Collins-Soper kernel from collinear parton correlators

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Outline

- **Inclusive DIS at the endpoint**

- **Off-lightcone collinear factorization**

- **Mellin space, rapidity evolution & the Collins-Soper kernel**
  - Collinear parton correlators
  - Collinear vacuum correlators

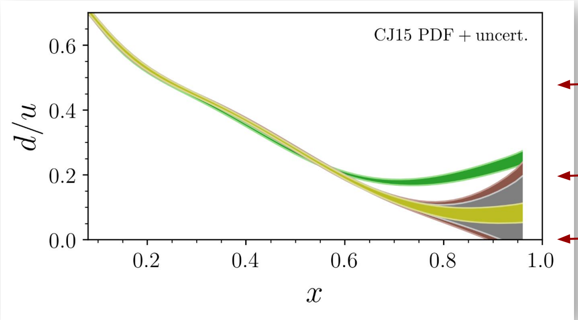
→ **New avenues for lattice calculation**

→ **Simple picture for large-x phenomenology**

Inclusive DIS at the endpoint

DIS at threshold (large Bjorken x)

- **Relevant for hadron structure**
  - d/u ratio & confinement
  - Positivity bounds
  - …



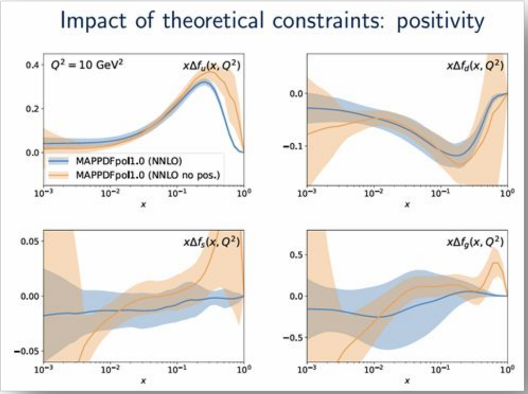
spin-flavor symmetry

hard gluons

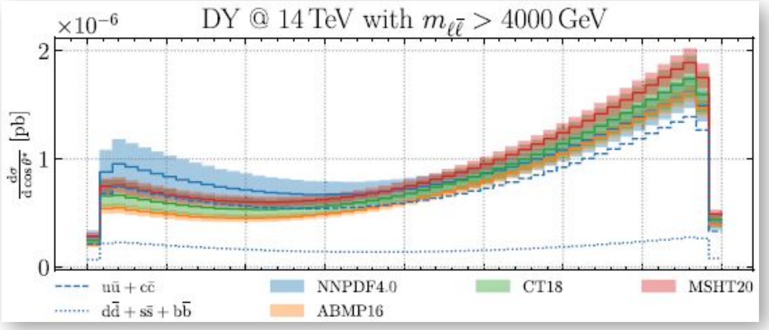
spin-½ quark + spin-0 diquark

Li, Accardi et al, PRD 109 (2024)

- **Large-x PDFs & forward LHC**
  - Precision needed for new physics
  - Especially at large rapidity / invariant mass



Nocera @ QCD evolution 2024



Ball et al, EPJC 82 (2022)

$W^2 = 4\ \mathrm{GeV}^2$

DIS at threshold (large Bjorken x)

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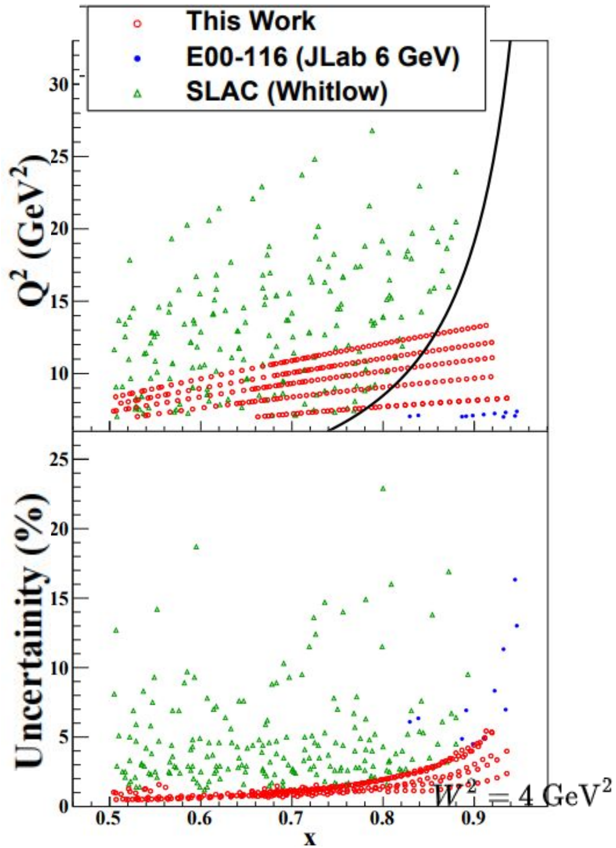
- **High-precision data from JLab 12**
  - p/D (Hall C, *2409.15236*)
  - BONUS 12 (soon!)
  - …



