Sensitivity to x-dependent GPDs from a combined analysis

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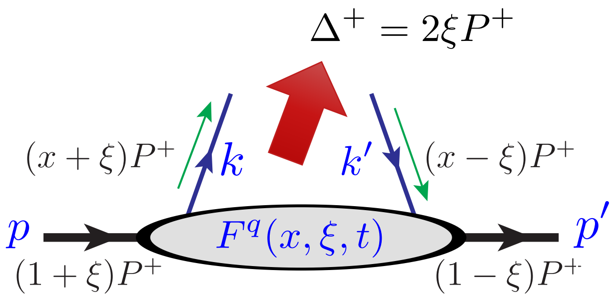
PRL 131 (2023) 161902
PRD 109 (2024) 074023
arXiv: 2409.06882
papers in preparation

Nov/19/2024
JLab

GPD and hadron structure

❑ Generalized parton distribution (GPD)



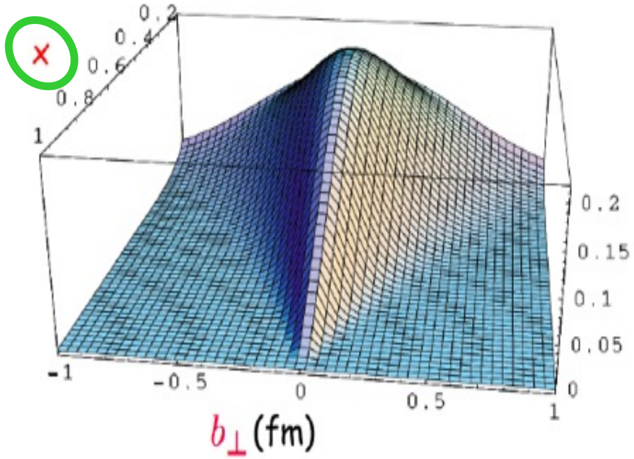
$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+ q(-z^-/2) | p \rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

❑ Tomography

$$f_i(x,\boldsymbol{b}_T) = \int d^2\boldsymbol{\Delta}_T\, e^{i\boldsymbol{\Delta}_T\cdot\boldsymbol{b}_T} F_i(x,0,-\Delta_T^2)$$

3D image

M. Burkardt, 2000, 2003



❑ Emergent hadron properties

$$\int_{-1}^{1} dx\, x\, F_i(x,\xi,t) \propto \langle p'|T_i^{++}(0)|p\rangle \qquad i = q, g$$

$$\Rightarrow \int_{-1}^{1} dx\, x\, H_i(x,\xi,t) = A_i(t) + \xi^2 D_i(t)$$
$$\int_{-1}^{1} dx\, x\, E_i(x,\xi,t) = B_i(t) - \xi^2 D_i(t)$$

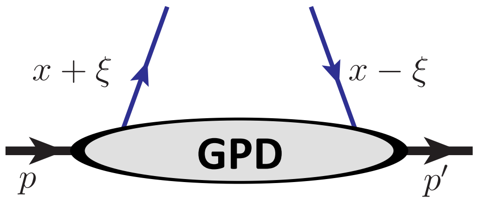
$$\Rightarrow J_i = \lim_{t\to 0}\int_{-1}^{1} dx\, x\,[H_i(x,\xi,t) + E_i(x,\xi,t)]$$

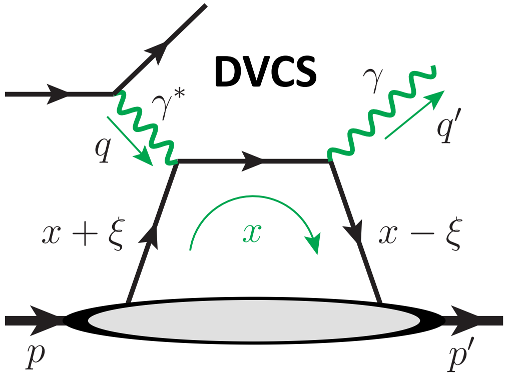
X.-D. Ji, 1997

Need full x-dependence at a substantial range of (t,ξ)!

x-dependence problem for GPD

❑ **Amplitude nature: exclusive processes**



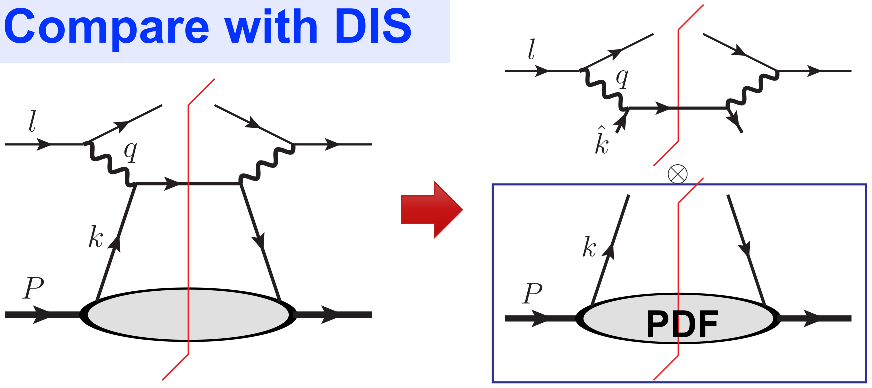


***x* ~ loop momentum**

$$iM \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

never pin down to some *x*

Compare with DIS

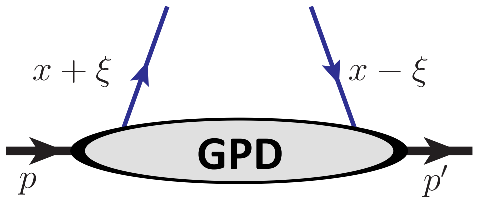


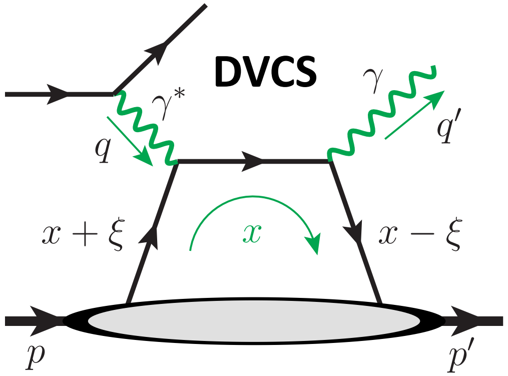
***cross section*: cut diagram**

$$\sigma_{\mathrm{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\, \hat{\sigma}(x/x_B)$$

x-dependence problem for GPD

❑ Amplitude nature: exclusive processes



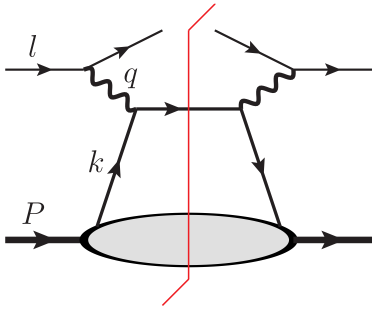


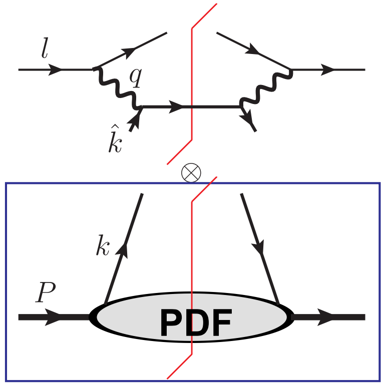
$x \sim$ loop momentum

$$i\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t)\cdot C(x,\xi;Q/\mu)$$

never pin down to some x

Compare with DIS





cross section: cut diagram

$$\sigma_{\mathrm{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\,\hat{\sigma}(x/x_B)$$

