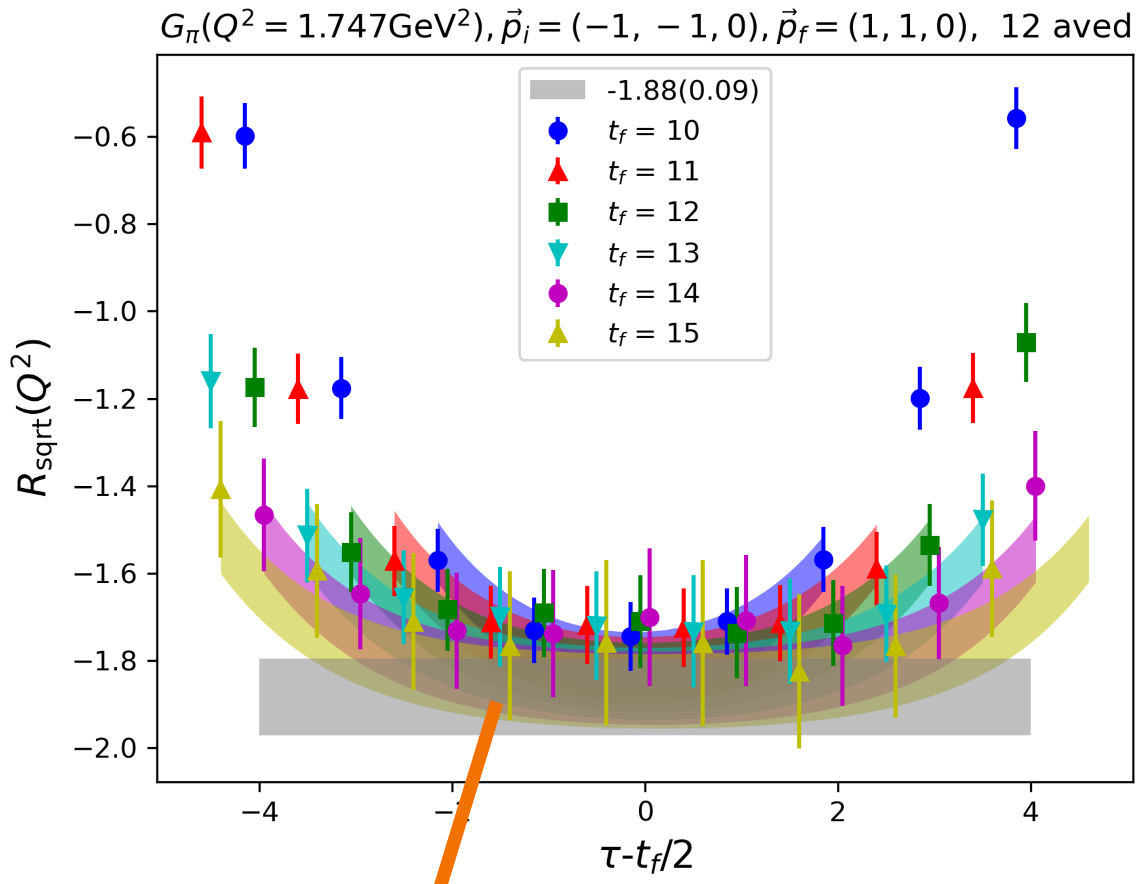

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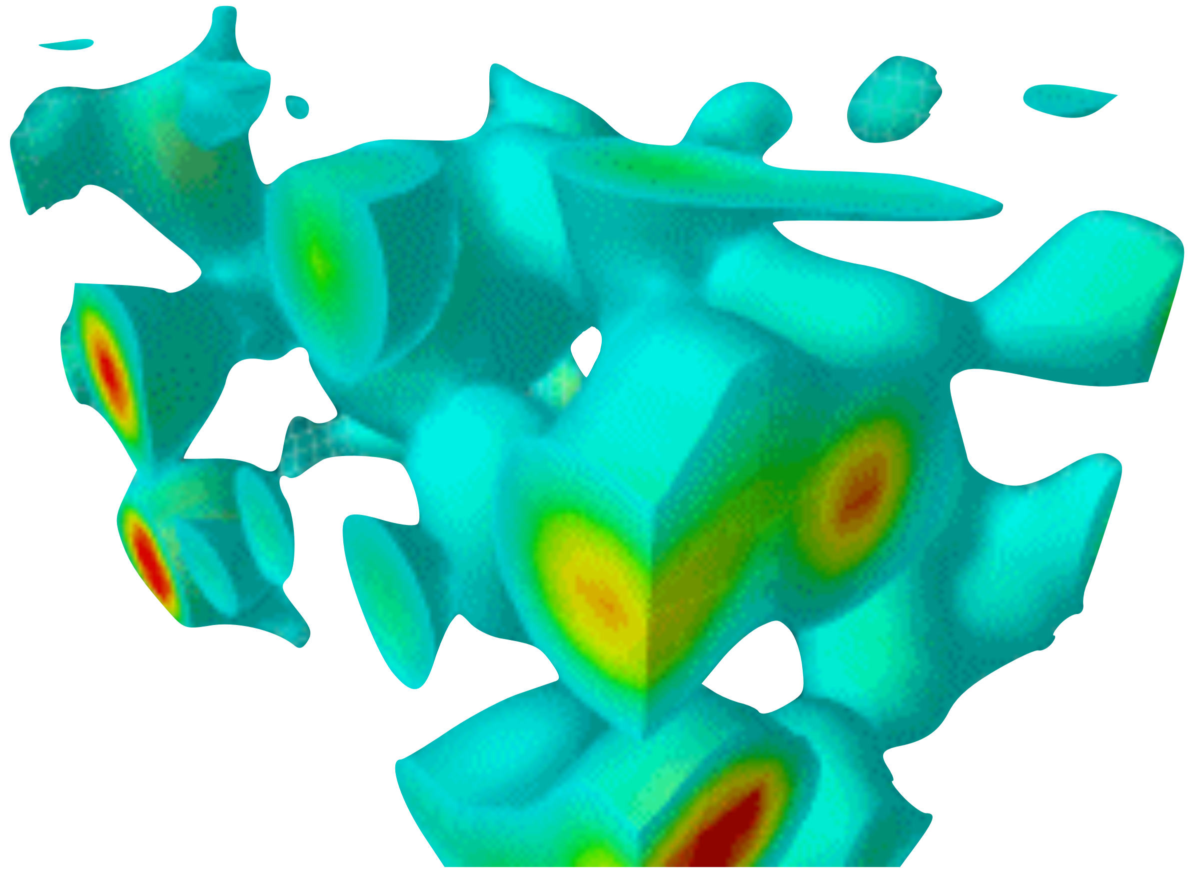

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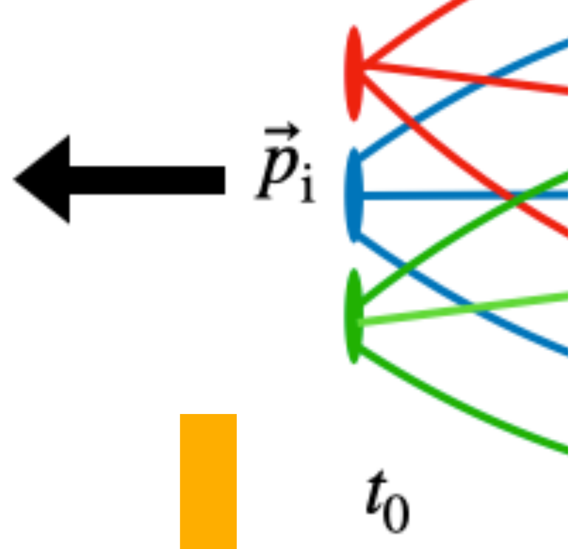
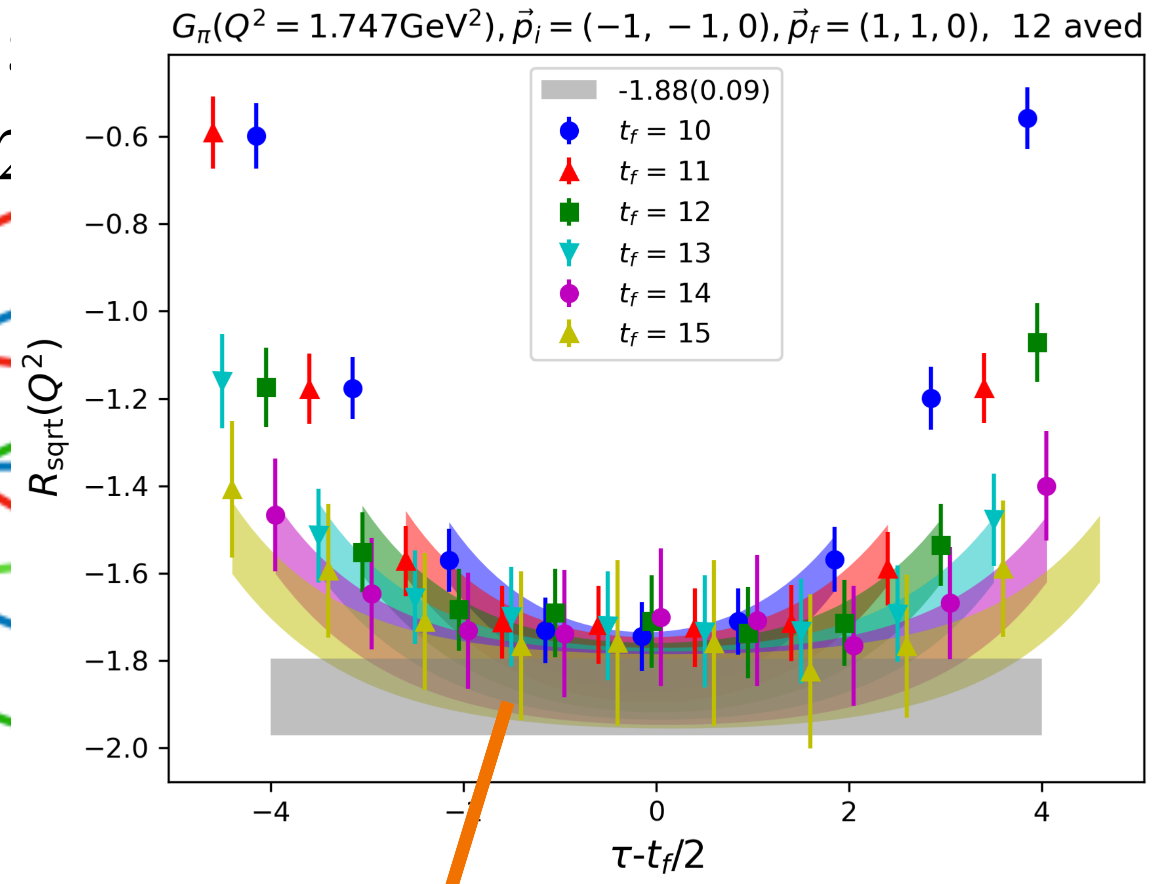
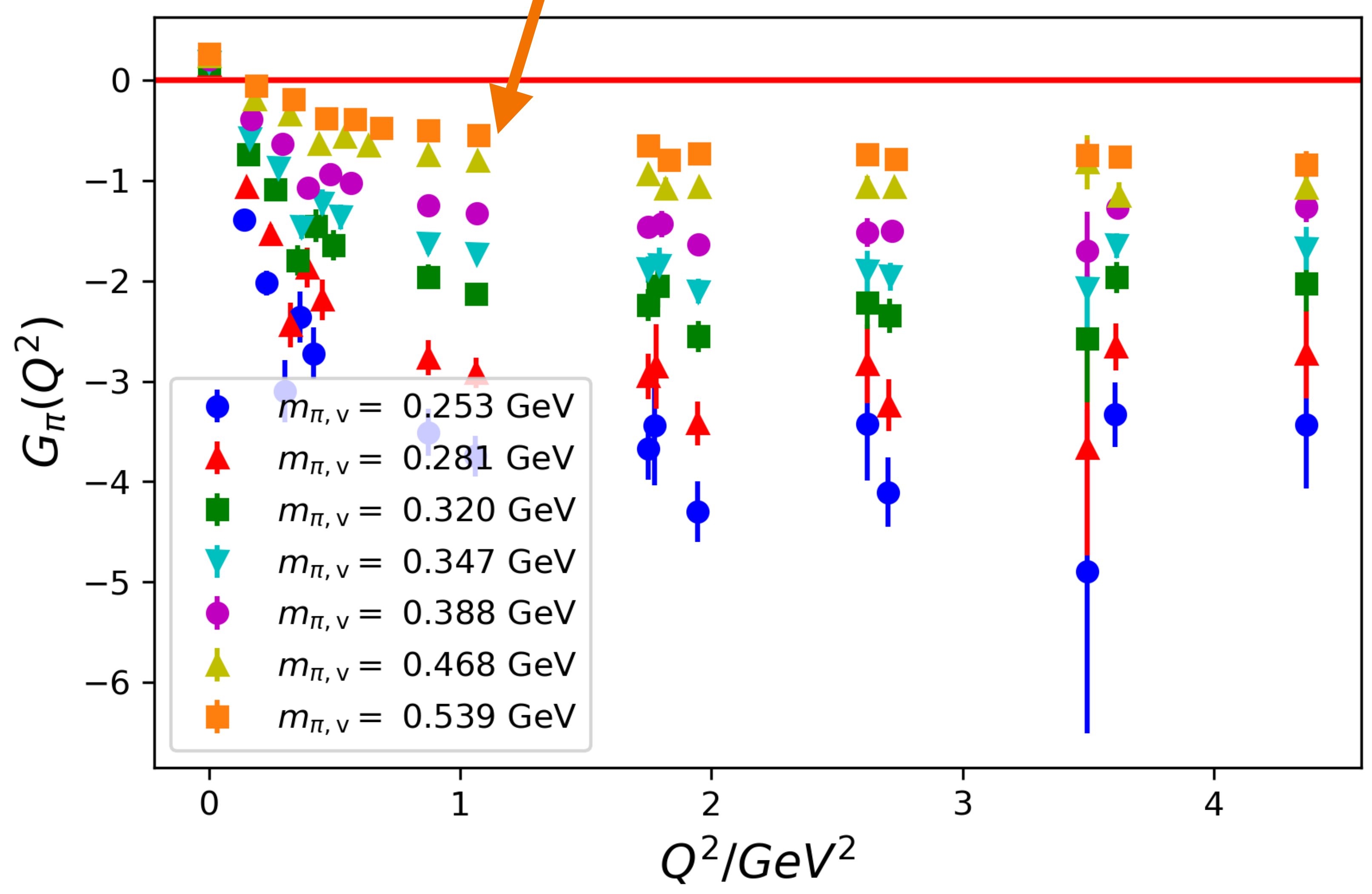




form factors $\mathscr{F}_{\mathrm{m,H}}(Q^2)$



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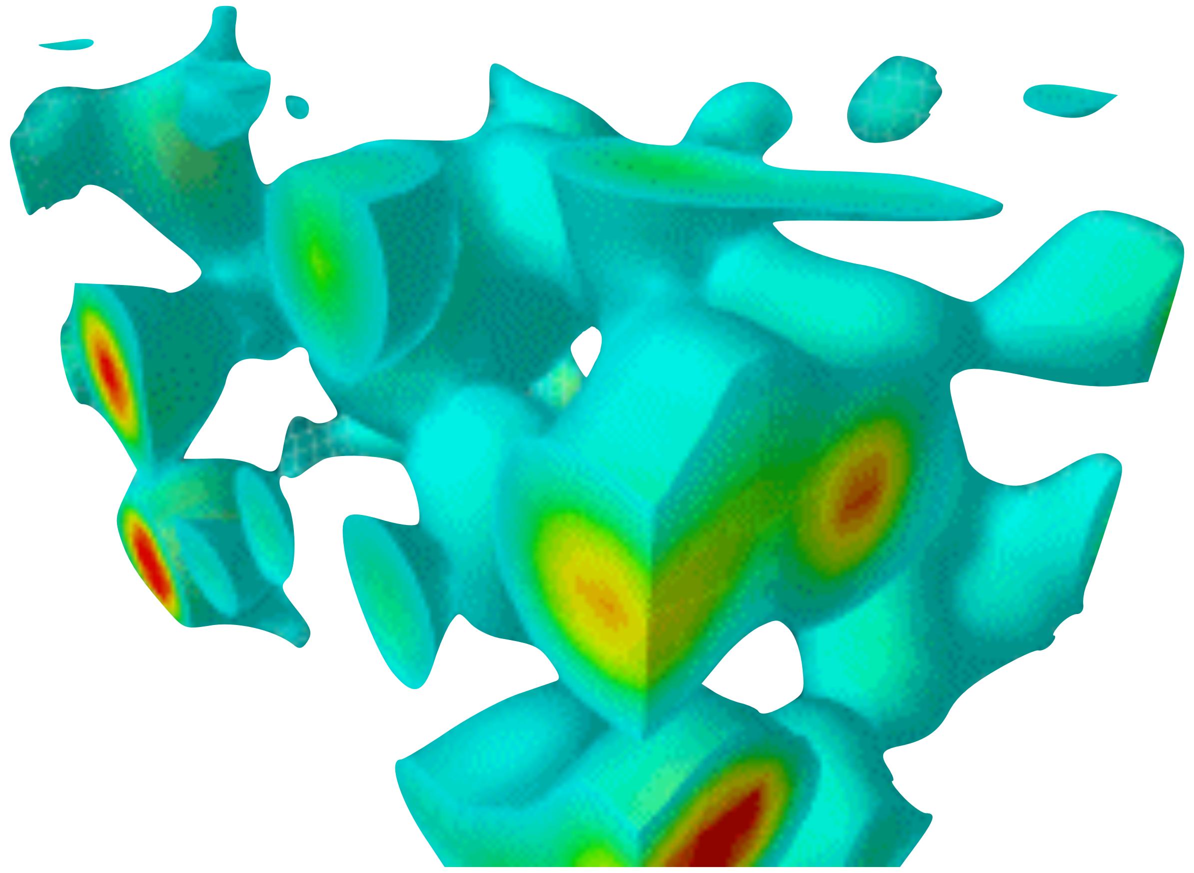

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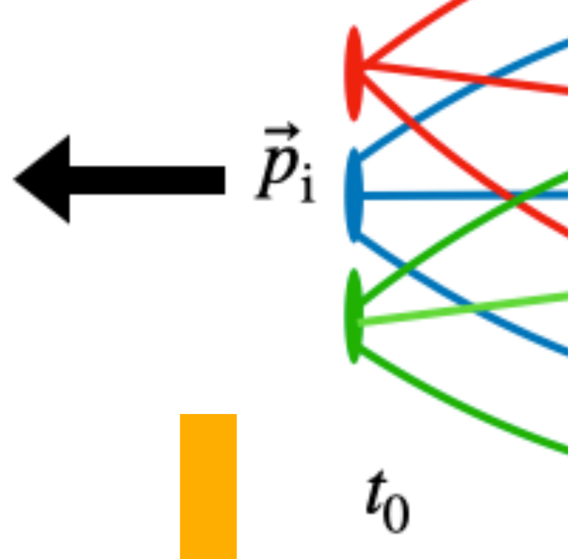
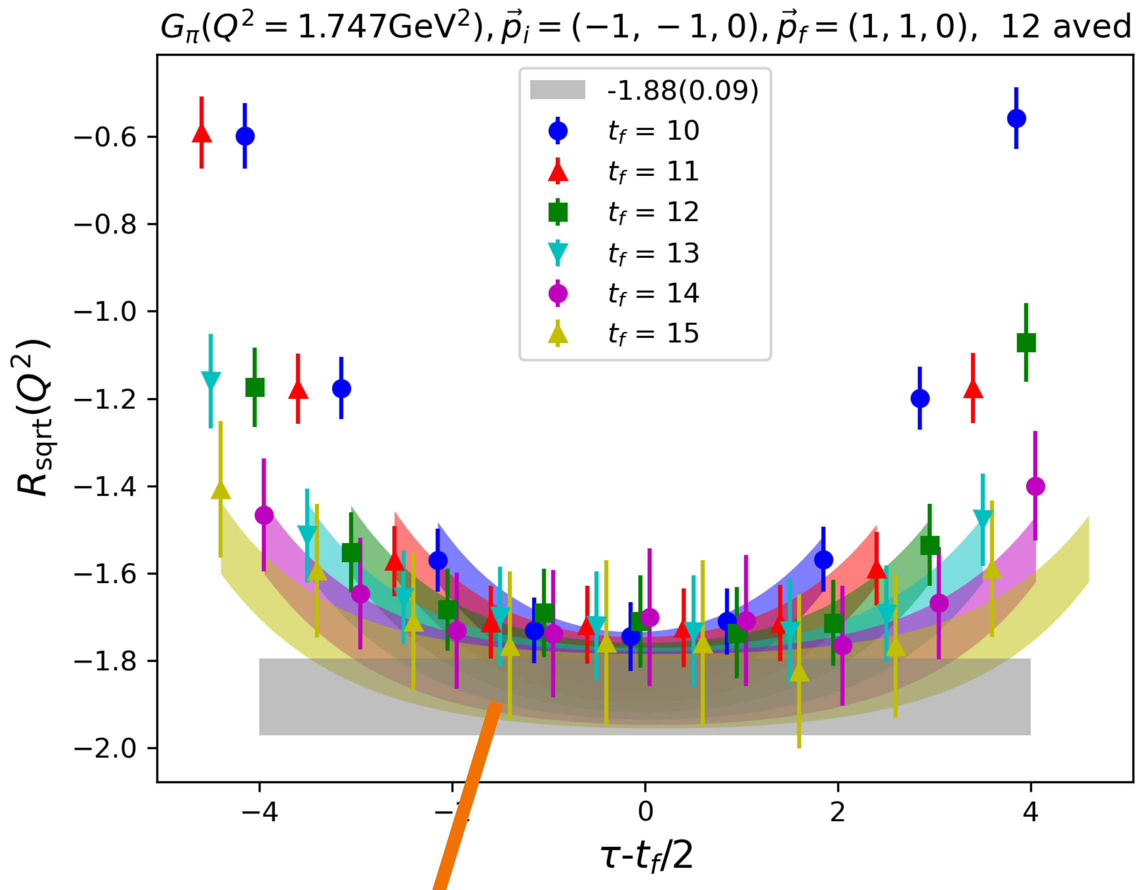
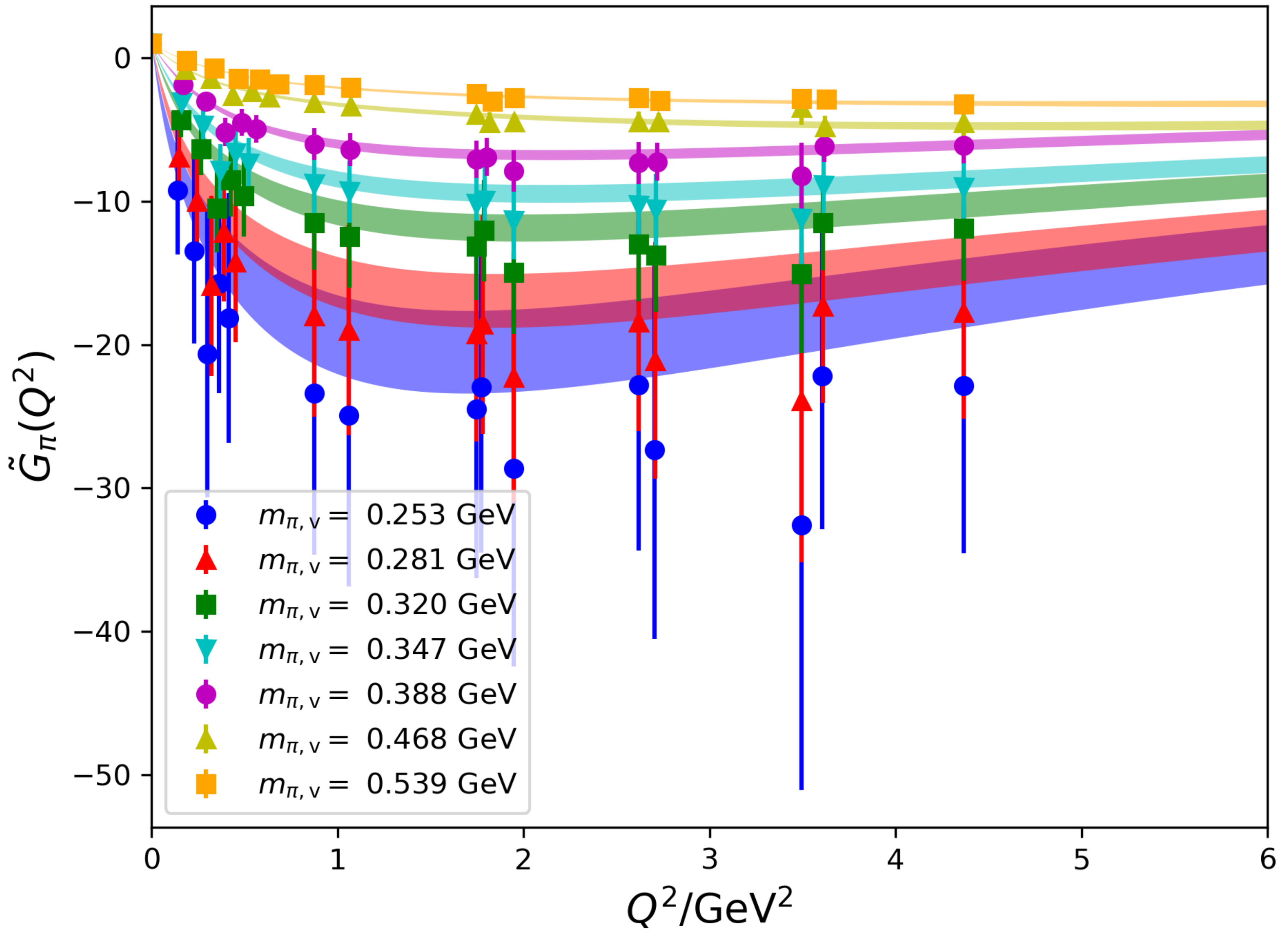




form factors $\mathscr{F}_{\mathrm{m,H}}(Q^2)$ z-exp fit



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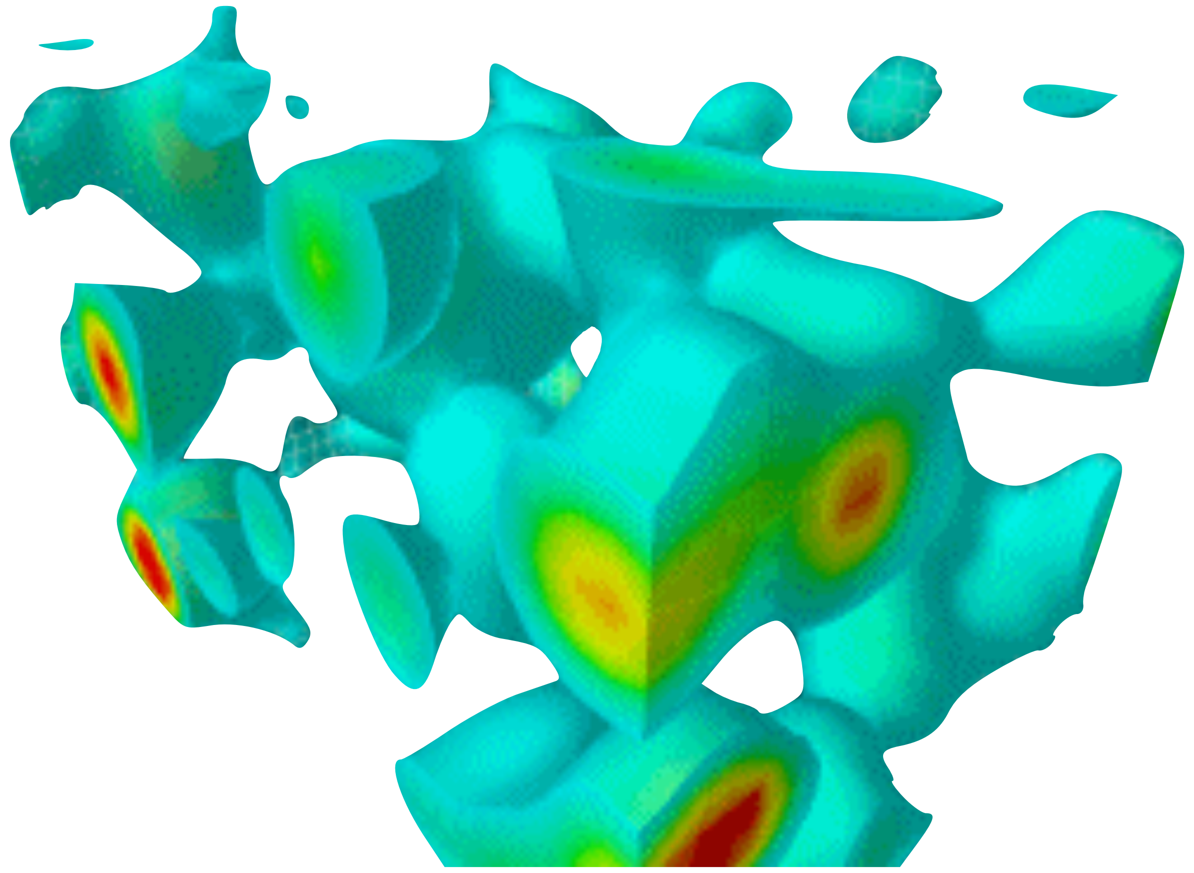