DIS at threshold (large Bjorken x)

- **Relevant for hadron structure**
  - d/u ratio & confinement
  - Positivity bounds
  - …
- **Large-x PDFs & forward LHC**
  - Precision needed for new physics
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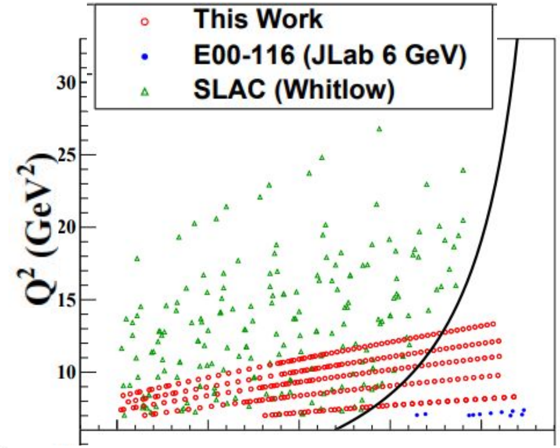
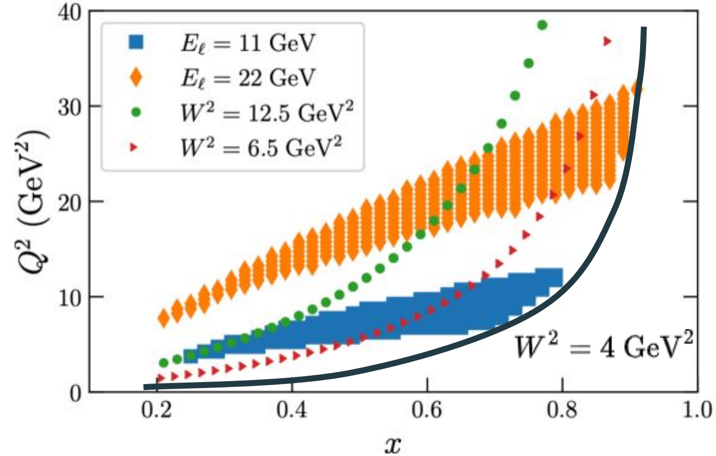
- **High-precision data from JLab 12 and 22**
  - p/D (Hall C, *2409.15236*)
  - BONUS 12 (soon!)
  - …

  $\longrightarrow \quad x \lesssim 0.9$!!





High-precision theory needs resummation

- **In QCD**
  - Summation of Large Corrections to Short Distance Cross-Sections *Sterman (1986)*
  - Resummation of the QCD Perturbative Series *Catani, Trentadue (1989)*
  - ...
- **In SCET**
  - Factorization and Momentum-Space Resummation in DIS *Becher, Neubert, Pecjak (2007)*
  - Rapidity Divergences and DIS in the Endpoint Region *Fleming, Labun (2012)*
  - Proper factorization in high-energy scattering near the endpoint *Chay, Kim (2013)*
  - ...
- **But some subtleties not yet fully explored**

→ ***Off-lightcone formalism – this talk!***

Accardi, Cerutti, Costa, Signori, Simonelli – very soon!

Open questions

- **Role of soft function**
  - Factorization in SCET

$$F_2^{\rm ns}(x,Q^2) = \sum_q e_q^2\, |C_V(Q^2,\mu)|^2\, Q^2 \int_x^1 d\xi\, J\Big(Q^2\,\frac{\xi-x}{x},\mu\Big)\, \phi_q^{\rm ns}(\xi,\mu)\,.$$

  - Compare with Sterman

$$F(x,Q^2) = |H_{\rm DI}(Q^2)|^2 \int_x^1 (\mathrm{d}y/y)\,\phi(y,Q^2) \int_0^{y-x} (\mathrm{d}w/[1-w])\, V(wQ)$$