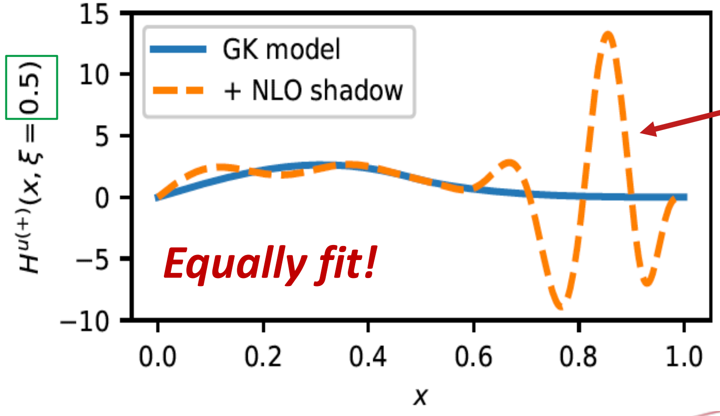
❑ Sensitivity to x: comes from $C(x,\xi;Q/\mu)$

$$C(x,\xi;Q/\mu) = T(Q/\mu)\cdot G(x,\xi) \propto \frac{1}{x-\xi+i\epsilon}\cdots$$

$$\Rightarrow\ i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F^{+}(x,\xi,t)}{x-\xi+i\epsilon} = \text{“}F_0(\xi,t)\text{”}$$

“moment”

“LO scaling”



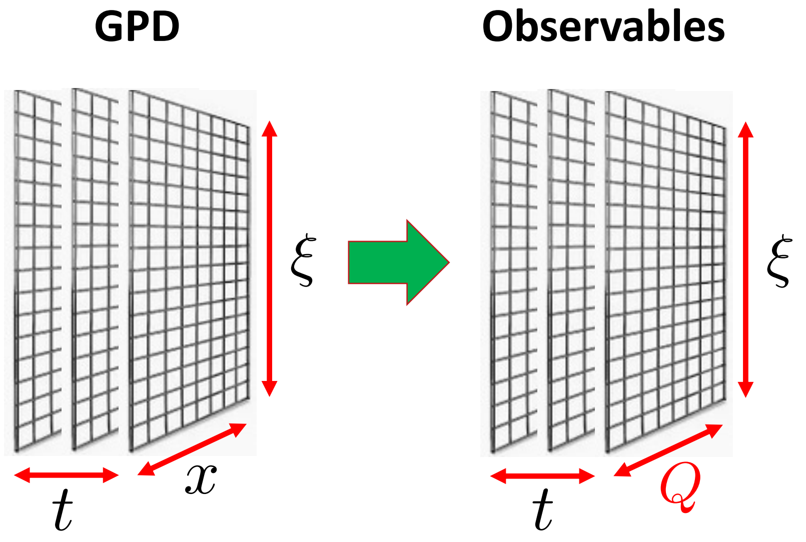
Shadow GPD

[Bertone et al. PRD `21]

Pixelation construction of GPD

Shadow GPDs make parametric method *biased*

→ **Construct GPDs from most flexible pixelation method**



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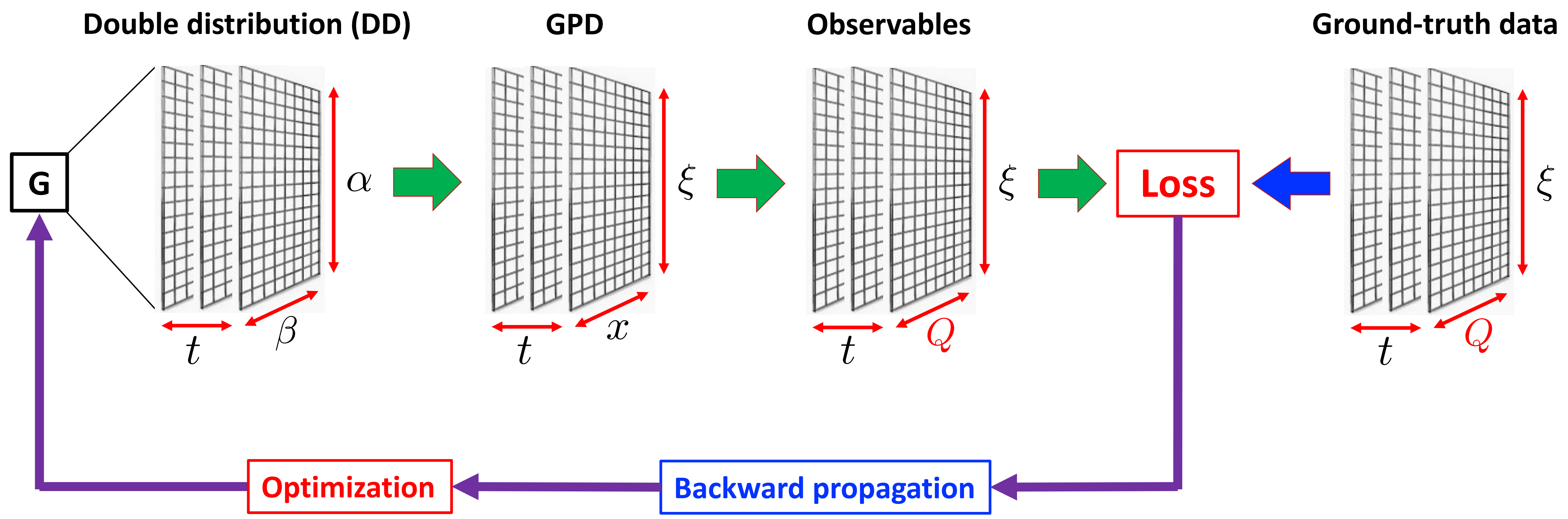
Construct GPDs from most flexible pixelation method



