DOE contracts:
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Systematic uncertainties from the implementation of higher twists in QCD analyses

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CTEQ-JLab Collaboration

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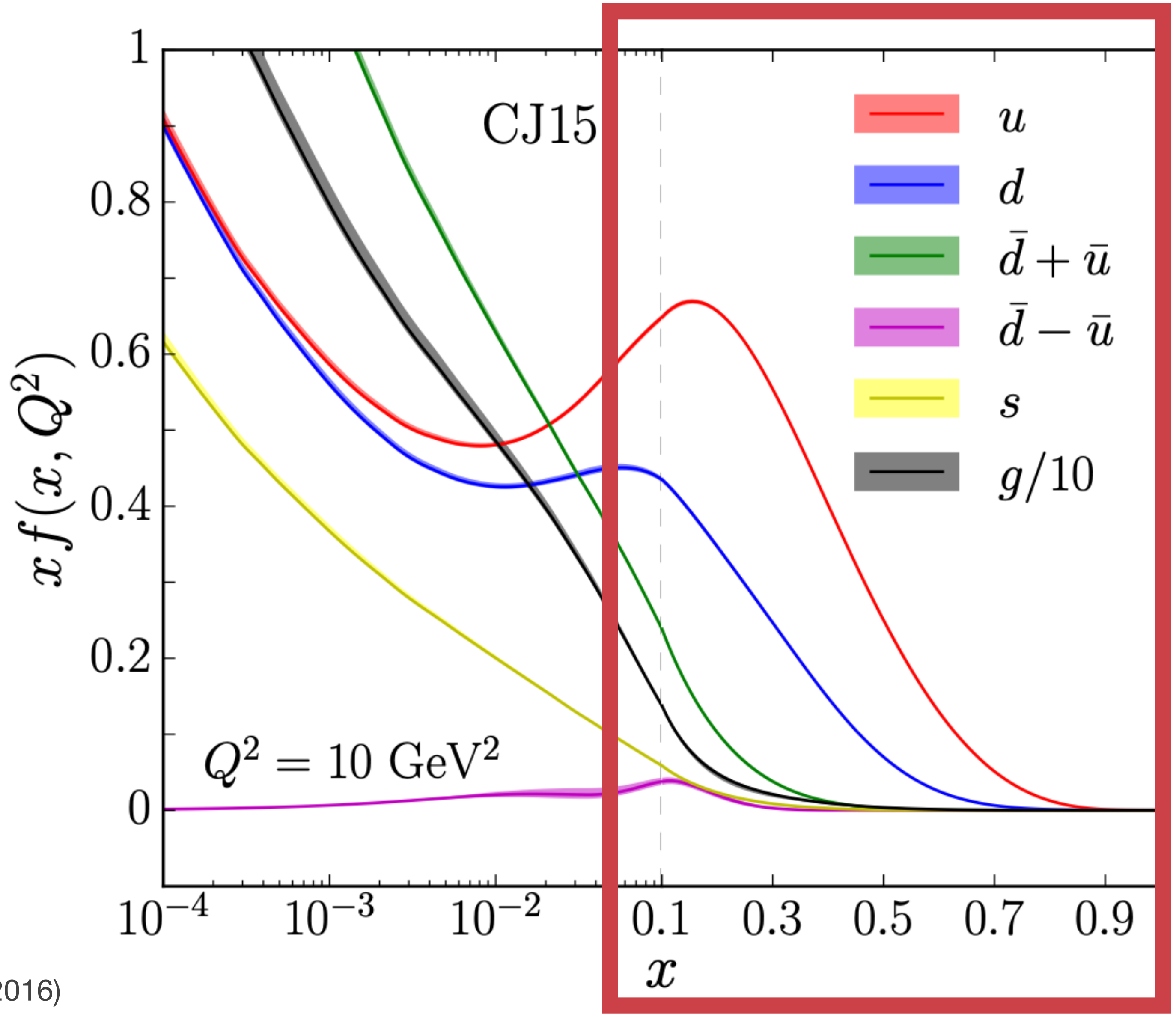
C.E. Keppel, W. Melnitchouk, P. Monaghan

Motivations

Understand the behaviour of PDFs in the large-x region



A. Accardi, et al., PRD 93 (2016)

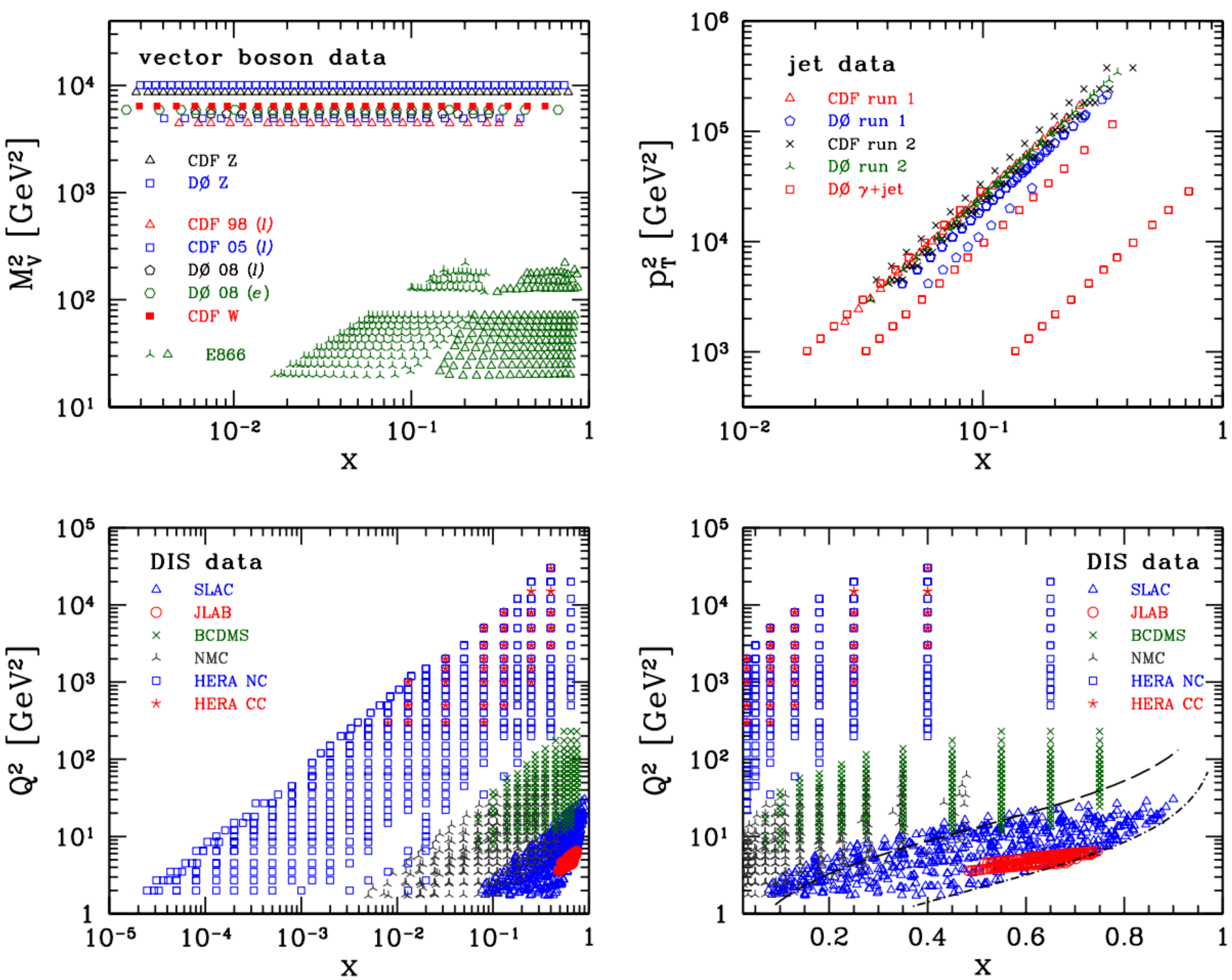
Main focus: $\frac{d}{u}$

Motivations

Which datasets do impose constraints on this region?

See, e.g., J. Owens, et al., PRD 87 (2013)

Main focus: $\frac{d}{u}$



u-quark

DIS on proton target
Drell — Yan data
…

d-quark

W-boson asymmetry
DIS on Deuterium targets
Proton-Tagged DIS (BONuS)
…

We have to deal with Deuterium target at large-x

Motivations

Which datasets do impose constraints on this region?

See, e.g., J. Owens, et al., PRD 87 (2013)

CJ global data set:

- 1000+ data points on Deuterium target
- high-x and low-Q^2
- $W^2 > 3\ \mathrm{GeV}^2$, $Q^2 > 1.69\ \mathrm{GeV}^2$

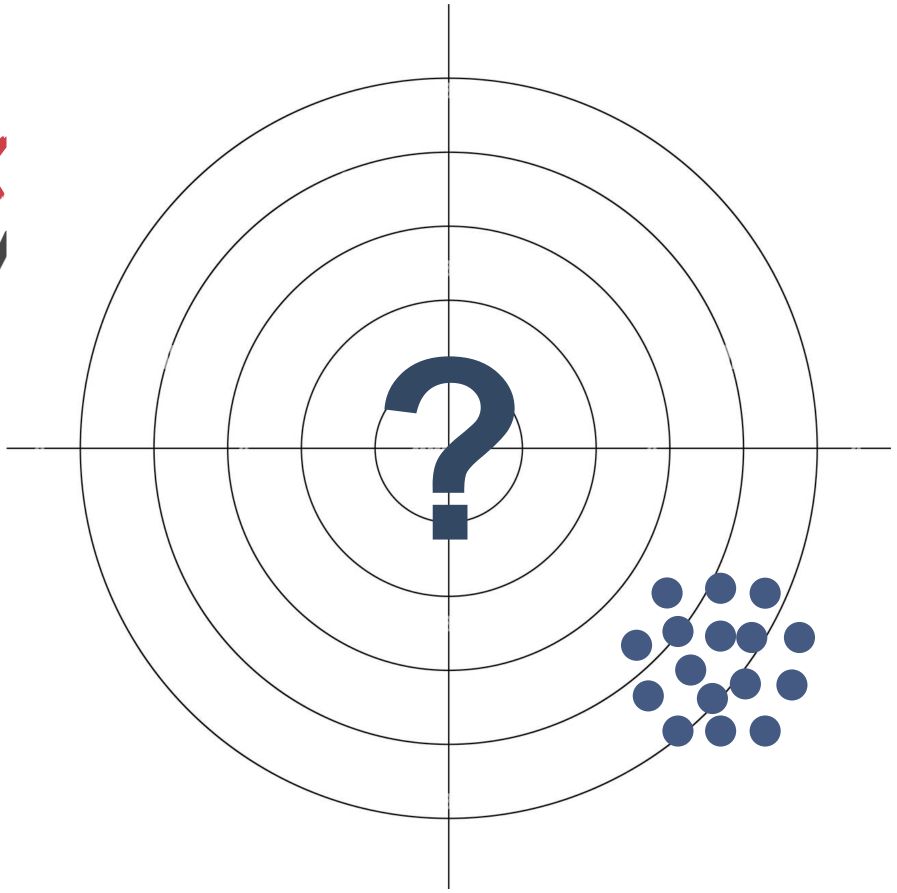
Nuclear corrections

TMC

Higher Twists

Precision ✗

Accuracy ✓

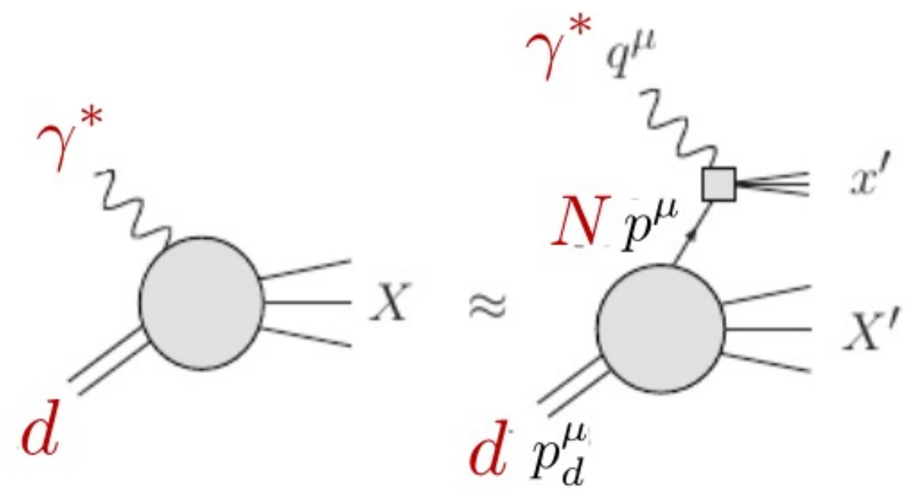


The choice of their implementation may be a source of systematic error

Deuterium: nuclear smearing

Nuclear impulse approximation

Melnitchouk, Schreiber, Thomas, PRD 49 (1994)
Kulagin, Piller, Weise, PRC 50 (1994)
Kulagin and Petti, NPA 765 (2006)



$$F_{2,D}(x_D, Q^2) = \int_{y_{Dmin}}^{y_{Dmax}} dy_D dp_T^2 f_{N/D}(y_D, p_T^2; \gamma) F_{2,N}\left(\frac{x_D}{y_D}, Q^2, p^2\right)$$