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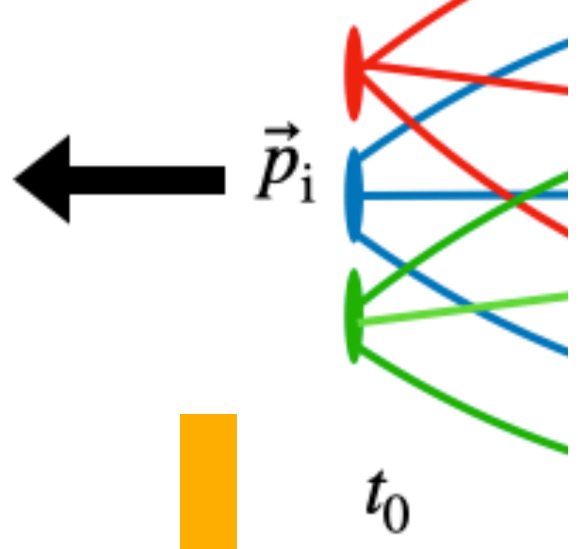

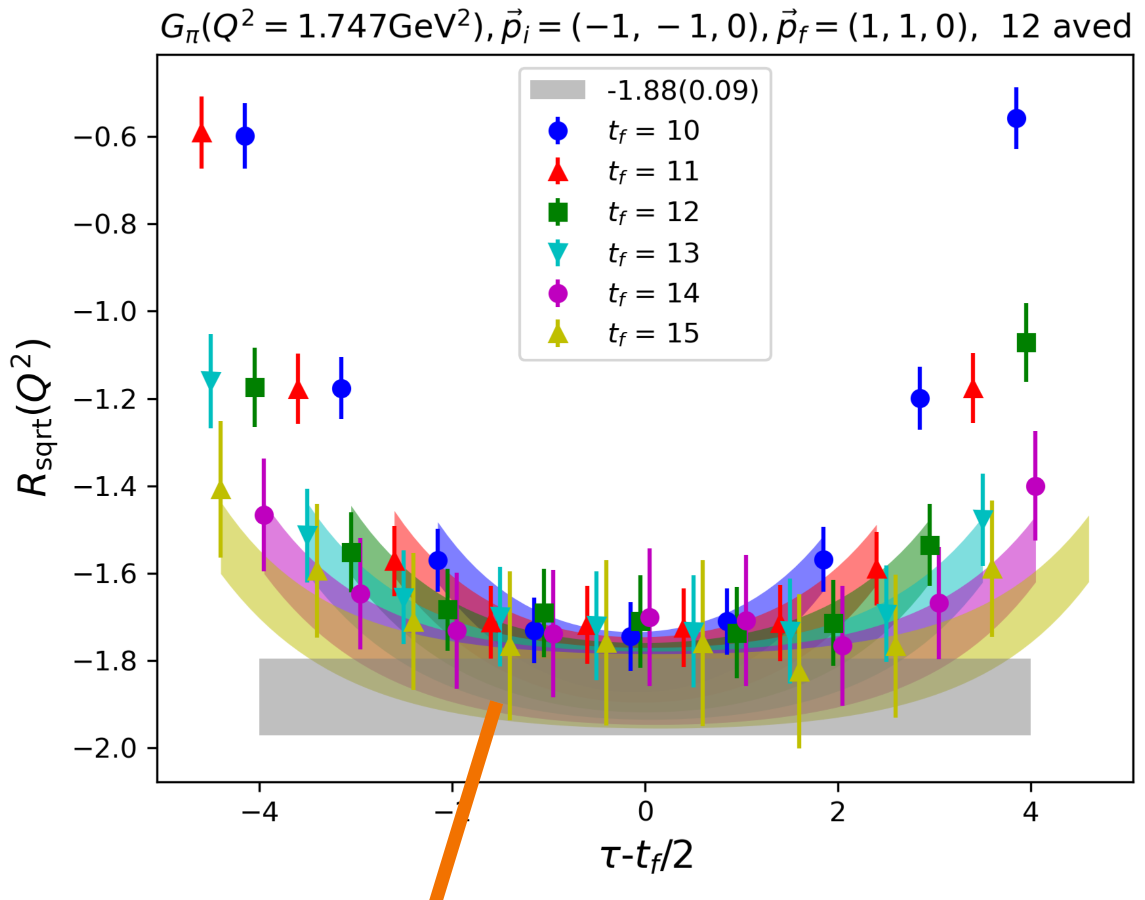

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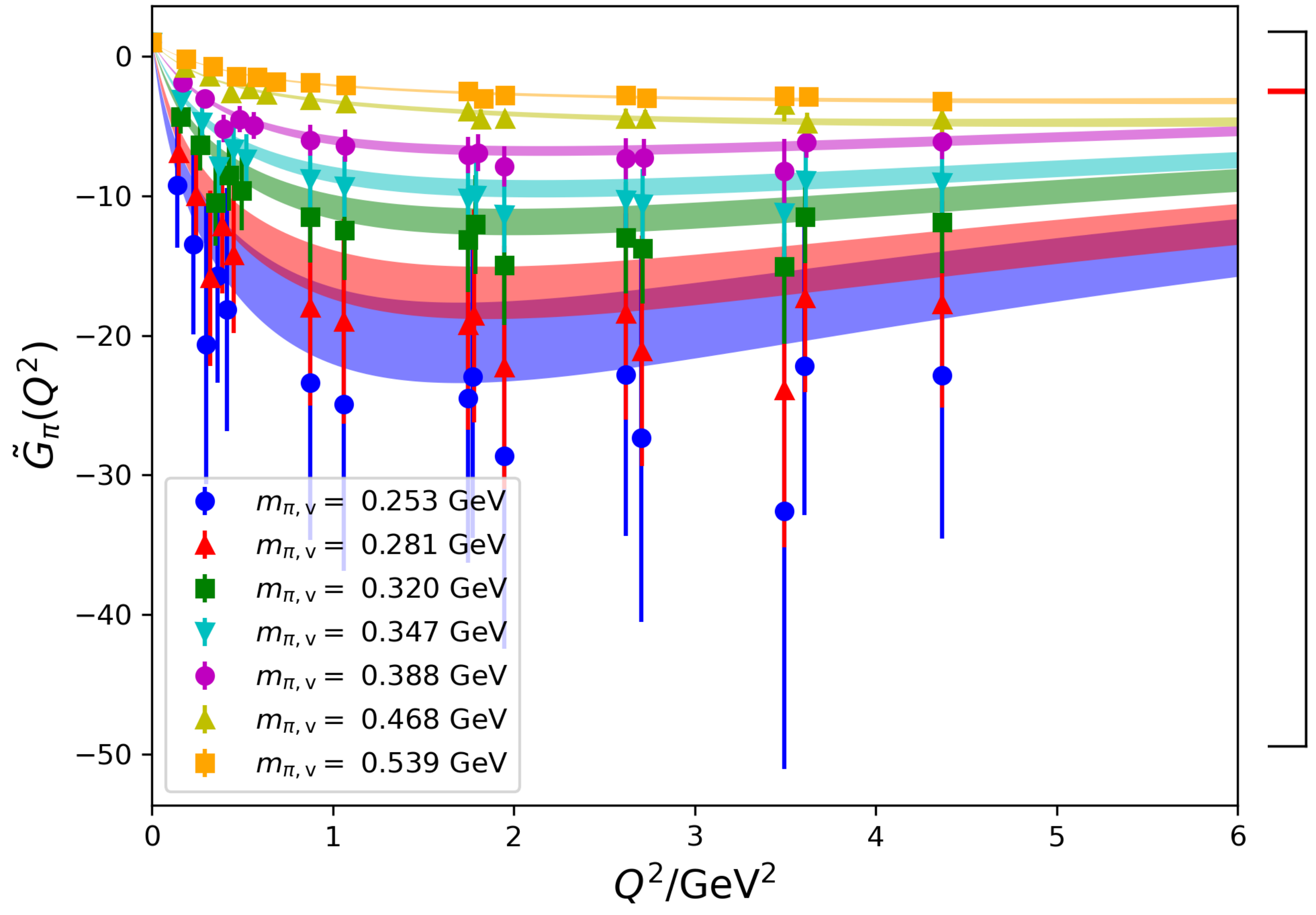

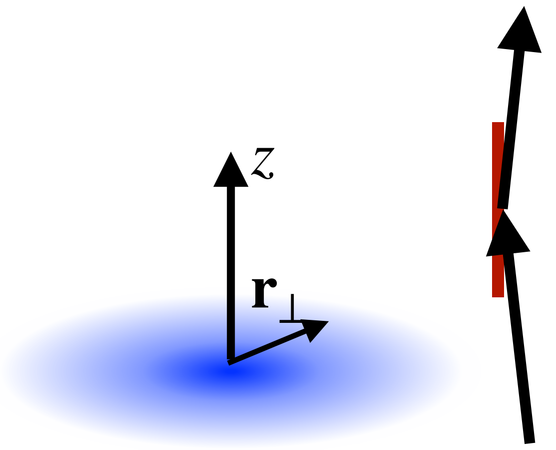

$$\rho_{\mathrm{H}}^{\mathrm{IMF}}(\mathbf{r}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}_\perp} \mathscr{F}_{\mathrm{m,H}}(Q^2)\Big|_{\mathbf{P}\cdot\mathbf{\Delta}=0}^{P_z\to\infty}$$

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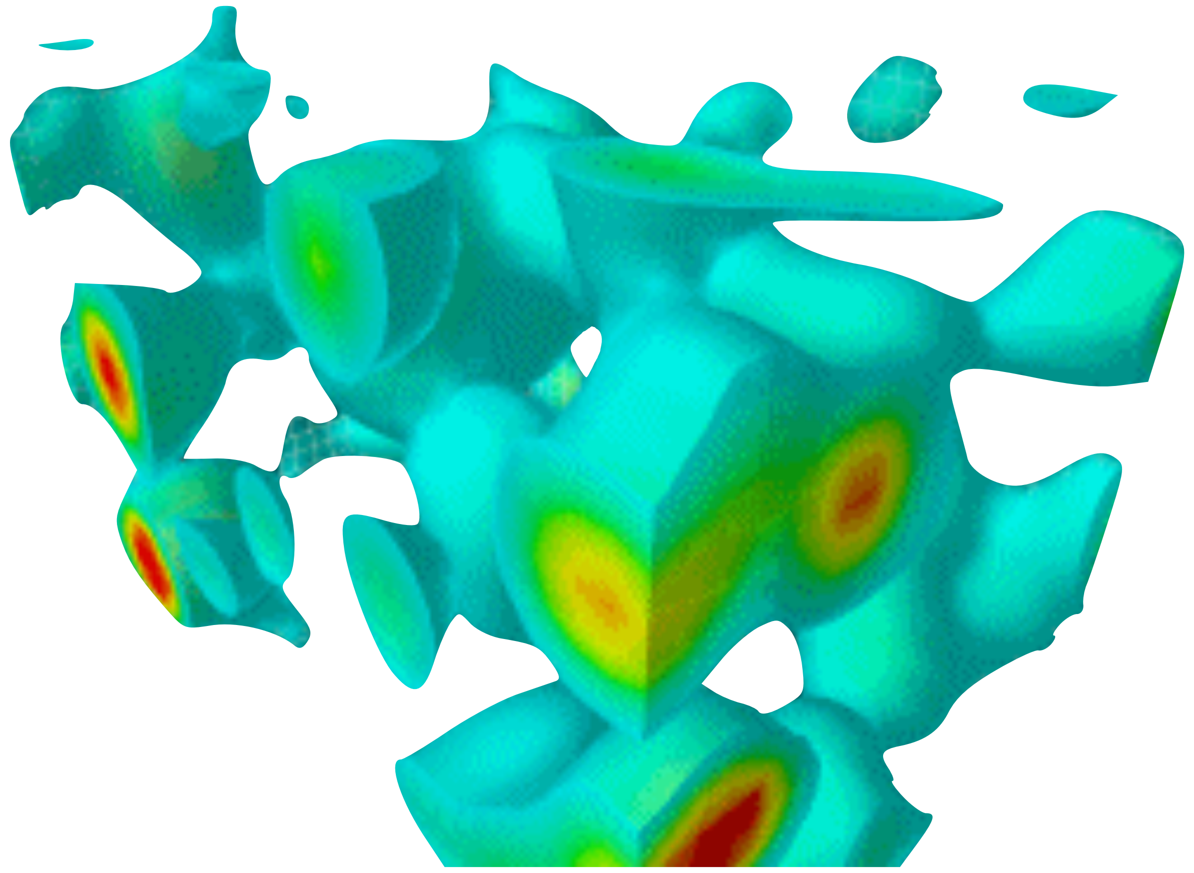

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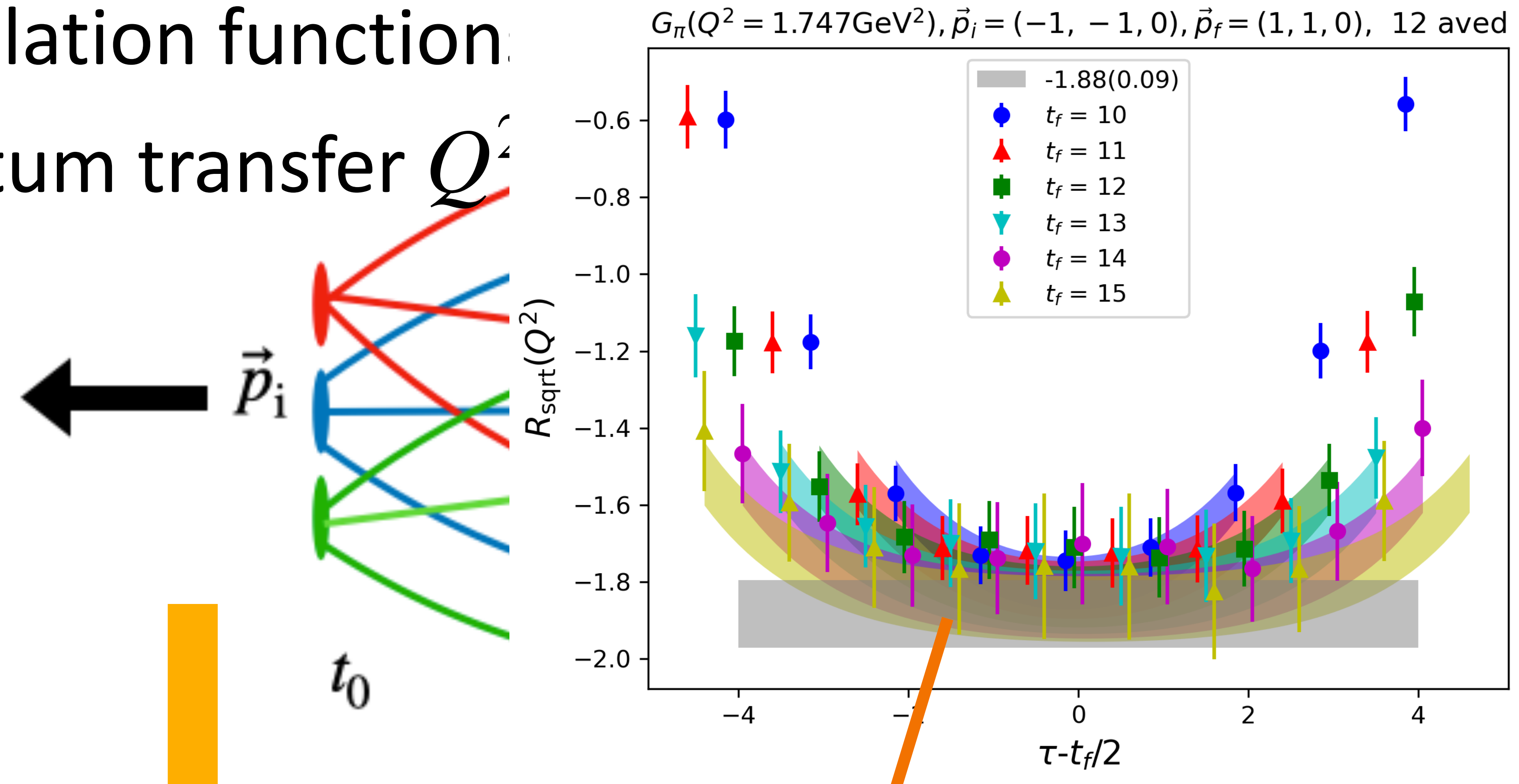

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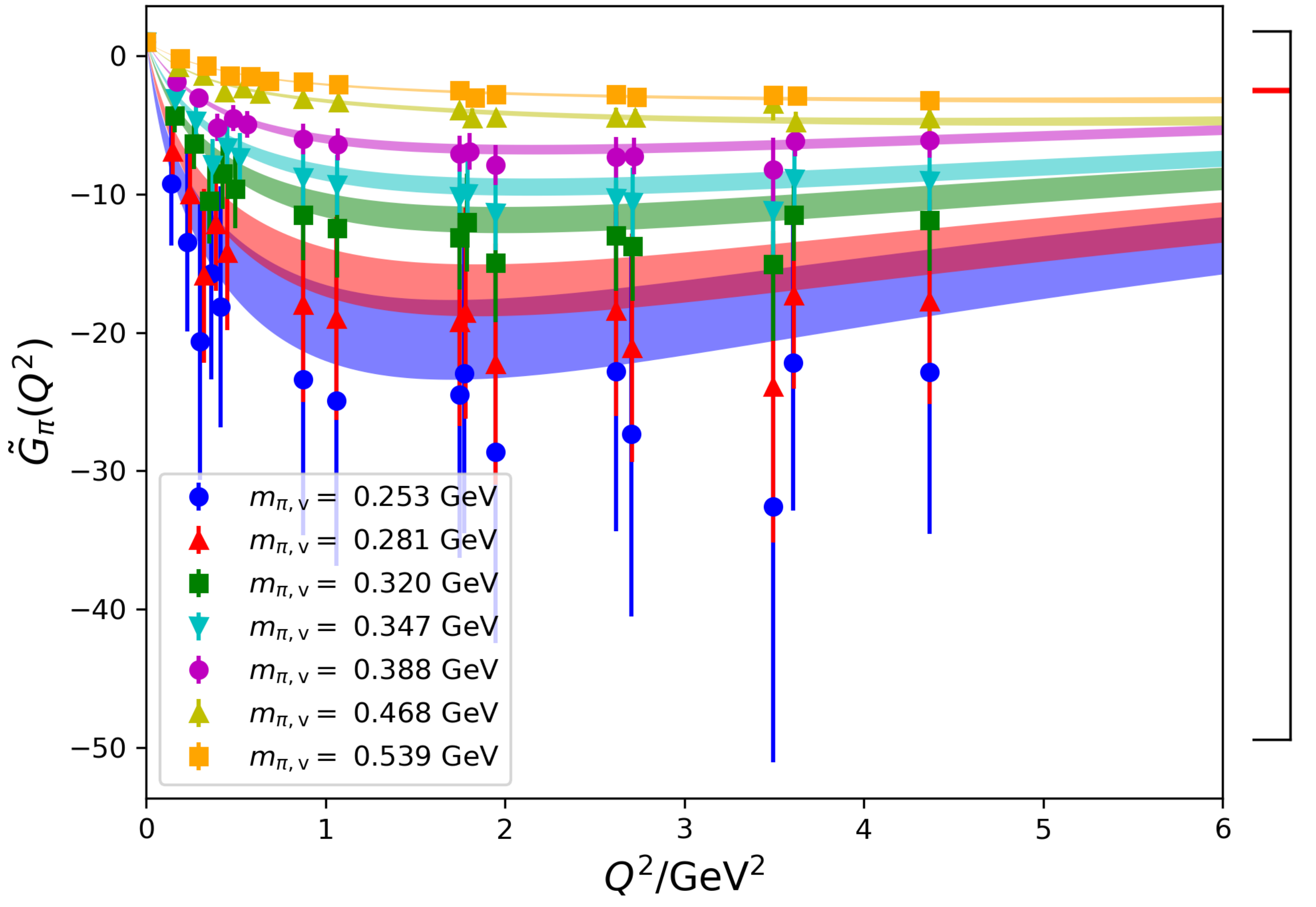