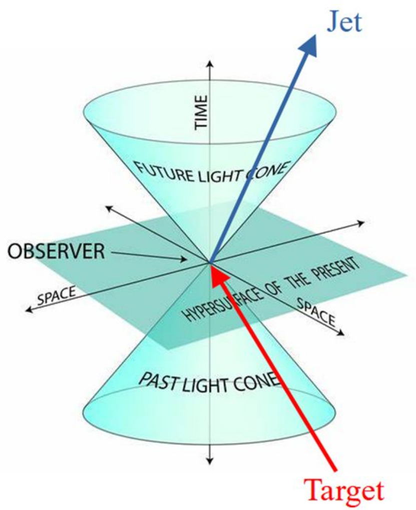
$$\times J[Q^2(y-x-w)/2x, Q] + O(1-x)^0. \qquad (3.13)$$

?

- **Difficulties with rapidity divergency**
  - “[the rapidity anomalous dimensions] reveal sensitivity to IR scales, which may signal a ***breakdown of factorization***” *Fleming, Labun (2012)* !?

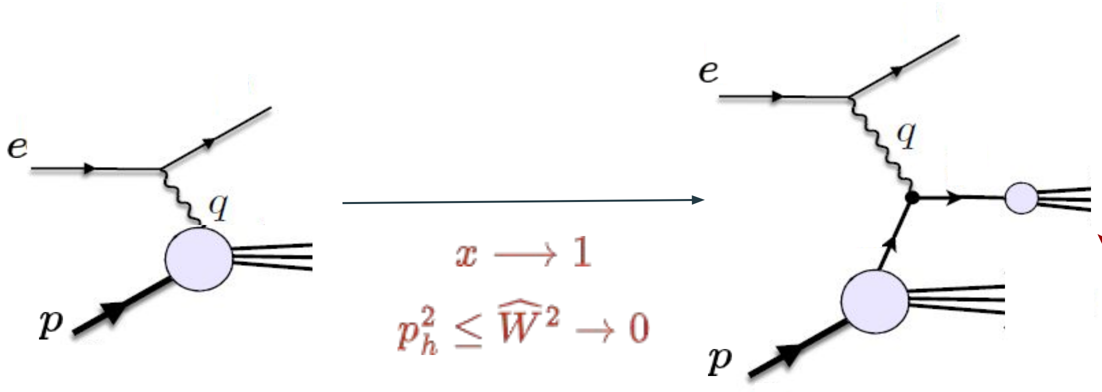
- **How to implement soft-collinear subtractions?**

Off-lightone collinear factorization



Kinematics and dynamics at large x

- Final state invariant mass $W^2 \to Q^2(1-x) + M^2$ is kinematically limited



aligned with scattered quark

$\sum_h |h\rangle\langle h| \neq \mathbb{1}$ when $p_h^2 \leq \widehat{W}^2$

  - **2-scale process:** Q^2 *and* $\widehat{W}^2 = Q^2(1-x)$

  - The **final state becomes more and more jet-like** as *x* increases

    $\longrightarrow$ Limited spread in transverse momentum

    $\longrightarrow$ Incomplete $\sum_h |h\rangle\langle h|$ with $p_h^2 \leq \widehat{W}^2$