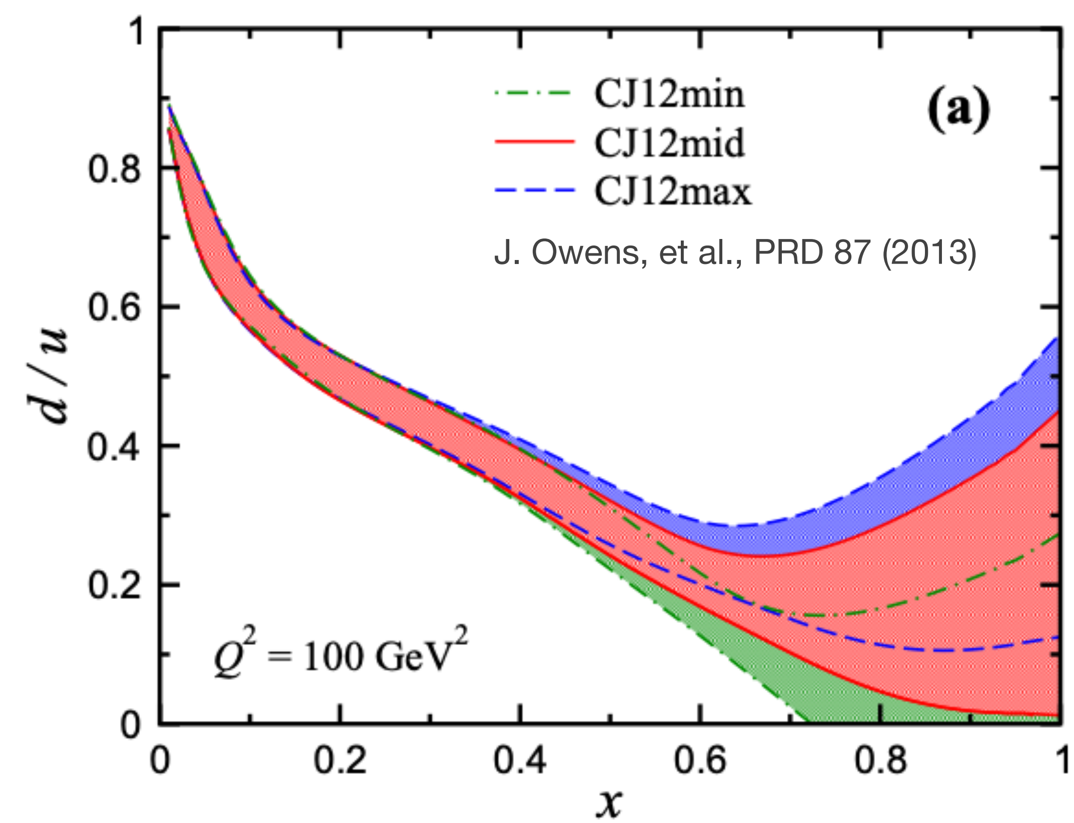
Smearing function:

Weak Binding approximation (WBA)

(Non) Relativistic approach

Nucleon wave function

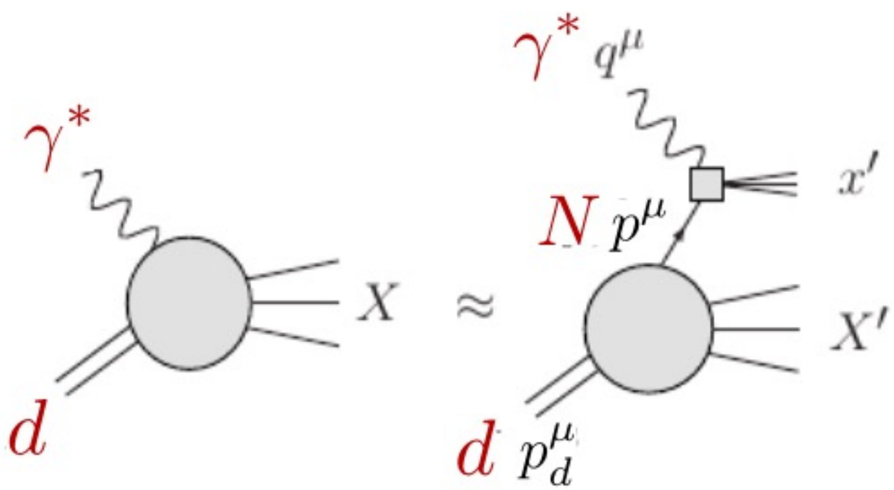
Nuclear binding and Fermi motion effects



Deuterium: off-shell corrections

Nuclear impulse approximation

Melnitchouk, Schreiber, Thomas, PRD 49 (1994)
Kulagin, Piller, Weise, PRC 50 (1994)
Kulagin and Petti, NPA 765 (2006)



$$F_{2,D}(x_D, Q^2) = \int_{y_{Dmin}}^{y_{Dmax}} dy_D dp_T^2 f_{N/D}(y_D, p_T^2; \gamma) F_{2,N}\left(\frac{x_D}{y_D}, Q^2, p^2\right)$$

Off-shell expansion (in nucleon virtuality p^2)

$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta f(x)\right]$$

$$F_{2N}(x, Q^2, p^2) = F_{2N}^{\text{free}}(x, Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta F(x)\right]$$

Kulagin, Piller, Weise, PRC 50 (1994)
Kulagin, Melnitchouk, et al., PRC 52 (1995)
Kulagin and Petti, NPA 765 (2006)

Free nucleon pdfs/SFs
$p^2 = m_N^2$

Off-shell function
(To be fitted)

Structure function
of a bound, off-shell nucleon

Off-shell function: parameterization

Off-shell corrections

$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta f(x)\right]$$

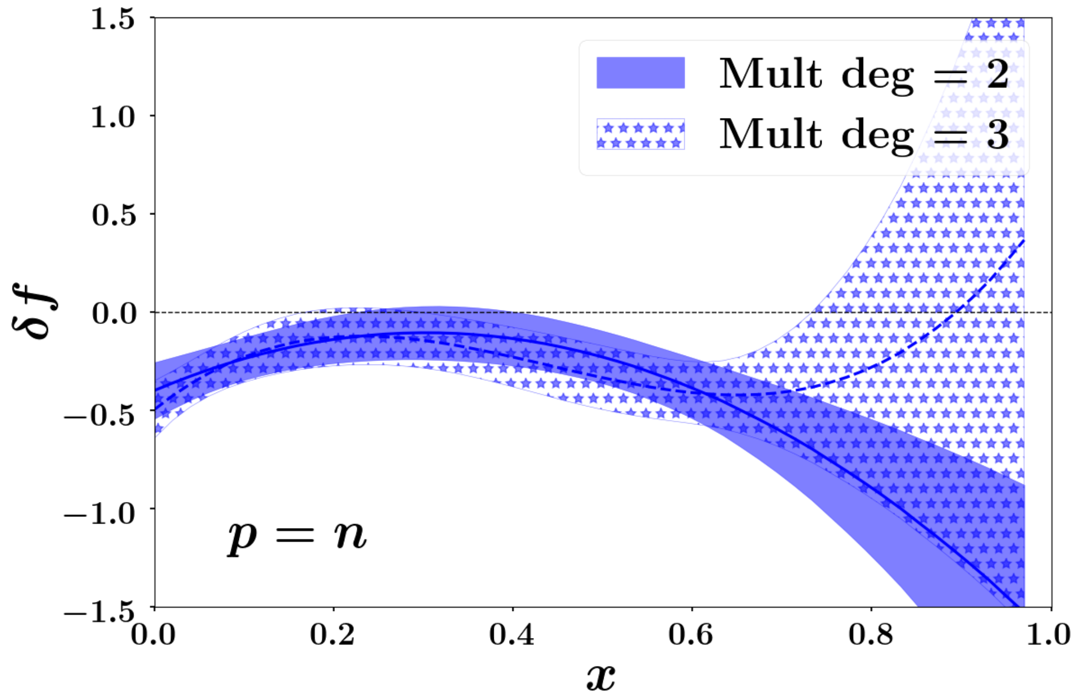
- KP-like model $\delta f^N = C(x - x_0)(x - x_1)(1 + x_0 - x)$

  \+ valence sum rule $\int_0^1 dx\, \delta f^N(x)\, [q(x) - \bar{q}(x)] = 0$

Kulagin and Petti, NPA 765 (2006)
Accardi, et al., PRD 93 (2016)
Accardi, et al., PRD 107 (2023)

- Polynomial model $\delta f(x) = \sum_n a_{off}^{(n)} x^n$

Alekhin, Kulagin, Petti, PRD 96 (2017)
Alekhin, Kulagin, Petti, PRD 105 (2022)
Alekhin, Kulagin, Petti, PRD 107 (2023)



$\Rightarrow$ Constrain power of CJ dataset only up to x = 0.6

Higher-Twist function

Higher Twist correction

Multiplicative (CJ fits)

$$F_2(x,Q^2) = F_2^{LT}(x,Q^2)\left(1+\frac{\boldsymbol{C}(x)}{Q^2}\right)$$

$$C(x) = a_{ht}^{(0)} x^{a_{ht}^{(1)}} (1 + a_{ht}^{(2)} x)$$

Additive

$$F_2 = F_2^{LT}(x,Q^2) + \frac{\boldsymbol{H}(x)}{Q^2}$$

$$H(x) = a_{ht}^{(0)} x^{a_{ht}^{(1)}} (1-x)^{a_{ht}^{(2)}} (1 + a_{ht}^{(3)} x)$$

they are related

$$F_2^{LT}(x,Q^2)\left(1+\frac{\boldsymbol{C}(x)}{Q^2}\right) = F_2^{LT}(x,Q^2) + F_2^{LT}(x,Q^2)\frac{C(x)}{Q^2}$$

$$= F_2^{LT}(x,Q^2) + \frac{\tilde{\boldsymbol{H}}(x,Q^2)}{Q^2}$$

Impact of HT on n/p ratio

Are experimental observables independent of the choice of the HT?

$$\frac{F_{2,n}}{F_{2,p}} = \frac{n}{p} \xrightarrow{x \to 1} \frac{4d+u}{4u+d} \simeq \frac{1}{4}$$

(extrapolation region)

Case 1: isospin-symmetric HT

Mult HT

$C_p(x) = C_n(x) = C(x)$

$$\frac{(4d+u)(1+C/Q^2)}{(4u+d)(1+C/Q^2)} \simeq \frac{1}{4}$$

No effect of HT

Add HT

$H_p(x) = H_n(x) = H(x)$

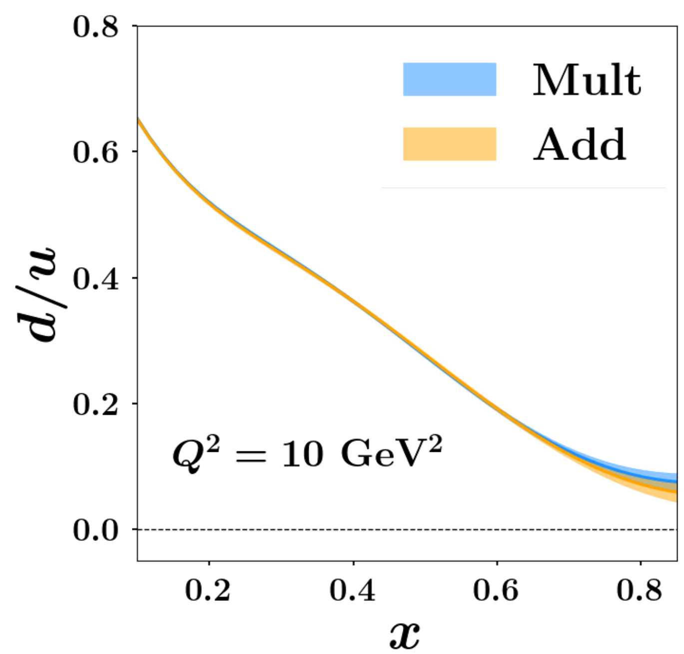
$$\frac{4d+u+H/Q^2}{4u+d+H/Q^2} \simeq \frac{1}{4} + 27\frac{H}{16uQ^2}$$

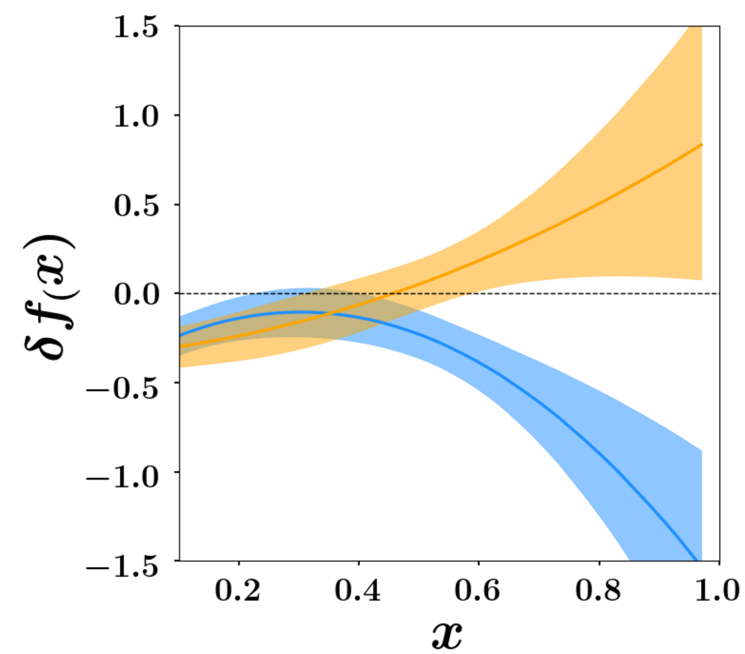
Strong effect of HT

Bias identified!!