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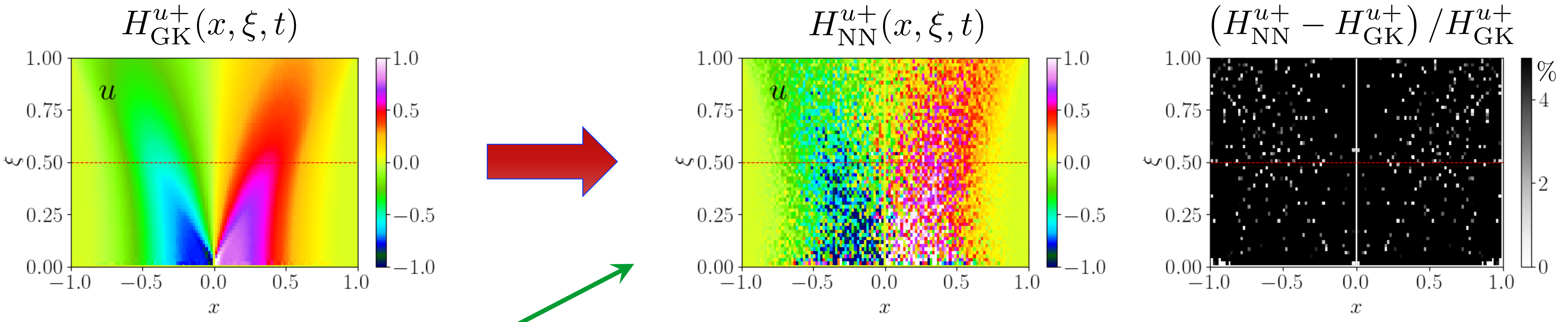
Reconstructing with only scaling moments

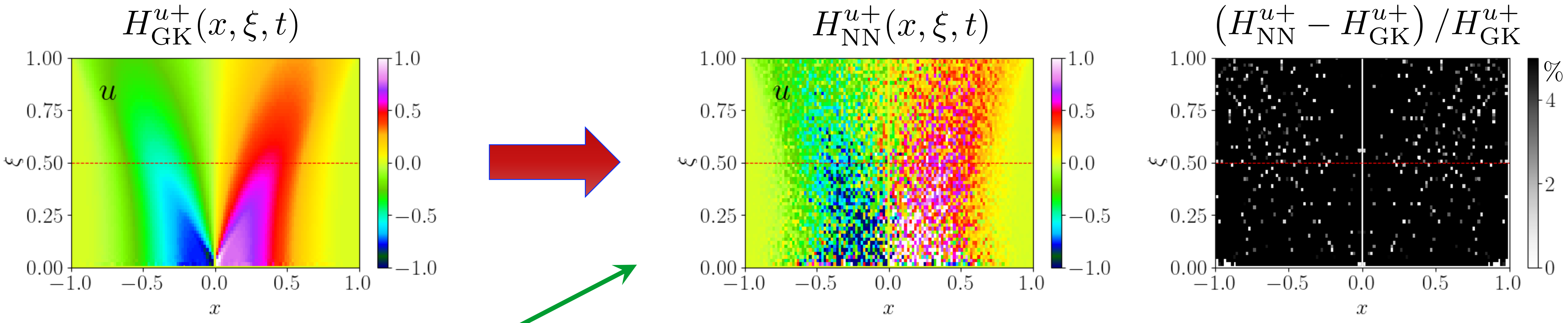


- **Neural network generator**

$$D^u_{\mathrm{NN}}(\beta,\alpha,t) = D^u_{\mathrm{GK}}(\beta,\alpha,t) * \left[1+\epsilon(\beta,\alpha,t)\right]$$

$$\epsilon(\beta,\alpha,t) \sim N(0,5)$$

Reconstructing with only scaling moments



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$$\epsilon(\beta,\alpha,t) \sim N(0,5)$$

- **Observable: DVCS moment**

$$M_0^{[H]}(\xi,t) = \int_{-1}^{1} dx \frac{H^+(x,\xi,t)}{x-\xi+i\epsilon}$$

- **Optimize with MSE loss**

$$L[H_{\rm GK}, H_{\rm NN}] = \sum_{\xi,t} \left| \frac{M_0^{[H_{\rm GK}]}(\xi,t) - M_0^{[H_{\rm NN}]}(\xi,t)}{r\cdot M_0^{[H_{\rm GK}]}(\xi,t)} \right|^2$$

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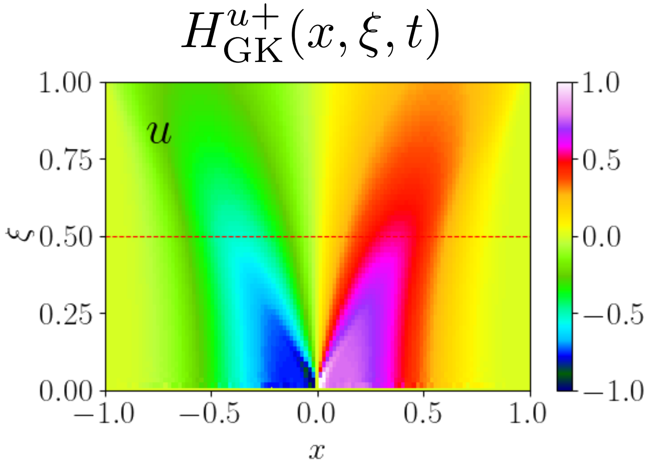
Reconstructing with only scaling moments