Kinematics and dynamics at large x

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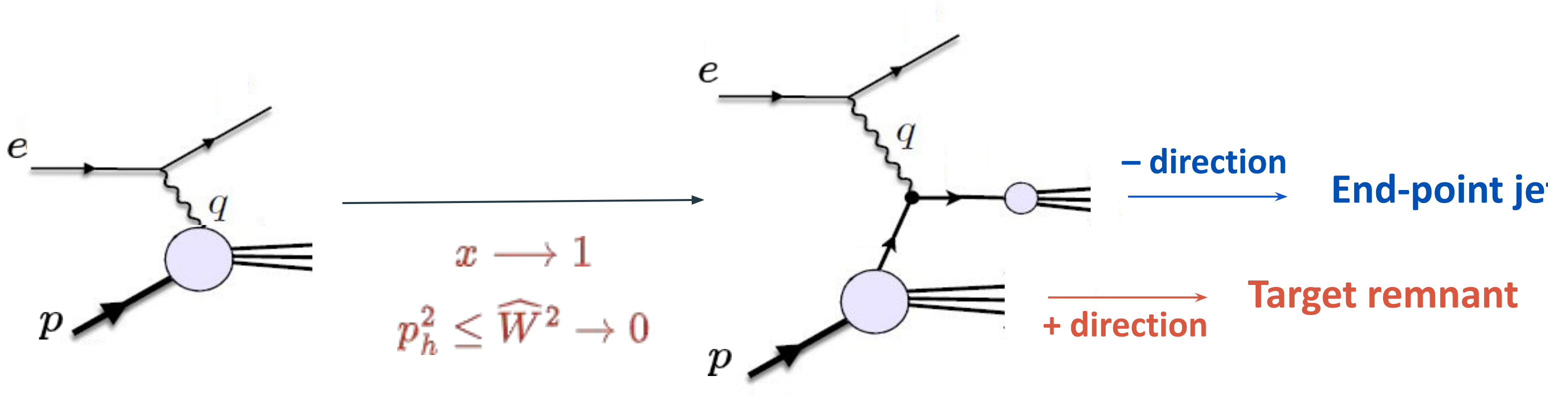
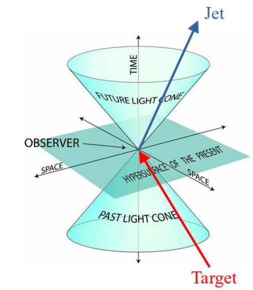
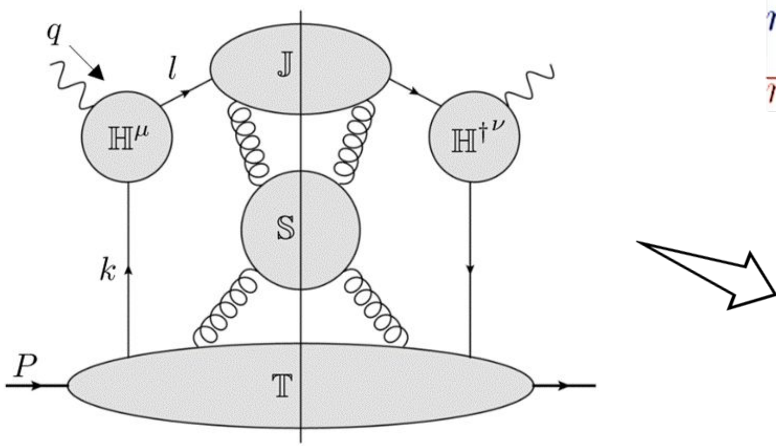


- **Neither inclusive nor semi-inclusive!**
  - $\longrightarrow$ Only a limited number of final state hadrons contribute to cross section
  - $\longrightarrow$ But we do not measure anything beside the scattered electron
- The process develops along **2 opposite light-cone directions like in SIDIS**
  - $\longrightarrow$ **Begs for a TMD-like approach**

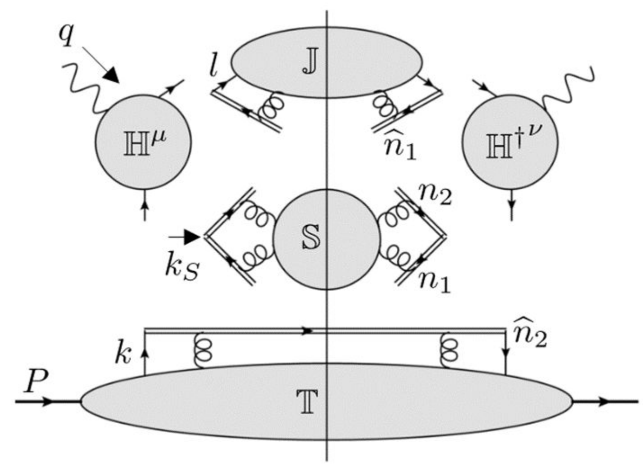
Off-lightcone collinear factorization



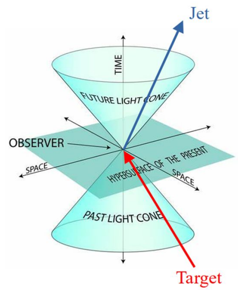
- At large *x*: TMD-like, but with inclusive jet function
  - **Wilson lines are tilted off the light cone**
    - ⟶ Gauge invariant rapidity regulators
    - ⟶ **Explicitly track, deal with rapidity effects**