Results in the CJ fitting framework

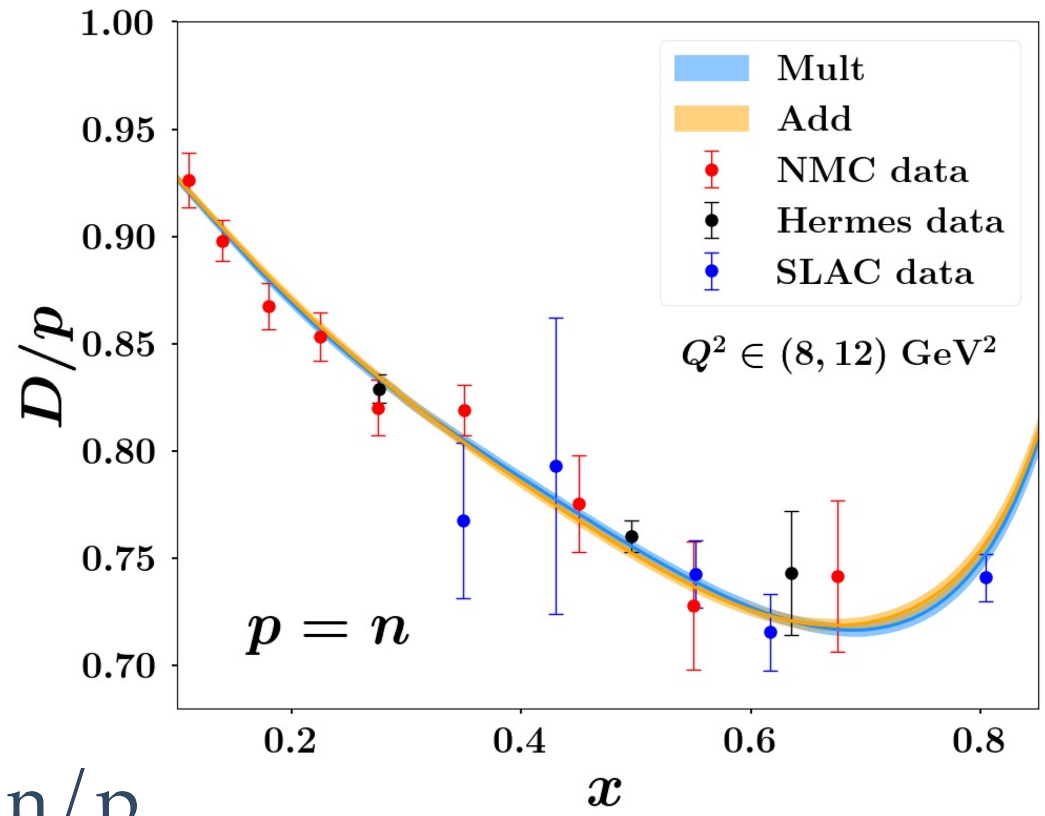
Case 1: isospin-symmetric HT

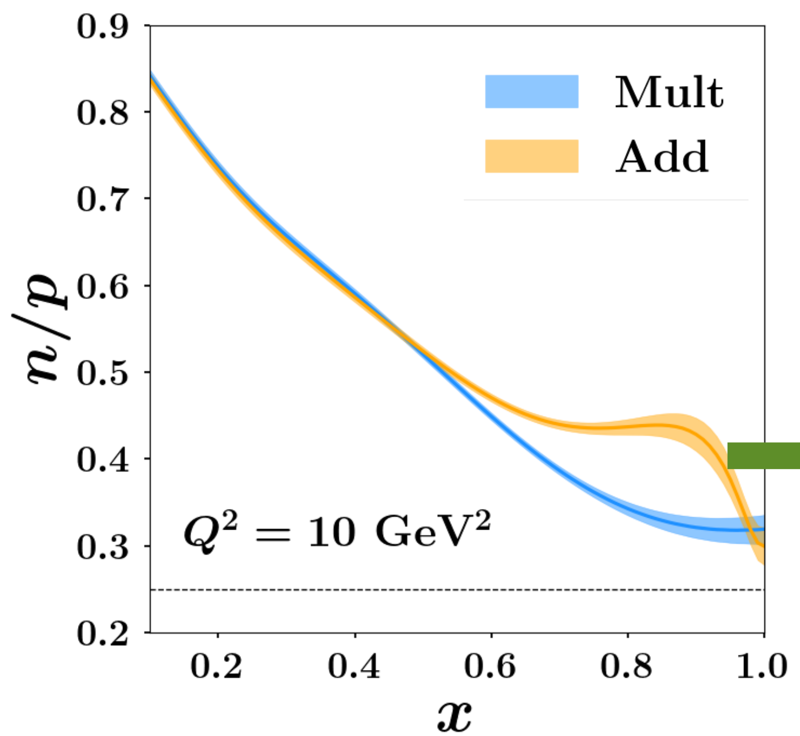




Bias identified

Off-shell compensates n/p



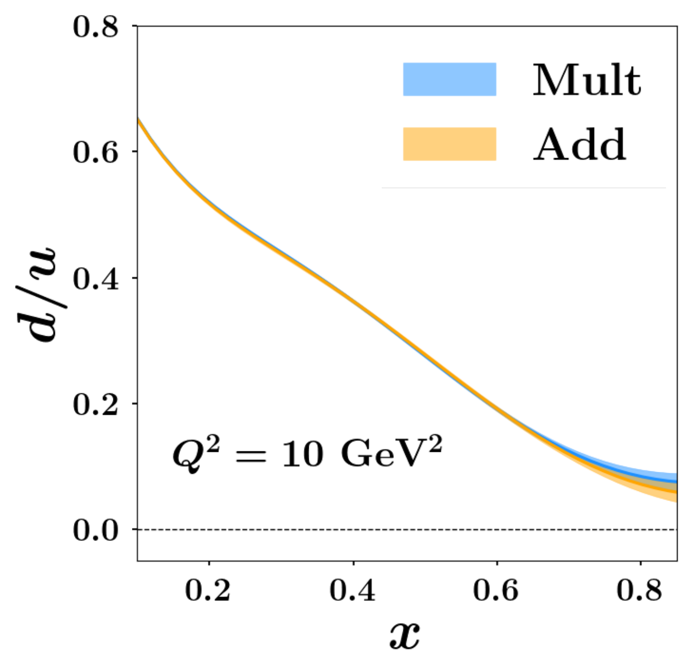


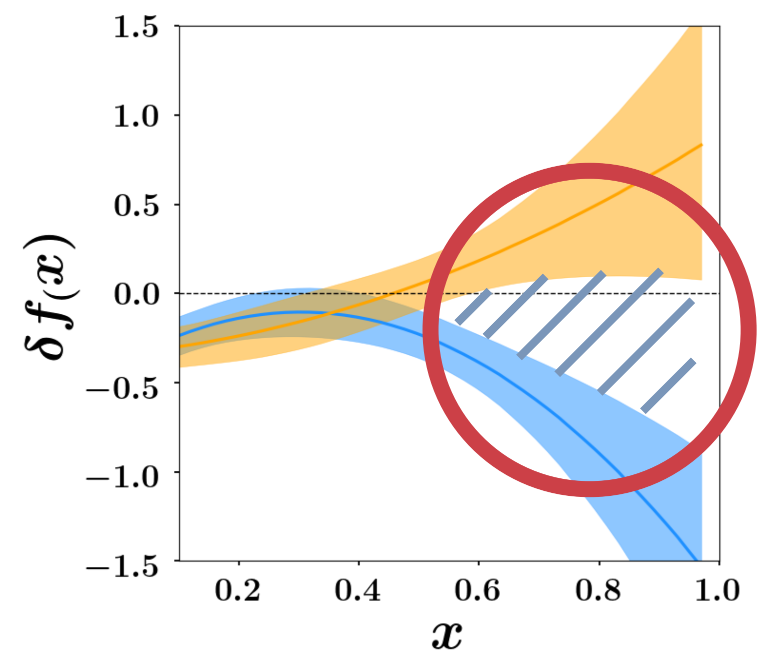
Artificially large n/p

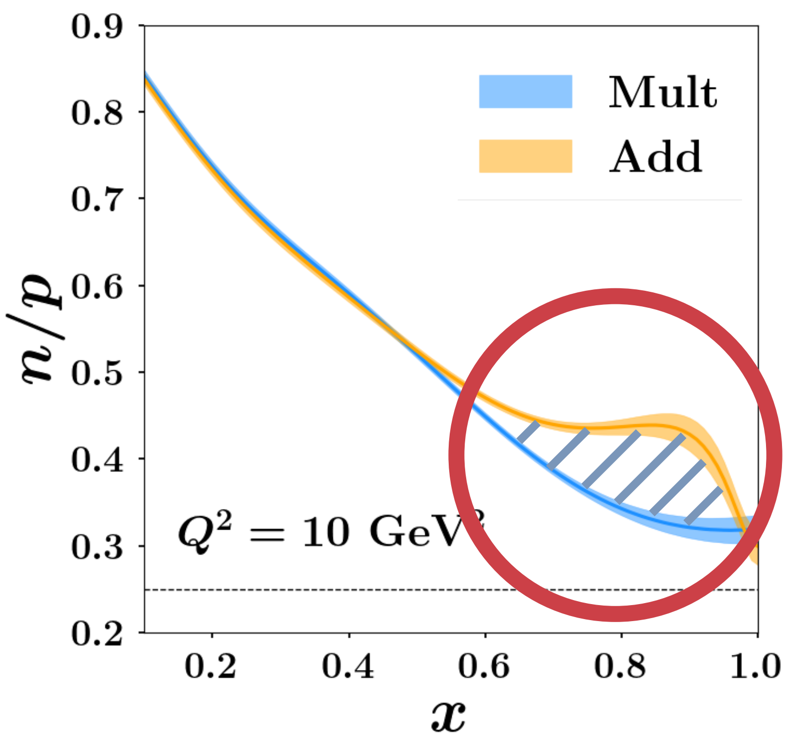
BUT smaller d/u than Mult

Results in the CJ fitting framework

Case 1: isospin-symmetric HT

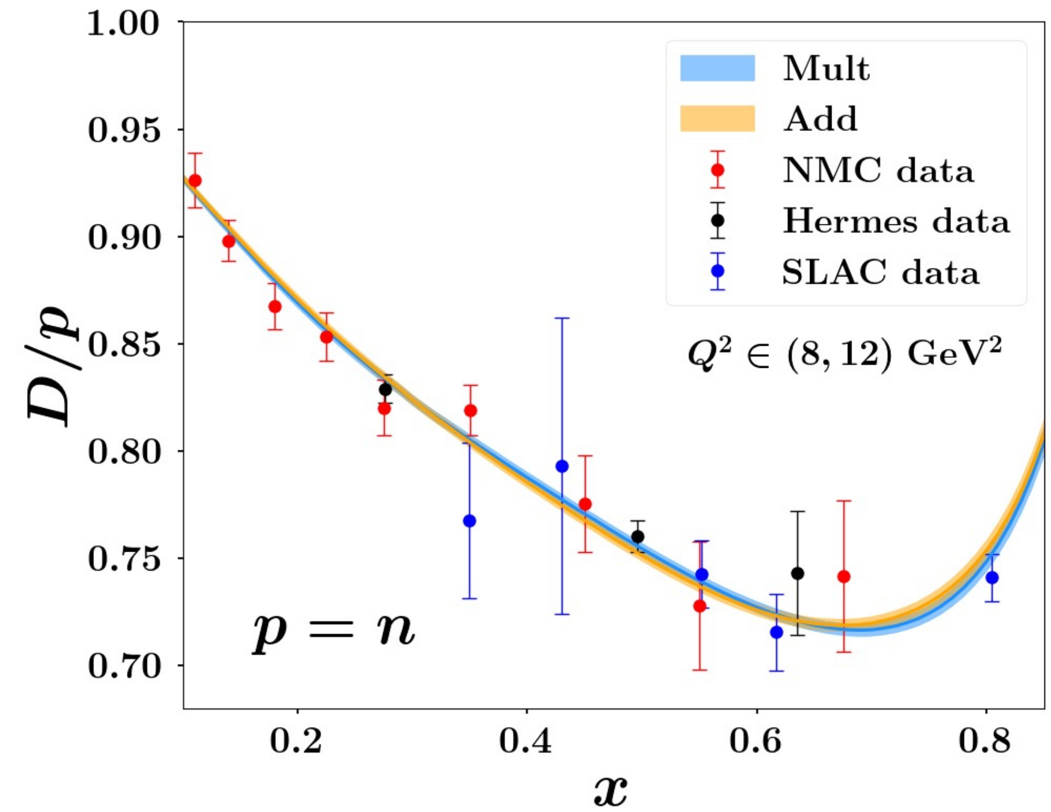






Bias identified

Off-shell compensates n/p



Systematic "implementation" uncertainty in a region of extrapolation

Impact of HT on n/p ratio

Are experimental observables independent of the choice of the HT?

$$\frac{n}{p} \xrightarrow{x \to 1} \frac{1}{4}$$

LT

Mult HT $C_p(x) = C_n(x) = C(x)$

Case 2: isospin-breaking HT

Add HT

$H_p(x) \neq H_n(x)$

$$\frac{u + H_n/Q^2}{4u + H_p/Q^2} \simeq \frac{1}{4} + 9\frac{4H_n - H_p}{16uQ^2}$$