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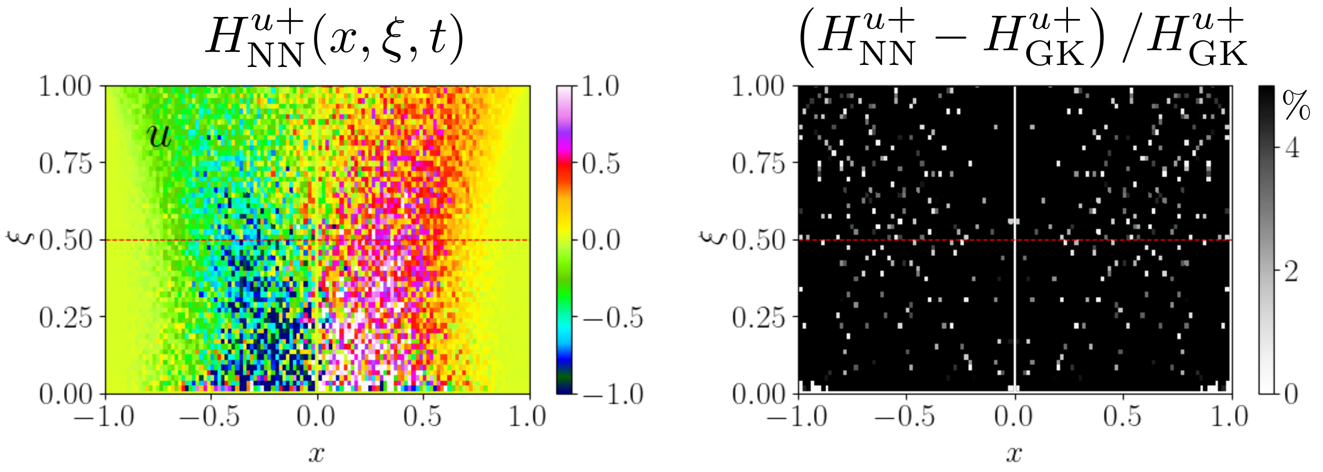
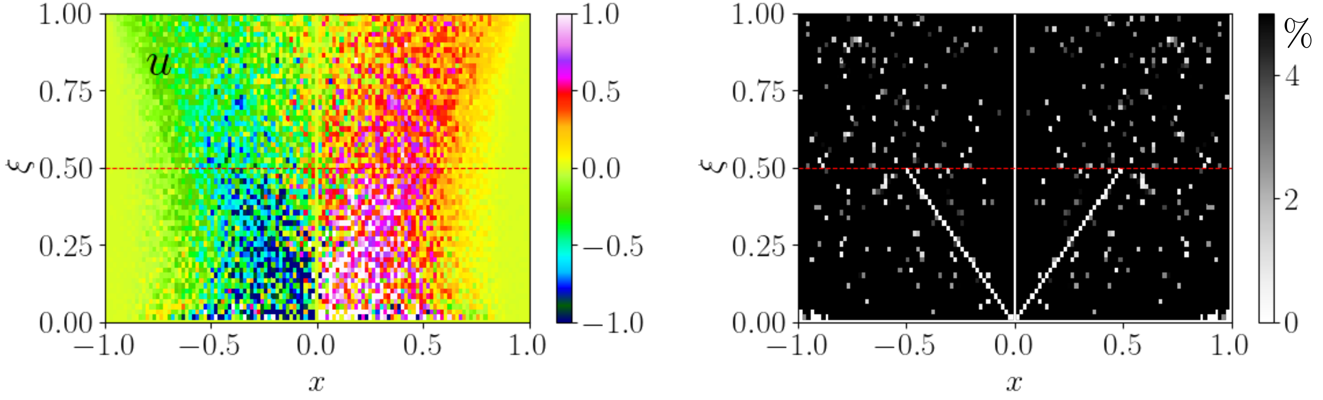


Training with DVCS moment $\xi \in [0, 0.5]$



Sensitivity surrounding the ridge.

How to understand the result?

$$M_0^{[H]}(\xi,t) = \int_{-1}^{1} dx \frac{H^+(x,\xi,t)}{x-\xi+i\epsilon} = P\int_{-1}^{1} dx H(x,\xi,t)\frac{2x}{x^2-\xi^2} - i\pi\left[H(\xi,\xi,t) - H(-\xi,\xi,t)\right]$$

Each pixel is independent (modulo DD to GPD conversion).

➢ Optimization process

Calculate observable and get loss

↓

Backward propagation

↓

$$\frac{\partial L[H_{\mathrm{GK}}, H_{\mathrm{NN}}]}{\partial H_{\mathrm{NN}}(x,\xi,t)} \propto \left[M_0^{[H_{\mathrm{GK}}]}(\xi,t) - M_0^{[H_{\mathrm{NN}}]}(\xi,t)\right] \cdot \frac{2x}{x^2-\xi^2}$$

(Real part only)

Optimization (gradient descent) (loops back to "Calculate observable and get loss")

$$H_{\mathrm{NN}}(x,\xi,t) \rightarrow H_{\mathrm{NN}}(x,\xi,t) - \mathrm{lr}\cdot\frac{\partial L[H_{\mathrm{GK}}, H_{\mathrm{NN}}]}{\partial H_{\mathrm{NN}}(x,\xi,t)}$$

How to understand the result?

$$M_0^{[H]}(\xi,t)=\int_{-1}^{1}dx\,\frac{H^+(x,\xi,t)}{x-\xi+i\epsilon}=P\int_{-1}^{1}dx H(x,\xi,t)\frac{2x}{x^2-\xi^2}-i\pi\left[H(\xi,\xi,t)-H(-\xi,\xi,t)\right]$$

Each pixel is independent (modulo DD to GPD conversion).

➢ Optimization process

Calculate observable and get loss

↓

Backward propagation

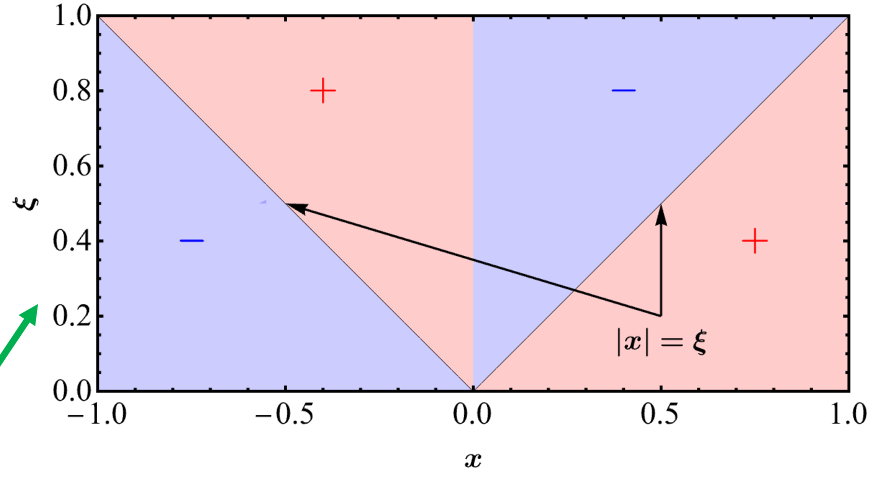
↓

$$\frac{\partial L[H_{\rm GK},H_{\rm NN}]}{\partial H_{\rm NN}(x,\xi,t)}\propto\left[M_0^{[H_{\rm GK}]}(\xi,t)-M_0^{[H_{\rm NN}]}(\xi,t)\right]\cdot\frac{2x}{x^2-\xi^2}$$

(Real part only)

Optimization (gradient descent)

$$H_{\rm NN}(x,\xi,t)\to H_{\rm NN}(x,\xi,t)-{\rm lr}\cdot\frac{\partial L[H_{\rm GK},H_{\rm NN}]}{\partial H_{\rm NN}(x,\xi,t)}$$



Tunning of each pixel is *deterministic*!

Determined by the sign of

$$M_0^{[H_{\rm GK}]}(\xi,t)-M_0^{[H_{\rm NN}]}(\xi,t)$$

in the *initial* input.

Keeps tunning until reaching a "solution"

→ shadow GPD!