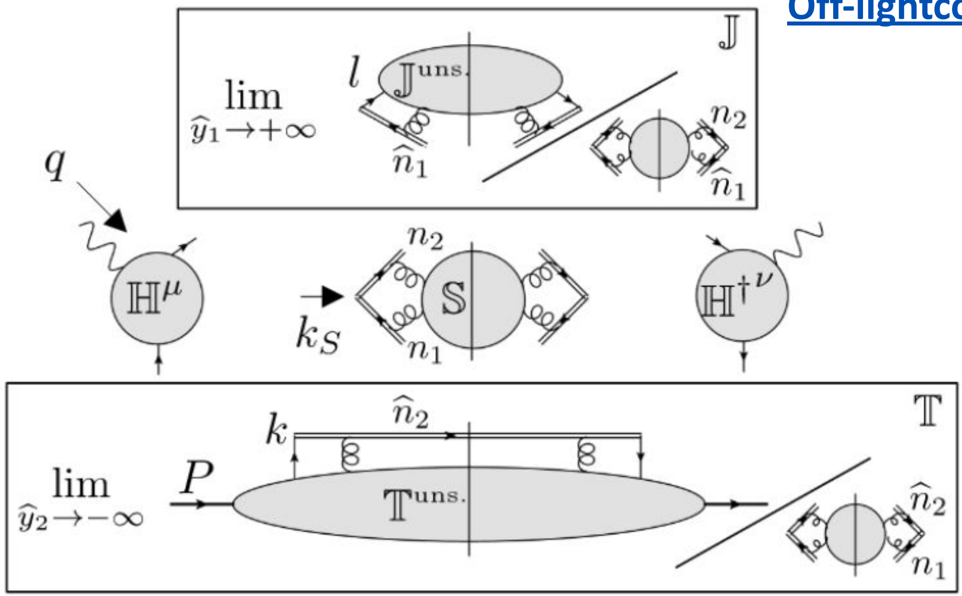
$$n = (1, 0, \vec{0}_T) \quad \mapsto \quad n_1 = (1, -e^{-2y_1}, \vec{0}_T),$$
$$\overline{n} = (0, 1, \vec{0}_T) \quad \mapsto \quad n_2 = (e^{2y_2}, 1, \vec{0}_T).$$



Off-lightcone collinear factorization

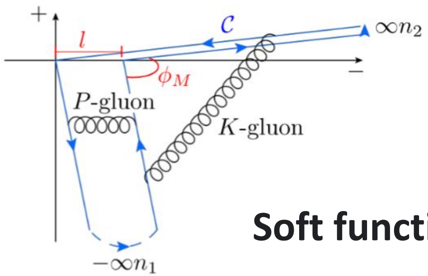


- At large *x*: TMD-like, but with <u>inclusive</u> jet function
  - **Soft-collinear subtractions**



Off-lightcone jet function $\mathcal{J}^{thr}$

$$\mathcal{J}^{thr} \sim \langle 0|\, \bar{\psi}\, W_{\hat{n}_1} \psi\, |0\rangle / S(\hat{n}_1, n_2)$$

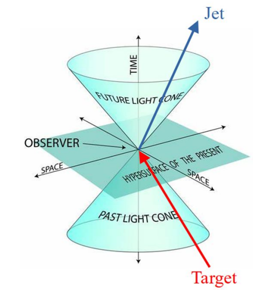


Soft function S

Off-lightcone target function ϕ^{thr}

$$\phi^{\text{thr}} \sim \langle p|\, \bar{\psi}\, W_{\hat{n}_1} \psi\, |p\rangle / S(n_1, \hat{n}_2)$$

Factorization theorem: x-space



- Two rapidity scales
  - y_1 : **target**, large and positive | y_2 : **jet**, large and negative

$$x\,W^{\mu\nu} = N_C\,\mathbf{u}^{\mu\nu}\,H(q^2)\sum_i \int_x^1 \frac{d\xi}{\xi}\int_0^{\xi-x} d\rho$$

$$\times\,\phi_i^{\rm thr.}(\xi, y_1)\,S\Big(\frac{\rho}{\xi}; y_1, y_2\Big)\,\mathcal{J}_i^{\rm thr.}\Big(\frac{\xi-x-\rho}{x}; y_2\Big)$$

$\frac{k^+}{p^+}$ hard parton mom. fraction

soft gluon mom. fraction $\frac{s^+}{p^+}$

jet fractional invariant mass $\frac{p_h^2}{Q^2}$

  - <u>Off-lightcone</u> **target function** & <u>**off-lightcone**</u> **jet function**
  - **Soft function appears naturally**, bridges the target-jet rapidity gap