$H_p(x) = H_n(x)$ → $\frac{1}{4} + 27\frac{H}{16uQ^2}$

$H_p(x) = 2H_n(x)$ → $\frac{1}{4} + 9\frac{H}{16uQ^2}$

n/p ratio is smaller

Mult HT

$C_p(x) \neq C_n(x)$

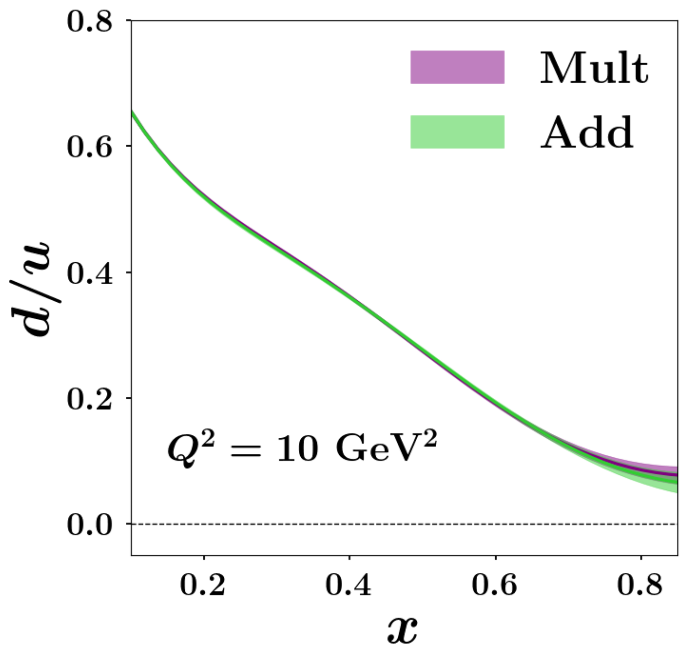
$$\frac{u + \tilde{H}_n/Q^2}{4u + \tilde{H}_p/Q^2}$$

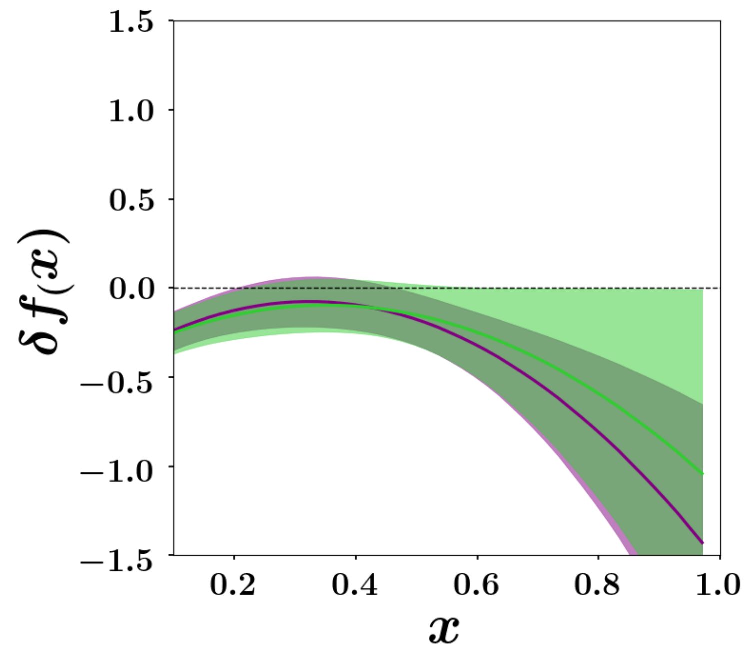
same as Add

Bias removed!

Results in the CJ fitting framework

Case 2: isospin-breaking HT

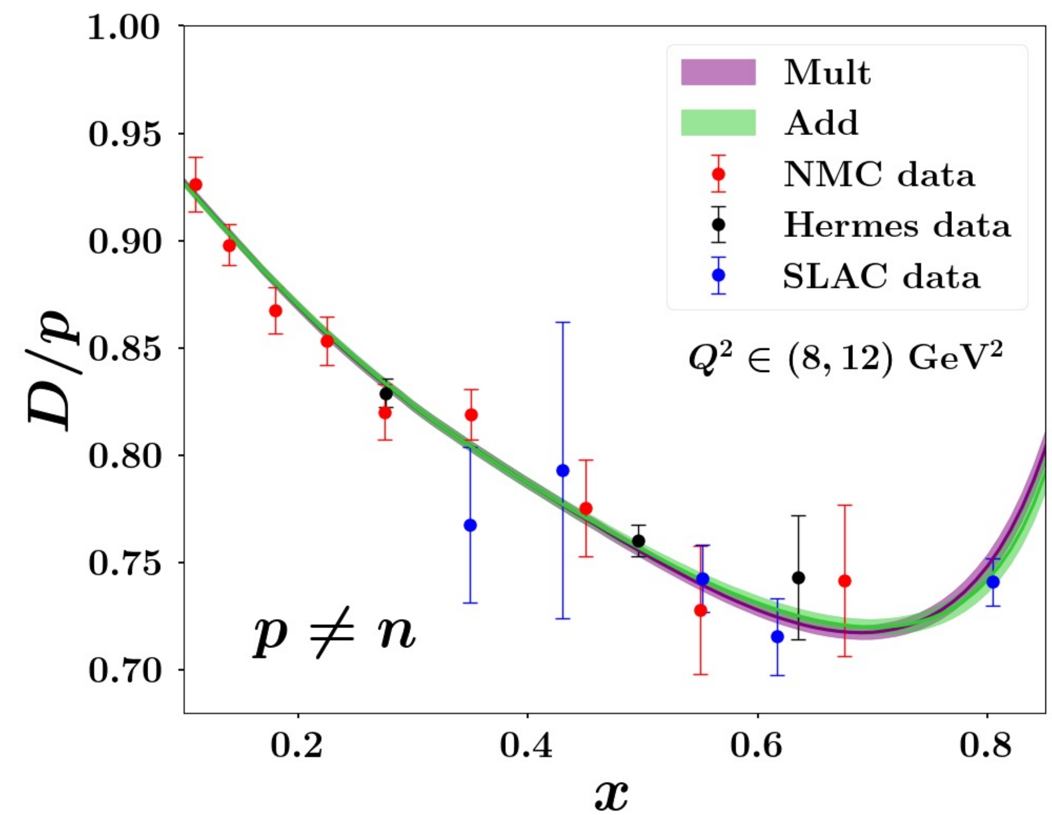


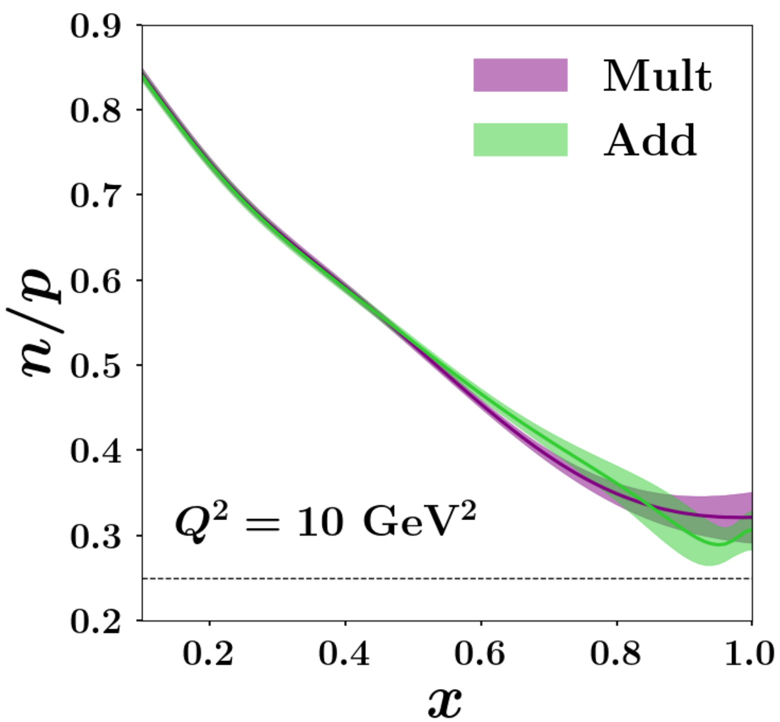


Bias removed

No need of compensation by off-shell

Theory expectations confirmed!



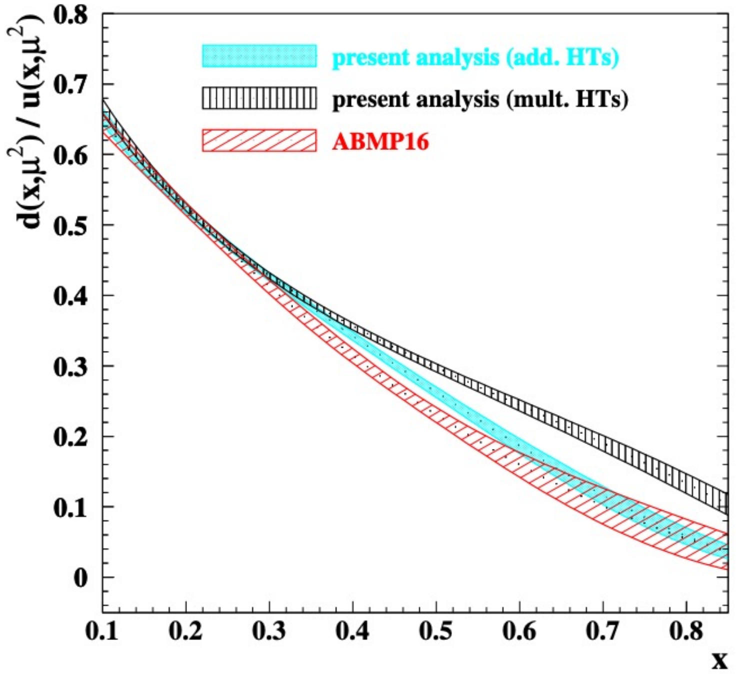


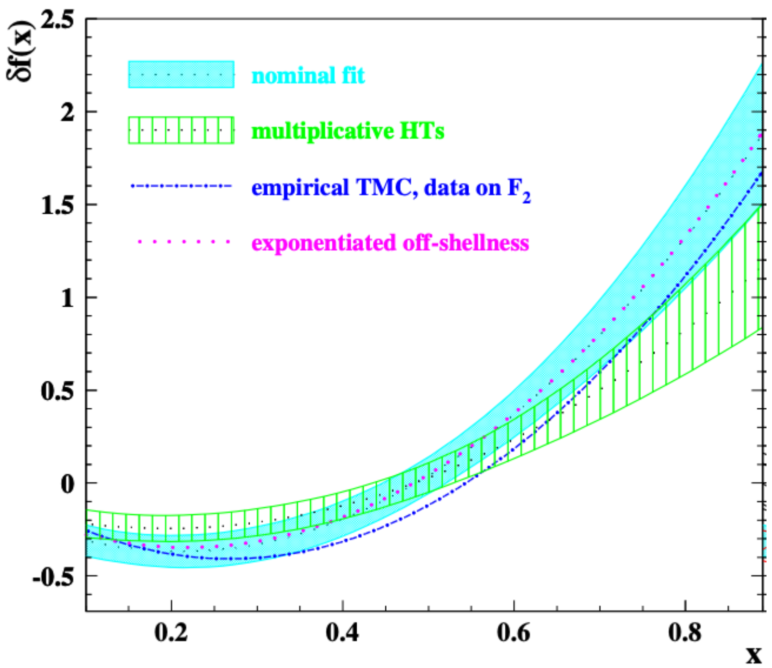
Compatible n/p

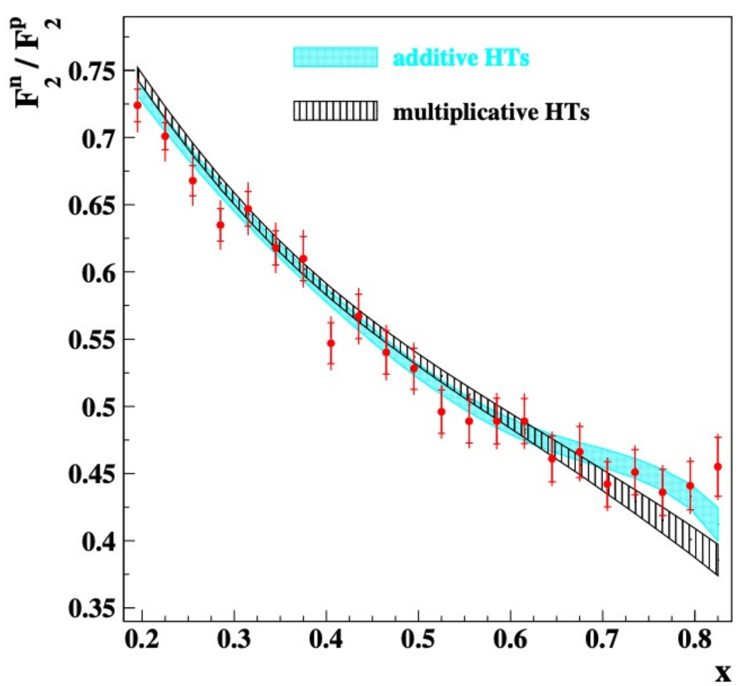
Comparison to other extractions

AKP

Alekhin, Kulagin, Petti, PRD 105 (2022)
Alekhin, Kulagin, Petti, PRD 107 (2023)