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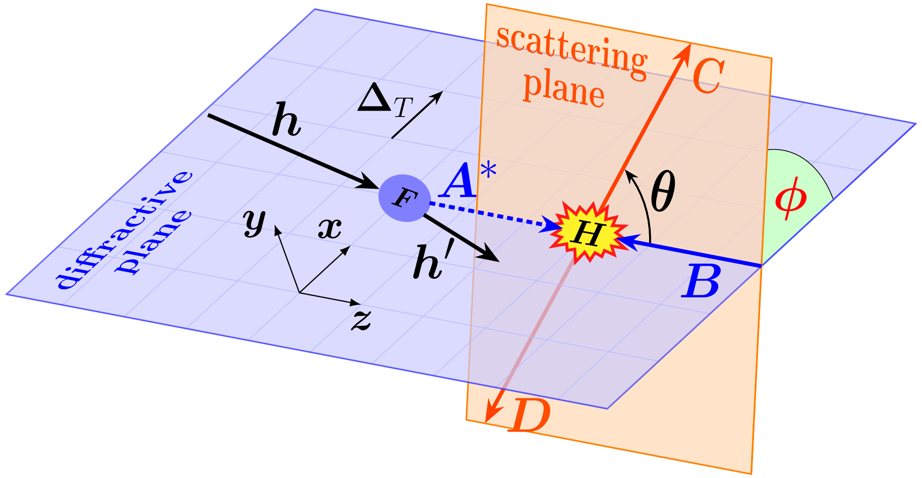
Types of x-sensitivity

[Qiu, Yu, PRD 107 (2023) 014007; Qiu, Sato, Yu, 2409.06882]



❑ x-sensitivity ⇔ 2 → 2 hard scattering

Kinematics:

1. $\hat{s} = 2\,\xi\,s\,/\,(1+\xi)$ ⟸ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⟺ x
3. ϕ ⟸ (A^*B) spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi} \cdot \int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

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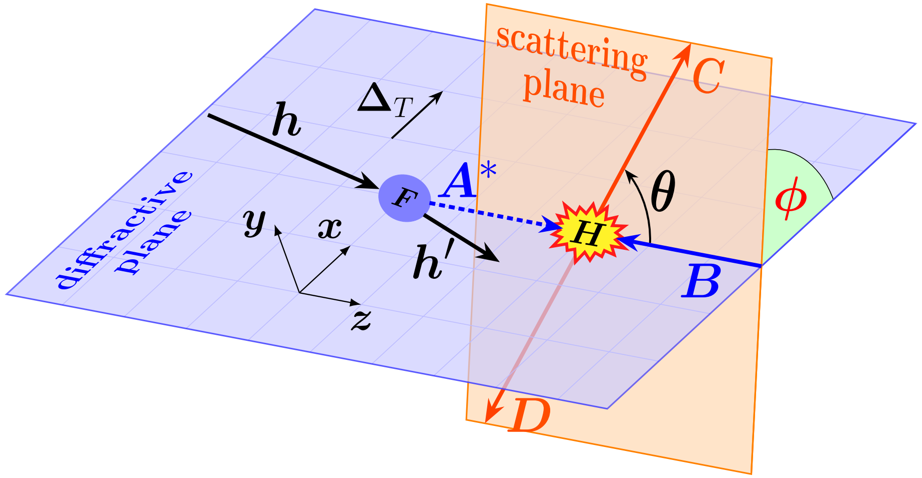

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[suppressing t and ξ dependence]

➢ **Moment-type sensitivity** $C(x;Q) = G(x)\cdot T(Q)$ ⟶ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ **Independent of Q. Scaling for F_G.**

⟶ **Inversion problem: *shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

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Types of x-sensitivity

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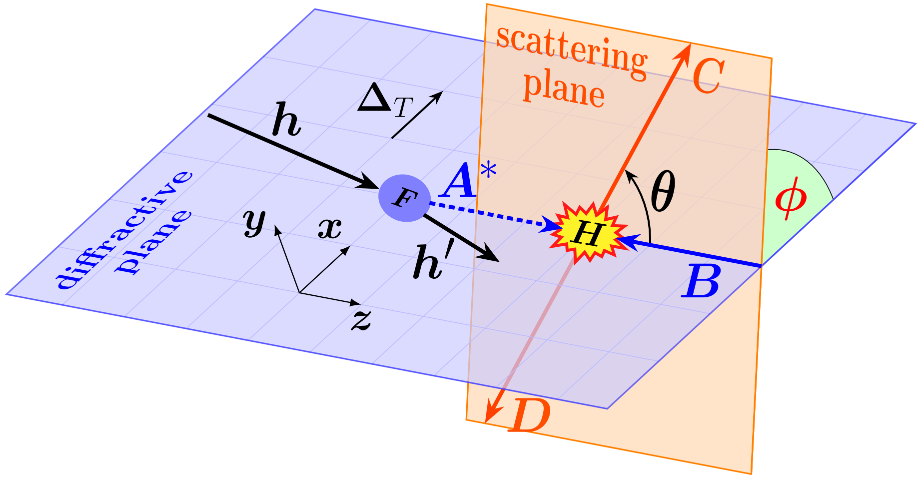

❑ x-sensitivity ⇔ 2 → 2 hard scattering

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➢ **Enhanced sensitivity** $C(x;Q) \neq G(x)\cdot T(Q)$ ⟹ $d\sigma/dQ \sim \left|C(x;Q)\otimes_x F(x,\xi,t)\right|^2$

Scaling *breaking* at LO

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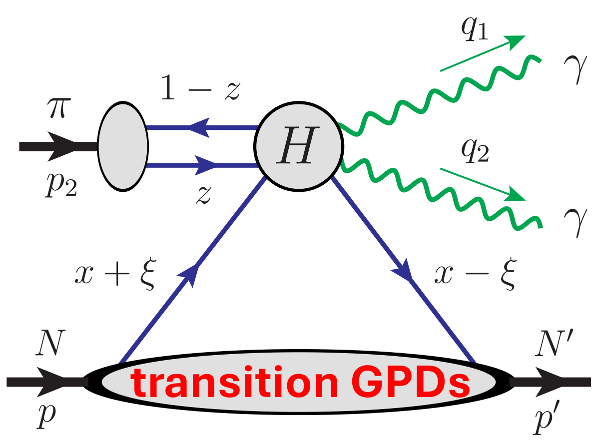
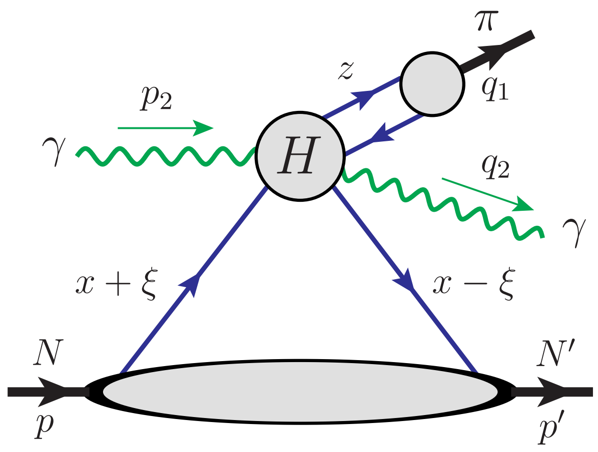
Two processes with enhanced x-sensitivity