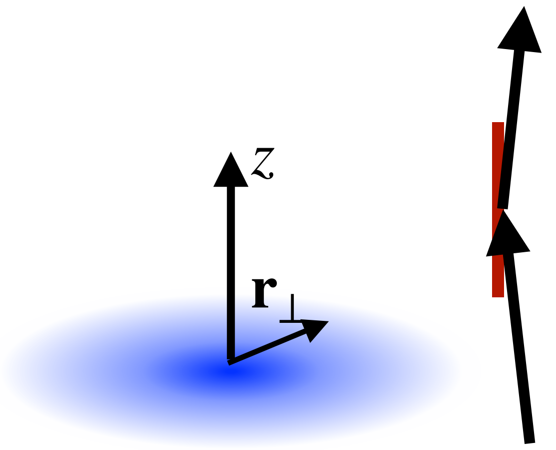

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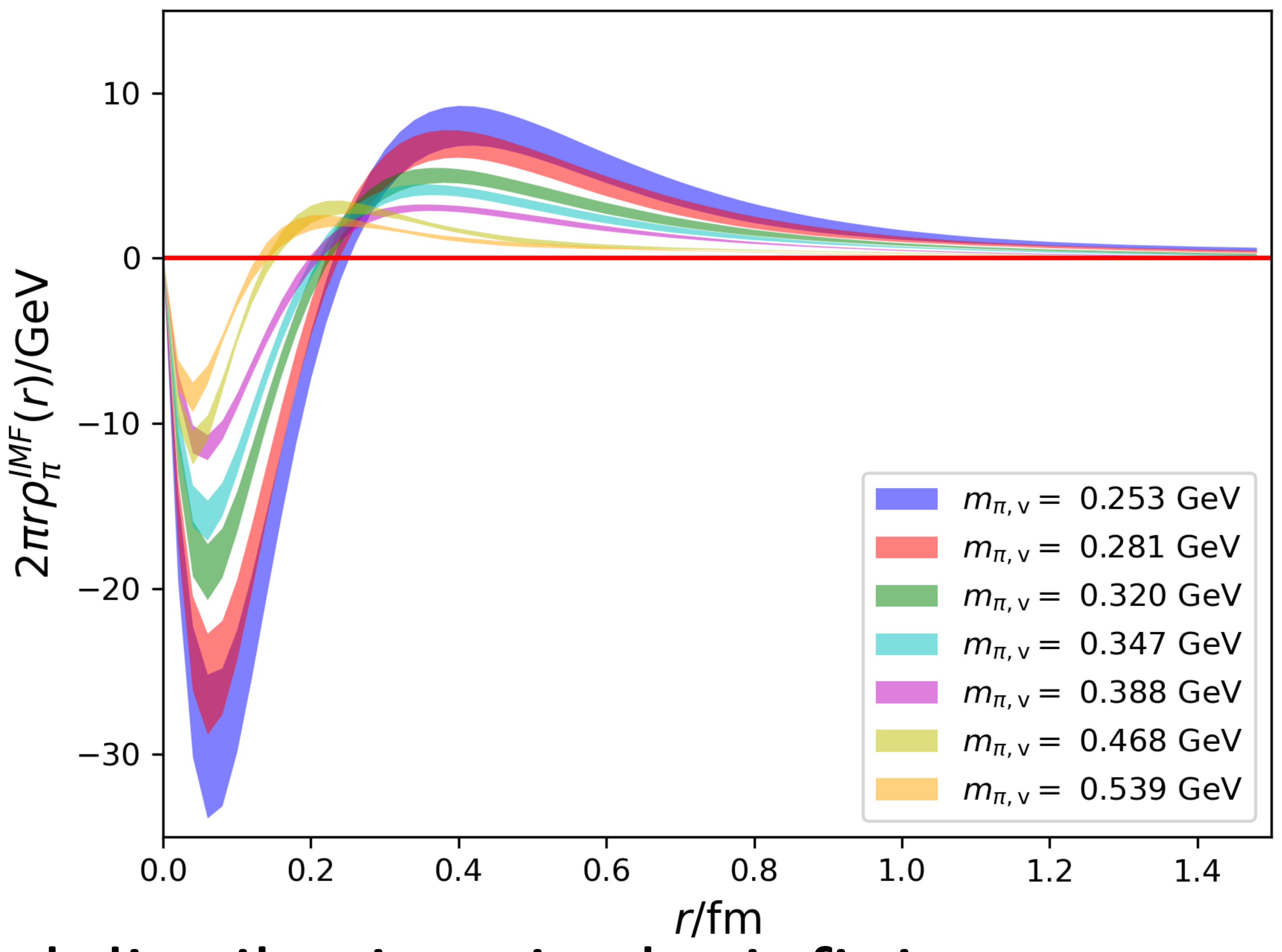
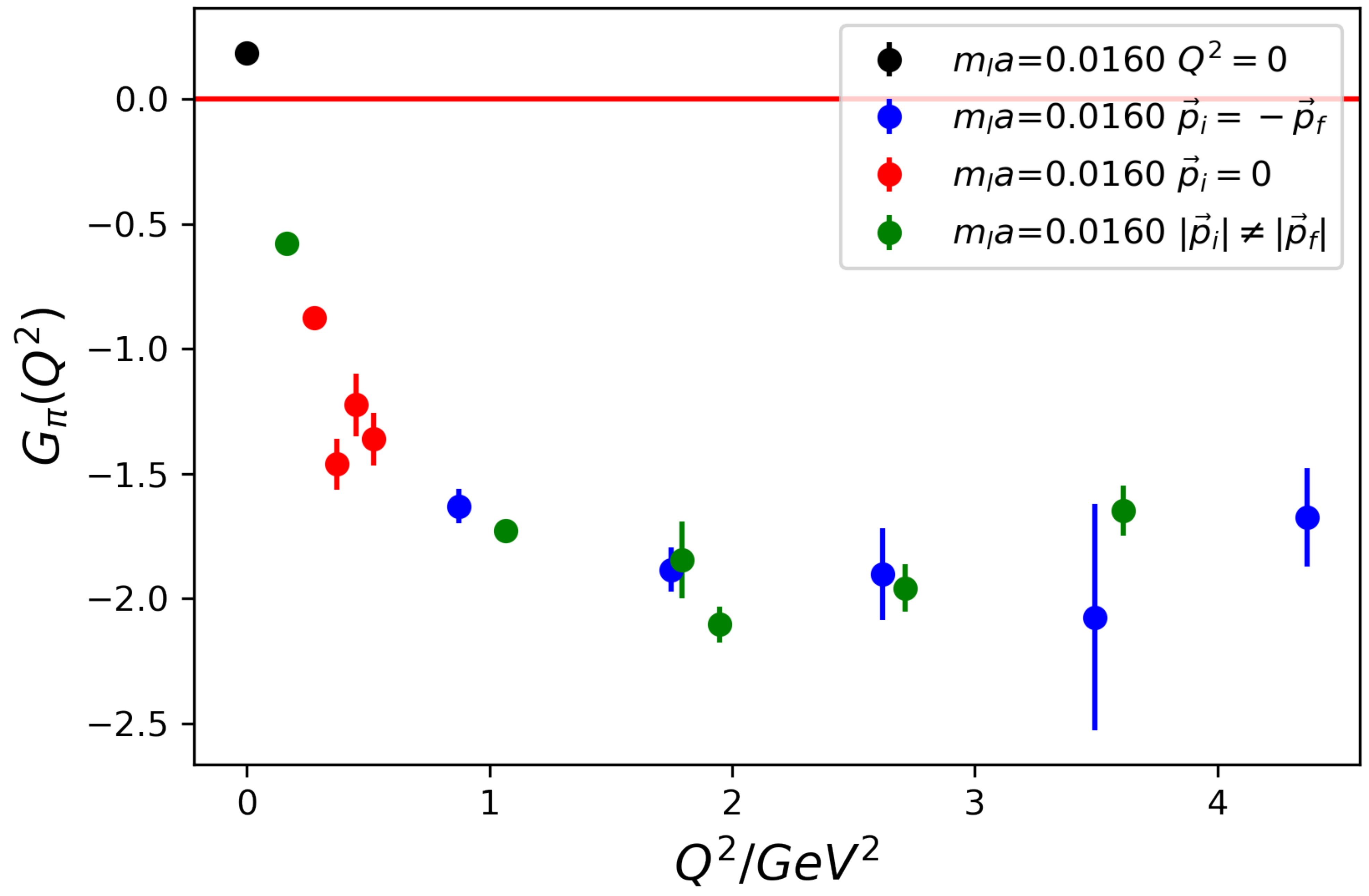
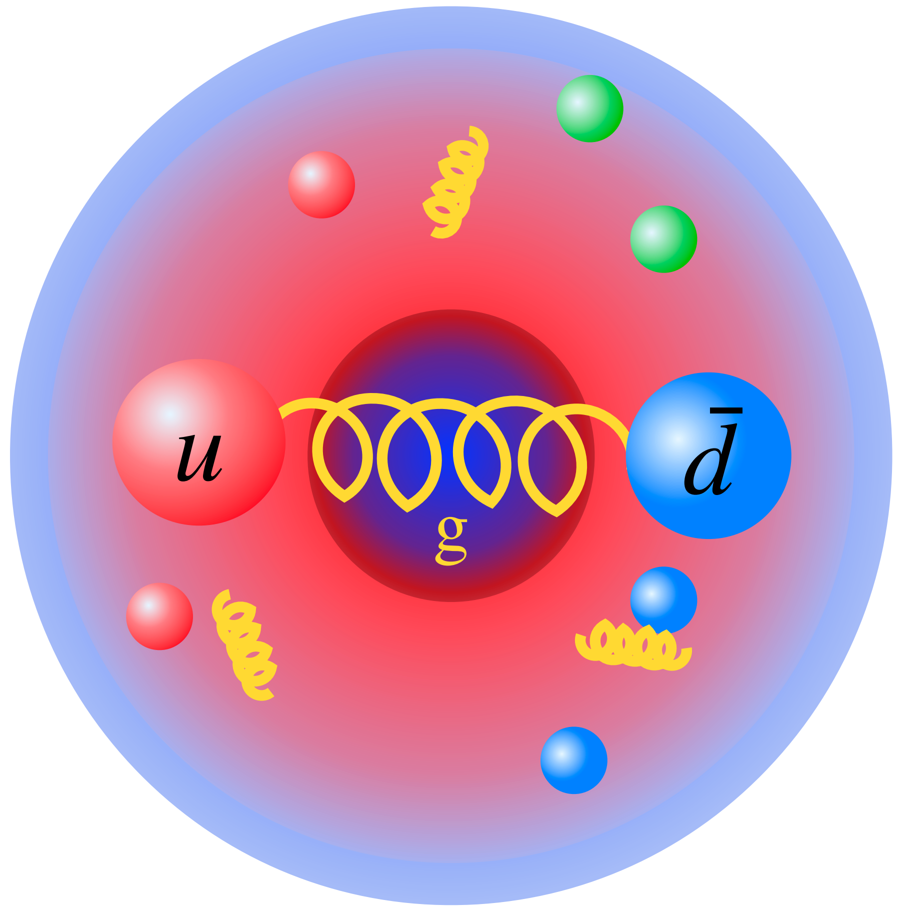
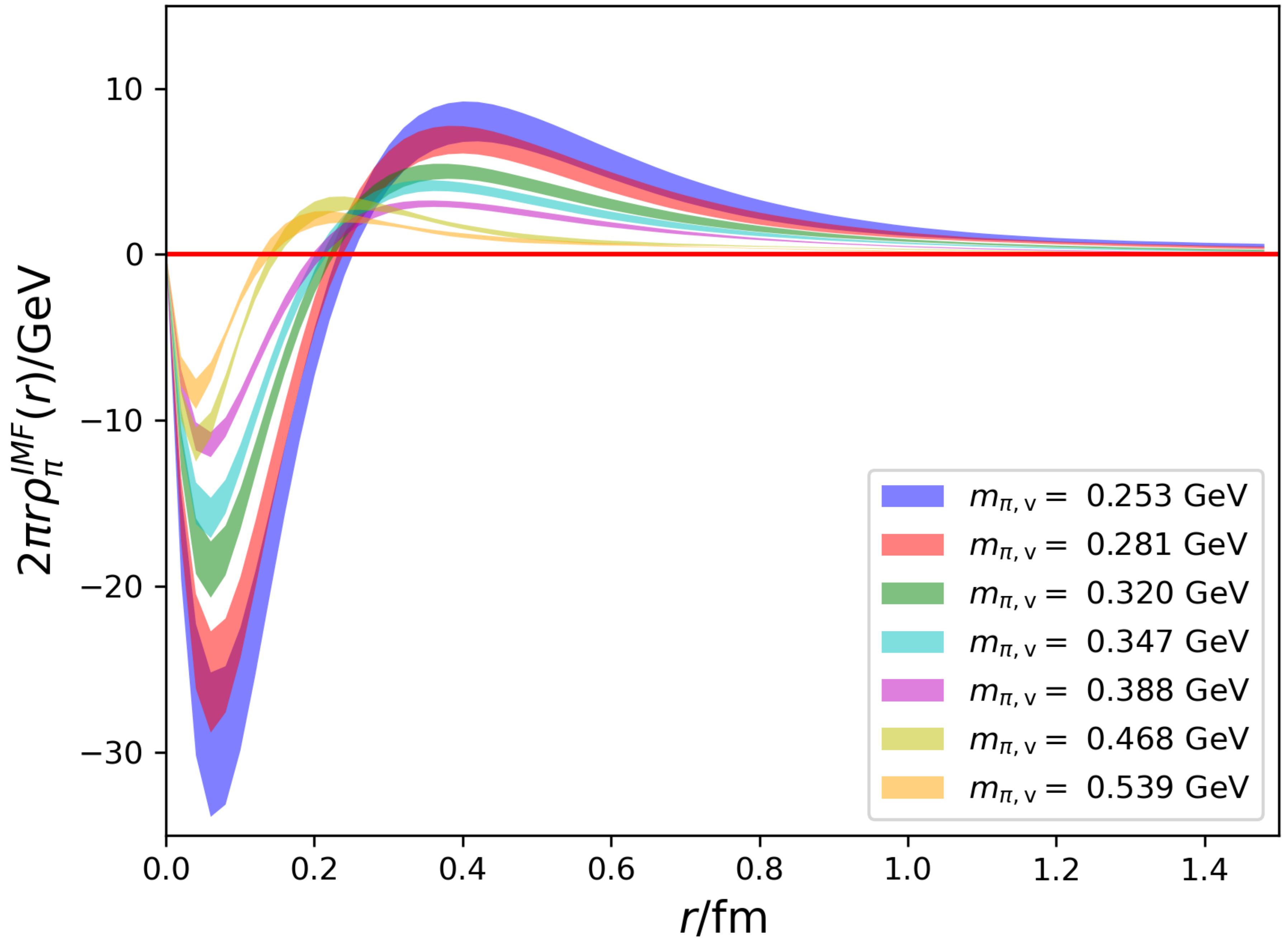


spatial distributions in the infinite-momentum frame (IMF).

Trace anomaly of the pion (glue part)







- **positive** at $Q^2 = 0\,\mathrm{GeV}^2$ (contribution to the pion mass from glue)
- **sign change** of glue trace anomaly form factor of the pion

- The predictions from chiral perturbation theory at small Q^2 region:

$$\mathscr{F}^{\mathrm{ChPT}}_{\mathrm{ta},\pi}(Q^2) \sim \frac{1}{2} - \frac{1}{2m_\pi^2} Q^2$$

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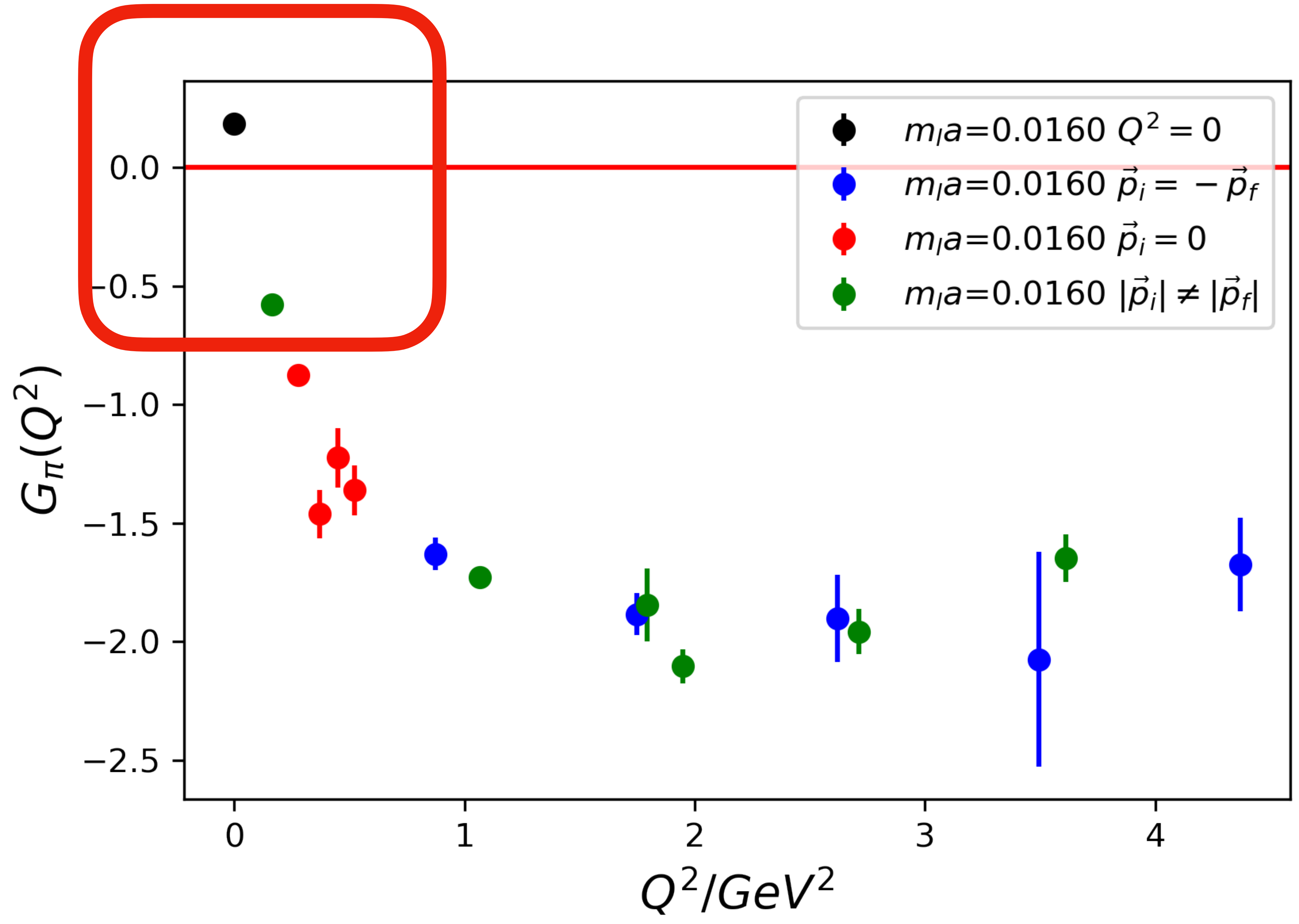

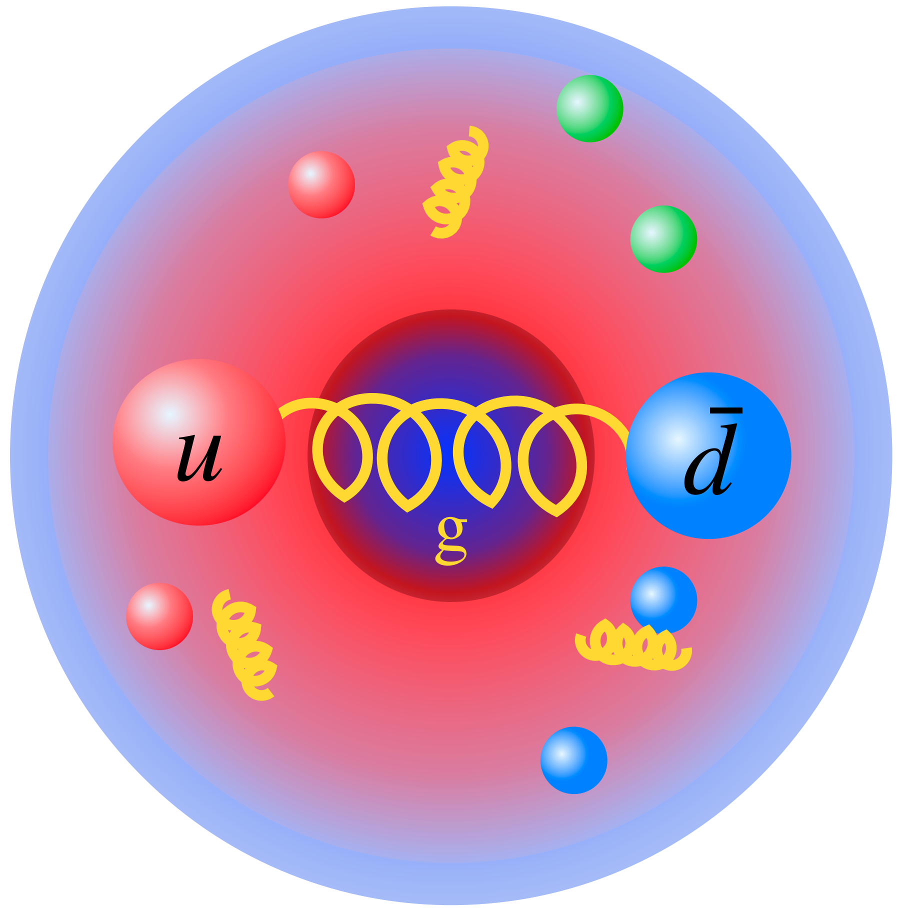

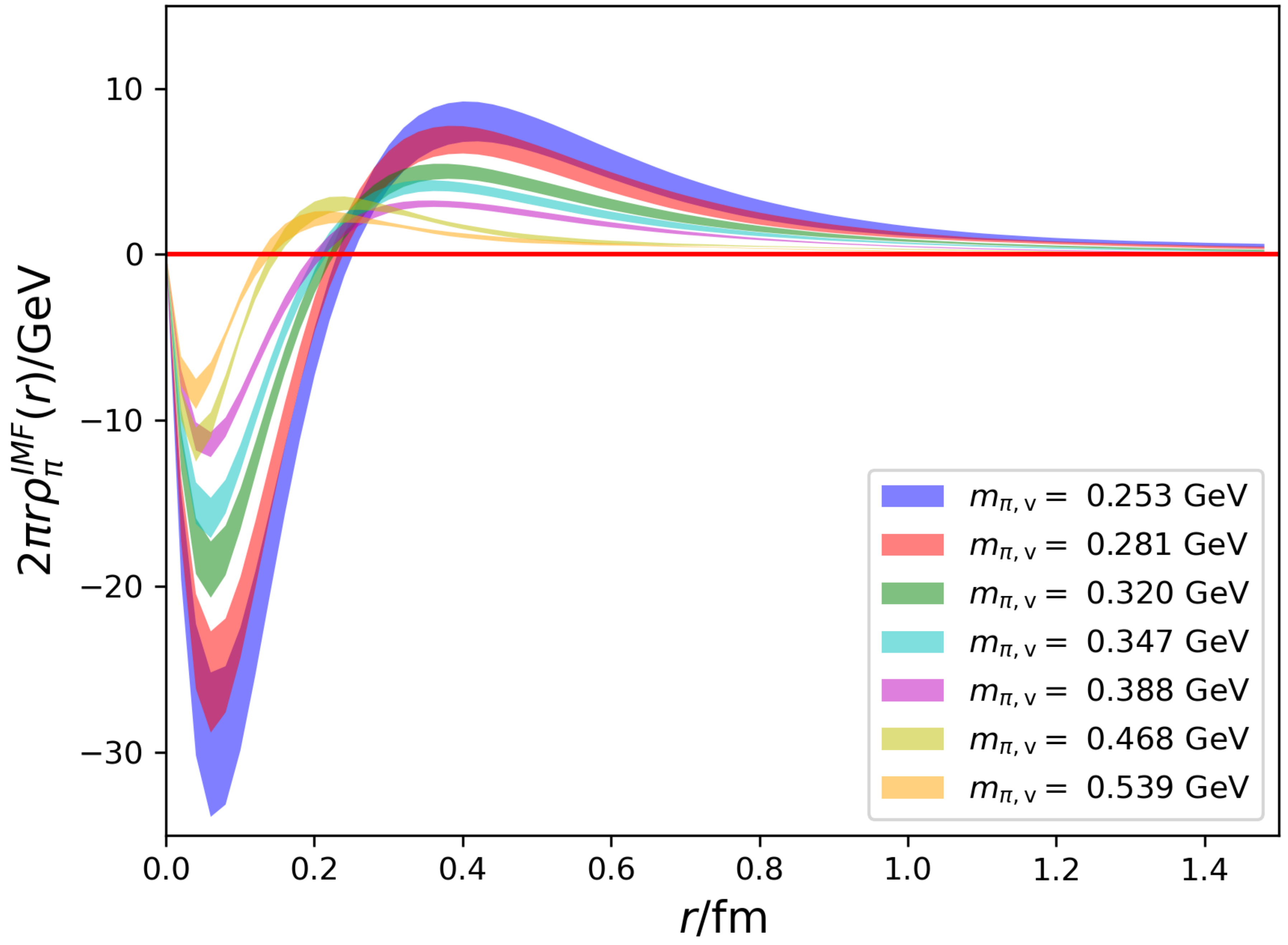

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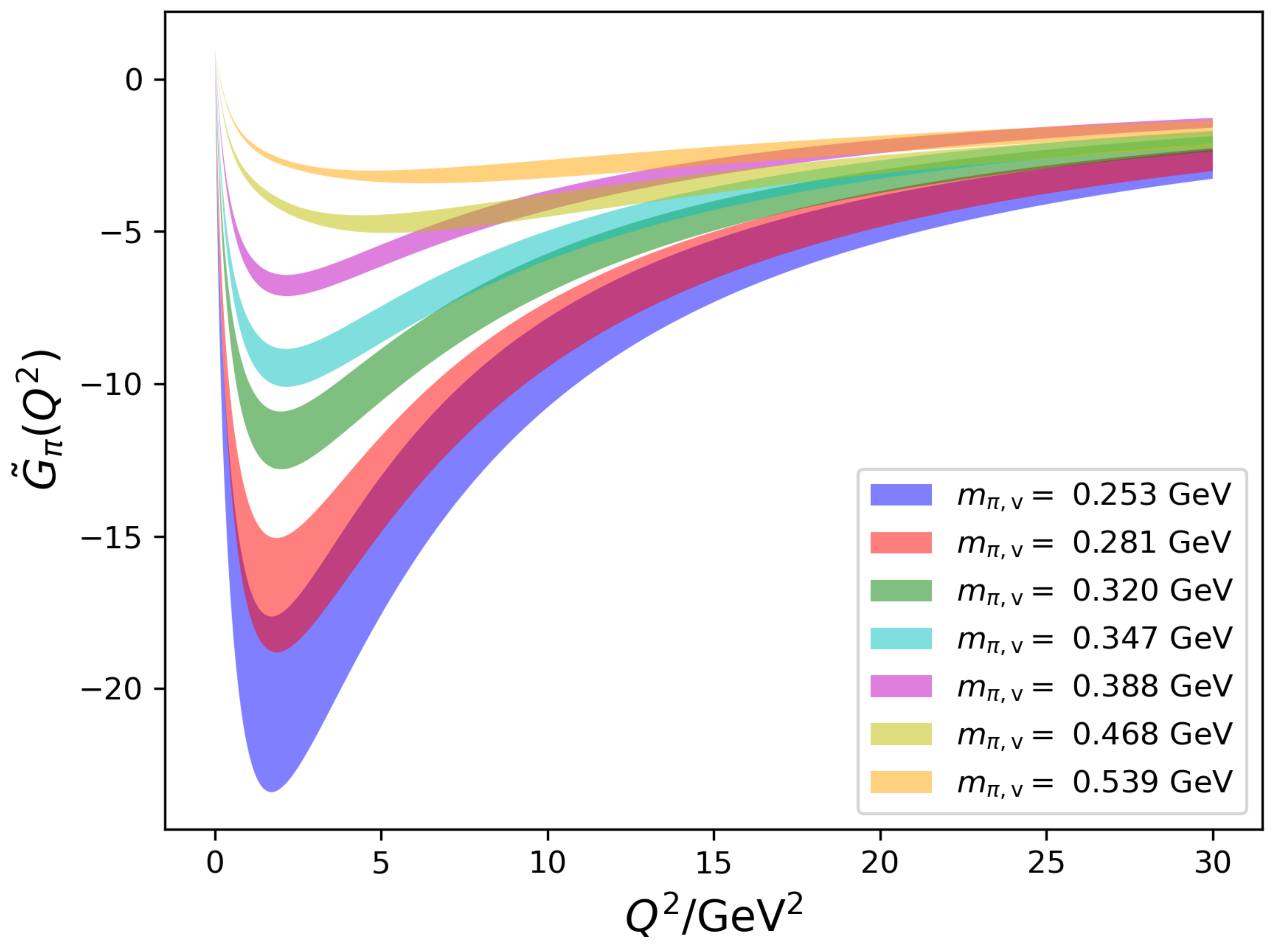
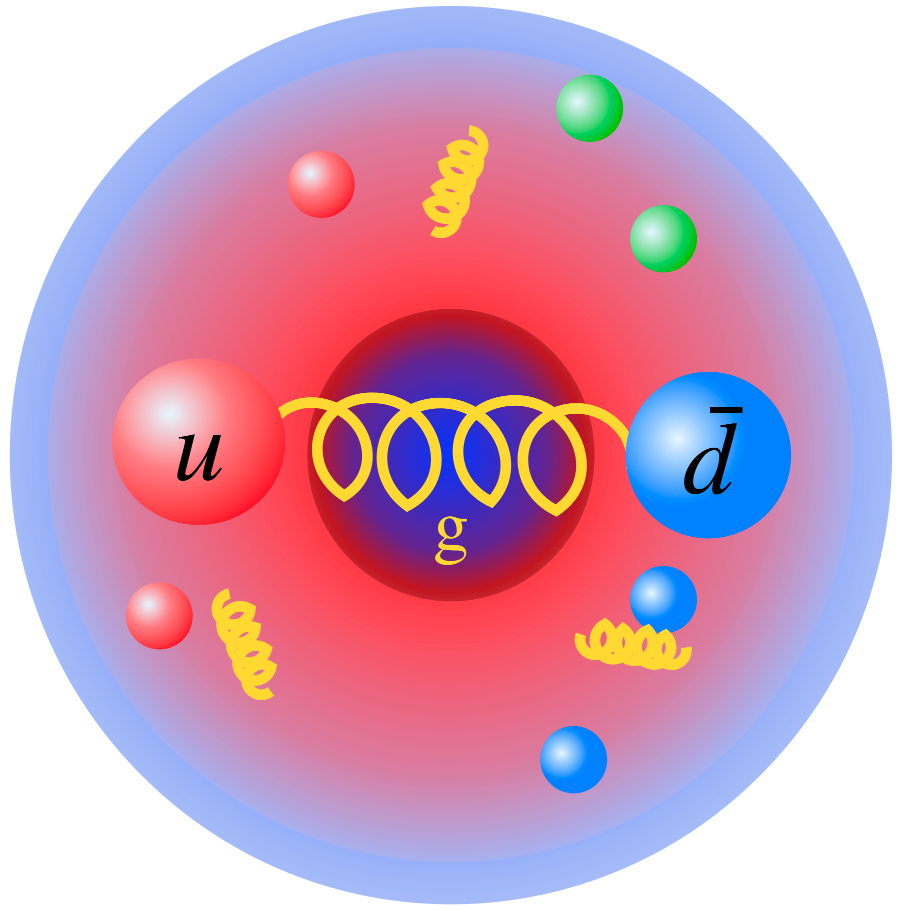
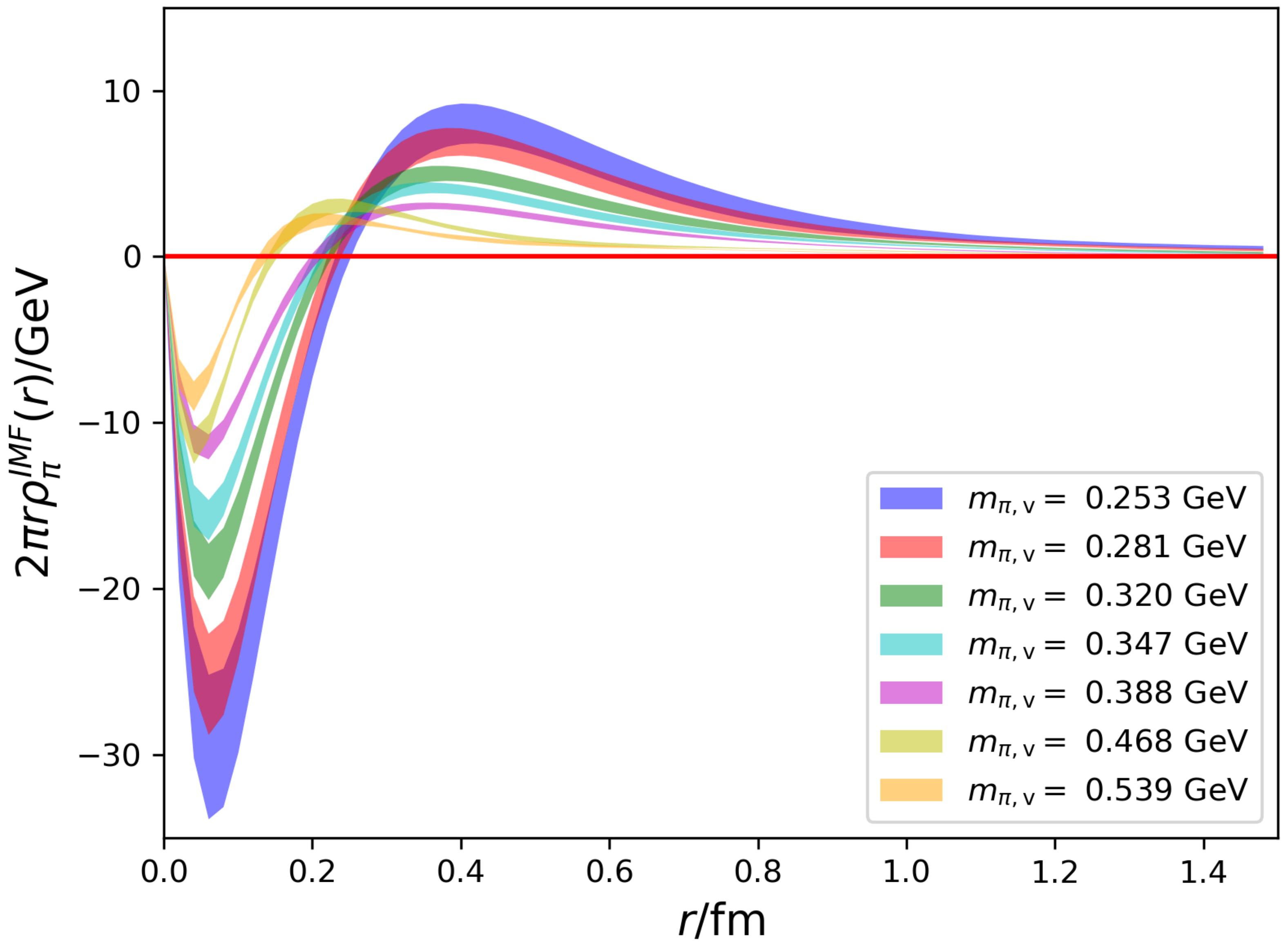
Trace anomaly of the pion (glue part)