Artificially large n/p
BUT smaller d/u than Mult

Comparison to other extractions

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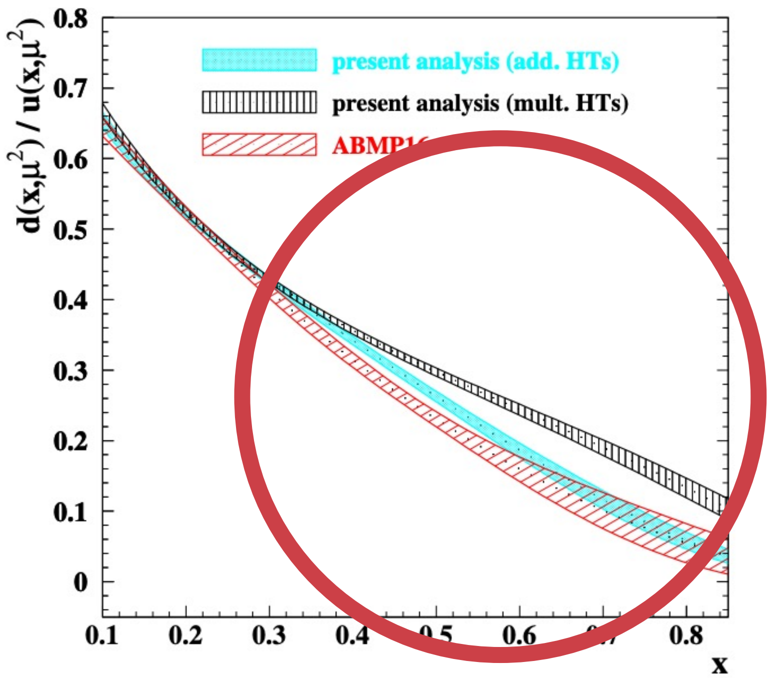
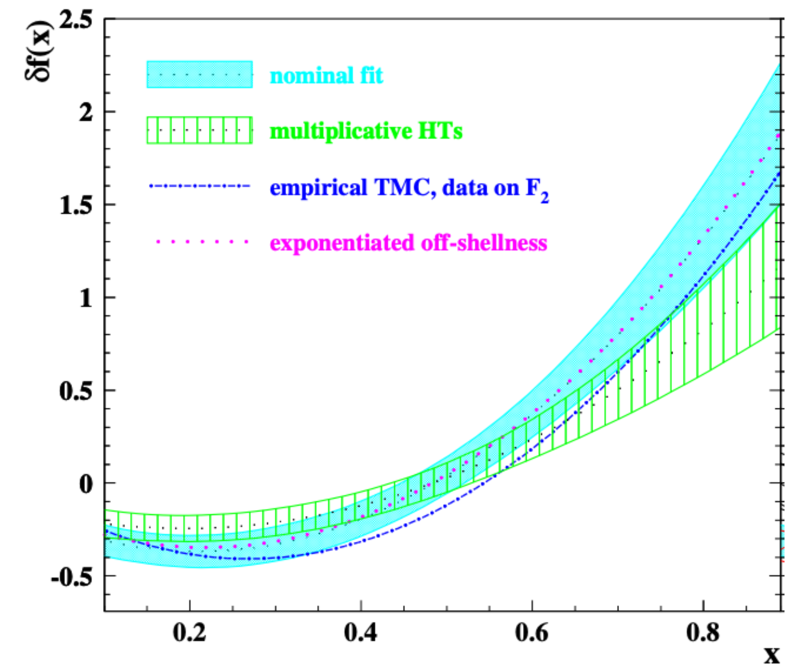
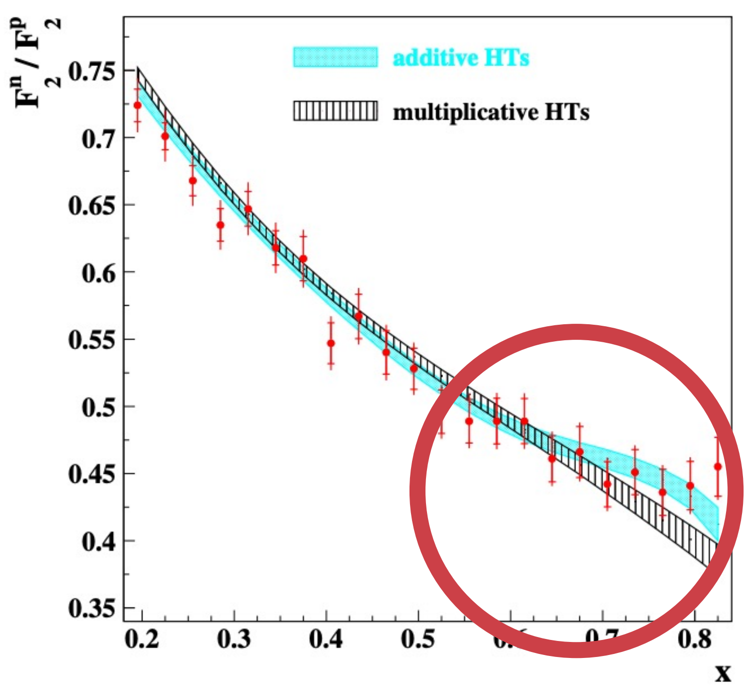






Systematic "implementation" uncertainty

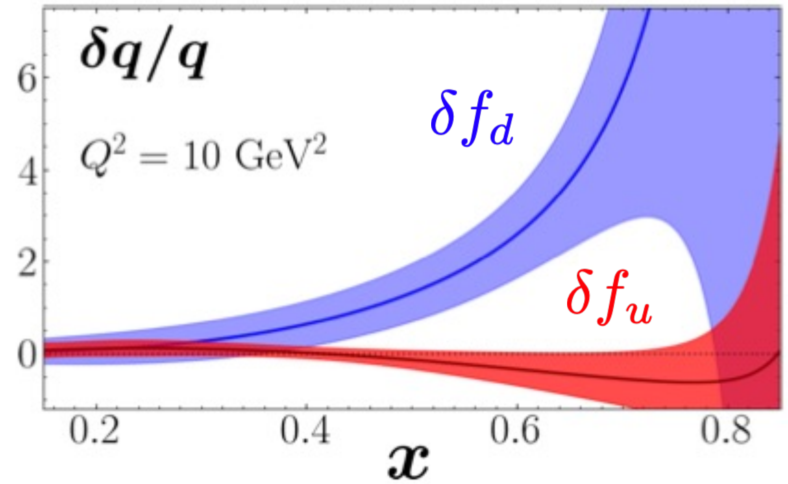
appears in both $\frac{n}{p}$ and $\frac{d}{u}$

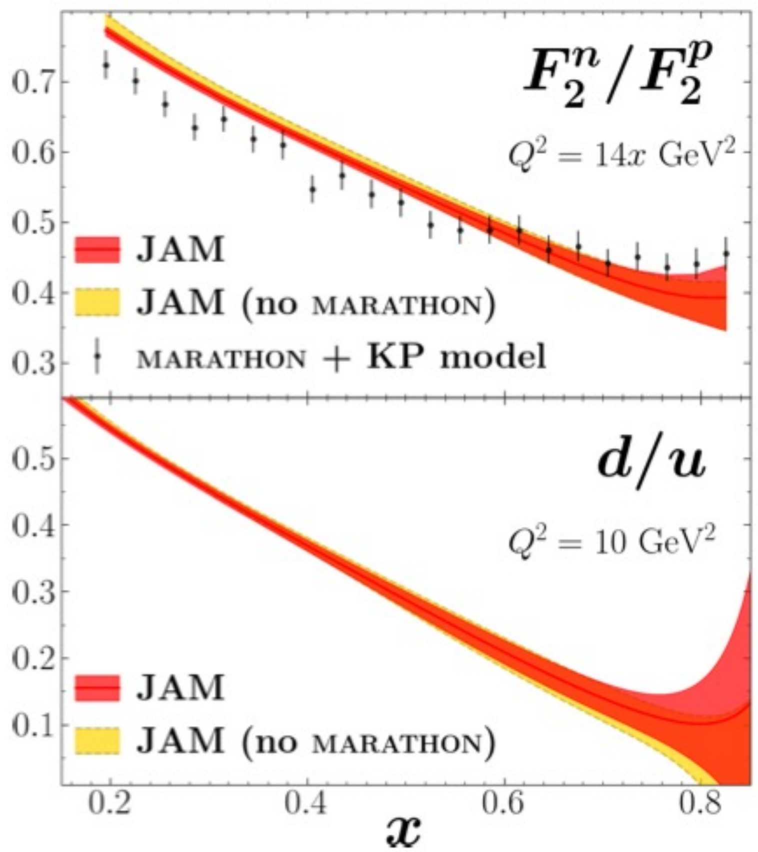
JAM results

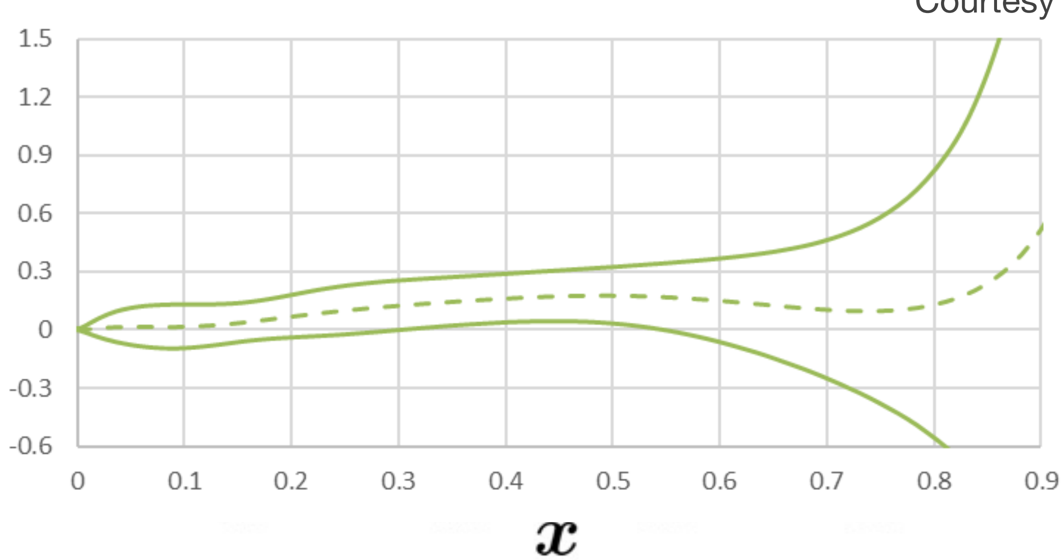
JAM *Fit including A=3 data* $\delta f_u \delta f_d$

JAM Collaboration, PRL 127 (2021)

Mult HT (p=n) as default choice





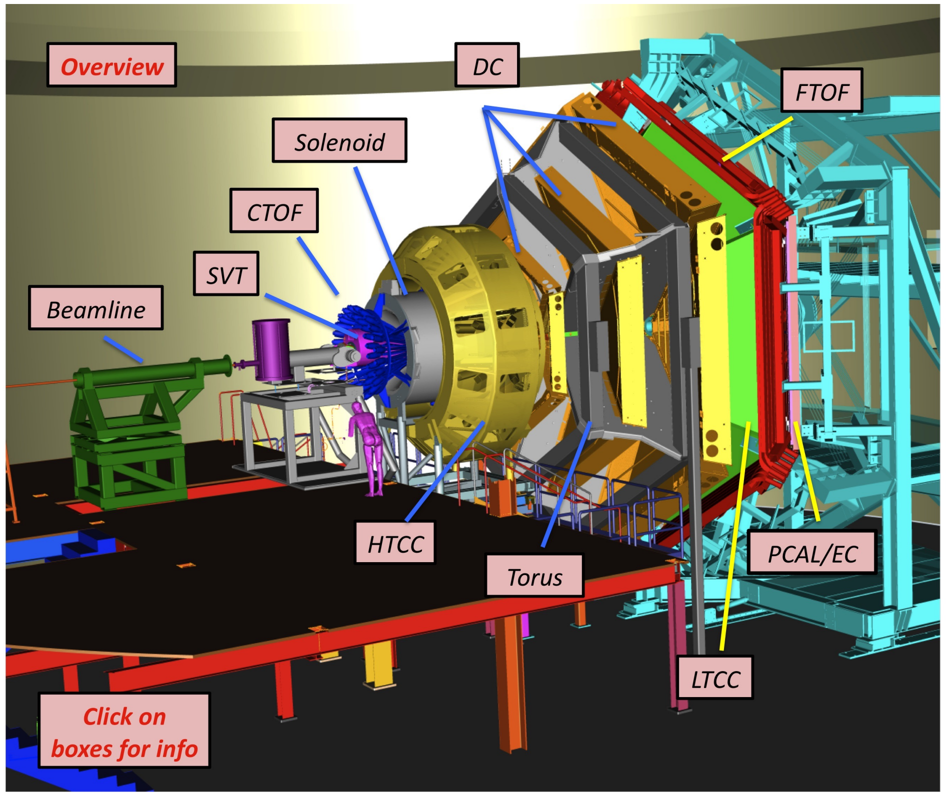


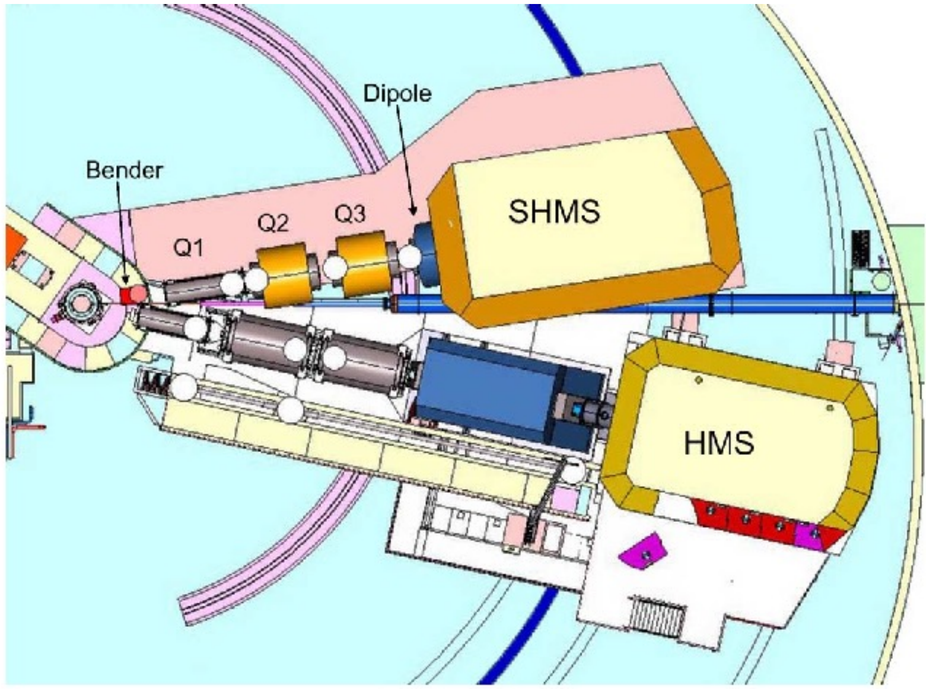
Courtesy of C. Cocuzza

$$\delta f(x)|_{\text{CJ-like}} = \frac{u\delta f_u + d\delta f_d}{u+d}$$

Compatible with CJ results

Need more information





CLAS12 (BoNUS12)