J-PARC, AMBER



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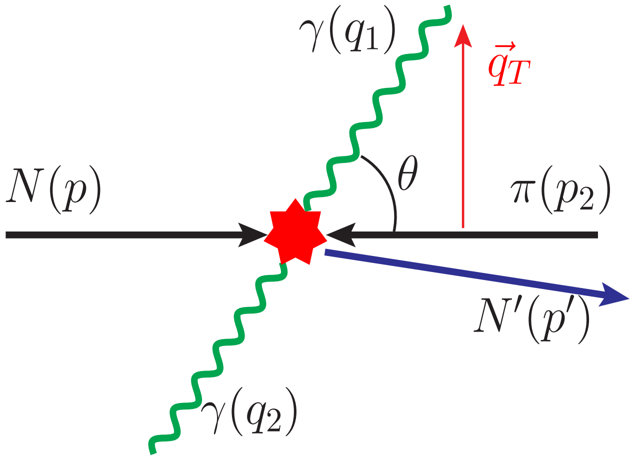
Qiu & Yu, JHEP 08 (2022) 103
Qiu & Yu, PRD 109 (2024) 074023

G. Duplancic et al., JHEP 11 (2018) 179
G. Duplancic et al., JHEP 03 (2023) 241
G. Duplancic et al., PRD 107 (2023), 094023
Qiu & Yu, PRD 107 (2023), 014007
Qiu & Yu, PRL 131 (2023), 161902

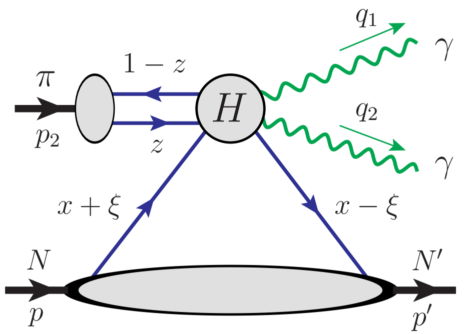
Enhanced x-sensitivity: (1) diphoton mesoproduction

[Qiu & Yu, JHEP 08 (2022) 103;
PRD 109 (2024) 074023]



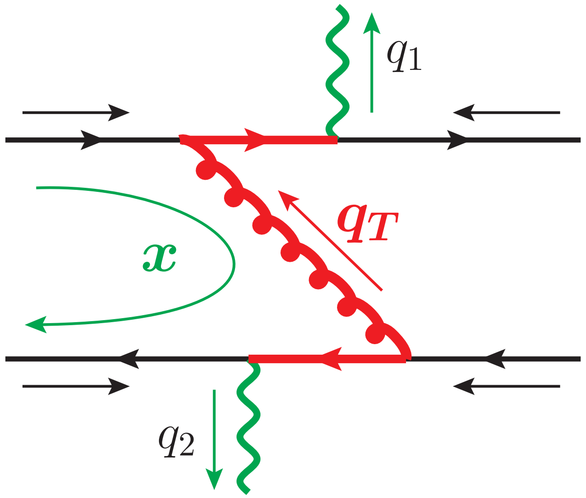
In addition to scaling integral

$$F_0(\xi,t) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\xi+i\epsilon}$$

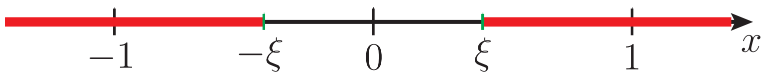


$i\mathcal{M}$ **also contains *non-scaling* integral**

$$I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\rho(z;\theta)+i\epsilon\,\mathrm{sgn}\left[\cos^2(\theta/2)-z\right]}$$

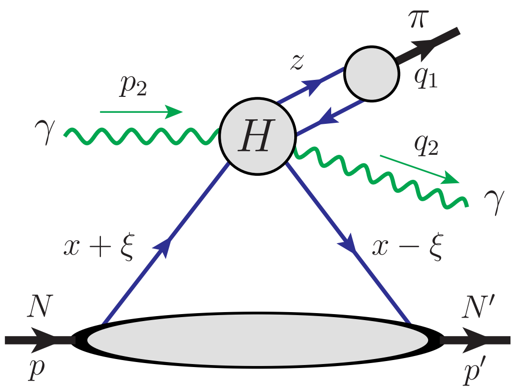


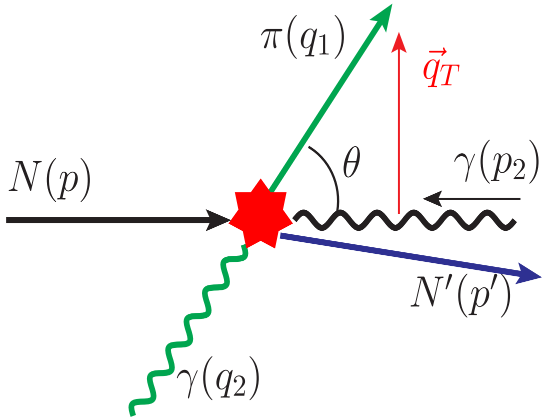
$$\rho(z;\theta) = \xi\cdot\left[\frac{1-z+\tan^2(\theta/2)\,z}{1-z-\tan^2(\theta/2)\,z}\right] \in (-\infty,-\xi]\cup[\xi,\infty)$$



Enhanced x-sensitivity: (2) γ-π pair photoproduction

[Qiu & Yu, PRL 131 (2023) 161902]