pion

mass







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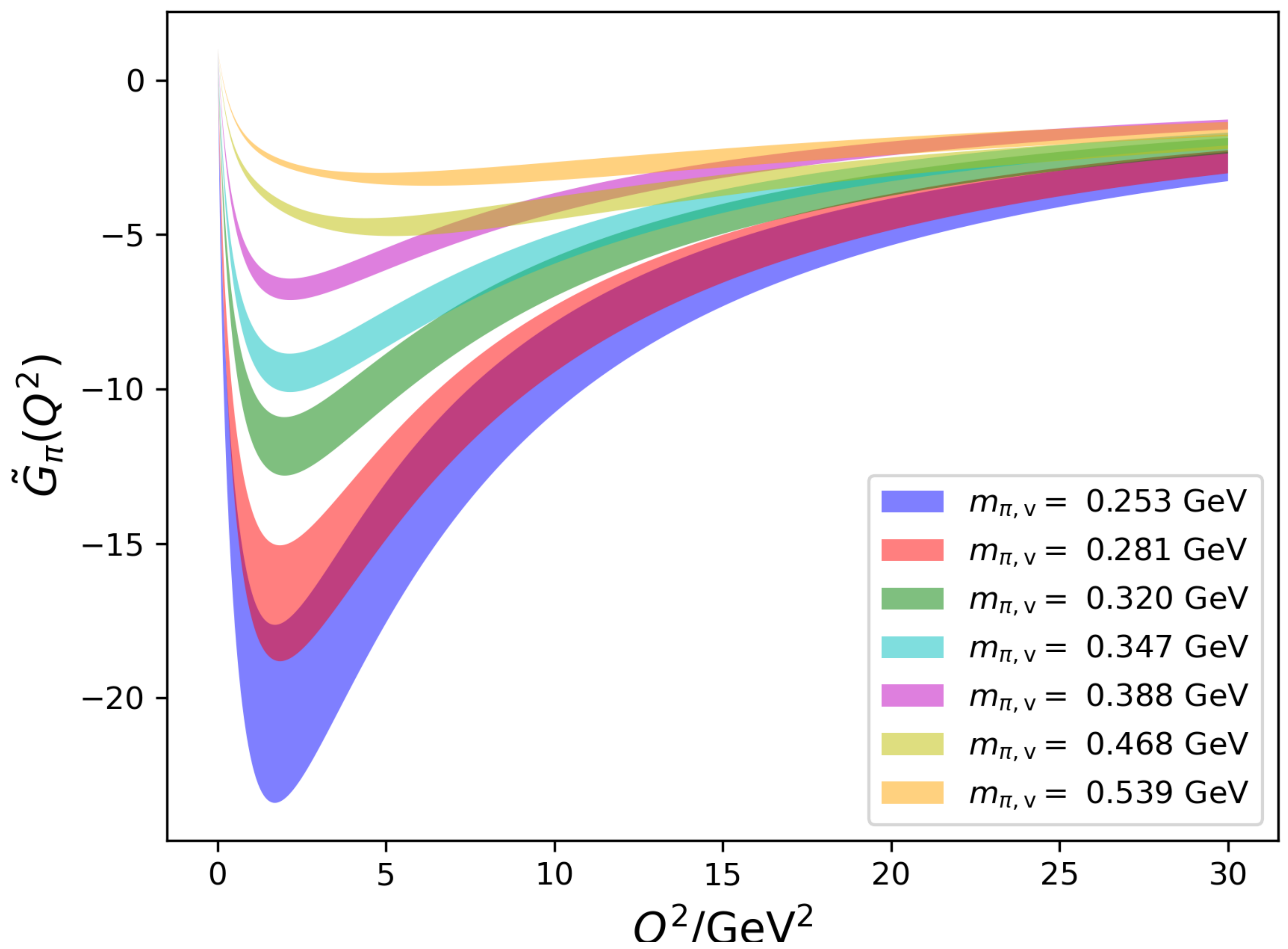
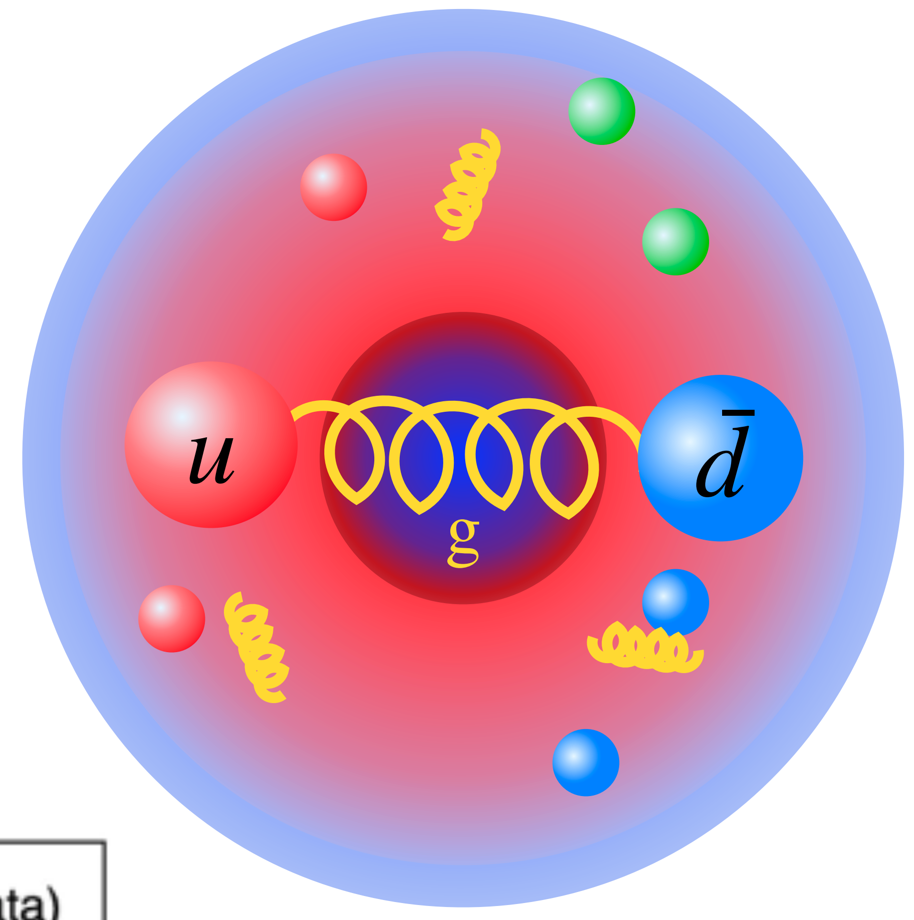
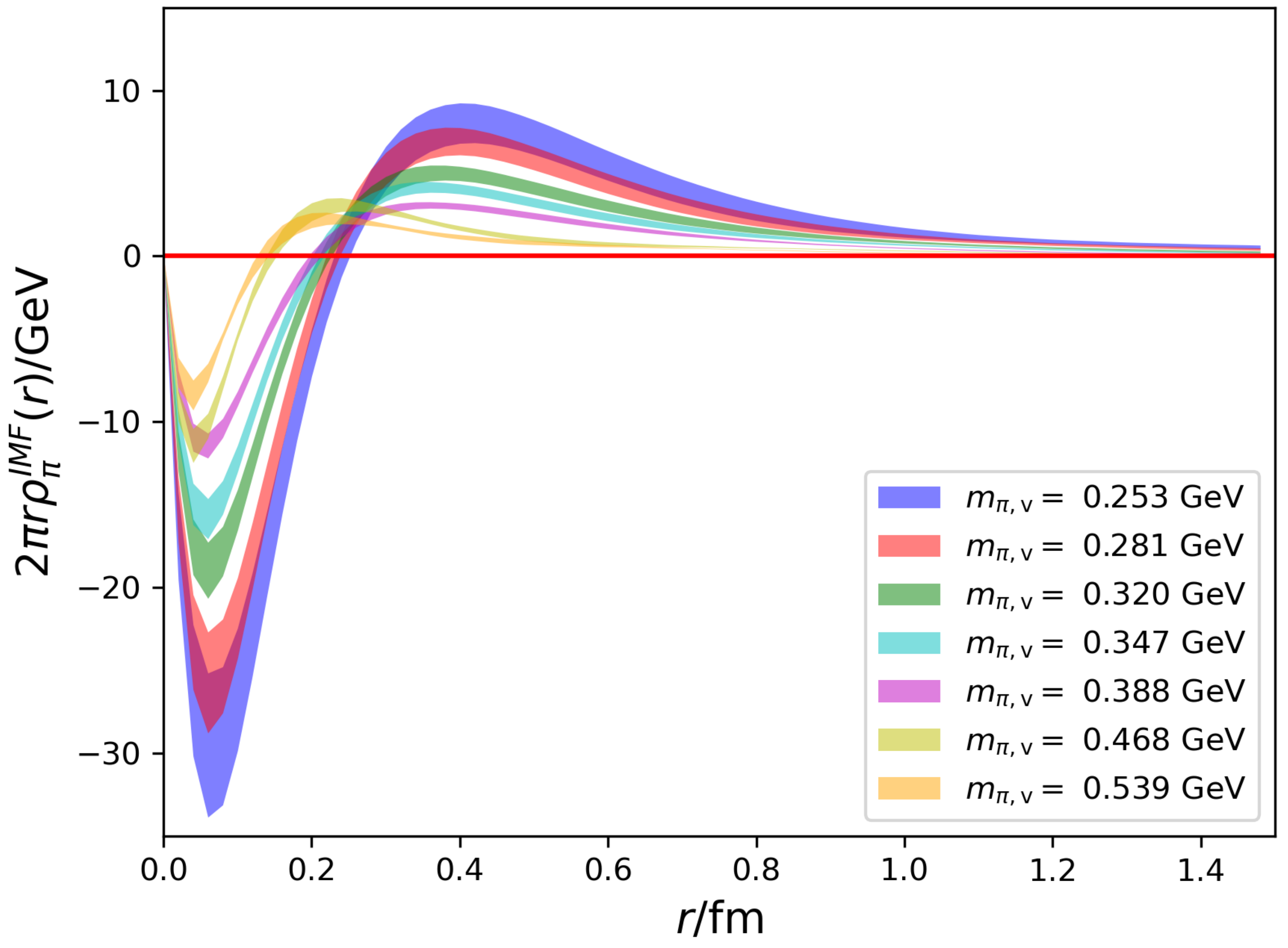
pion

mass

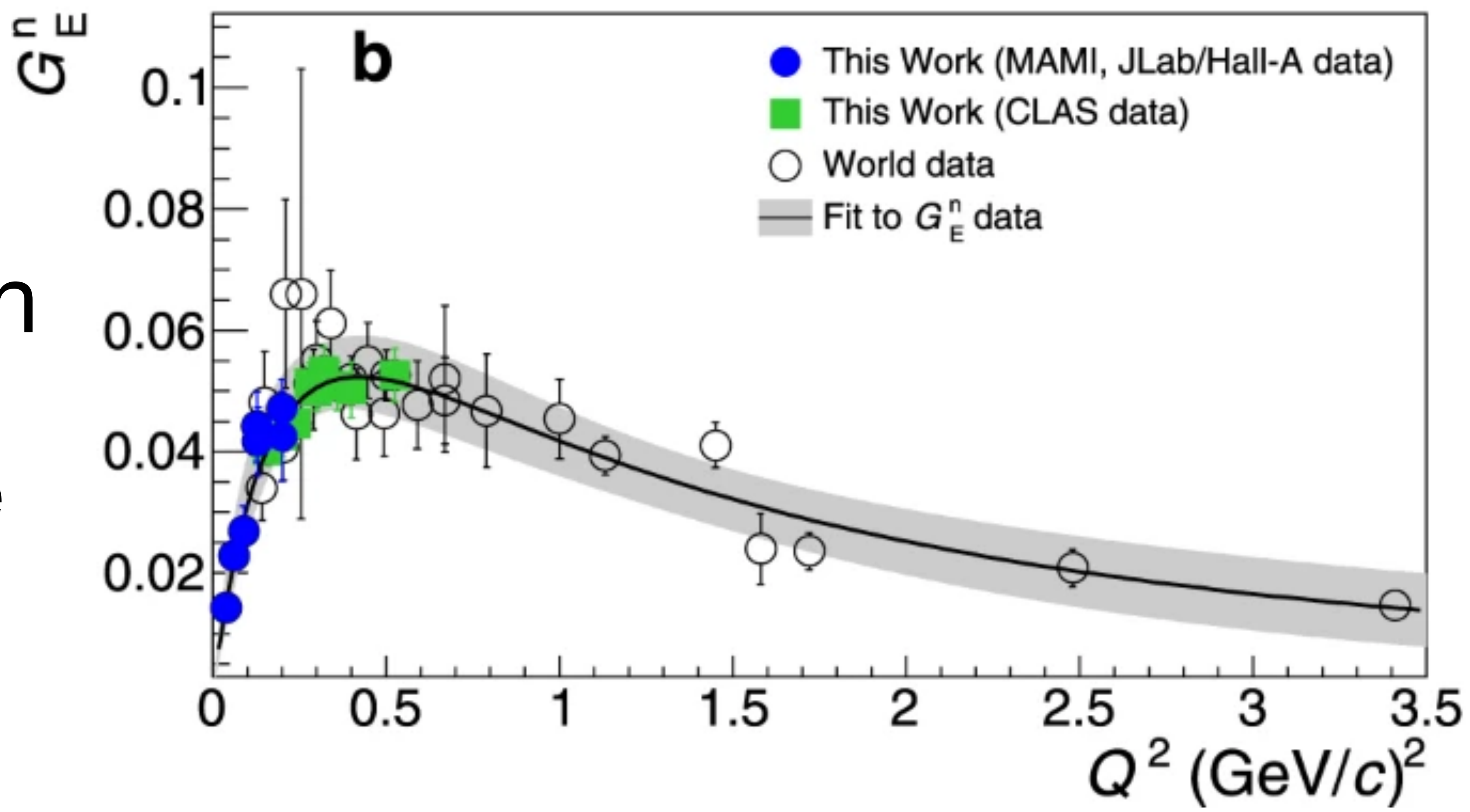






neutron

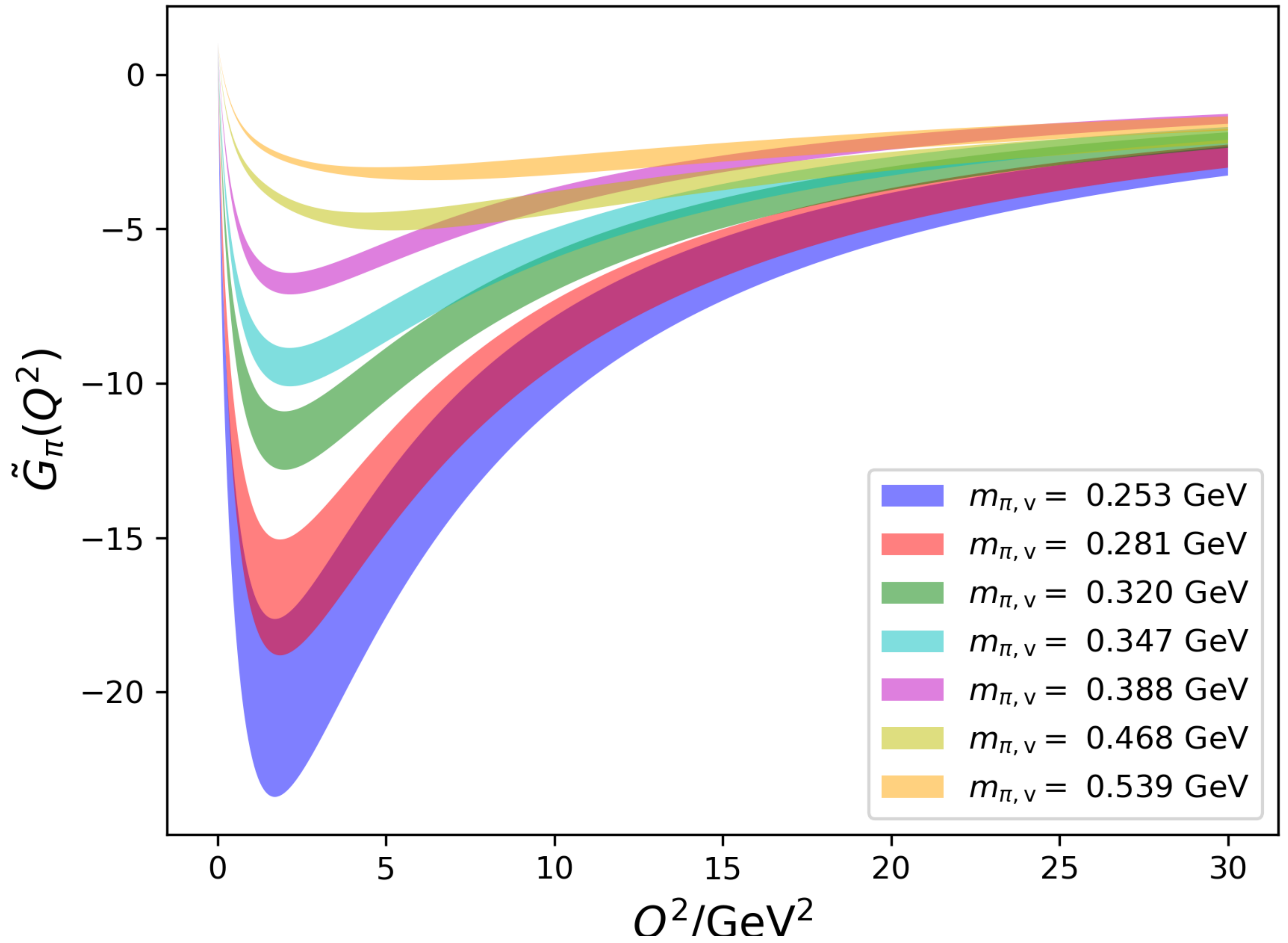
charge

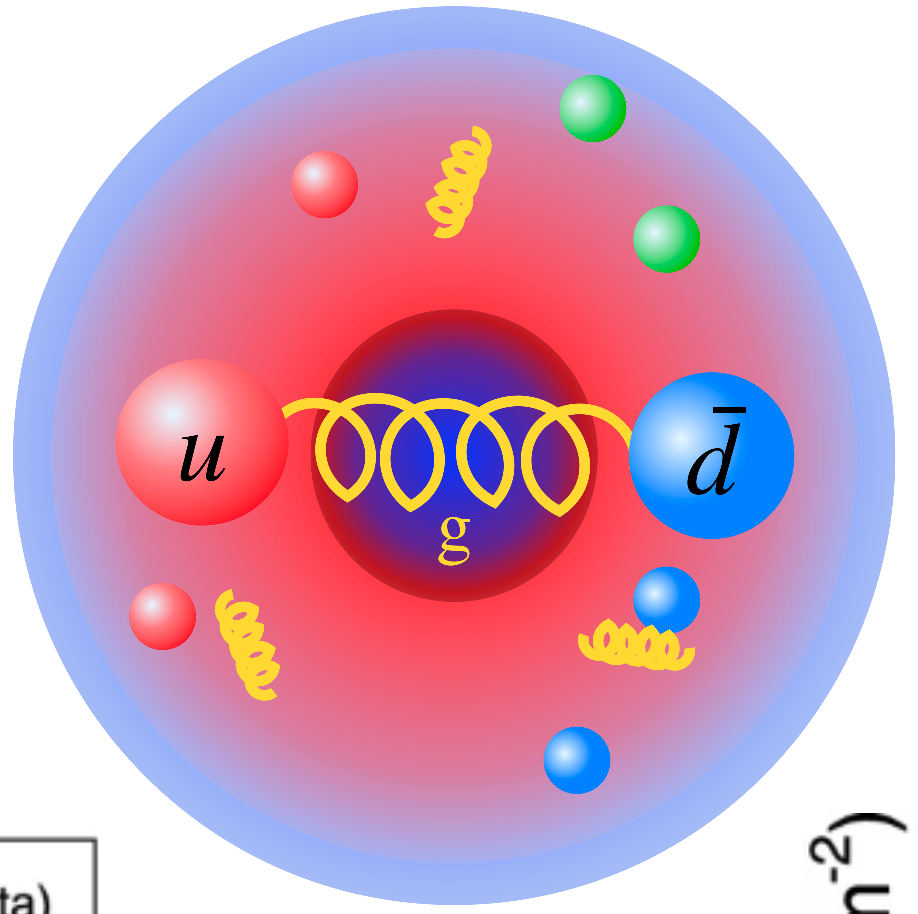


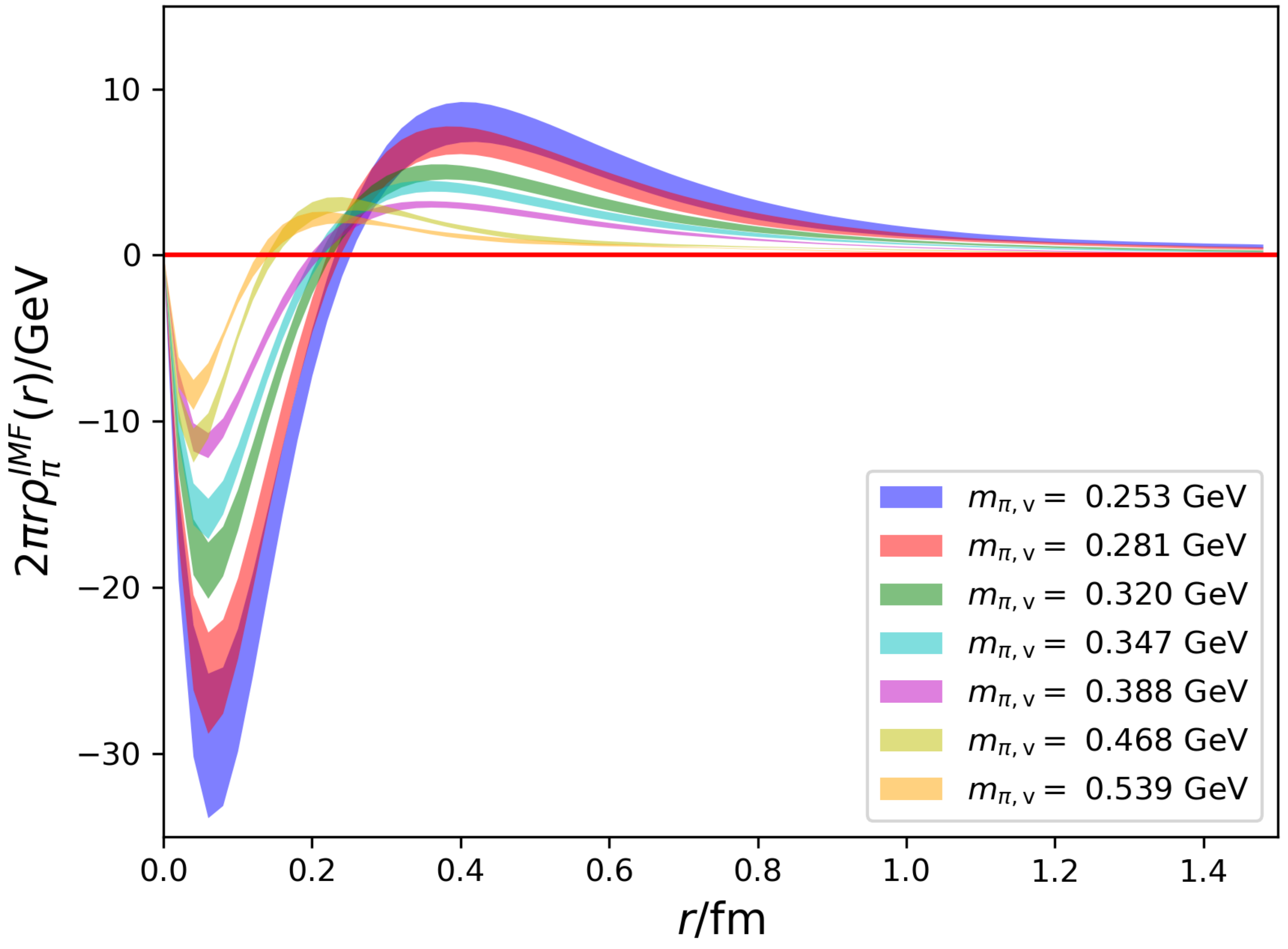
Trace anomaly of the pion (glue part)