$$e + d \rightarrow e' + p + X$$

Hall C

$$e + p/D \rightarrow e' + X$$

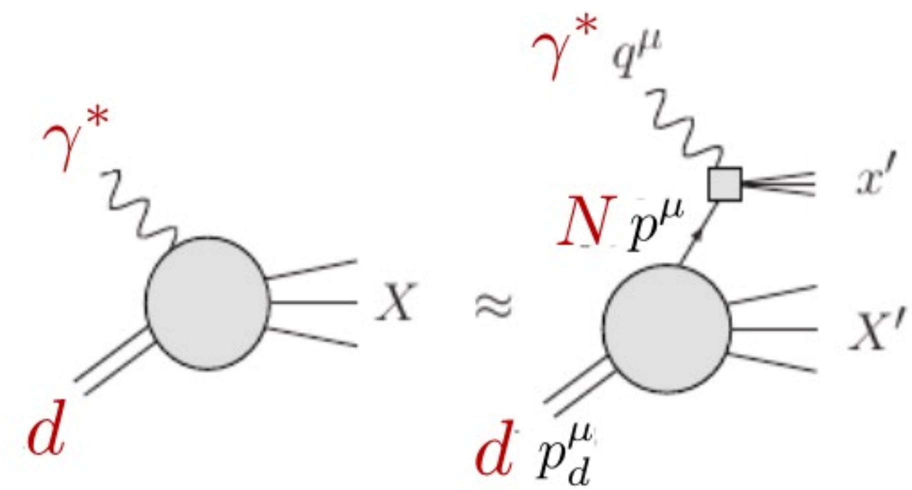
Biswas, et al., 2409.15236

New experimental data in the large-x region are needed to understand the correct interconnection of d/u, n/p ratios and off-shell corrections

Deuterium: off-shell corrections

Nuclear impulse approximation

Melnitchouk, Schreiber, Thomas, PRD 49 (1994)
Kulagin, Piller, Weise, PRC 50 (1994)
Kulagin and Petti, NPA 765 (2006)



$$F_{2,D}(x_D, Q^2) = \int_{y_{Dmin}}^{y_{Dmax}} dy_D dp_T^2 f_{N/D}(y_D, p_T^2; \gamma) F_{2,N}\left(\frac{x_D}{y_D}, Q^2, p^2\right)$$

Off-shell expansion (in nucleon virtuality p^2)

$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta f(x)\right]$$

Kulagin, Piller, Weise, PRC 50 (1994)
Kulagin, Melnitchouk, et al., PRC 52 (1995)
Kulagin and Petti, NPA 765 (2006)

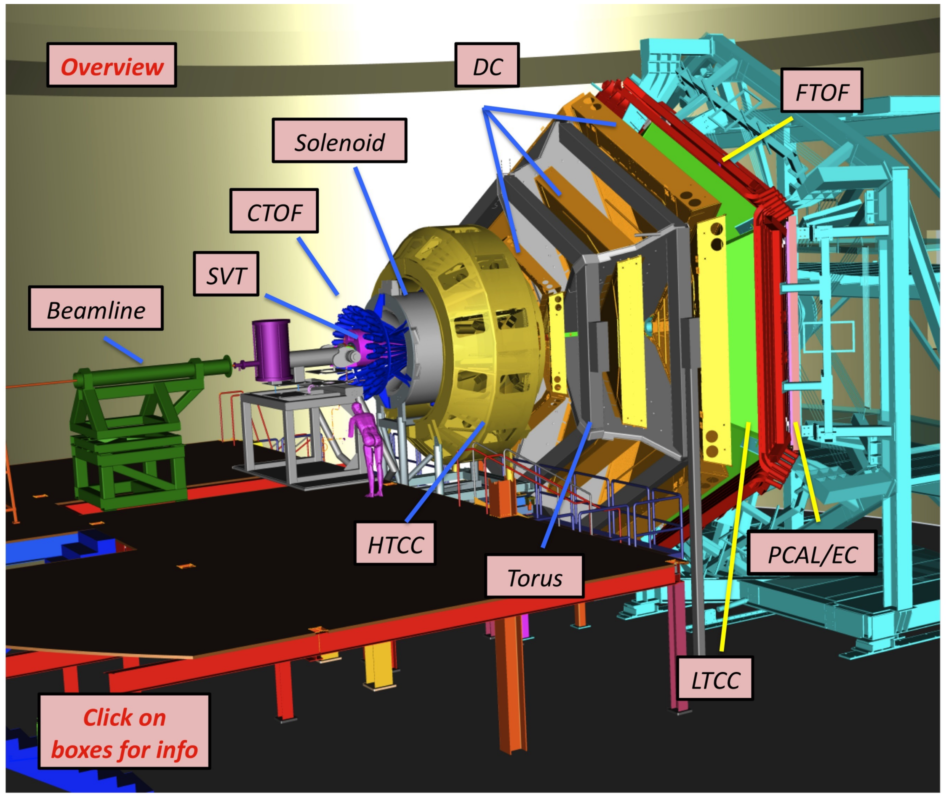
This is an ASSUMPTION

Is it possible to verify it?

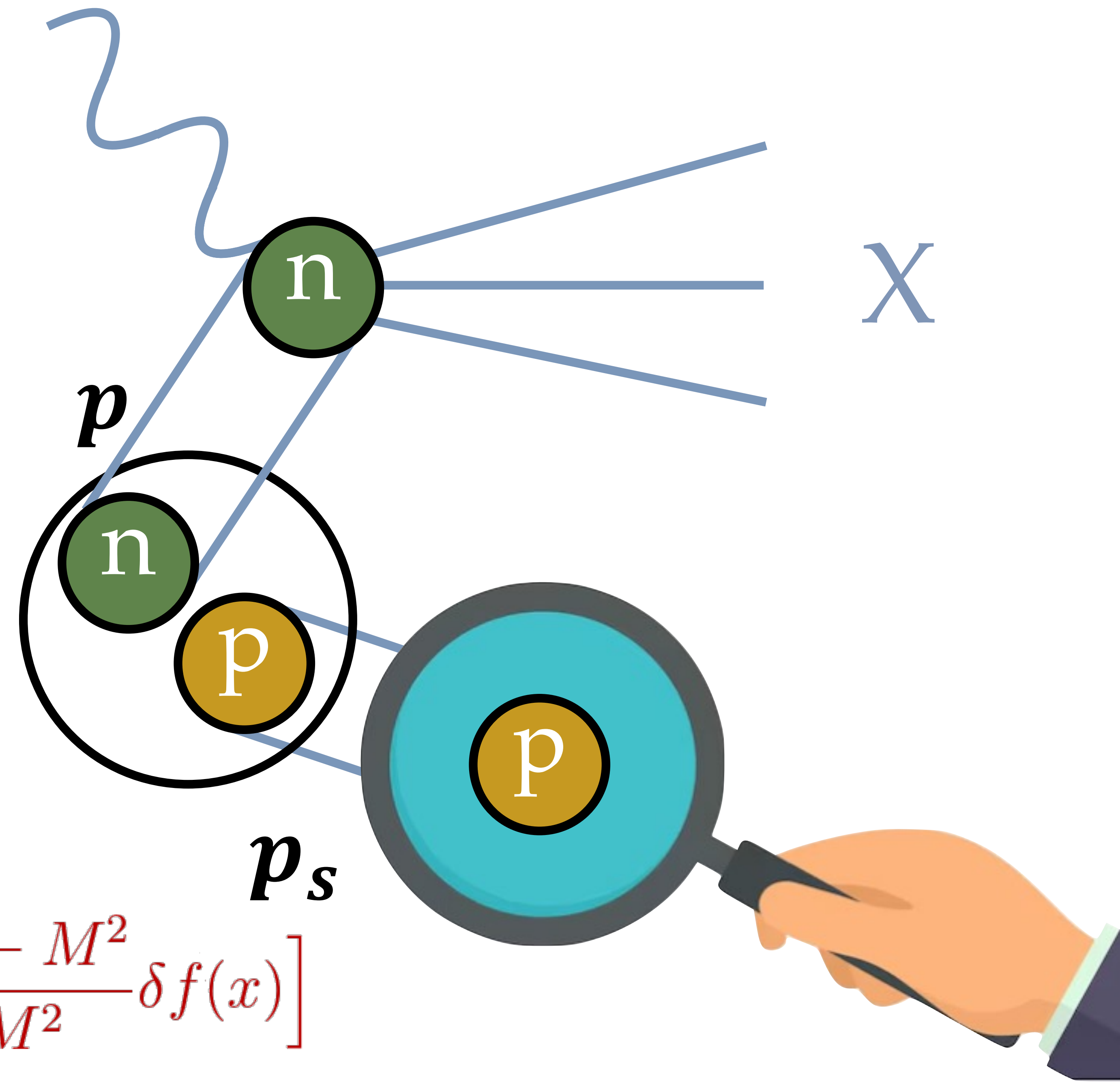
Structure function

of a bound, off-shell nucleon

Need more information



CLAS12 (BoNUS12)

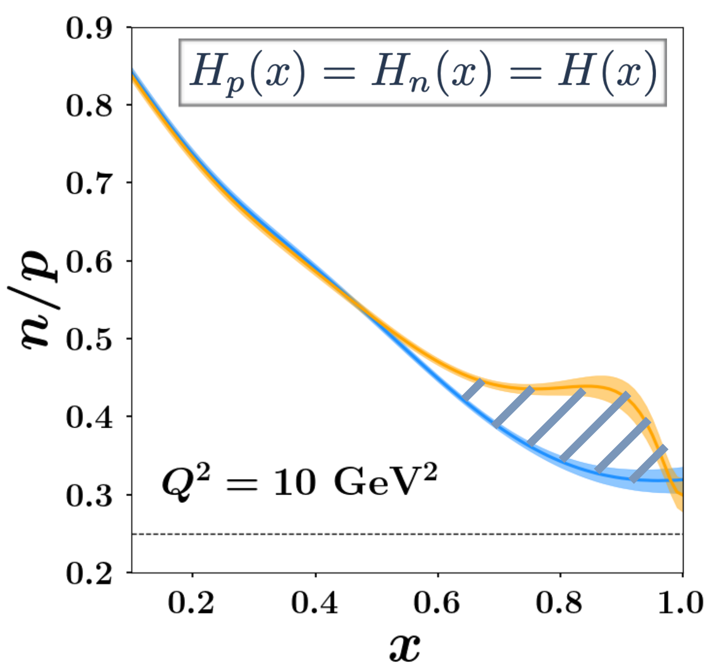


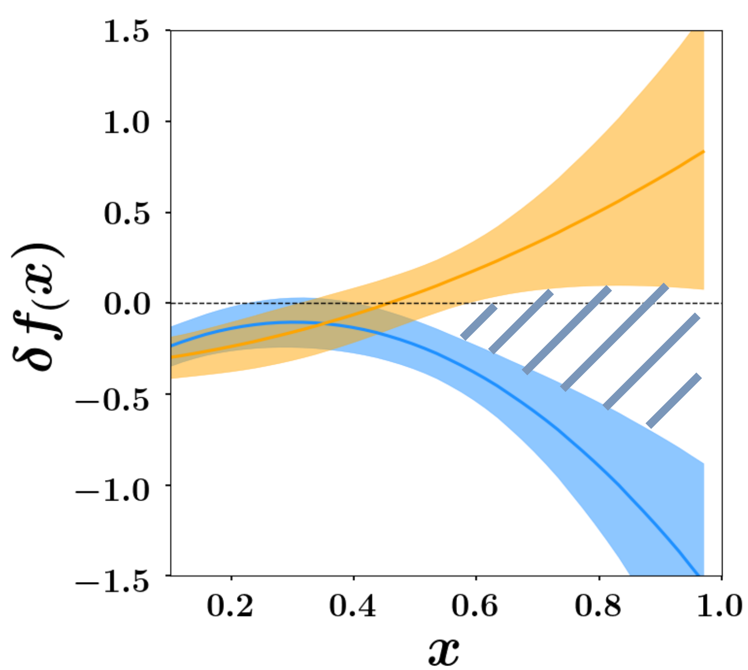
$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta f(x)\right]$$