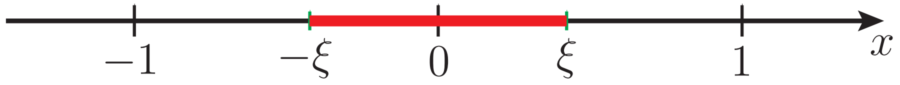




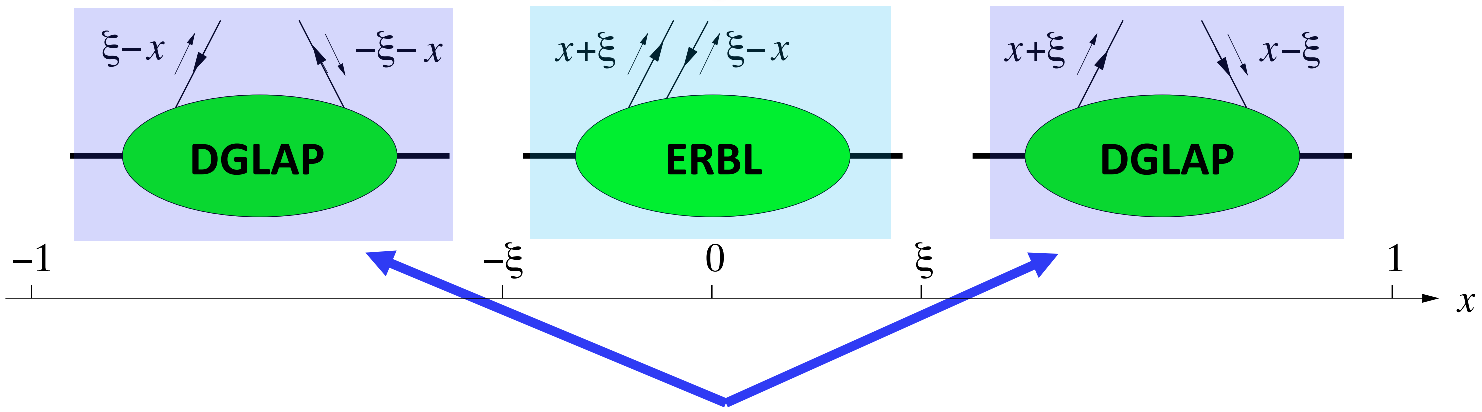
$i\mathcal{M}$ **also contains the *non-scaling* integral**

$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$



Complementary sensitivity

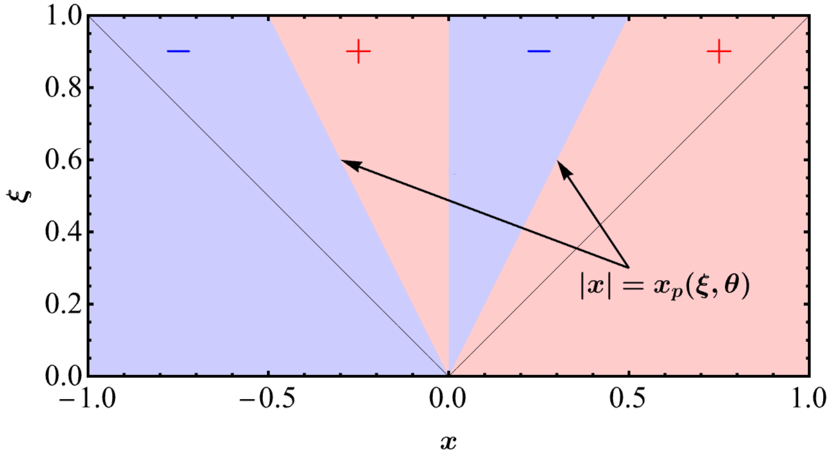


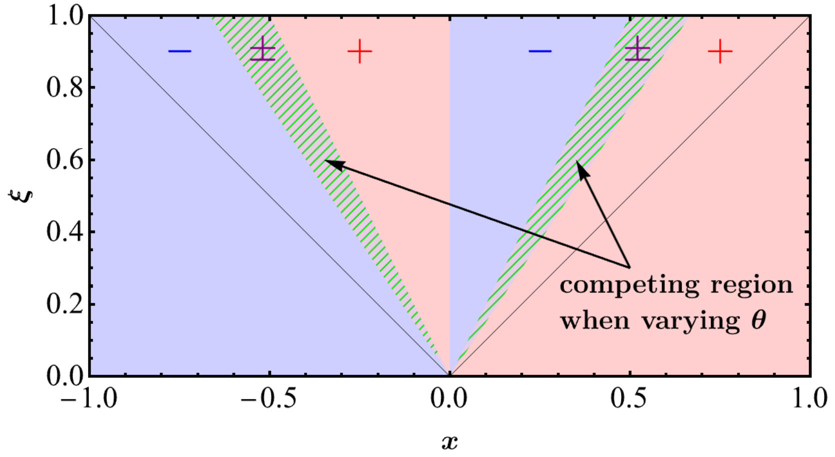
$N\,\pi \to N'\,\gamma\,\gamma$

Improve with *non*-scaling integrals

Non-scaling integral

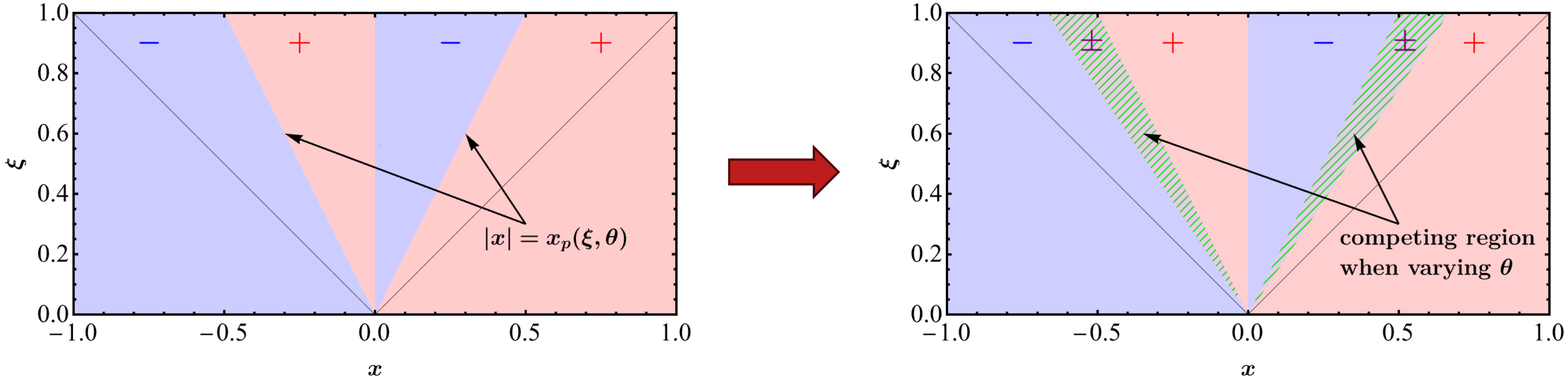
$$M_{\theta}^{[H]}(\xi,t) = \int_{-1}^{1} dx \frac{H(x,\xi,t)}{x - x_p(\xi,\theta) + i\epsilon} \quad \Rightarrow \quad \frac{\partial L}{\partial H(x,\xi,t)} \propto \frac{2x}{x^2 - x_p^2}$$