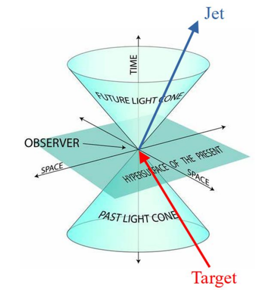
Mellin space and the CS kernel

large x $\longrightarrow$ large N

functions get a hat

$\bar{y}_0$

Factorization theorem 1



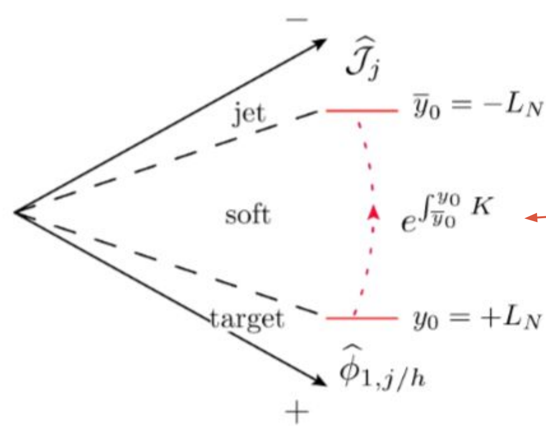
- **Choose symmetric rapidity scales** like in **TMDs**
  - $y_0 = +L_N$: **target initial rap.** | $\bar{y}_0 = -L_N$: **jet initial rap.**

$$W^{\mu\nu} = N_C\, \mathbf{u}^{\mu\nu}\, H(\mu, Q) \sum_j \int_0^1 dx\, x^{N-1}$$

$$\times\, \widehat{\phi}_j^{\text{thr}}(N, \mu, y_0)\, e^{\int_{\bar{y}_0}^{y_0} dy\, K(a_S(\mu), L_N + y)}\, \widehat{\mathcal{J}}_j^{\text{thr}}(N, \mu, \bar{y}_0)$$

Collins-Soper kernel in inclusive DIS !!

$$L_N = \log \frac{\mu}{\sqrt{2}P^+} N e^{\gamma_E}$$



Soft $S = \exp(\int K)$
bridges the rapidity gap

Rapidity evolution → CS kernel

- **The analogy with the TMD case keeps on**
  - **Square root definition** for subtractions $\widehat{\mathcal{J}}^{\rm sqrt}(y_n) = \lim_{\substack{\hat{y}_1 \to +\infty \\ \hat{y}_2 \to -\infty}} \widehat{\mathcal{J}}^{\rm uns}(\hat{y}_1) \sqrt{\frac{\widehat{S}(y_n, \hat{y}_2)}{\widehat{S}(\hat{y}_1, \hat{y}_2)\widehat{S}(\hat{y}_1, y_n)}}$
  - Absorbs the CS kernel

$$\widehat{\phi}^{\rm thr}(N,\mu,y_0)e^{\int_{\bar{y}_0}^{y_0} dy\, K(a_S, L_N + y)}\widehat{\mathcal{J}}^{\rm thr}(N,\mu,\bar{y}_0) = \widehat{\phi}^{\rm sqrt}(N,\mu,y_n)\widehat{\mathcal{J}}^{\rm sqrt}(N,\mu,y_n)$$

  - And **rapidity evolution becomes simpler**

Collinear vacuum correlator

$$\frac{\partial}{\partial y_n}\log \widehat{\mathcal{J}}^{\,\rm sqrt}(N,\mu,y_n) = +K(a_S(\mu), L_N + y_n),$$
$$\frac{\partial}{\partial y_n}\log \widehat{\phi}^{\,\rm sqrt}(N,\mu,y_n) = -K(a_S(\mu), L_N + y_n).$$

→ 2 new ways of **extracting CS-kernel from lattice**

Collinear proton correlator

Factorization theorem 2

- **Choose both rapidity scales equal to the jet one**
  - $\bar{y}_0 = y_0 = -L_N$
  - Like evolving the target to the jet's rapidity

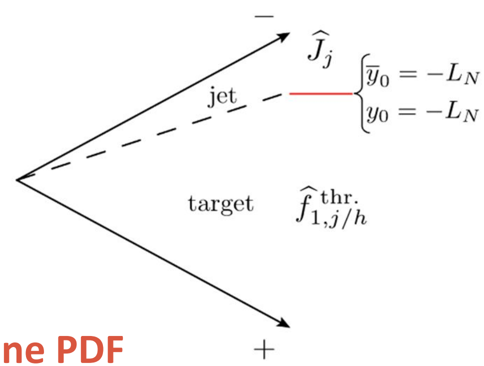
"Usual" lightcone jet function

Accardi, Costa, Signori (2017-2023)

$$\widehat{\mathcal{J}}_i^{\rm sqrt}(N,\mu,-L_N) = C(a_S(\mu),L_N)\,\widehat{J}_j(N;\mu),$$