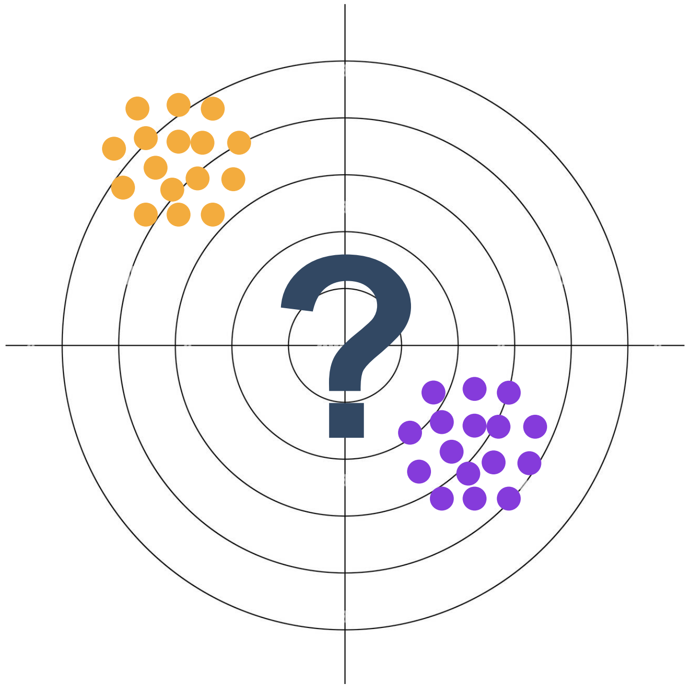
Experimental data differential on the off-shell proton virtuality p_s^2
would allow us to pin down the off-shell correction in a more clean way

TAKE-HOME message

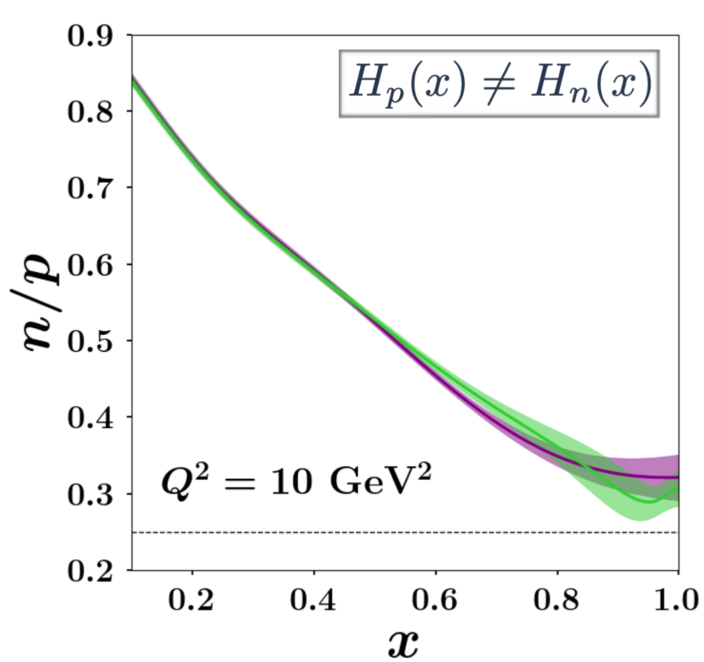
Case 1: isospin-symmetric HT

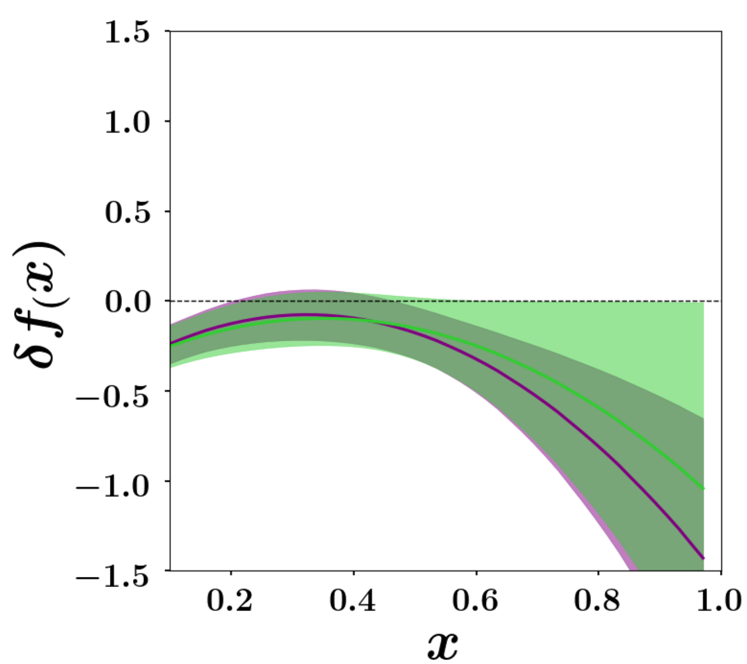


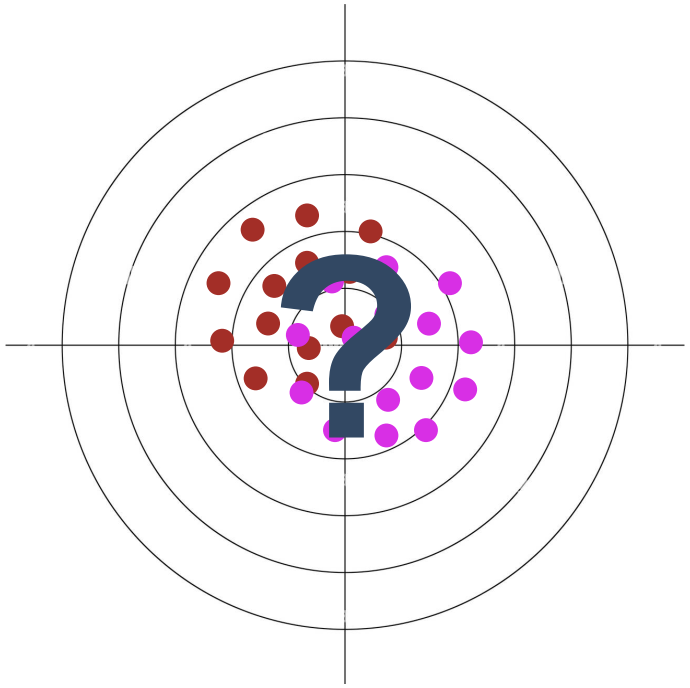




Case 2: isospin-breaking HT

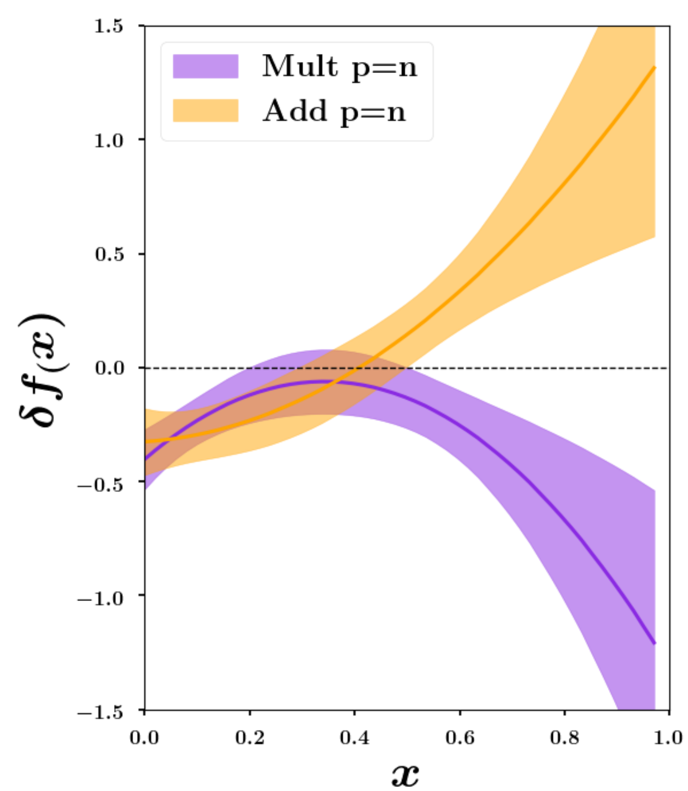
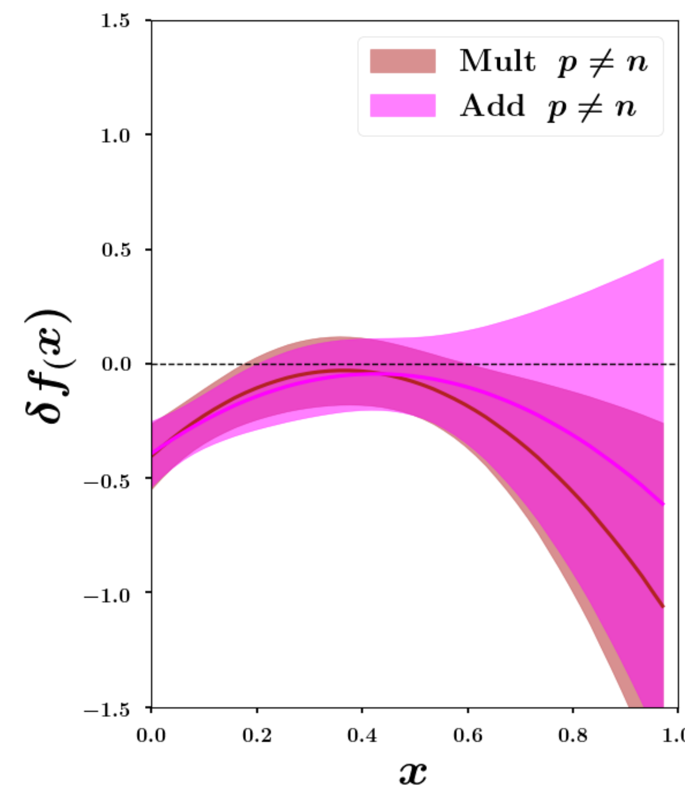






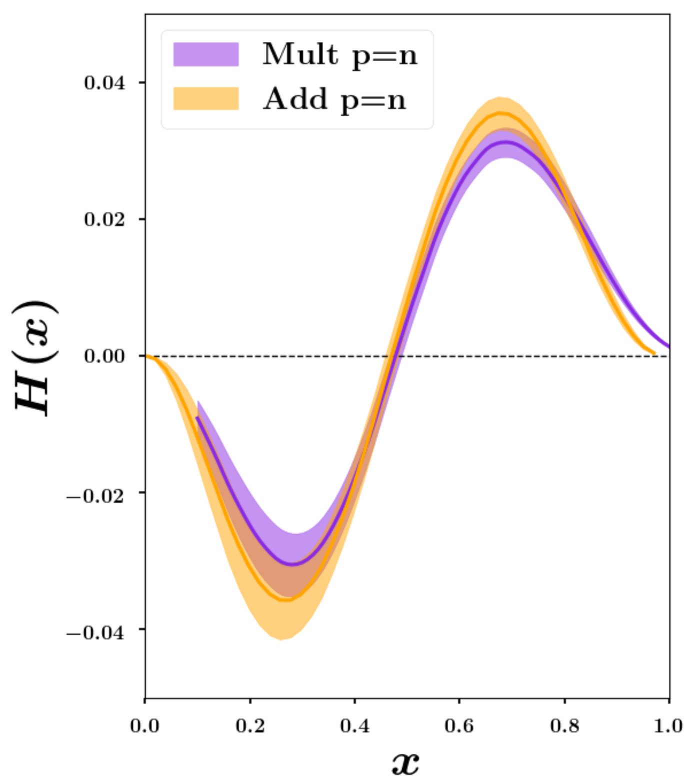
Backup

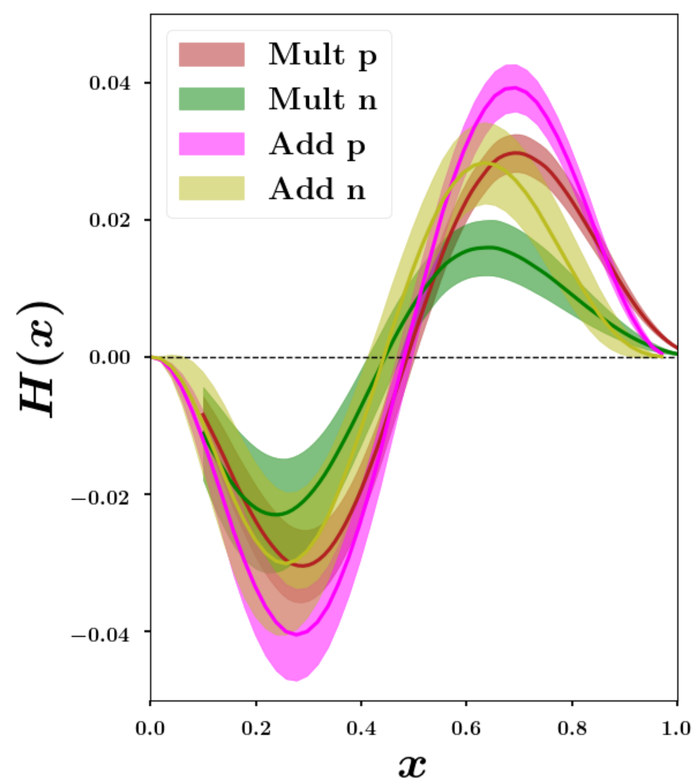
Off-shell table

higher-twist \ isospin	Symmetric	Non symmetric
Multiplicative		
Additive		

Higher-Twist table

isospin / higher-twist	Symmetric	Non symmetric
Multiplicative	$\tilde{H} = F_{2,N}(x, Q^2) H_{\text{Mult}}(x)$	$\delta\tilde{H} = F_{2,N}(x, Q^2)\delta H_{\text{Mult}}(x)$
Additive		