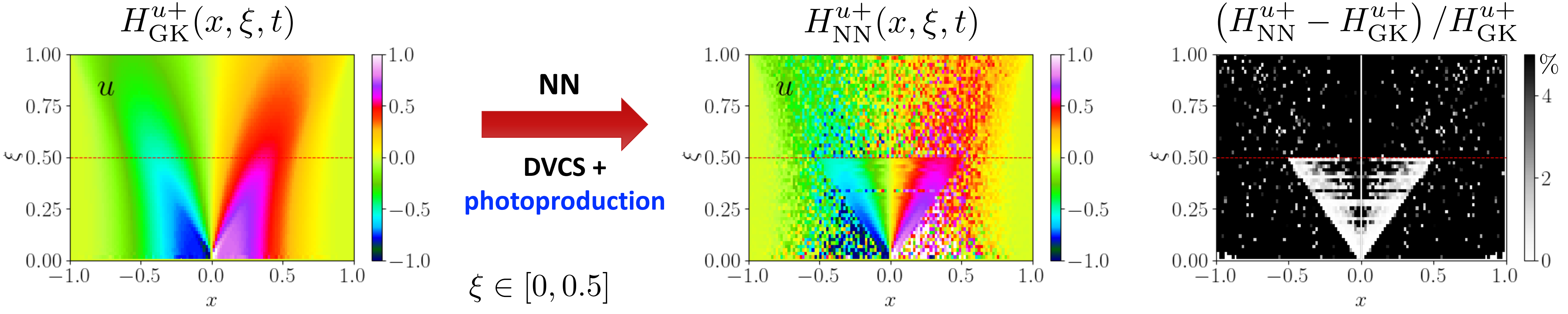




Improve with *non*-scaling integrals

Non-scaling integral $M_\theta^{[H]}(\xi,t) = \int_{-1}^{1} dx \frac{H(x,\xi,t)}{x - x_p(\xi,\theta) + i\epsilon}$ → $\frac{\partial L}{\partial H(x,\xi,t)} \propto \frac{2x}{x^2 - x_p^2}$





$\xi \in [0, 0.5]$

$\cos\theta \in [-0.95, 0.95]$

Compare with constraints from Lattice observables

- ***Good lattice cross section*** **method** [Ma & Qiu, PRL 120 (2018) 2, 022003]

$$\langle p|\mathcal{O}_n(z)|p\rangle = \sum_i \int_{-1}^{1} \frac{dx}{x} \underbrace{f_i\left(x,\mu^2\right)}_{\text{PDF}} \underbrace{C_n^i\left(z^2, x\omega, \mu^2\right)}_{\text{perturbative}} + \mathcal{O}\left(z^2\Lambda^2_{\text{QCD}}, z^2p^2\right)$$

← **Applies at small distance z**

↑ **Non-local operator**

Compare with constraints from Lattice observables

- **Good lattice cross section method** [Ma & Qiu, PRL 120 (2018) 2, 022003]

$$\langle p|\mathcal{O}_n(z)|p\rangle = \sum_i \int_{-1}^{1} \frac{dx}{x} \underbrace{f_i\left(x,\mu^2\right)}_{\text{PDF}} \underbrace{C_n^i\left(z^2, x\omega, \mu^2\right)}_{\text{perturbative}} + \mathcal{O}\left(z^2\Lambda_{\rm QCD}^2, z^2p^2\right)$$

← **Applies at small distance *z***

↑ **Non-local operator**

- **Extending to GPD** [Qiu & Yu, unpublished]

$$\langle p'|\mathcal{O}_n(z)|p\rangle = \frac{1}{2z\cdot P}\bar{u}(p')\left\{\sum_i \int_{-1}^{1} \frac{dx}{x}\left[(\gamma\cdot z)H_i\left(x,\xi,t\right) + \frac{i\sigma^{z\Delta}}{2m}E_i\left(x,\xi,t\right)\right] C_n^i\left(z^2, x\omega, \xi\omega\right)\right\} u(p) + \mathcal{O}\left(z^2\Lambda_{\rm QCD}^2, z^2P^2, z^2t\right)$$

$$P = (p+p')/2, \quad \omega = P\cdot z$$

Compare with constraints from Lattice observables

❑ *Good lattice cross section* method [Ma & Qiu, PRL 120 (2018) 2, 022003]

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Applies at small distance z

Non-local operator: $\mathcal{O}_n(z)$

❑ Extending to GPD [Qiu & Yu, unpublished]

$$\langle p'|\mathcal{O}_n(z)|p\rangle = \frac{1}{2z\cdot P}\bar{u}(p')\left\{\sum_i \int_{-1}^{1} \frac{dx}{x}\left[(\gamma\cdot z)H_i\left(x,\xi,t\right) + \frac{i\sigma^{z\Delta}}{2m}E_i\left(x,\xi,t\right)\right] C_n^i\left(z^2, x\omega, \xi\omega\right)\right\} u(p)$$
$$+ \mathcal{O}\left(z^2\Lambda_{\rm QCD}^2, z^2P^2, z^2t\right)$$

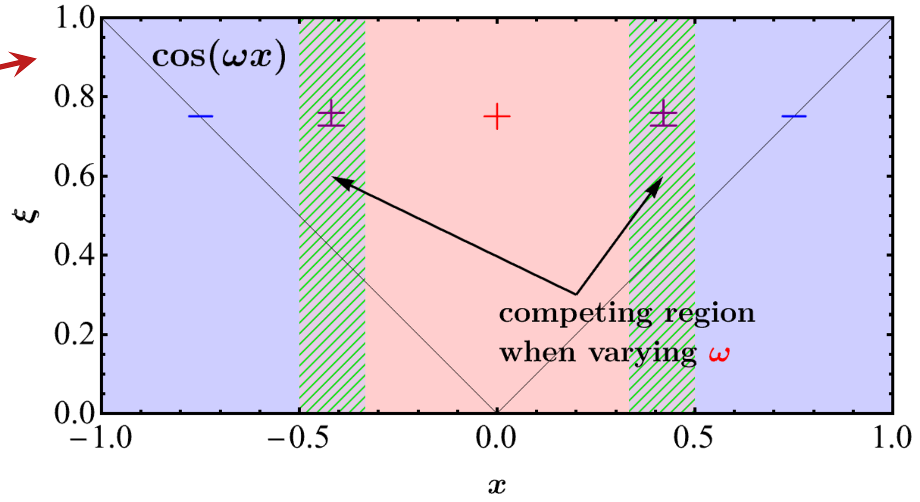
$$P = (p+p')/2, \quad \omega = P\cdot z$$

For example,

$$\mathcal{O}_V(z) = z^2\left[\bar{\psi}_q\gamma^\mu\psi_{q'}\right](z/2)\left[\bar{\psi}_{q'}\gamma_\mu\psi_q\right](-z/2)$$

$$\Rightarrow \quad C_V^0(z) = \frac{i}{2\pi^2}e^{ix\omega} = \frac{i}{2\pi^2}\left[\cos(x\omega) + i\sin(x\omega)\right]$$

- $(P = 2\ \text{GeV},\ z = 0.3\ \text{fm}) \implies \omega \simeq \pi$
- $(P = 3\ \text{GeV},\ z = 0.3\ \text{fm}) \implies \omega \simeq 1.5\pi$



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Summary

- Extracting x-dependence of GPDs has difficulty from exclusiveness
- Pixelation + NN provides a way to *visualize* the fitting process
- DVCS (and similar processes) mostly constrain the ridge $x = \pm\xi$
- *Non-scaling* integrals are needed to constrain other regions
- *Lattice* observables provide useful complementary constraints

⇒ global analysis

Thank you!