$$\widehat{\phi}_j^{\rm sqrt}(N,\mu,-L_N) = \frac{\widehat{f}_j(N,\mu)}{C(a_S(\mu),L_N)}.$$

$$C = 1 + a_S C_F \frac{\pi^2}{6} + \ldots$$



Usual lightcone PDF

Collins, Soper, Sterman, 1986

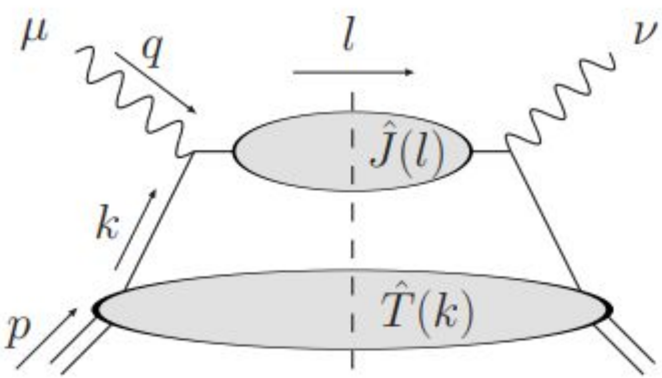
- C factors cancel, no CS kermel, lightcone formula!!

$$W^{\mu\nu} = N_C\,\mathbf{u}^{\mu\nu}\,H(\mu,Q)\,\sum_j \int_0^1 dx\,x^{N-1}\,\widehat{f}_j(N,\mu)\,\widehat{J}_j(N,\mu)$$

Like in SCET !

Factorization theorem 2

- Simple picture: “jet mass corrections” *Accardi, Qiu JHEP (2008)*



Phase-space limitations

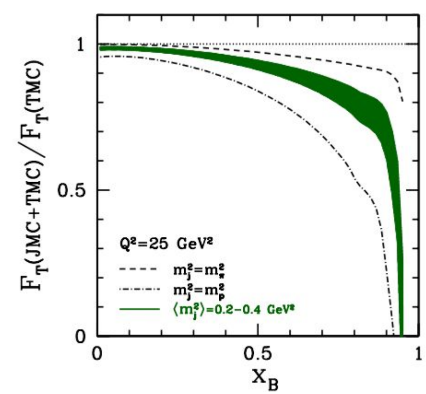
$$F_2(x,Q^2) = \int_0^{Q^2\frac{1-x}{x}} dl^2\, J(l^2)\, F_2^{(0)}\Big(x_N\big(1+\frac{l^2}{Q^2}\big), Q^2\Big)$$

Jet mass corrections

- Phenomenology to be further developed

  Accardi, Cerutti, Costa, SImonelli, Signori, in prep.

  - Q^2 evolution
  - Matching to small x
  - Models for $J(l^2)$



Summary and Perspectives

With off-lightcone factorization:

- **The CS-kernel is universal in any 2 opposite lightcone directions process:**
  - DIS at threshold
    $\longrightarrow$ also DY, e+e-
  - SIDIS, and also thrust observables in e+e-, …

- **Global fits of CS kernel ?**
  - Evaluated at different values of its variable
  - Increased sensitivity, range

- **Lattice calculation of CS kernel**
  - From rapidity divergence of **off-lightcone collinear** correlators
  - 2 possibilities: **proton** or **vacuum correlators**

New method for CS extraction from lattice

- **Inclusive DIS is special:**
  - Collins Soper kernel cancels if target evolved until jet
  - Recovers SCET factorization theorem

Pheno analysis of JLab 12/22 data

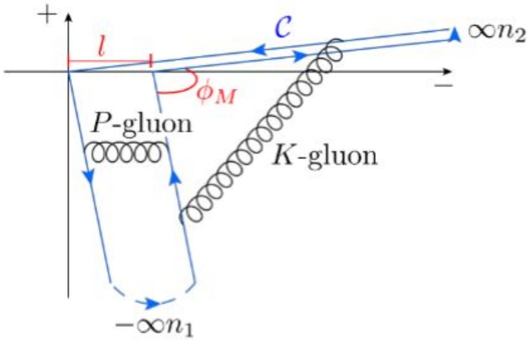
Thank you!