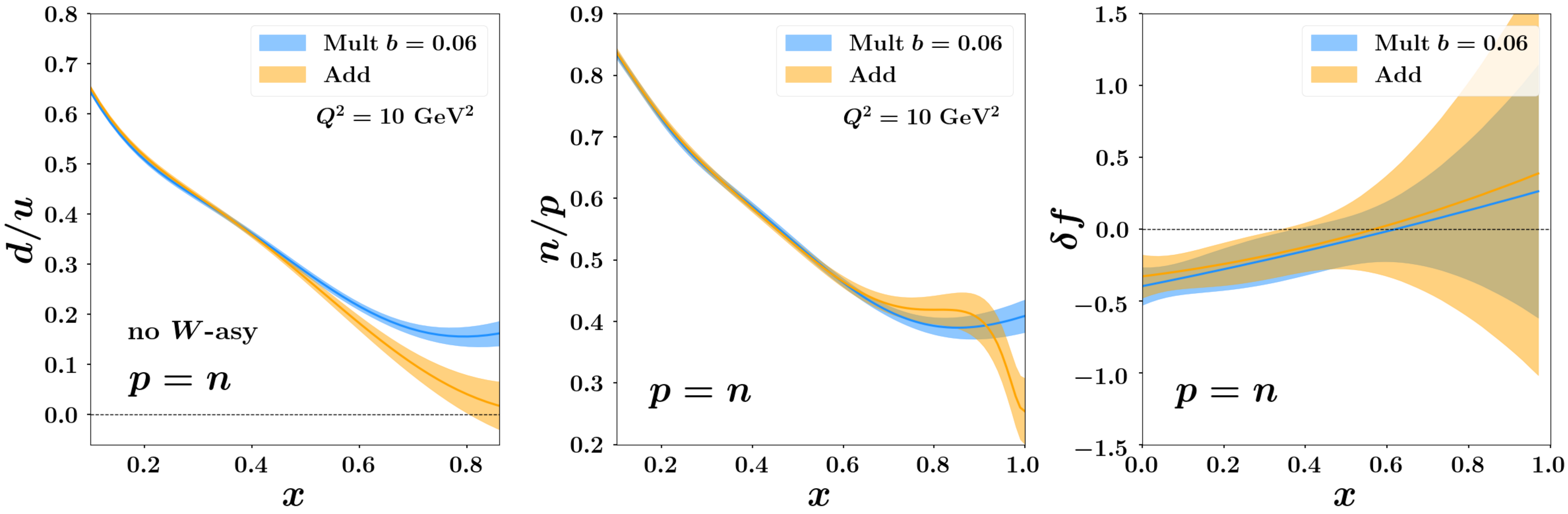




Other results

Case 1: isospin-symmetric HT

d/u artificially increased



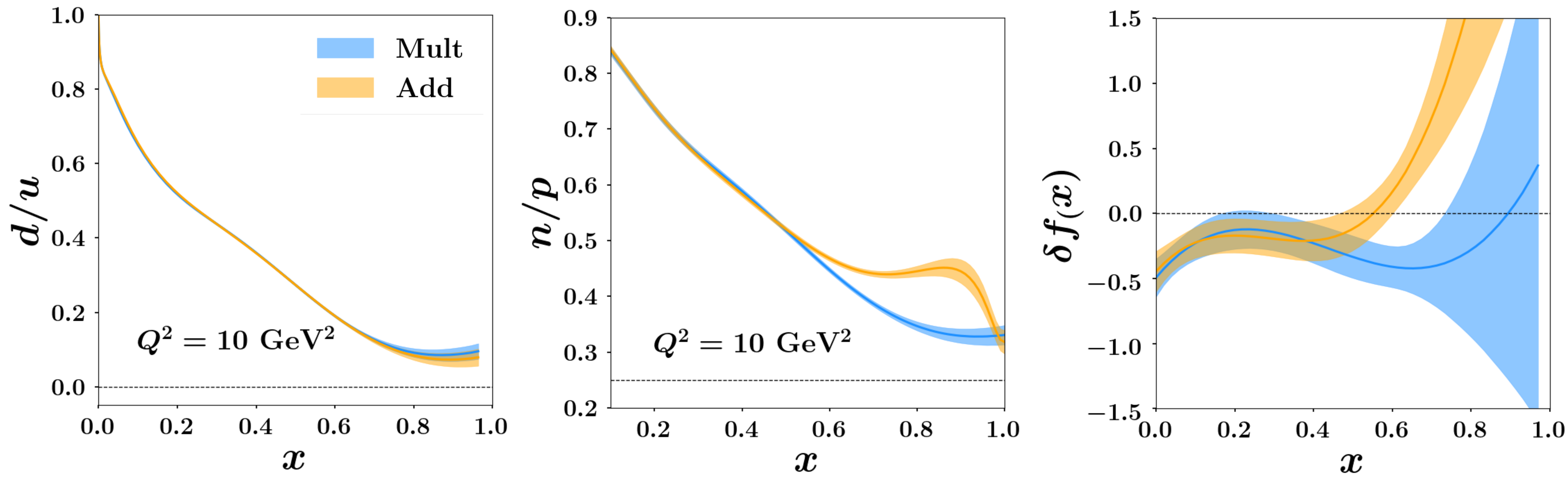
Higher d/u to absorb bias

Disfavored by DIS on deuteron and W-boson asymmetry data

Other results

Case 1: isospin-symmetric HT

Off-shell function: polynomial of 3^{rd} degree



Similar to nominal result

Some implementation differences

Theoretical choices →

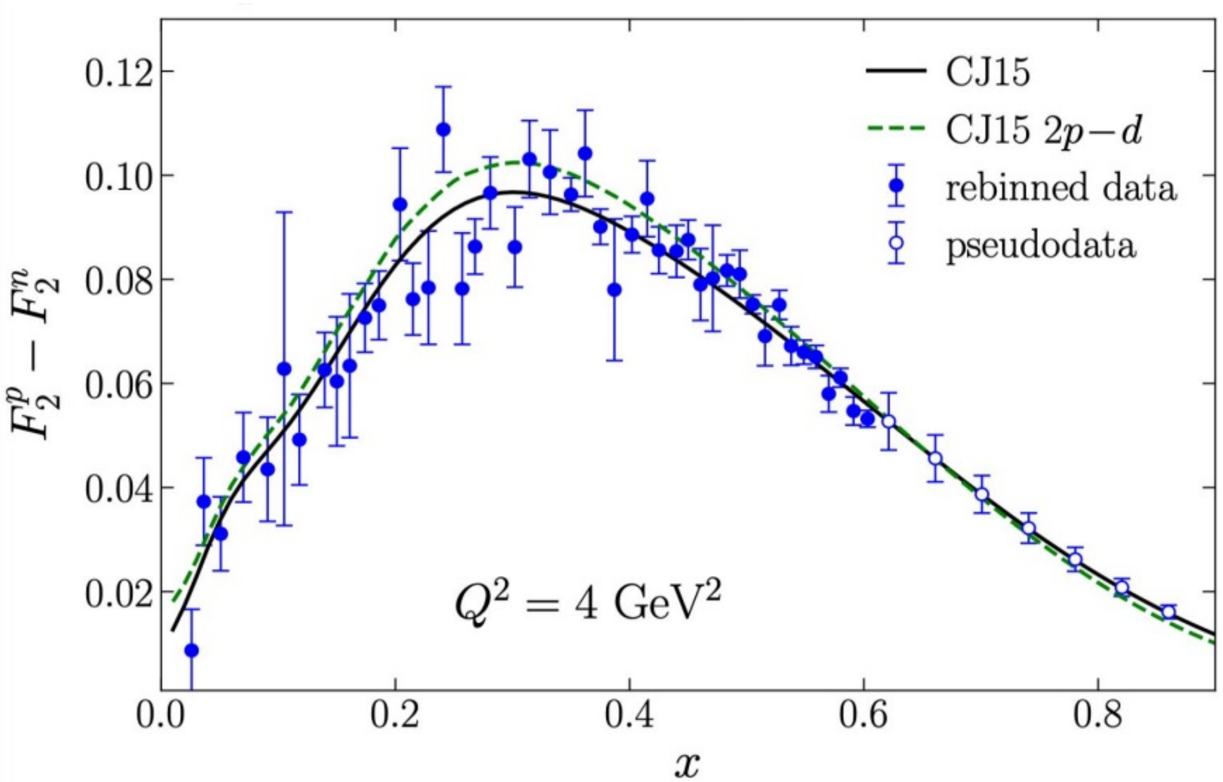
Corrections (increasing-x) ↓

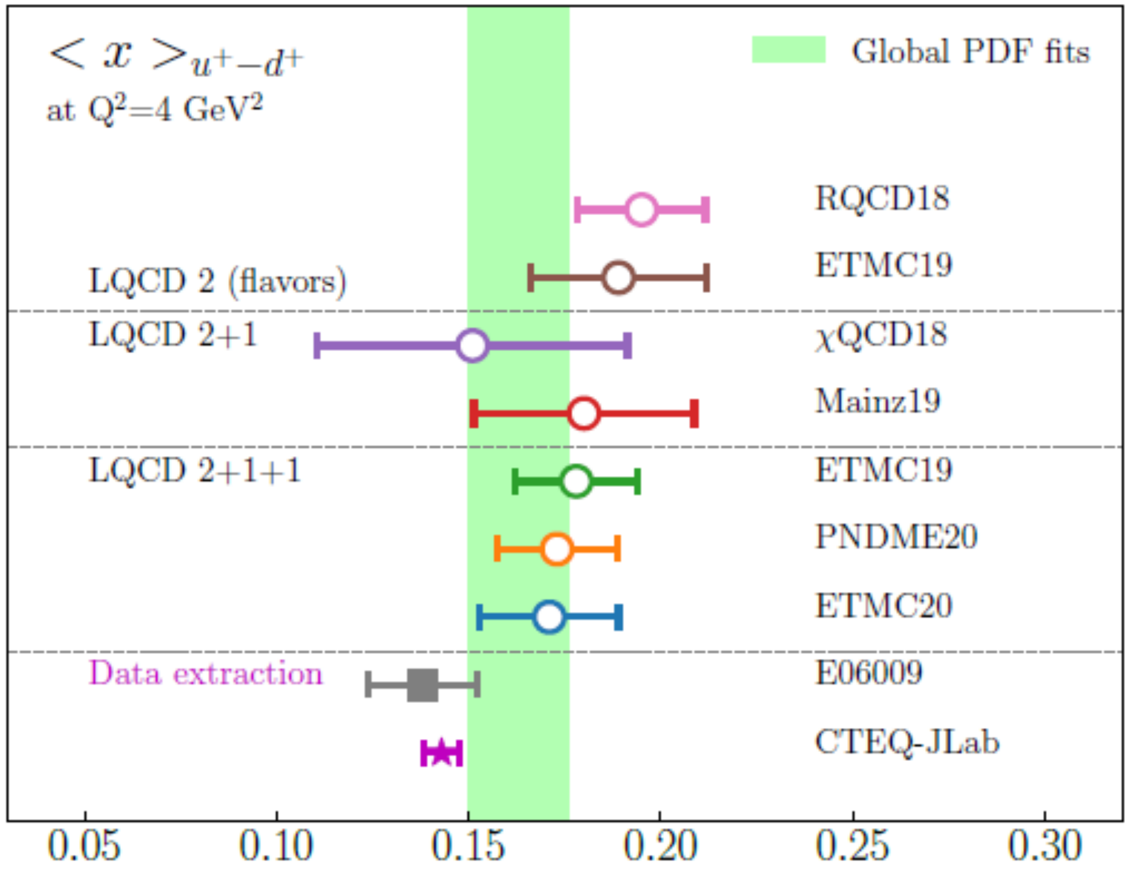
	KP	AKP	CJ15	AKP-like
shadowing	yes	yes (which one?)	MST x<0.1	(same)
smearing	Paris	AV18	AV18 x>0.1	(same)
pi-cloud	yes	yes	----	----
TMC	GP O(Q4)?	GP O(Q4)??	GP approx.	(same)
HT	H (p=n ??)	H (p=n)	C (p=n)	H & C, p=n & p!=n
HT(x)	??	5 pt. spline	parametrized	parametrized
off-shell	O(p2-M2)	O(p2-M2)	O(p2-M2)	(same)
df(x)	factorized	polyn. 2nd/3rd	factorized + sum rule	polyn. 2nd/3rd
pi thresh.	yes	yes	----	----

Application: non-singlet moments

$$M_2^{p-n}(Q^2) = \int_0^1 dx \frac{\xi^3}{x^3}\left[\frac{3+9r+8r^2}{20}\right] F_2^{p-n}(x,Q^2)$$

$$\frac{3}{C_2} M_2^{p-n} = \langle x \rangle_{u^+-d^+} + \text{ HT}$$





Li, Accardi, MC, Fernando et al., PRD 109 (2024)

- **x<0.01**: Regge theory
- **0.01<x<0.6**: Exp. data
- **x>0.6**: CJ15 model

$$\langle x \rangle_{u^+-d^+} = \int_0^1 dx x[u(x) + \bar{u}(x) - d(x) - \bar{d}(x)]$$