pion

mass

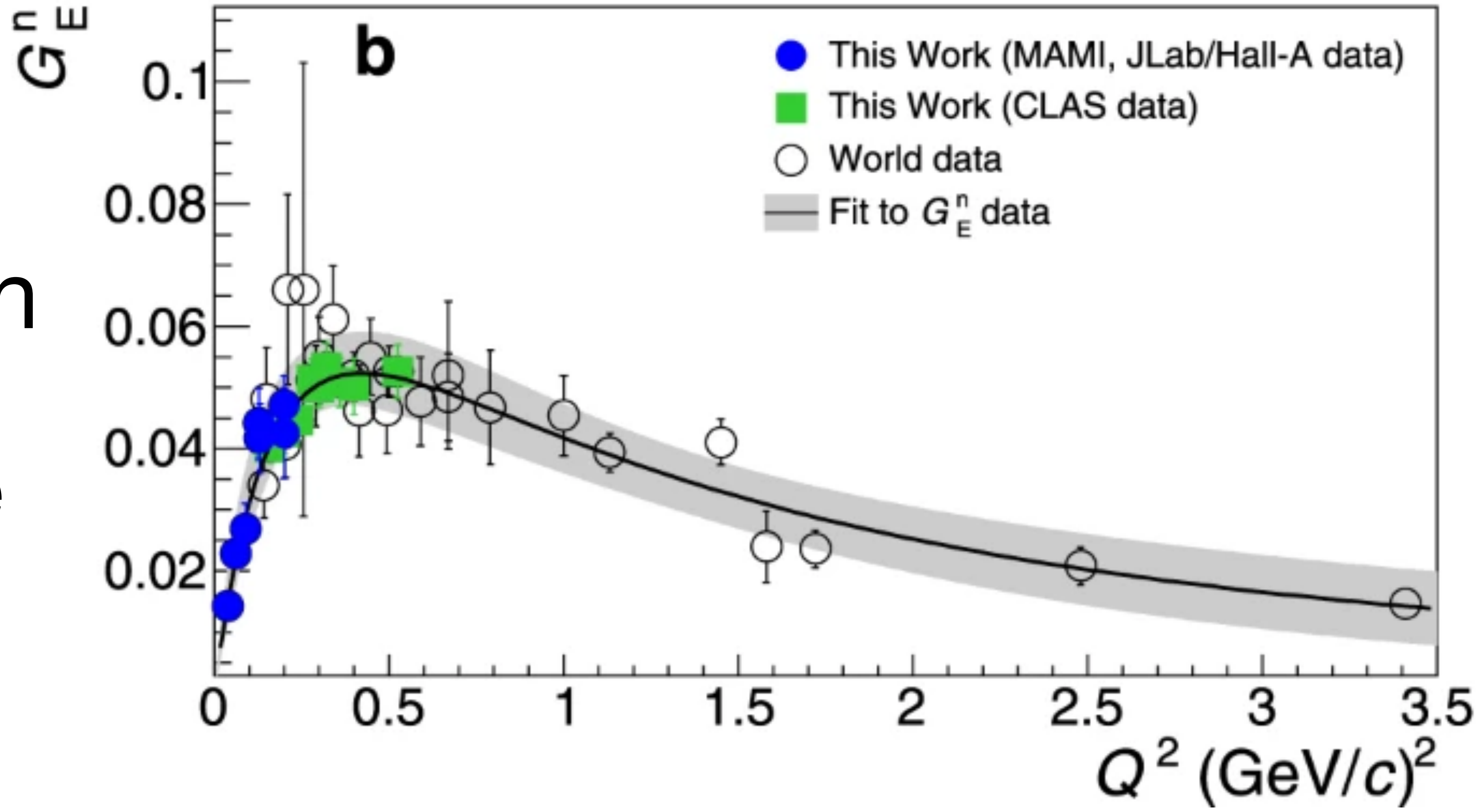


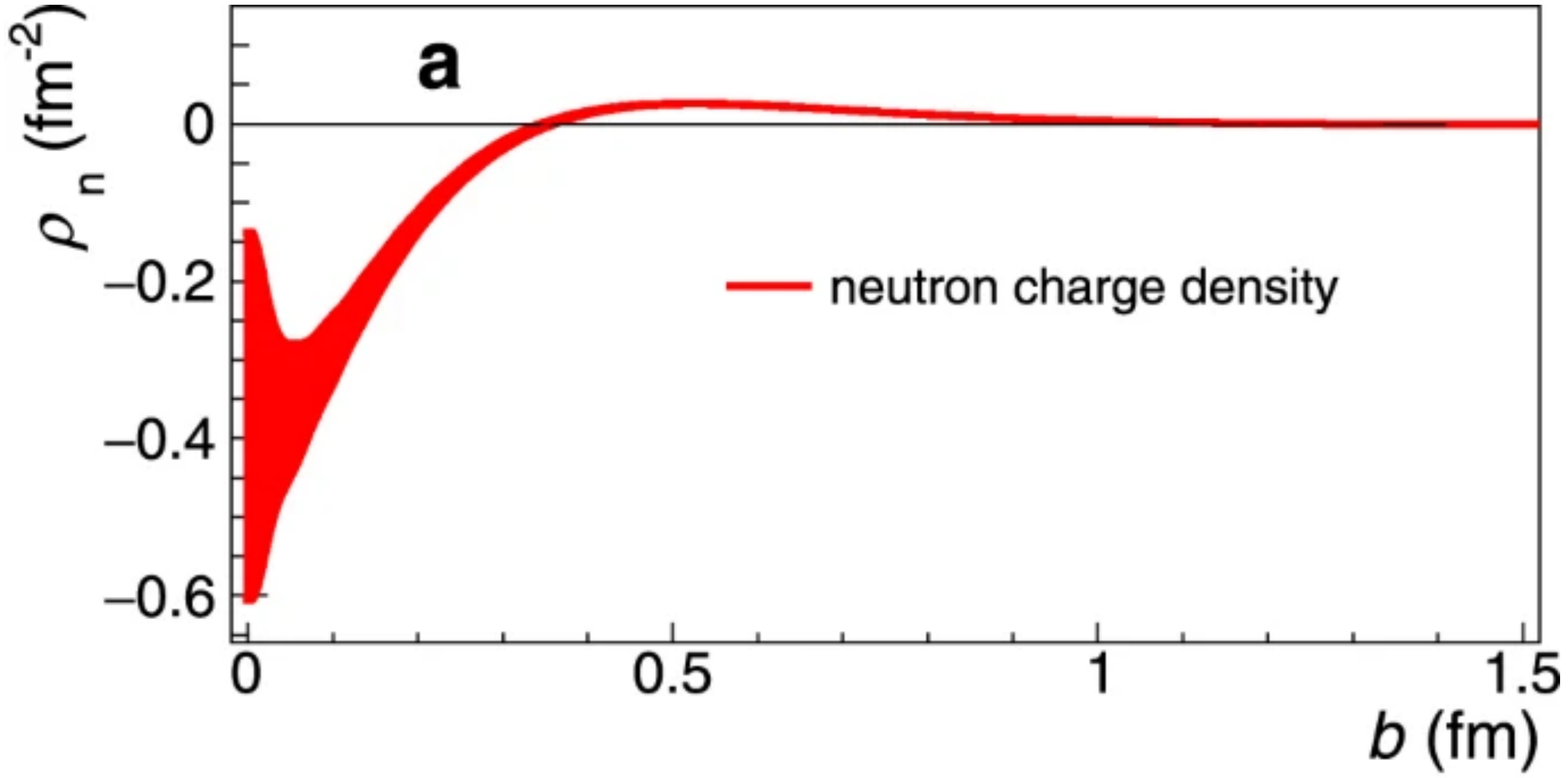




neutron

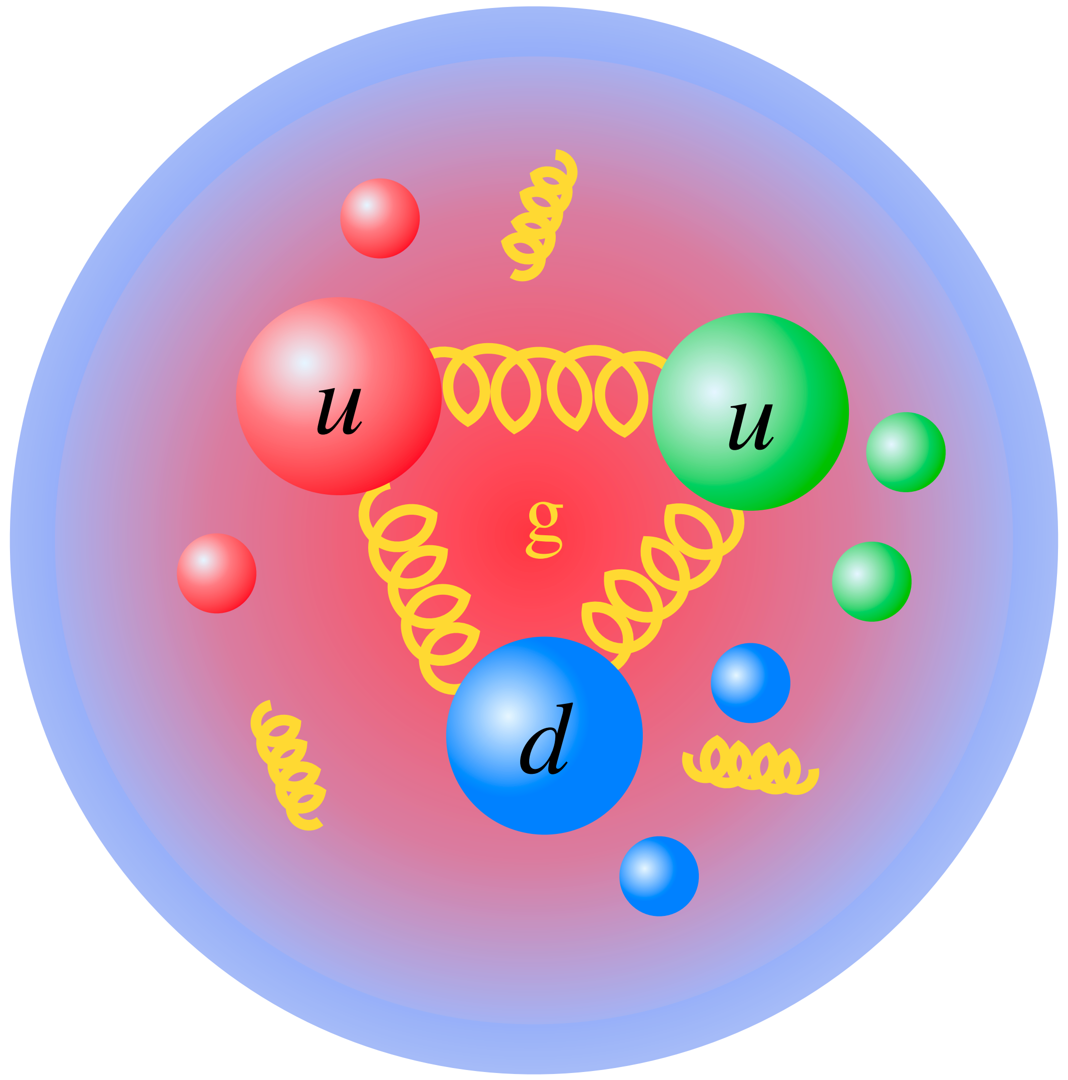
charge

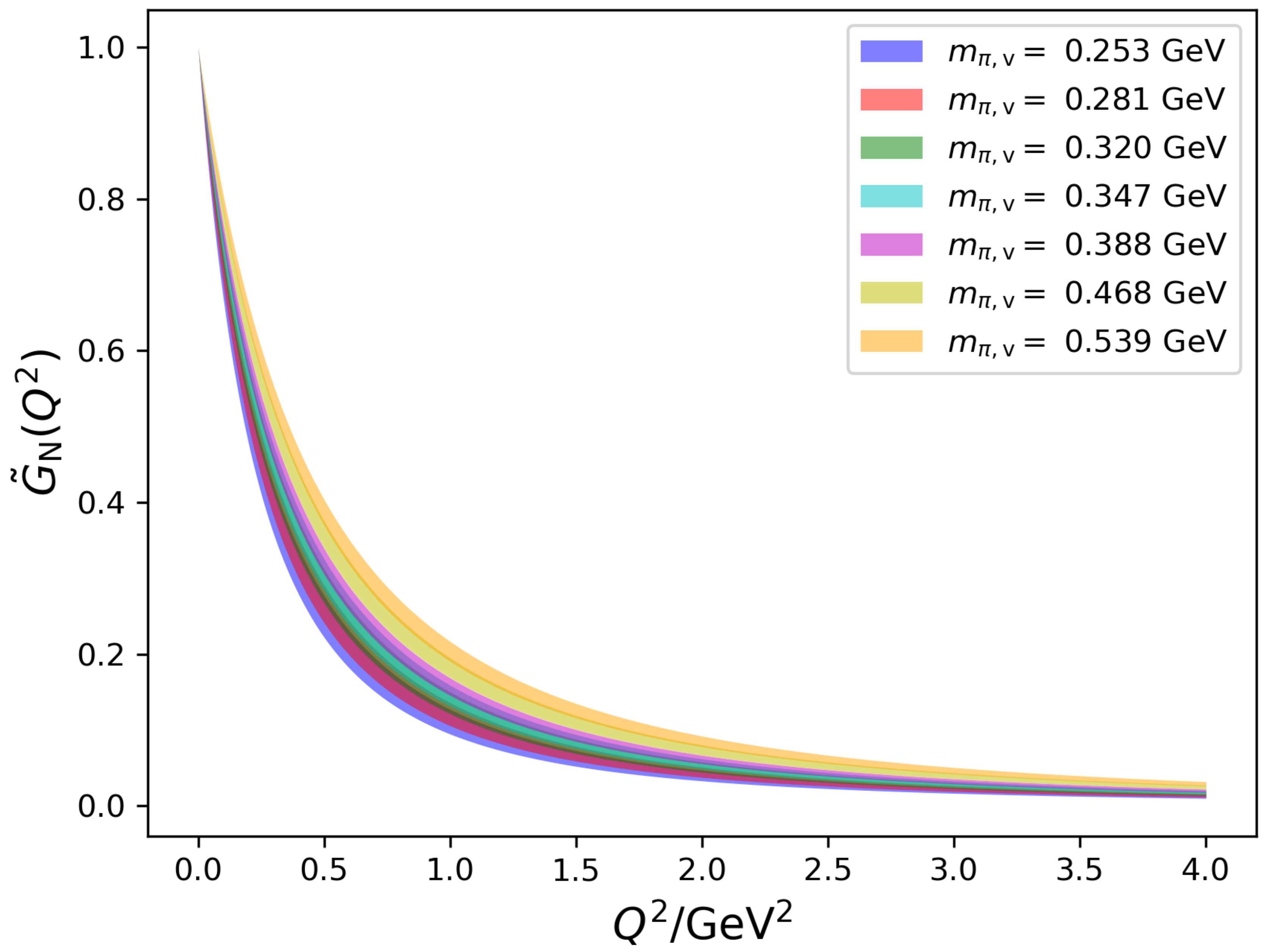


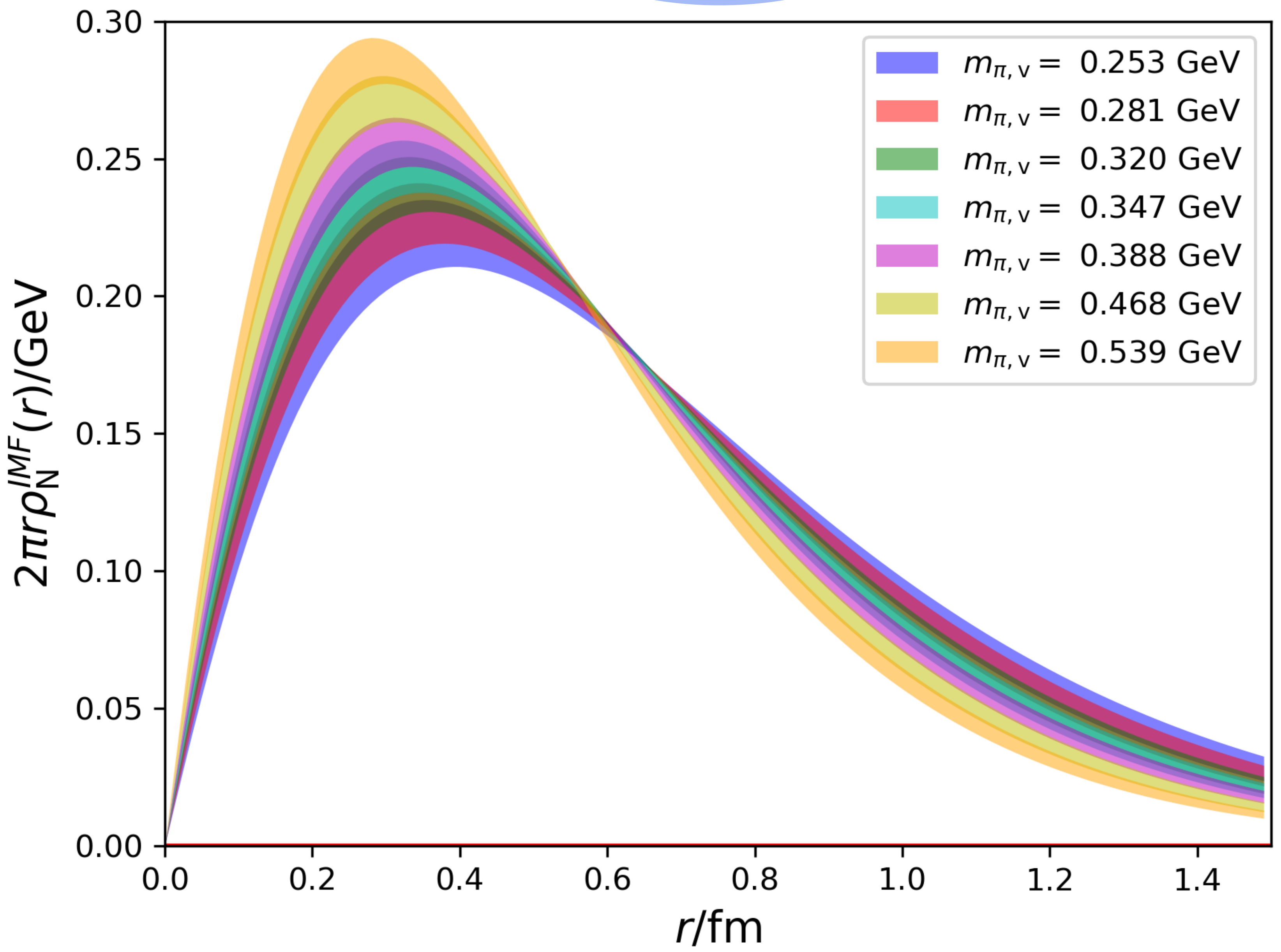


Atac, H., Constantinou, M., Meziani, ZE. et al. Measurement of the neutron charge radius and the role of its constituents. *Nat Commun* 12, 1759 (2021)

Trace anomaly of the nucleon (glue part)







$$\langle r^2 \rangle_{\mathrm{m}}(\mathrm{H}) = -6 \left. \frac{d\mathscr{F}_{\mathrm{m,H}}(Q^2)}{dQ^2} \right|_{Q^2 \to 0} \sim \langle r_g^2 \rangle_{\mathrm{ta}}(\mathrm{H}) = -6 \left. \frac{dG_{\mathrm{H}}(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

$$R_{\mathrm{m}} \simeq R_g \equiv \sqrt{\langle r_g^2 \rangle_{\mathrm{ta}}^{\mathrm{phys}}(\mathrm{N})} = 0.89(10)(07)\,\mathrm{fm}$$

Trace anomaly form factors and GFF

X. Ji, arXiv:2102.07830 [hep-ph]
K.-F. Liu, arXiv:2302.11600 [hep-ph]

Trace Anomaly Form Factors

$$T^\mu_\mu = \sum_f m_f \overline{\psi}_f \psi_f + \underbrace{\left[\sum_f m_f \gamma_m(g) \overline{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^2\right]}_{(T^\mu_\mu)^a \textit{ trace anomaly, RG invariant}}$$

$$1 = \int \frac{d^3p}{(2\pi)^3} |p\rangle \frac{m}{E_p} \langle p|, \quad |p\rangle = \sqrt{\frac{E_p}{m}} a_p^+ |\Omega\rangle$$

Energy-momentum Tensor

$$T^{\mu\nu} = \hat{T}^{\mu\nu} + \bar{T}^{\mu\nu}$$

Trace part: $\frac{\eta_{\mu\nu}}{d} T^\alpha_\alpha$ → $A_{q,g}$, $\bar{C}_{q,g}$

Traceless part: $(T^{\mu\nu} - \frac{\eta_{\mu\nu}}{d} T^\alpha_\alpha)$ → $A_{q,g}$, $B_{q,g}$, $D_{q,g}$

Tong, et al., Physics Letters B 823, 136751 (2021)

Pefkou, et al., Phys. Rev. D 105, 054509 (2022)

Hackett, et al., *Phys.Rev.Lett.* 132 (2024) 25, 251904

Y. Hatta, arXiv:1810.05116 [hep-ph]

Gravitational Form Factors

moments of Generalized Parton Distribution (GPD)

$$\langle P'|(T^{\mu\nu}_{q,g})|P\rangle/2m_N = \bar{u}(P')\Big[A_{q,g}(Q^2)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(Q^2)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2m_N} + D_{q,g}(Q^2)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{m_N} + \bar{C}_{q,g}(Q^2) m_N \eta^{\mu\nu}\Big]u(P)$$

$$|p\rangle = \sqrt{2E_p} a_p^+ |\Omega\rangle$$

$$\langle p', \mathbf{s}'|T^\mu_\mu|p, \mathbf{s}\rangle = m_N \mathscr{F}_{m,N}(Q^2) \bar{u}(p', \mathbf{s}') u(p, \mathbf{s})$$

$$\mathscr{F}_{m,H}(Q^2) = \mathscr{F}_{ta,H}(Q^2) + \mathscr{F}_{\sigma,H}(Q^2)$$

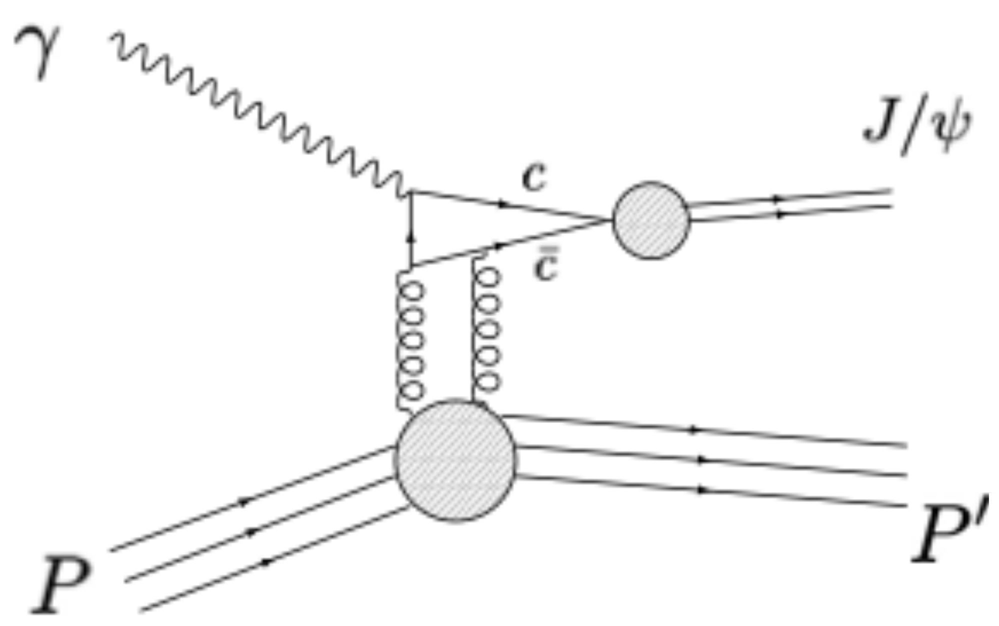
$$\langle P'|T^\mu_\mu|P\rangle/2m_N = \bar{u}(P')\Big[(A(Q^2)m_N - B(Q^2)\frac{Q^2}{4m_N} + 3D(Q^2)\frac{Q^2}{m_N}\Big]u(P)$$

$$\langle r^2\rangle_m(H) \sim \langle r^2\rangle_{ta} = -6\left(\frac{dA(Q^2)}{dQ^2} + \frac{3D(0)}{M^2} - \frac{d\mathscr{F}_\sigma(Q^2)}{dQ^2}\right)$$