Backup

Operator definitions - 1

$$\widehat{S}(N,\mu,y_1,y_2) = Z_S(\varepsilon,\mu,y_1-y_2)\frac{\mathrm{Tr}_C}{N_C}\langle 0|\mathcal{P}\exp\left\{-ig_0\int_{\mathcal{C}(l,\phi_M)} dx^\mu A_\mu^{(0)}(x)\right\}|0\rangle + \mathcal{O}(1/N)$$

$$= \exp\left\{\int_{y_2}^{y_1+i\frac{\pi}{2}} dy\, K(a_S(\mu),L_N+y) + \frac{1}{2}\left[P\left(a_S(\mu),L_N+y_1+i\frac{\pi}{2}\right) + P\left(a_S(\mu),L_N+y_2\right)\right] + \mathcal{O}(e^{-2y_1},e^{2y_2})\right\} + \mathcal{O}(1/N).$$



$$\mathcal{J}_j^{\mathrm{uns.}}(\mu,l^2/Q^2,\widehat{y}_1) = Z_J^{\mathrm{uns.}}(\varepsilon,\mu,l^2/Q^2,\widehat{y}_1)\frac{Q^2}{2\pi l^-}\int\frac{d^4\omega}{(2\pi)^4}e^{-il\cdot\omega}\,\mathrm{Disc}\{\,\mathrm{Tr}\langle 0|\gamma^-\overline{\psi}_j^{(0)}(\omega)W(0\to\omega)\psi_j^{(0)}(0)|0\rangle\}$$

$$\phi_{1,j/h}^{\mathrm{uns.}}(\xi,\mu,\widehat{y}_2) = Z_T(\varepsilon,\xi,\mu,\widehat{y}_2)\int\frac{d\rho^-}{2\pi}e^{-i\xi P^+\rho^-}\,\mathrm{Tr}\langle P|\overline{\psi}_j^{(0)}(\rho)W(0\to\rho)\frac{\gamma^+}{2}\psi_j^{(0)}(0)|P\rangle,$$

Operator definitions - 2

$$\widehat{\mathcal{J}}_j^{\text{thr.}}(N,\mu,y_2) = \lim_{\widehat{y}_1\to+\infty} \frac{\widehat{\mathcal{J}}_j^{\text{uns.}}(N,\mu,\widehat{y}_1)}{\widehat{S}_J(N,\mu,\widehat{y}_1,y_2)}$$

$$= \widehat{\mathcal{J}}_j^{\text{thr.}}(N,\mu,-L_N)\exp\{-\int_{y_2}^{-L_N} dy\, K(a_S(\mu),L_N+y) - \frac{1}{2}P(a_S(\mu),L_N+y_2)\} + \mathcal{O}(1/N)$$

$$\widehat{\phi}_{1,j/h}^{\text{thr.}}(N,\mu,y_1) = \lim_{\widehat{y}_2\to-\infty} \frac{\widehat{\phi}_{1,j/h}^{\text{uns.}}(N,\mu,\widehat{y}_2)}{\widehat{S}_T(N,\mu,y_1,\widehat{y}_2)}$$

$$= \widehat{\phi}_{1,j/h}^{\text{thr.}}(N,\mu,+L_N)\exp\{-\int_{L_N}^{y_1+i\frac{\pi}{2}} dy\, K(a_S`(\mu),L_N+y) - \frac{1}{2}P(a_S(\mu),L_N+y_1+i\frac{\pi}{2})\} + \mathcal{O}(1/N).$$

Operator definitions - 3

$$\widehat{\mathcal{J}}_j^{\mathrm{sqrt.}}(N,\mu,y_n) = \lim_{\substack{\widehat{y}_1\to+\infty \\ \widehat{y}_2\to-\infty}} \widehat{\mathcal{J}}_j^{\mathrm{uns.}}(N,\mu,\widehat{y}_1)\sqrt{\frac{\widehat{S}(N,\mu,y_n-i\frac{\pi}{2},\widehat{y}_2)}{\widehat{S}(N,\mu,\widehat{y}_1,\widehat{y}_2)\widehat{S}(N,\mu,\widehat{y}_1,y_n)}} + \mathcal{O}(1/N),$$

$$\widehat{\phi}_{1,j/h}^{\mathrm{sqrt.}}(N,\mu,y_n) = \lim_{\substack{\widehat{y}_1\to+\infty \\ \widehat{y}_2\to-\infty}} \widehat{\phi}_{1,j/h}^{\mathrm{uns.}}(N,\mu,\widehat{y}_2)\sqrt{\frac{\widehat{S}(N,\mu,\widehat{y}_1,y_n)}{\widehat{S}(N,\mu,\widehat{y}_1,\widehat{y}_2)\widehat{S}(N,\mu,y_n-i\frac{\pi}{2},\widehat{y}_2)}} + \mathcal{O}(1/N).$$