Mass radius of the nucleon

- Our result with lattice QCD

$$R_\mathrm{m} \simeq R_g \equiv \sqrt{\langle r_g^2 \rangle_\mathrm{ta}^\mathrm{phys}(\mathrm{N})} = 0.89(10)(07)\,\mathrm{fm}$$



$$\mathcal{M}_{\gamma P \to \psi P}(t) = -Qe\, c_2\, \frac{16\pi^2 M}{b}\, \langle P'|T|P\rangle.$$

- A direct dipole fit to the recent GlueX Collaboration (experimental) data:

$$R_\mathrm{m} \equiv \sqrt{\langle R_\mathrm{m}^2 \rangle} \simeq R_g = 0.55(3)\,\mathrm{fm}$$

D. E. Kharzeev, Phys. Rev. D 104, 054015 (2021),arXiv:2102.00110
A. Ali et al. (GlueX), Phys. Rev. Lett. 123, 072001 (2019), arXiv:1905.10811

- A recent ***lattice*** calculation of the quark and glue GFF

$$R_\mathrm{m} \equiv \sqrt{\langle r^2 \rangle_\mathrm{m}^\mathrm{GFF}(\mathrm{N})} = 1.038(98)\,\mathrm{fm}$$

Pefkou, et al., Phys. Rev. D 105, 054509 (2022) Pefkou, Private communication
Hackett, et al., *Phys.Rev.Lett.* 132 (2024) 25, 251904

- A ***holographic GFF*** calculation with lattice input:

$$R_\mathrm{m} \simeq R_g = 0.926(8)\,\mathrm{fm}$$

K. A. Mamo and I. Zahed, Phys. Rev. D 106, 086004 (2022), arXiv:2204.08857 [hep-ph]

- Using the quark (from ***lattice***) and the glue GFF from ***fitting the near-threshold*** J/Ψ ***production*** at $\xi > 0$

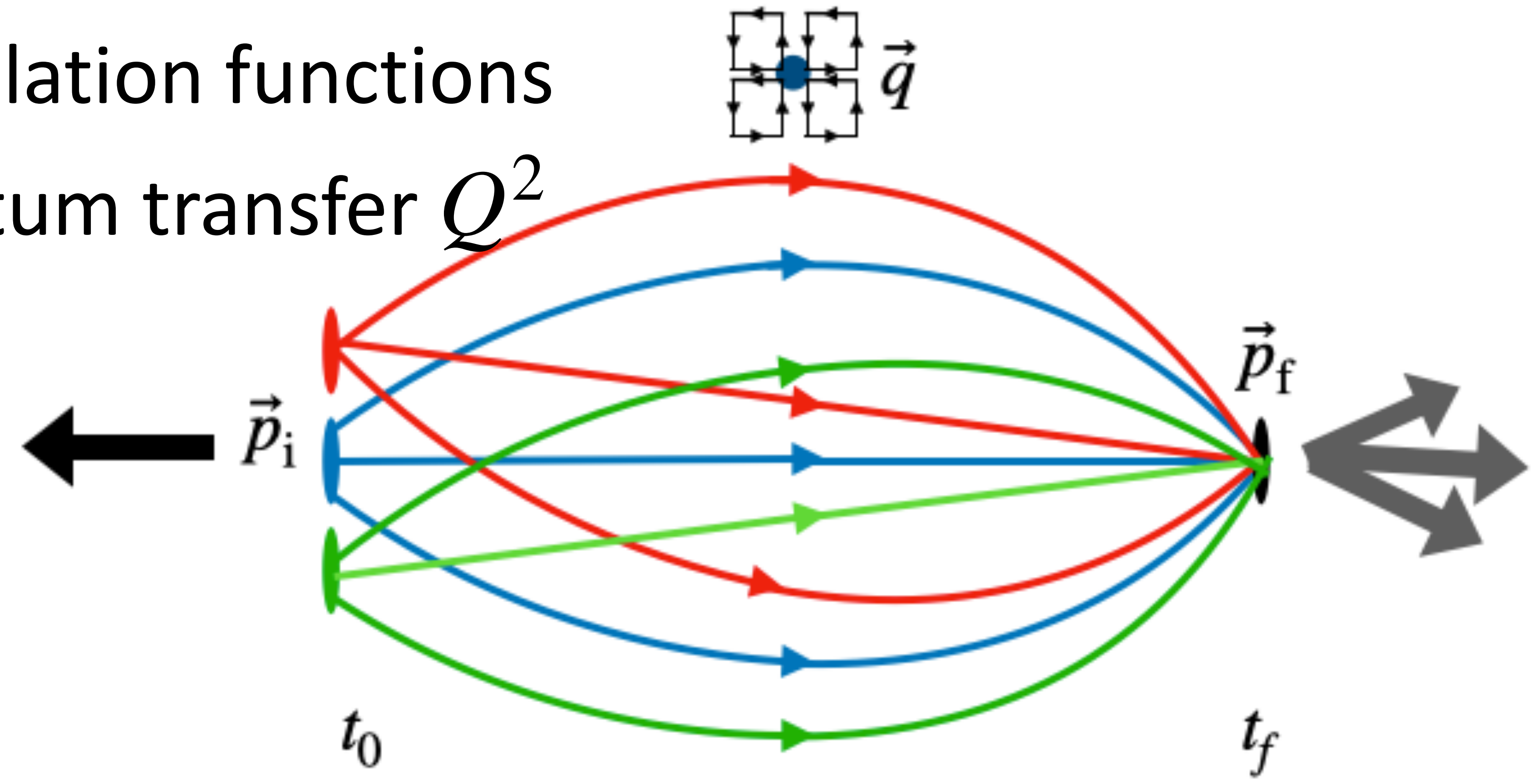
$$R_\mathrm{m} \simeq R_g = 1.20(13)\,\mathrm{fm}$$

Y. Guo, X. Ji, Y. Liu, and J. Yang, Phys. Rev. D 108, 034003 (2023), arXiv:2305.06992 [hep-ph]

Systematic uncertainties?

3pt/2pt correlation functions

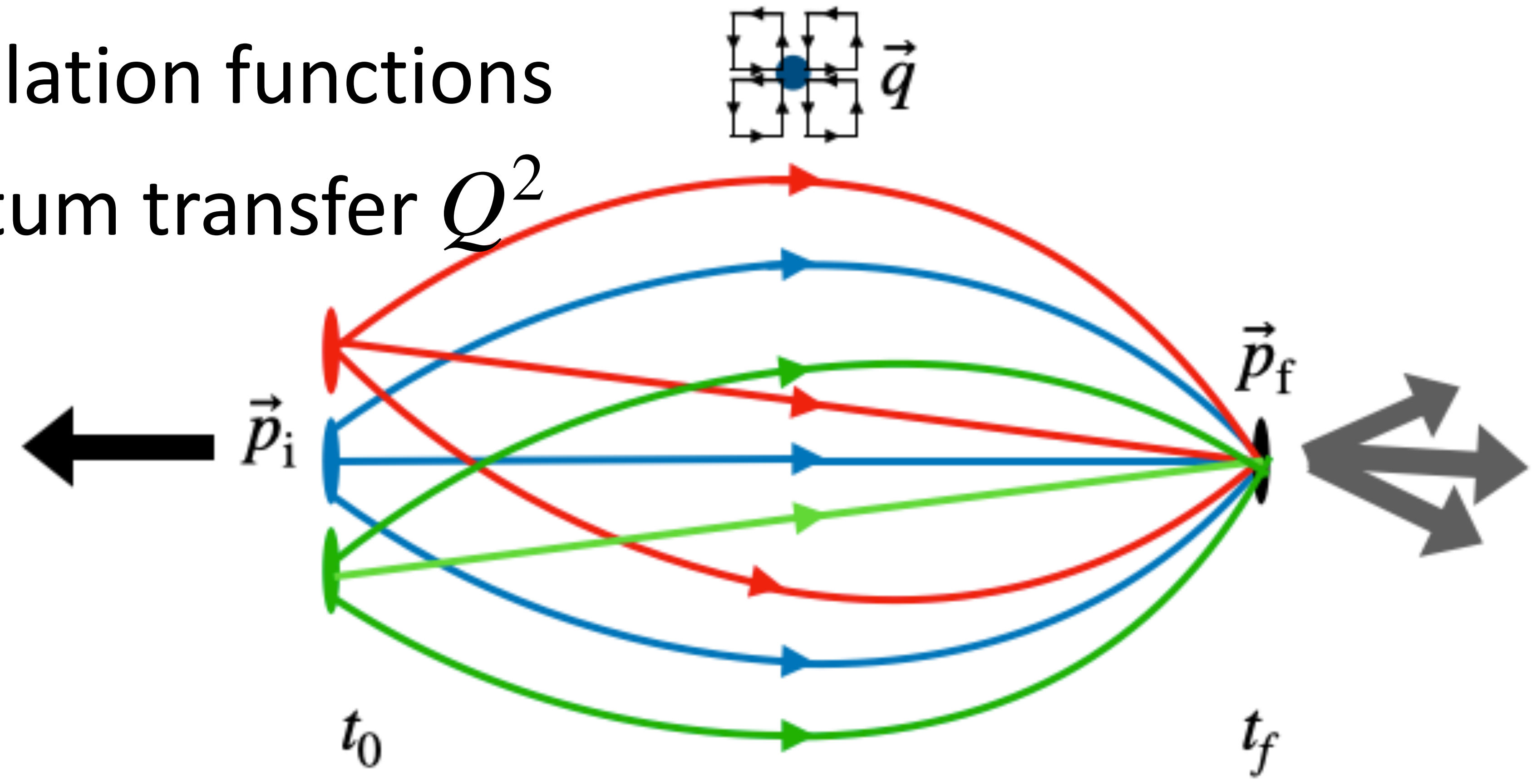
with momentum transfer Q^2

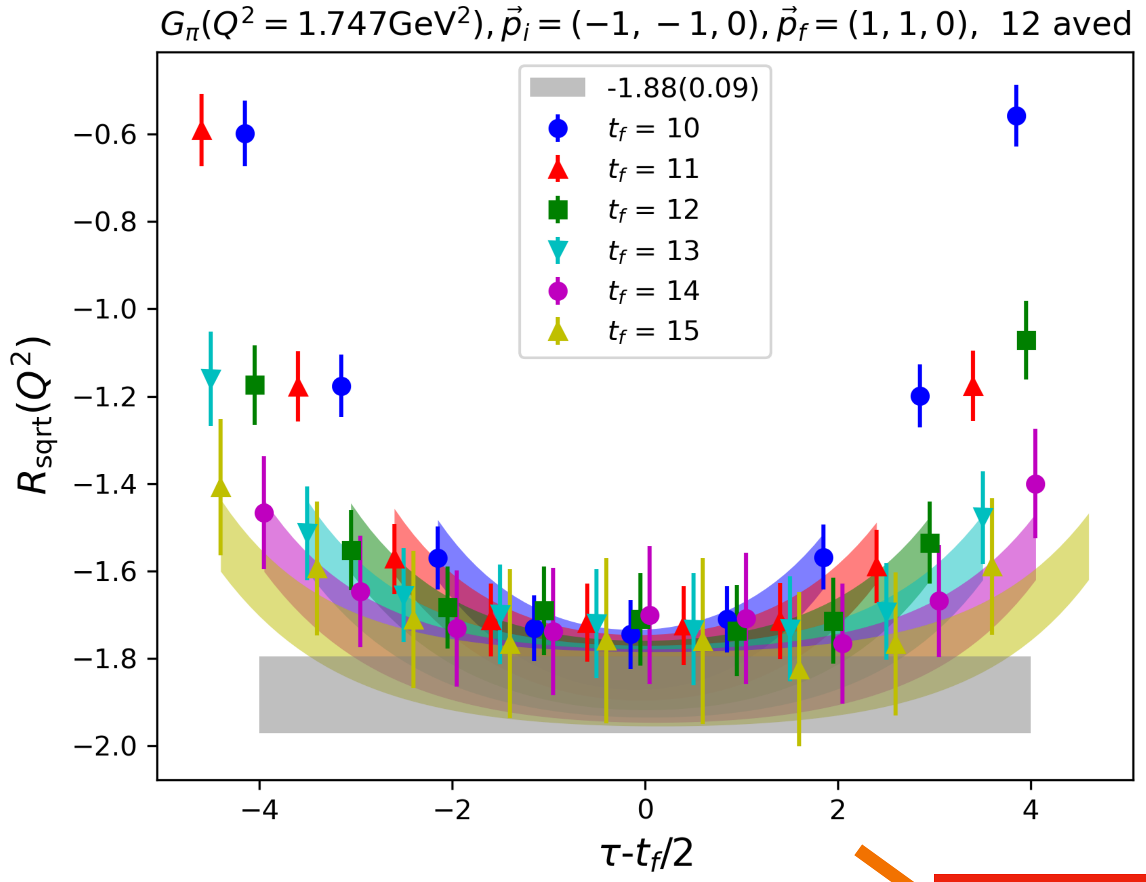


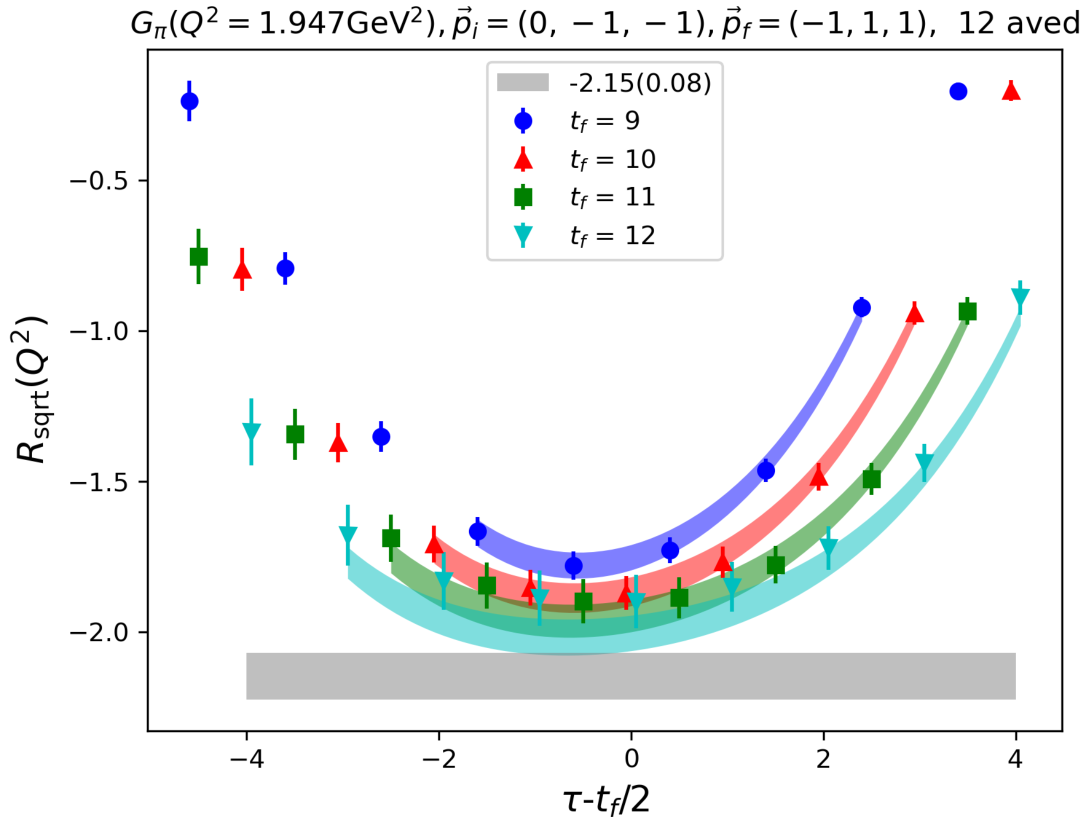
Systematic uncertainties?

3pt/2pt correlation functions

with momentum transfer Q^2

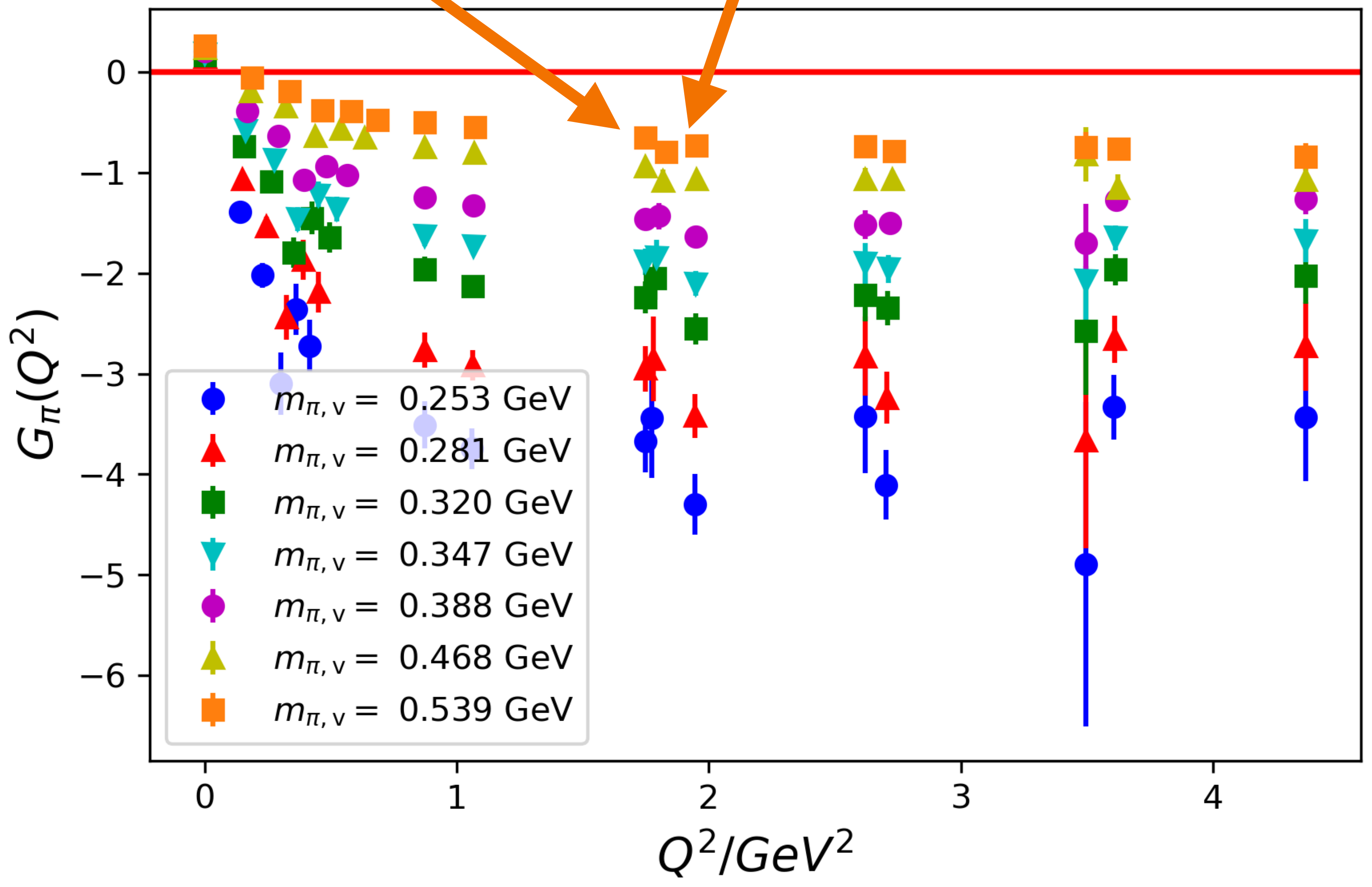






excited-state contaminations

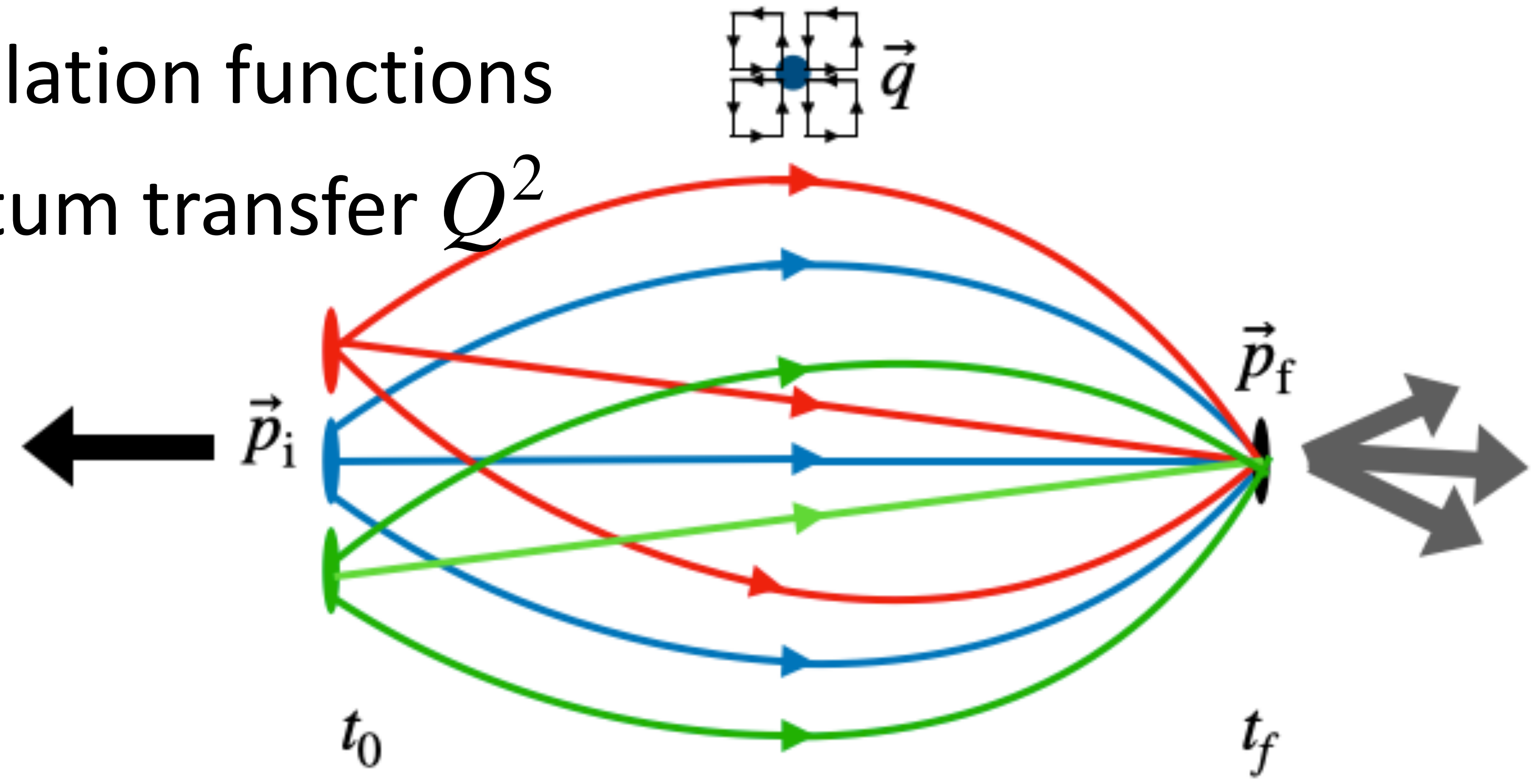
form factors $\mathscr{F}_{\mathrm{m,H}}(Q^2)$

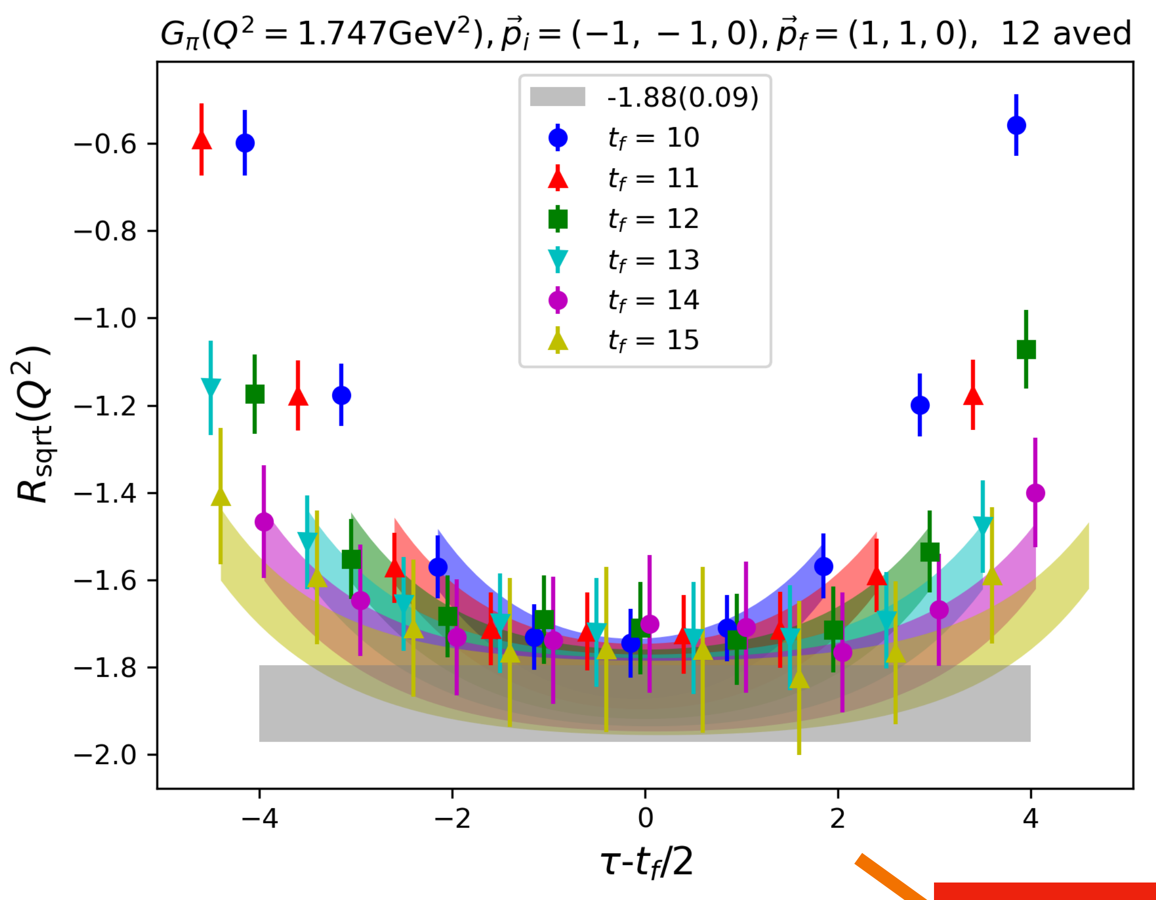


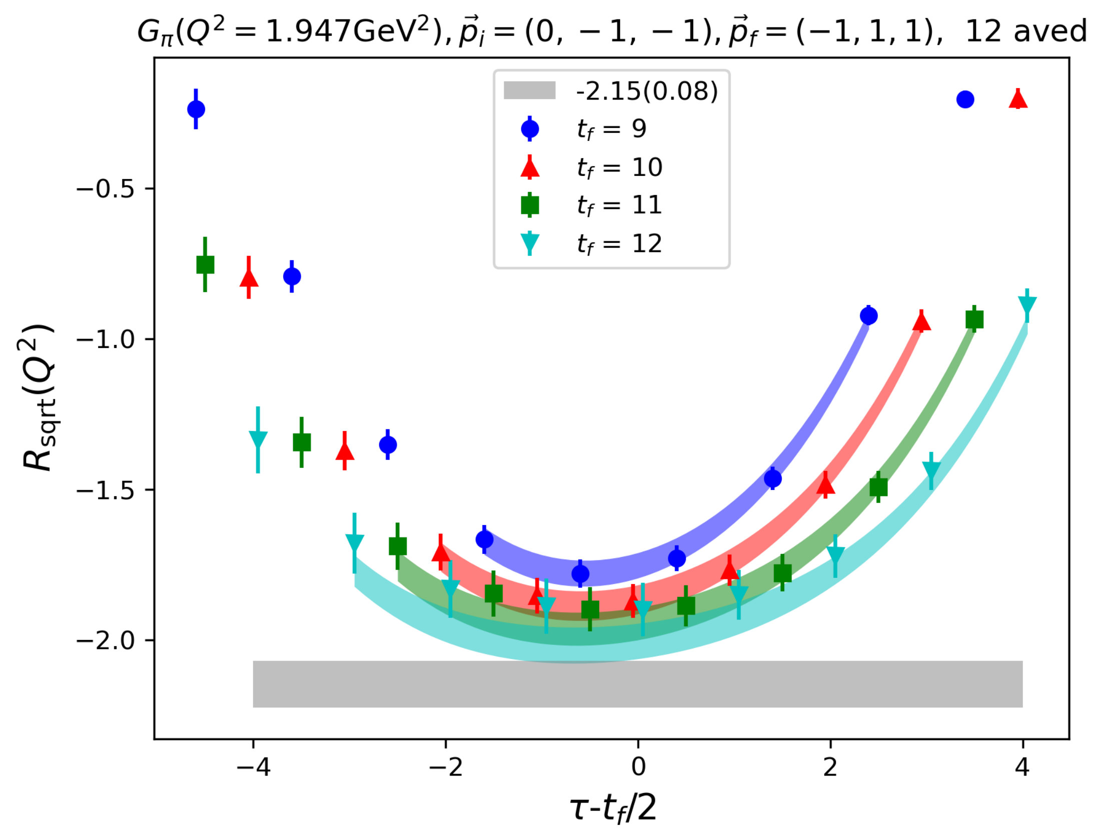
Systematic uncertainties?

3pt/2pt correlation functions

with momentum transfer Q^2

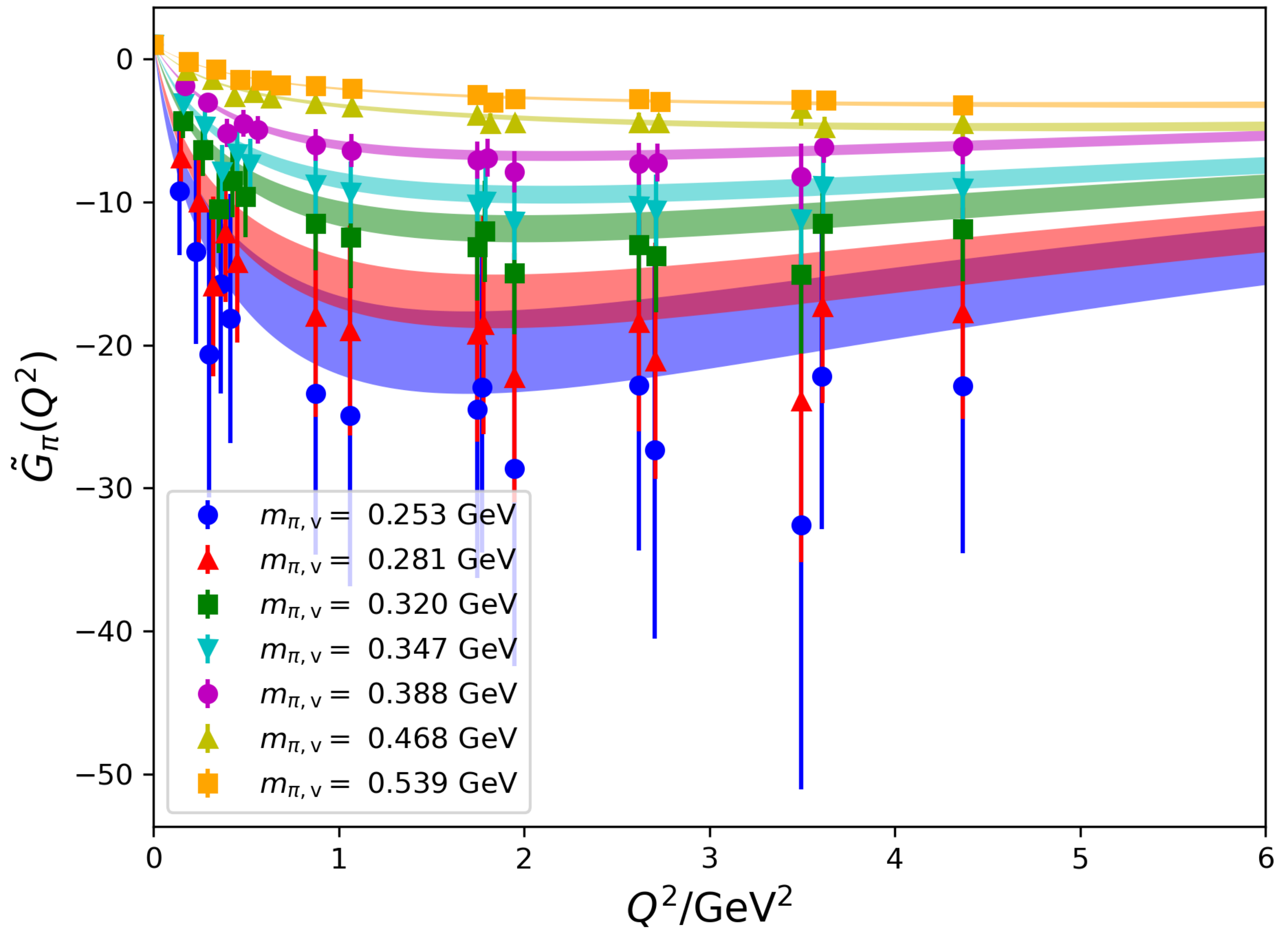






excited-state contaminations

form factors $\mathscr{F}_{\mathrm{m,H}}(Q^2)$



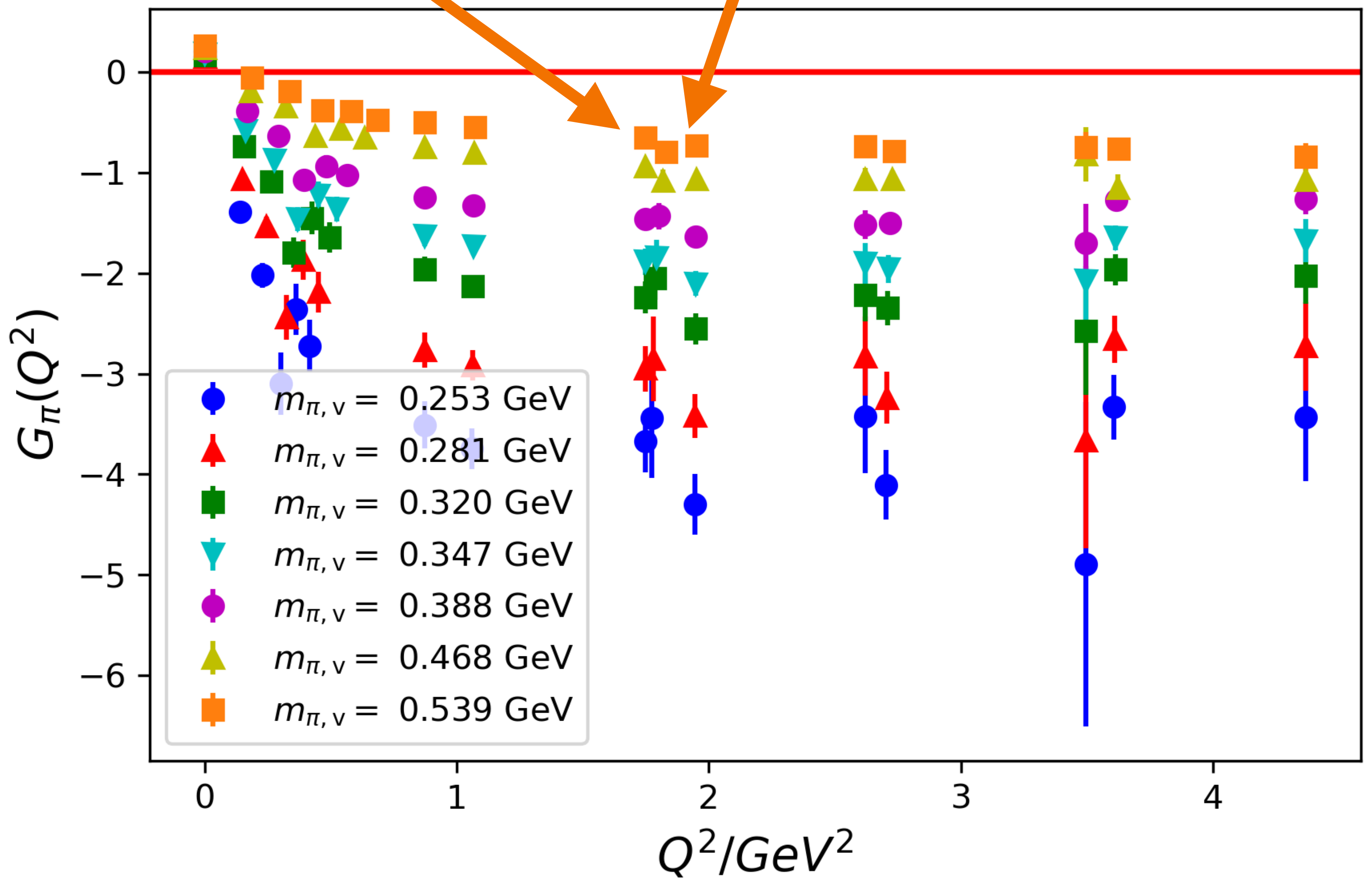
z-expasion fit

$$\tilde{G}_\mathrm{H}(Q^2) = \sum_{k=0}^{k_\mathrm{max}} a_k z^k,$$

$$z(t, t_\mathrm{cut}, t_0) = \frac{\sqrt{t_\mathrm{cut} - t} - \sqrt{t_\mathrm{cut} - t_0}}{\sqrt{t_\mathrm{cut} - t} + \sqrt{t_\mathrm{cut} - t_0}}, t = -Q^2$$

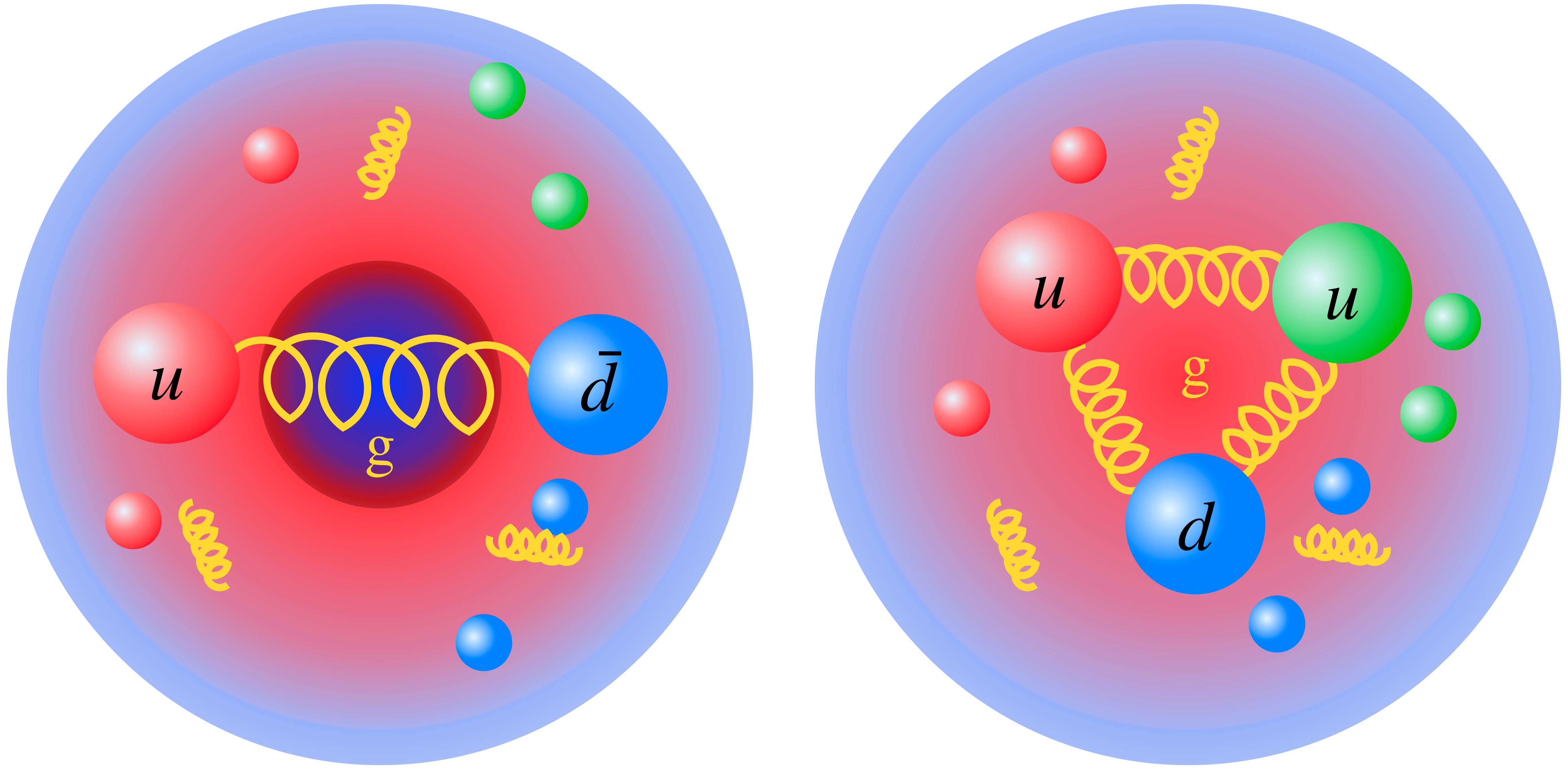
[R. J. Hill and G. Paz, Phys. Rev. D 82, 113005 (201(

form factors $\mathscr{F}_{\mathrm{m,H}}(Q^2)$



Conclusion and outlook

- We have calculated the trace anomaly form factors of the EMT(glue part) with lattice QCD to **reveal the mass distribution within hadrons**:



$$R_{\mathrm{m}} \simeq R_g \equiv \sqrt{\langle r_g^2 \rangle_{\mathrm{ta}}^{\mathrm{phys}}(\mathrm{N})} = 0.89(10)(07)\ \mathrm{fm}$$

$$T^{\mu}_{\mu} = \underbrace{\sum_f m_f \overline{\Psi}_f \Psi_f}_{\text{the } \sigma \text{ term}} + \underbrace{\frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\Psi}_f \Psi_f}_{\langle (T^{\mu}_{\mu})_{\mathrm{a}} \rangle \text{ trace anomaly, RG invariant}}$$

m_π^{phys} $\quad a \to 0$

Outlook

- In the future, we will include the quark part for the pion, nucleon and ρ meson.
- Calculations on lattice ensembles with physical quark masses and smaller lattice spacings for extrapolations to the continuum limit.

- **Better ways to control/estimate:**
  - **excited-state contaminations.**
  - **uncertainties from the fitting of the form factors.**

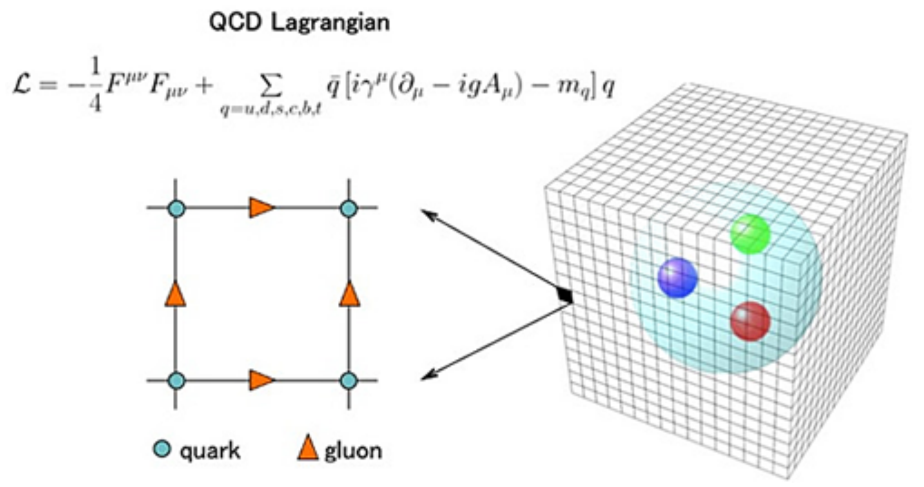
Backup Slides

Numerical setup with lattice QCD

- Observables from the path integral (Euclidean):

$$\langle O \rangle = \frac{\int \mathscr{D}\phi O e^{-S_\mathrm{E}(\phi)}}{\int \mathscr{D}\phi e^{-S_\mathrm{E}(\phi)}} \longleftarrow \quad Z_E(J) = \int \mathscr{D}\phi e^{-\int d^4 x_E \mathscr{L}_E}$$

(partition function in statistical mechanics)



- Ensemble average: with distribution $\propto e^{-S_\mathrm{E}(\phi)}$

$$\langle O \rangle \simeq \frac{1}{N}\sum_N O_i \pm \mathcal{O}(\sqrt{N})$$

- Generate with Monte-Carlo methods with lattice actions $S_\mathrm{E}^\mathrm{lat}(\phi)$:
  2+1-flavor domain-wall fermion configurations with Iwasaki gauge action by RBC-UKQCD collab.

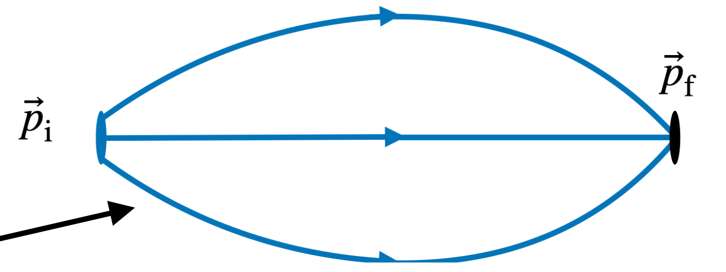
Input parameters
$a, m_q, \ldots, \alpha_s, \ldots$

Ensemble	$L^3 \times T$	a (fm)	L(fm)	m_π (MeV)	N_conf
24I	$24^3 \times 64$	0.1105(3)	2.65	340	788

E.g. n-point correlation functions

$$\langle \Omega | \mathrm{T}\phi(x_1)\ldots\phi(x_n) | \Omega \rangle$$

↓ Wick's theorem



Renormalization → hadron spectrum, matrix elements …, at $m_{\mathrm{v},q}$

$$S_F(y,j,b;x,i,a) = (D_\mathrm{ov}^{-1})^{y,j,b}_{x,i,a}$$

Overlap fermion with valence quark masses $m_{\mathrm{v},q}$

Renormalization on the lattice

The trace anomaly emerges with the lattice regulation after renormalization:

$$T^\mu_\mu = \underbrace{\sum_f m_f \overline{\psi}_f \psi_f}_{\text{the } \sigma \text{ term}} + \underbrace{\frac{\beta(g)}{2g} F^2 + \sum_f \gamma_m(g) m_f \overline{\psi}_f \psi_f}_{\langle (T^\mu_\mu)_a \rangle \text{ trace anomaly,RG invariant}}$$

S. Caracciolo, G. Curci, P. Menotti, and A. Pelissetto, Annals of Physics 197, 119 (1990)

H. Makino and H. Suzuki, Progress of Theoretical and Experimental Physics 2014, 063B02 (2014)

M. Dalla Brida, L. Giusti, and M. Pepe, JHEP 04, 043 (2020), arXiv:2002.06897 [hep-lat]

Renormalization method: based on the mass sum rule

F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

- $\frac{\beta(g)}{2g}$ and $\gamma_m(g)$ are independent of the hadron state
- Solve the mass sum rule equations for pseudo-scalar(π) and vector meson(ρ) at one valence mass:

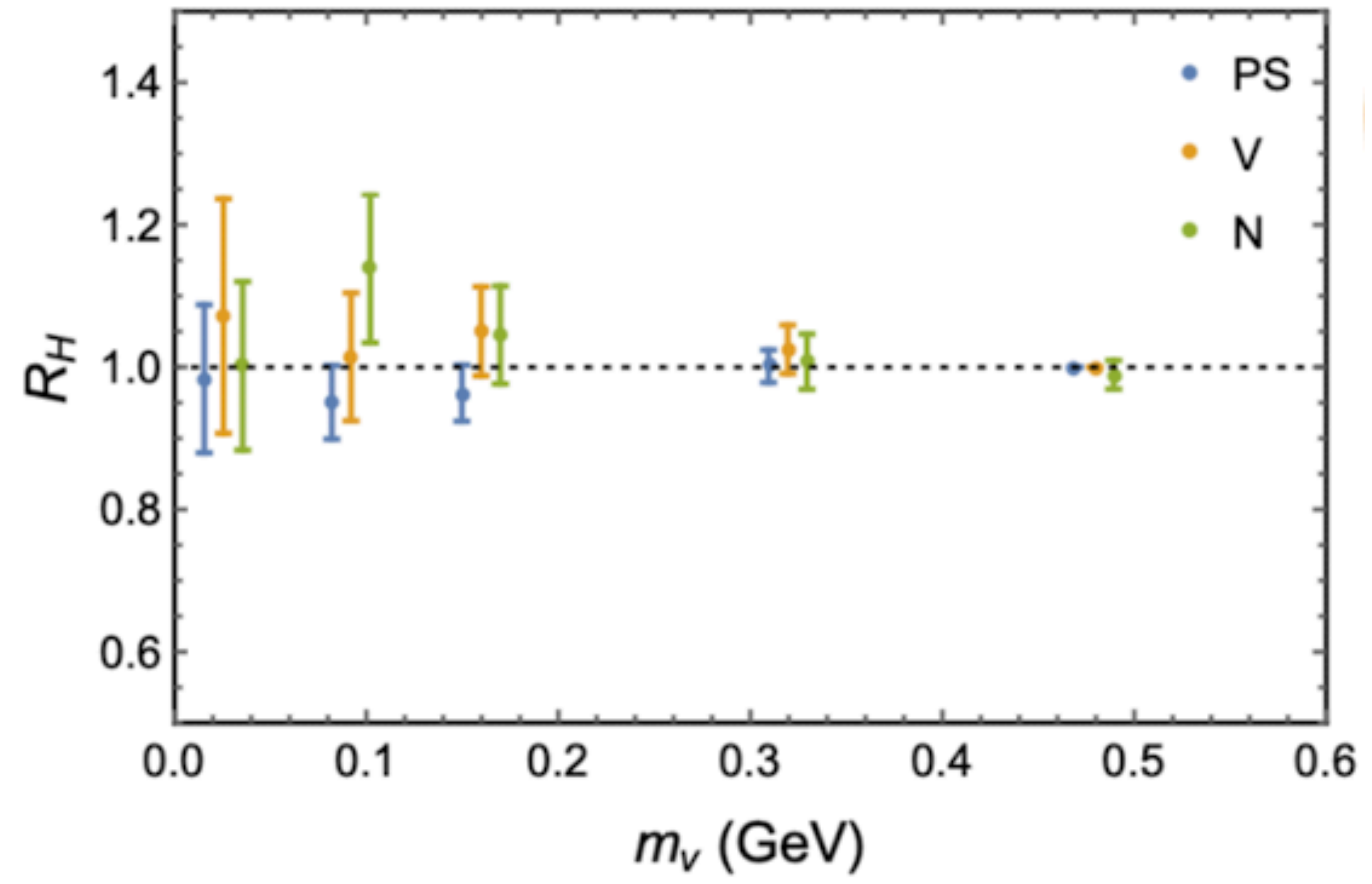
$$M_{\rm PS} - (1+\gamma_m)\langle H_m \rangle_{\rm PS} - \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm PS} = 0,$$

$$M_{\rm V} - (1+\gamma_m)\langle H_m \rangle_{\rm V} - \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm V} = 0$$

and obtain the bare $\frac{\beta(g)}{2g}$ and $\gamma_m(g)$

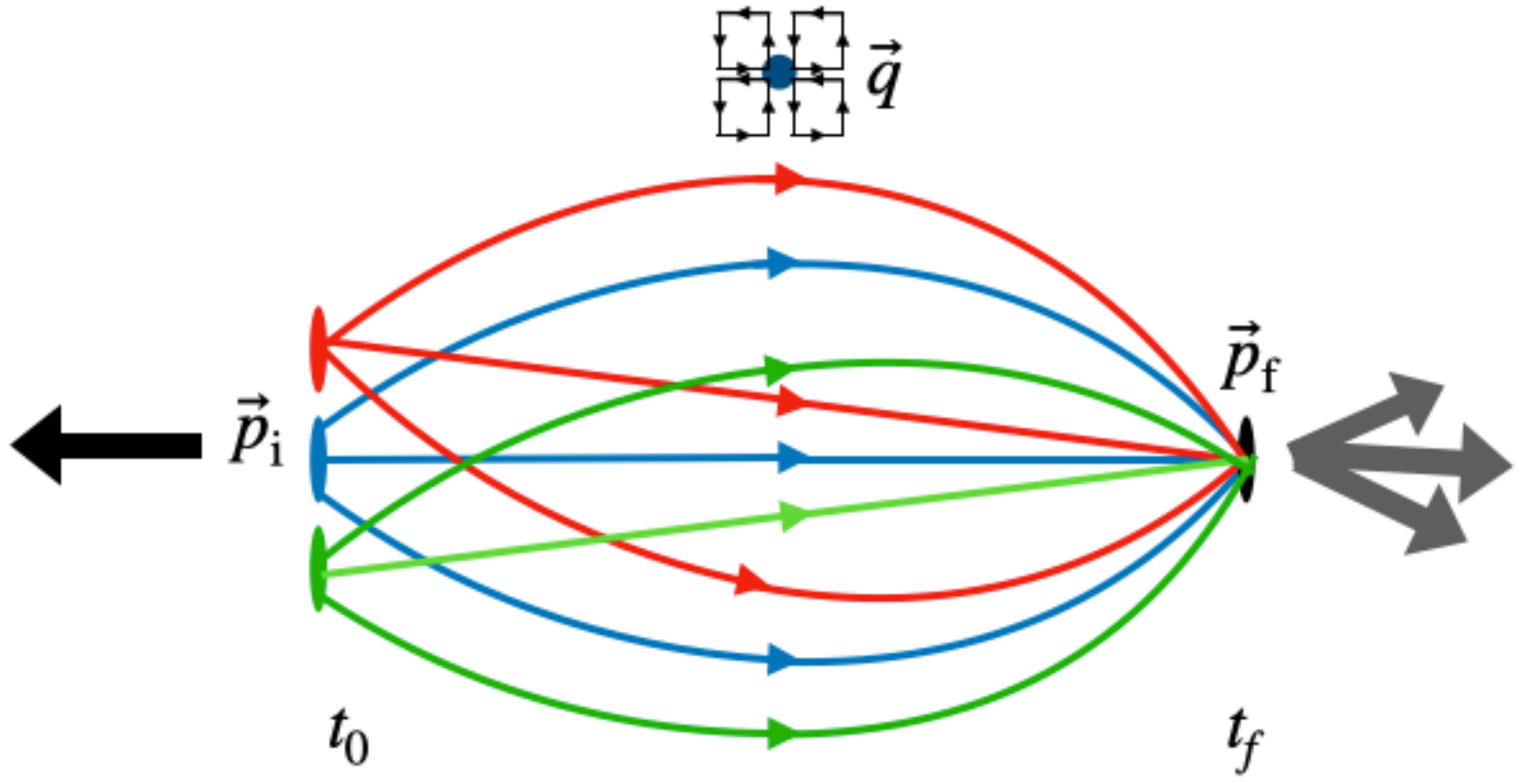
- Verify the assumptions: sum rule satisfied for other masses →Mixing with lower dimensional operators is negligible

$$R_H = \left[(1+\gamma_m)\langle H_m \rangle_{\rm H} + \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm H}\right] / m_{\rm H} \sim 1$$



Joint fit of the two- and three point functions

Three-point functions

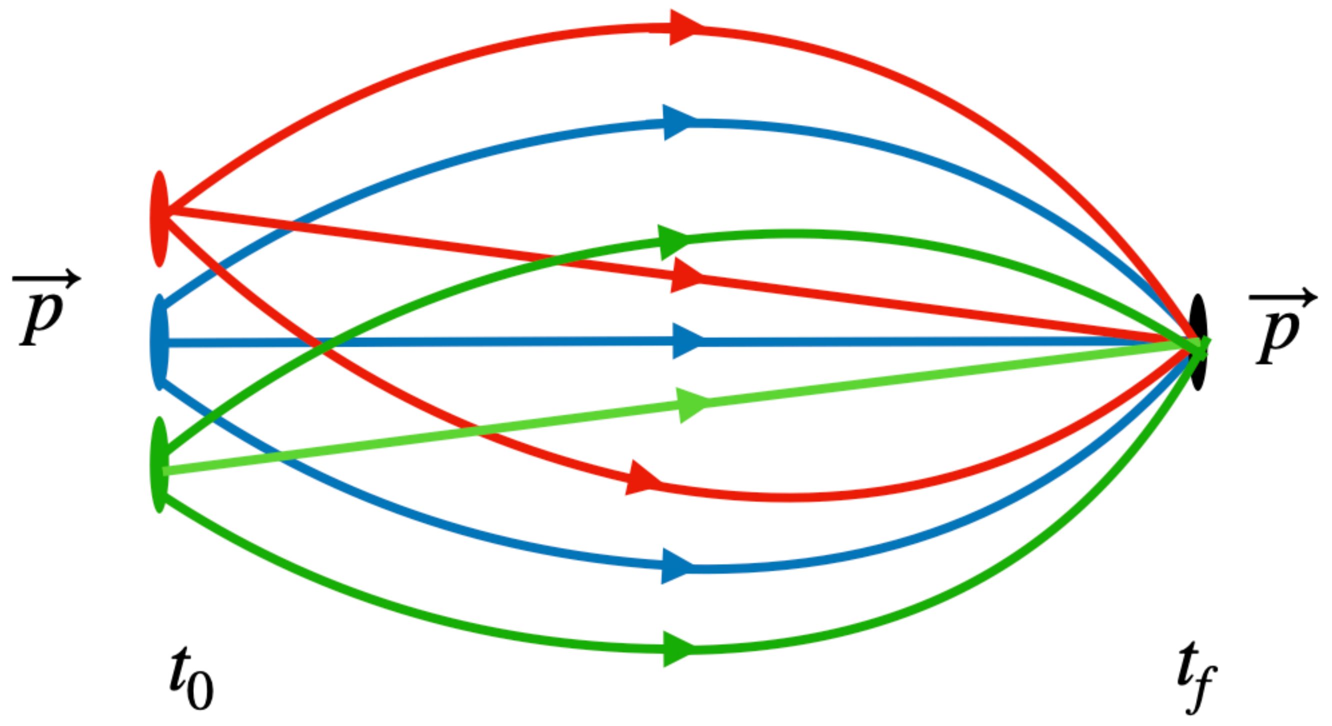


$$C_{\mathrm{H,3pt}}(\vec{p}_i,\vec{p}_f,t,\tau) = m_{\mathrm{H}}\, G_{\mathrm{H}}(Q^2)\, \mathscr{K}_{\mathrm{H,3pt}}(p_i,p_f) Z_{\vec{p}_i} Z_{\vec{p}_f} e^{-E_i\tau - E_f(t-\tau)}$$

$$+C_1 e^{-E_i^1\tau - E_f(t-\tau)} + C_2 e^{-E_i\tau - E_f^1(t-\tau)} + C_3 e^{-E_i^1\tau - E_f^1(t-\tau)}$$

3pt-2pt joint fit $\rightarrow G_{\mathrm{H}}(Q^2)$

Two-point functions

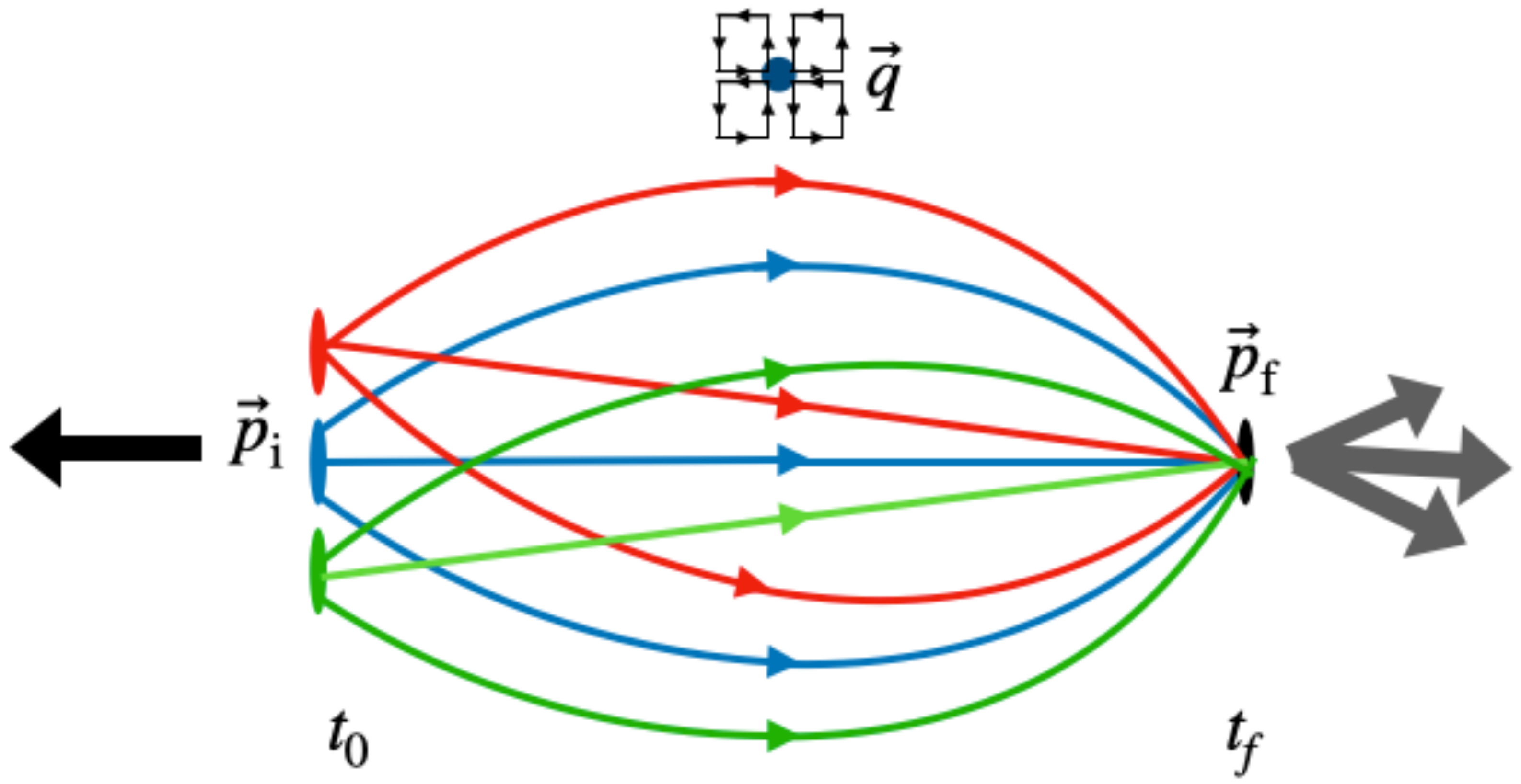


$$C_{\mathrm{H,2pt}}(t) = \mathscr{K}_{\mathrm{H,2pt}}(E_i) Z^2_{\vec{p}_i}[e^{-E_i t} + e^{-E_i(T-t)}] + A_1 e^{-E_i^1 t}$$

$$C_{\mathrm{H,2pt}}(t) = \mathscr{K}_{\mathrm{H,2pt}}(E_f) Z^2_{\vec{p}_f}[e^{-E_f t} + e^{-E_f(T-t)}] + A_1 e^{-E_f^1 t}$$

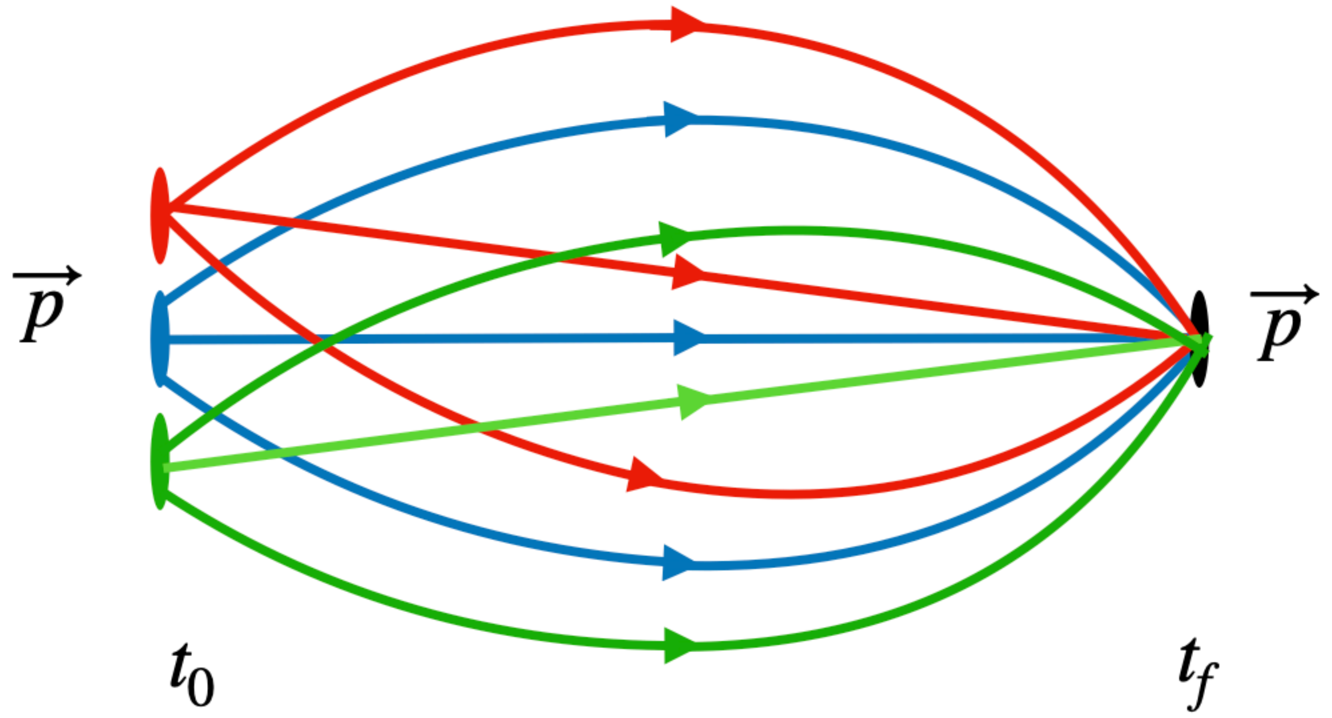
Ratio of the two- and three point functions

Three-point functions



$$R_{\mathrm{sqrt}}(t,\tau;\vec{p}_i,\vec{p}_f) = \frac{C_{\mathrm{H},3pt}(t,\tau;\vec{p}_i,\vec{p}_f)}{C_{\mathrm{H},2pt}(t;\vec{p}_f)} \times \sqrt{\frac{C_{\mathrm{H},2pt}(t-\tau;\vec{p}_i)C_{\mathrm{H},2pt}(t;\vec{p}_f)C_{\mathrm{H},2pt}(\tau;\vec{p}_f)}{C_{\mathrm{H},2pt}(t-\tau;\vec{p}_f)C_{\mathrm{H},2pt}(t;\vec{p}_i)C_{\mathrm{H},2pt}(\tau;\vec{p}_i)}} \Bigg/ \left[\frac{m_\mathrm{H}\mathscr{K}_{\mathrm{H},3pt}(p_i,p_f)}{\sqrt{\mathscr{K}_{\mathrm{H},2pt}(p_i)\mathscr{K}_{\mathrm{H},2pt}(p_f)}}\right]$$

Two-point functions



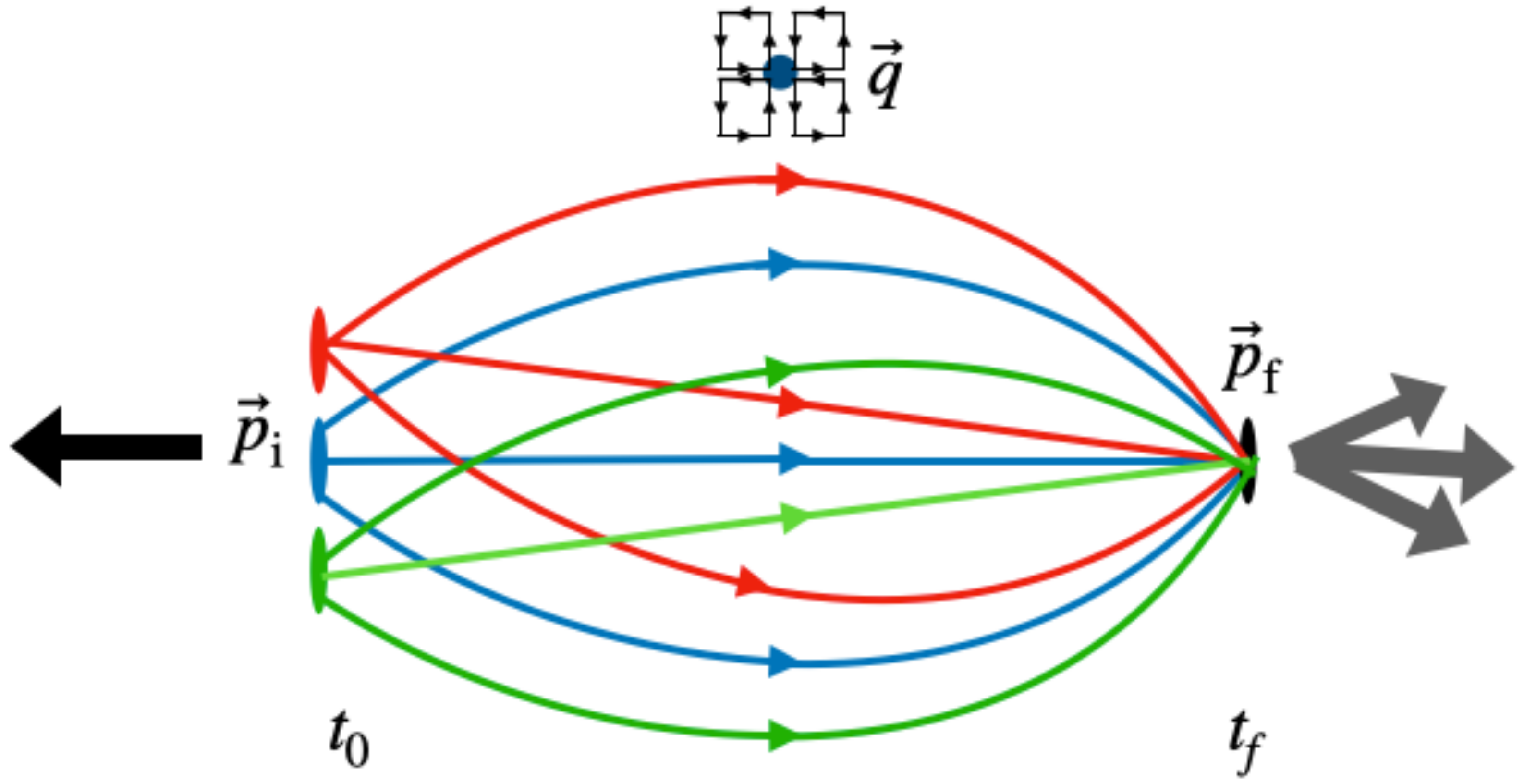
$$R_{\mathrm{sqrt}} \sim G_\mathrm{H}(Q^2) + C_1'' e^{-\Delta E_i^1 \tau} + C_2'' e^{-\Delta E_f^1 (t-\tau)} + C_3'' e^{-\Delta E_i^1 \tau - \Delta E_f^1 (t-\tau)}$$

$$\xrightarrow{t \gg \tau \gg 0} G_\mathrm{H}(Q^2)$$

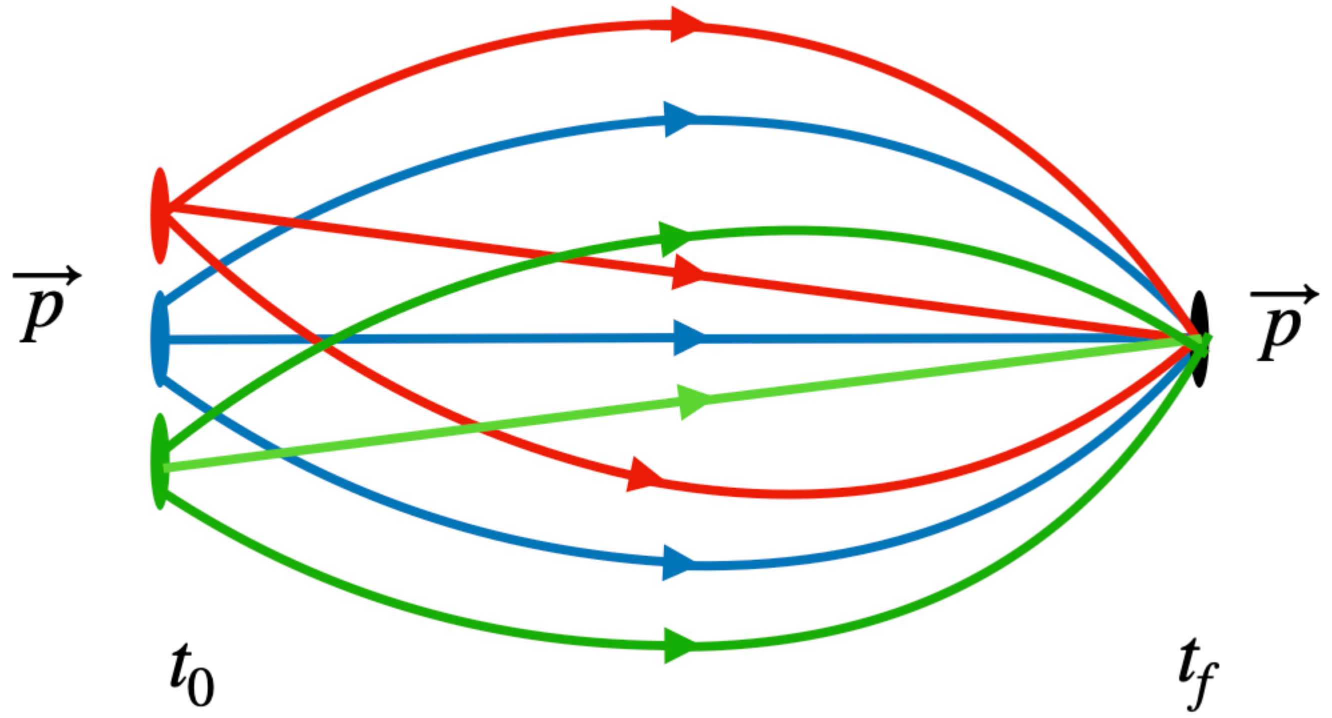
Compare it with the results from 3pt-2pt joint fit

Extract form factors from the two- and three point functions

Three-point functions



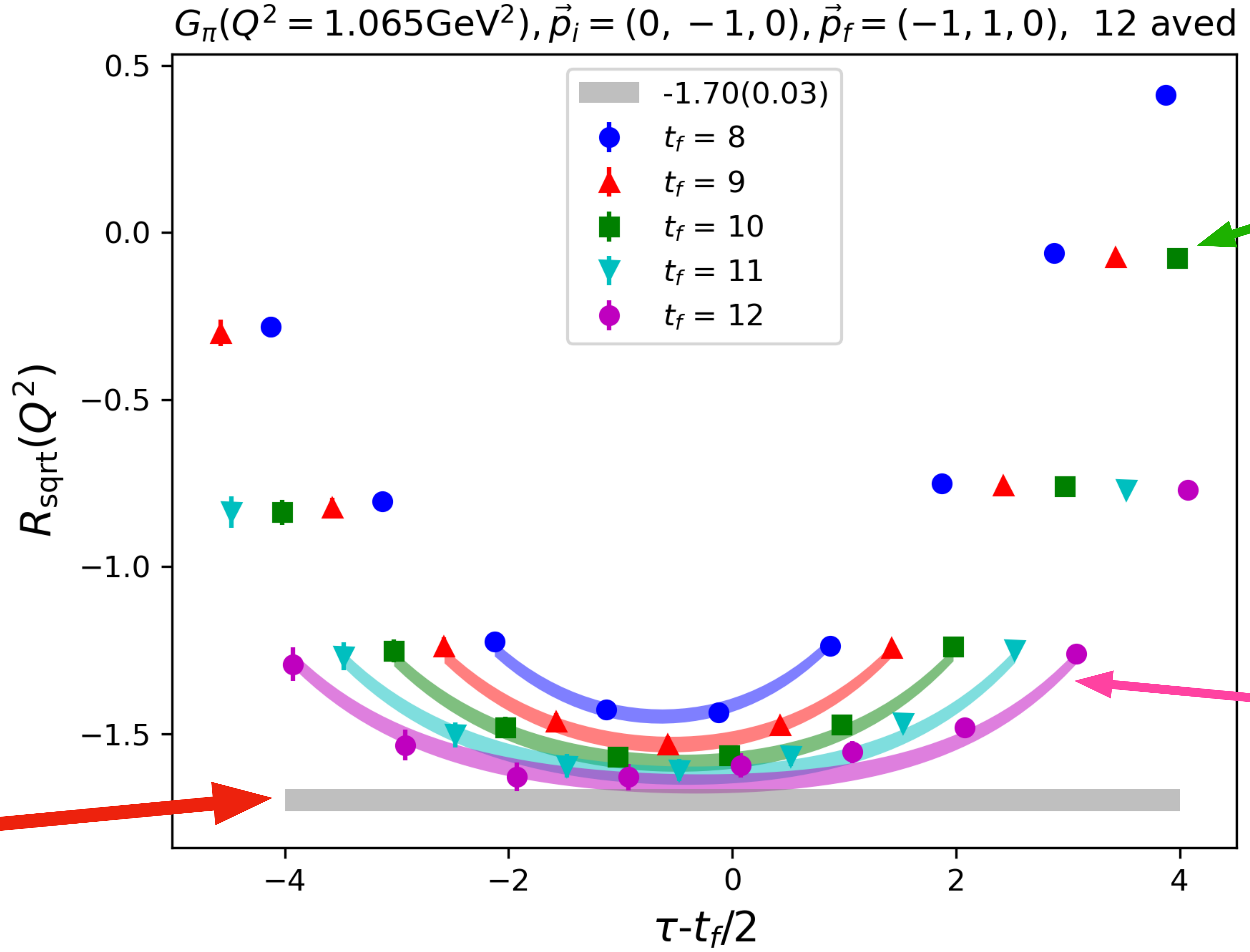
Two-point functions



$$R_\mathrm{sqrt} \sim G_\mathrm{H}(Q^2) + C_1'' e^{-\Delta E_i^1 \tau} + C_2'' e^{-\Delta E_f^1 (t-\tau)} + C_3'' e^{-\Delta E_i^1 \tau - \Delta E_f^1 (t-\tau)}$$

$$\xrightarrow{t \gg \tau \gg 0} G_\mathrm{H}(Q^2)$$

Compare it with the results from 3pt-2pt joint fit



R_sqrt from data

R_sqrt from joint fit

$G_\mathrm{H}(Q^2)$

Extract form factors from the two- and three point functions

- source-at-rest: $|\vec{p}_\mathrm{i}| = 0$ with $\vec{q} = \vec{p}_\mathrm{f}$
- back-to-back: $\vec{p}_\mathrm{f} = -\vec{p}_\mathrm{i}$ with $\vec{q} = 2\vec{p}_\mathrm{f}$
- near-back-to-back: $\vec{p}_\mathrm{f} \neq -\vec{p}_\mathrm{i}, \vec{p}_\mathrm{f} \,\&\, -\vec{p}_\mathrm{i} \sim \vec{q}/2$